Title: 12/13 PSI - Gravitational Physics Review Lecture 6

Date: Feb 04, 2013 09:00 AM

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Abstract:


Derivation of black hole solns

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For the "warped" metric

$$ds^2 = A^2(r)\, g_{\mu\nu}\, dx^\mu dx^\nu - dr^2$$

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$$ds^2 = A^2(r)\, g_{\mu\nu}\, dx^\mu dx^\nu - dr^2 \qquad (\text{DIM } D)$$

$$\left.\begin{aligned} \underset{\sim}{\theta}^a{}_b &= \underset{\sim}{\theta}_0{}^a{}_b \\ \underset{\sim}{\theta}^a{}_r &= \frac{A'}{A}\,\underset{\sim}{\omega}^a \end{aligned}\right\} \quad \begin{aligned} R^r{}_r &= (D-1)\,A''/A \\ R^\mu{}_\nu &= \frac{1}{A^2} R_0{}^\mu{}_\nu + \left[(D-2)\frac{A'^2}{A^2} + \frac{A''}{A}\right]\delta^\mu_\nu \end{aligned}$$

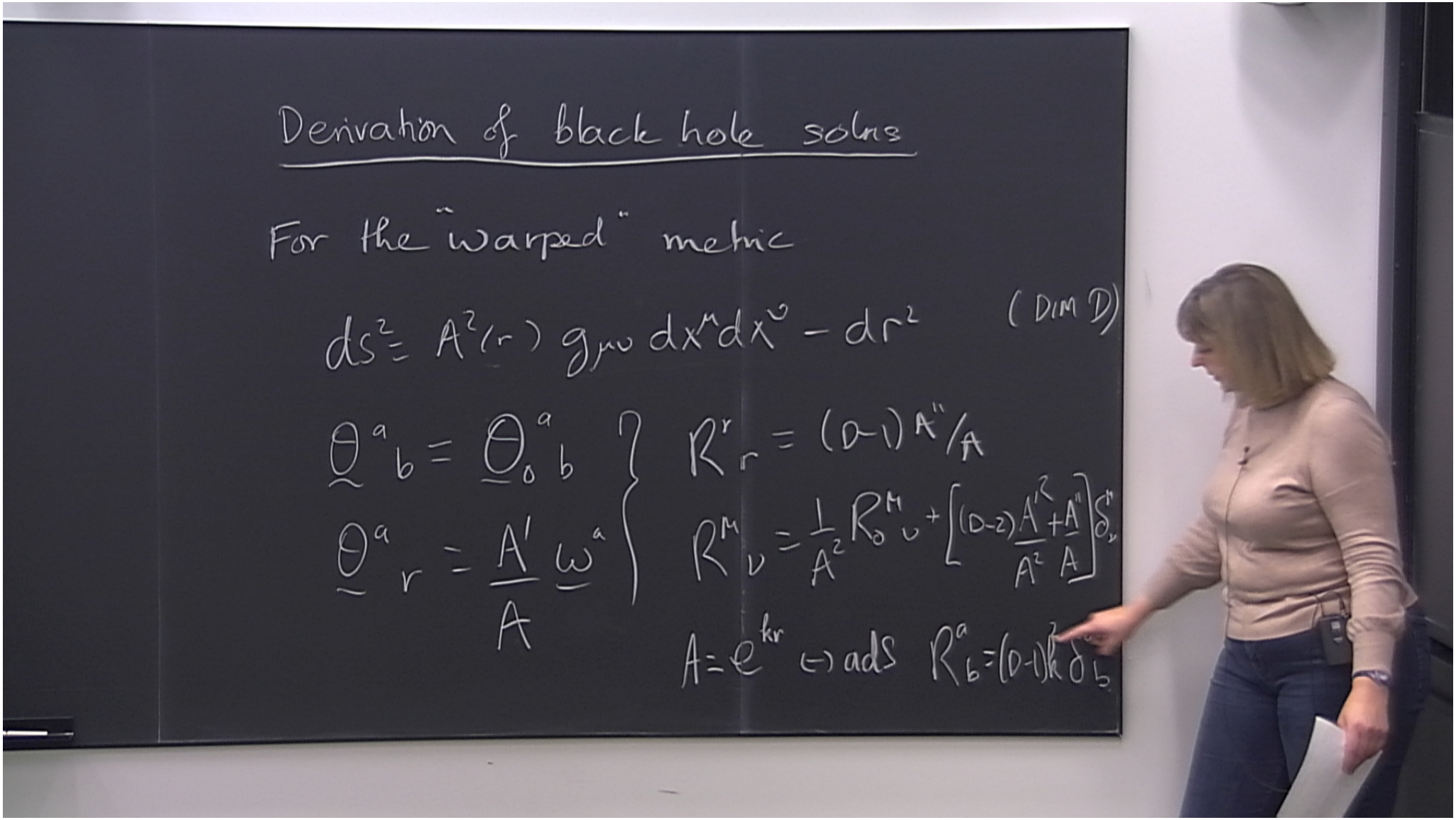
Derivation of black hole solns

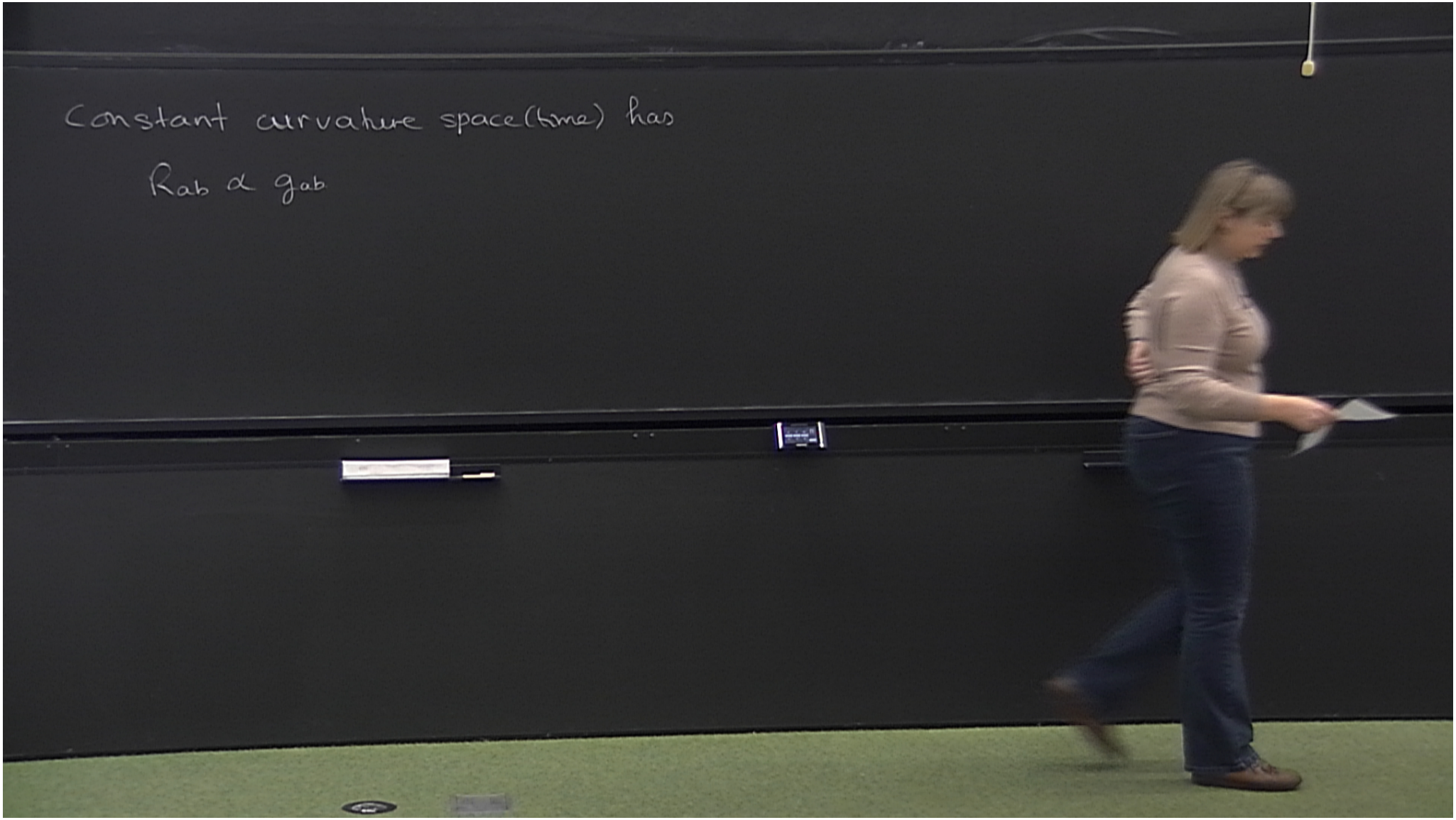
For the "warped" metric

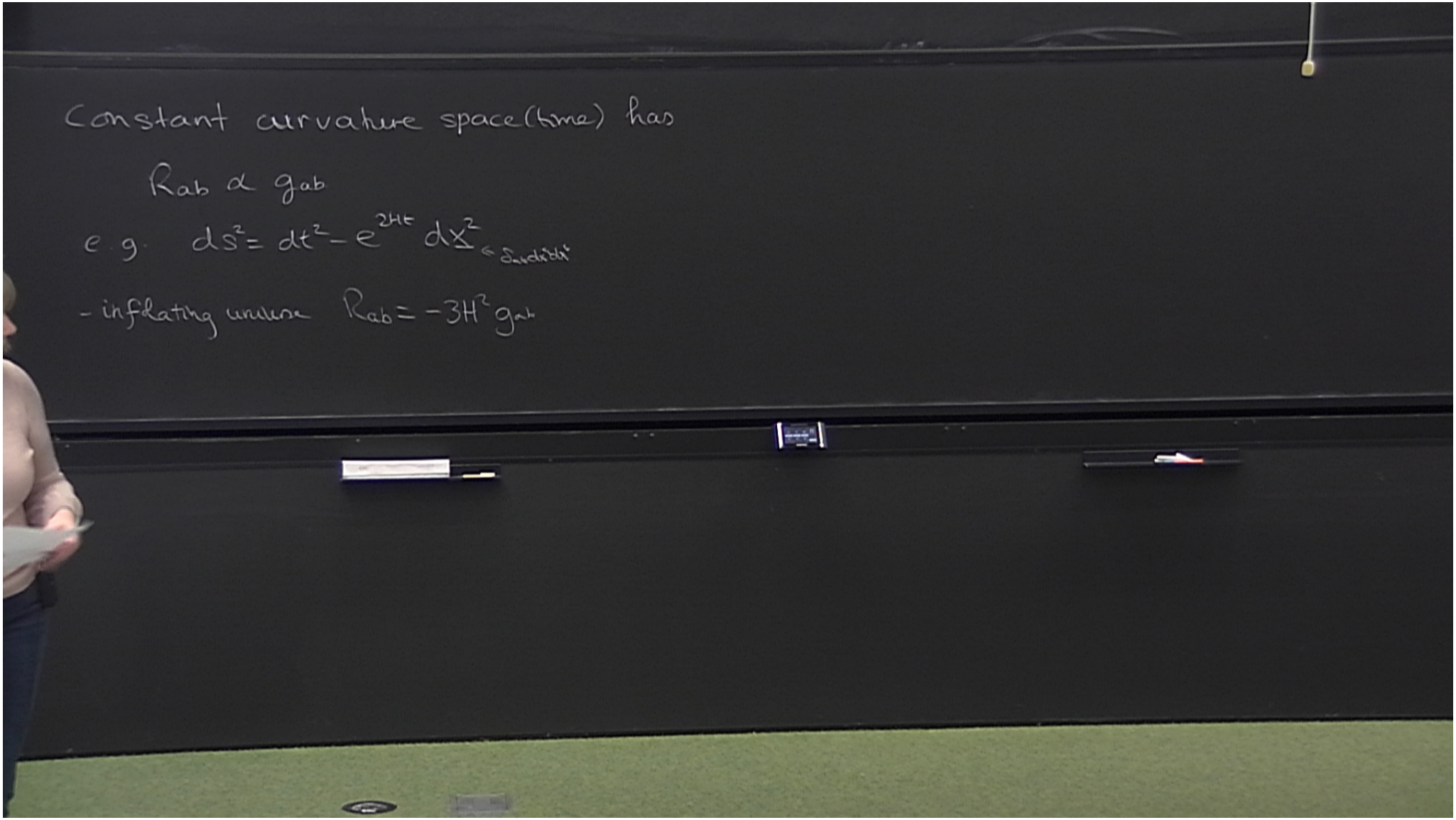
$$ds^2 = A^2(r)\, g_{\mu\nu}\, dx^\mu dx^\nu - dr^2 \qquad (\text{DIM } D)$$

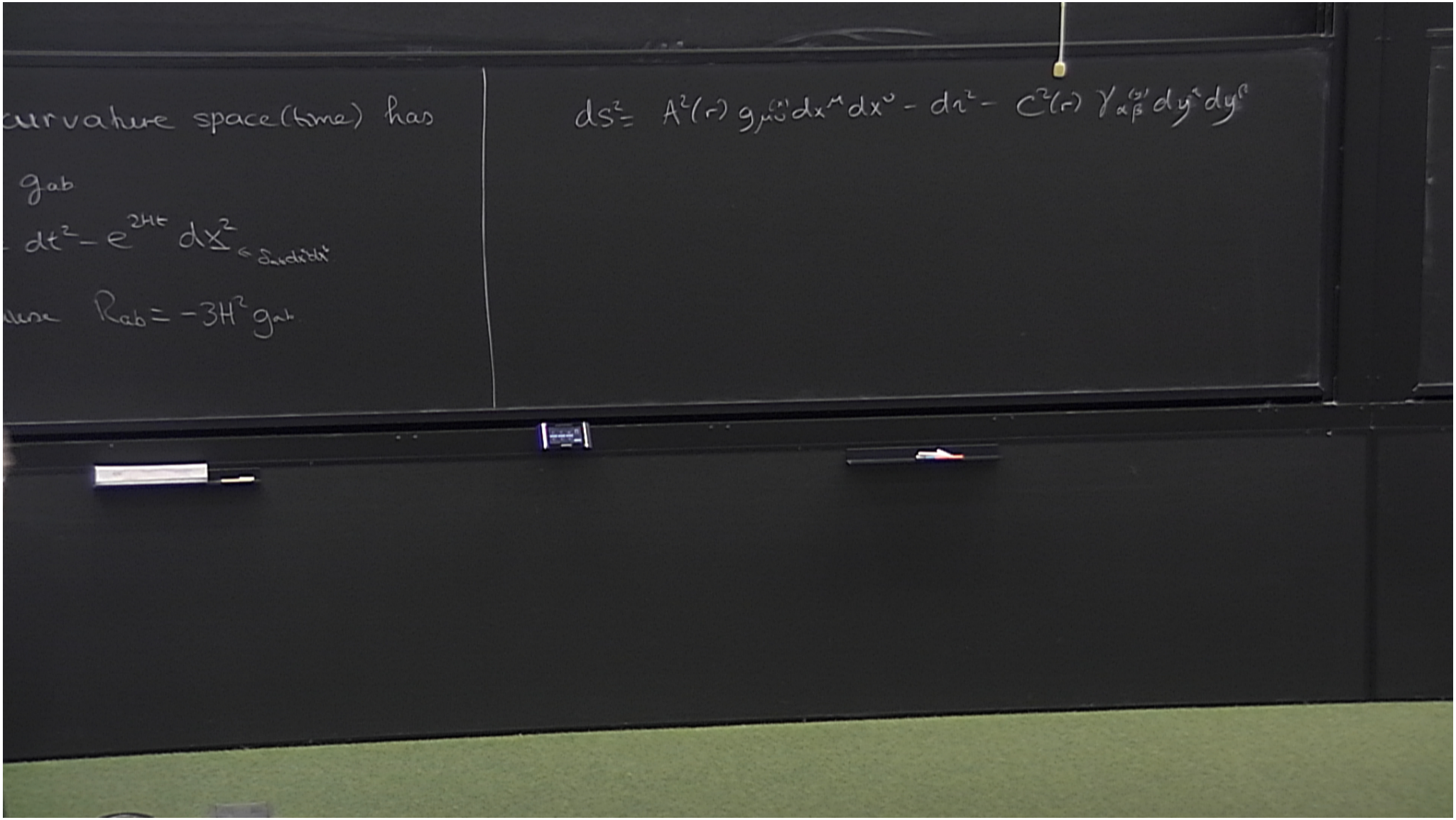
$$\left.\begin{aligned} \underset{\sim}{\theta}^a{}_b &= \underset{\sim}{\theta}_0{}^a{}_b \\ \underset{\sim}{\theta}^a{}_r &= \frac{A'}{A}\,\underset{\sim}{\omega}^a \end{aligned}\right\} \quad \begin{aligned} R^r{}_r &= (D-1)\,A''/A \\ R^\mu{}_\nu &= \frac{1}{A^2} R_0{}^\mu{}_\nu + \left[(D-2)\frac{A'^2}{A^2} + \frac{A''}{A}\right]\delta^\mu_\nu \end{aligned}$$

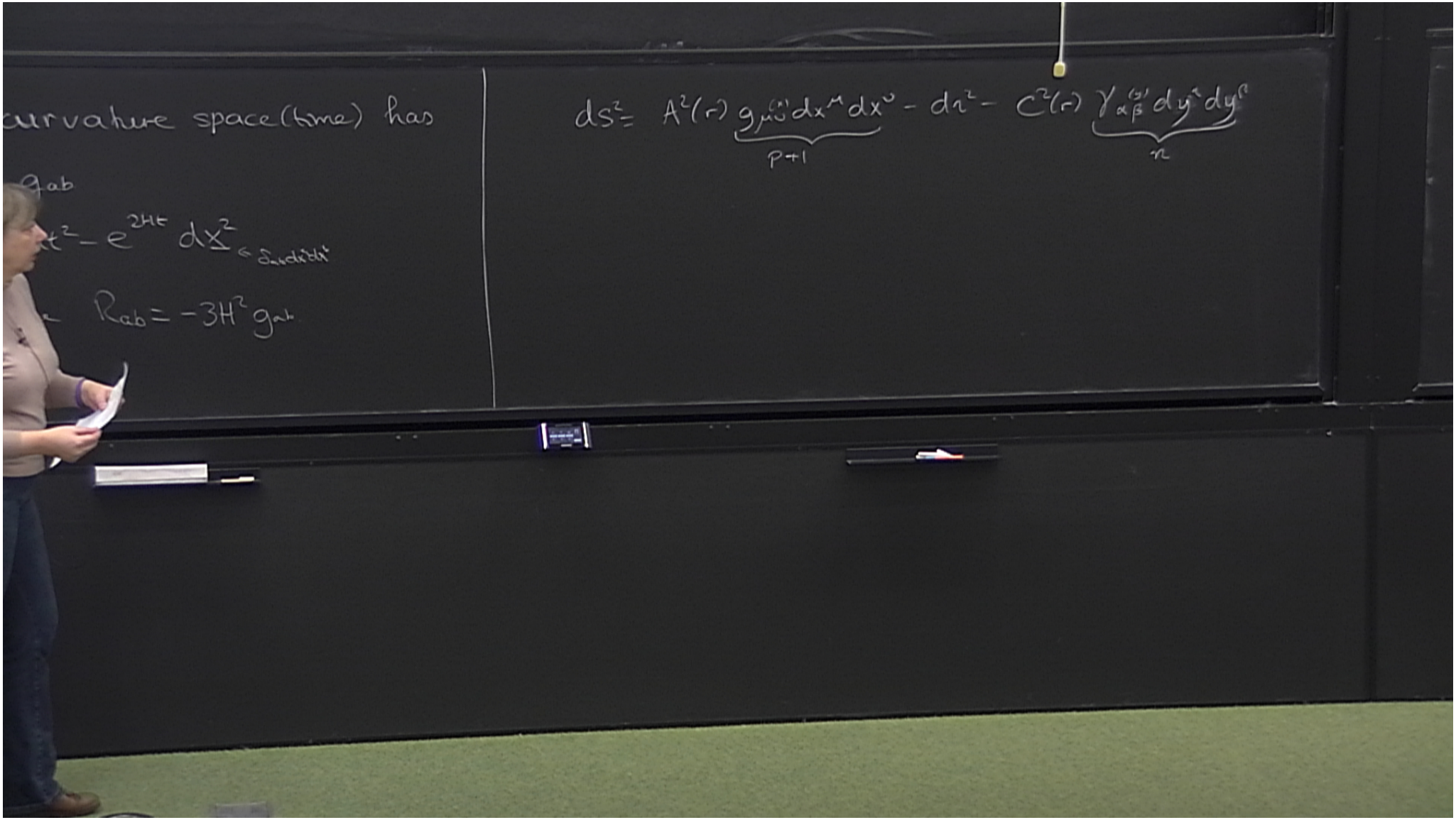
$A = e^{kr} \hookrightarrow$ ads $\quad R^a{}_b = (D-1)k^2\,\delta^a_b$

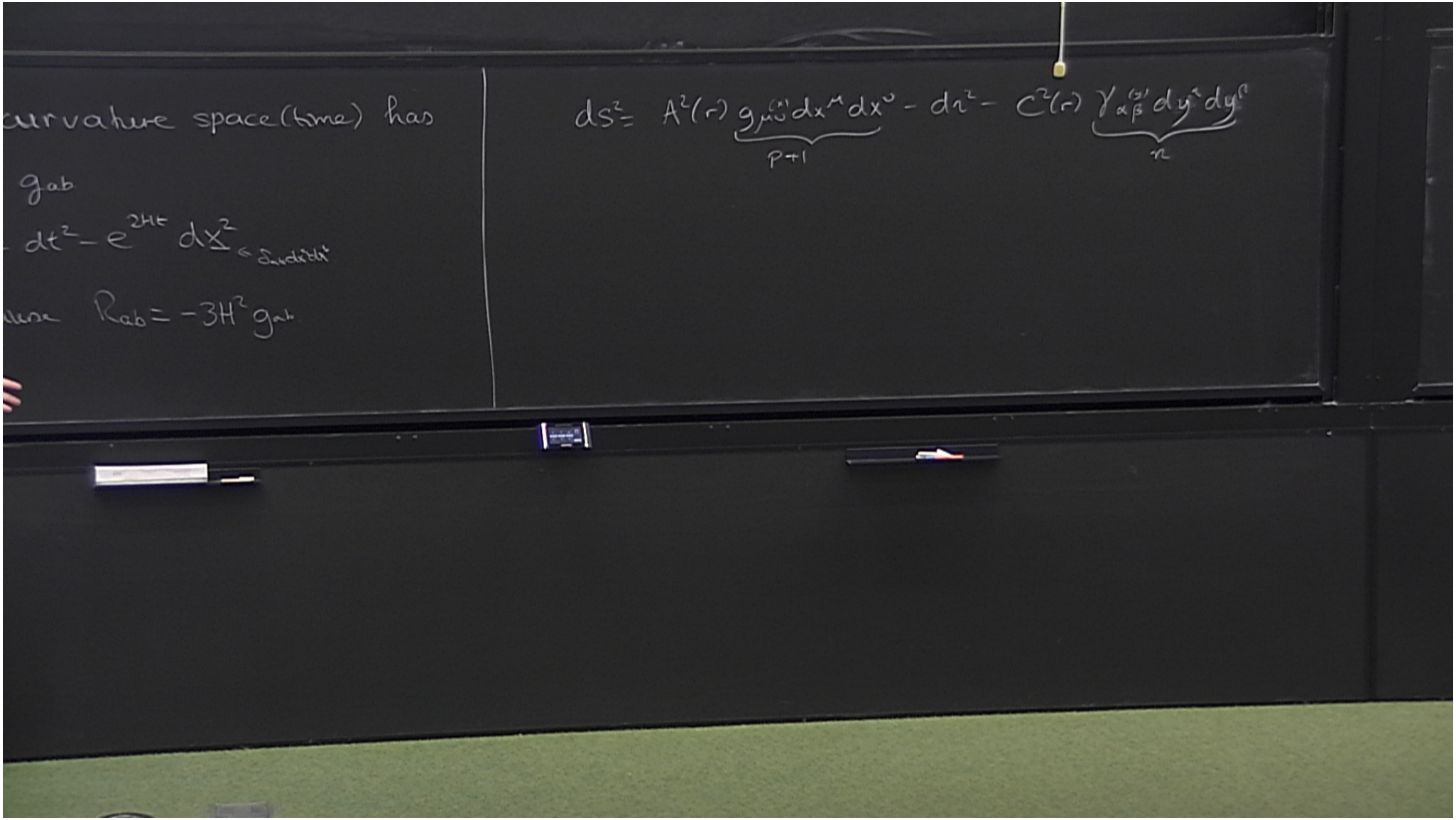

Derivation of black hole solns
For the "warped" metric
$ds^2 = A^2(r)\, g_{\mu\nu}\, dx^\mu dx^\nu - dr^2$ (DIM D)
$\underset{\sim}{\theta}^a{}_b = \underset{0}{\theta}^a{}_b$
$\underset{\sim}{\theta}^a{}_r = \frac{A'}{A}\underset{\sim}{\omega}^a$
$R^r{}_r = (D-1)A''/A$
$R^\mu{}_\nu = \frac{1}{A^2}R^\mu_0{}_\nu + \left[(D-2)\frac{A'^2}{A^2} + \frac{A''}{A}\right]\delta^\mu_\nu$
$A = e^{kr}$ ↪ ads $R^a{}_b = (D-1)k^2\delta^a_b$

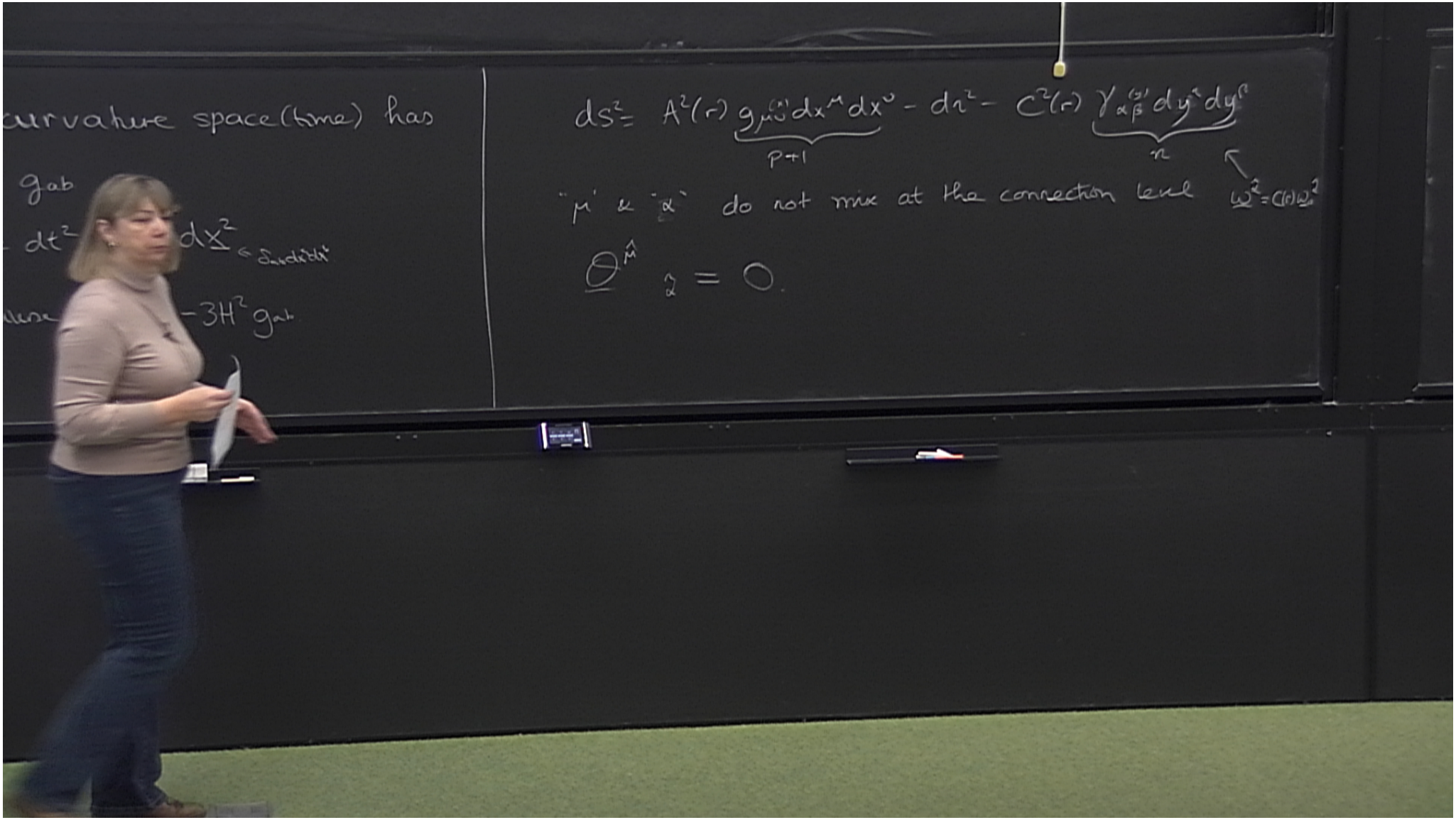

Constant curvature space(time) has
$R_{ab} \propto g_{ab}$

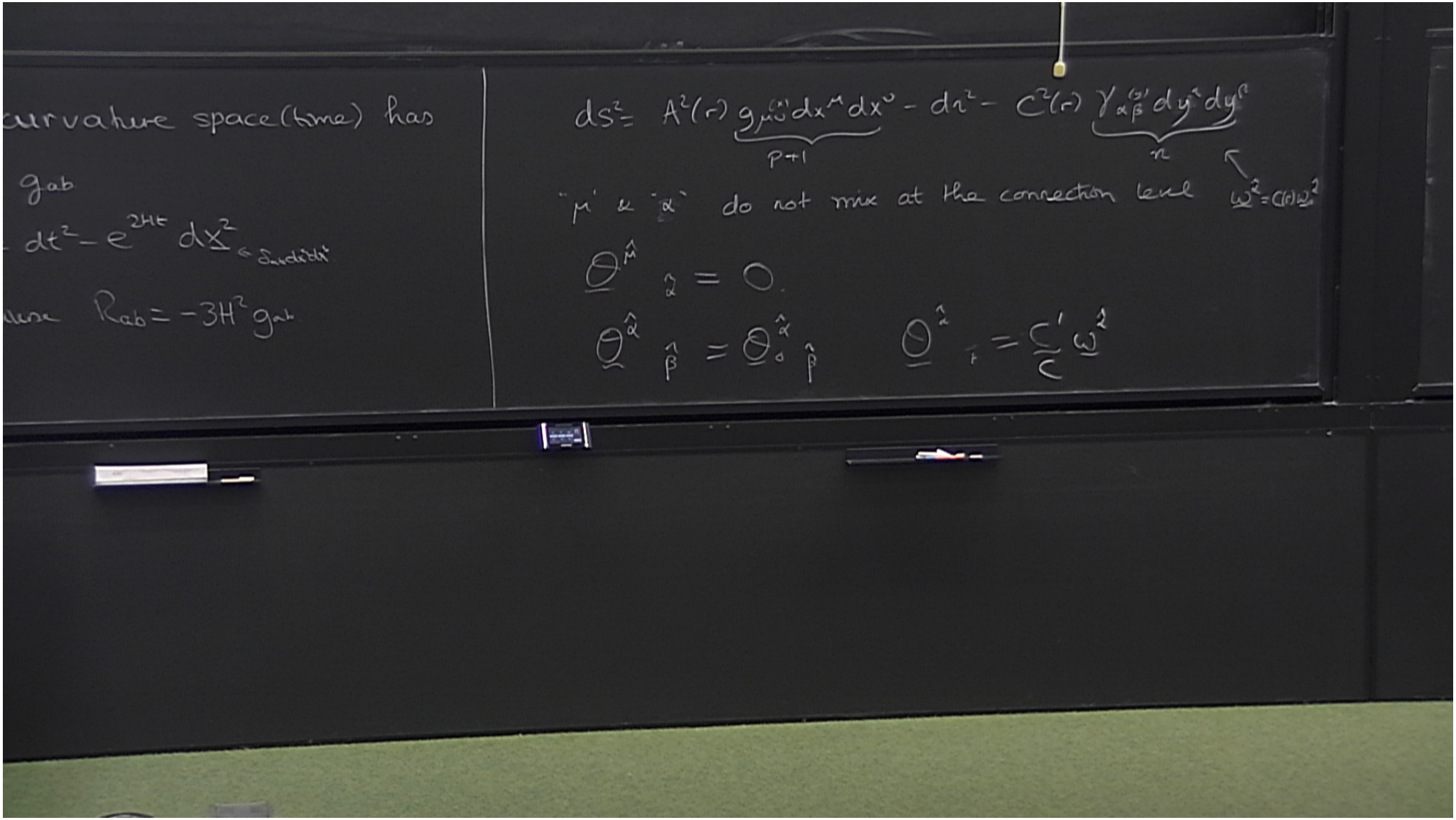

Constant curvature space(time) has
$R_{ab} \propto g_{ab}$
e.g. $ds^2 = dt^2 - e^{2Ht} dx^2$ ← $\delta_{ij} dx^i dx^j$
- inflating universe $R_{ab} = -3H^2 g_{ab}$

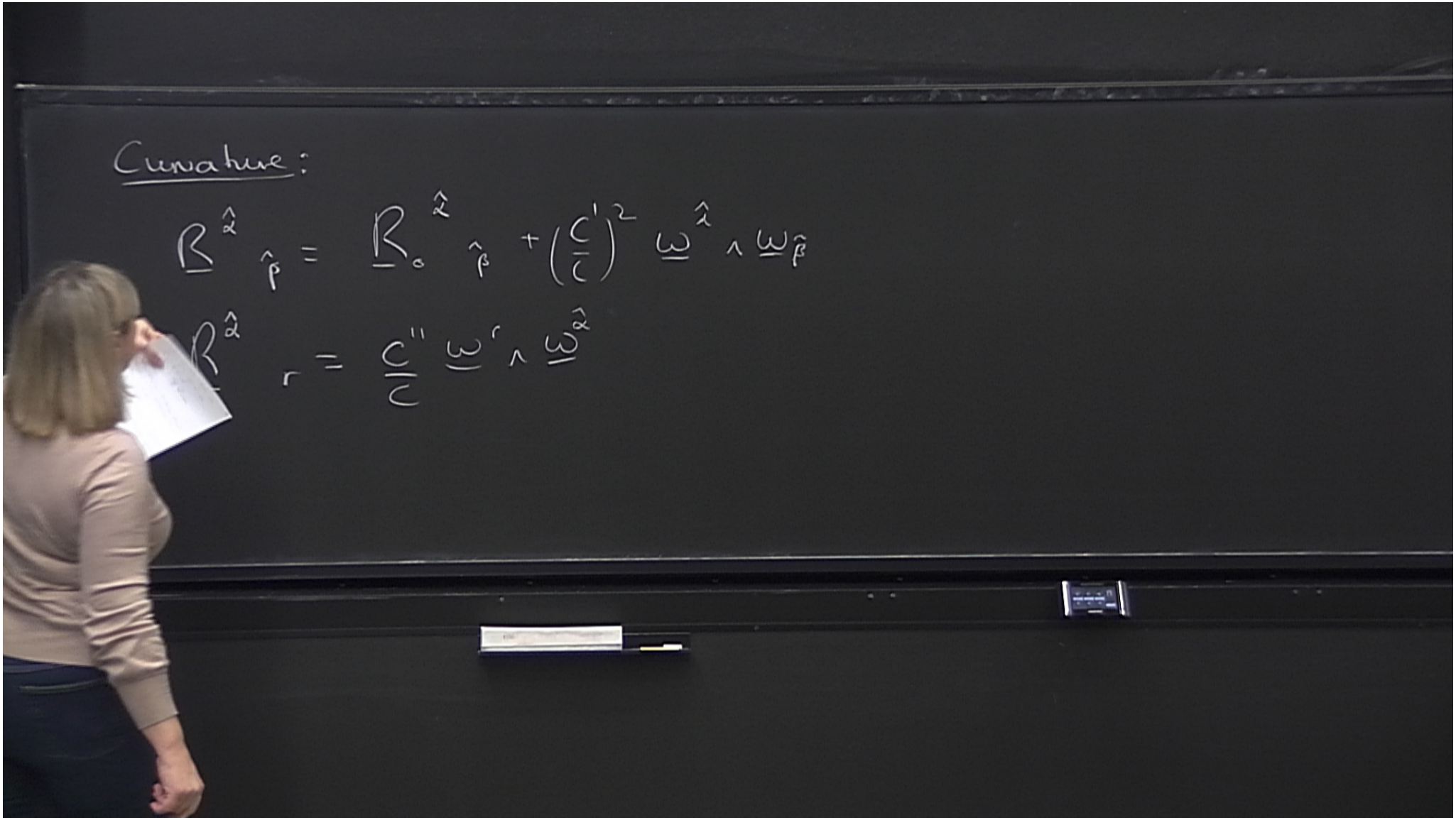

curvature space(time) has
g_{ab}
$dt^2 - e^{2Ht} dx^2$
$R_{ab} = -3H^2 g_{ab}$
$ds^2 = A^2(r) g_{\mu\nu} dx^\mu dx^\nu - dr^2 - C^2(r) \gamma_{\alpha\beta} dy^\alpha dy^\beta$


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p+1
n


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g_{ab}
dt^2
dx^2
$-3H^2 g_{ab}$
$ds^2 = A^2(r)\, g_{\mu\nu}\, dx^\mu dx^\nu - dr^2 - C^2(r)\, \gamma_{\alpha\beta}\, dy^\alpha dy^\beta$
p+1
n
"μ" & "α" do not mix at the connection level
$\Omega^2 = C(r)\omega^2$
$\Theta^{\hat{\mu}}{}_{\hat{\alpha}} = 0$


curvature space(time) has
g_{ab}
$dt^2 - e^{2Ht} dx^2$
$R_{ab} = -3H^2 g_{ab}$
$ds^2 = A^2(r) g_{\mu\nu} dx^\mu dx^\nu - dr^2 - C^2(r) \gamma_{\alpha\beta} dy^\alpha dy^\beta$
p+1
n
"μ" & "α" do not mix at the connection level
$\omega^{\hat{r}}_{\hat{r}} = 0$
$\omega^{\hat{\alpha}}_{\hat{\beta}} = \omega^{\hat{\alpha}}_{0\,\hat{\beta}}$
$\omega^{\hat{r}}_{\hat{t}} = \frac{C'}{C} \omega^{\hat{t}}$

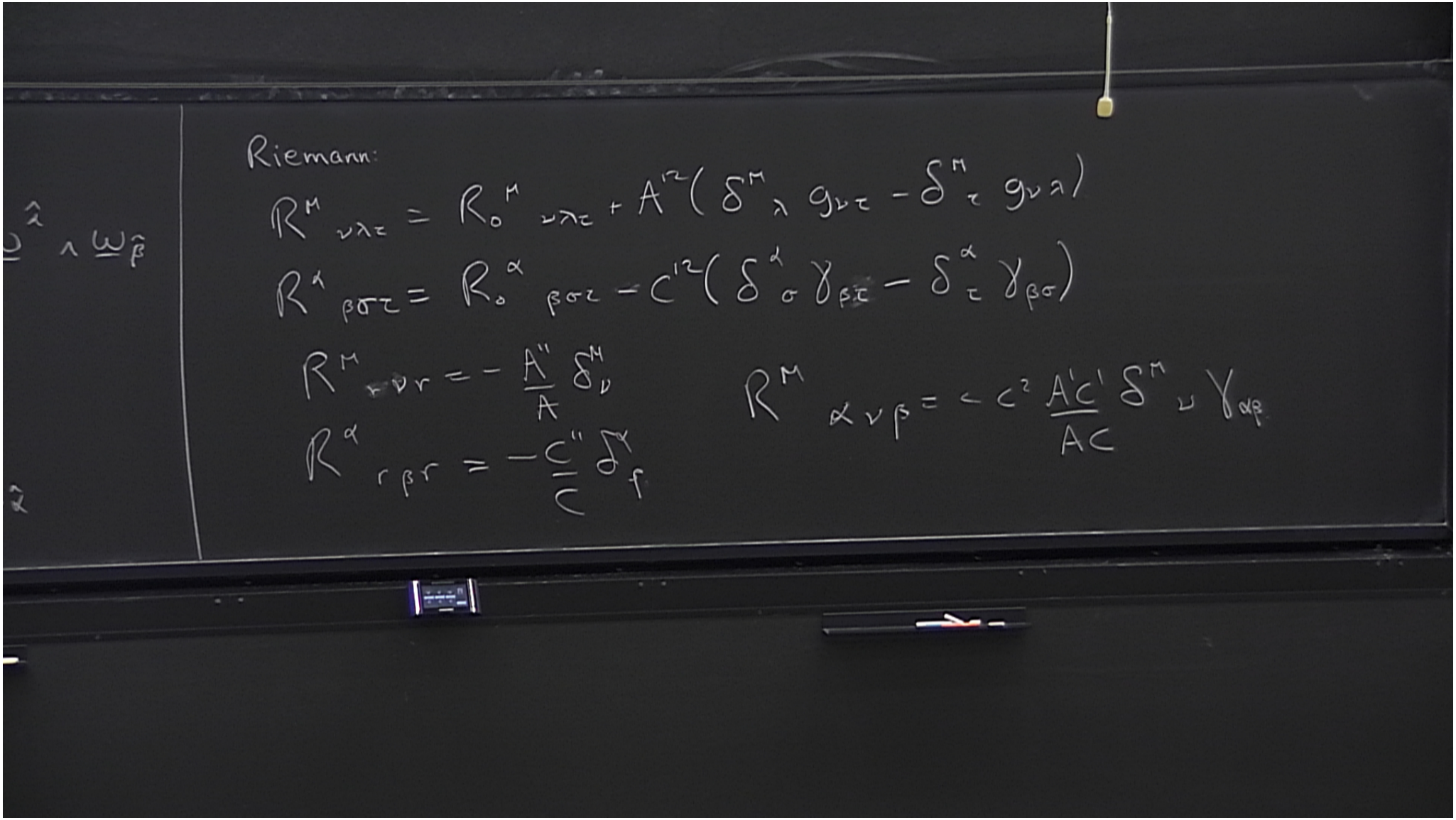

Curvature:
$\underline{R}^{\hat{\alpha}}{}_{\hat{\beta}} = \underline{R}_0{}^{\hat{\alpha}}{}_{\hat{\beta}} + \left(\frac{c'}{c}\right)^2 \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}_{\hat{\beta}}$
$\underline{R}^{\hat{\alpha}}{}_{r} = \frac{c''}{c} \underline{\omega}^{r} \wedge \underline{\omega}^{\hat{\alpha}}$

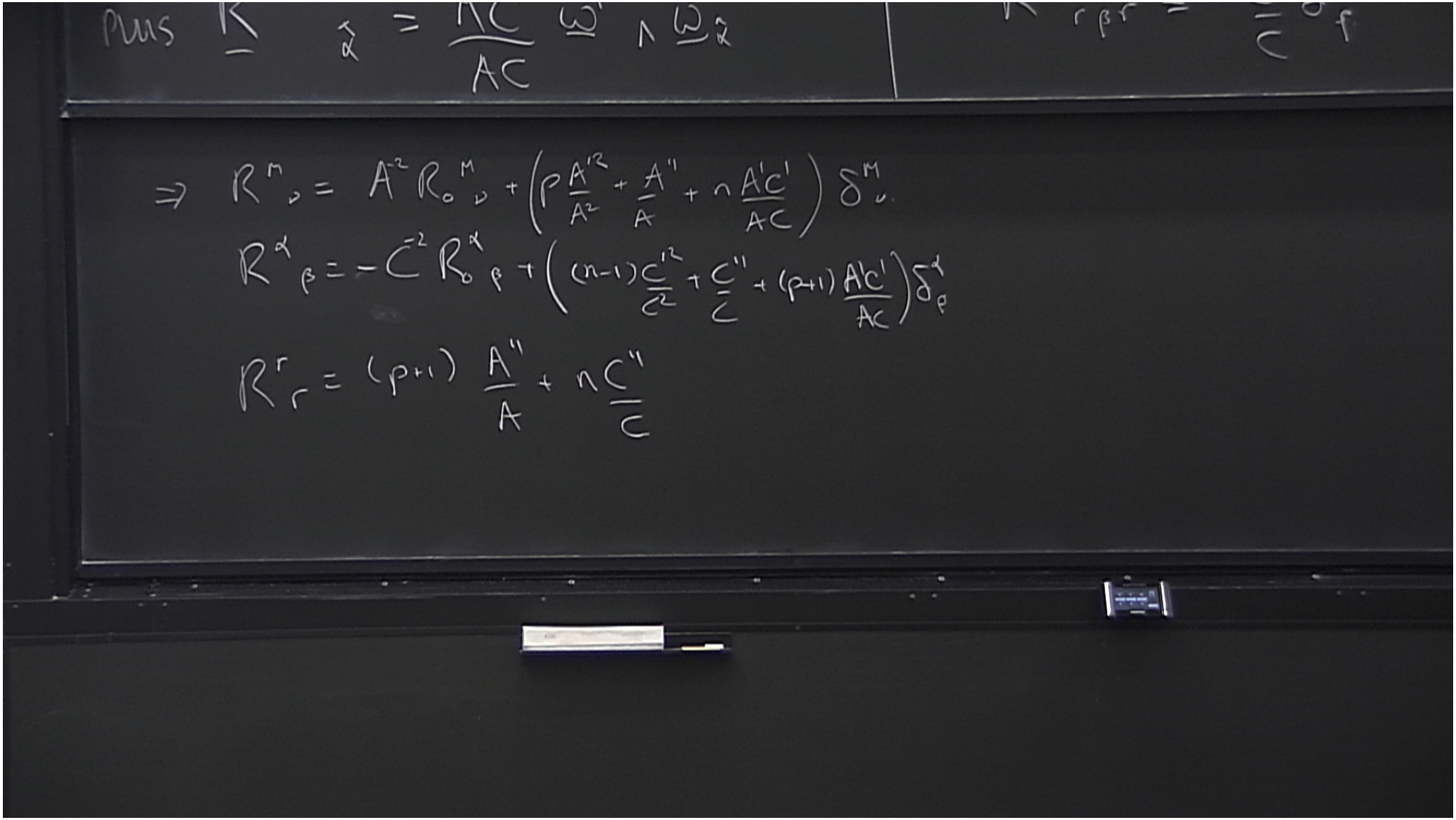
Curvature:

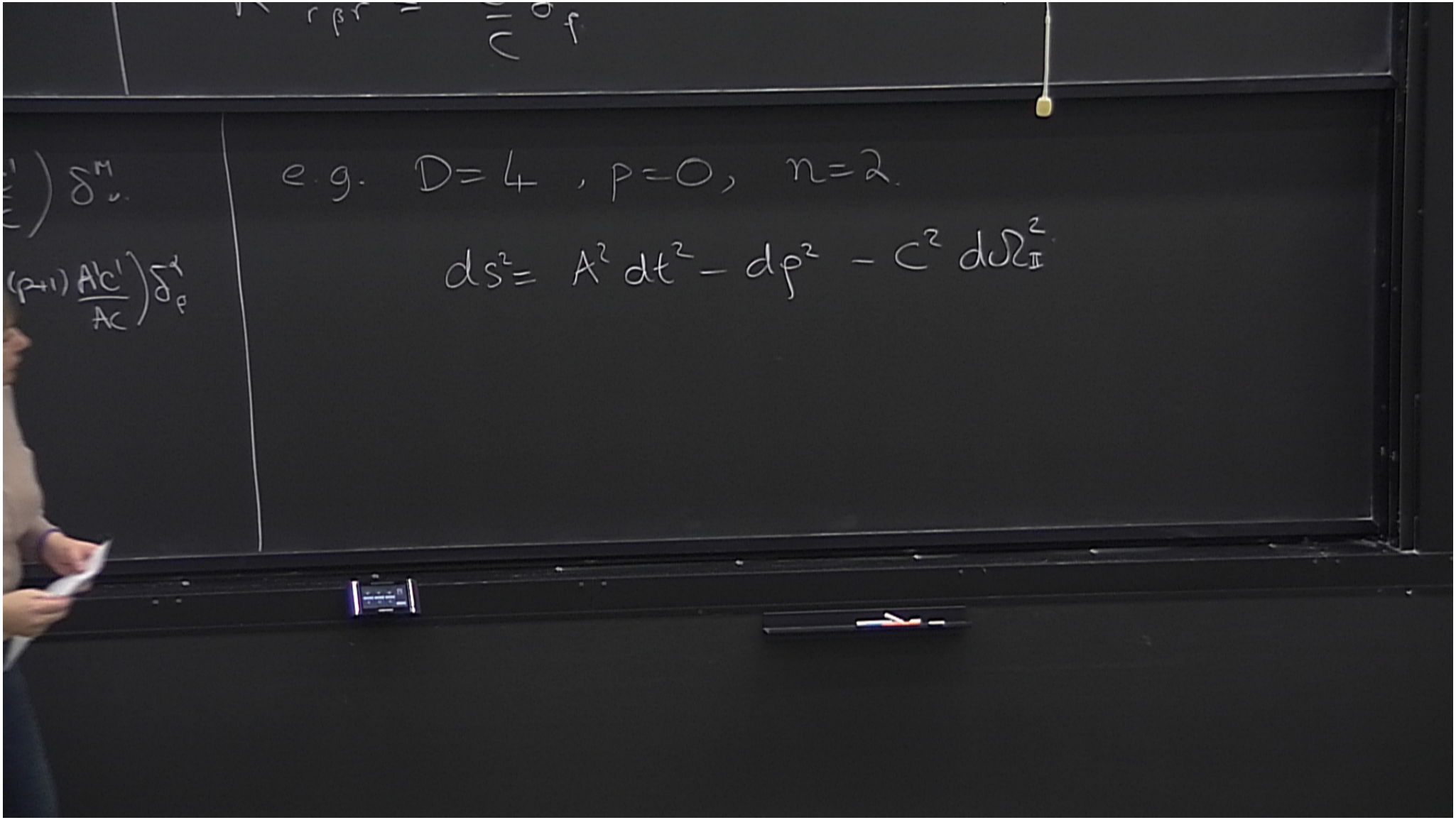
$$\underline{R}^{\hat{\alpha}}{}_{\hat{\beta}} = \underline{R}_0{}^{\hat{\alpha}}{}_{\hat{\beta}} + \left(\frac{C'}{C}\right)^2 \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}^{\hat{\beta}}$$

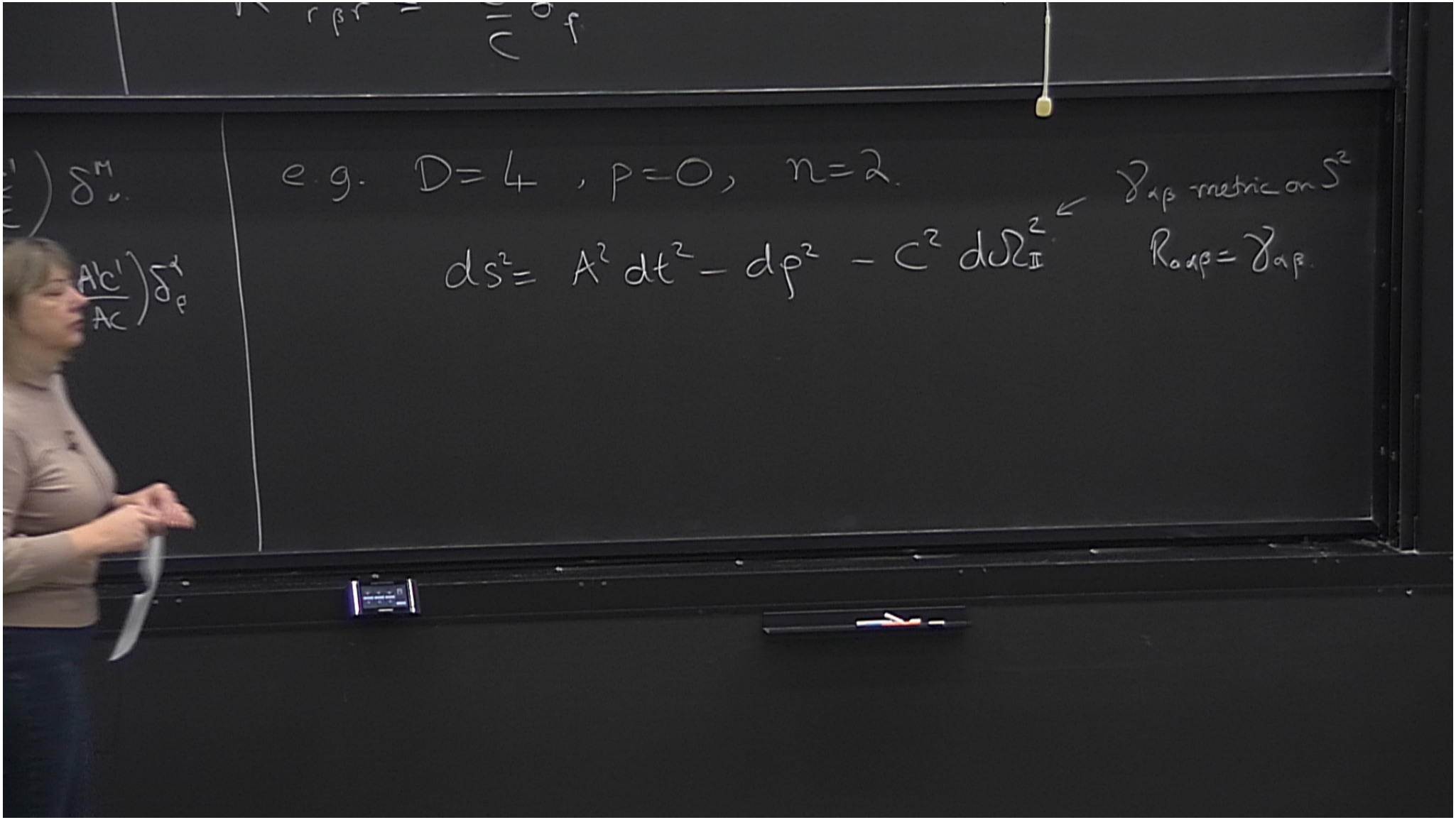
$$\underline{R}^{\hat{\alpha}}{}_{r} = \frac{C''}{C} \underline{\omega}^{r} \wedge \underline{\omega}^{\hat{\alpha}}$$

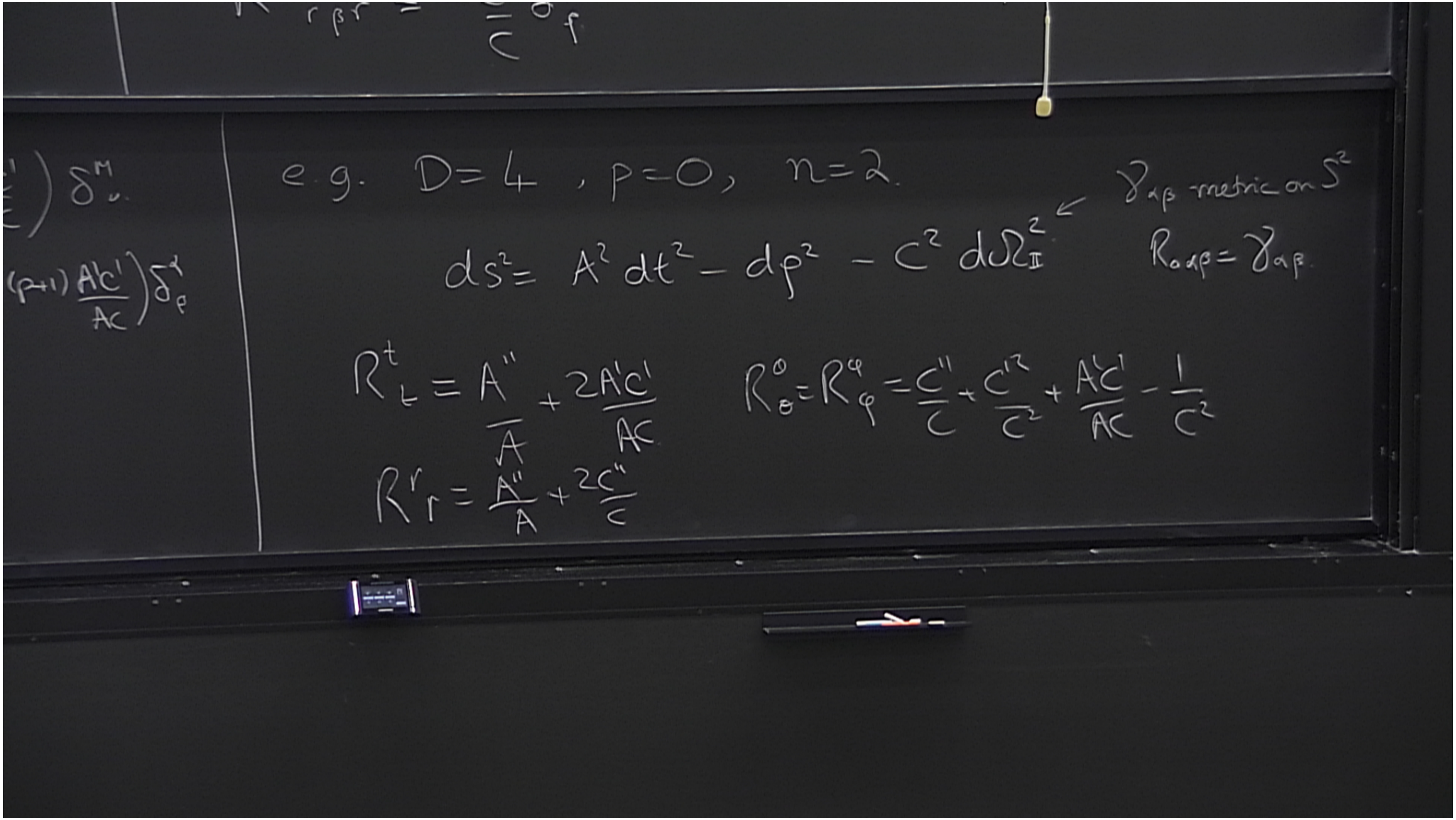
Plus $\underline{R}^{\hat{\alpha}}{}_{\hat{\alpha}} = \frac{A'C'}{AC} \underline{\omega}^{\hat{\alpha}} \wedge \underline{\omega}^{\hat{\alpha}}$

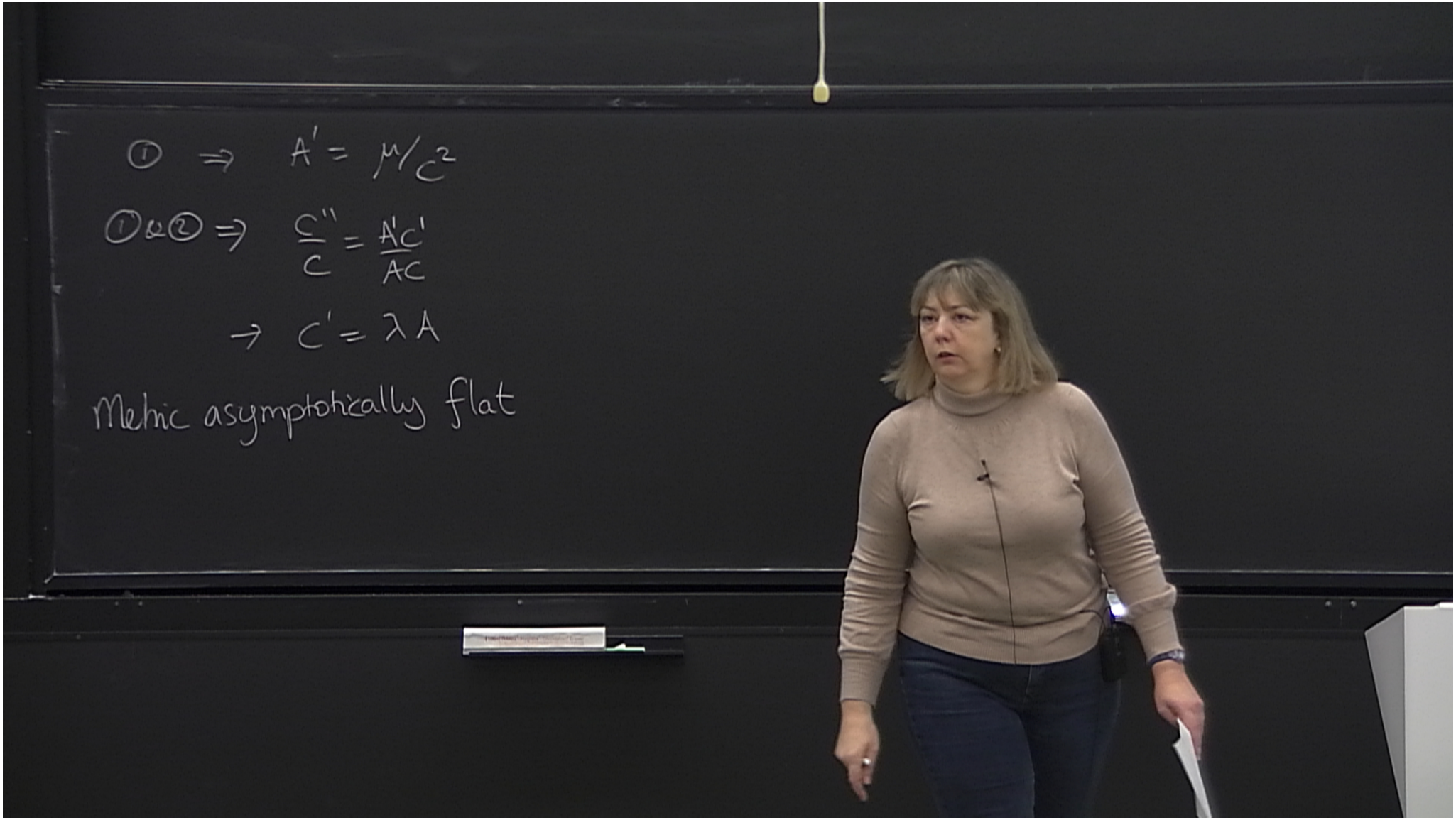

Riemann:
$R^{\mu}{}_{\nu\lambda\tau} = R_0{}^{\mu}{}_{\nu\lambda\tau} + A^{12}(\delta^{\mu}_{\lambda} g_{\nu\tau} - \delta^{\mu}_{\tau} g_{\nu\lambda})$
$R^{\alpha}{}_{\beta\sigma\tau} = R_0{}^{\alpha}{}_{\beta\sigma\tau} - C^{12}(\delta^{\alpha}_{\sigma}\gamma_{\beta\tau} - \delta^{\alpha}_{\tau}\gamma_{\beta\sigma})$
$R^{\mu}{}_{\nu r} = -\frac{A''}{A}\delta^{\mu}_{\nu}$
$R^{\alpha}{}_{r\beta r} = -\frac{C''}{C}\delta^{\alpha}_{\beta}$
$R^{\mu}{}_{\alpha\nu\beta} = -C^2\frac{A'C'}{AC}\delta^{\mu}_{\nu}\gamma_{\alpha\beta}$

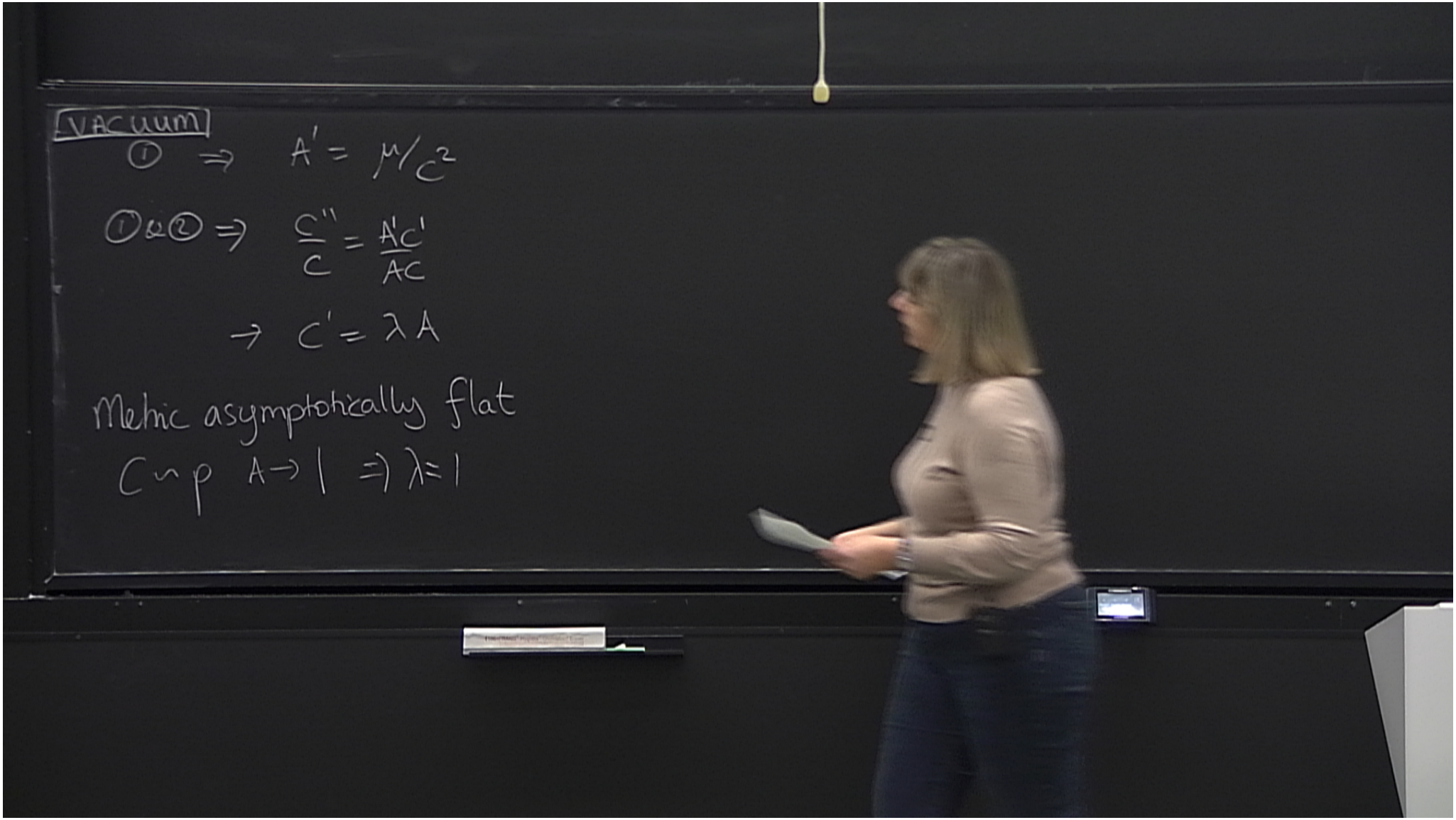

Plus
$\Rightarrow R^{\mu}{}_{\nu} = A^{-2} R_0{}^{\mu}{}_{\nu} + \left(\rho \frac{A'^2}{A^2} + \frac{A''}{A} + n \frac{A'C'}{AC}\right) \delta^{\mu}_{\nu}$
$R^{\alpha}{}_{\beta} = -C^{-2} R_0{}^{\alpha}{}_{\beta} + \left((n-1)\frac{C'^2}{C^2} + \frac{C''}{C} + (\rho+1)\frac{A'C'}{AC}\right)\delta^{\alpha}_{\beta}$
$R^{r}{}_{r} = (\rho+1)\frac{A''}{A} + n\frac{C''}{C}$

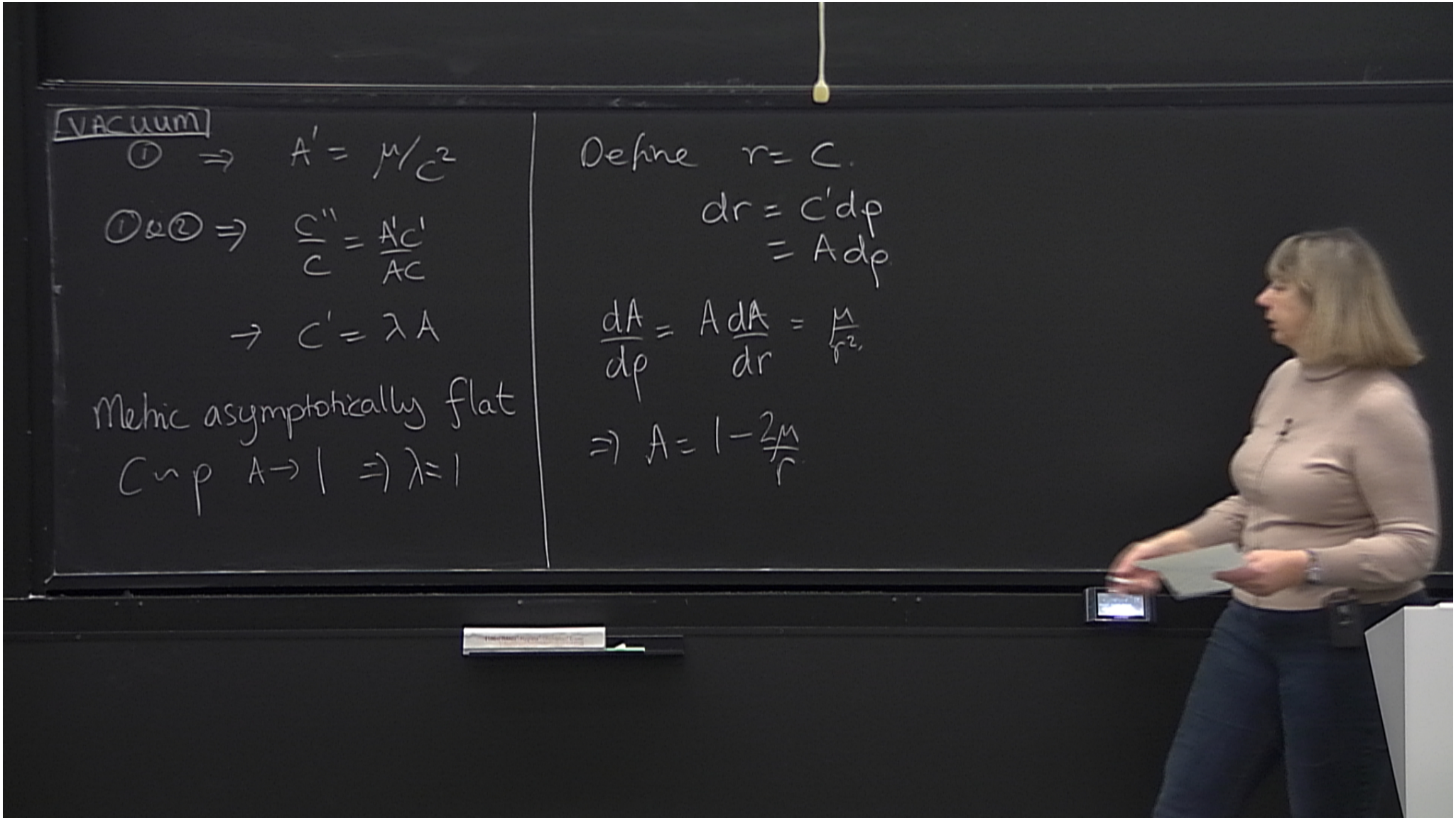

e.g. D=4, p=0, n=2.
$ds^2 = A^2 dt^2 - dp^2 - C^2 d\Omega_{II}^2$


e.g. D=4, p=0, n=2
$\gamma_{\alpha\beta}$ metric on S^2
$ds^2 = A^2 dt^2 - dp^2 - C^2 d\Omega_{II}^2$
$R_{\alpha\beta} = \gamma_{\alpha\beta}$


e.g. D=4, p=0, n=2
$\gamma_{\alpha\beta}$ metric on S^2
$ds^2 = A^2 dt^2 - dp^2 - C^2 d\Omega_{II}^2$
$R_{\alpha\beta} = \gamma_{\alpha\beta}$
$R^t_t = \frac{A''}{A} + \frac{2A'C'}{AC}$
$R^r_r = \frac{A''}{A} + \frac{2C''}{C}$
$R^\theta_\theta = R^\varphi_\varphi = \frac{C''}{C} + \frac{C'^2}{C^2} + \frac{A'C'}{AC} - \frac{1}{C^2}$


① ⇒ $A' = \mu/c^2$
①&② ⇒ $\frac{C''}{C} = \frac{A'C'}{AC}$
→ $C' = \lambda A$
Mchic asymptotically flat


VACUUM
① ⇒ $A' = \mu/c^2$
①&② ⇒ $\frac{C''}{C} = \frac{A'C'}{AC}$
→ $C' = \lambda A$
Metric asymptotically flat
Cusp $A \to 1$ ⇒ $\lambda = 1$


VACUUM
① ⇒ A' = μ/c²
①&② ⇒ C''/C = A'C'/AC
→ C' = λA
Metric asymptotically flat
C∼ρ A→1 ⇒ λ=1
Define r = C.
dr = C'dρ
= Adρ
dA/dρ = A dA/dr = μ/r²
⇒ A = 1 − 2μ/r

VACUUM

① $\Rightarrow$ $A' = \mu/c^2$

①&② $\Rightarrow$ $\frac{C''}{C} = \frac{A'C'}{AC}$

$\rightarrow C' = \lambda A$

Metric asymptotically flat

$C \sim \rho$ $A \rightarrow 1$ $\Rightarrow \lambda = 1$

Define $r = C$.

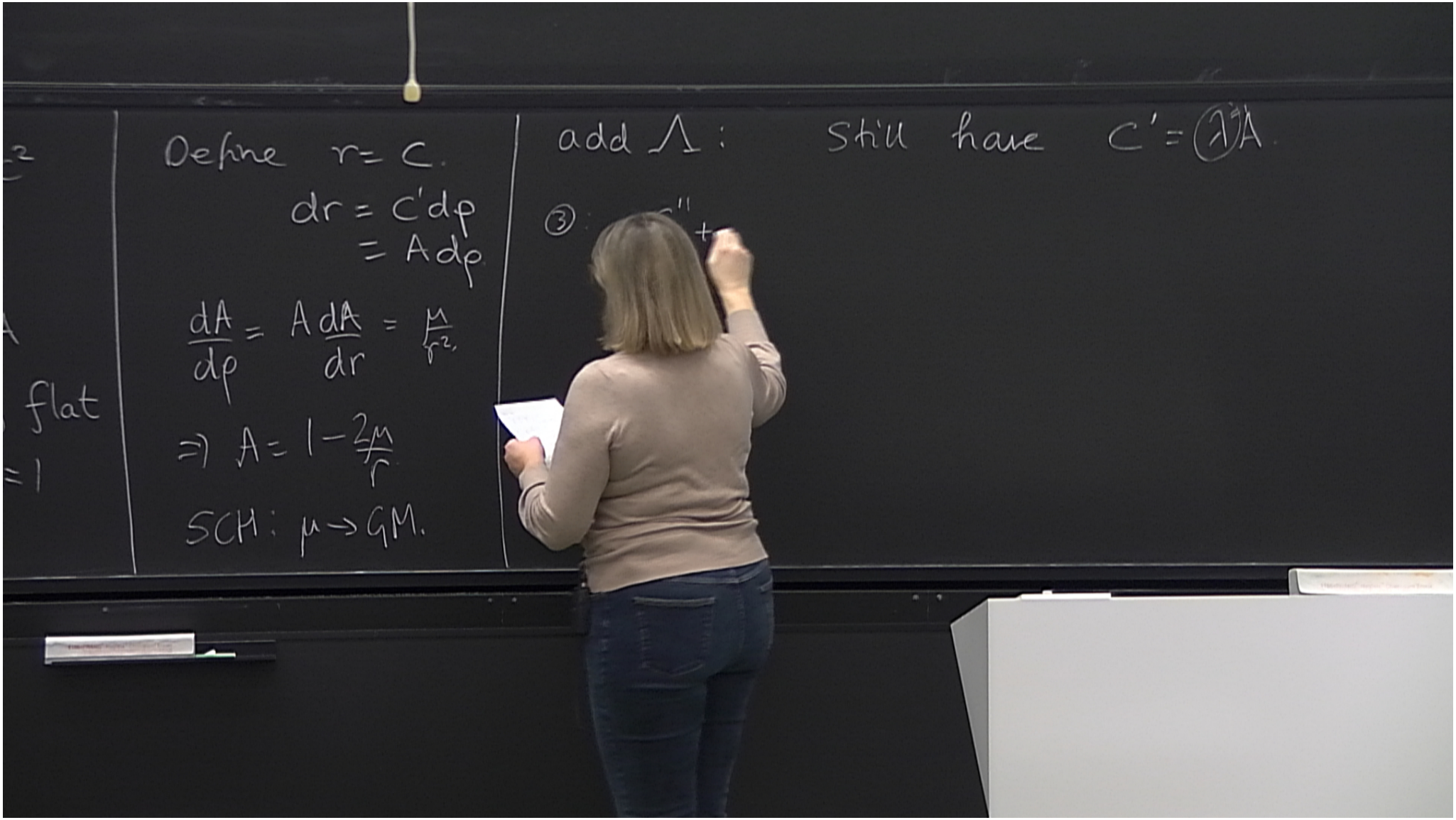
$dr = C' d\rho$

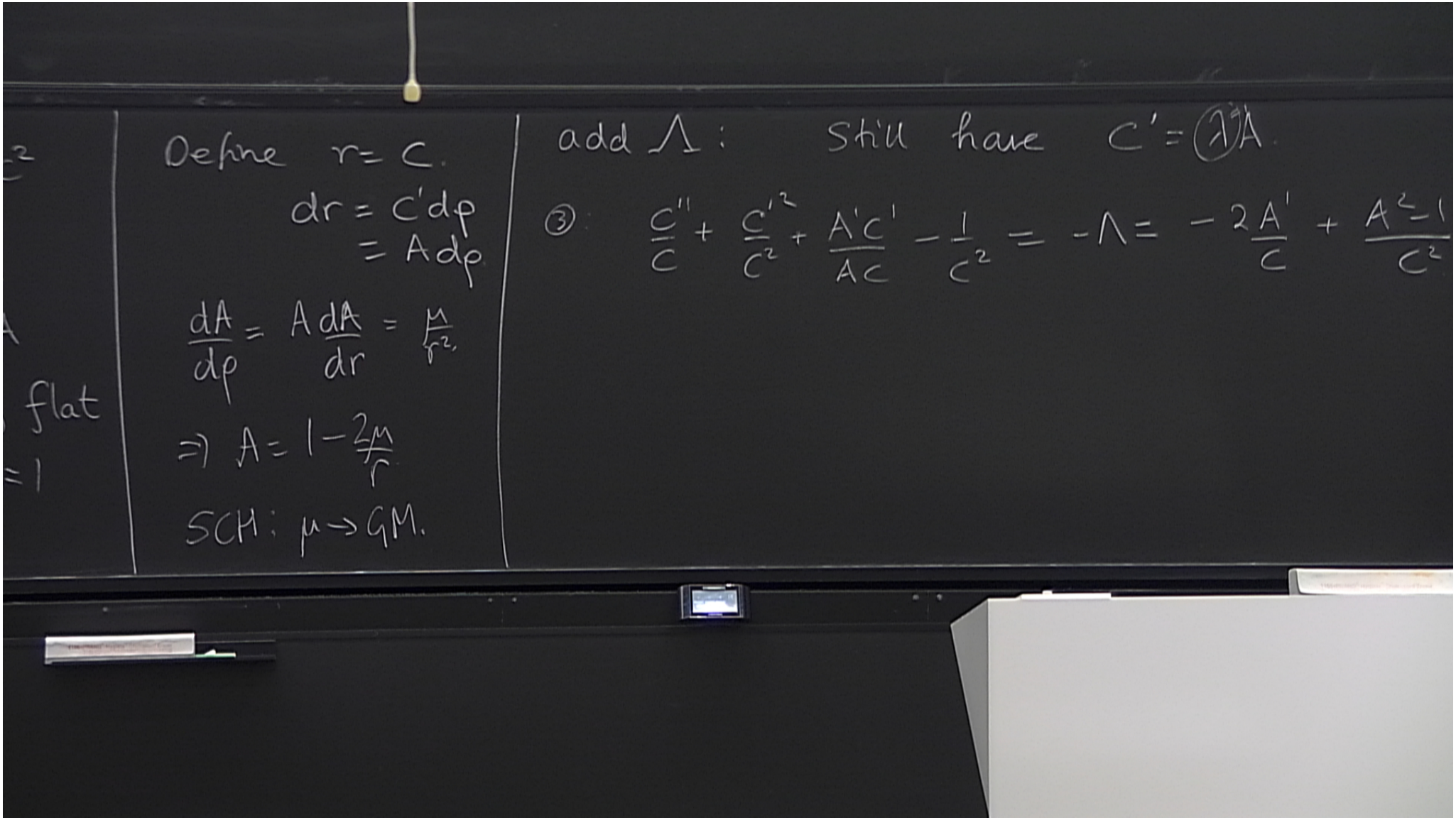
$= A\, d\rho$

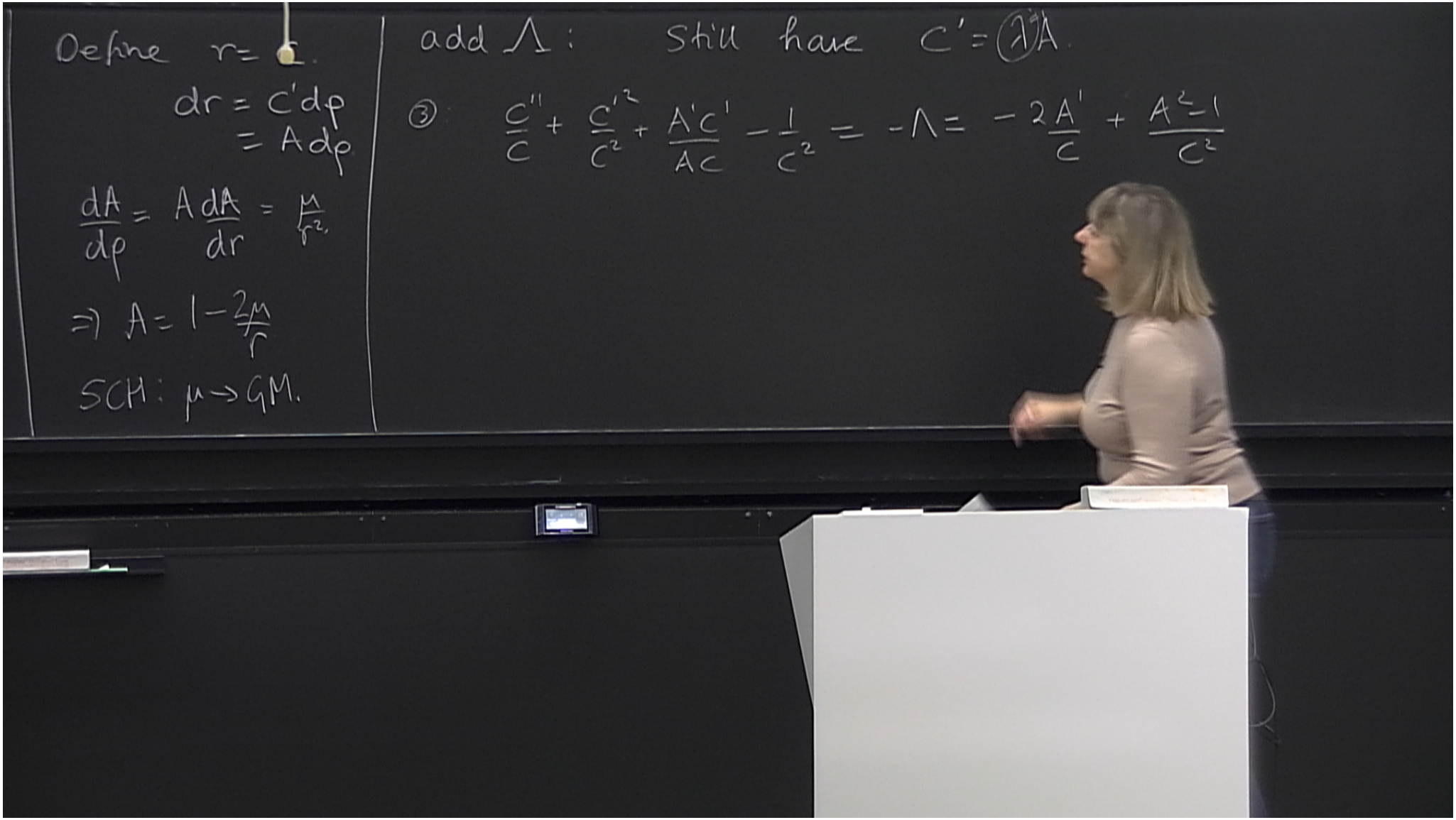
$\frac{dA}{d\rho} = A \frac{dA}{dr} = \frac{\mu}{r^2}$

$\Rightarrow A = 1 - \frac{2\mu}{r}$

SCH: $\mu \rightarrow GM$.


flat
Define $r = c$.
$dr = c'dp$
$= A dp$
$\frac{dA}{dp} = A\frac{dA}{dr} = \frac{\mu}{r^2}$
$\Rightarrow A = 1 - \frac{2\mu}{r}$
SCH: $\mu \to GM$.
add Λ: still have $c' = (1)A$.
③


flat
=1
Define r = C.
dr = C'dρ
= A dρ
dA/dρ = A dA/dr = μ/r²
⇒ A = 1 − 2μ/r
SCH: μ → GM.
add Λ: still have C' = (1)A
③ C''/C + C'²/C² + A'C'/AC − 1/C² = −Λ = −2A'/C + A²−1/C²


Define $r = \frac{1}{\rho}$.
$dr = C' d\rho$
$= A d\rho$
$\frac{dA}{d\rho} = A\frac{dA}{dr} = \frac{\mu}{r^2}$
$\Rightarrow A = 1 - \frac{2\mu}{r}$
SCH: $\mu \to GM$.
add Λ: Still have $C' = \frac{1}{2}A$.
③ $\frac{C''}{C} + \frac{C'^2}{C^2} + \frac{A'C'}{AC} - \frac{1}{C^2} = -\Lambda = -\frac{2A'}{C} + \frac{A^2-1}{C^2}$

Define $r = \rho$.

$dr = C' d\rho$
$= A d\rho$

$\frac{dA}{d\rho} = A \frac{dA}{dr} = \frac{\mu}{r^2}$

$\Rightarrow A = 1 - \frac{2\mu}{r}$

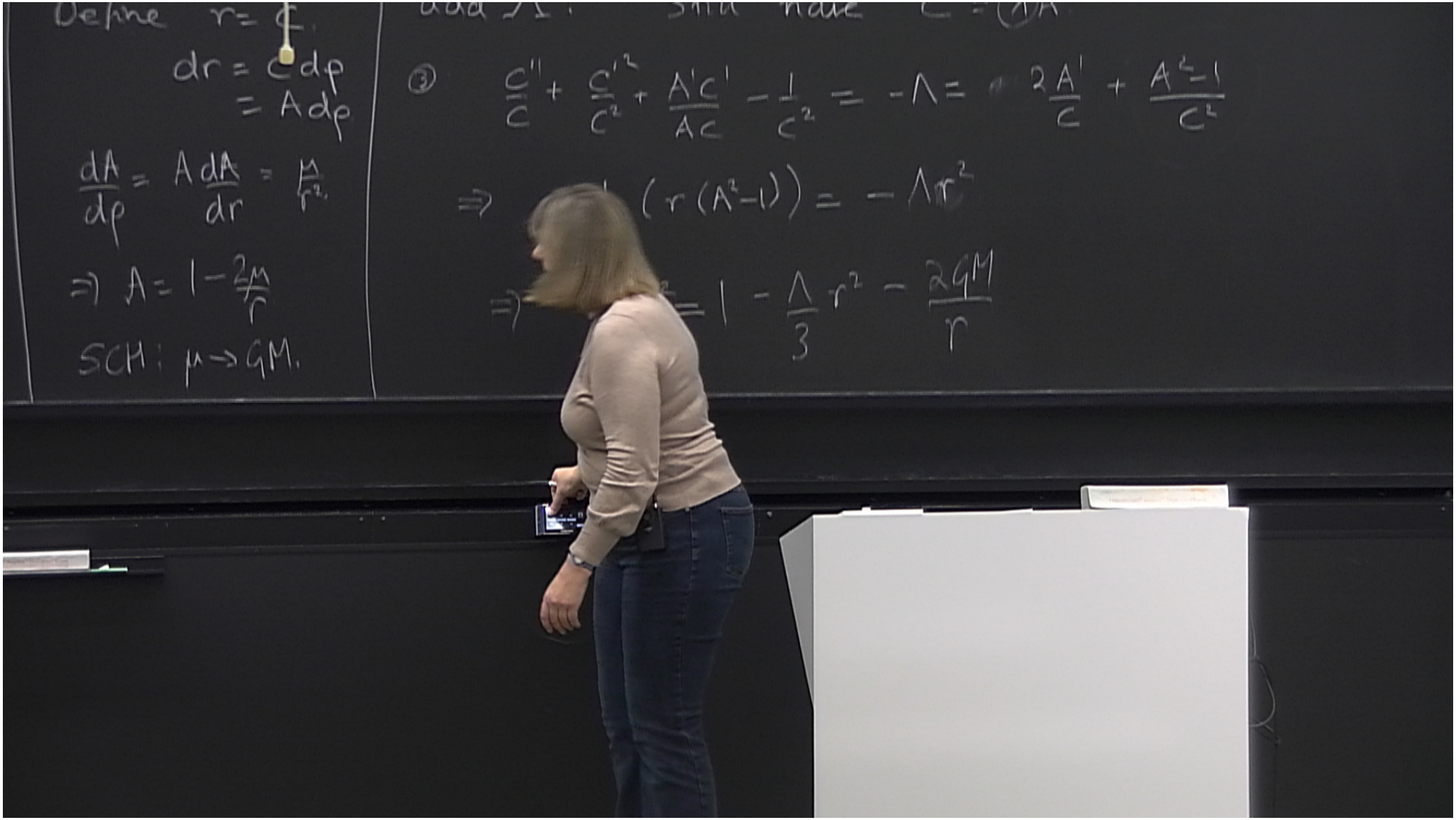
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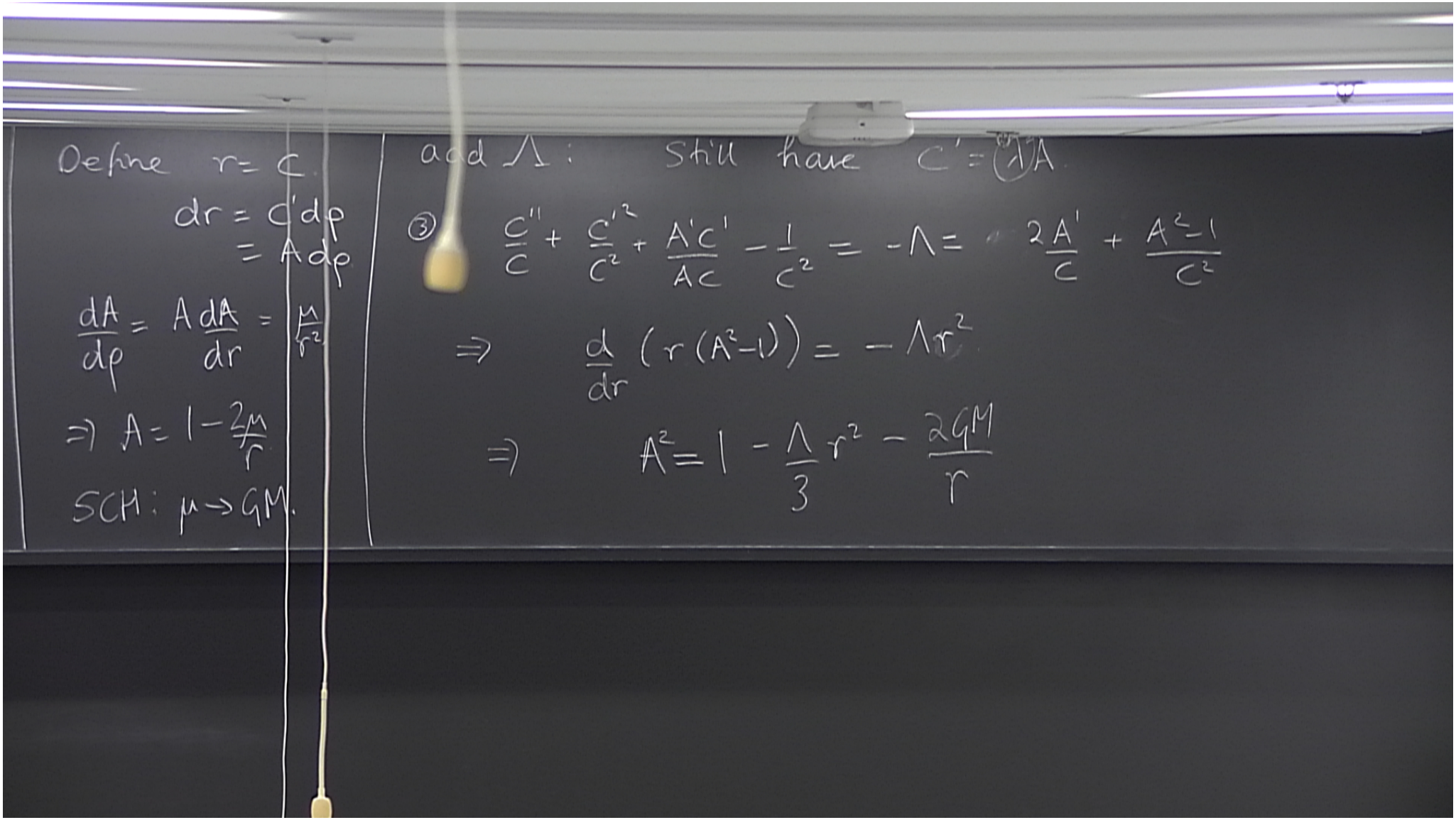
add Λ: Still have $C' = (1)A$

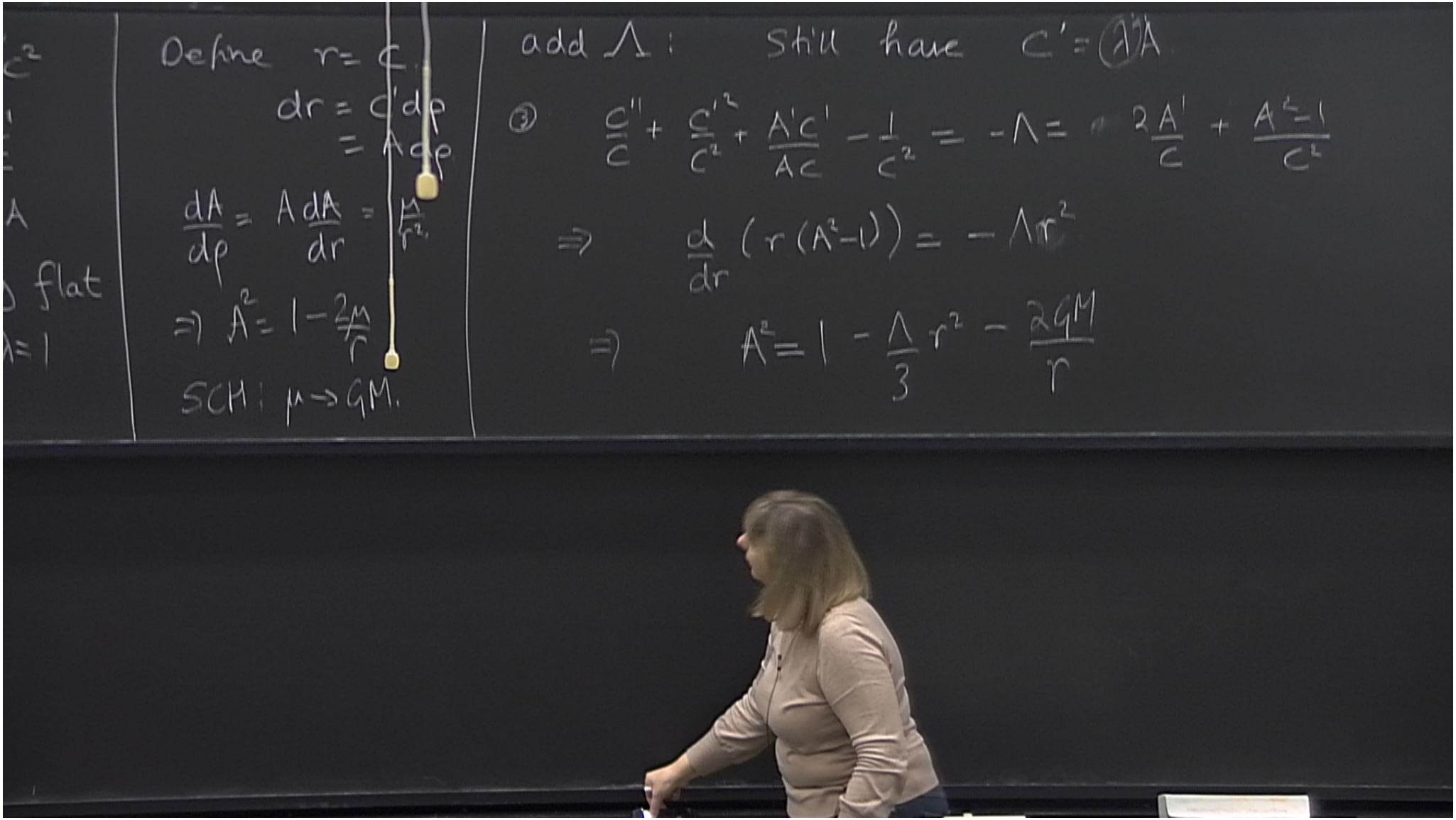
(3) $\frac{C''}{C} + \frac{C'^2}{C^2} + \frac{A'C'}{AC} - \frac{1}{C^2} = -\Lambda = \frac{2A'}{C} + \frac{A^2-1}{C^2}$

$\Rightarrow \frac{d}{dr}(r(A^2-1)) = -\Lambda C^2$




Define r = ζ
dr = C dρ
= A dρ
dA/dρ = A dA/dr = μ/r²
⇒ A = 1 − 2μ/r
SCH: μ → GM.
③
C''/C + C'²/C² + A'C'/AC − 1/C² = −Λ = 2A'/C + (A²−1)/C²
⇒ (r(A²−1)) = −Λr²
⇒ = 1 − Λ/3 r² − 2GM/r


Define $r = C$
$dr = C' d\rho$
$= A d\rho$
$\frac{dA}{d\rho} = A\frac{dA}{dr} = \frac{\mu}{r^2}$
$\Rightarrow A = 1 - \frac{2\mu}{r}$
SCH: $\mu \to GM$.
add Λ: still have $C' = (A)A$
$\frac{C''}{C} + \frac{C'^2}{C^2} + \frac{A'C'}{AC} - \frac{1}{C^2} = -\Lambda = \frac{2A'}{C} + \frac{A^2-1}{C^2}$
$\Rightarrow \frac{d}{dr}(r(A^2-1)) = -\Lambda r^2$
$\Rightarrow A^2 = 1 - \frac{\Lambda}{3}r^2 - \frac{2GM}{r}$


Define r = C.
dr = C' dρ
= A dρ
dA/dρ = A dA/dr = μ/r²
⇒ A² = 1 − 2μ/r
Sch: μ → GM.
add Λ: Still have C' = (A')A
③ C''/C + C'²/C² + A'C'/AC − 1/C² = −Λ = 2A'/C + (A²−1)/C²
⇒ d/dr (r(A²−1)) = −Λr²
⇒ A² = 1 − (Λ/3) r² − 2GM/r
flat

$\Lambda \to 0$ SCH

$GM \to 0$ (a) DS.

in static co-ords

$$ds^2 = \left(1 - \frac{\Lambda}{3} r^2\right) dt^2 - \frac{dr^2}{1 - \frac{\Lambda}{3} r^2} - r^2 d\Omega_{II}^2$$

$\Lambda \to 0$ SCH

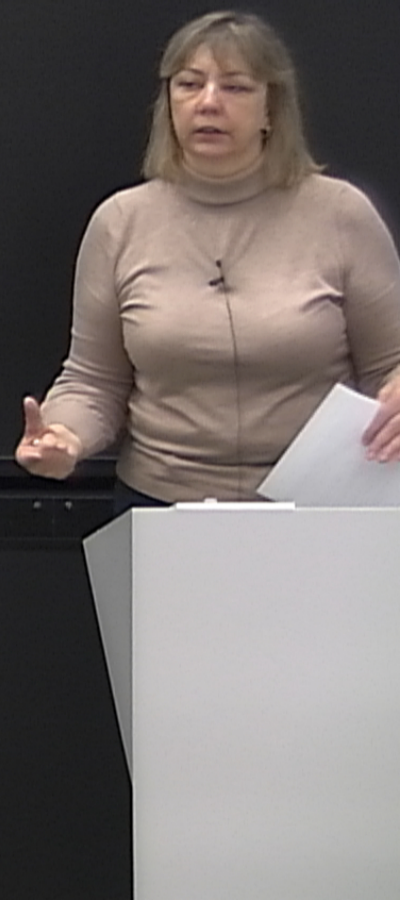
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$r \to \sqrt{\frac{3}{\Lambda}}$, $g_{tt} \to 0$ $g_{rr} \to \infty$.

Cosmological Event Horizon



$\Lambda \to 0$ SCH

$GM \to 0$ (a) DS.

in static co-ords

$$ds^2 = \left(1 - \frac{\Lambda}{3} r^2\right) dt^2 - \frac{dr^2}{1 - \frac{\Lambda}{3} r^2} - r^2 d\Omega_{II}^2$$

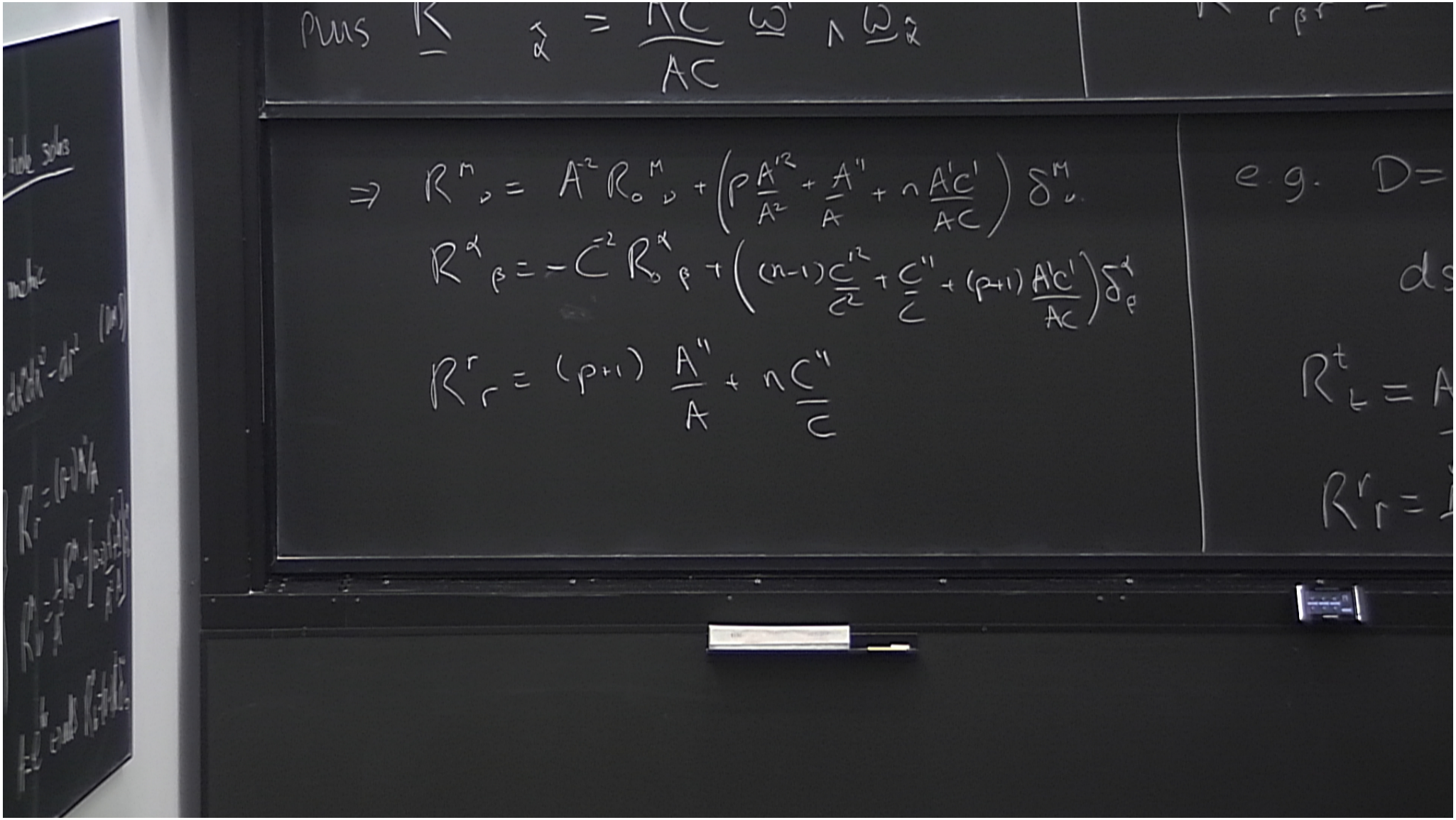
$r \to \sqrt{\frac{3}{\Lambda}}$, $g_{tt} \to 0$ $g_{rr} \to \infty$.

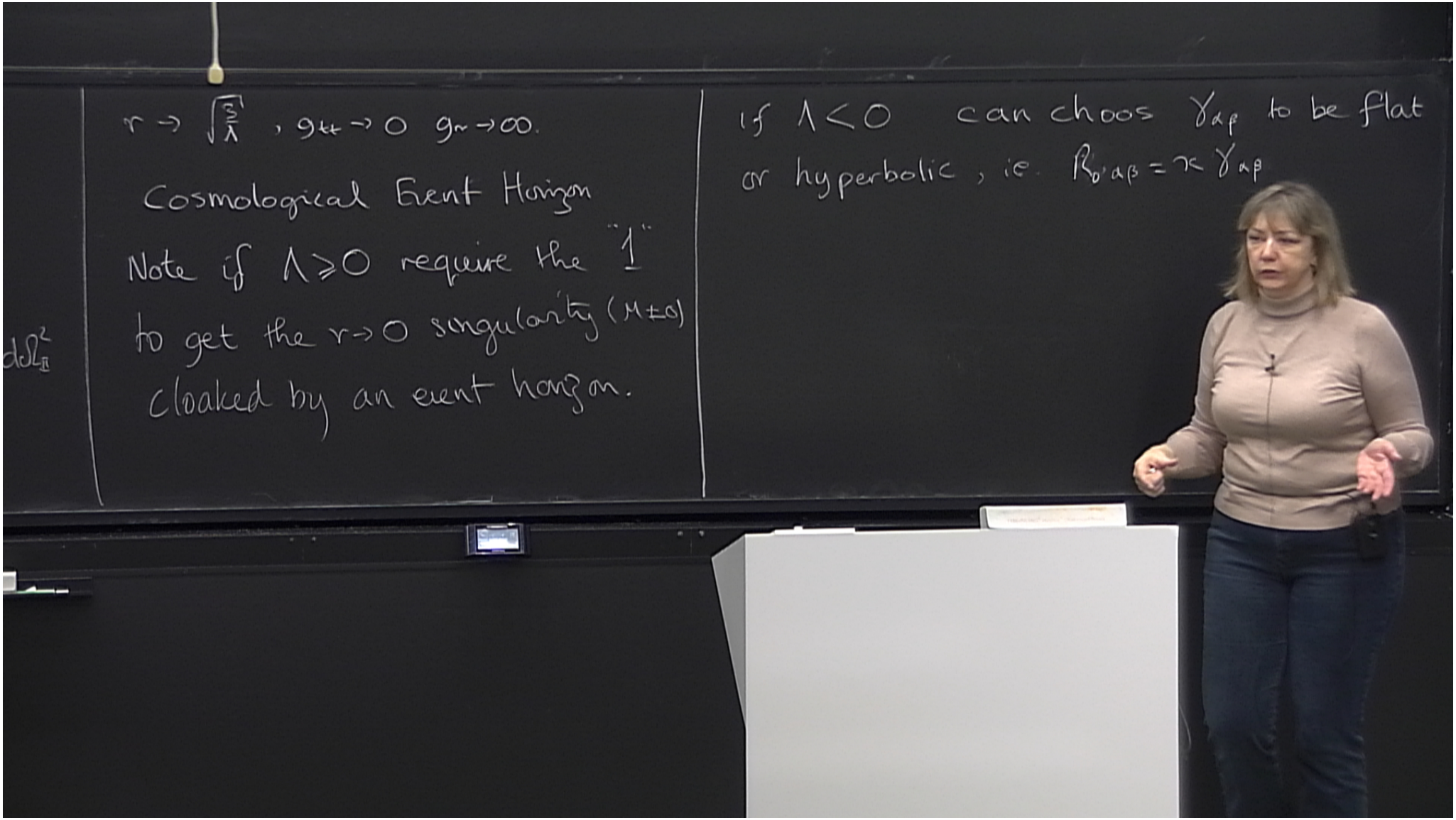
Cosmological Event Horizon

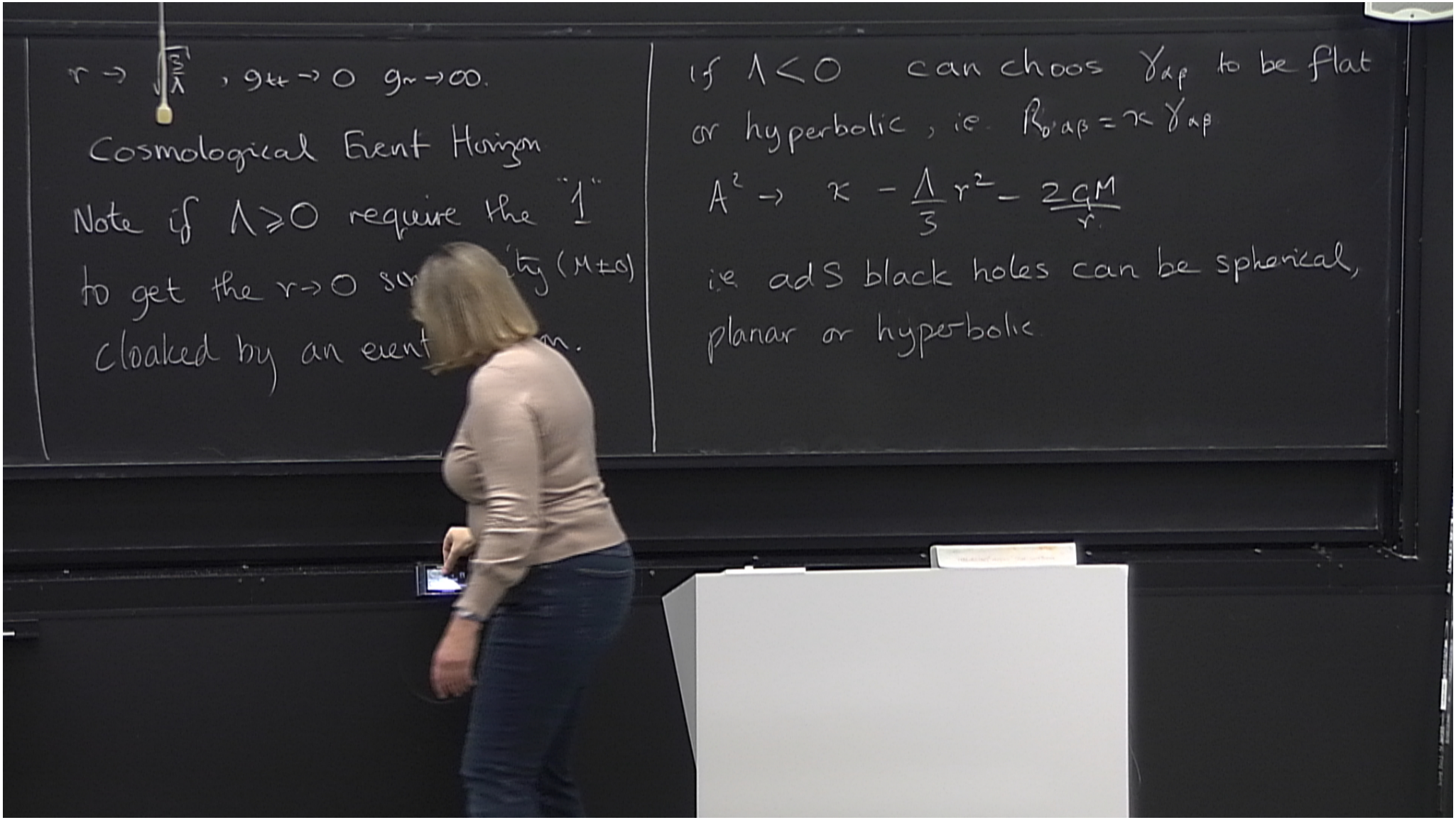
Note if $\Lambda \geq 0$ require the "1"

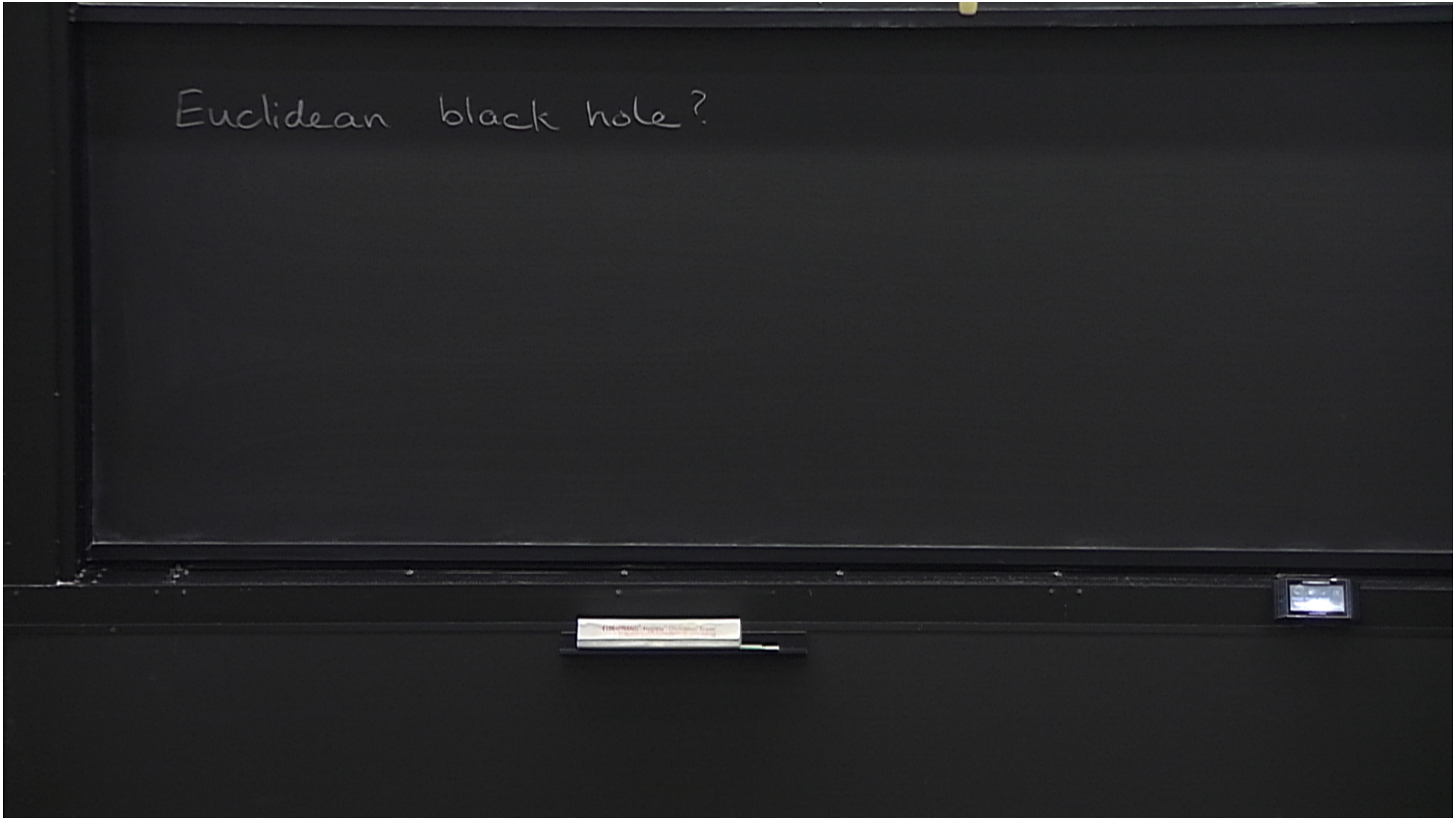
to get the $r \to 0$ singularity ($M \neq 0$)

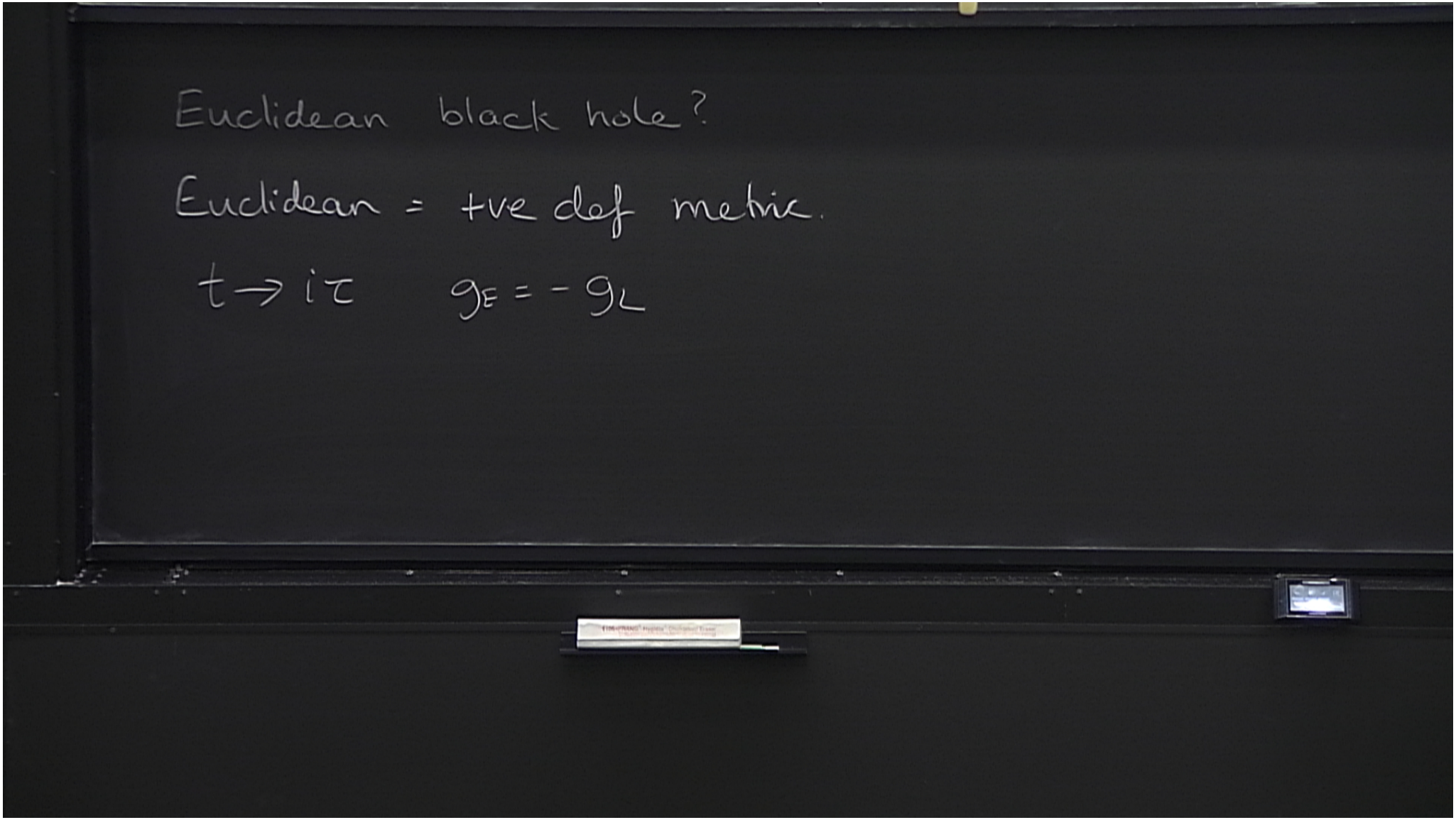
cloaked by an event horizon.

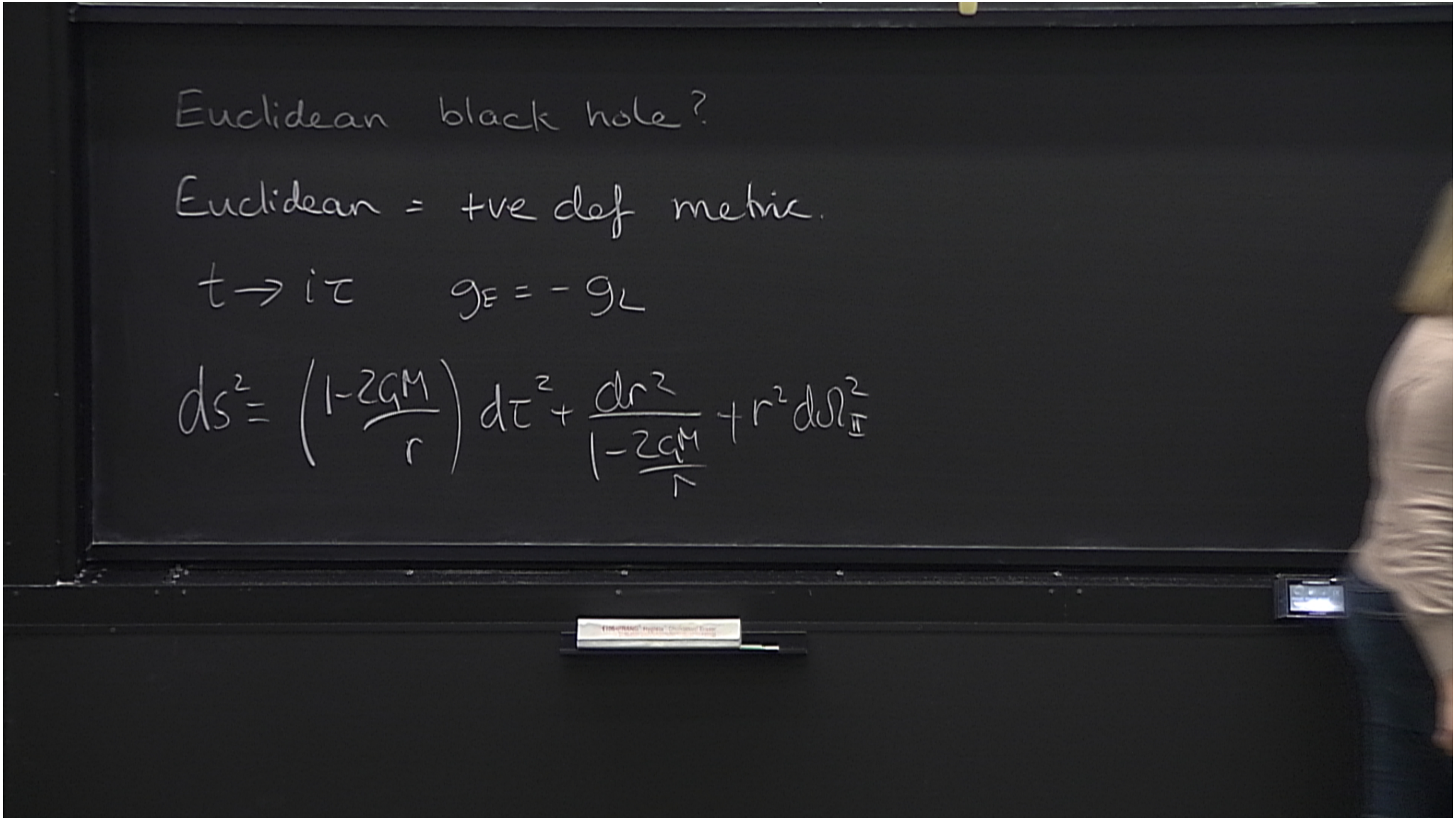

Plus $\underline{R}^{\cdot}_{\alpha} = \frac{AC}{AC}\,\underline{\omega}' \wedge \underline{\omega}_{\alpha}$
$\Rightarrow R^{\mu}{}_{\nu} = A^{-2} R_0{}^{\mu}{}_{\nu} + \left(\rho\frac{A'^2}{A^2} + \frac{A''}{A} + n\frac{A'C'}{AC}\right)\delta^{\mu}_{\nu}$
$R^{\alpha}{}_{\beta} = -C^{-2} R_0{}^{\alpha}{}_{\beta} + \left((n-1)\frac{C'^2}{C^2} + \frac{C''}{C} + (\rho+1)\frac{A'C'}{AC}\right)\delta^{\alpha}_{\beta}$
$R^{r}{}_{r} = (\rho+1)\frac{A''}{A} + n\frac{C''}{C}$
e.g. D=
ds
$R^{t}{}_{t} = A$
$R^{r}{}_{r} =$

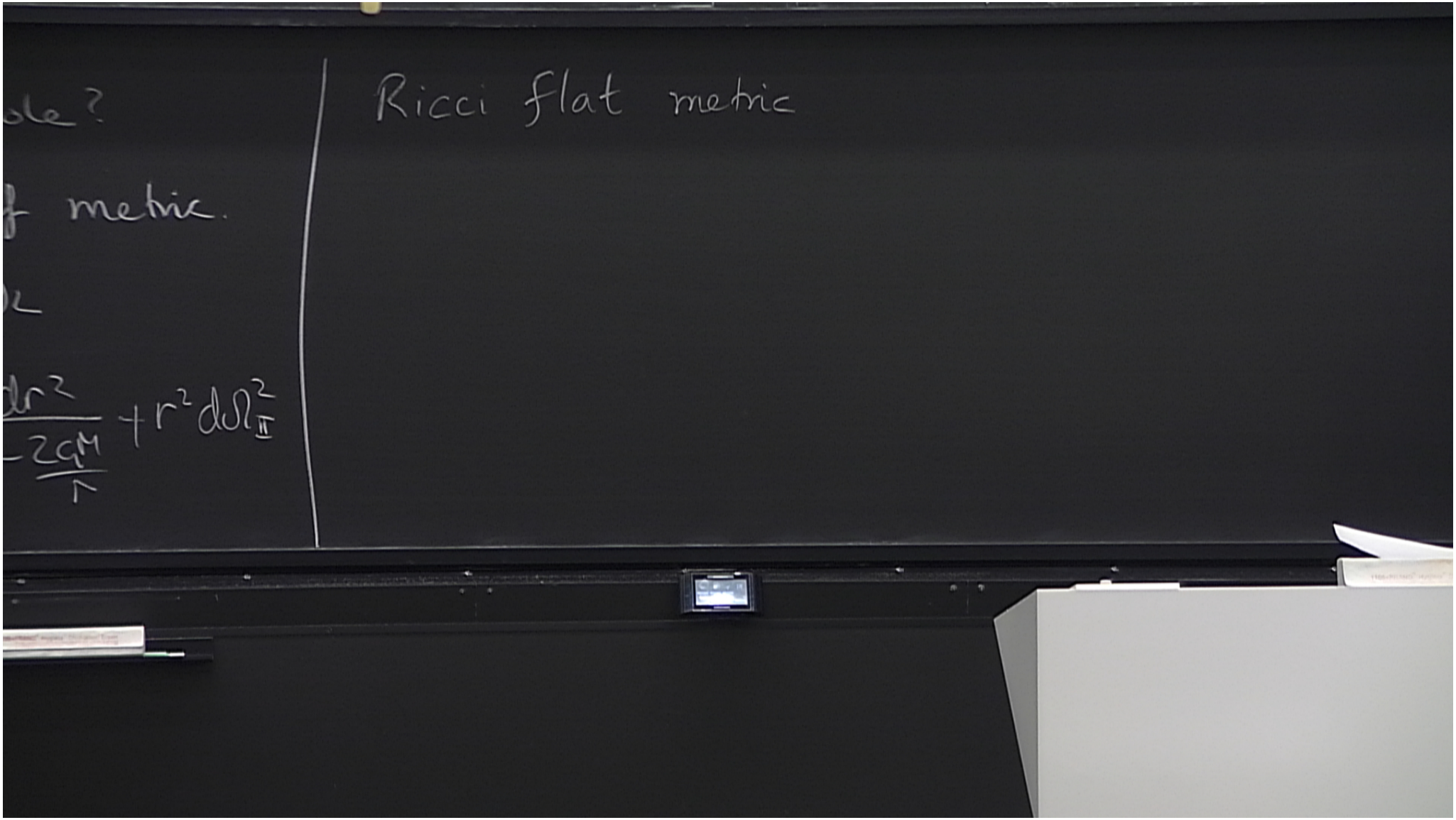

$r \to \sqrt{\frac{3}{\Lambda}}$, $g_{tt} \to 0$ $g_{rr} \to \infty$.
Cosmological Event Horizon
Note if $\Lambda \geq 0$ require the "1"
to get the $r \to 0$ singularity ($M \neq 0$)
cloaked by an event horizon.
if $\Lambda < 0$ can choos $\gamma_{\alpha\beta}$ to be flat
or hyperbolic, ie. $R_{0'\alpha\beta} = \pi \gamma_{\alpha\beta}$
$d\Omega_{II}^2$

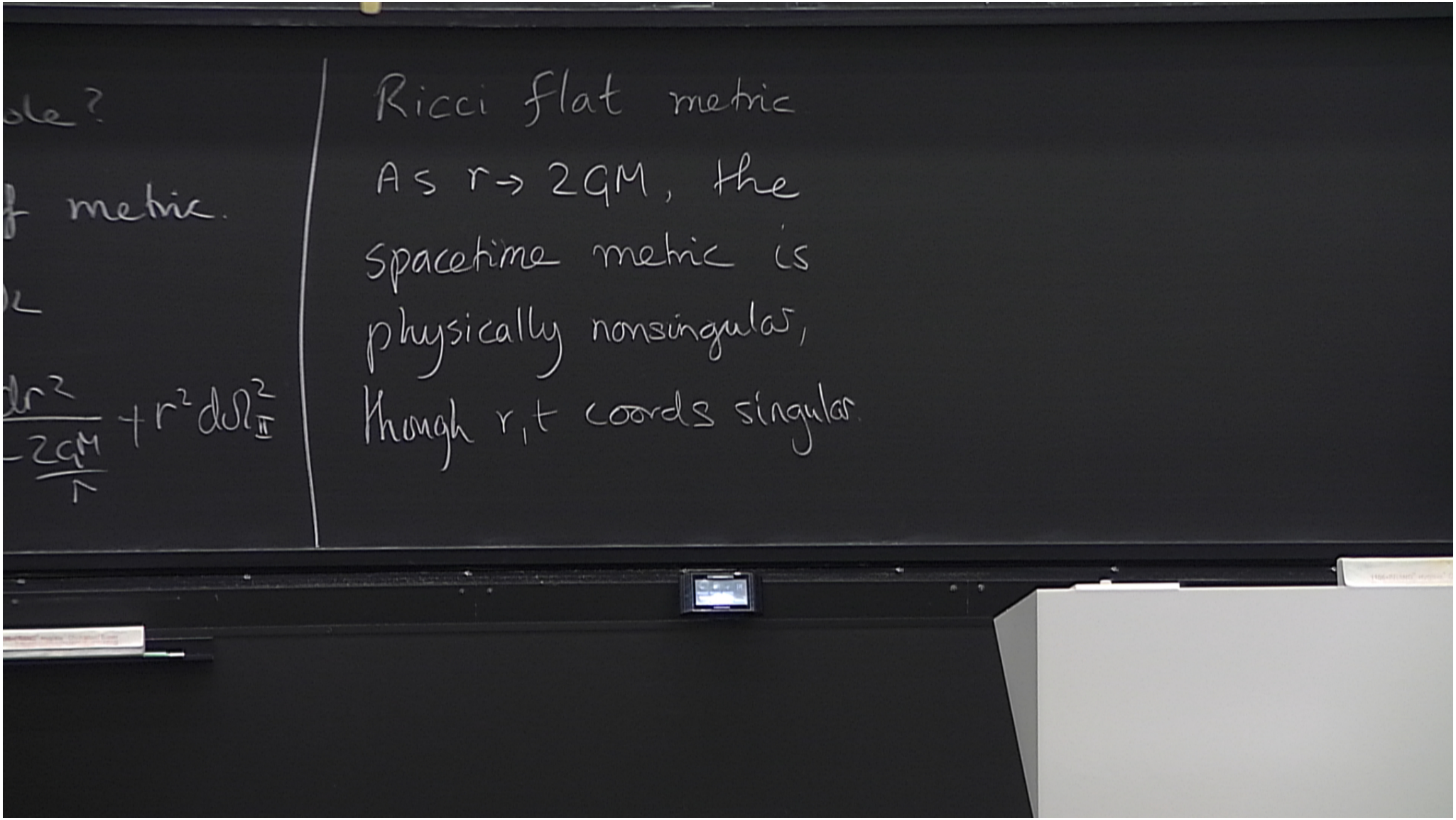

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$A^2 \to \kappa - \frac{\Lambda}{3} r^2 - \frac{2GM}{r}$
ie adS black holes can be sphorical,
planar or hyperbolic

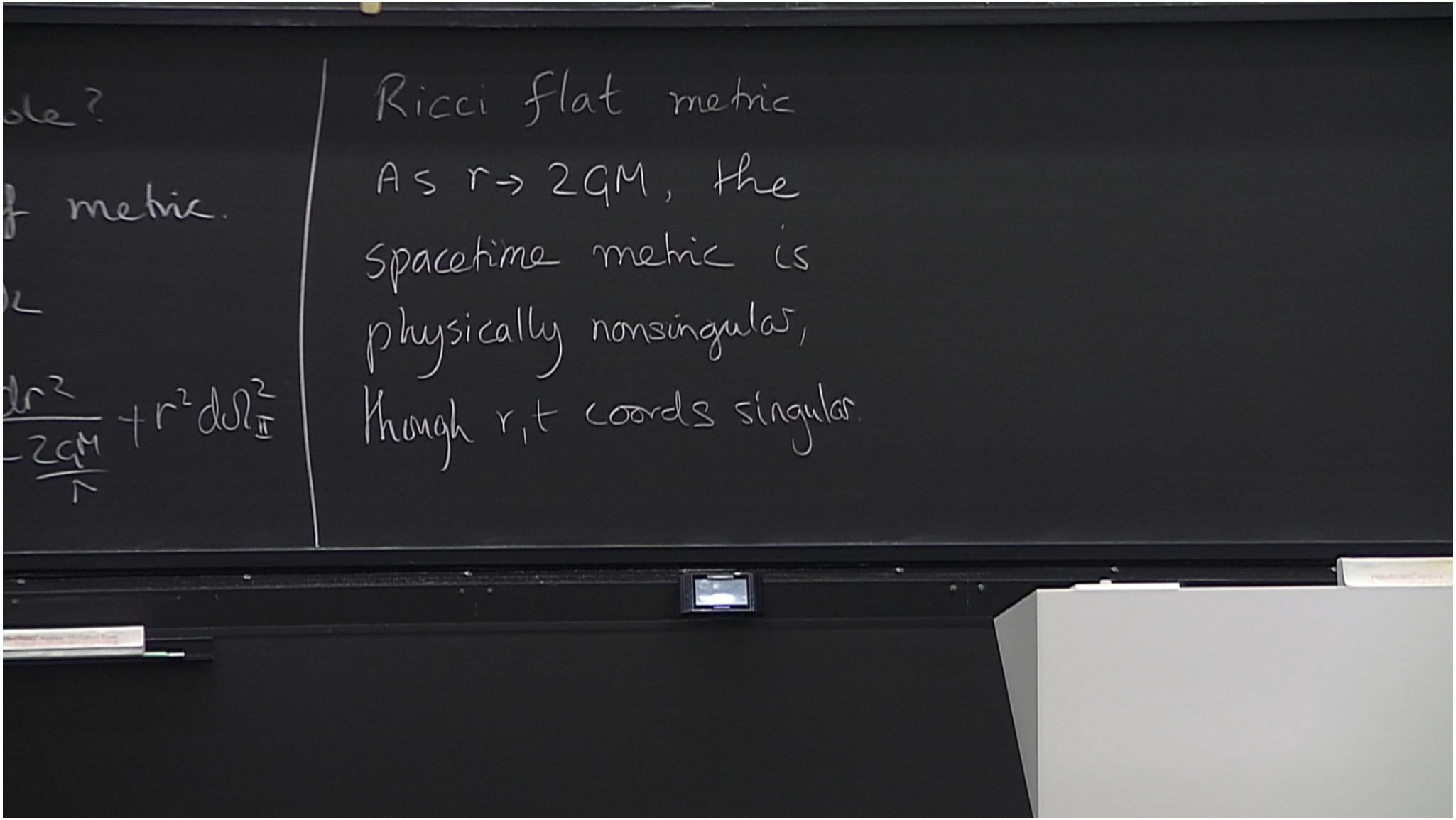

Euclidean black hole?

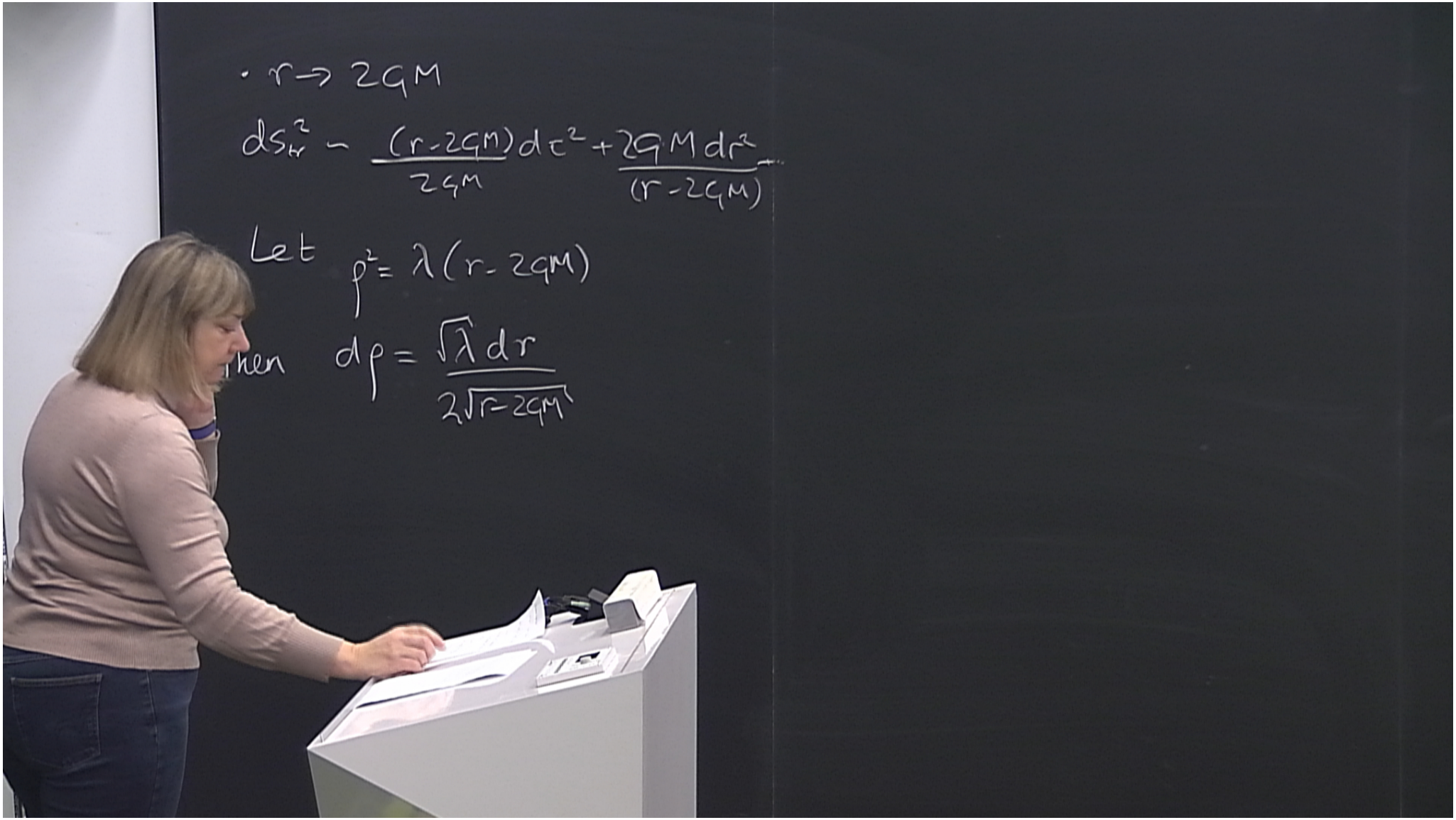

Euclidean black hole?
Euclidean = +ve def metric.
$t \to it$ $g_E = -g_L$

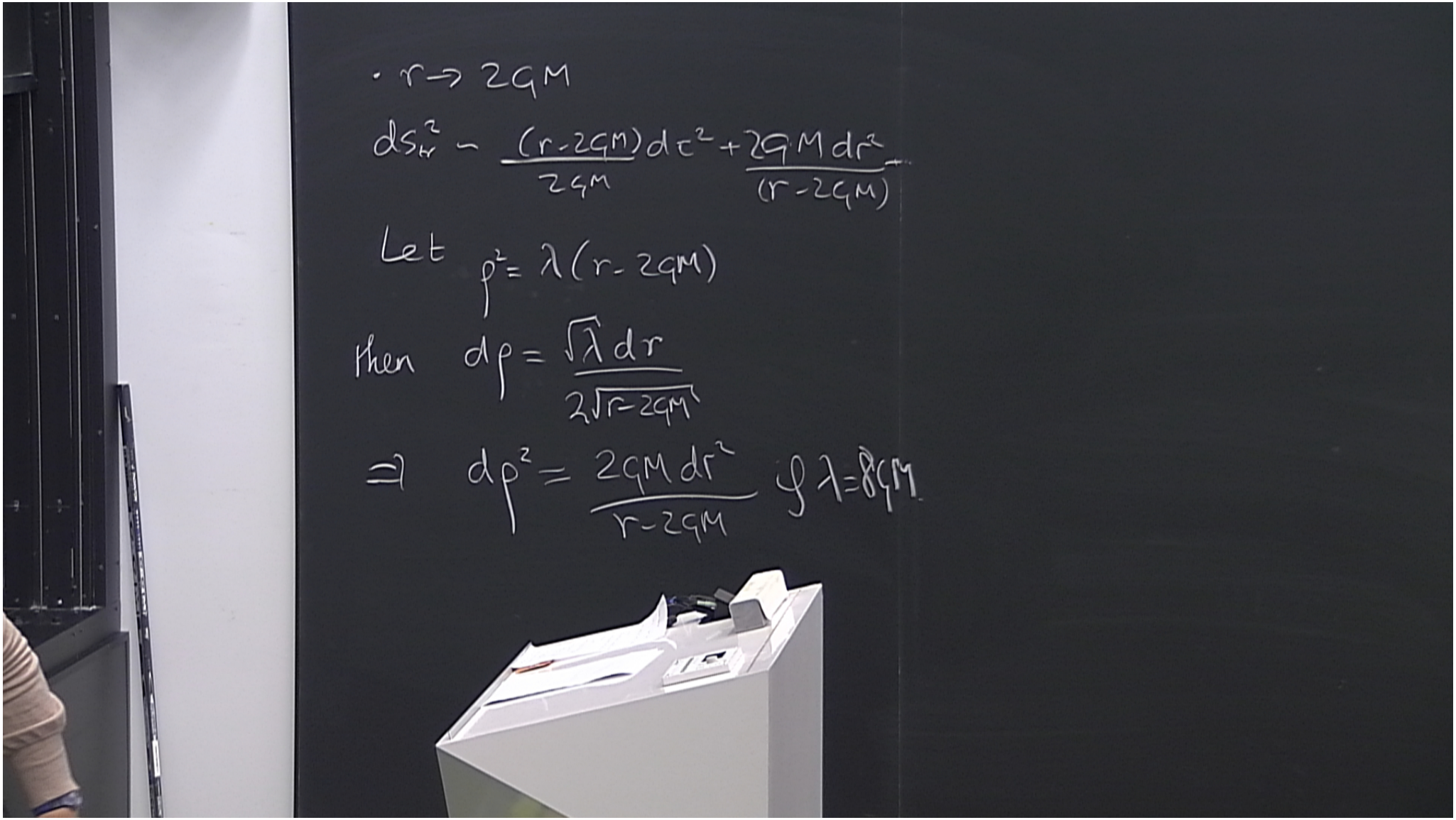

Euclidean black hole?
Euclidean = +ve def metric.
$t \to i\tau$ $g_E = -g_L$
$ds^2 = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2 d\Omega_{II}^2$

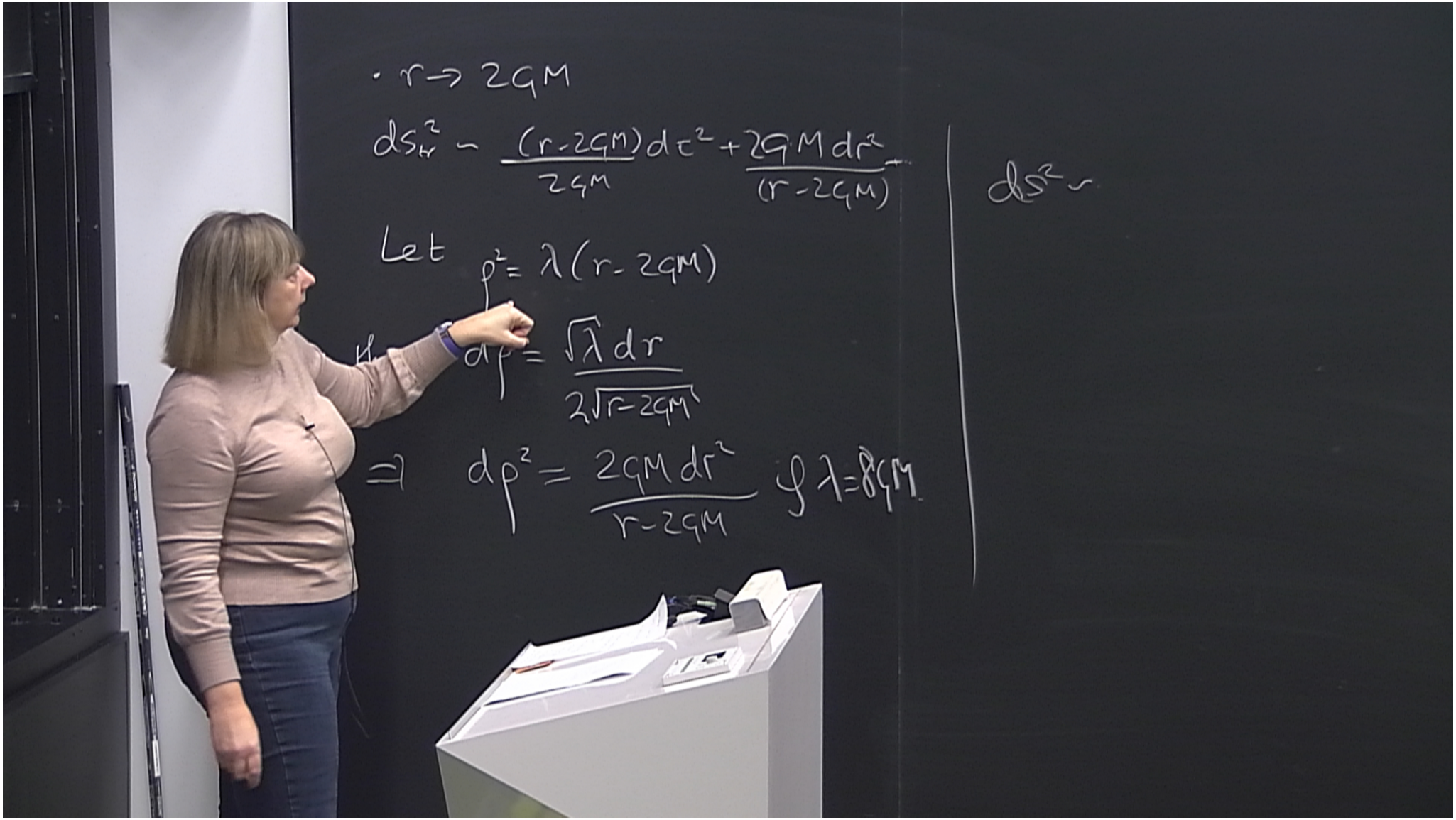

ole?
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Ricci flat metric

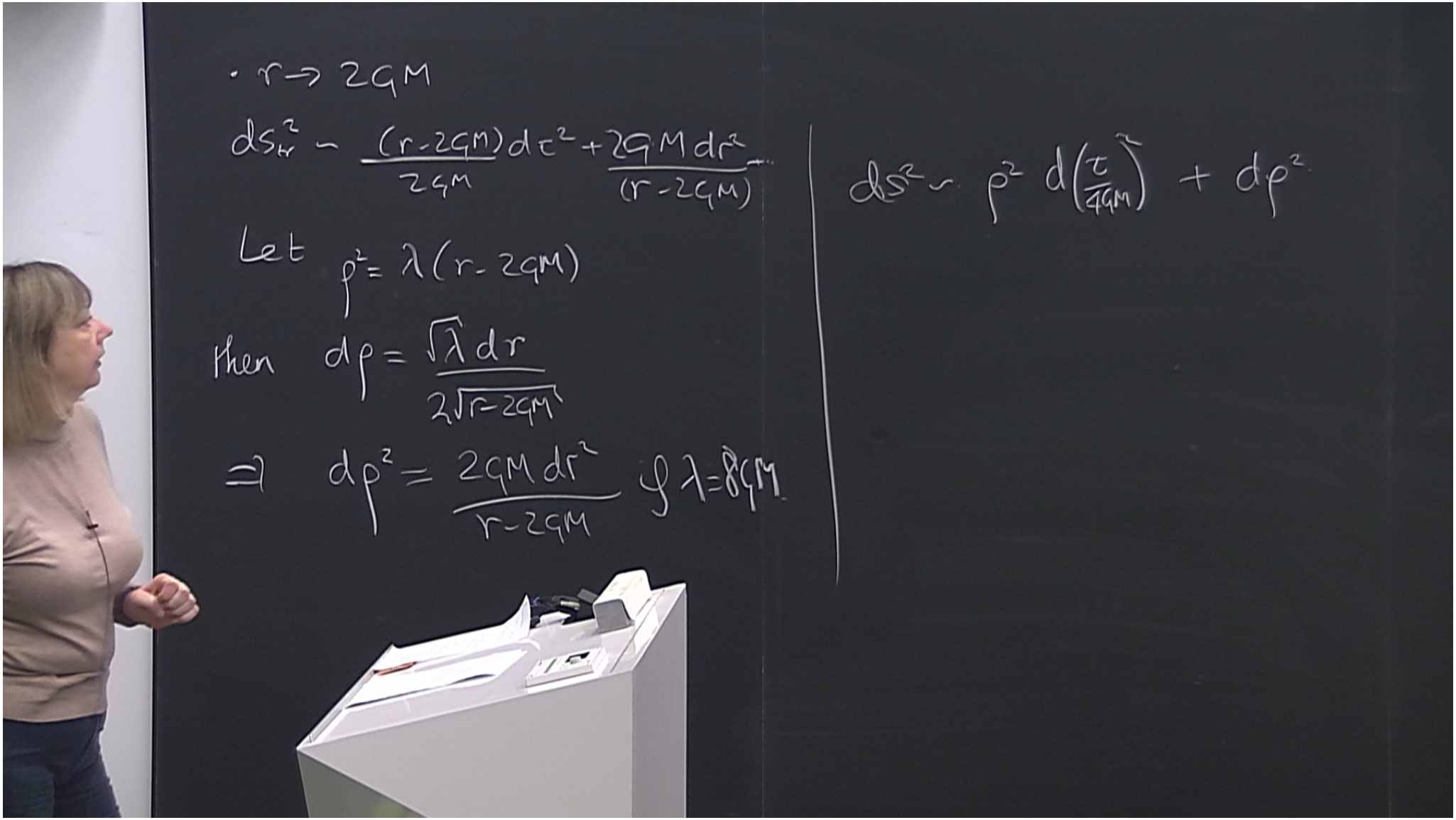

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As $r \to 2GM$, the
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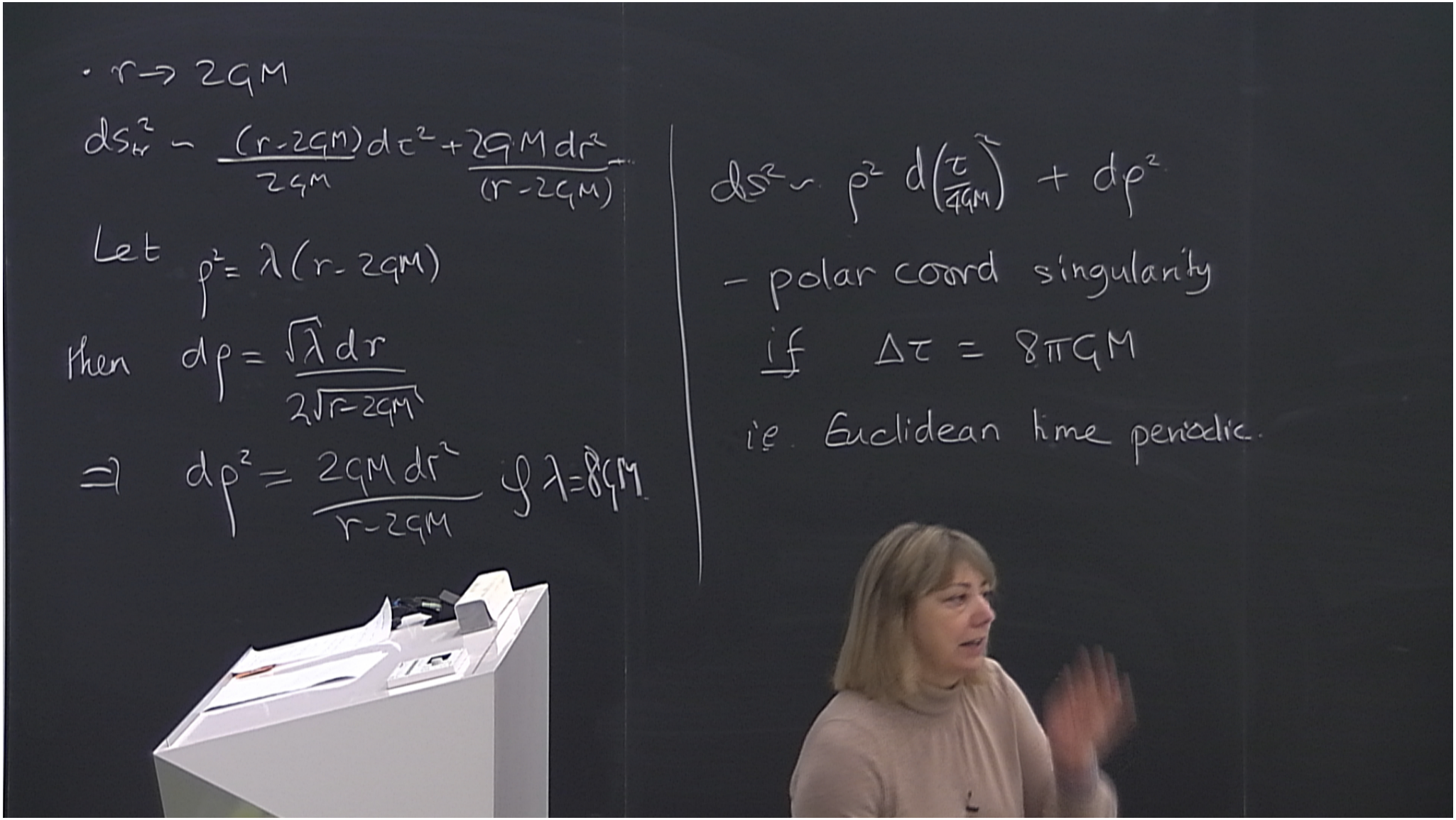

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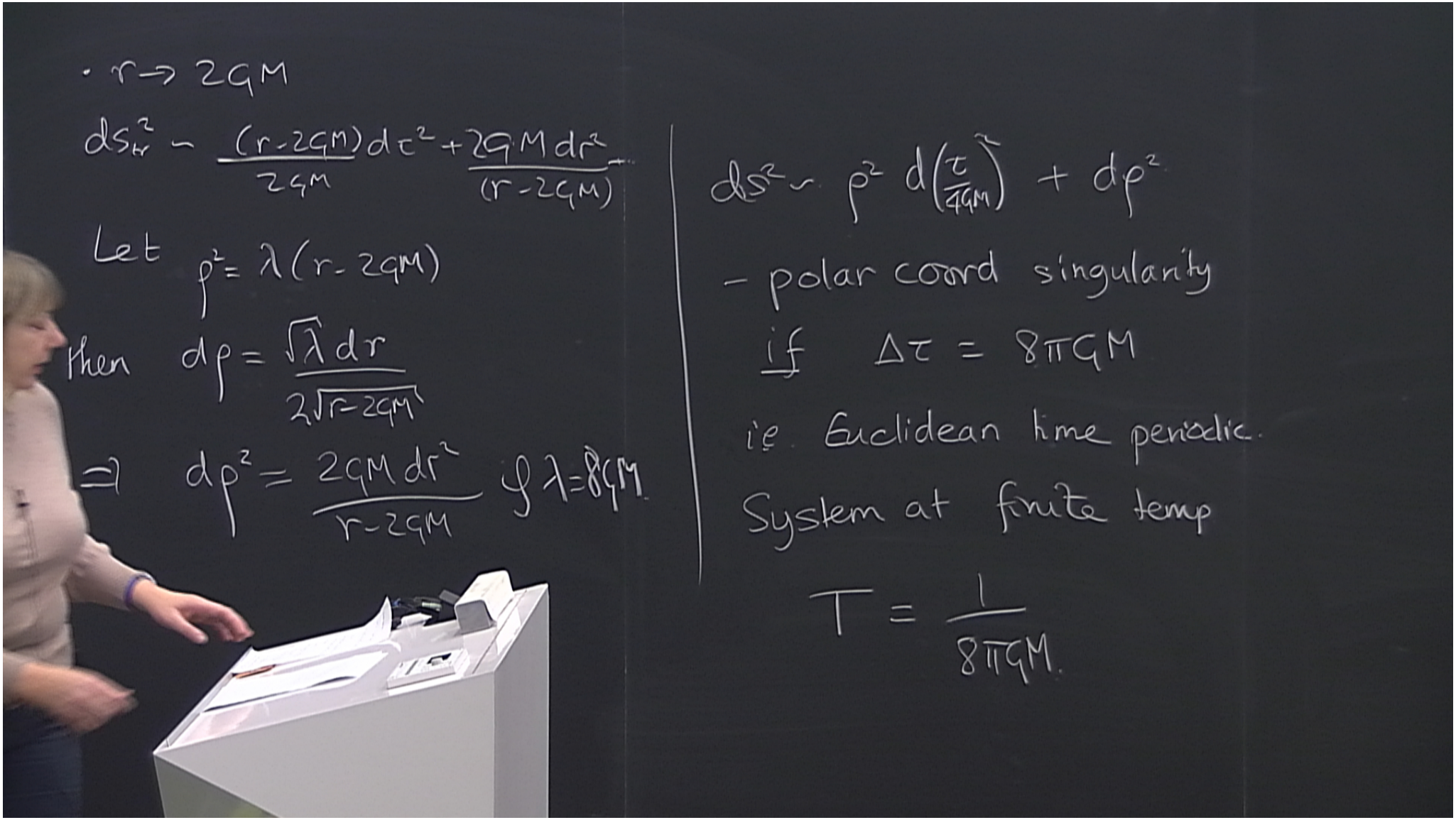

· r → 2GM
$ds^2_{Kr} \sim \frac{(r-2GM)}{2GM} dt^2 + \frac{2GM\, dr^2}{(r-2GM)} + \ldots$
Let $\rho^2 = \lambda(r-2GM)$
then $d\rho = \frac{\sqrt{\lambda}\, dr}{2\sqrt{r-2GM}}$

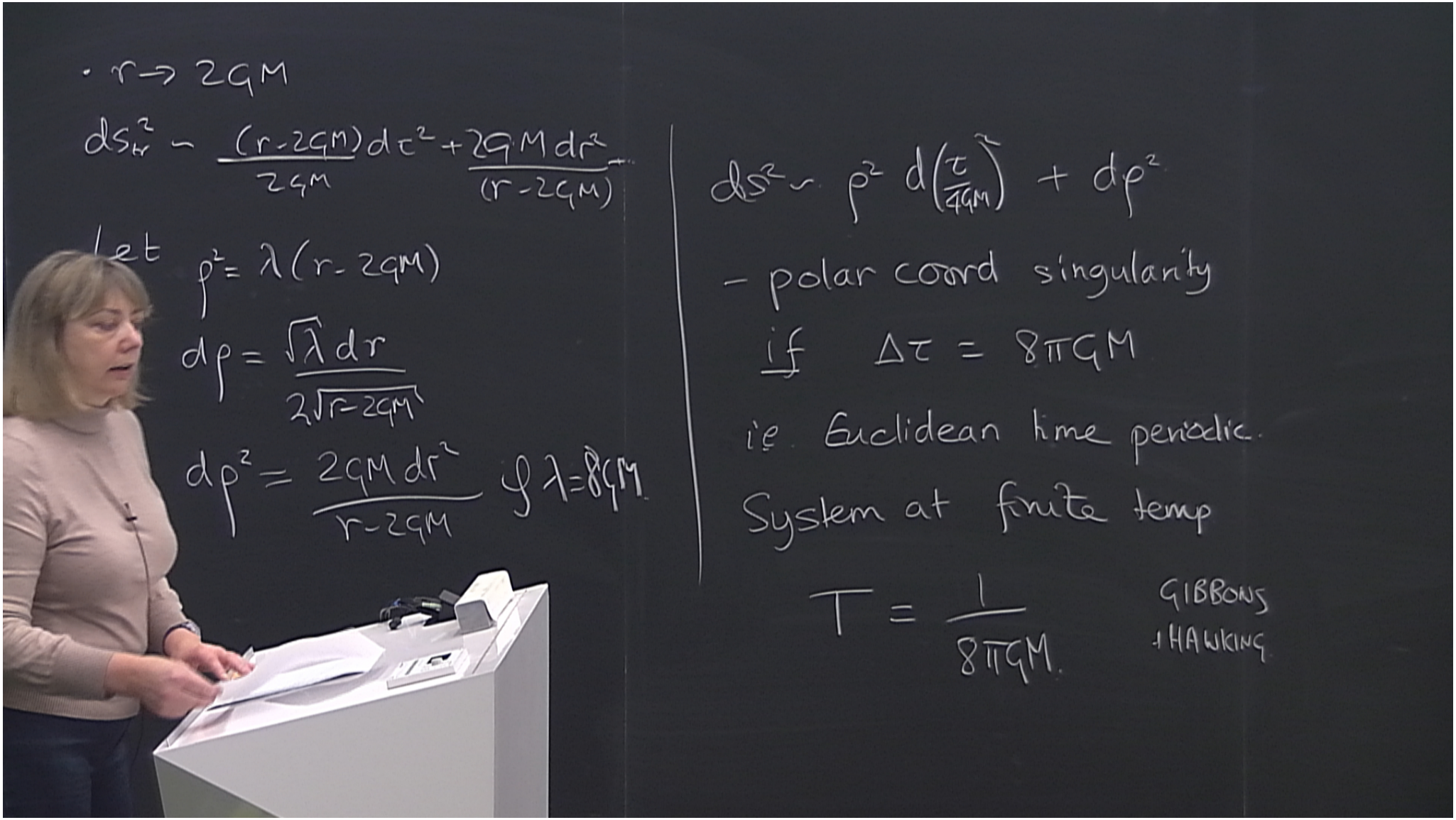

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- polar coord singularity
if $\Delta\tau = 8\pi GM$
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GIBBONS + HAWKING

• $r \to 2GM$

$$ds_{Kr}^2 \sim \frac{(r-2GM)}{2GM} dt^2 + \frac{2GM\, dr^2}{(r-2GM)} + \ldots$$

Let $\rho^2 = \lambda(r-2GM)$

Then $d\rho = \frac{\sqrt{\lambda}\, dr}{2\sqrt{r-2GM}}$

$$\Rightarrow d\rho^2 = \frac{2GM\, dr^2}{r-2GM} \quad \text{if } \lambda = 8GM$$



$$ds^2 \sim \rho^2 d\left(\frac{\tau}{4GM}\right)^2 + d\rho^2$$

– polar coord singularity

if $\Delta\tau = 8\pi GM$

i.e. Euclidean time periodic.

System at finite temp

$$T = \frac{1}{8\pi GM} = \frac{\hbar c^3}{8\pi GM k_B}$$

GIBBONS + HAWKING

$ds^2 \sim \rho^2 d\left(\frac{\theta}{4GM}\right) + d\rho^2$

- polar coord singularity

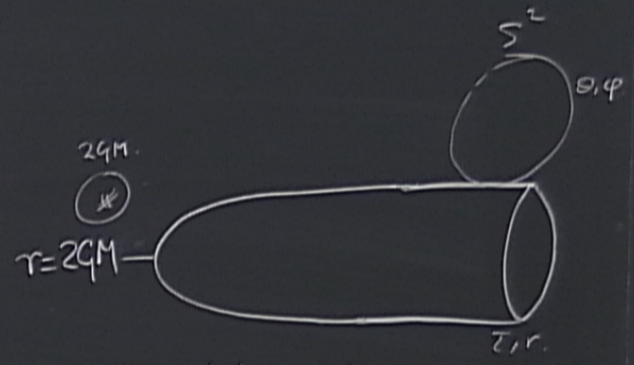
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GIBBONS + HAWKING



Euclidean black hole cigar

$S^2 \times \mathbb{R}^2$