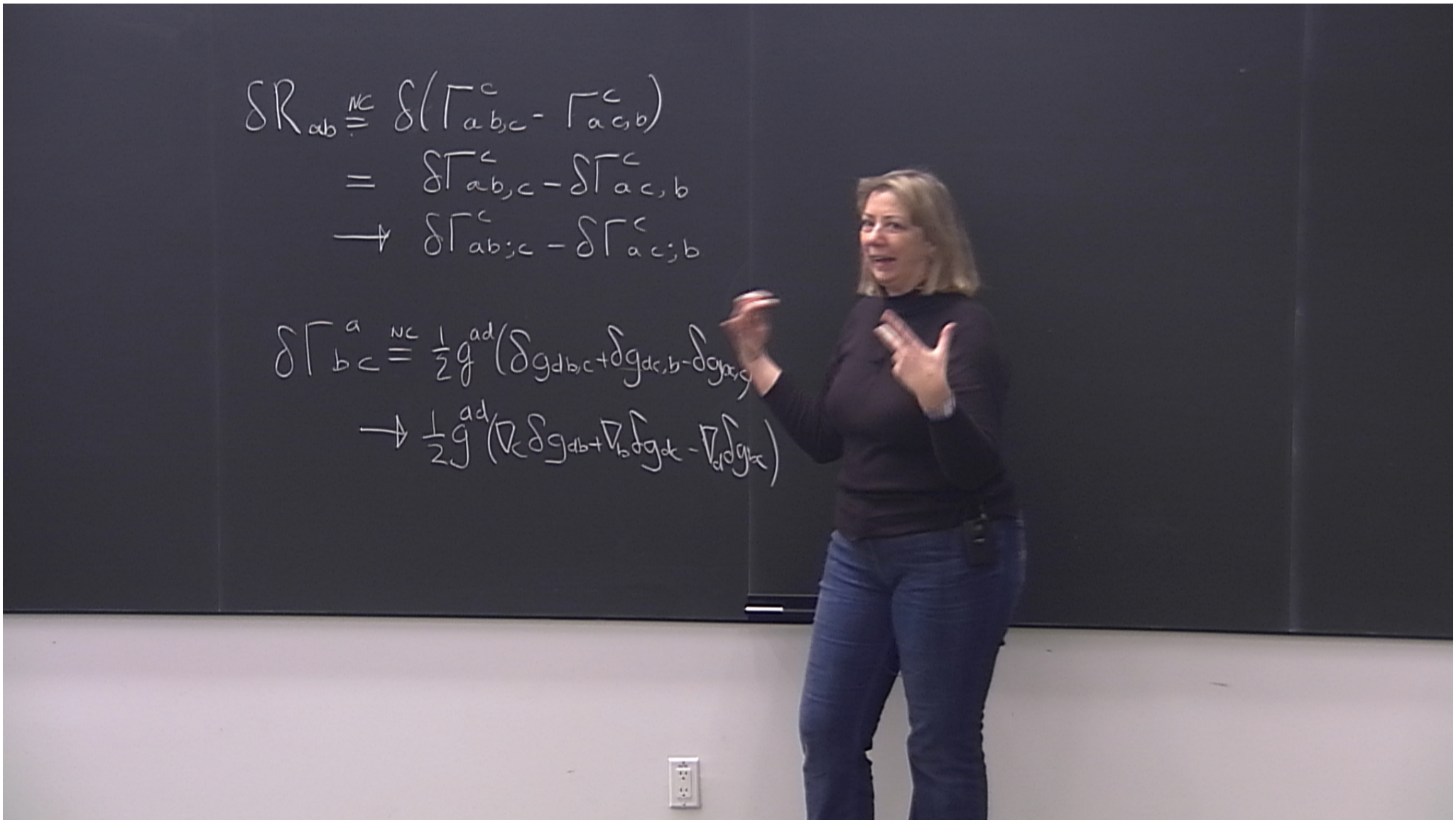
Title: 12/13 PSI - Gravitational Physics Review Lecture 8

Date: Feb 06, 2013 09:00 AM

URL: http://pirsa.org/13020064

Abstract:


$\delta R_{ab} \overset{NC}{=} \delta(\Gamma^c_{ab,c} - \Gamma^c_{ac,b})$
$= \delta\Gamma^c_{ab,c} - \delta\Gamma^c_{ac,b}$
$\rightarrow \delta\Gamma^c_{ab;c} - \delta\Gamma^c_{ac;b}$
$\delta\Gamma^a_{bc} \overset{NC}{=} \frac{1}{2} g^{ad}(\delta g_{db,c} + \delta g_{dc,b} - \delta g_{bc,d}$
$\rightarrow \frac{1}{2} g^{ad}(\nabla_c \delta g_{db} + \nabla_b \delta g_{dc} - \nabla_d \delta g_{bc})$

$$\delta R_{ab} \overset{NC}{=} \delta\left(\Gamma^{c}_{ab,c} - \Gamma^{c}_{ac,b}\right)$$

$$= \delta\Gamma^{c}_{ab,c} - \delta\Gamma^{c}_{ac,b}$$

$$\rightarrow \delta\Gamma^{c}_{ab;c} - \delta\Gamma^{c}_{ac;b} \checkmark$$

$$\delta\Gamma^{a}_{bc} \overset{NC}{=} \frac{1}{2} g^{ad}\left(\delta g_{db,c} + \delta g_{dc,b} - \delta g_{bc,d}\right)$$

$$\rightarrow \frac{1}{2} g^{ad}\left(\nabla_c \delta g_{db} + \nabla_b \delta g_{dc} - \nabla_d \delta g_{bc}\right) \checkmark$$

Nonperturbative solutions

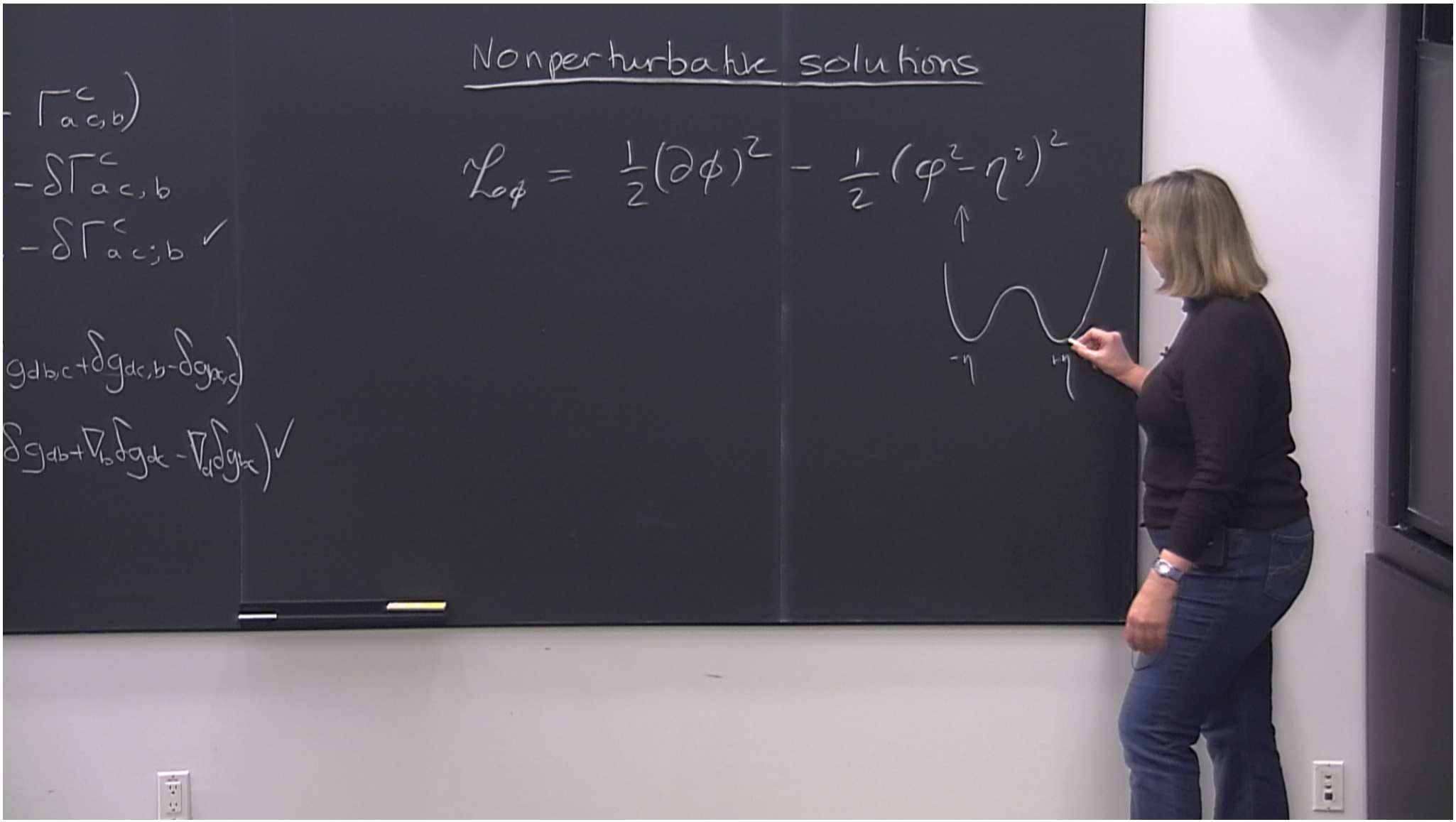
$$\delta R_{ab} \overset{NC}{=} \delta\left(\Gamma^c_{ab,c} - \Gamma^c_{ac,b}\right)$$

$$= \delta\Gamma^c_{ab,c} - \delta\Gamma^c_{ac,b}$$

$$\rightarrow \delta\Gamma^c_{ab;c} - \delta\Gamma^c_{ac;b} \checkmark$$

$$\delta\Gamma^a_{bc} \overset{NC}{=} \frac{1}{2} g^{ad}\left(\delta g_{db,c} + \delta g_{dc,b} - \delta g_{bc,d}\right)$$

$$\rightarrow \frac{1}{2} g^{ad}\left(\nabla_c \delta g_{db} + \nabla_b \delta g_{dc} - \nabla_d \delta g_{bc}\right) \checkmark$$


Nonperturbative solutions
$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\varphi^2 - \eta^2)^2$
$-\eta$
$+\eta$
$- \Gamma^c_{ac,b})$
$- \delta\Gamma^c_{ac,b}$
$- \delta\Gamma^c_{ac;b}$
$g_{db,c} + \delta g_{dc,b} - \delta g_{bc,d})$
$\delta g_{db} + \nabla_b \delta g_{dc} - \nabla_d \delta g_{bc})$

$- \Gamma^c_{ac,b})$

$- \delta\Gamma^c_{ac,b}$

$- \delta\Gamma^c_{ac;b}$ ✓

$g_{db,c} + \delta g_{dc,b} - \delta g_{bc,d})$

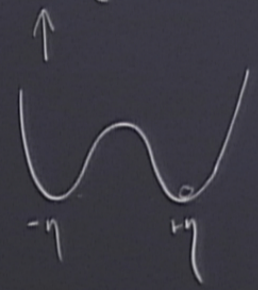
$\delta g_{db} + \nabla_b \delta g_{dc} - \nabla_d \delta g_{bc})$ ✓

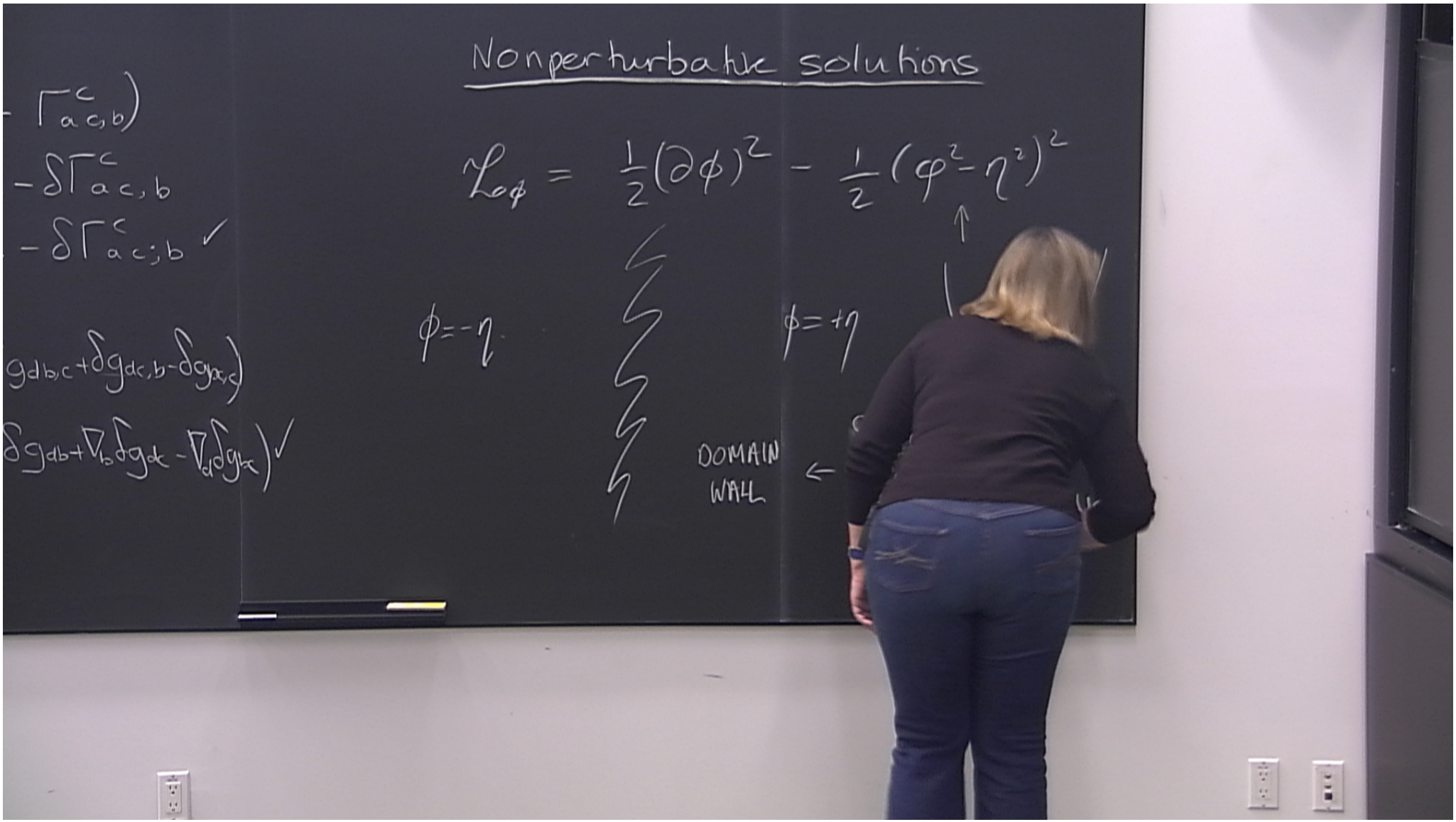
Nonperturbative solutions

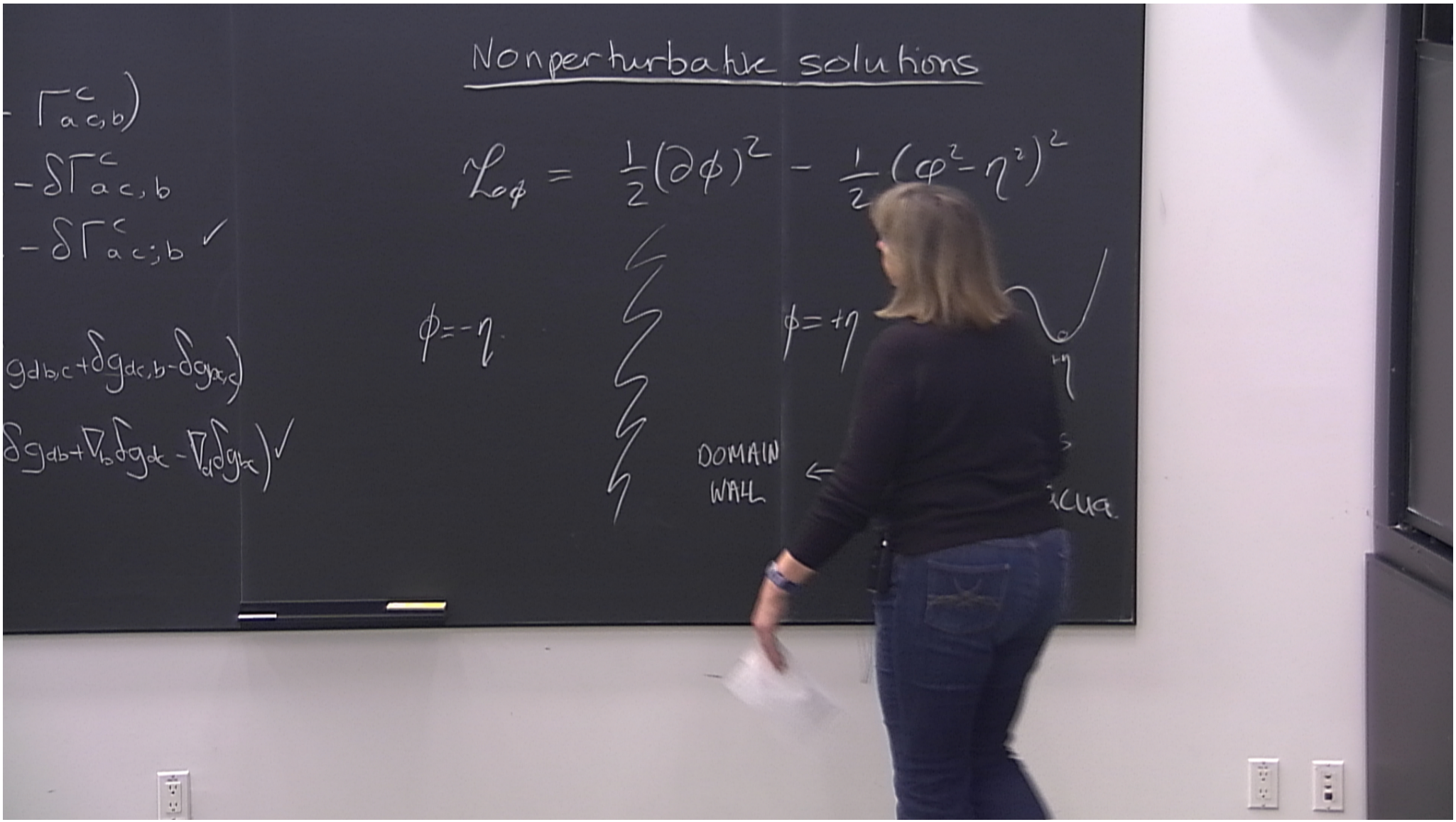
$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\varphi^2 - \eta^2)^2$$

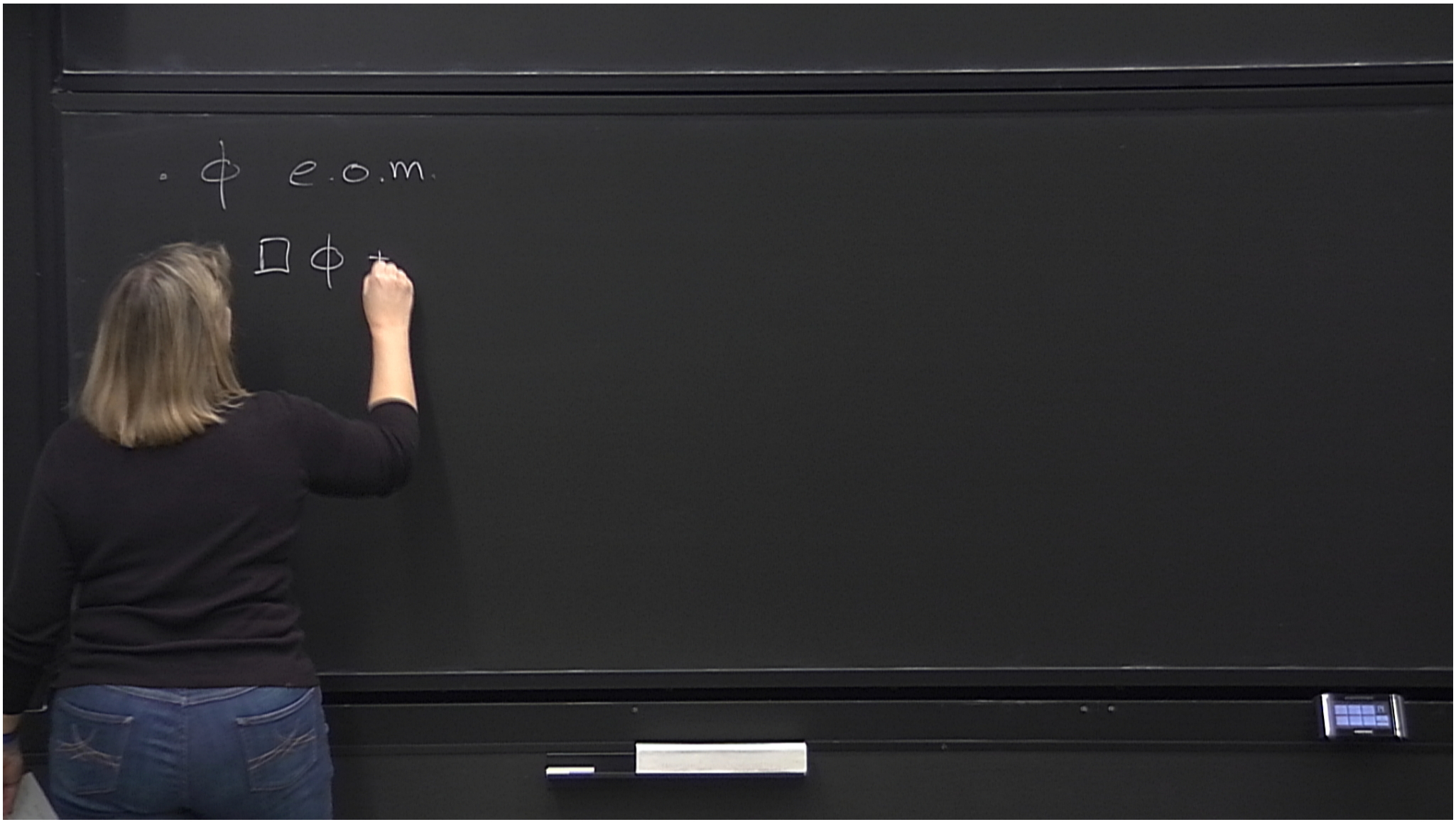
$\phi = -\eta$

$\phi = +\eta$




Nonperturbative solutions
$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\varphi^2 - \eta^2)^2$
$\phi = -\eta$
$\phi = +\eta$
DOMAIN WALL


Nonperturbative solutions
$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\phi^2-\eta^2)^2$
$\phi=-\eta$
$\phi=+\eta$
DOMAIN WALL

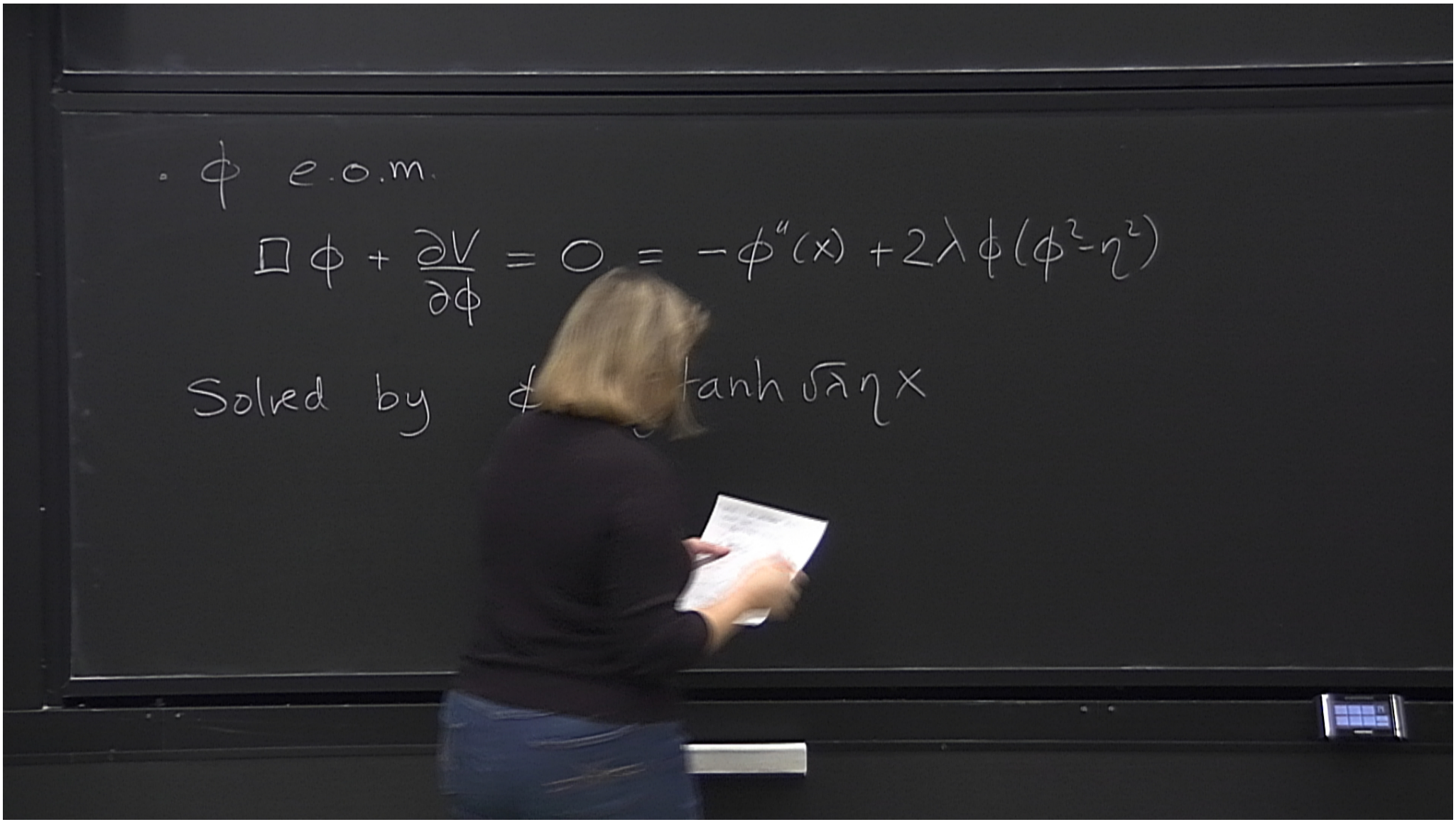

ϕ e.o.m.
$\Box\phi +$

- ϕ e.o.m.

$$\Box\phi + \frac{\partial V}{\partial \phi} = 0 = -\phi''(x) + 2\lambda\phi(\phi^2 - \eta^2)$$

- ϕ e.o.m.

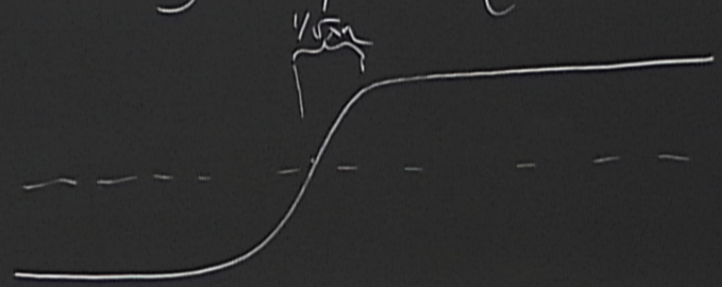
$$\Box\phi + \frac{\partial V}{\partial \phi} = 0 = -\phi''(x) + 2\lambda\phi(\phi^2 - \eta^2)$$

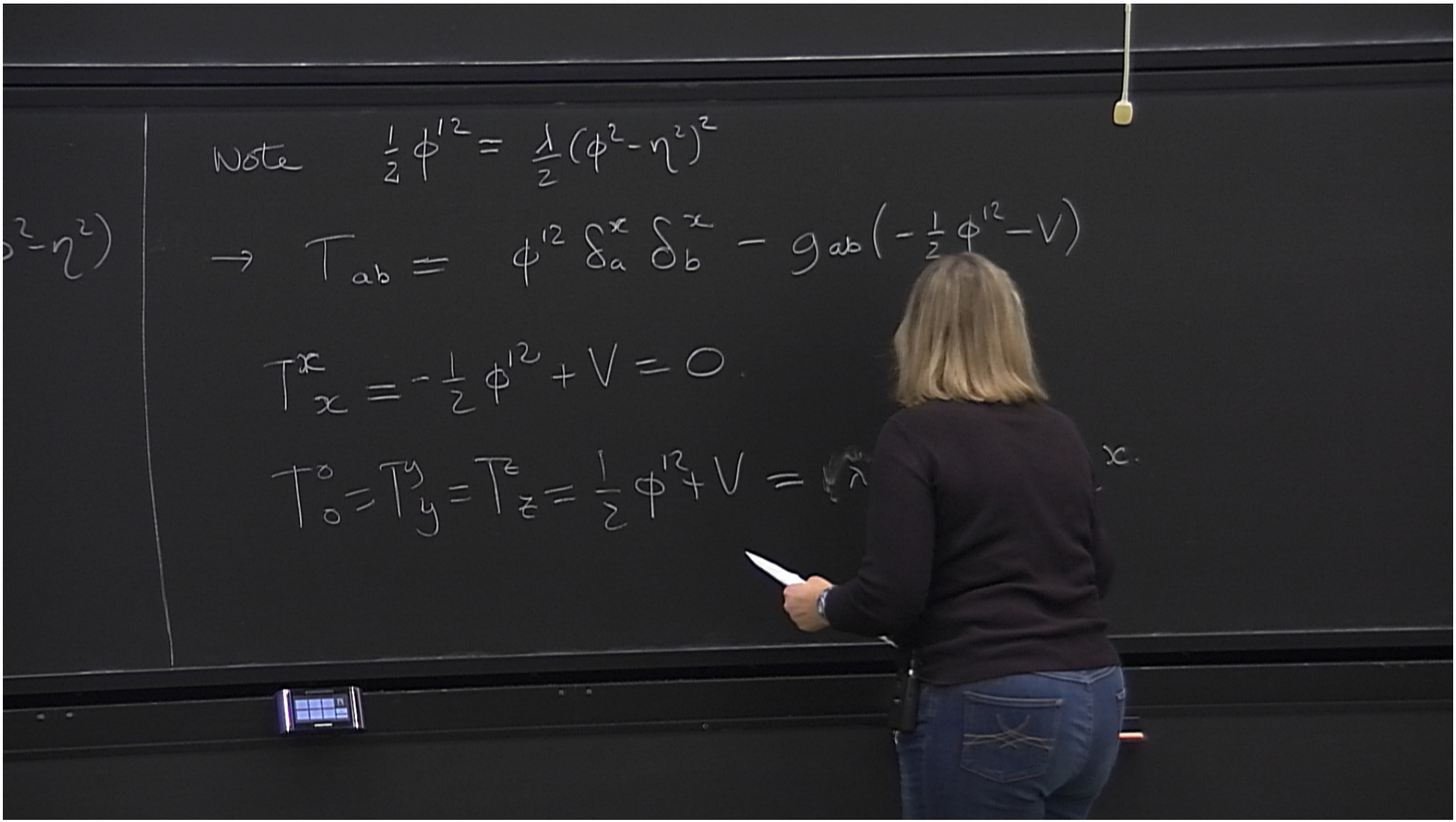

• φ e.o.m.
□φ + ∂V/∂φ = 0 = −φ''(x) + 2λφ(φ²−η²)
Solved by φ tanh √λ η x

- ϕ e.o.m.

$$\Box\phi + \frac{\partial V}{\partial \phi} = 0 = -\phi''(x) + 2\lambda\phi(\phi^2 - \eta^2)$$

Solved by $\phi = \eta \tanh \sqrt{\lambda}\,\eta x$



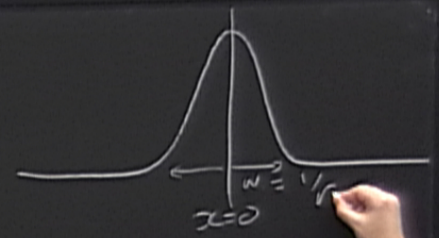

Note $\frac{1}{2}\phi'^2 = \frac{\lambda}{2}(\phi^2-\eta^2)^2$
$\rightarrow T_{ab} = \phi'^2 \delta^x_a \delta^x_b - g_{ab}(-\frac{1}{2}\phi'^2 - V)$
$T^x{}_x = -\frac{1}{2}\phi'^2 + V = 0$
$T^0{}_0 = T^y{}_y = T^z{}_z = \frac{1}{2}\phi'^2 + V =$

$$-2\lambda\phi(\phi^2-\eta^2)$$

$$\times$$

Note $\frac{1}{2}\phi'^2 = \frac{\lambda}{2}(\phi^2-\eta^2)^2$

$$\rightarrow \quad T_{ab} = \phi'^2 \delta^x_a \delta^x_b - g_{ab}\left(-\tfrac{1}{2}\phi'^2 - V\right)$$



$$T^x_x = -\tfrac{1}{2}\phi'^2 + V = 0$$

$$T^0_0 = T^y_y = T^z_z = \tfrac{1}{2}\phi'^2 + V = \lambda\eta^4 \operatorname{sech}^4 \sqrt{\lambda}\,\eta x$$

$$T^x_x = -\frac{1}{2}\phi'^2 + V = 0$$

$$T^0_0 = T^y_y = T^z_z = \frac{1}{2}\phi'^2 + V = \lambda\eta^4 \operatorname{sech}^4 \sqrt{\lambda}\eta x$$

Transverse E-M vanishes, parallel cpt $\propto g_{\mu\beta}$

$- \Gamma^c_{ac,b})$

$- \delta\Gamma^c_{ac,b}$

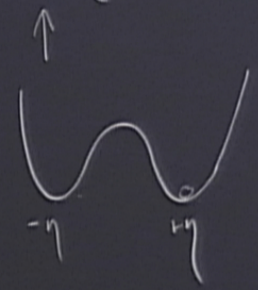
$- \delta\Gamma^c_{ac;b}$ ✓

$g_{db,c} + \delta g_{dc,b} - \delta g_{bc,d})$

$\delta g_{db} + \nabla_b \delta g_{dc} - \nabla_d \delta g_{bc})$ ✓

Nonperturbative solutions

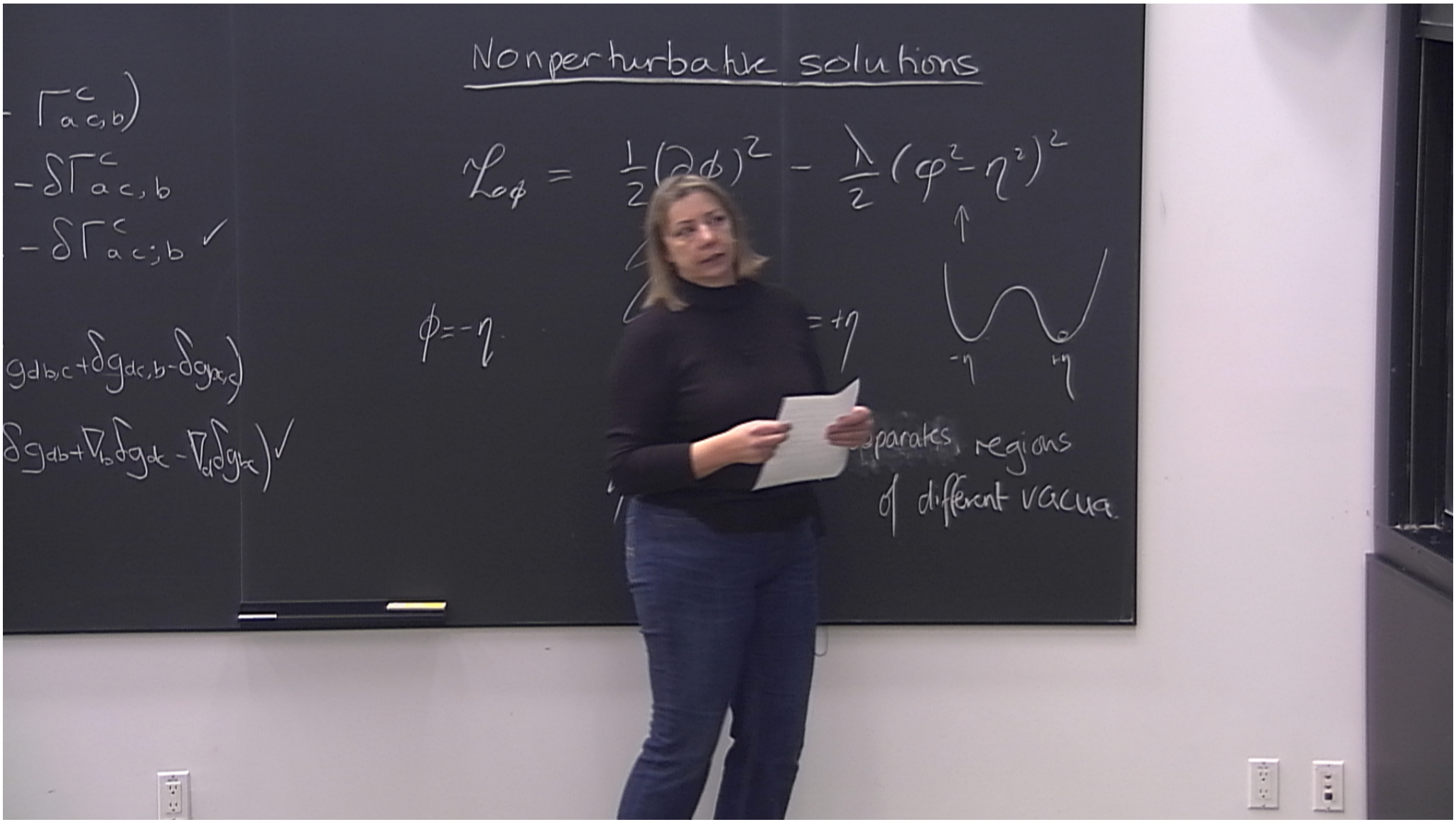
$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2 - \eta^2)^2$$



$\phi = -\eta$

$\phi = +\eta$

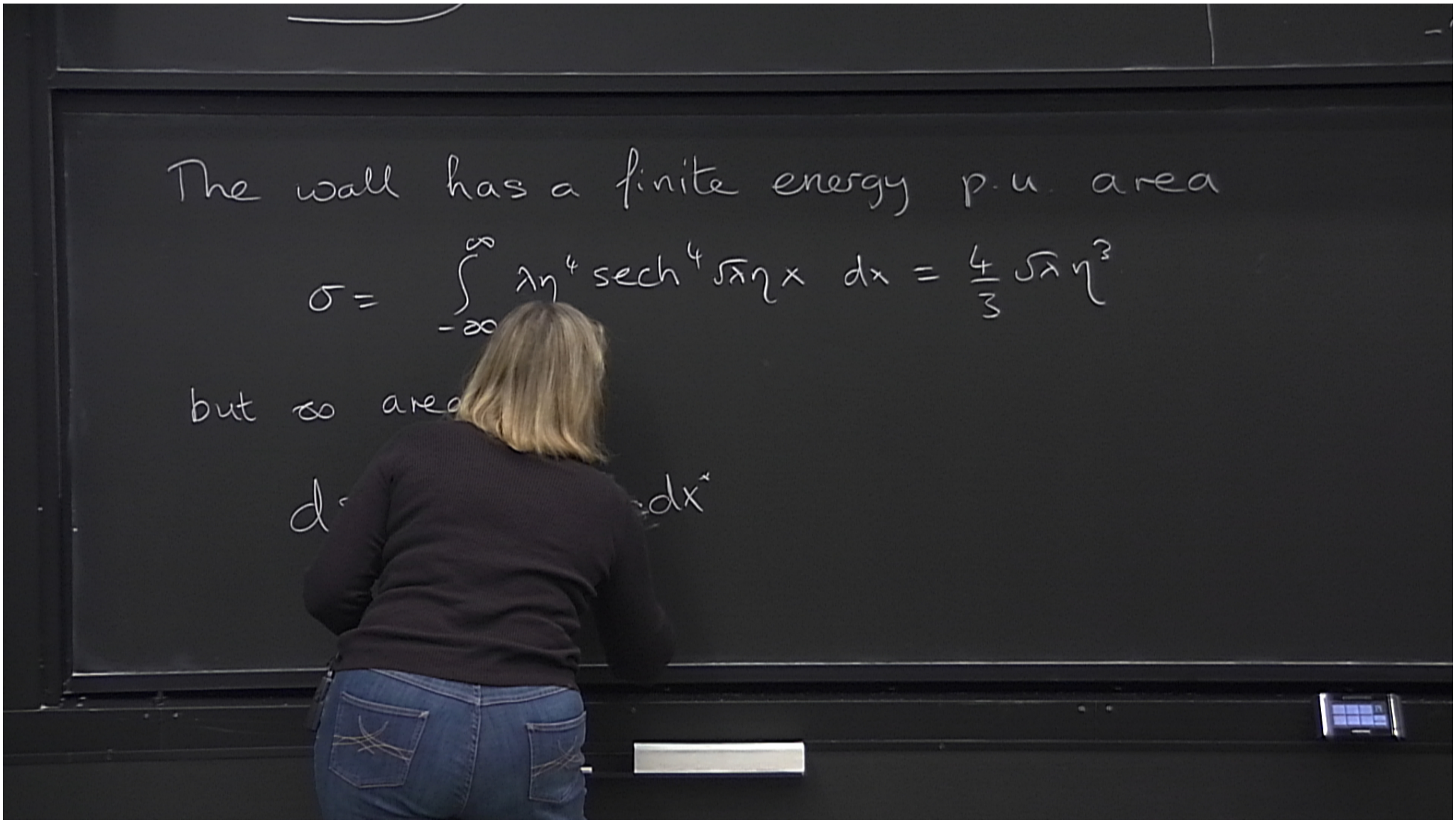
DOMAIN WALL ← Separates regions of different vacua


Nonperturbative solutions
$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2 - \eta^2)^2$
$\phi = -\eta$
$= +\eta$
$-\eta$
$+\eta$
separates regions
of different vacua

The wall has a finite energy p.u. area

$$\sigma = \int_{-\infty}^{\infty} \lambda \eta^4 \operatorname{sech}^4 \sqrt{\lambda}\eta x \, dx = \frac{4}{3}\sqrt{\lambda}\eta^3$$

but ∞ area!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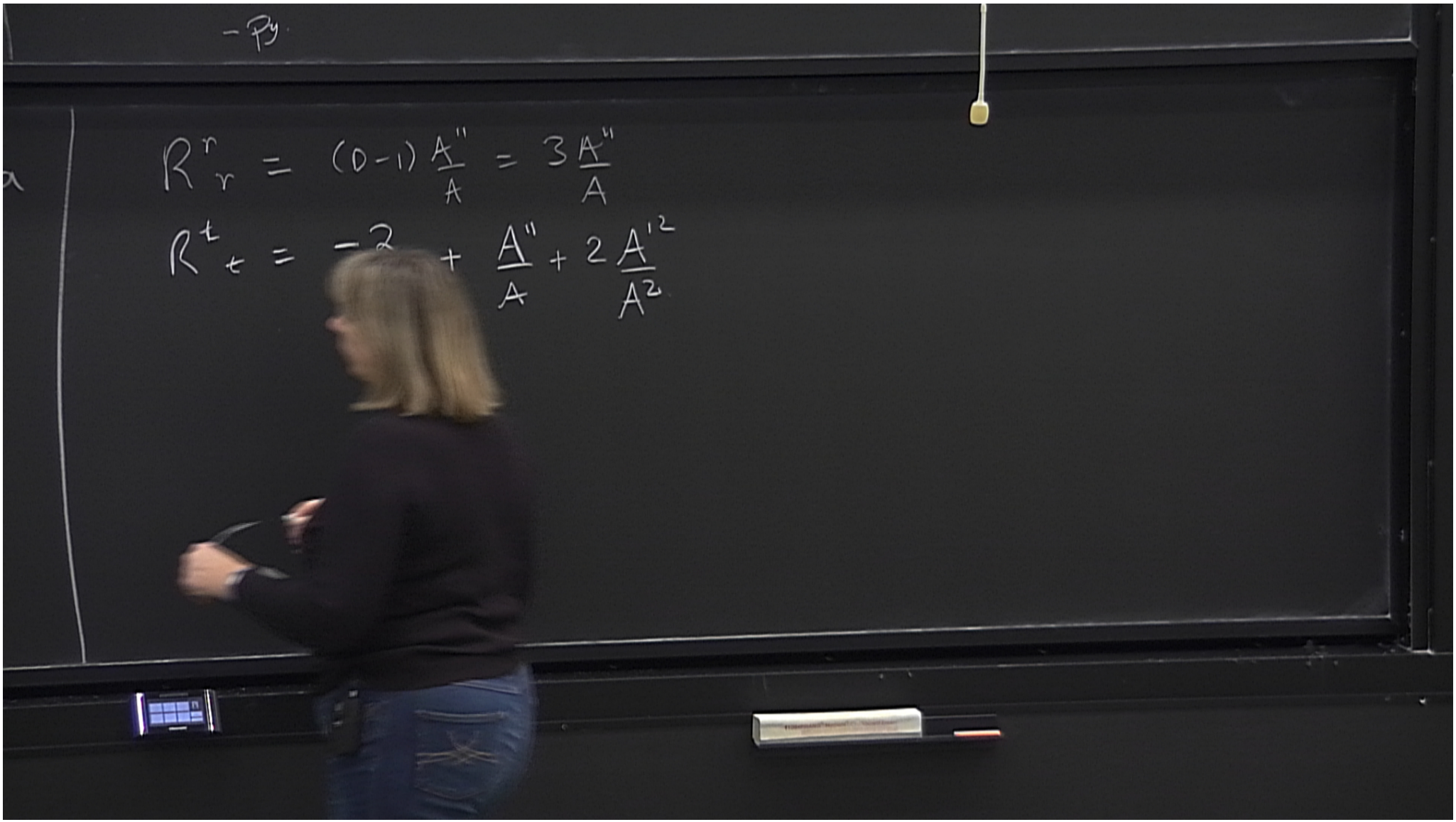
$$\sigma = \int_{-\infty}^{\infty} \lambda \eta^4 \operatorname{sech}^4 \sqrt{\lambda}\eta x \, dx = \frac{4}{3}\sqrt{\lambda}\eta^3$$

but ∞ area!

$$ds^2 = A^2(r)\, \gamma_{\alpha\beta}\, dx^\alpha dx^\beta - dr^2.$$

de-Sitter.


$R^r{}_r = (0-1)\frac{A''}{A} = 3\frac{A''}{A}$
$R^t{}_t = -2 + \frac{A''}{A} + 2\frac{A'^2}{A^2}$

$-P_y$

$$R^r{}_r = (0-1)\frac{A''}{A} = 3\frac{A''}{A}$$

$$R^t{}_t = -\frac{2}{\ell^2 A^2} + \frac{A''}{A} + 2\frac{A'^2}{A^2}$$

The wall has a finite energy p.u. area

$$\sigma = \int_{-\infty}^{\infty} \lambda \eta^4 \operatorname{sech}^4 \sqrt{\lambda}\eta x \, dx = \frac{4}{3}\sqrt{\lambda}\eta^3$$

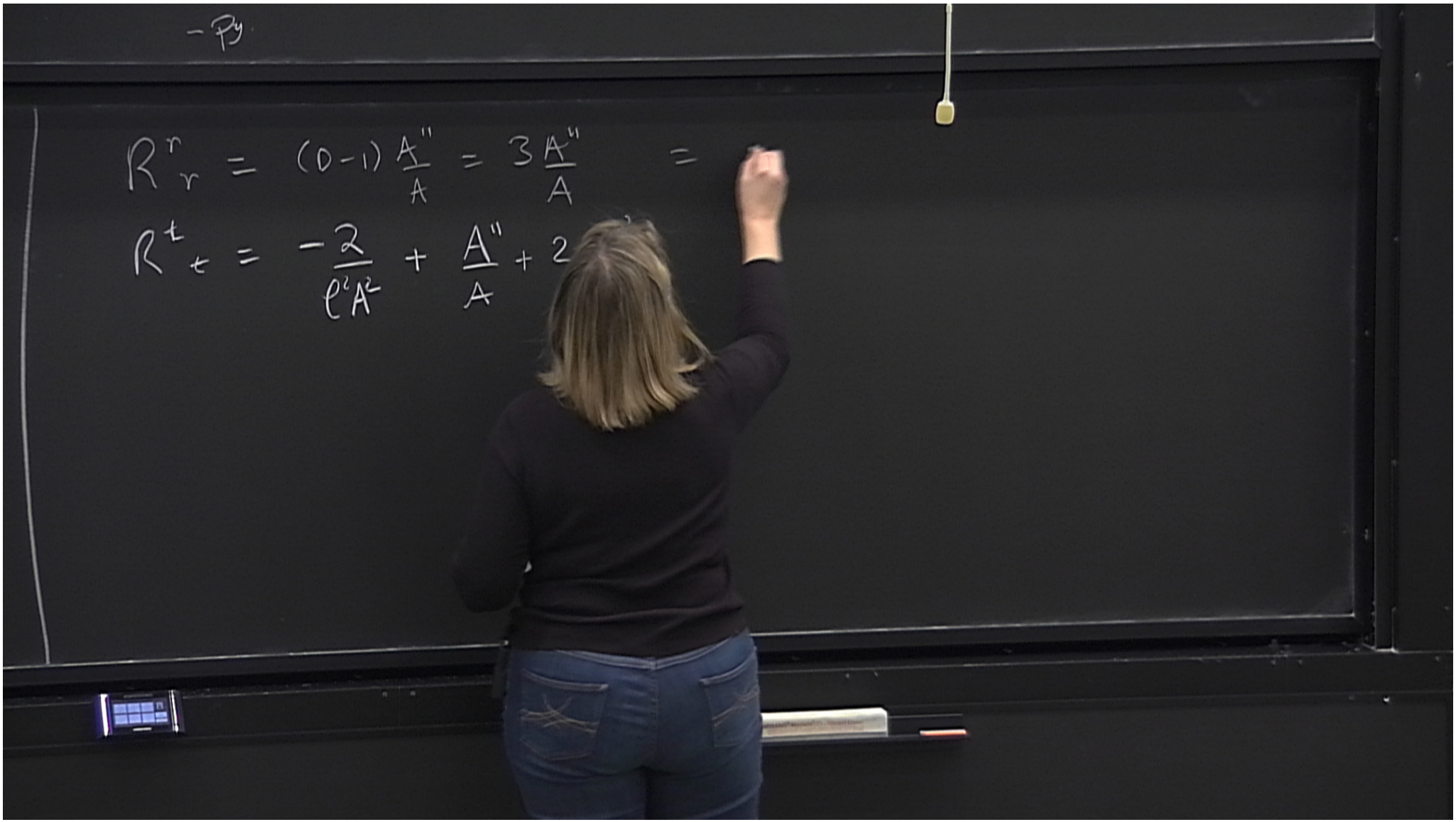
but ∞ area!

$$ds^2 = A^2(r)\, \gamma_{\alpha\beta}\, dx^\alpha dx^\beta - dr^2$$

de-Sitter $\quad R_{0\alpha\beta} = -\frac{2}{\ell^2}\gamma_{\alpha\beta}$

$R^r{}_r$

$R^t{}_t$


$-P_y$
$R^r{}_r = (D-1)\frac{A''}{A} = 3\frac{A''}{A} =$
$R^t{}_t = -\frac{2}{\ell^2 A^2} + \frac{A''}{A} + 2$

$-P_y$

$$R^r{}_r = (D-1)\frac{A''}{A} = 3\frac{A''}{A} = 8\pi G\left(T^r{}_r - \frac{1}{2}T\right) \quad \Big\| \quad 8\pi G\eta^2 \cdot (\lambda\eta^2 \ldots (r)$$

$$R^t{}_t = -\frac{2}{\ell^2 A^2} + \frac{A''}{A} + 2\frac{A'^2}{A^2} = 8\pi G\left(T^0{}_0 - \frac{1}{2}T\right)$$

$-P_y$

$$R^r{}_r = (0-1)\frac{A''}{A} = 3\frac{A''}{A} = 8\pi G\left(T^r{}_r - \frac{1}{2}T\right)$$

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$$8\pi G\eta^2 \cdot (\lambda\eta)^2 \cdot f_m(r)$$

$$\uparrow$$

$$\left(\frac{\eta}{M_P}\right)^2$$

$-P_y$

$$R^r{}_r = (D-1)\frac{A''}{A} = 3\frac{A''}{A} = 8\pi G\left(T^r{}_r - \frac{1}{2}T\right)$$

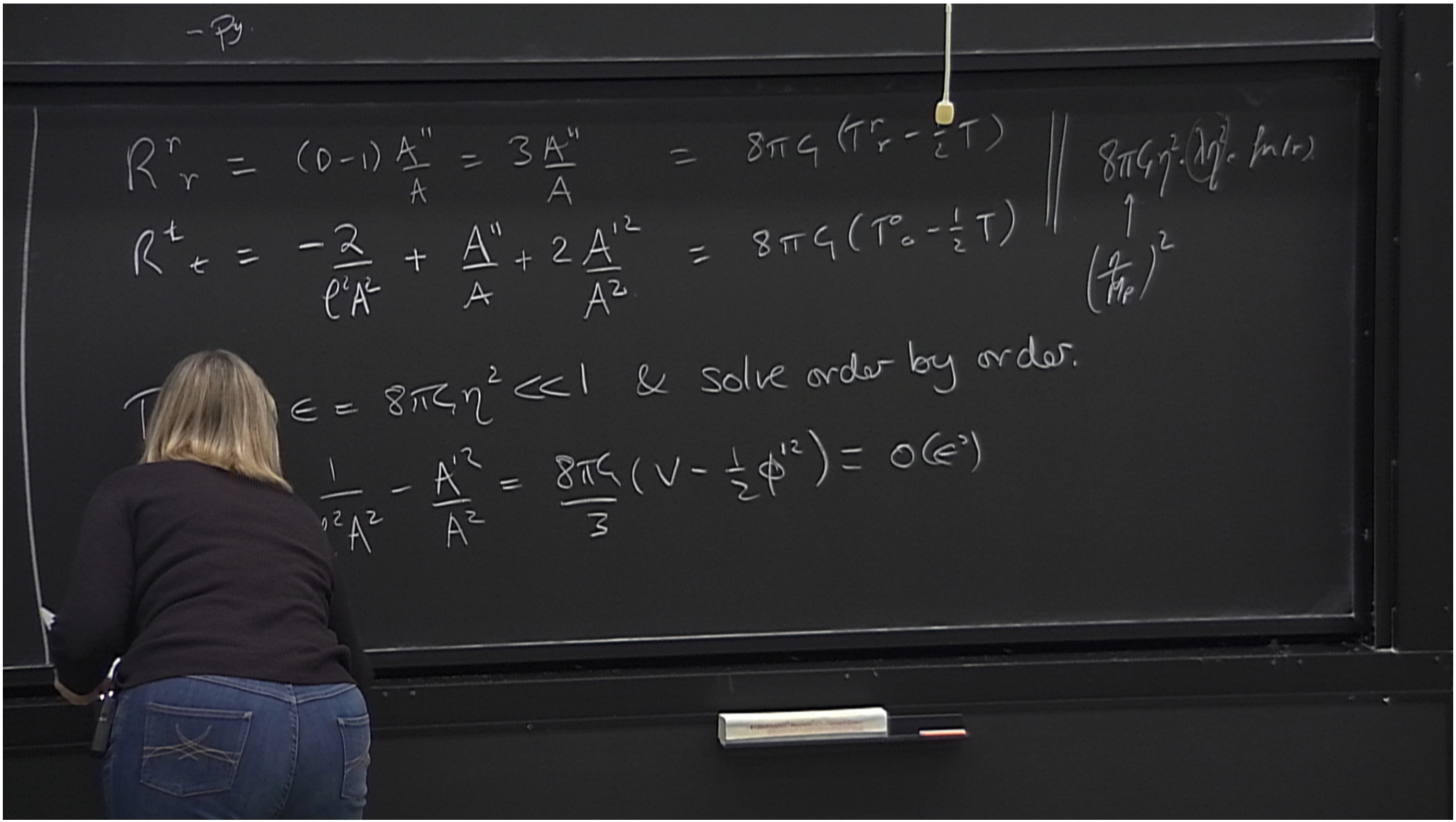
$$R^t{}_t = -\frac{2}{\ell^2 A^2} + \frac{A''}{A} + 2\frac{A'^2}{A^2} = 8\pi G\left(T^0{}_0 - \frac{1}{2}T\right)$$

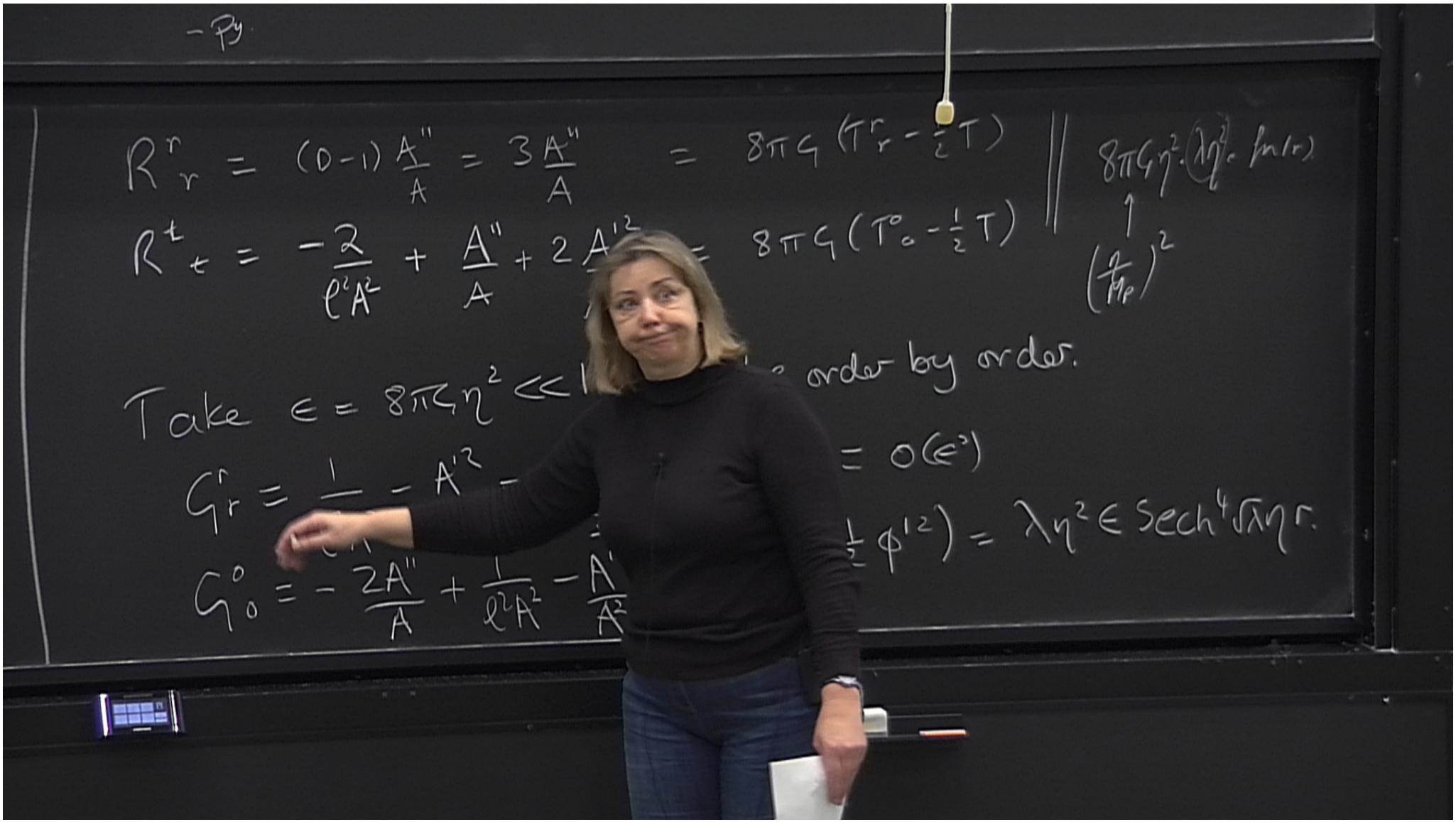
$$8\pi G\eta^2 \cdot (\lambda\eta^2) \cdot f_m(r)$$

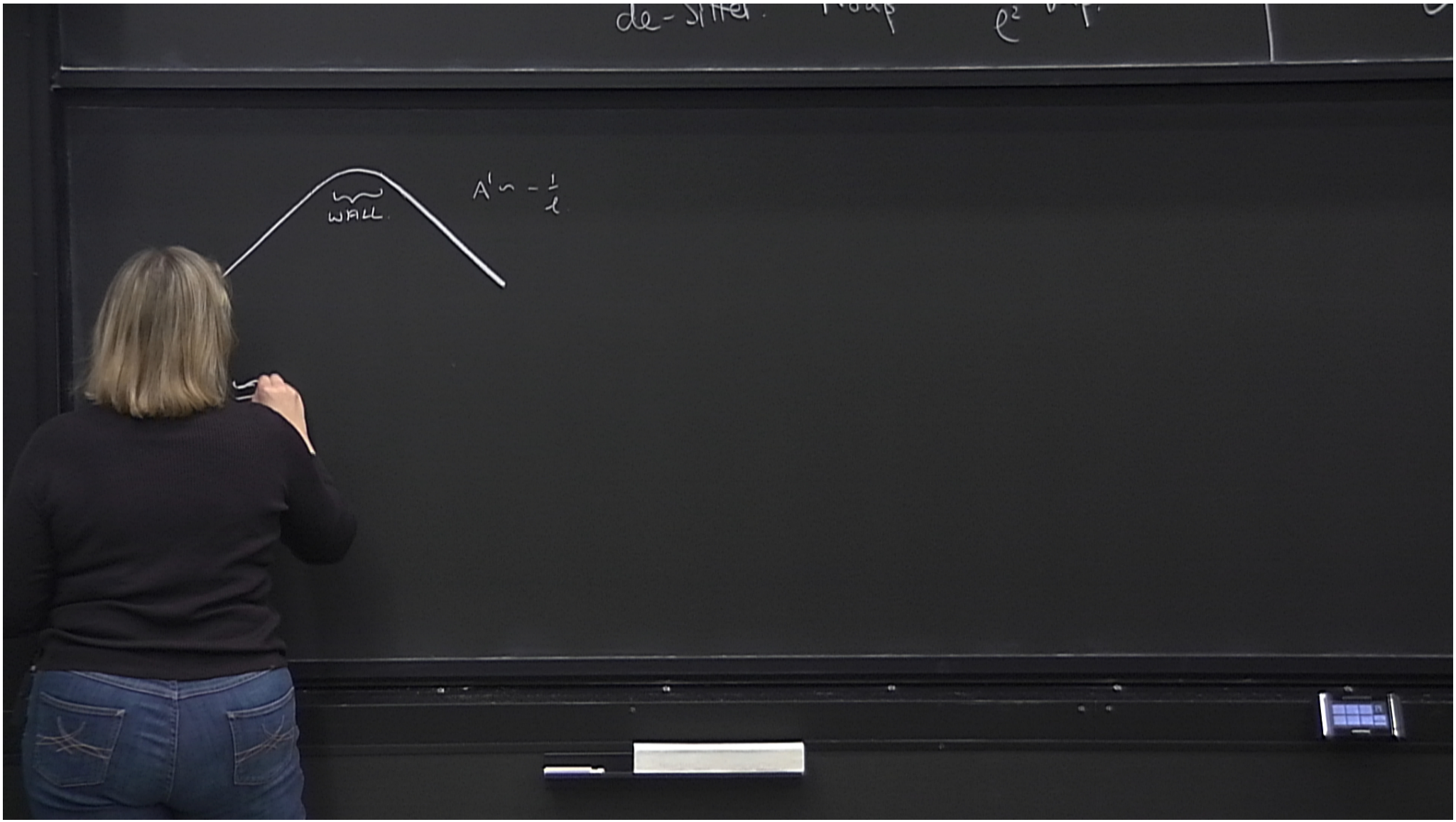
$$\uparrow$$

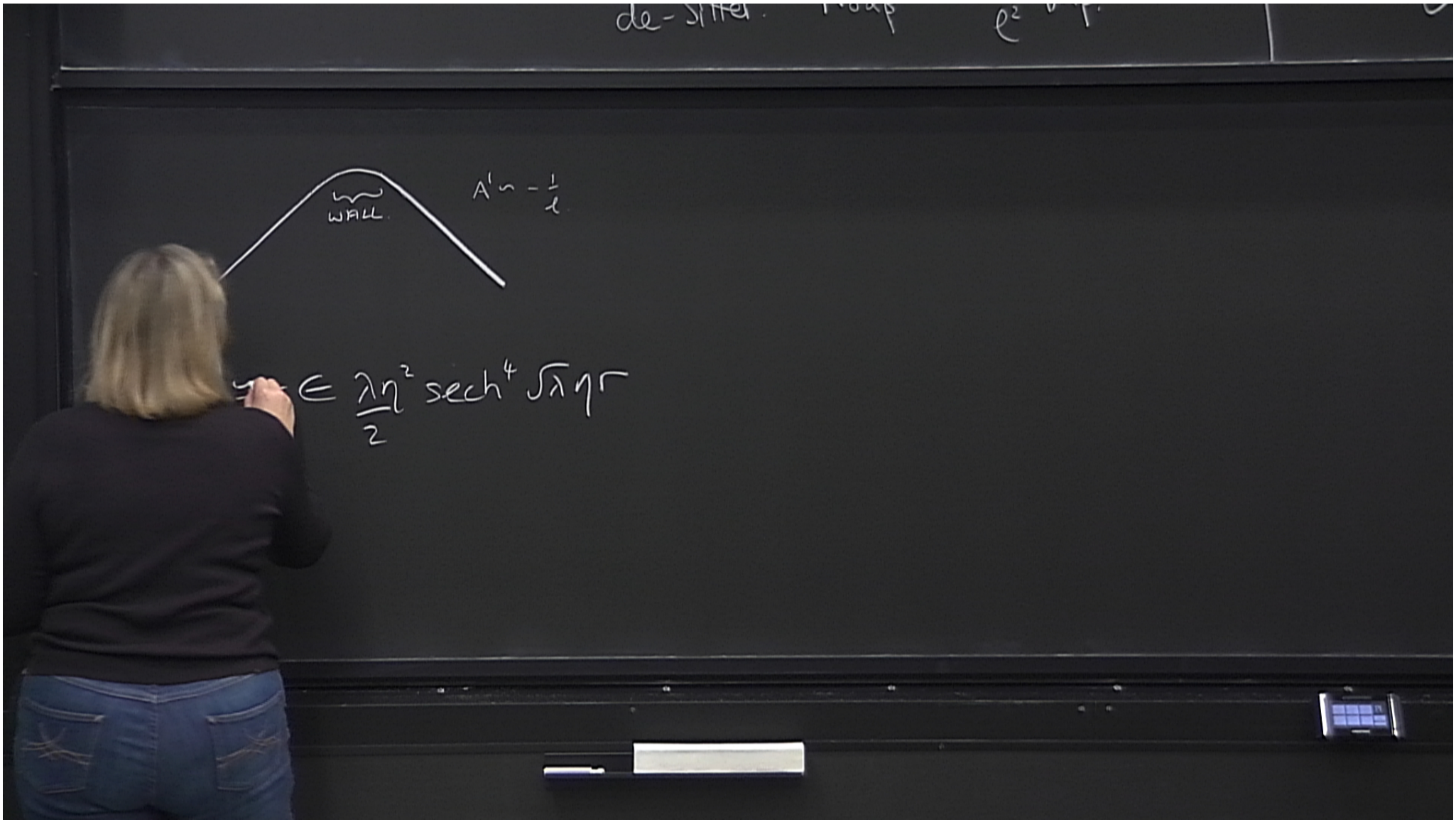
$$\left(\frac{\eta}{M_P}\right)^2$$

Take $\epsilon = 8\pi G\eta^2 \ll 1$ & solve order by order.


$R^r{}_r = (D-1)\frac{A''}{A} = 3\frac{A''}{A} = 8\pi G\left(T^r{}_r - \frac{1}{2}T\right)$
$R^t{}_t = -\frac{2}{\ell^2 A^2} + \frac{A''}{A} + 2\frac{A'^2}{A^2} = 8\pi G\left(T^0{}_0 - \frac{1}{2}T\right)$
$8\pi G\eta^2 \cdot (\lambda\eta)^2 \cdot f_m(r)$
$\left(\frac{\eta}{M_P}\right)^2$
$\epsilon = 8\pi G\eta^2 \ll 1$ & solve order by order.
$\frac{1}{\ell^2 A^2} - \frac{A'^2}{A^2} = \frac{8\pi G}{3}\left(V - \frac{1}{2}\phi'^2\right) = O(\epsilon^2)$


$R^r{}_r = (D-1)\frac{A''}{A} = 3\frac{A''}{A} = 8\pi G\,(T^r{}_r - \frac{1}{2}T)$
$R^t{}_t = -\frac{2}{\ell^2 A^2} + \frac{A''}{A} + 2A'^2 = 8\pi G\,(T^0{}_0 - \frac{1}{2}T)$
$8\pi G\eta^2 \cdot (\lambda\eta^2) \cdot \mathrm{fn}(r)$
$\left(\frac{\eta}{M_P}\right)^2$
Take $\epsilon = 8\pi G\eta^2 \ll 1$
order by order.
$G^r{}_r = \frac{1}{\ell^2 A^2} - A'^2 - \ldots = O(\epsilon^2)$
$\frac{1}{2}\phi'^2) = \lambda\eta^2 \epsilon\, \mathrm{sech}^4 \sqrt{\lambda}\eta r.$
$G^0{}_0 = -\frac{2A''}{A} + \frac{1}{\ell^2 A^2} - \frac{A'}{A^2}$


WALL
A' ~ -1/ℓ


de-Sitter
WALL
$A' \sim -\frac{1}{\ell}$
$\in \frac{\lambda\eta^2}{2} \operatorname{sech}^4 \sqrt{\lambda}\eta r$

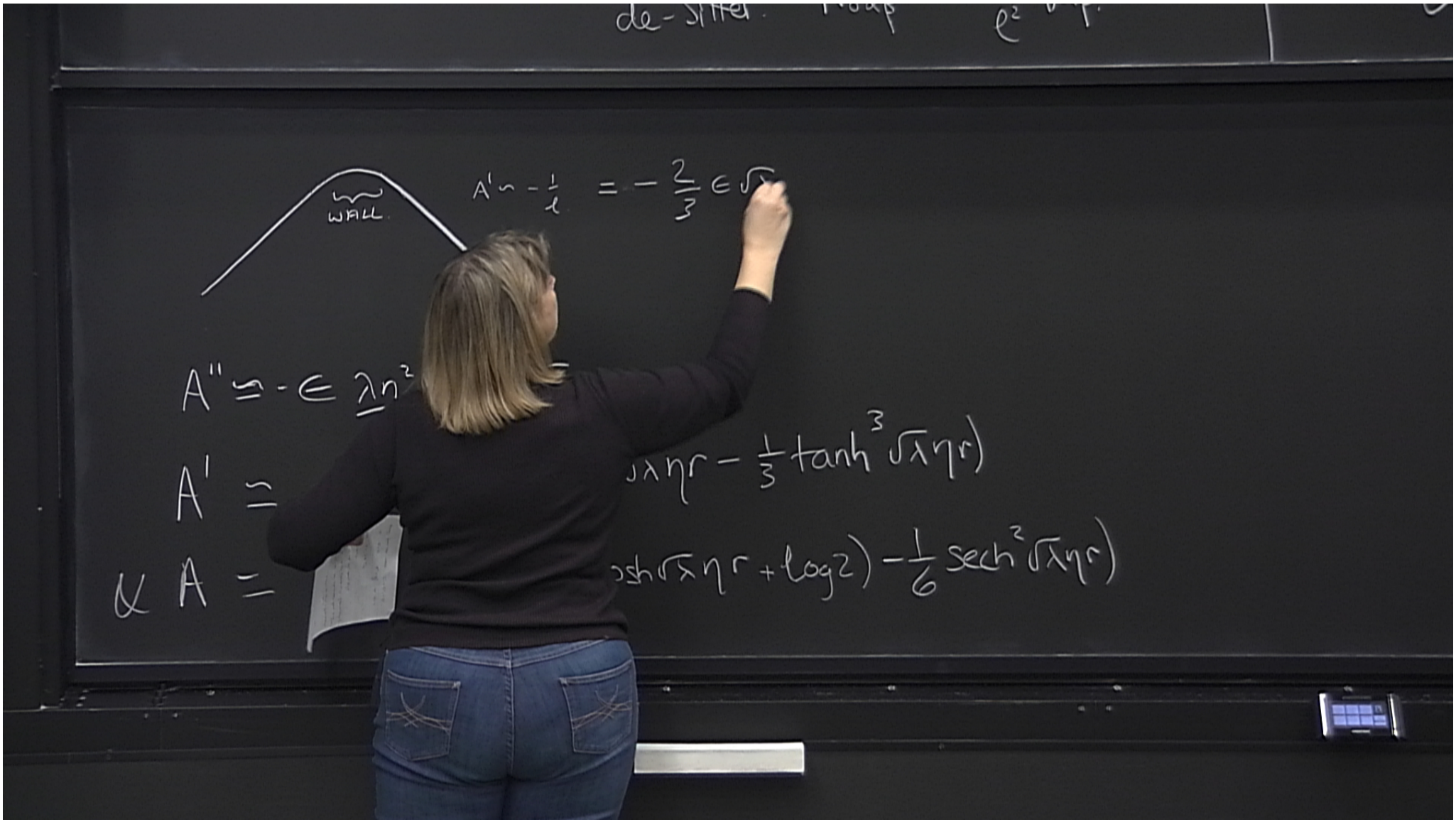
de-Sitter

$A' \sim -\frac{1}{\ell}$

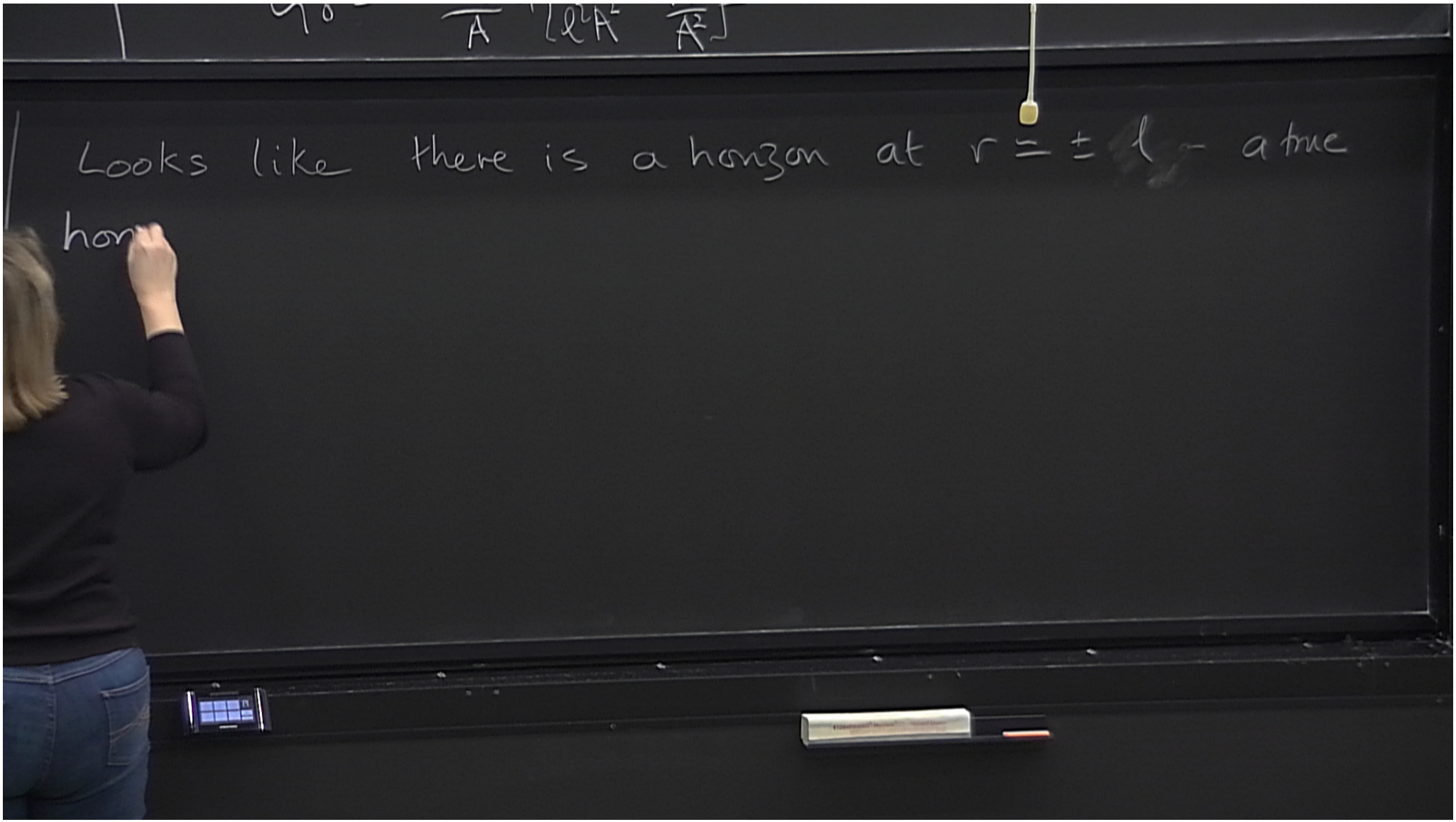
$$A'' \simeq -\epsilon \frac{\lambda \eta^2}{2} \operatorname{sech}^4 \sqrt{\lambda}\eta r$$

$$A' \simeq -\epsilon \sqrt{\lambda}\eta \left(\tanh \sqrt{\lambda}\eta r - \frac{1}{3} \tanh^3 \sqrt{\lambda}\eta r \right)$$

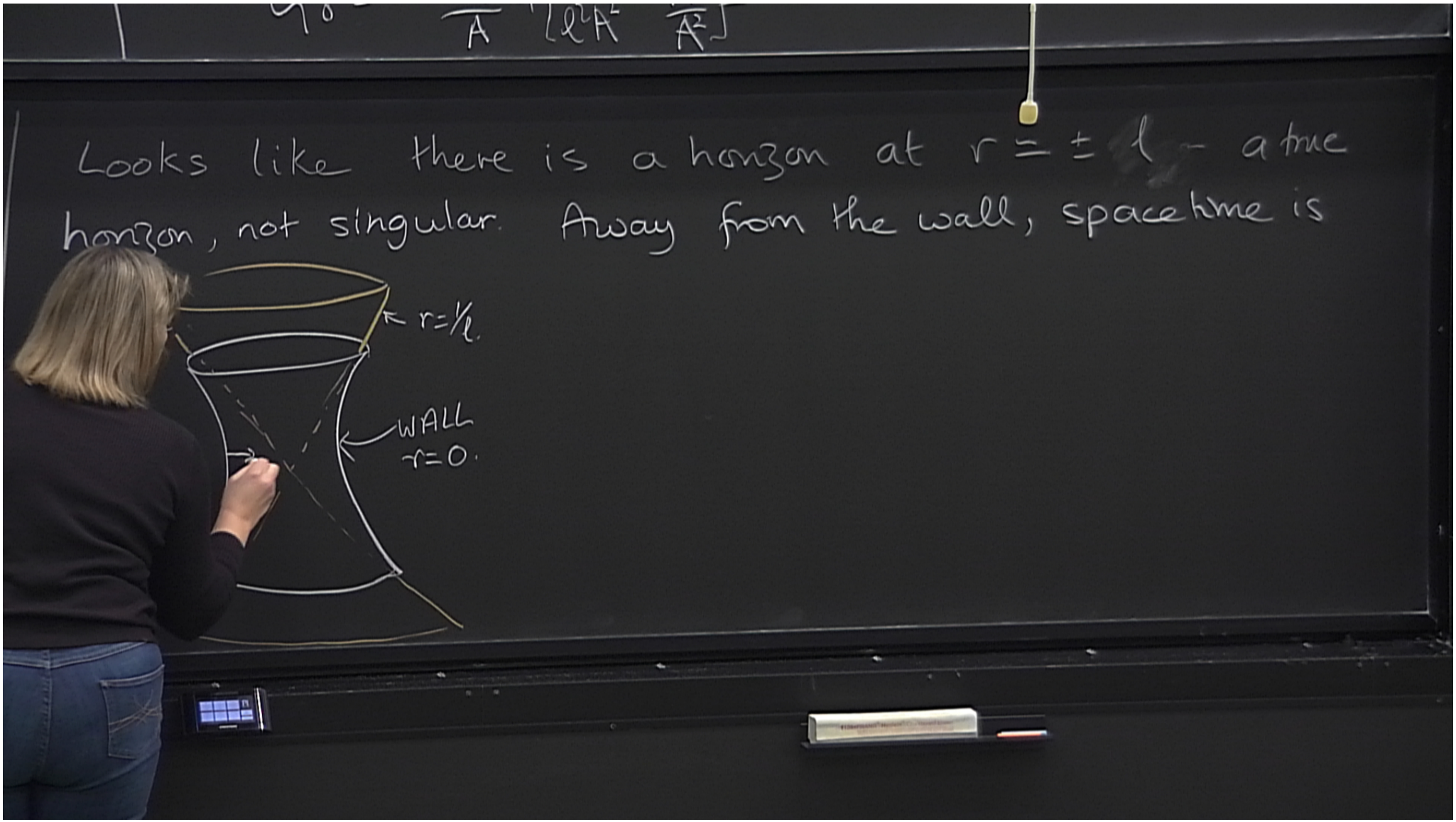
$$\& \; A \simeq 1 - \epsilon \left(\frac{2}{3} \left(\log \cosh \sqrt{\lambda}\eta r + \log 2 \right) - \frac{1}{6} \operatorname{sech}^2 \sqrt{\lambda}\eta r \right)$$


WALL
A'~ -1/ℓ = -2/3 ∈ √λ
A'' ≃ -∈ λn²
A' ≃
A ≃
tanh³ √ληr)
sech² √ληr)

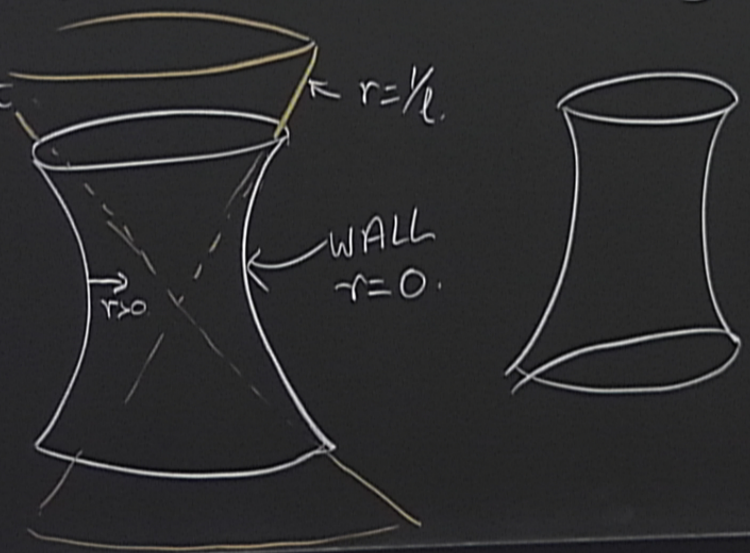
Looks like there is a horizon at $r = \pm \ell$


Looks like there is a horizon at r = ± l - a true
hor

Looks like there is a horizon at $r = \pm l$ – a true horizon, not singular.


Looks like there is a horizon at r = ± l - a true
horizon, not singular. Away from the wall, spacetime is
r=1/l.
WALL
r=0.

Looks like there is a horizon at $r = \pm \ell$ – a true horizon, not singular. Away from the wall, spacetime is flat



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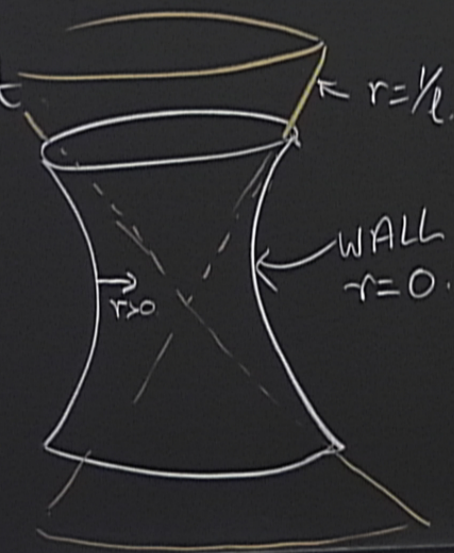
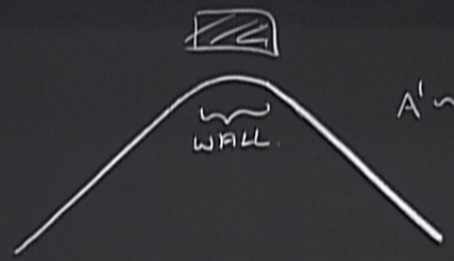


t = const

de-Sitter



$$A' \sim -\frac{1}{\ell} = -\frac{2}{3}\epsilon\sqrt{\lambda}\eta$$

$$A'' \simeq -\epsilon\frac{\lambda\eta^2}{2}\operatorname{sech}^4\sqrt{\lambda}\eta r$$

$$A' \simeq -\epsilon\sqrt{\lambda}\eta\left(\tanh\sqrt{\lambda}\eta r - \frac{1}{3}\tanh^3\sqrt{\lambda}\eta r\right)$$

$$\propto A \simeq 1 - \epsilon\left(\frac{2}{3}\left(\log\cosh\sqrt{\lambda}\eta r + \log 2\right) - \frac{1}{6}\operatorname{sech}^2\sqrt{\lambda}\eta r\right)$$

Looks lik
horizon, not
flat

$\frac{}{A}\left[\ell^2 A^2 \quad \frac{}{A^2}\right]$

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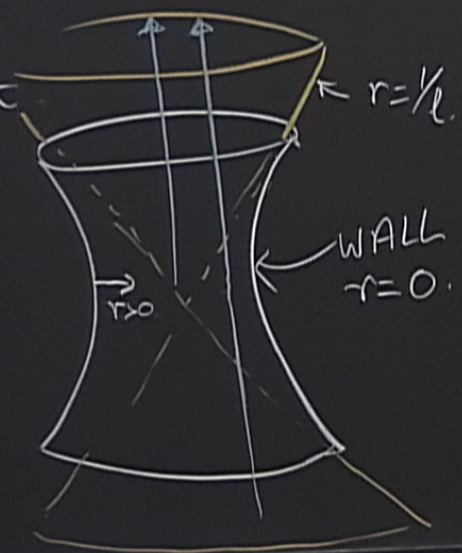

t = const

$\frac{}{A}\left[\ell^2 A^2 \quad \frac{}{A^2}\right]$

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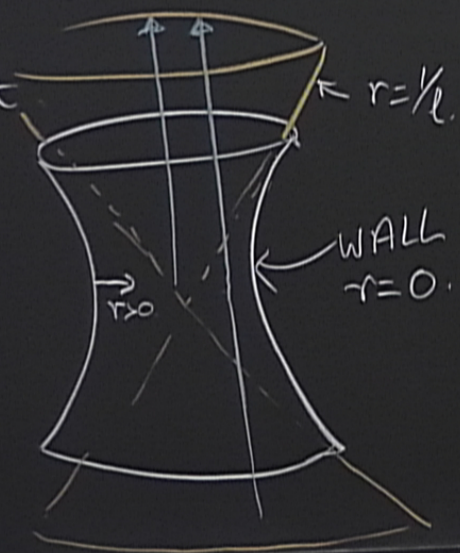
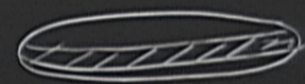


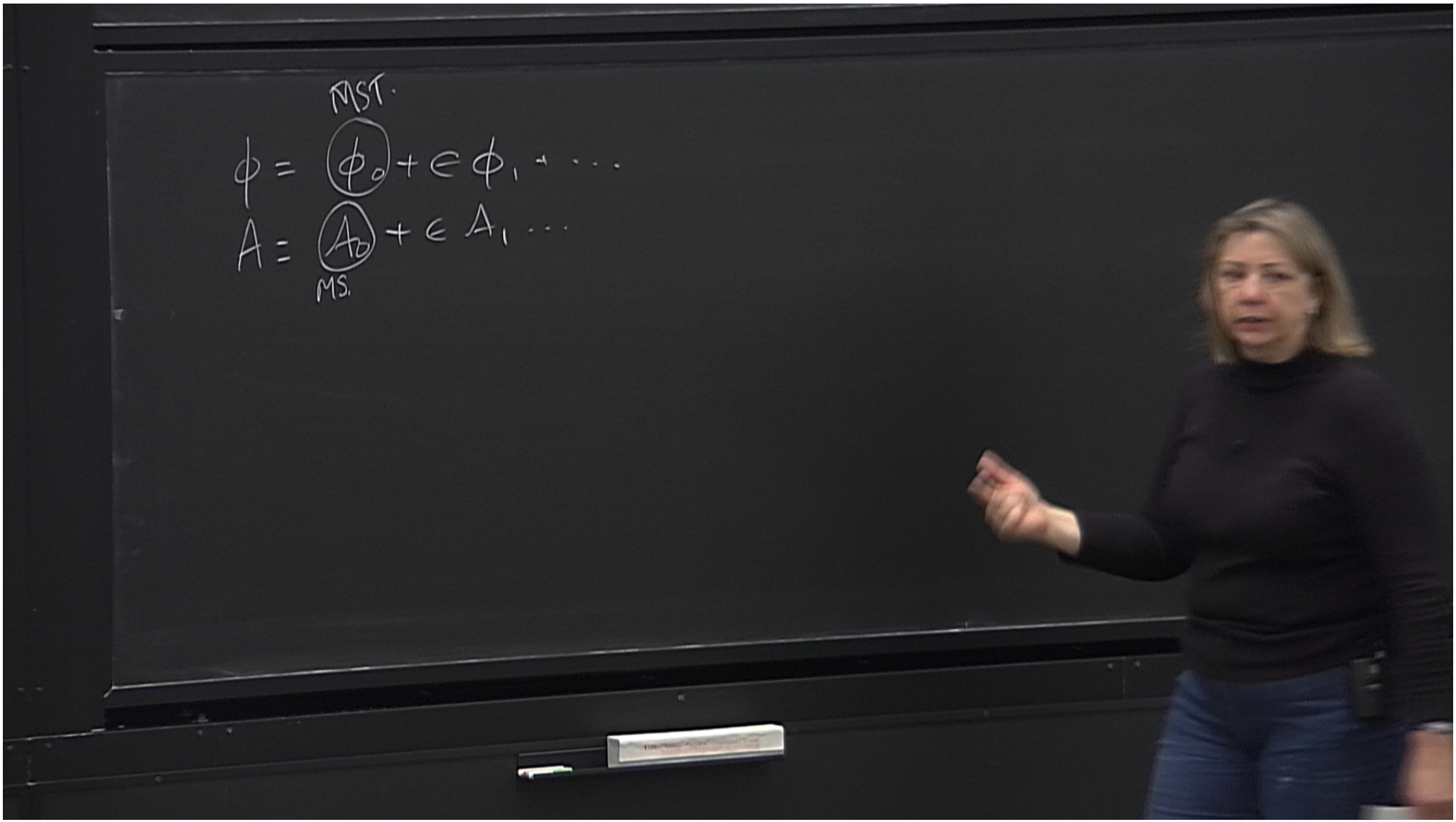
t = const

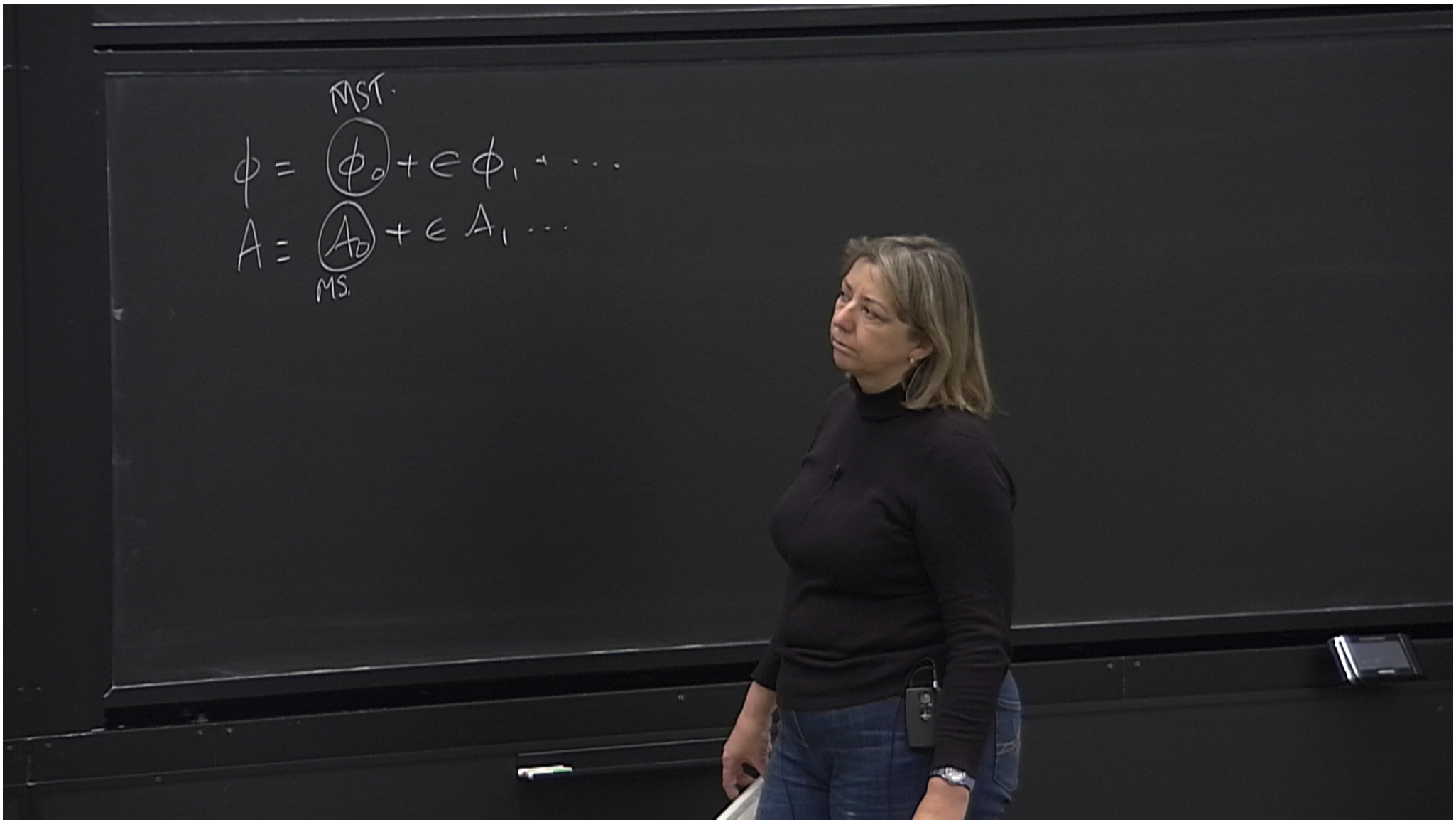


DOMAIN WALL

violates SEC

· observers accelerate away from wall


MST.
$\phi = \phi_0 + \epsilon \phi_1 + \cdots$
$A = A_0 + \epsilon A_1 \cdots$
MS.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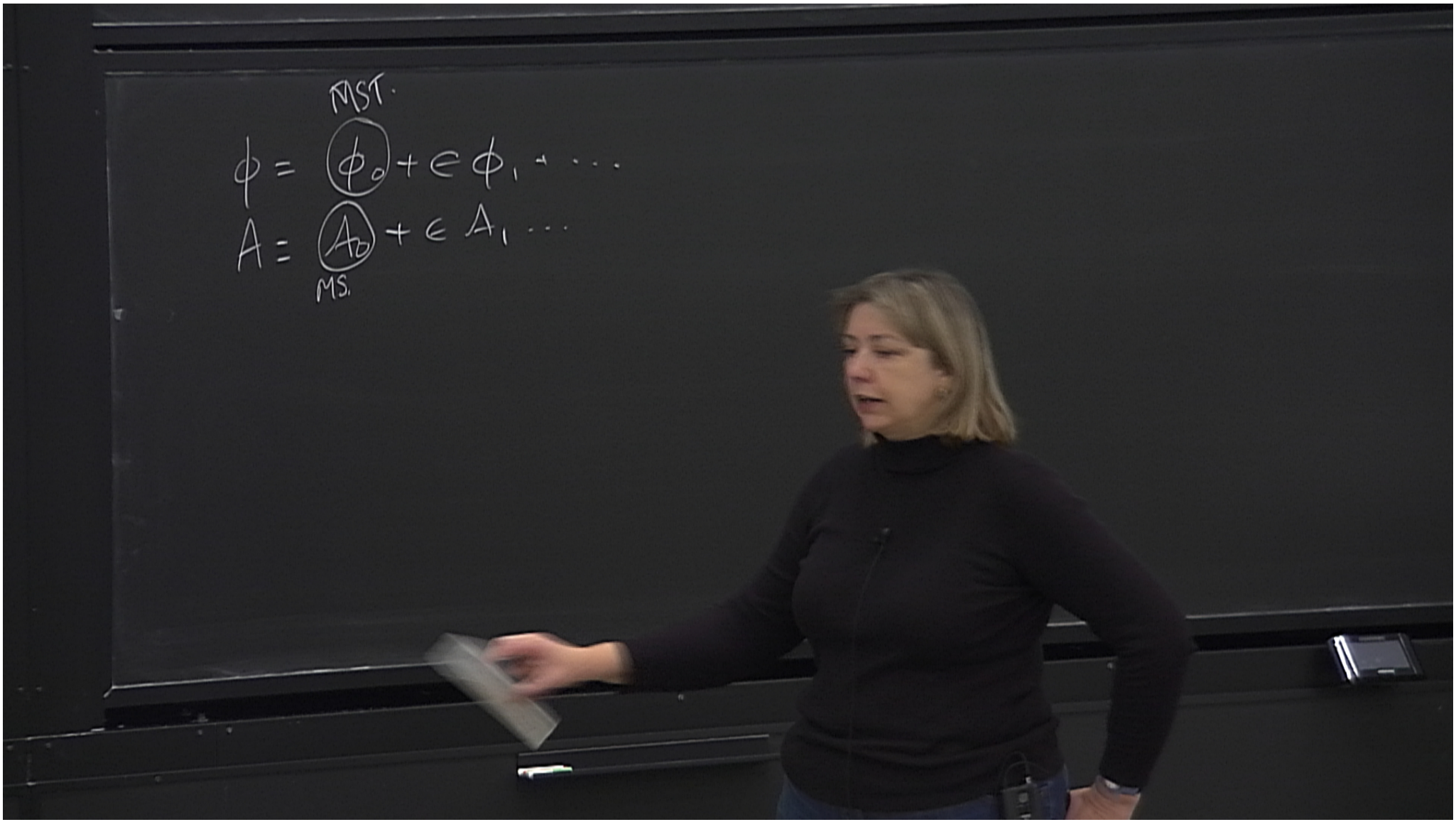
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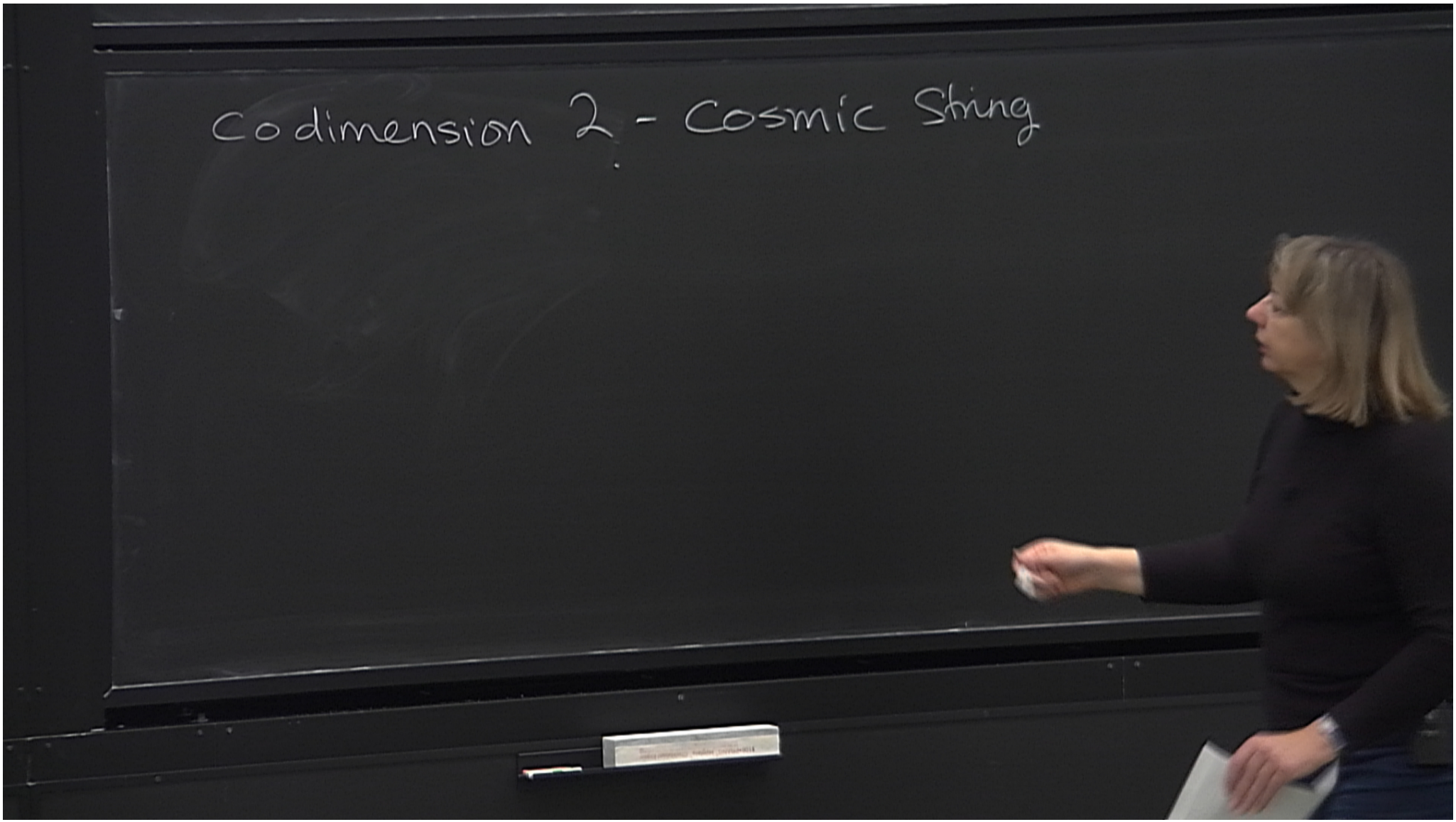

Codimension 2 - Cosmic String

Codimension 2 - Cosmic String

Abelian Higgs model

$$\mathcal{L} = \frac{1}{2}|D\Phi|^2 - \frac{1}{4}F^2 - \frac{\lambda}{8}(|\Phi|^2 - \eta^2)^2$$

Codimension 2 - Cosmic String

Abelian Higgs model

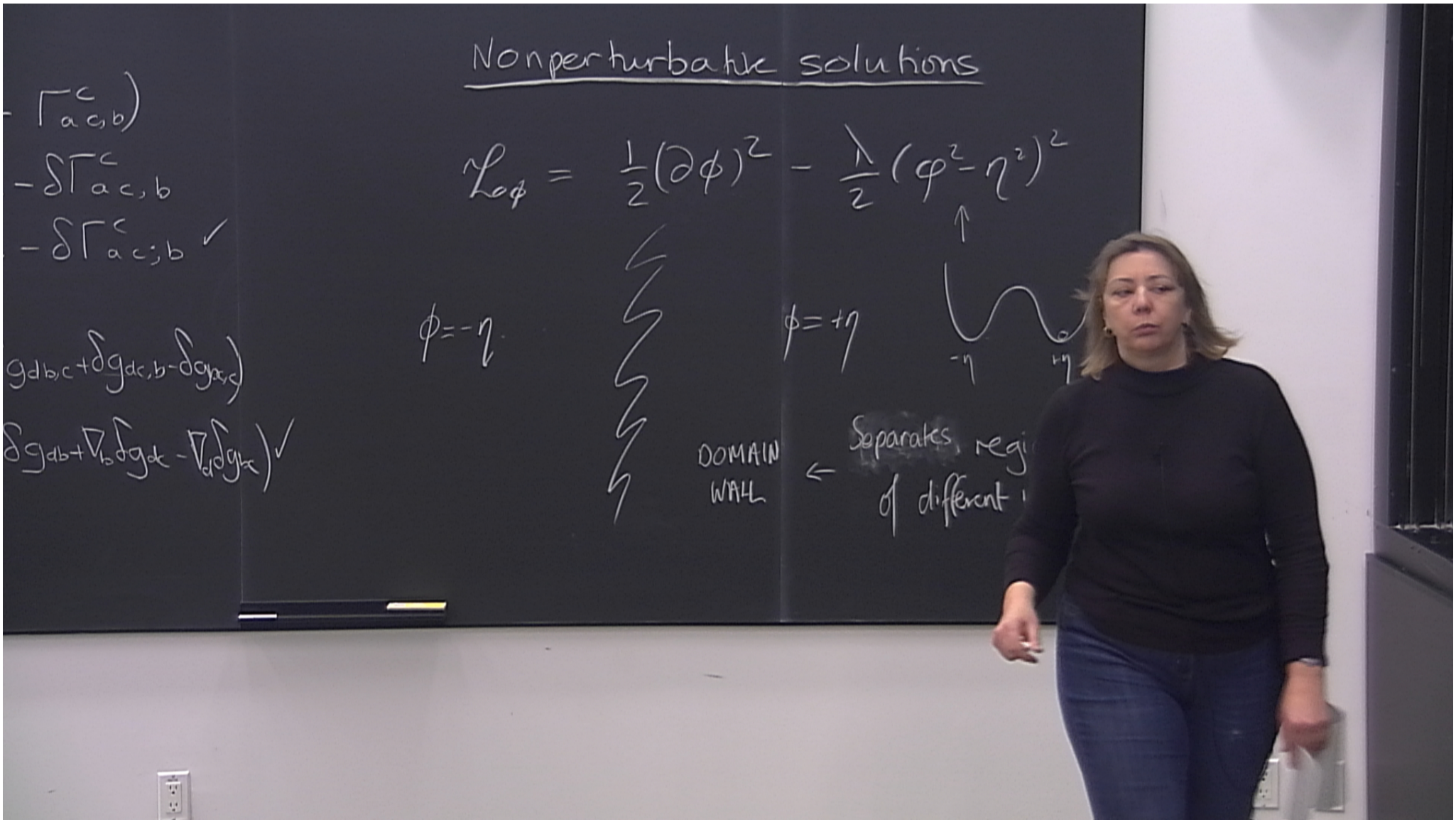
$$\mathcal{L} = \frac{1}{2}|D\Phi|^2 - \frac{1}{4}F^2 - \frac{\lambda}{8}(|\Phi|^2 - \eta^2)^2$$

$$\Phi = f e^{i\chi}$$

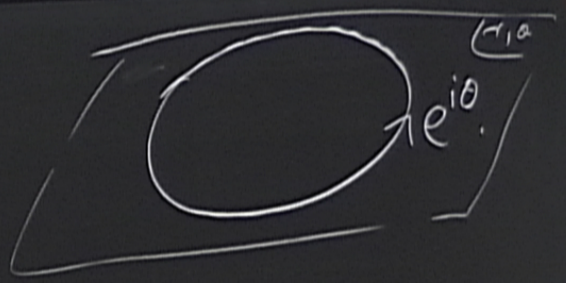
$$A_\mu = \frac{1}{e}(P_\mu - \partial_\mu \chi)$$

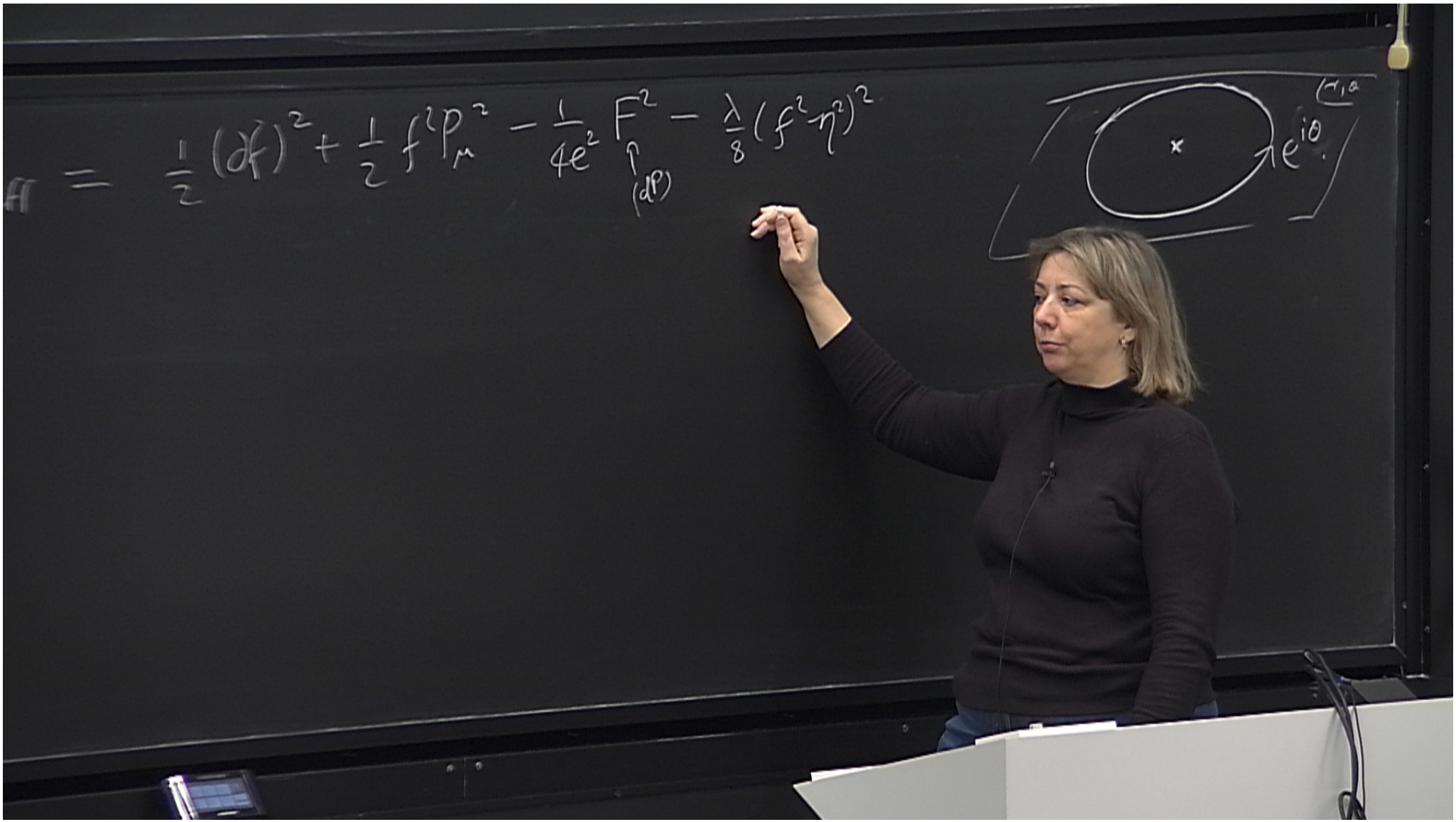
$$\mathcal{L}_{eff} = \frac{1}{2}(\partial f)$$

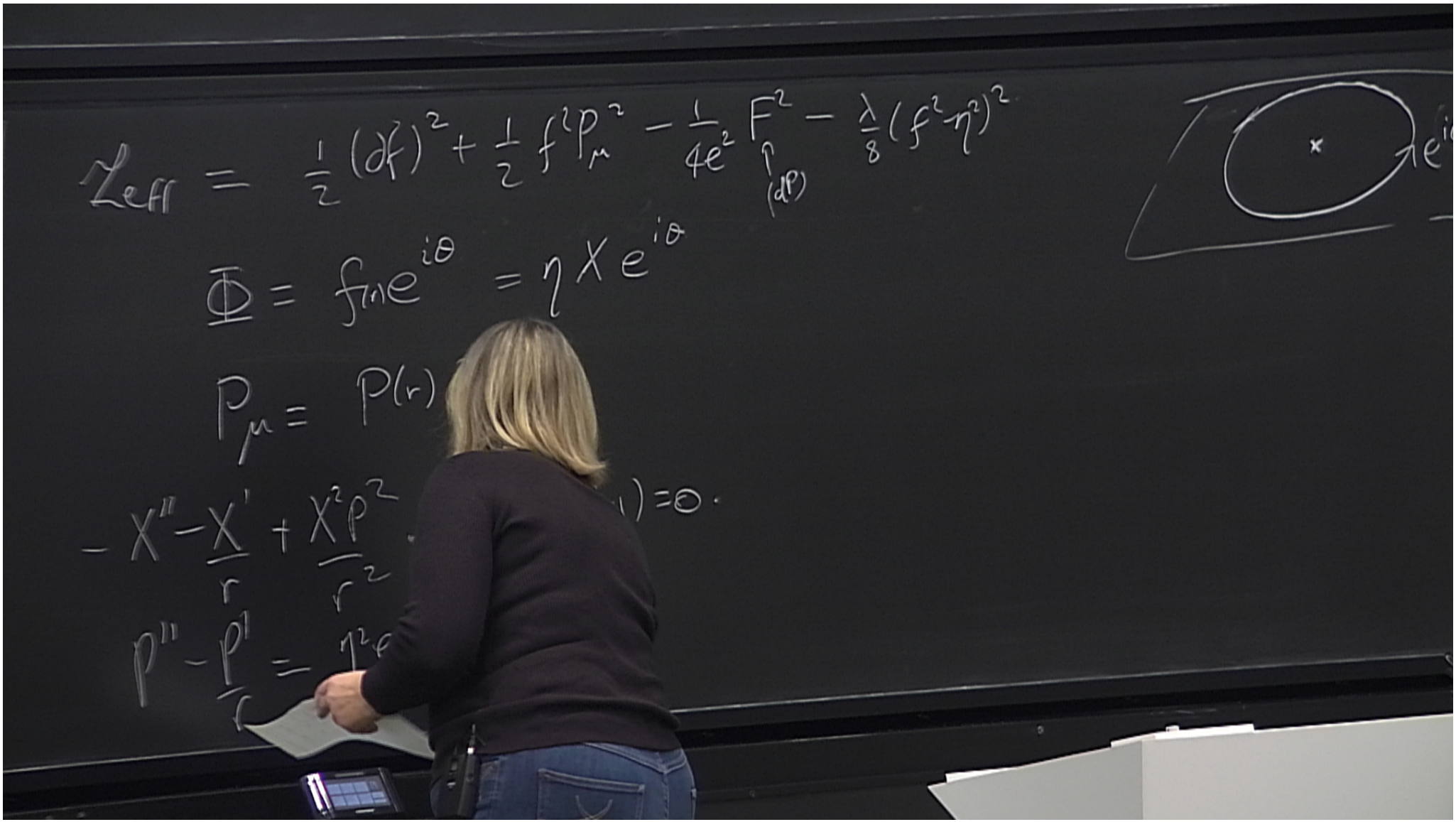
$$\mathcal{L}_{eff} = \frac{1}{2}(\partial f)^2 + \frac{1}{2} f^2 P_\mu^2 - \frac{1}{4e^2} \underset{(dP)}{F^2} - \frac{\lambda}{8}(f^2 - \eta^2)^2$$

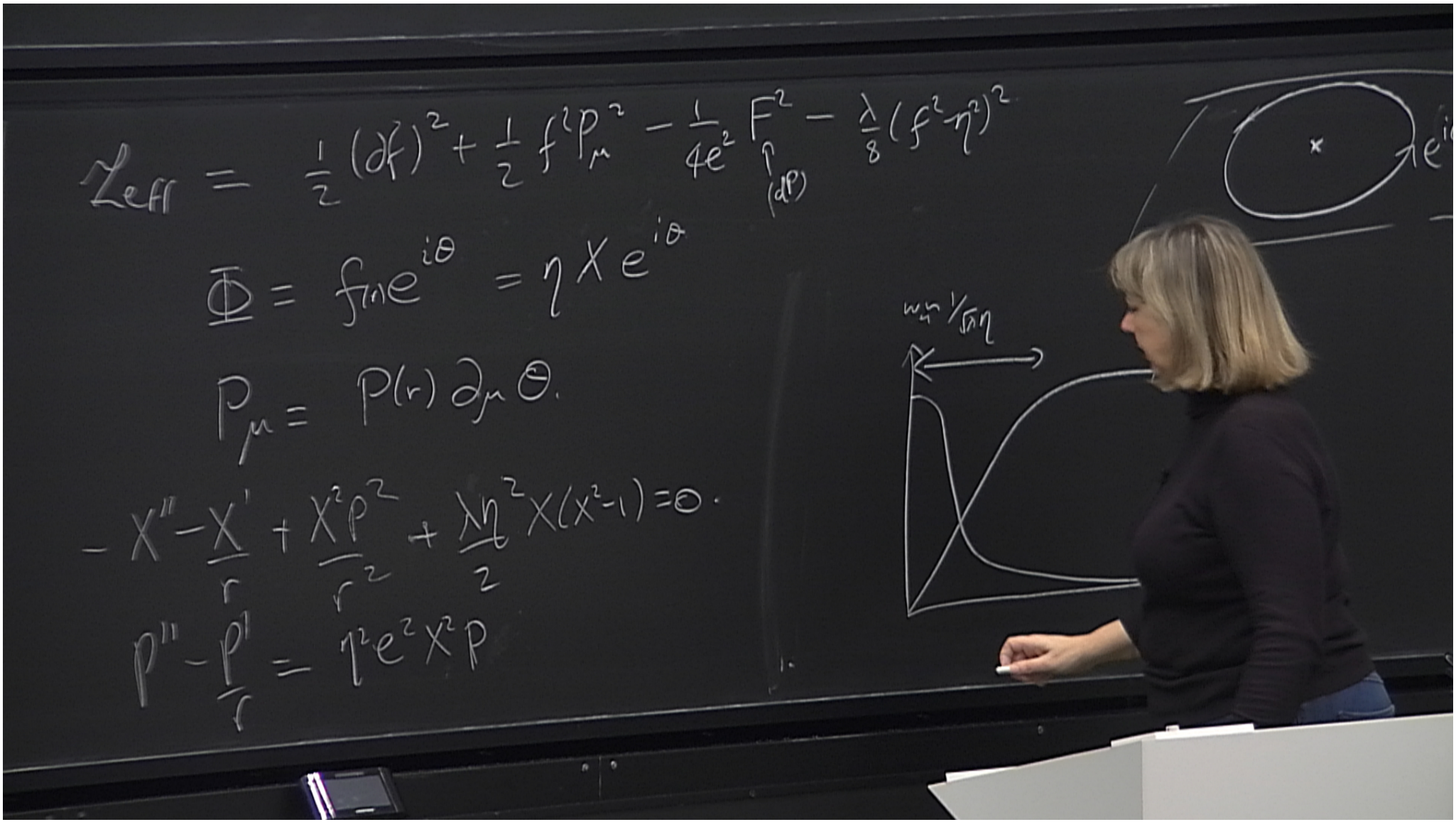

Nonperturbative solutions
$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2 - \eta^2)^2$
$\phi = -\eta$
$\phi = +\eta$
$-\eta$
$+\eta$
DOMAIN WALL
Separates regi
of different

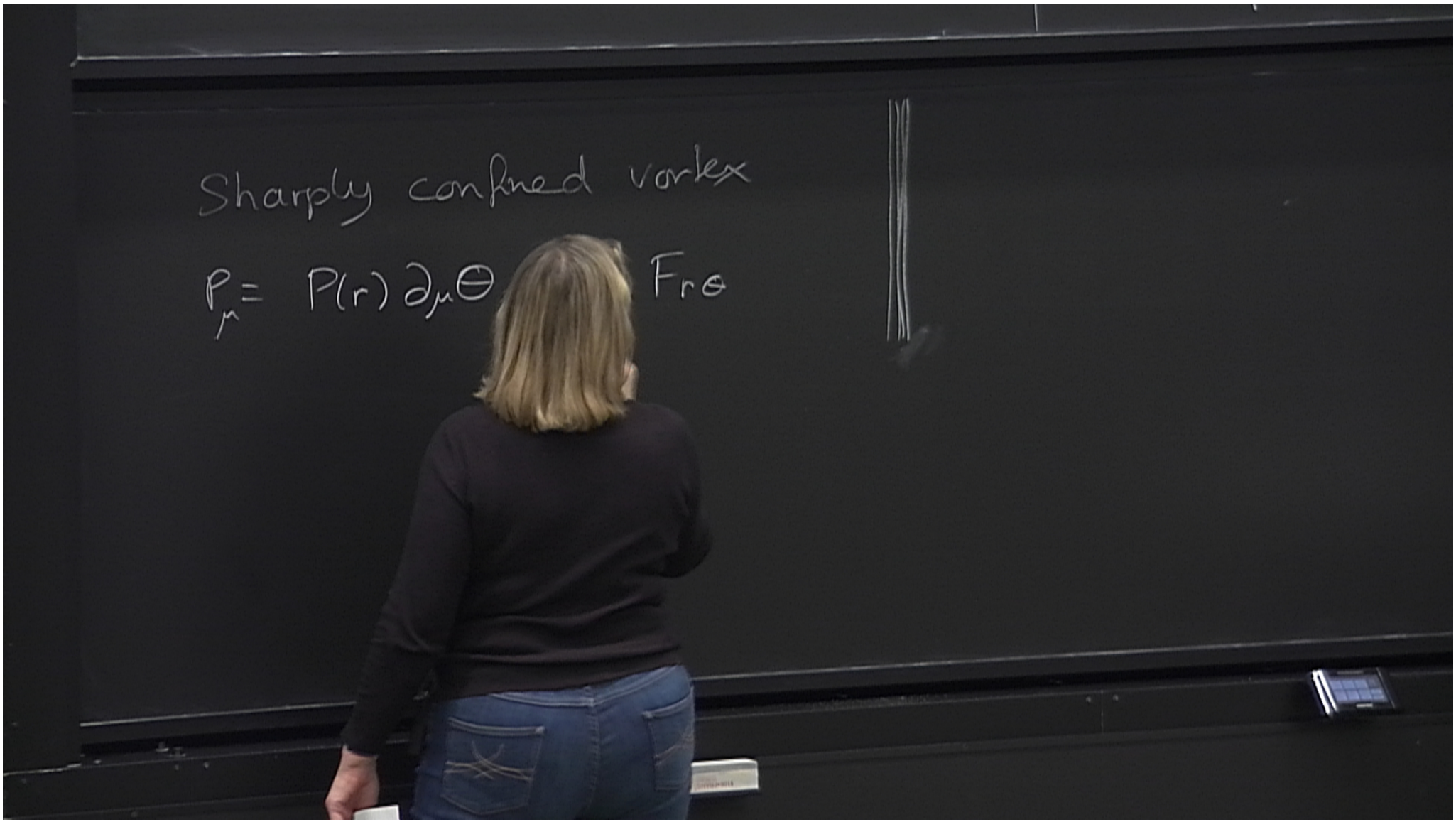
$$\mathcal{L}_{eff} = \frac{1}{2}(\partial f)^2 + \frac{1}{2} f^2 P_\mu^2 - \frac{1}{4e^2} \underset{(dP)}{F^2} - \frac{\lambda}{8}(f^2 - \eta^2)^2$$




$\mathcal{L}_{eff} = \frac{1}{2}(\partial f)^2 + \frac{1}{2} f^2 P_\mu^2 - \frac{1}{4e^2} F^2 - \frac{\lambda}{8}(f^2 - \eta^2)^2$
(dP)
$e^{i\theta}$


$\mathcal{L}_{eff} = \frac{1}{2}(\partial f)^2 + \frac{1}{2} f^2 P_\mu^2 - \frac{1}{4e^2} F^2_{(dP)} - \frac{\lambda}{8}(f^2-\eta^2)^2$
$\Phi = f_1 e^{i\theta} = \eta X e^{i\theta}$
$P_\mu = P(r)$
$-X'' - \frac{X'}{r} + \frac{X^2 P^2}{r^2}$
$P'' - \frac{P'}{r} = \eta^2$

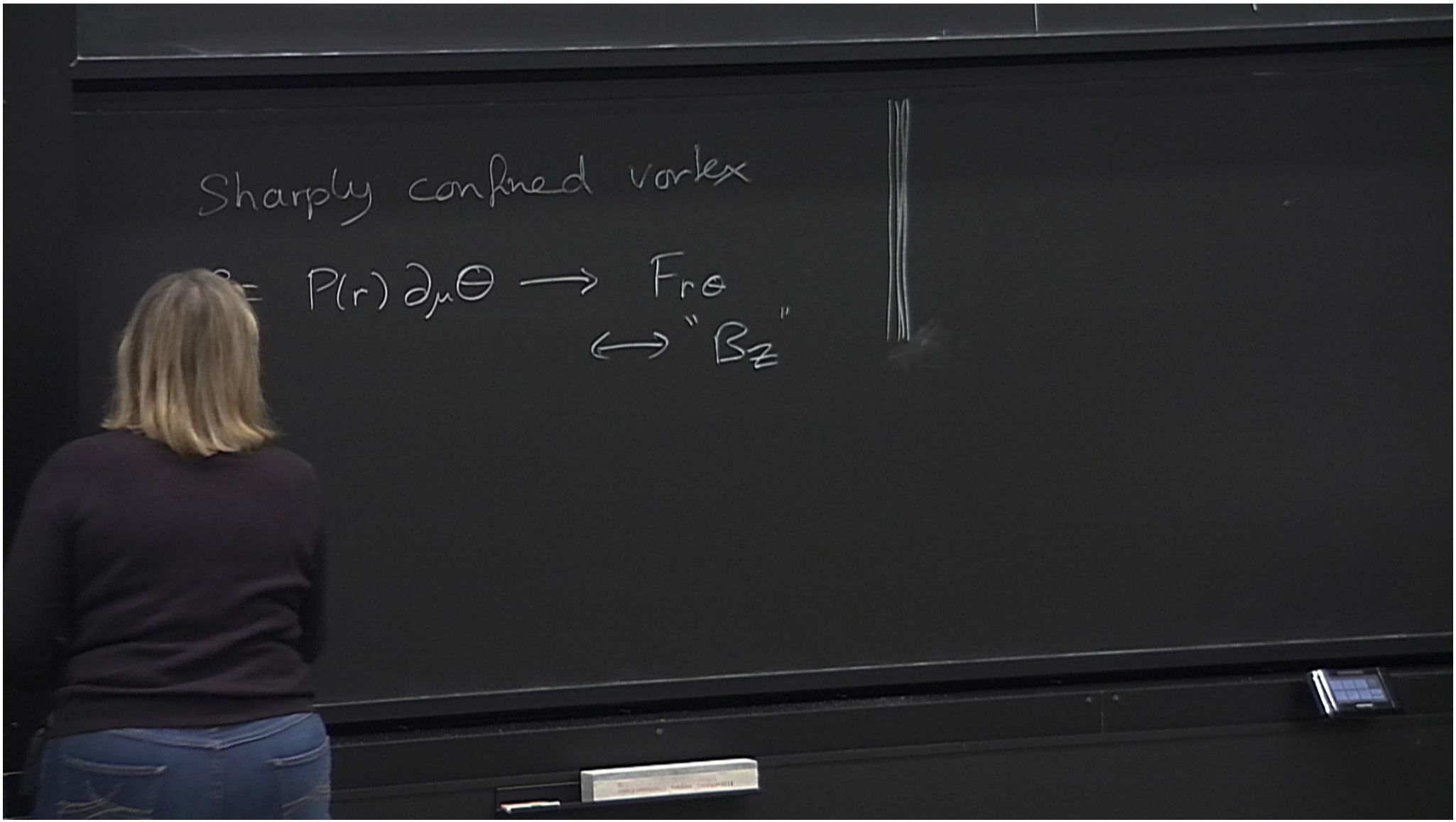

$\mathcal{L}_{eff} = \frac{1}{2}(\partial f)^2 + \frac{1}{2} f^2 P_\mu^2 - \frac{1}{4e^2} F^2_{(dP)} - \frac{\lambda}{8}(f^2-\eta^2)^2$
$\Phi = f_\pi e^{i\theta} = \eta X e^{i\theta}$
$P_\mu = P(r)\,\partial_\mu \theta.$
$-X'' - \frac{X'}{r} + \frac{X^2 P^2}{r^2} + \frac{\lambda\eta^2}{2} X(X^2-1) = 0.$
$P'' - \frac{P'}{r} = \eta^2 e^2 X^2 P$
$m_\pi \sim 1/\sqrt{\lambda}\eta$

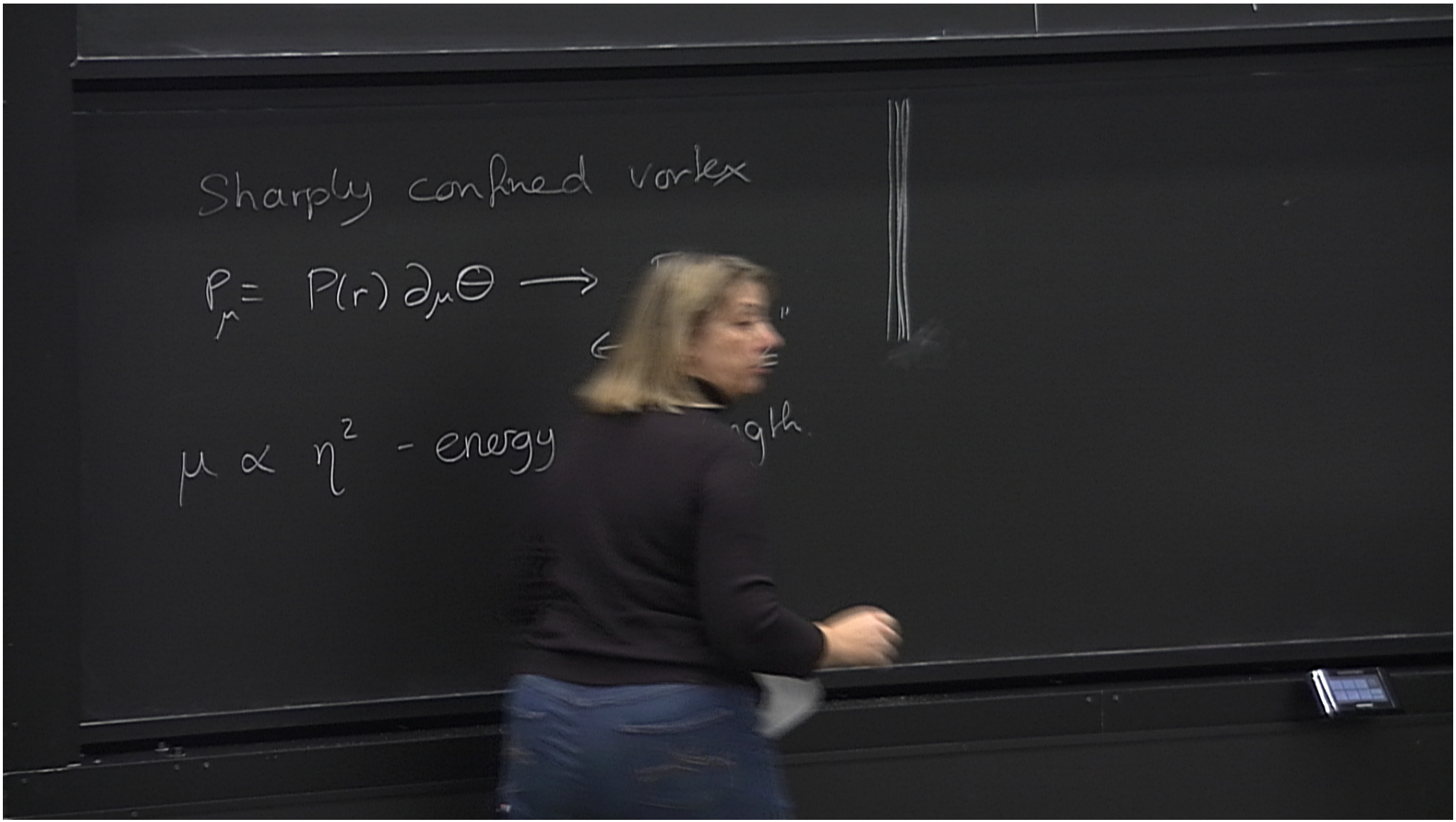

Sharply confined vortex
$P_\mu = P(r)\,\partial_\mu \Theta$
$F_{r\theta}$

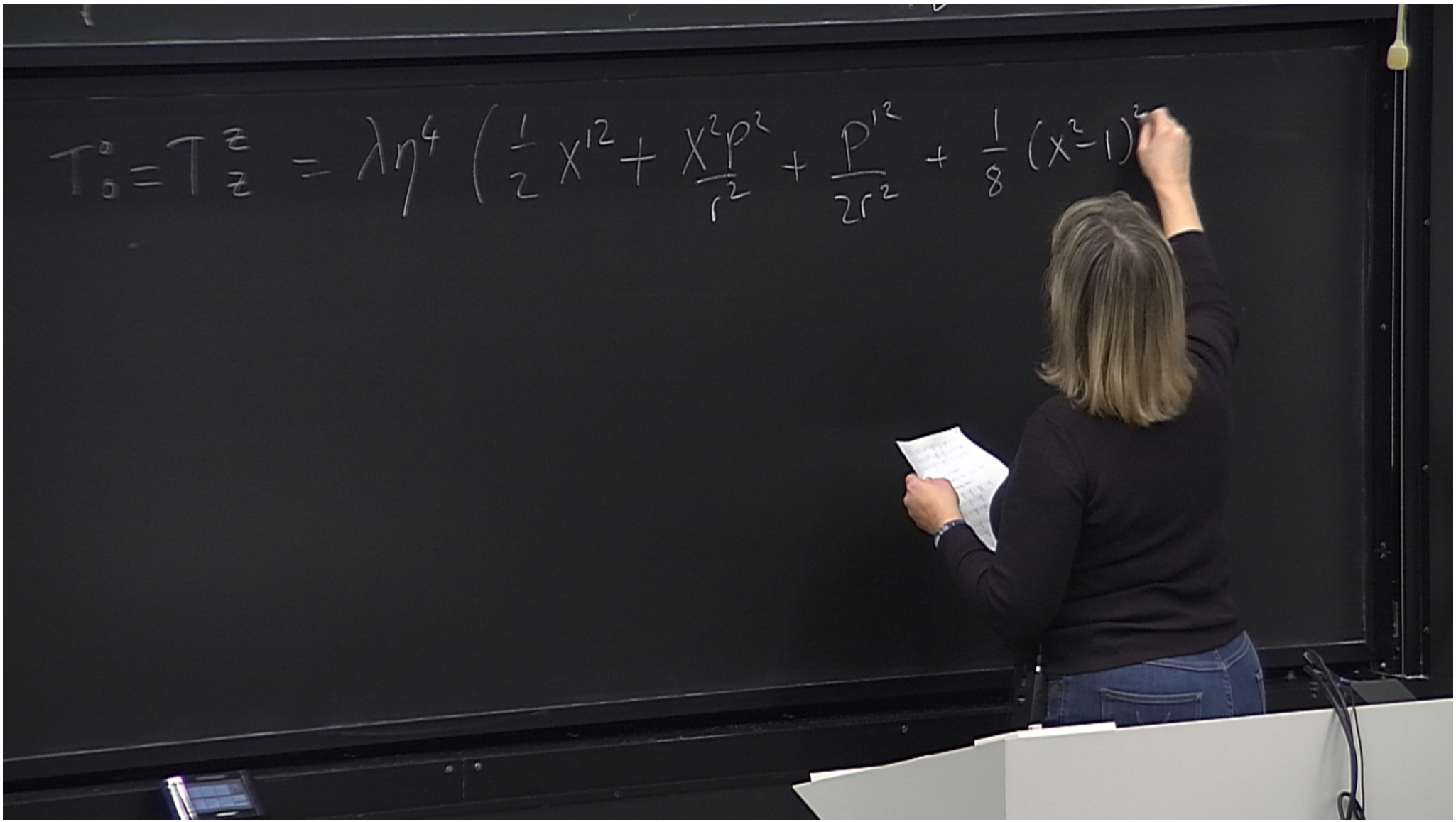
Sharply confined vortex

$$P_\mu = P(r)\,\partial_\mu \Theta \longrightarrow F_{r\theta}$$

$$\longleftrightarrow \text{"}B_z\text{"}$$


Sharply confined vortex
$= P(r)\,\partial_\mu \Theta \longrightarrow F_{r\theta}$
$\longleftrightarrow$ "B_z"


Sharply confined vortex
$P_\mu = P(r)\,\partial_\mu \Theta \longrightarrow$
$\mu \propto \eta^2$ - energy


$T^0_0 = T^z_z = \lambda\eta^4 \left(\frac{1}{2} X'^2 + \frac{X^2 p^2}{r^2} + \frac{p'^2}{2r^2} + \frac{1}{8}(x^2-1)^2 \right.$

$$ds^2 = A^2(r)(dt^2 - dz^2) - dr^2 - C^2 d\theta^2$$

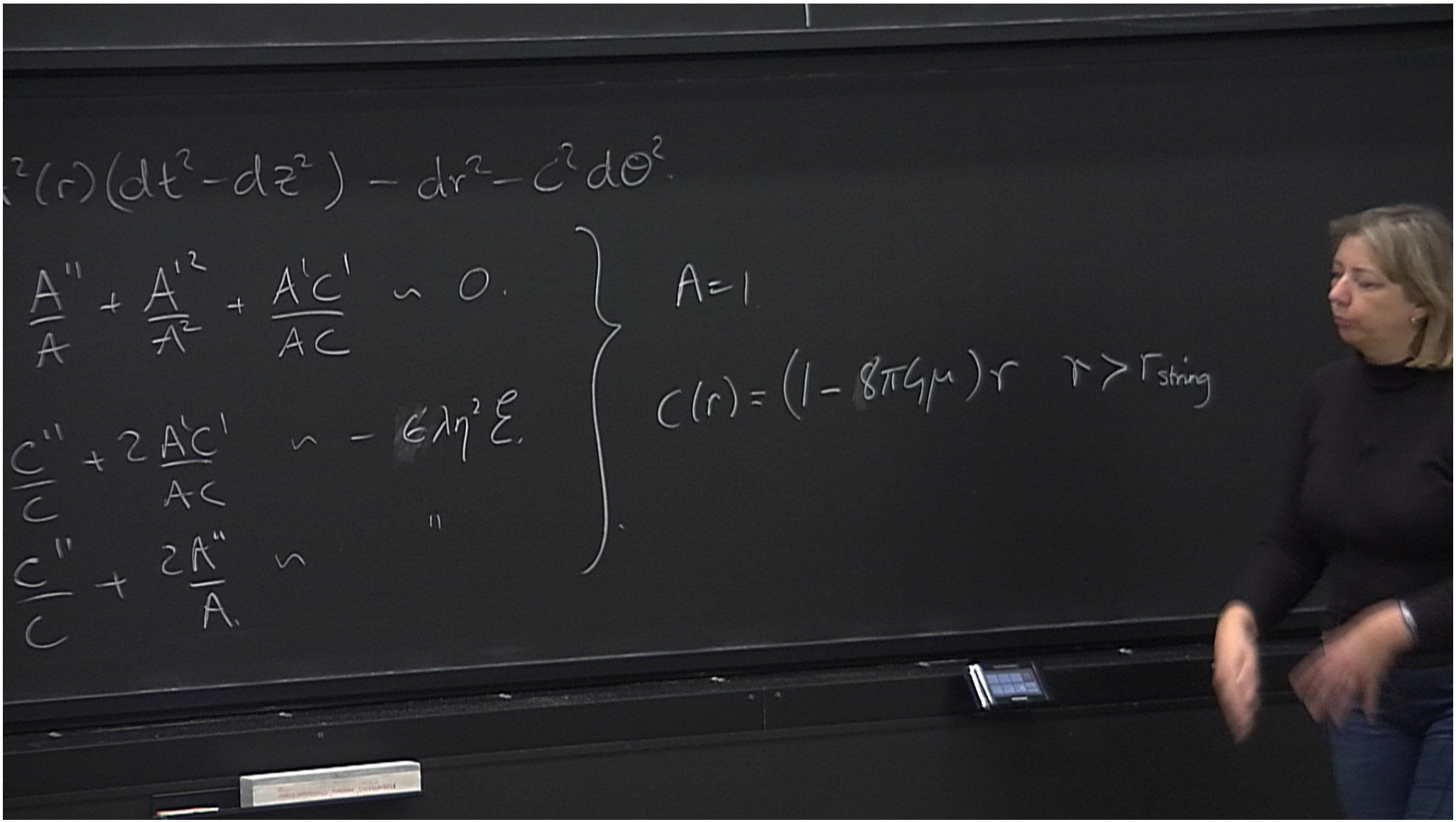
$$R^t_t = \frac{A''}{A} + \frac{A'^2}{A^2} + \frac{A'C'}{AC}$$

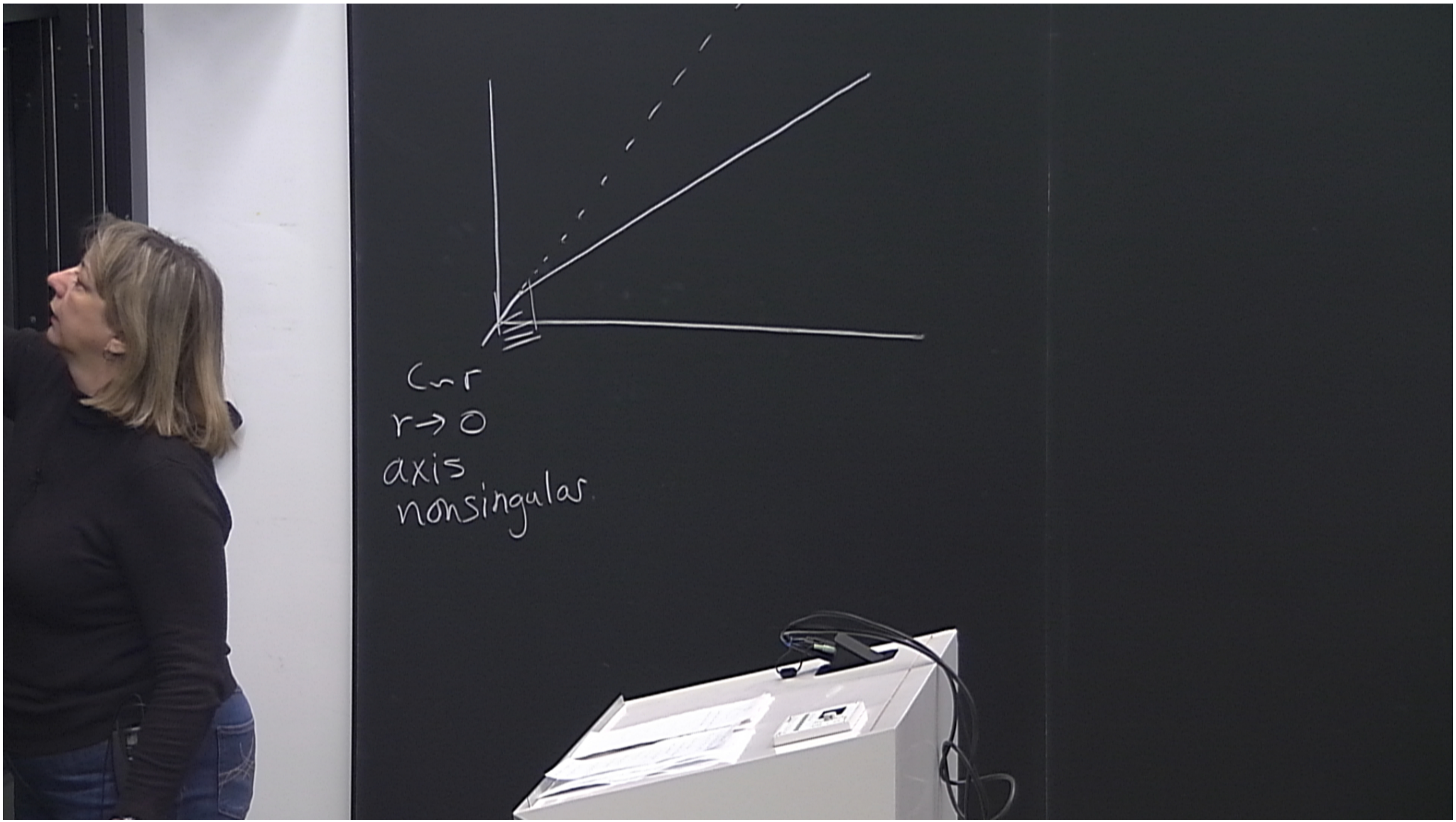
$$R^\theta_\theta = \frac{C''}{C} + 2\frac{A'C'}{AC}$$

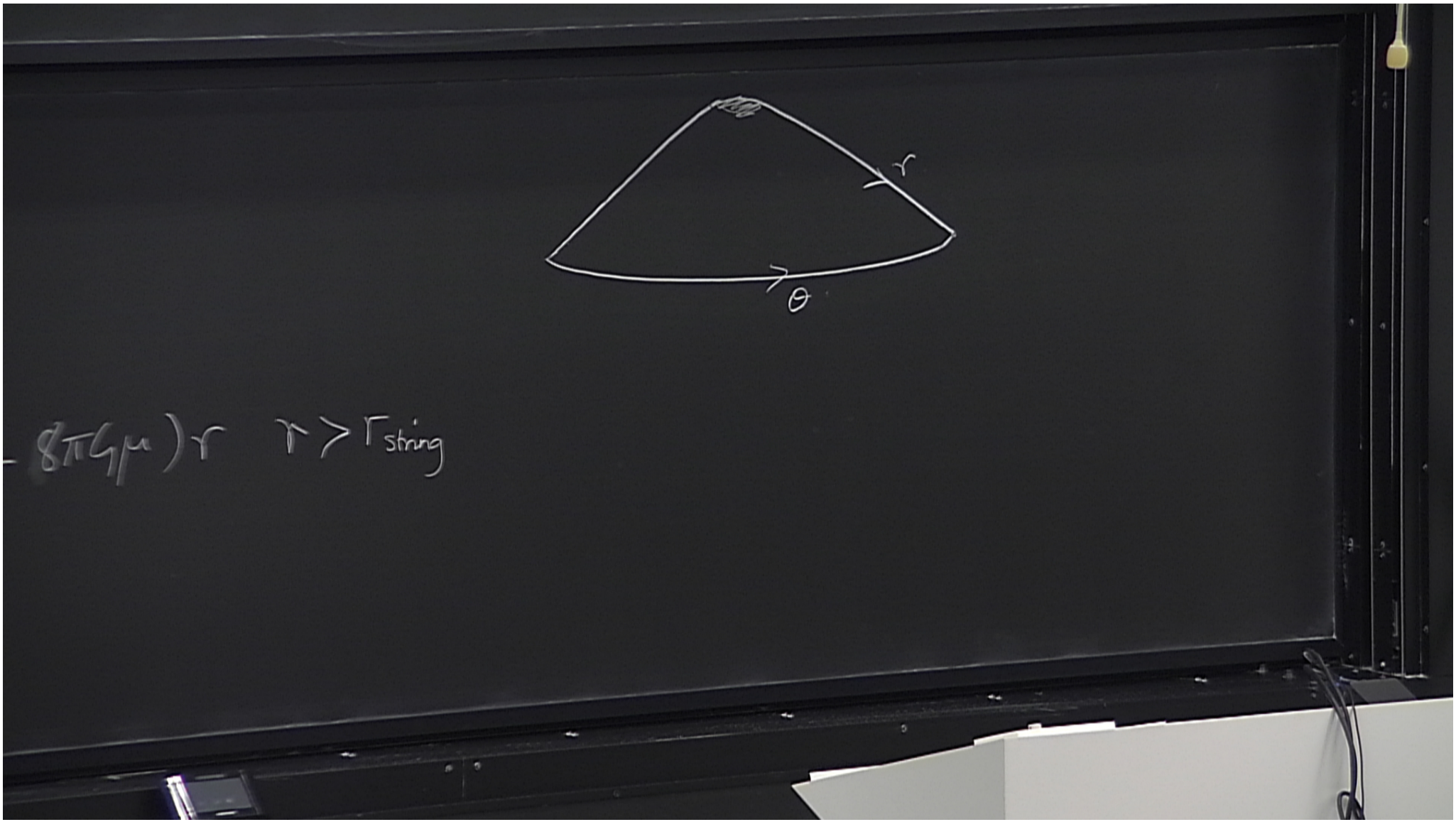
$$R^r_r = \frac{C''}{C} + 2\frac{A''}{A}.$$

$$ds^2 = A^2(r)(dt^2 - dz^2) - dr^2 - C^2 d\theta^2.$$

$$\left.\begin{aligned}
R^t_t &= \frac{A''}{A} + \frac{A'^2}{A^2} + \frac{A'C'}{AC} \sim 0. \\
R^\theta_\theta &= \frac{C''}{C} + 2\frac{A'C'}{AC} \sim -6\lambda\eta^2\mathcal{E}. \\
R^r_r &= \frac{C''}{C} + 2\frac{A''}{A} \sim \quad '' \quad .
\end{aligned}\right\} \quad A = 1$$


$^2(r)(dt^2 - dz^2) - dr^2 - C^2 d\theta^2$
$\frac{A''}{A} + \frac{A'^2}{A^2} + \frac{A'C'}{AC} \sim 0$
$\frac{C''}{C} + 2\frac{A'C'}{AC} \sim -6\lambda\eta^2\mathcal{E}$
$\frac{C''}{C} + 2\frac{A''}{A} \sim$
$A = 1$
$C(r) = (1 - 8\pi G\mu)r \quad r > r_{string}$


$C \sim r$
$r \to 0$
axis
nonsingular


r
θ
- 8πGμ) r r > r_string