

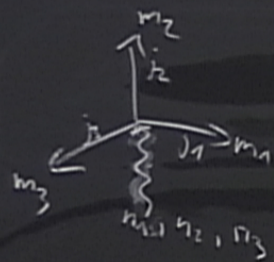
Title: 12/13 PSI - Quantum Gravity Review Lecture 15

Date: Feb 15, 2013 10:15 AM

URL: <http://pirsa.org/13020058>

Abstract:

Gauß constraints

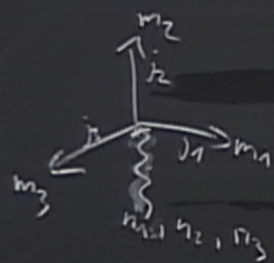


$$\sum_{p_i} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \bigotimes_{i=1}^3 |j_i, m_i, p_i\rangle$$

"invariant tensor"

• has norm 1

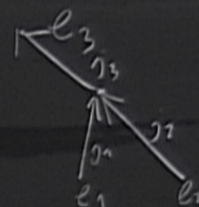
Gauß constraints

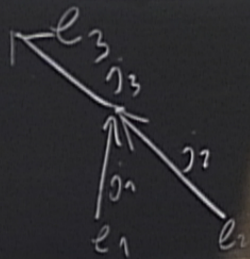


$$\sum_{P_i} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \bigotimes_{i=1}^3 |j_i, m_i, p_i\rangle$$

"invariant tensor"

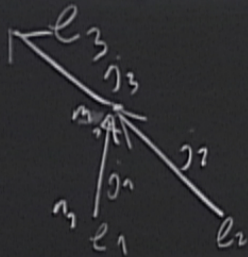
• has norm 1



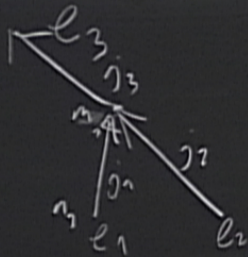


$$\begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & p_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle$$

$$|j_3, m_3, p_3\rangle$$

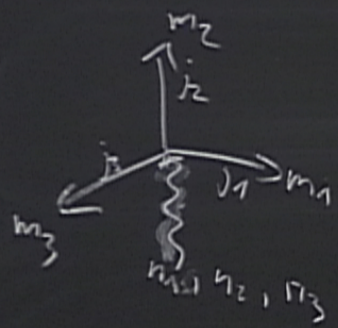


$$\sum_{q_1, q_2, P_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & P_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle |j_3, m_3, P_3\rangle$$



$$\sum_{q_1, q_2, P_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & P_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle |j_3, m_3, P_3\rangle$$

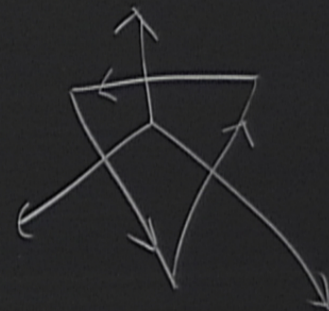
Gauß constraints

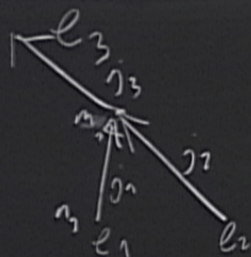


$$\sum_{P_i} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \bigotimes_{i=1}^3 |j_i, m_i, p_i\rangle$$

"invariant tensor"

• has norm 1





$$\sum_{q_1, q_2, p_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & p_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle$$

$$|j_3, m_3, p_3\rangle$$

S^2

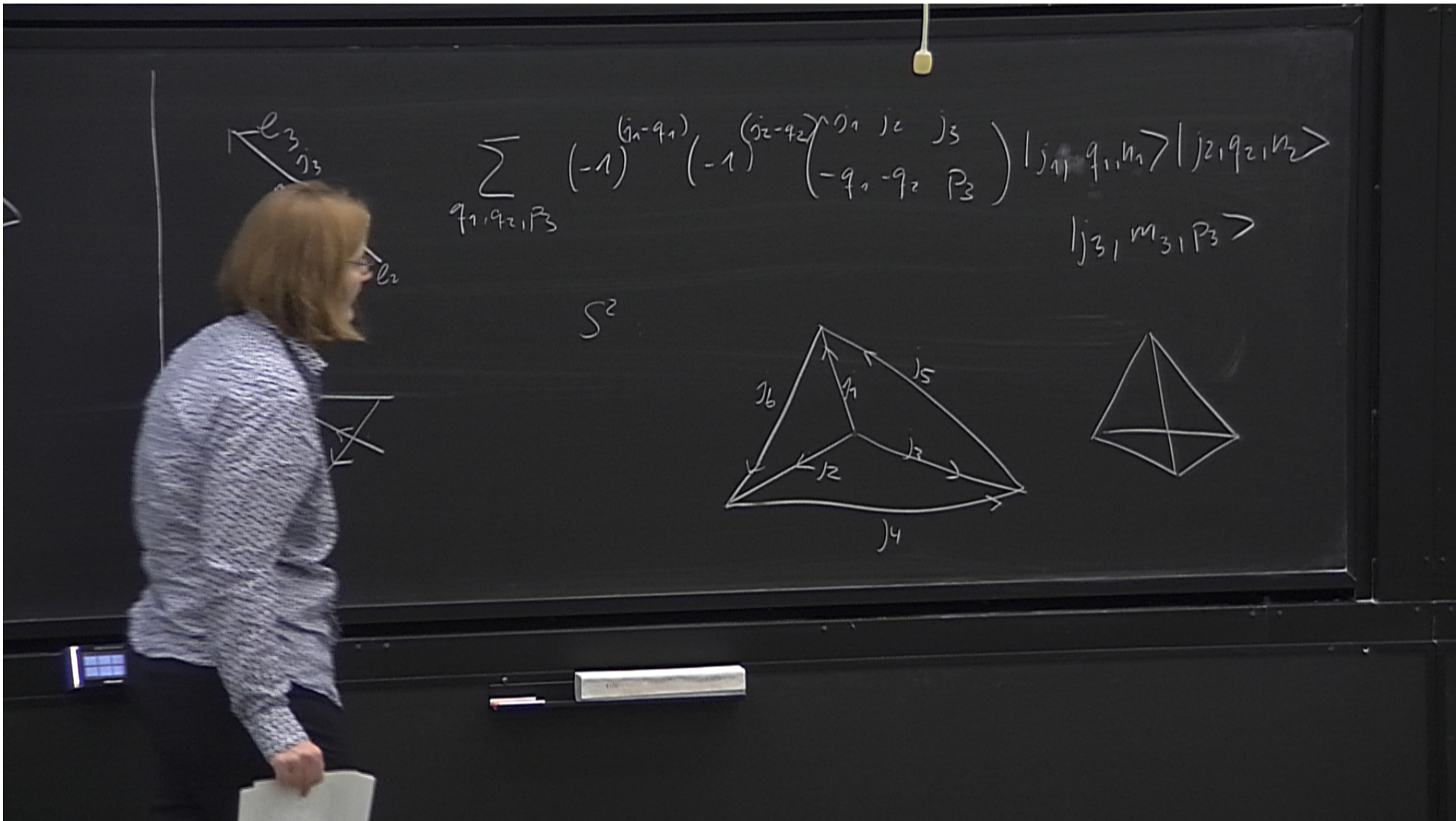


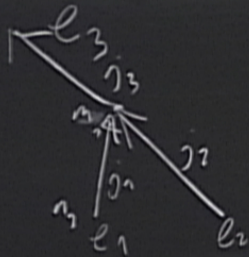


$$\sum_{q_1, q_2, p_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & p_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle |j_3, m_3, p_3\rangle$$

S^2

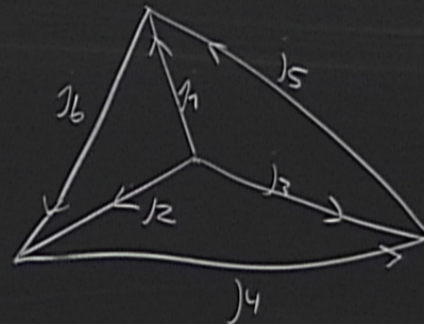
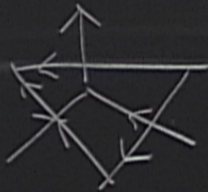


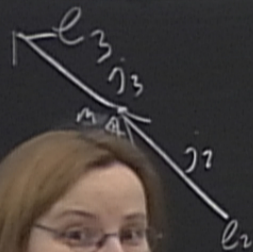




$$\sum_{q_1, q_2, p_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & p_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle |j_3, m_3, p_3\rangle$$

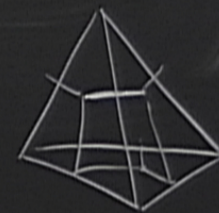
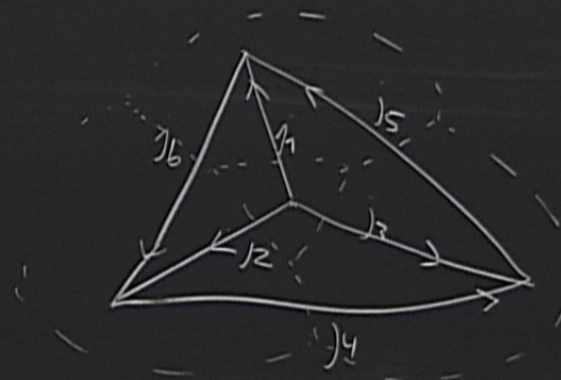
S^2

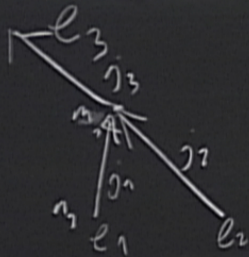




$$\sum_{q_1, q_2, p_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & p_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle |j_3, m_3, p_3\rangle$$

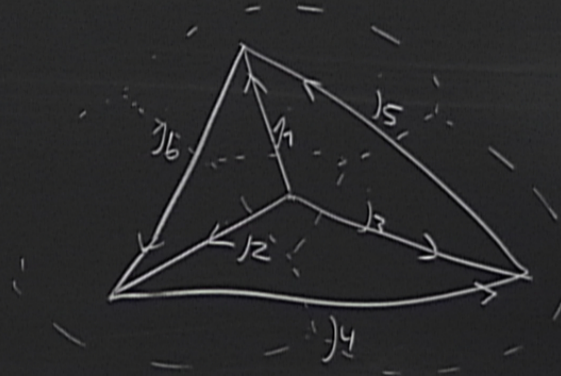
S^2





$$\sum_{q_1, q_2, p_3} (-1)^{(j_1 - q_1)} (-1)^{(j_2 - q_2)} \begin{pmatrix} j_1 & j_2 & j_3 \\ -q_1 & -q_2 & p_3 \end{pmatrix} |j_1, q_1, m_1\rangle |j_2, q_2, m_2\rangle |j_3, m_3, p_3\rangle$$

S^2



$$\text{Tetr} \begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} := \sum_{p_1 \dots p_6} \sum_{q_1 \dots q_6} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} (-1)^{\sum_{i=1}^6 (j_i - q_i)} ($$

$$\begin{aligned}
 \text{Tetr} \begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} &:= \\
 \sum_{p_1 \dots p_6} \sum_{q_1 \dots q_6} &\begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} (-1)^{\sum_{i=1}^6 (j_i - q_i)} \begin{pmatrix} j_1 & j_5 & j_6 \\ -q_1 & -q_5 & p_6 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -q_2 & -q_6 & p_4 \end{pmatrix} \\
 &\begin{pmatrix} j_3 & j_4 & j_5 \\ -q_3 & -q_4 & p_5 \end{pmatrix} \bigotimes_{i=1}^6 |j_i, q_i, p_i\rangle
 \end{aligned}$$

Flatness constraints

$$F_{ab}^i = 0$$

Flatness constraints

$$\tilde{F}_{ab} = 0$$

E-Dyn (Abelian group)



h_r^{Abelian}

$$= \exp\left(-\int_r A_a \dot{\gamma}^a(s) ds\right) = \exp\left(-\int_S F_{ab} dx^a dx^b\right) \approx$$

Flatness constraints

$$\vec{F}_{ab} = 0$$

E-Dyn (Abelian group)

$$\textcircled{S} \quad h_r^{\text{Abelian}} = \exp\left(-\int_r A_a \dot{\gamma}^a(s) ds\right) = \exp\left(-\int_S F_{ab} dx^a dx^b\right) \approx \mathbb{1} - \int_S F$$

Constraints

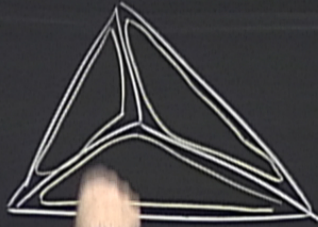
$$F = 0 \quad \Rightarrow \quad \boxed{h_{\gamma} = \underline{11}}$$

←

an group)

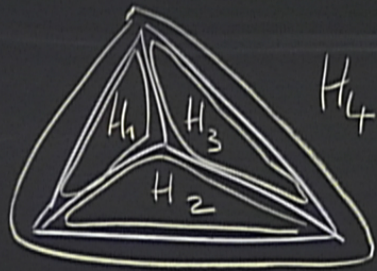
$$\exp\left(-\int_{\gamma} A_a \dot{\gamma}^a(s) ds\right) = \exp\left(-\int_S F_{ab} dx^a dx^b\right) \approx \underline{11} - \int_S F_{ab} dx^a dx^b$$

$$F = dA$$



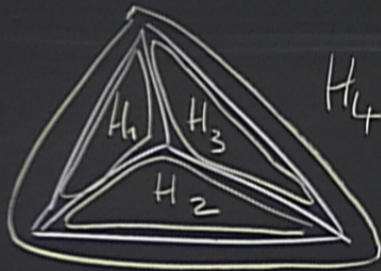
$$F = dA$$

Solve flatness



$$F = dA$$

Solve flatness



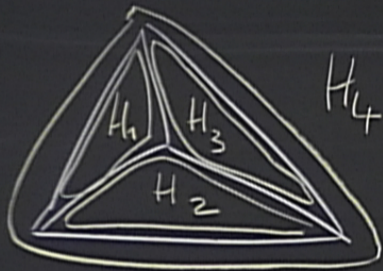
H_4

$$\delta(H_1) \delta(H_2) \delta(H_3) \delta(H_4)$$



$$F = dA$$

Solve flatness



$$\delta(H_1) \delta(H_2) \delta(H_3) \delta(H_4)$$

↑
δ-fct on group

$$\int \delta(h) f(h) dh = f(1)$$

(Homework)

Integral form of the Branch identity

(H_k)

$$H_1 = H_2 = H_3 = \underline{11}$$

$$\Rightarrow H_4 = \underline{11}$$

$f(1)$

Integral form of the Branch identity

$$H_1 = H_2 = H_3 = \underline{11}$$

$$\Rightarrow H_4 = \underline{11}$$

$$H_4 = g_2 H_1 H_3 H_2 g_2^{-1}, \quad H_4 = g_6 g_5 g_4$$

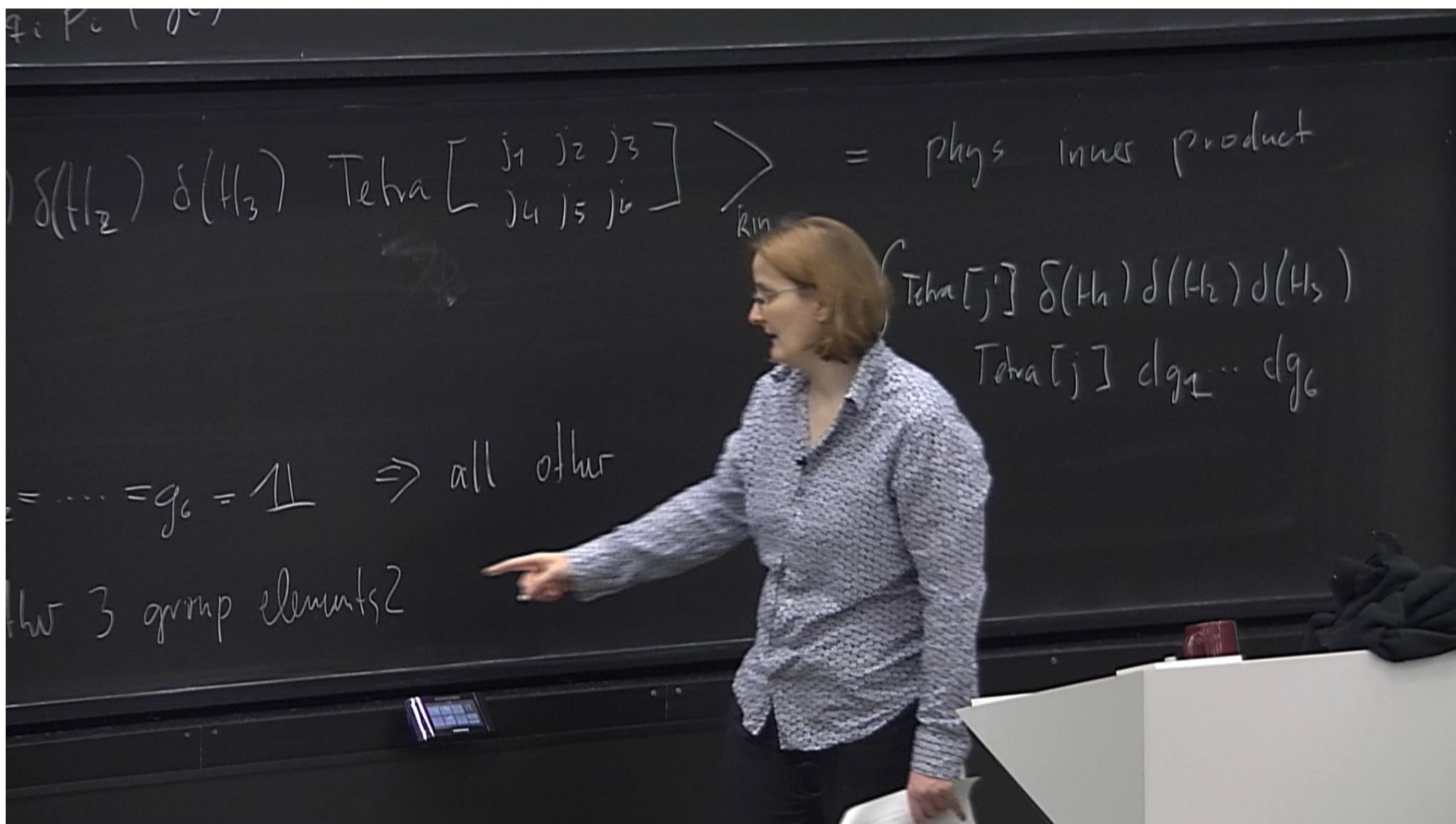
$$H_1 = g_2^{-1} g_6 g_1 \quad H_2 = g_3^{-1} g_4 g_2 \quad H_3 = g_1^{-1} g_5 g_3$$

$$\langle \text{Tetra} \begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix}, \delta(t_1) \delta(t_2) \delta(t_3) \text{Tetra} \begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} \rangle$$

$$\langle \text{Tetra} \begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix} \rangle, \quad \delta(h_1) \delta(h_2) \delta(h_3) \text{Tetra} \begin{bmatrix} j_1 \\ j_4 \end{bmatrix}$$

Motivation for Result:

- flatness: one solution $g_1 = g_2 = \dots = g_6 = 11$
- What about the integration over the other 3 group elements



$g_i = P_i (0)$

$$\delta(H_2) \delta(H_3) \text{Tetra} \begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} \rangle_{\text{kin}} = \text{phys inner product}$$

$$\int \text{Tetra}[j'] \delta(H_4) \delta(H_2) \delta(H_3) \text{Tetra}[j] dg_4 \dots dg_6$$

$= \dots = g_6 = 11 \Rightarrow$ all other solution can be obtained by $SH(2)$ gauge trafo's.

for 3 group elements

$=g_c = 11 \Rightarrow$ all other solutions can be obtained by SU(2) gauge trafo's.

group elements $\Rightarrow$ integration over gauge orbits (Gauß constraint) $\Rightarrow$ can be ignored

$$\begin{pmatrix} j_1 & j_5 & j_6 \\ q_1 & q_5 & p_6 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -q_2 & & \end{pmatrix}$$

$$|P_i\rangle$$

$$(g_c)$$

$$\left\{ \begin{matrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{matrix} \right\} = \sum_{P_1 \dots P_6} (-1)^{\sum_{i=1}^6 (j_i - P_i)}$$

6j-symbol

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ P_1 & P_2 & P_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -P_1 & -P_5 & P_6 \end{pmatrix}$$

$$\begin{pmatrix} j_2 & j_6 & j_4 \\ -P_2 & -P_6 & P_4 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ -P_3 & -P_4 & P_5 \end{pmatrix}$$

$$F_{\perp} = dA$$

Transition amplitude between two tetrahedral

$$F_{\perp} = dA$$

Transition amplitude between two tetra

$$\langle \text{Proj} [\text{Tetra} \begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix}], \text{Proj} [\text{Tetra} \begin{bmatrix} j_1 & \dots \\ \dots & j_n \end{bmatrix}] \rangle_{\text{phys}}$$

$$F = dA$$

Transition amplitude between two tetrahedra

$$\langle P_{10} [\text{Tetra} \begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix}], P_{16j} [\text{Tetra} \begin{bmatrix} j_1 & \dots \\ \dots & j_6 \end{bmatrix}] \rangle_{\text{phys}} =$$

$$F = dA$$

Transition amplitude between two tetrahedra

$$\langle P_{10} [\text{Tetra} \begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix}] , P_{16} [\text{Tetra} \begin{bmatrix} j_1 & \dots \\ \dots & j_6 \end{bmatrix}] \rangle$$

two tetrahedral SNW

$$\left[\begin{array}{c} j_1 \\ \vdots \\ j_6 \end{array} \right] \rangle_{\text{phys}} = \left(\prod_{l=1}^6 \sqrt{d_l} \right) \left\{ \begin{array}{ccc} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{array} \right\} \left(\prod_{l=1}^6 \sqrt{d_l} \right) \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$$

an just set $g_i = \underline{11}$

$$\Rightarrow D_{q_i p_i}^{j_i}(\underline{11}) = \delta_{q_i p_i}$$

Tetra $\begin{bmatrix} j_1 & j_2 \\ \dots & j_6 \end{bmatrix} \rangle_{\text{phys}}$
 proj.

~



$$= \sqrt{\prod_i |g_i|} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

THING

$$\Rightarrow D_{q, p_i}^{j_i}(\underline{11}) = \delta_{q, p_i}$$

$$= \sqrt{\prod_i |j_i|} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

CREATION
PROBABILITY

~Ponzano Regge model