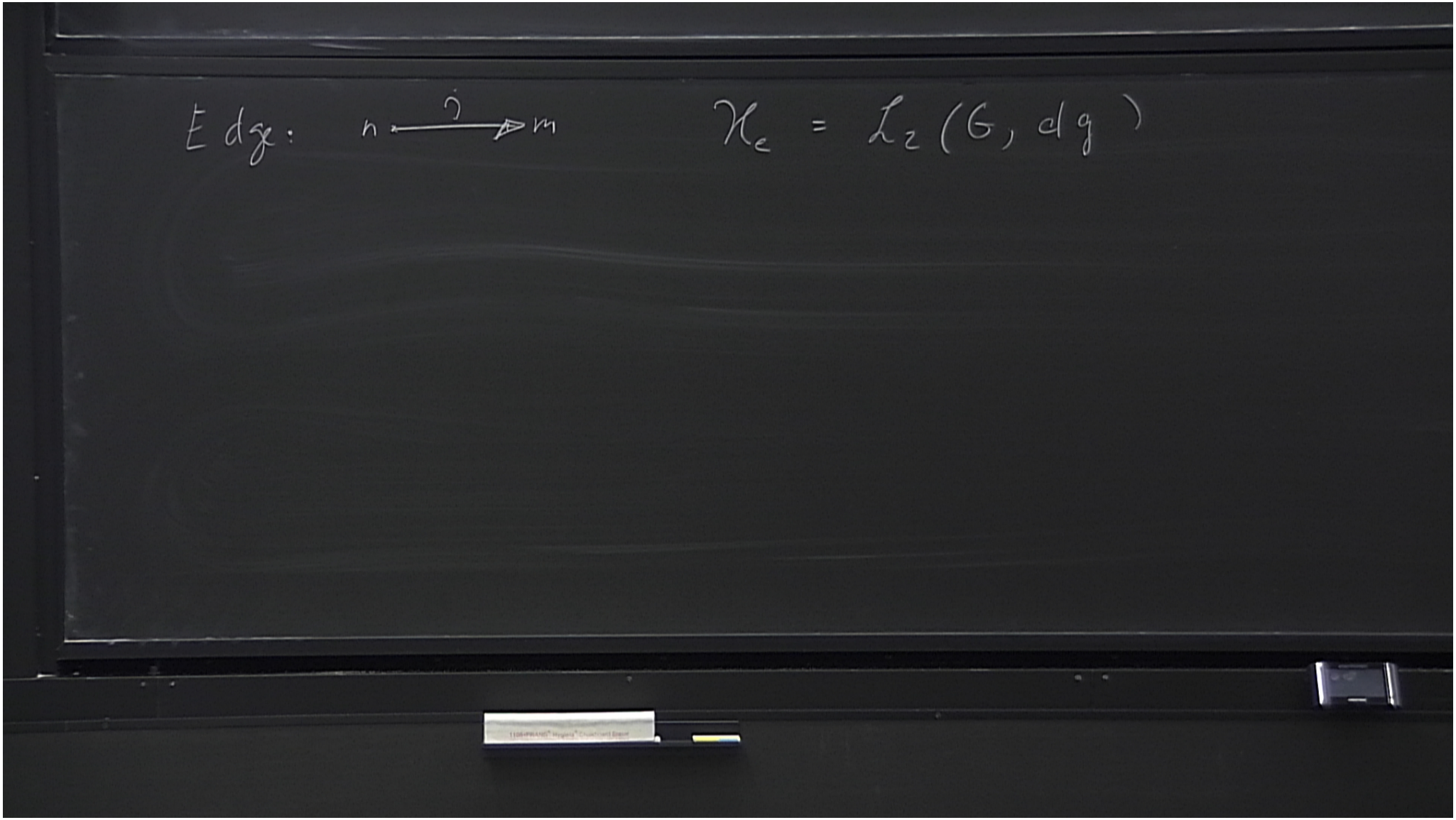
Title: 12/13 PSI - Quantum Gravity Review Lecture 14

Date: Feb 14, 2013 10:15 AM

URL: http://pirsa.org/13020057

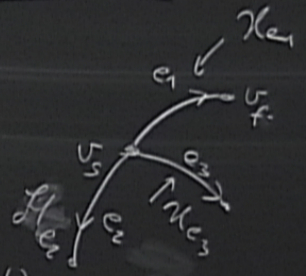
Abstract:


Edge: n → m
$\mathcal{H}_e = \mathcal{L}_2(G, dg)$

Edge: $n \xrightarrow{i} m$ $\quad \mathcal{H}_e = L_2(G, dg)$

Solving the Gauß constraints

Graphs:



$e : [0,1] \to \Sigma$

Γ : Graph, edges (oriented), vert. us

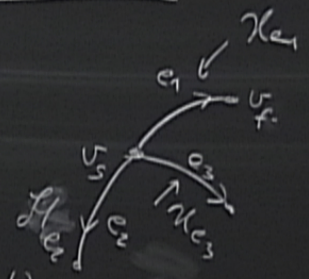
Edge: $n \xrightarrow{e} m$ $\quad \mathcal{H}_e = L_2(G, dg)$

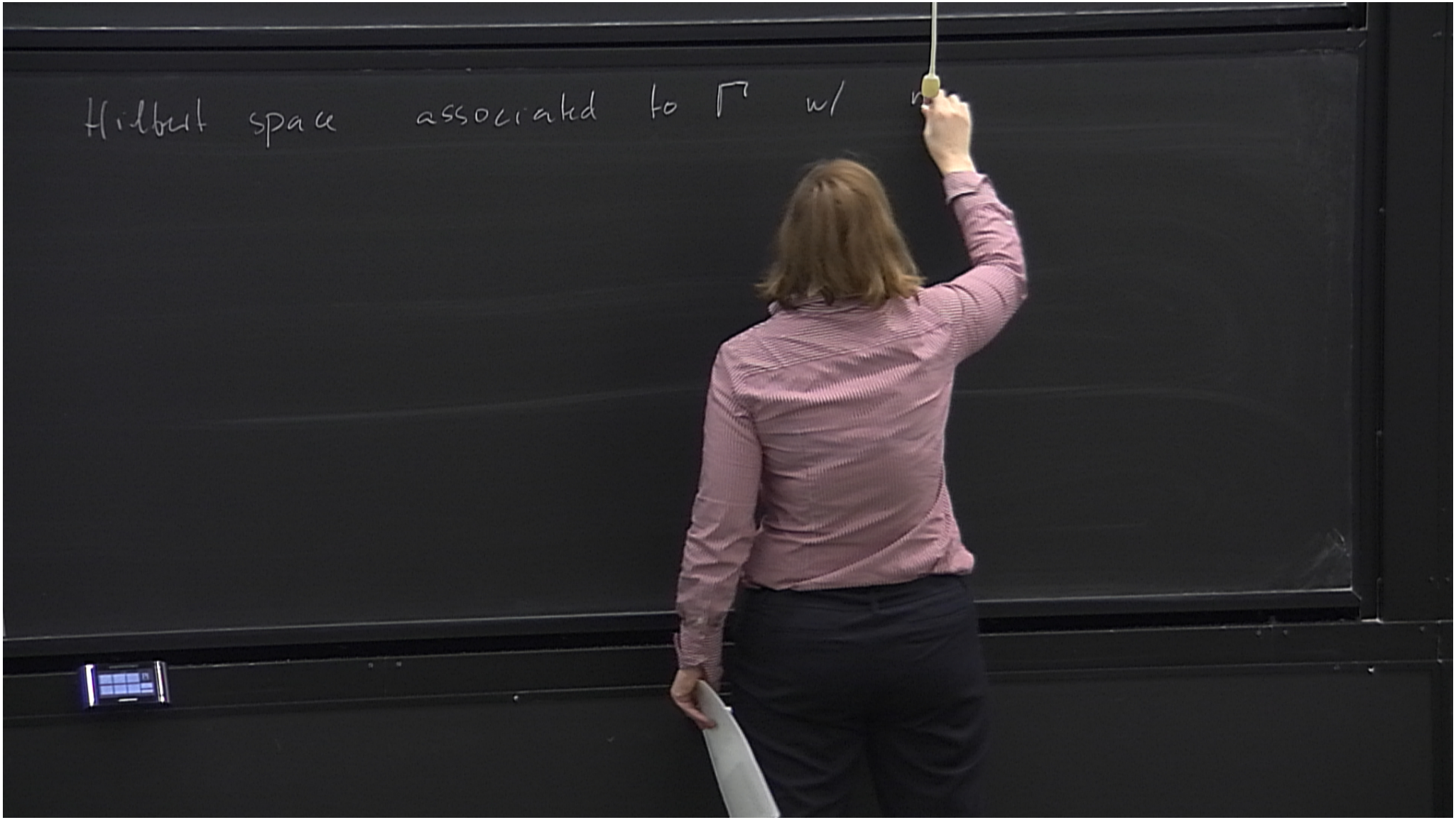
Solving the Gauß constraints

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$e : [0,1] \to \Sigma$

Γ : Graph, edges (oriented), vert. us




Hilbert space associated to Γ w/

Hilbert space associated to Γ w/ n edges $e_1, \dots, e_n$

$$\mathcal{H}_\Gamma := \mathcal{H}_{e_1} \otimes \mathcal{H}_{e_2} \otimes \dots \otimes \mathcal{H}_{e_n}$$

Hilbert space associated to Γ w/ n edges $e_1, \dots, e_n$

$$\mathcal{H}_\Gamma := \mathcal{H}_{e_1} \otimes \mathcal{H}_{e_2} \otimes \cdots \otimes \mathcal{H}_{e_n}$$

$\rightarrow \quad \psi(g_1, g_2, \dots, g_n)$

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$$\langle \psi_1, \psi_2 \rangle_\Gamma = \int \overline{\psi_1(g_1, \dots g_n)}\ \psi_2(g_1, \dots, g_n)\ dg_1 \cdots dg_n$$

Hilbert space associated to Γ w/ n edges $e_1, \dots, e_n$

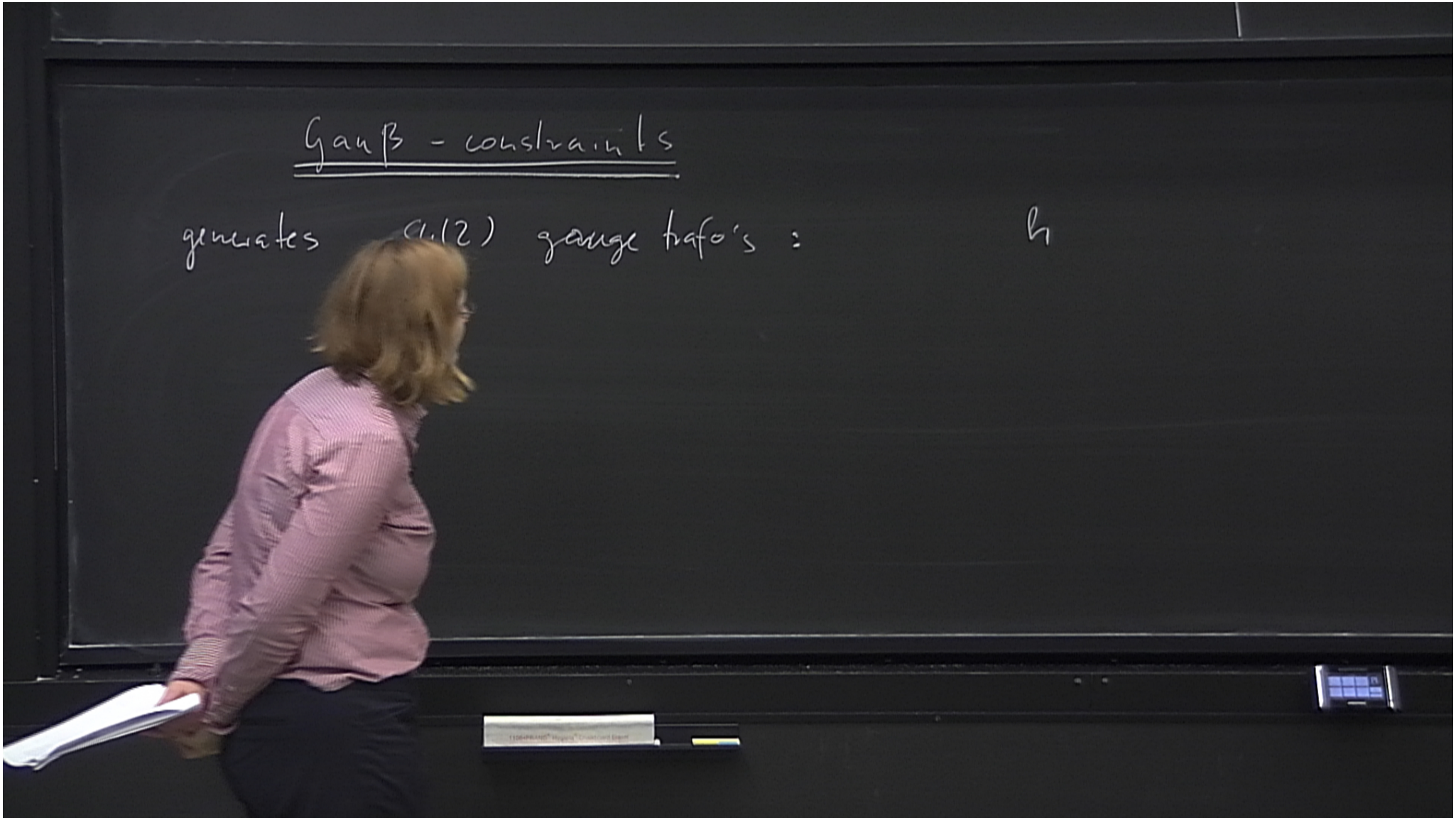
$$\mathcal{H}_\Gamma := \mathcal{H}_{e_1} \otimes \mathcal{H}_{e_2} \otimes \cdots \otimes \mathcal{H}_{e_n}$$

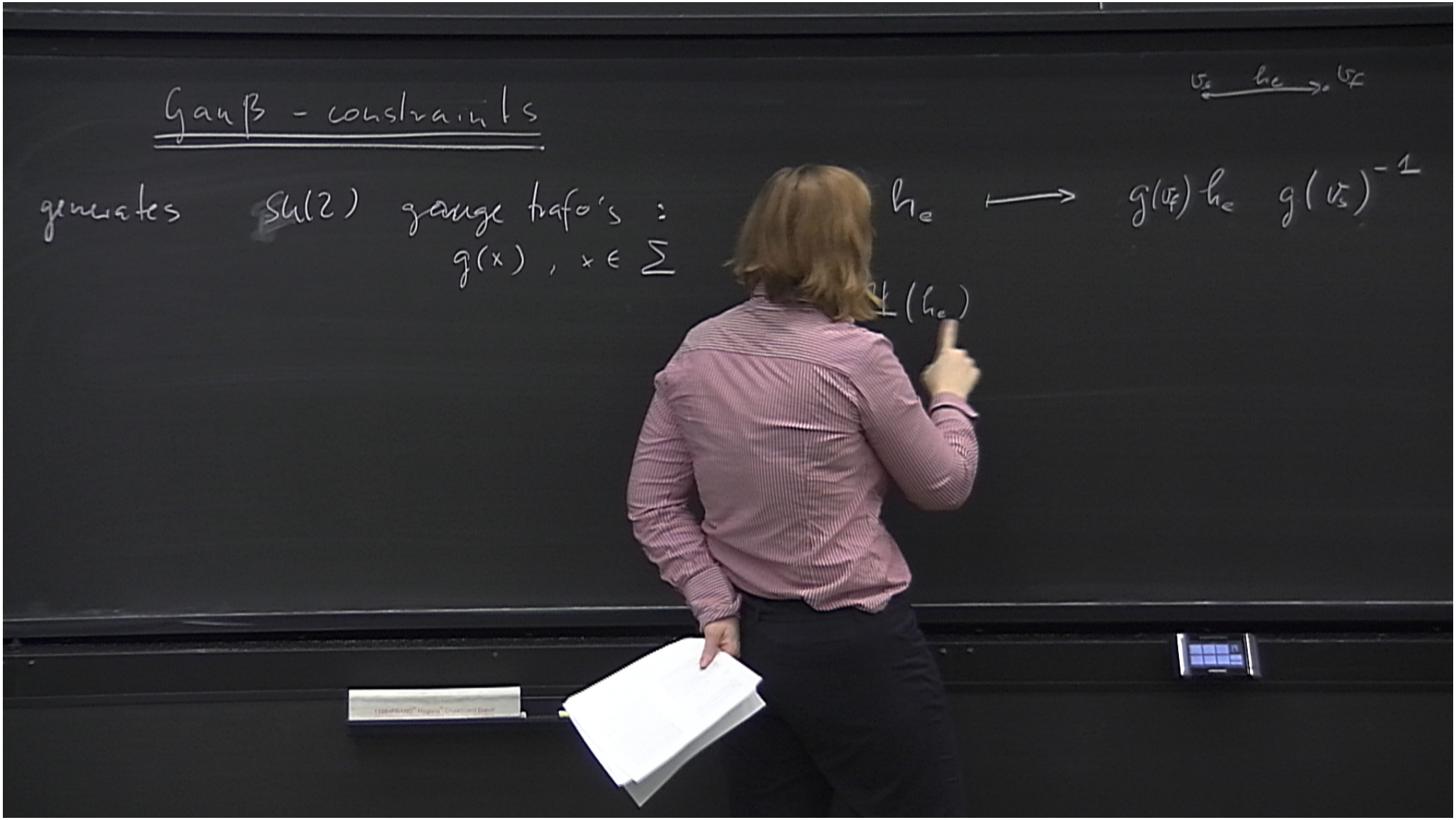
$$H = \mathcal{L}_2(\mathbb{R}, dx)$$

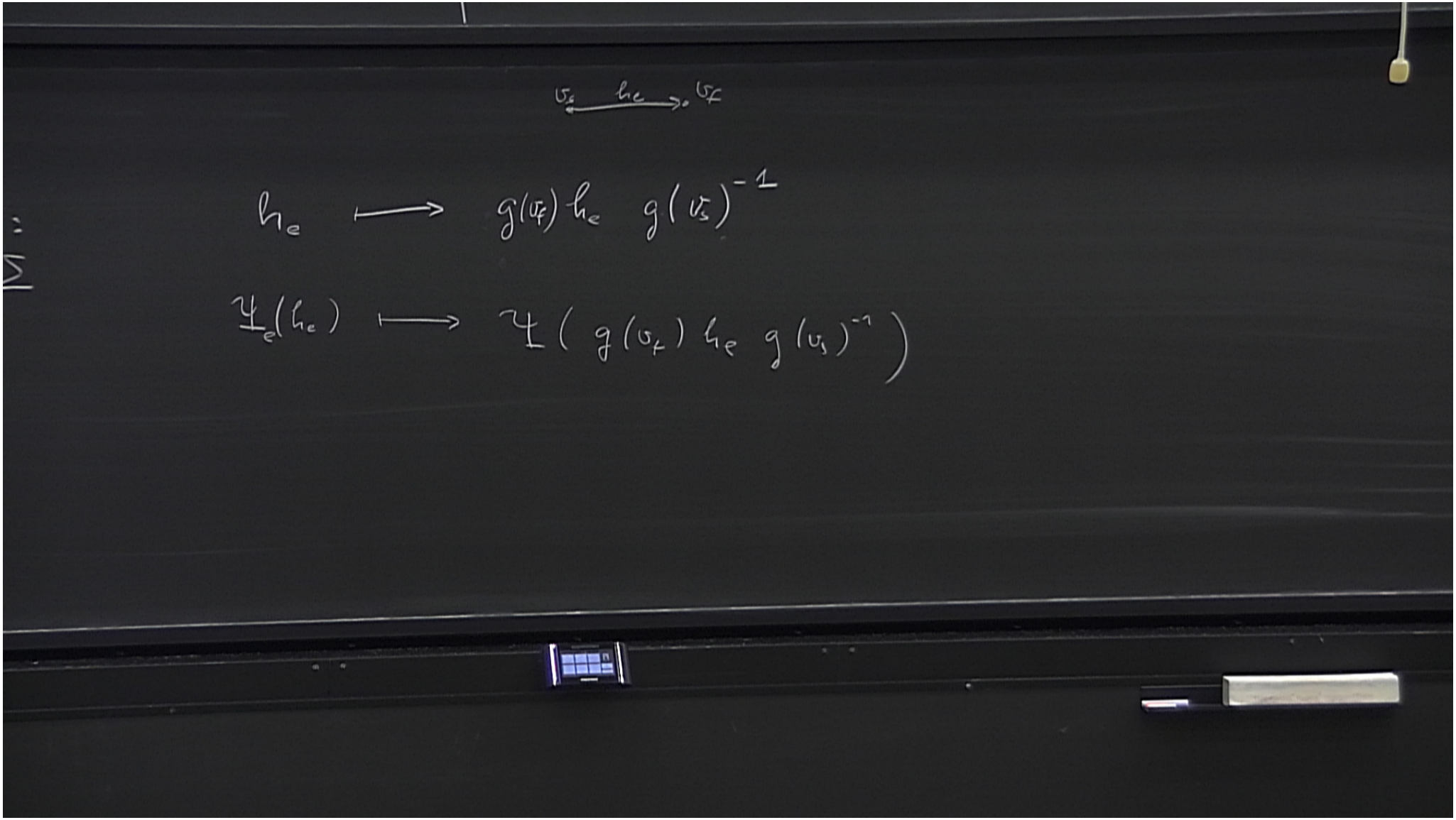
$$H \otimes H \simeq \mathcal{L}_2(\mathbb{R}^2, dx\,dy)$$

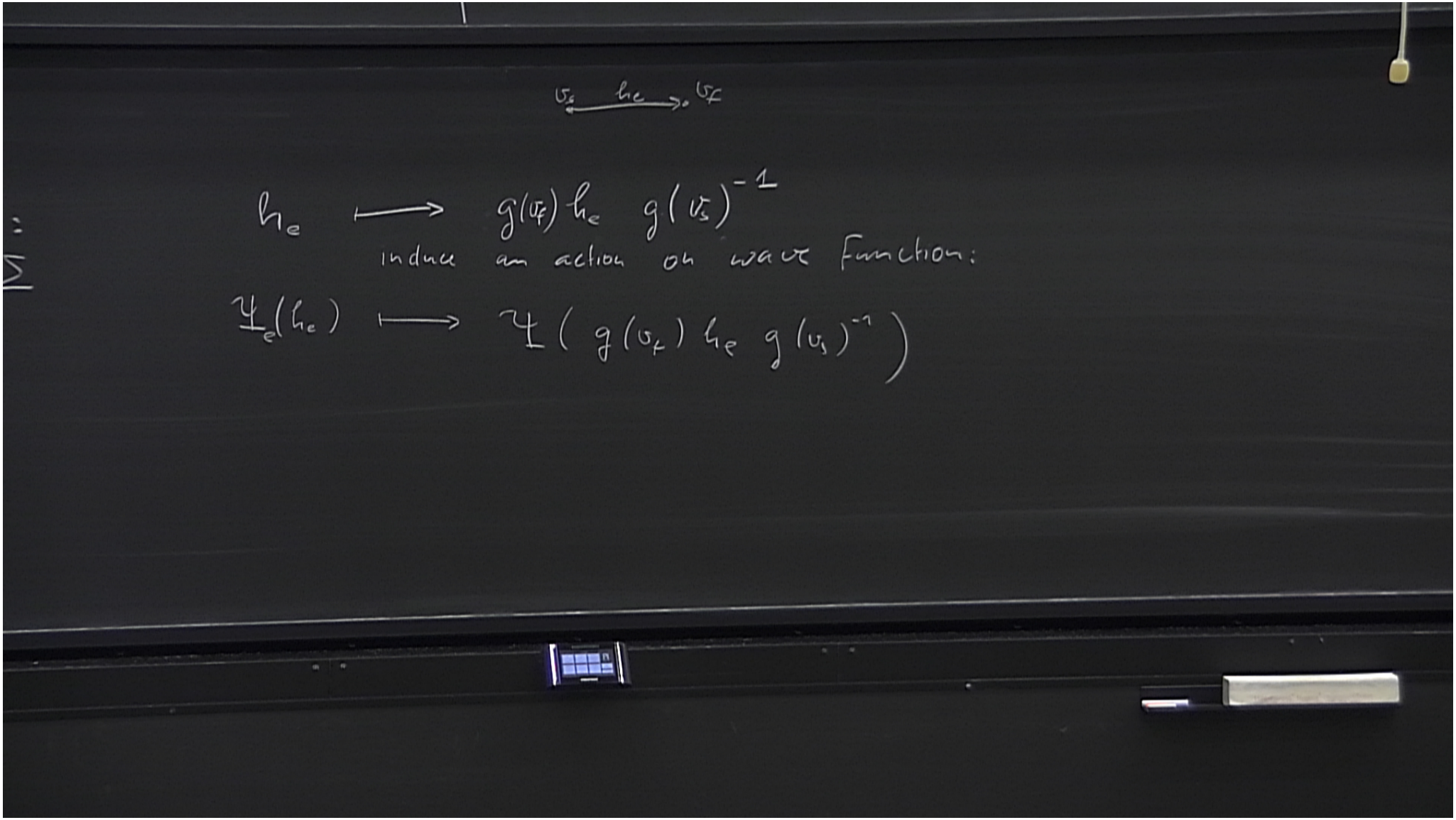
$$\rightarrow \quad \psi(g_1, g_2, \dots, g_n)$$

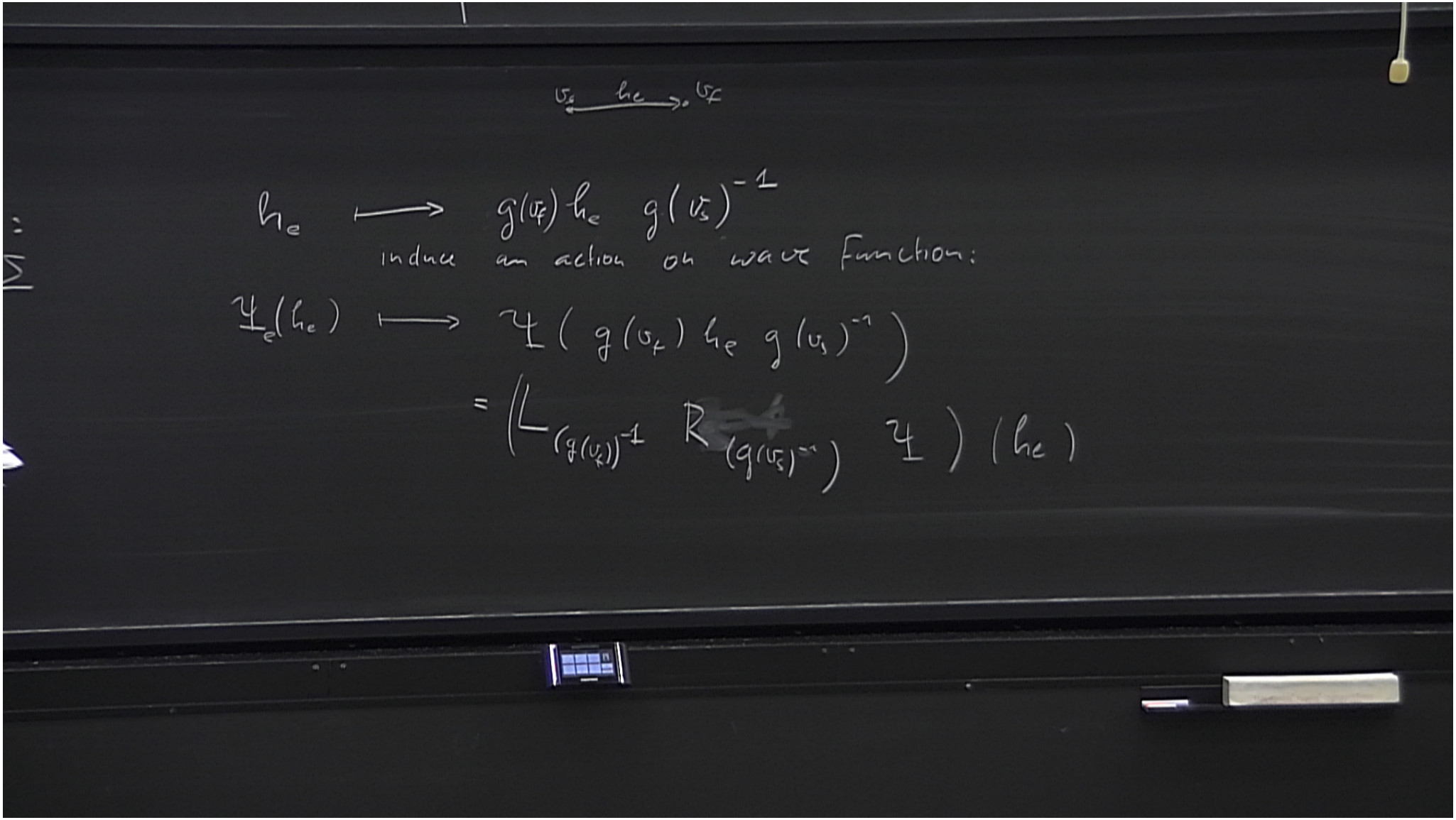
$$\langle \psi_1, \psi_2 \rangle_\Gamma = \int \overline{\psi_1(g_1, \dots, g_n)}\, \psi_2(g_1, \dots, g_n)\, dg_1 \cdots dg_n$$

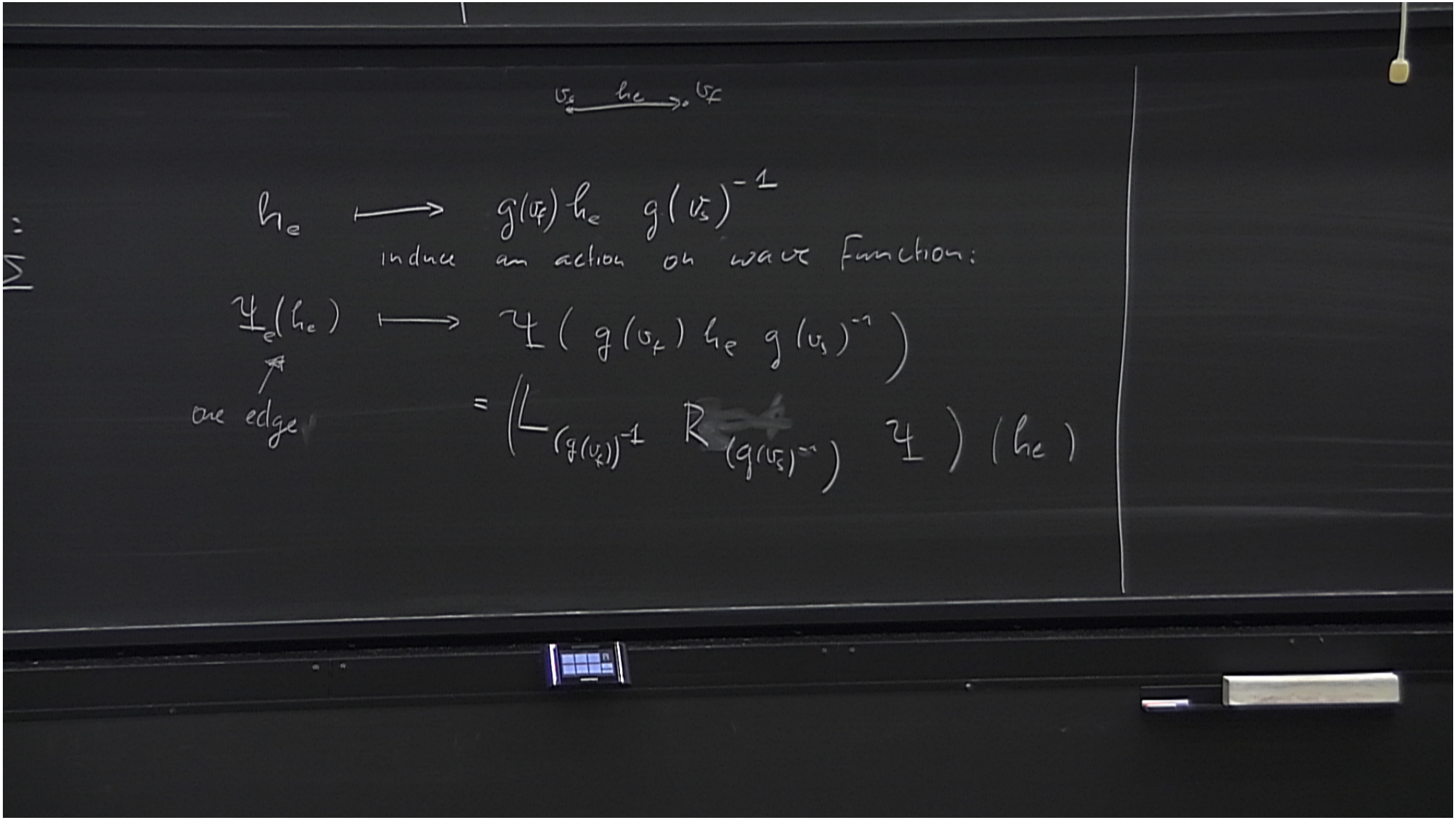

Gauß - constraints
generates SU(2) gauge trafo's :

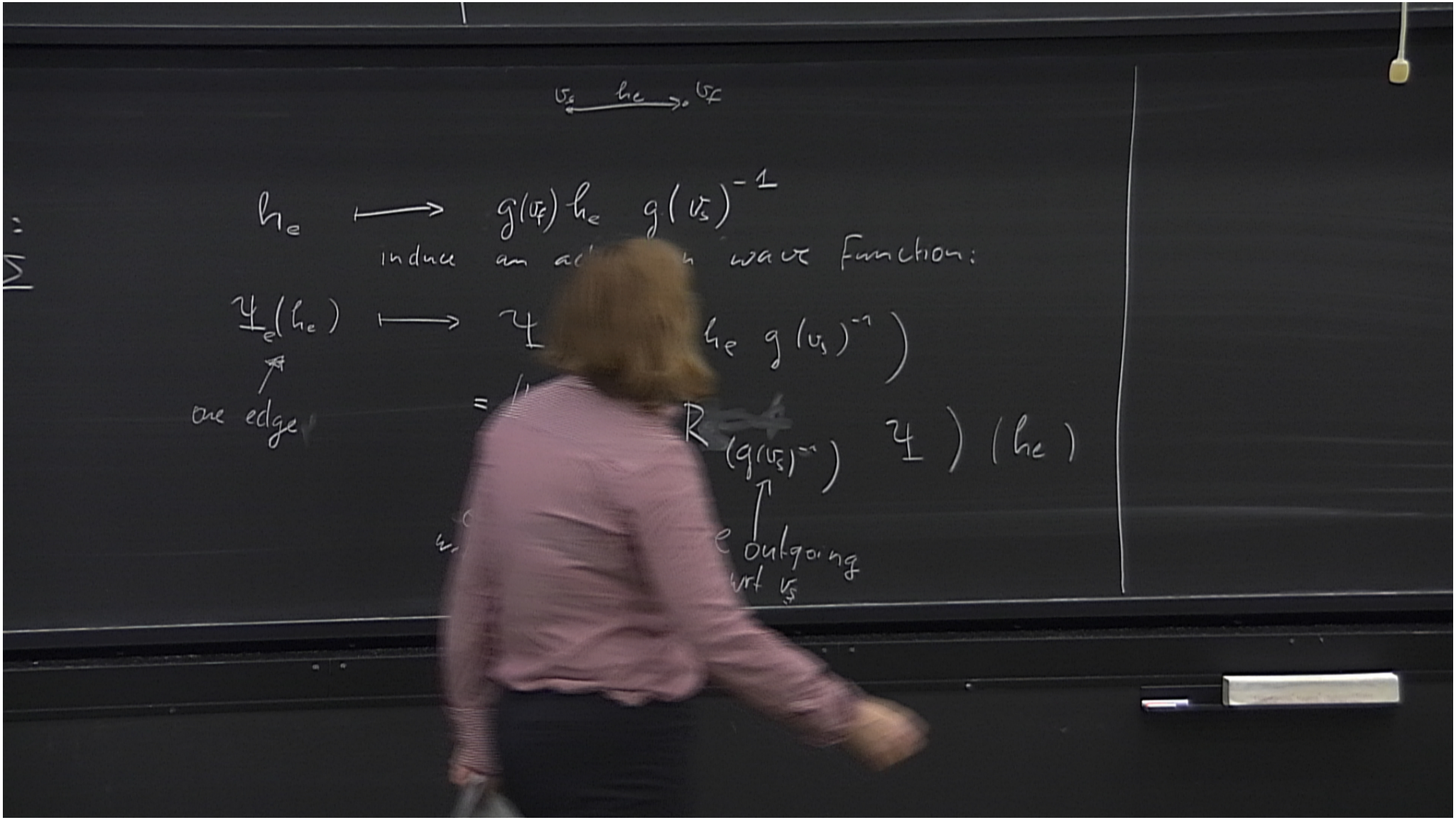

Gauß - constraints
generates SU(2) gauge trafo's :
g(x) , x ∈ Σ
h_e ⟶ g(v_f) h_e g(v_s)^{-1}
v_s h_e v_f


$v_s \xrightarrow{h_e} v_t$
$h_e \longmapsto g(v_t)\, h_e\, g(v_s)^{-1}$
Σ:
$\mathcal{Y}_e(h_e) \longmapsto \mathcal{Y}\left(g(v_t)\, h_e\, g(v_s)^{-1}\right)$


v_s h_e v_t
$h_e \longmapsto g(v_t)\, h_e\, g(v_s)^{-1}$
induce an action on wave function:
$\Psi_e(h_e) \longmapsto \Psi\left(g(v_t)\, h_e\, g(v_s)^{-1}\right)$


$v_s \xrightarrow{h_e} v_t$
$h_e \longmapsto g(v_t)\, h_e\, g(v_s)^{-1}$
induce an action on wave function:
$\Psi_e(h_e) \longmapsto \Psi\left(g(v_t)\, h_e\, g(v_s)^{-1}\right)$
$= \left(L_{(g(v_t))^{-1}} R_{(g(v_s)^{-1})}\, \Psi\right)(h_e)$


v_s h_e v_t
$h_e \longmapsto g(v_t)\, h_e\, g(v_s)^{-1}$
induce an action on wave function:
$\Psi_e(h_e) \longmapsto \Psi(g(v_t)\, h_e\, g(v_s)^{-1})$
one edge
$= (L_{(g(v_t))^{-1}} R_{(g(v_s)^{-1})} \Psi)(h_e)$


v_s h_e v_t
$h_e \longmapsto g(v_t)\, h_e\, g(v_s)^{-1}$
induce an ac n wave Function:
$\Psi_e(h_e) \longmapsto \Psi$ $h_e\, g(v_s)^{-1}$)
= /
R $(g(v_s)^{-1})$ Ψ) (h_e)
one edge
e outgoing
rt v_s

$v_s \xrightarrow{h_e} v_t$

$h_e \mapsto g(v_t)\, h_e\, g(v_s)^{-1}$

action on wave function:

$\psi(h_e) \mapsto \psi(g(v_t)\, h_e\, g(v_s)^{-1})$

$(L_{g(v_t)^{-1}} R_{g(v_s)^{-1}} \psi)(h_e)$

e outgoing wrt v_s

At one vertex v:

action of gauge trafo at v.

$\psi(h_1, \dots, h_n) \mapsto \left(\bigotimes_{e: v = v_f(e)} L_g\right) \otimes$

$v_s \xrightarrow{h_e} v_t$

$(v_t)\, h_e\, g(v_s)^{-1}$

action on wave function:

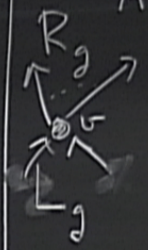
$\psi(g(v_t)\, h_e\, g(v_s)^{-1})$

$L_{(g(v_t))^{-1}}\ R_{(g(v_s)^{-1})}\ \psi)(h_e)$

following

e outgoing wrt v_s

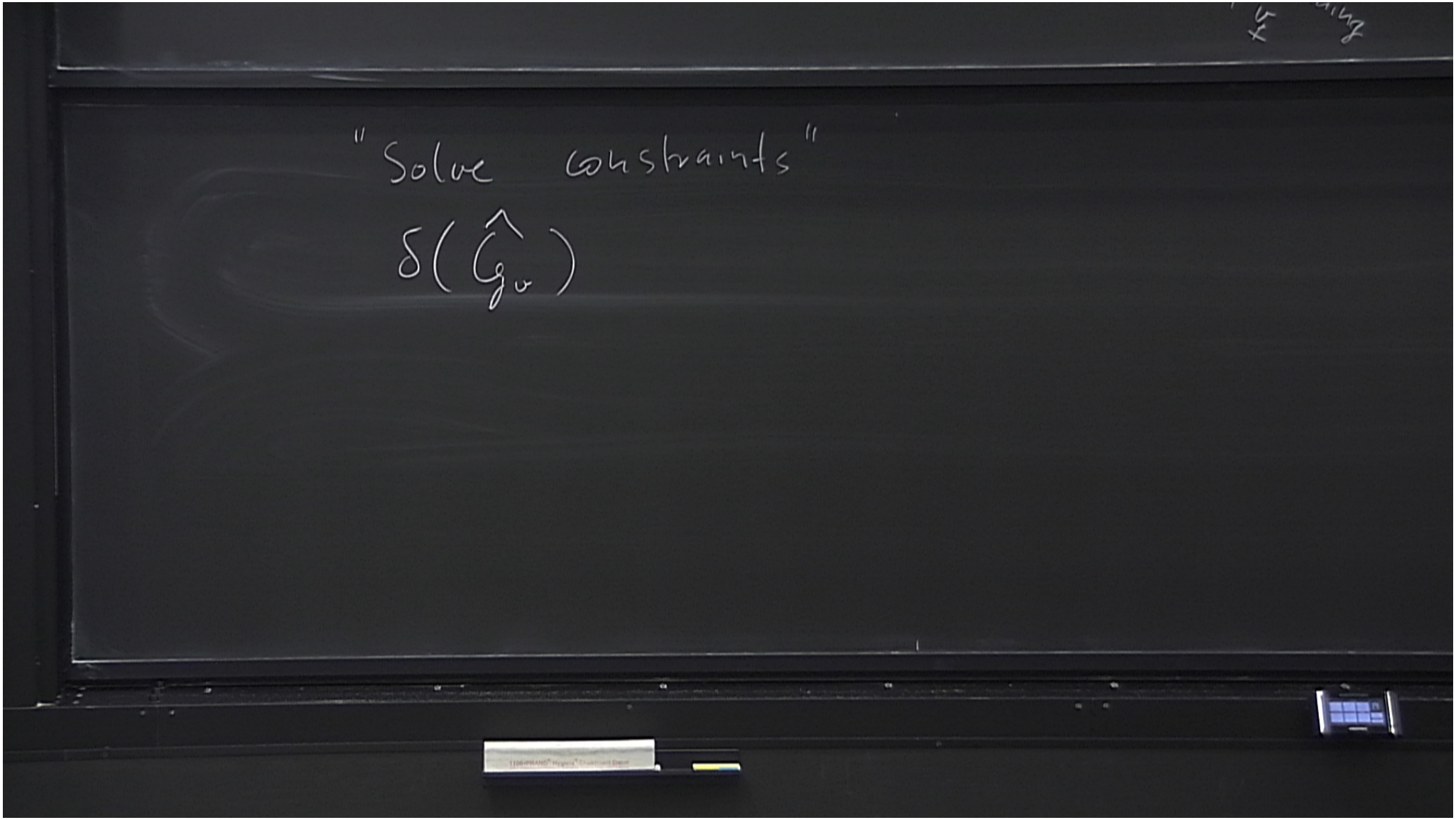
At one vertex v:



action of gauge trafo at v.

$\psi(h_1, \dots, h_n)$

$\mapsto \Big[\Big(\bigotimes_{e: v = v_t(e)} L_g\Big) \otimes \Big(\bigotimes_{e: v = v_s(e)} R_g\Big)\ \psi\Big](h_1, \dots, h_n)$


"Solve Constraints"
$\delta(\hat{G}_{\mu\nu})$

...nts

...uge trafo's :
$g(x)$, $x \in \Sigma$

$v_s \xrightarrow{h_e} v_t$

$$h_e \longmapsto g(v_t)\, h_e\, g(v_s)^{-1}$$

induce an action on wave function:

$$\psi_e(h_e) \longmapsto \psi\left(g(v_t)\, h_e\, g(v_s)^{-1}\right)$$

$$= \left(L_{(g(v_t))^{-1}}\, R_{(g(v_s)^{-1})}\, \psi\right)(h_e)$$

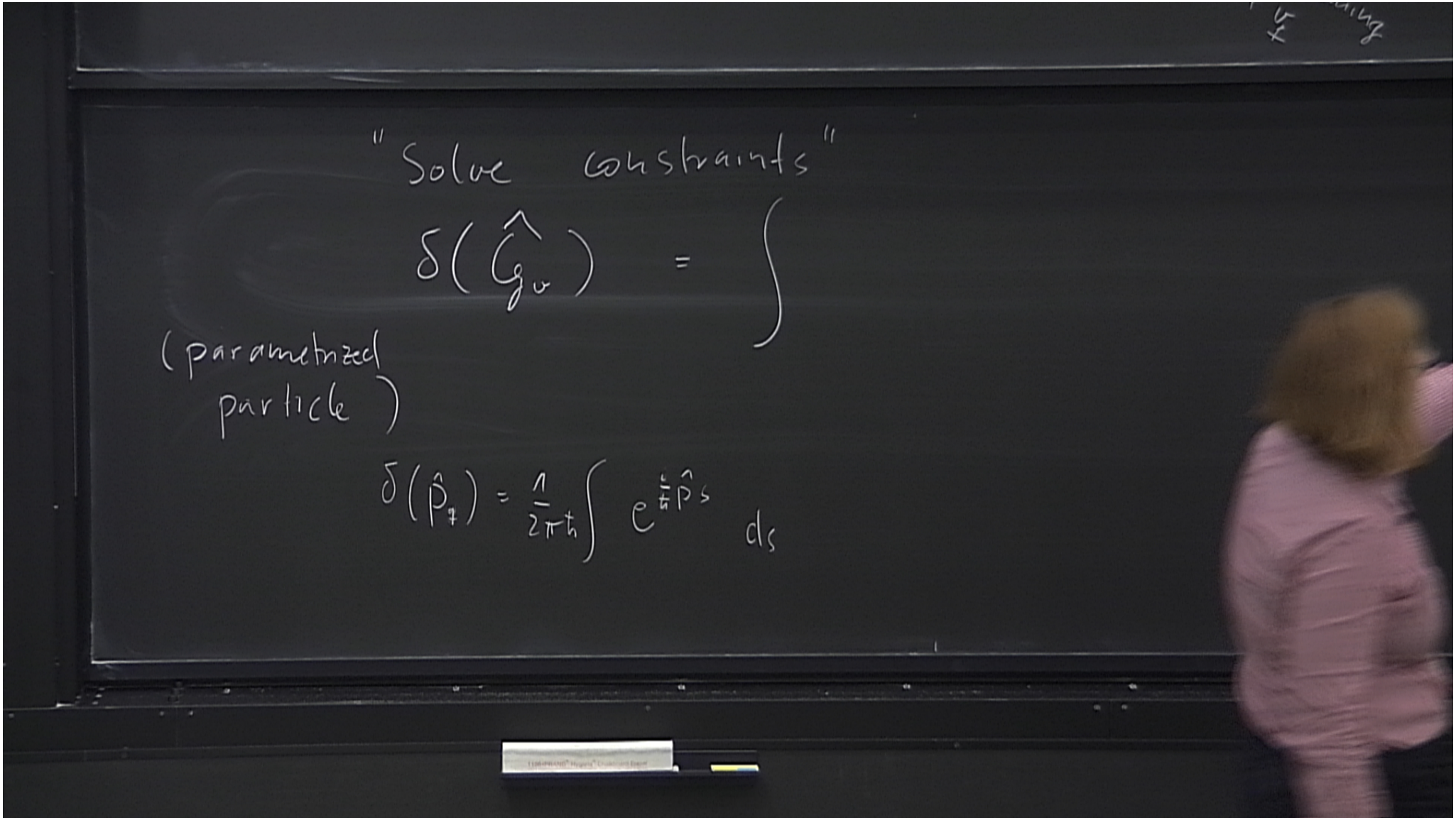
ψ_e: one edge

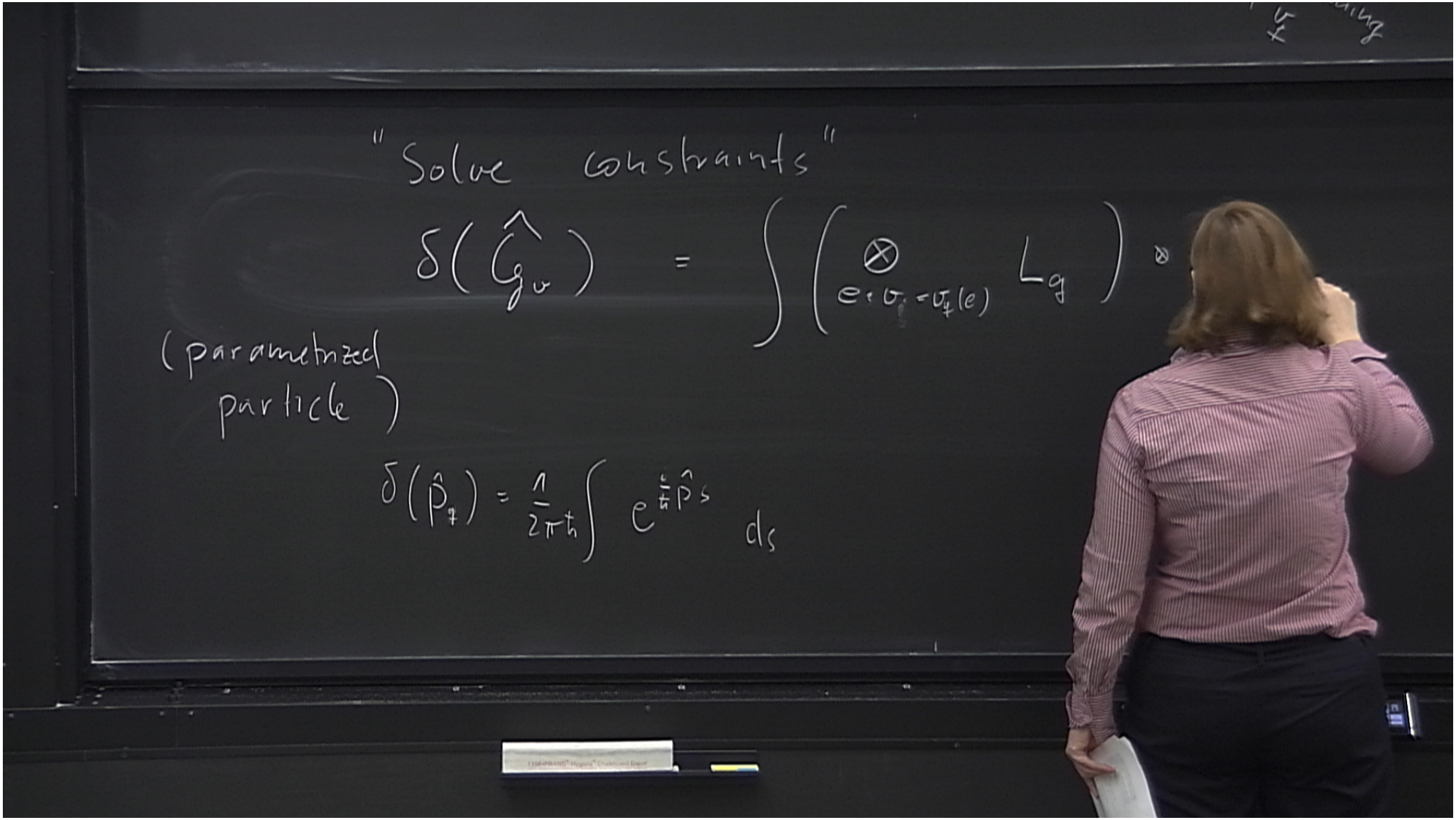
e incoming w.r.t v_t

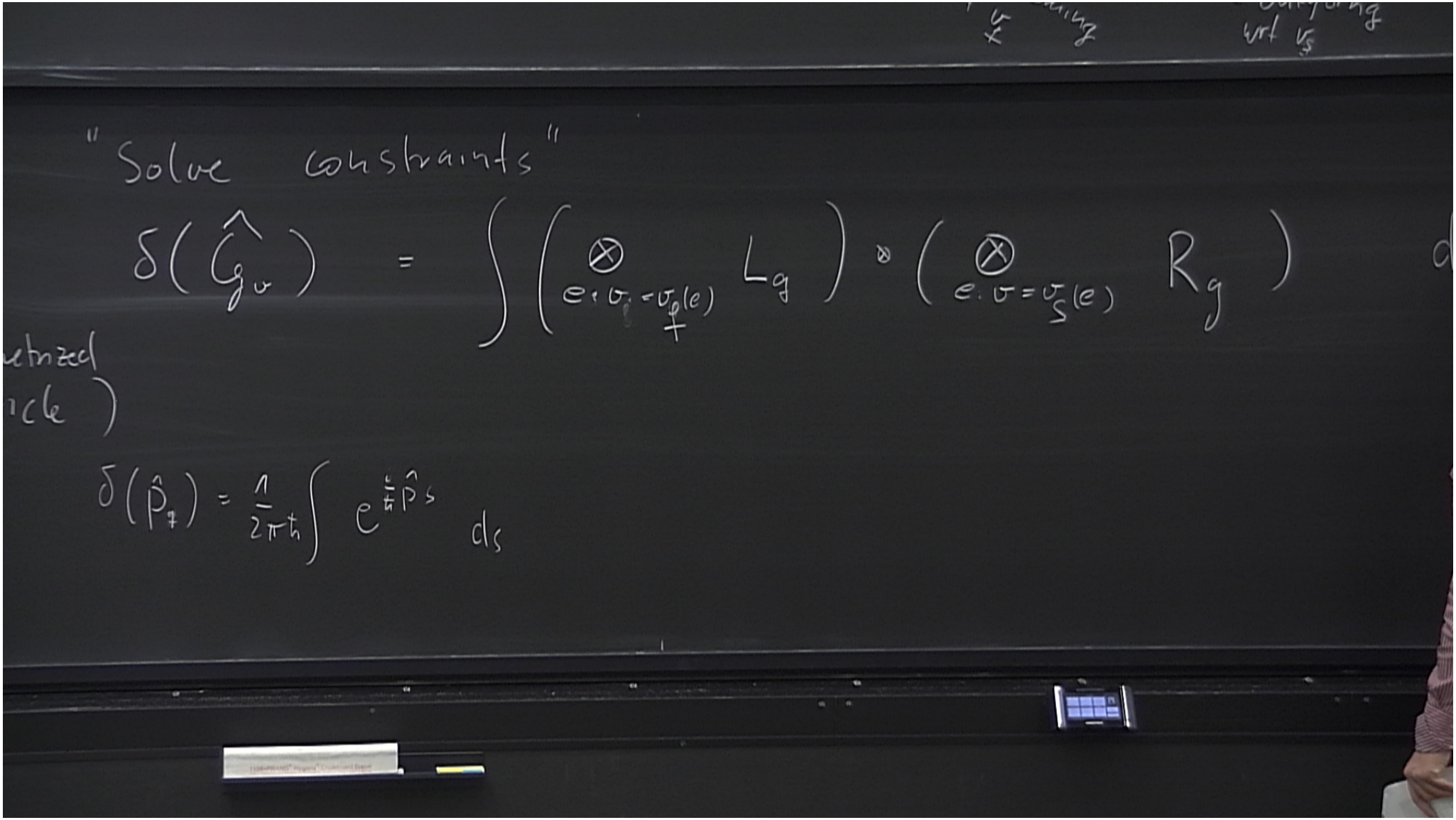
e outgoing wrt v_s

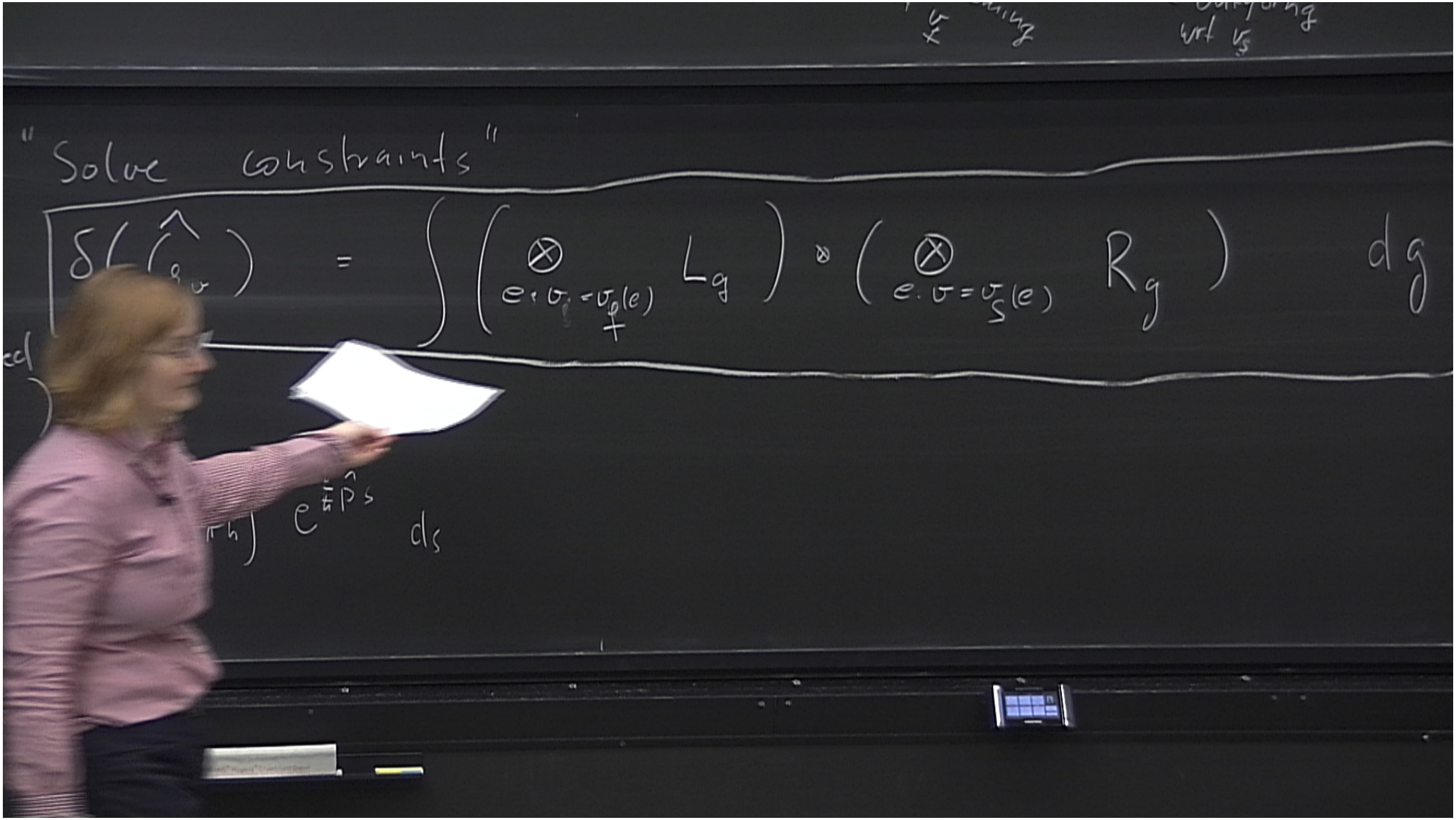


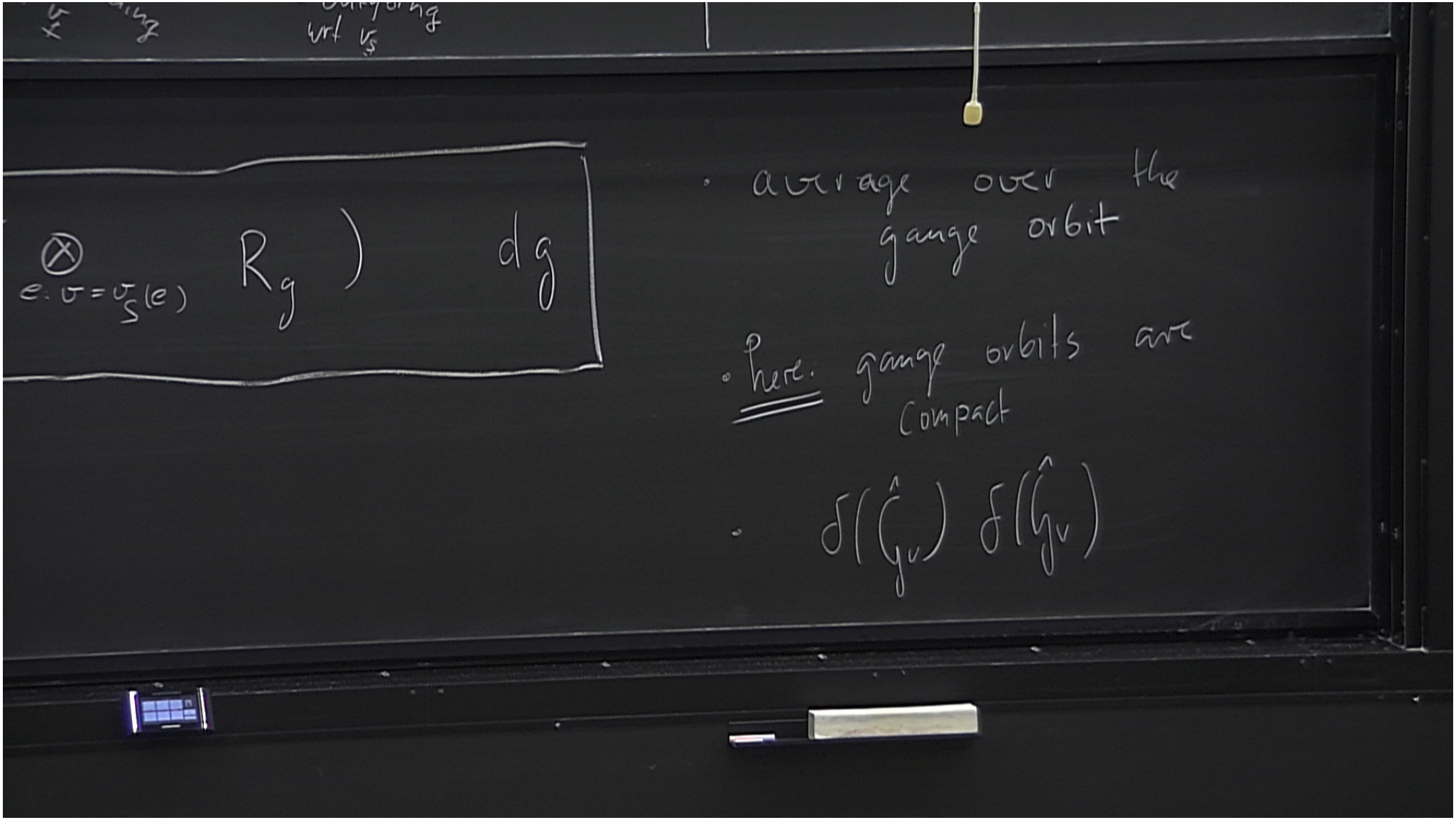
...onstraints"

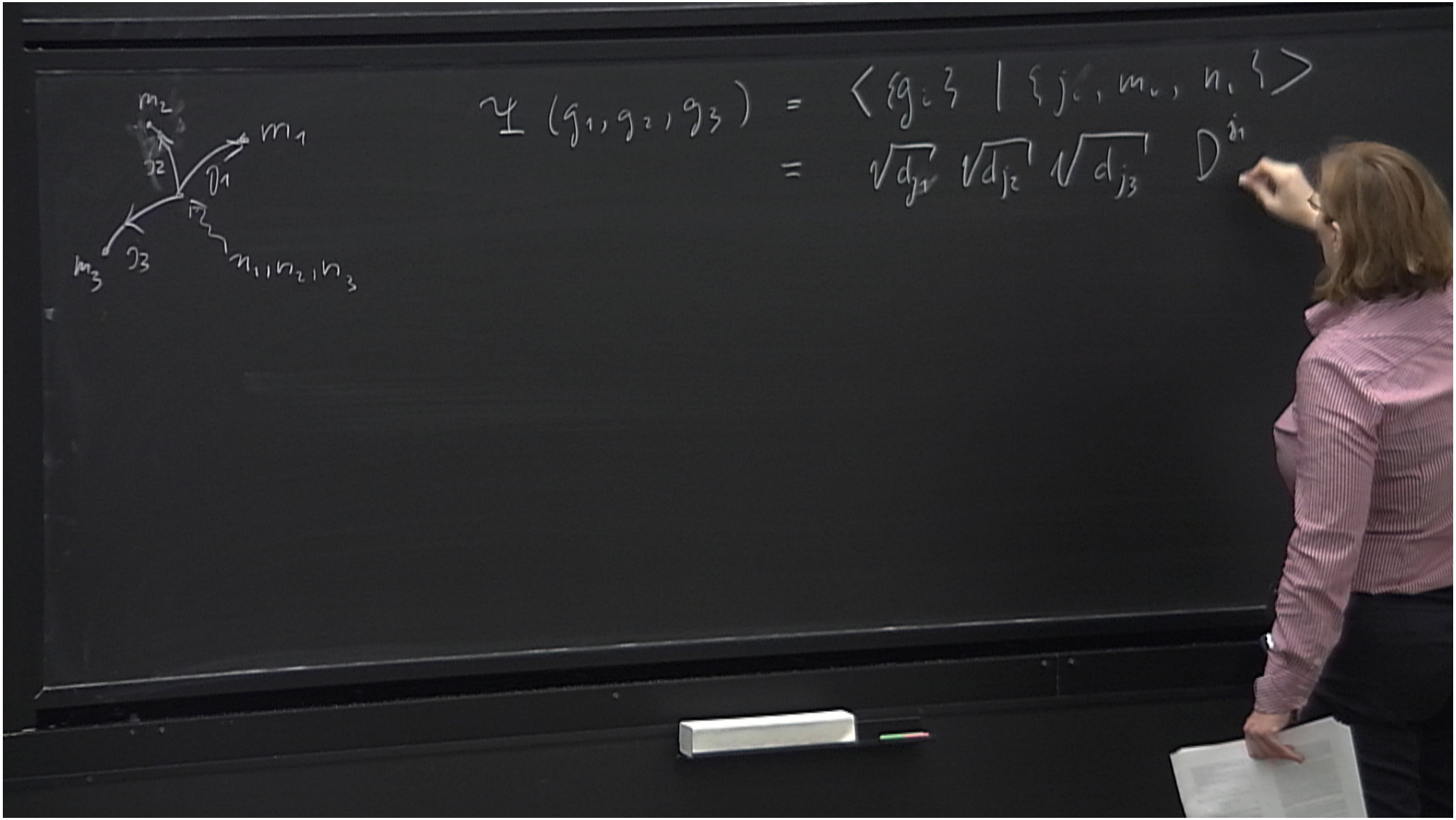

"Solve constraints"
$\delta(\hat{C}_\sigma) = \int$
(parametrized particle)
$\delta(\hat{p}_\tau) = \frac{1}{2\pi\hbar}\int e^{\frac{i}{\hbar}\hat{p}s}\, ds$

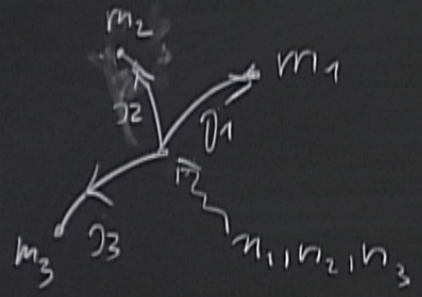

"Solve constraints"
$\delta(\hat{C}_v) = \int \left(\bigotimes_{e:v_i = v_t(e)} L_g \right) \otimes$
(parametrized particle)
$\delta(\hat{p}_t) = \frac{1}{2\pi\hbar} \int e^{\frac{i}{\hbar} \hat{p} s} \, ds$


"Solve constraints"
$\delta(\hat{G}_v) = \int \left(\bigotimes_{e: v_t(e)=v} L_g \right) \otimes \left(\bigotimes_{e: v_s(e)=v} R_g \right)$
$\delta(\hat{p}_\pi) = \frac{1}{2\pi\hbar} \int e^{\frac{i}{\hbar}\hat{p}s}\, ds$


"Solve constraints"
$\delta(\hat{\quad}) = \int \left(\bigotimes_{e:v=v_t(e)} L_g \right) \otimes \left(\bigotimes_{e:v=v_s(e)} R_g \right) dg$
$e^{\pm i \hat{p}_s}$
ds


$\bigotimes_{e.v=v_S(e)} R_g \Big) \, dg$
• average over the gauge orbit
• Here. gauge orbits are compact
• $\delta(\hat{\zeta}_v)\,\delta(\hat{\zeta}_v)$

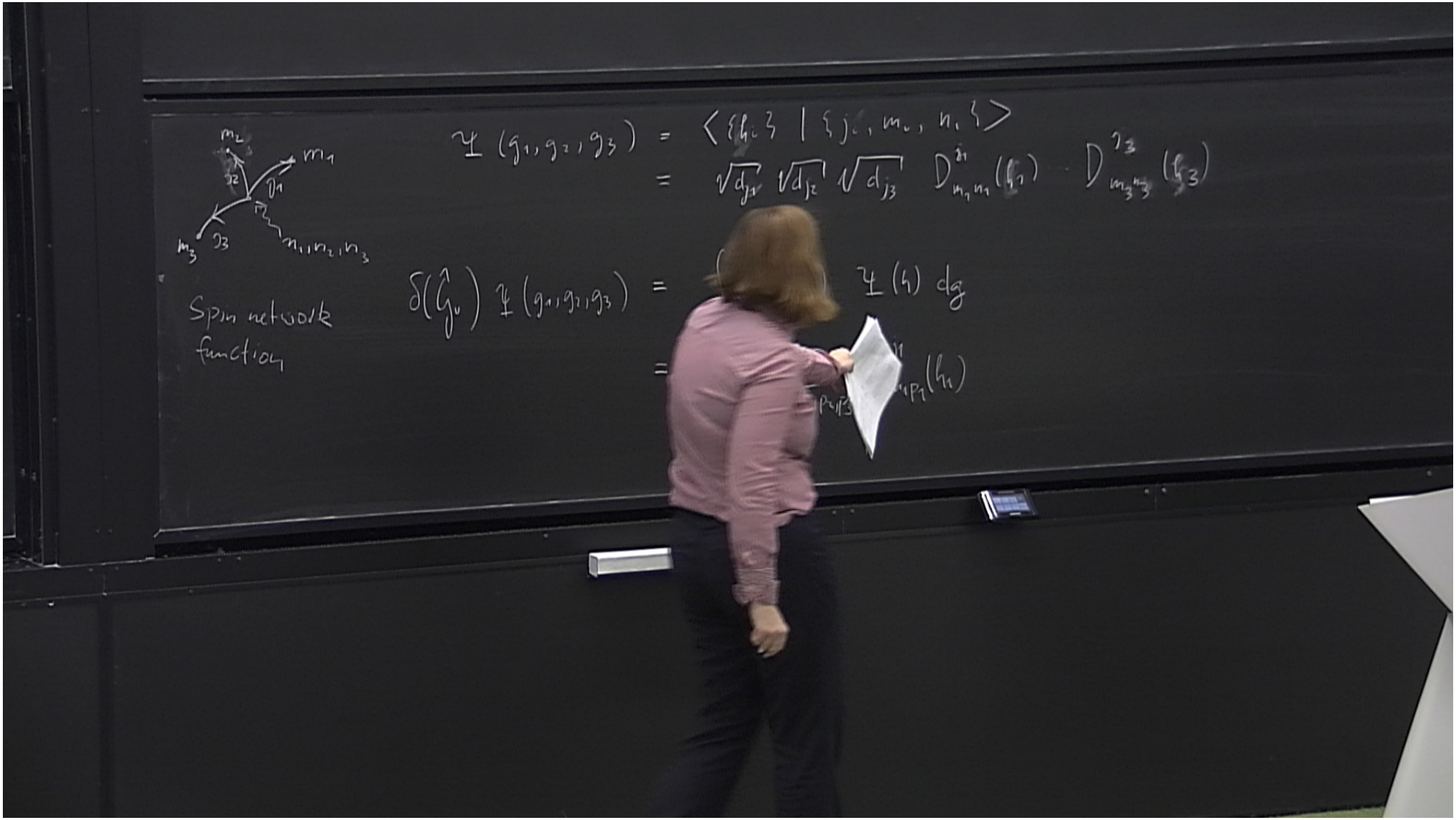

m_2
m_1
j_2
j_1
m_3
j_3
n_1, n_2, n_3
$\Psi(g_1, g_2, g_3) = \langle \{g_i\} | \{j_i, m_i, n_i\} \rangle$
$= \sqrt{d_{j_1}} \sqrt{d_{j_2}} \sqrt{d_{j_3}} D^{j_1}$

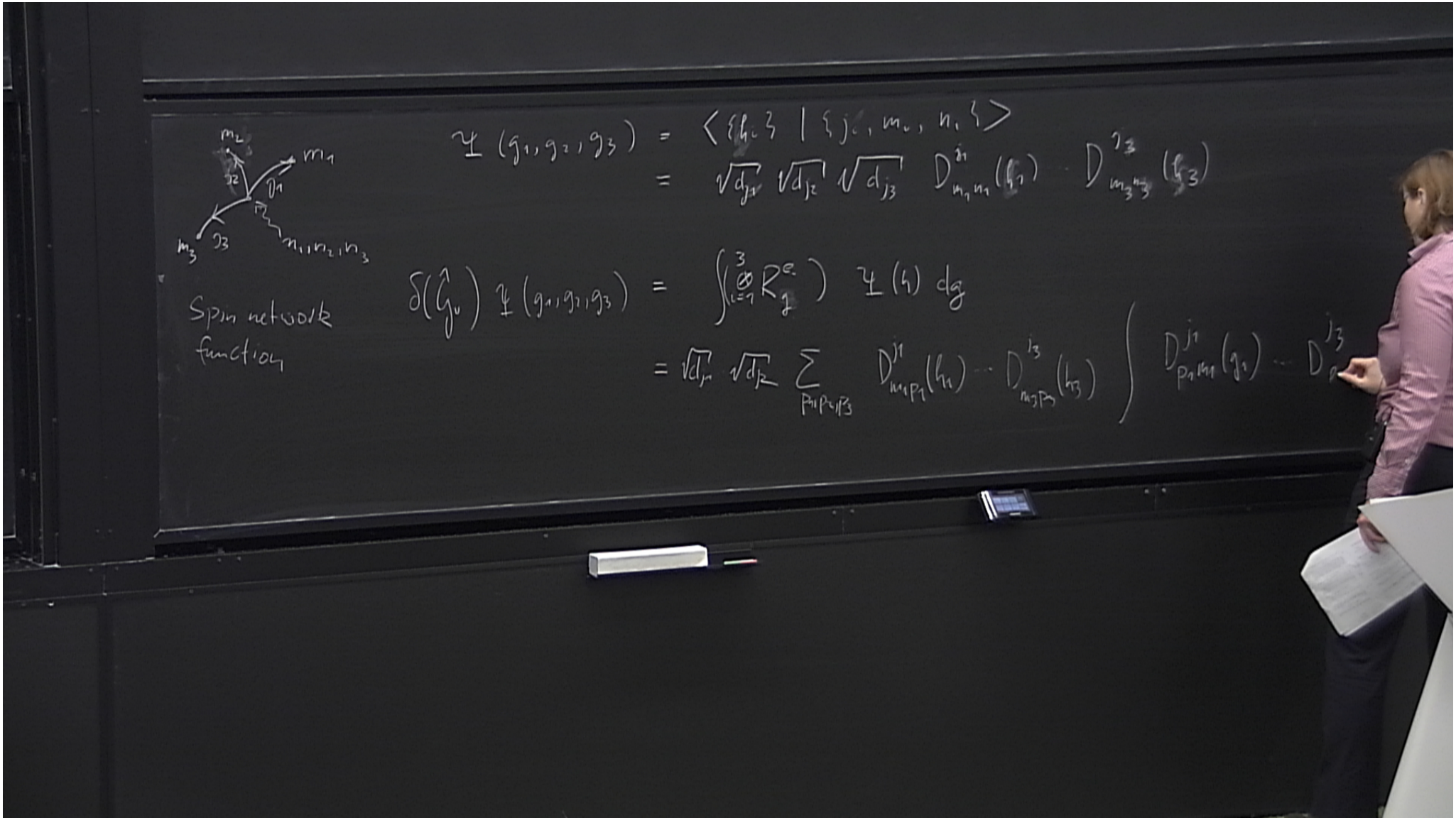


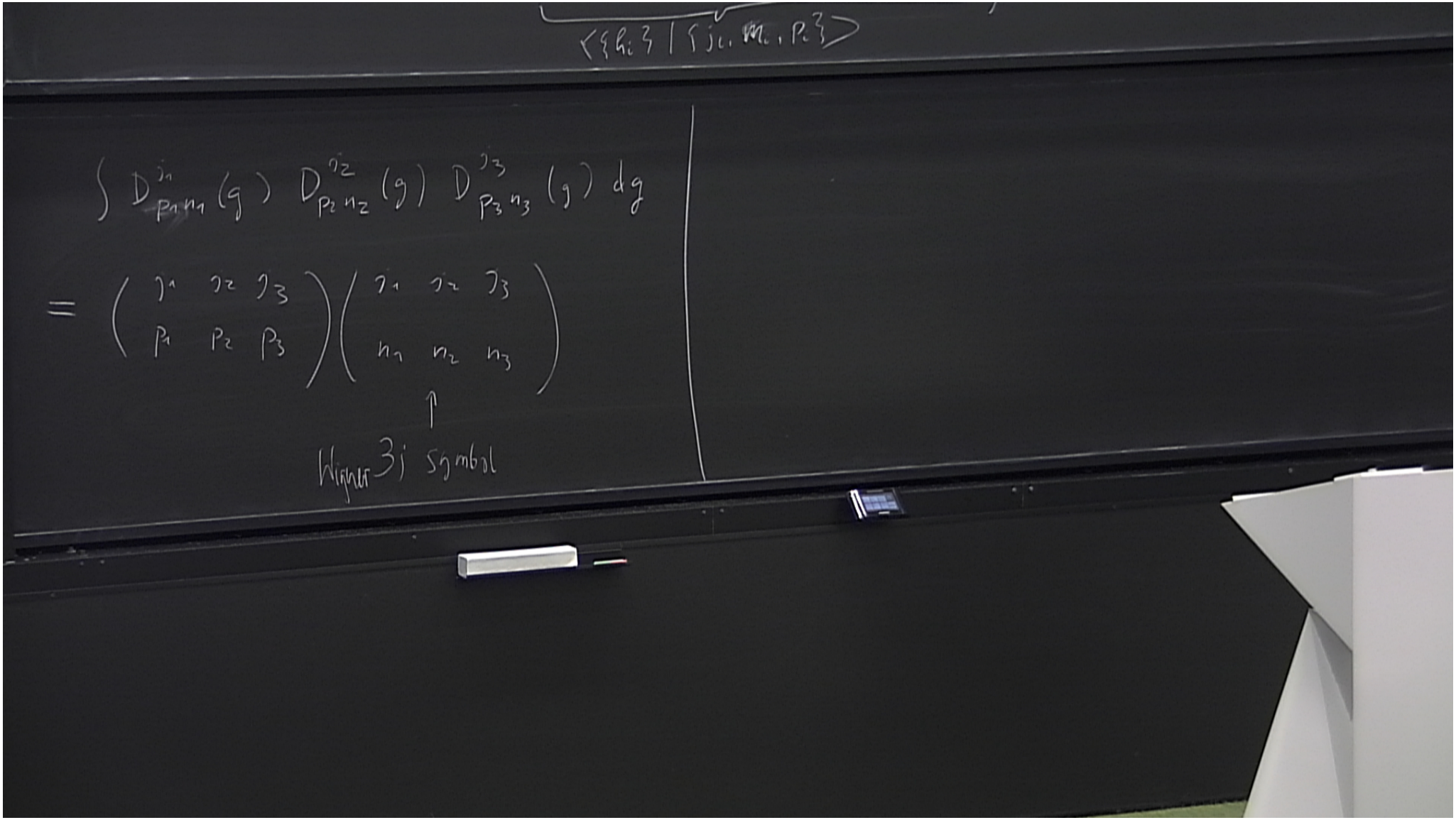
$$\Psi(g_1, g_2, g_3) = \langle \{g_i\} \mid \{j_v, m_v, n_v\} \rangle$$
$$= \sqrt{d_{j_1}}\sqrt{d_{j_2}}\sqrt{d_{j_3}}\, D^{j_1}_{m_1 n_1}(g_1)$$

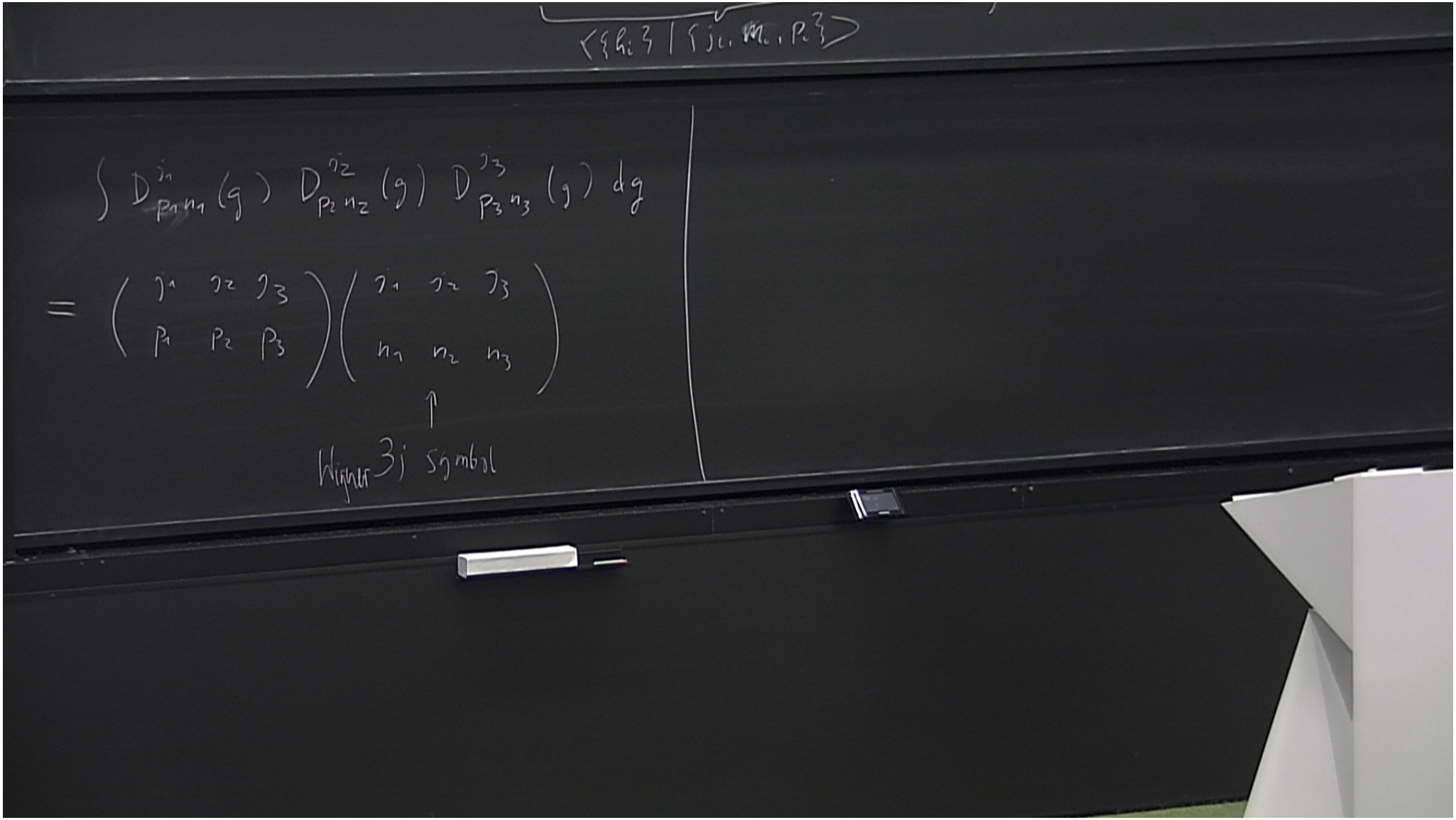
Spin network function

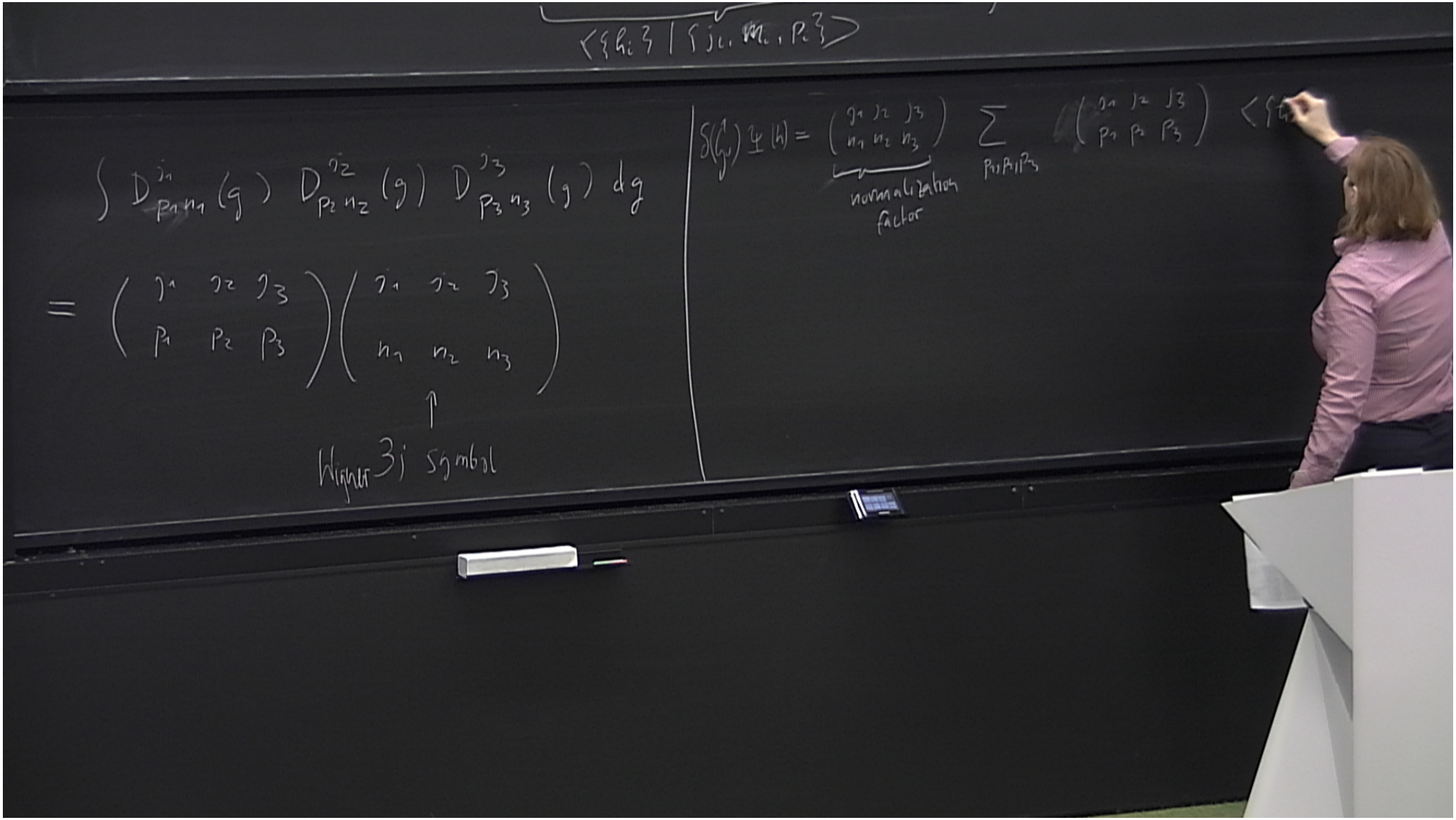
$$\delta(\hat{G}_v)\,\Psi(g_1, g_2, g_3) =$$

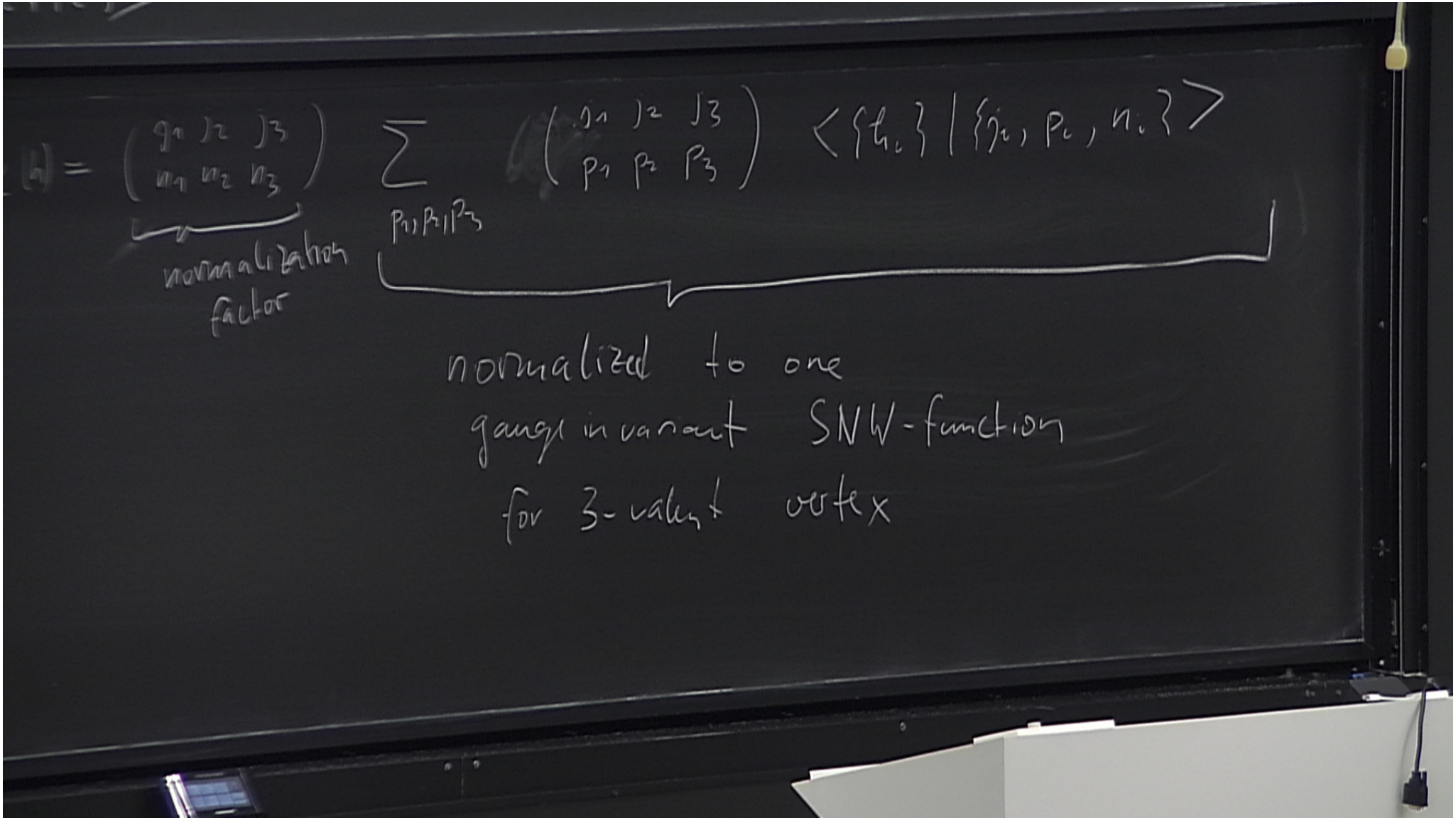

Spin network function

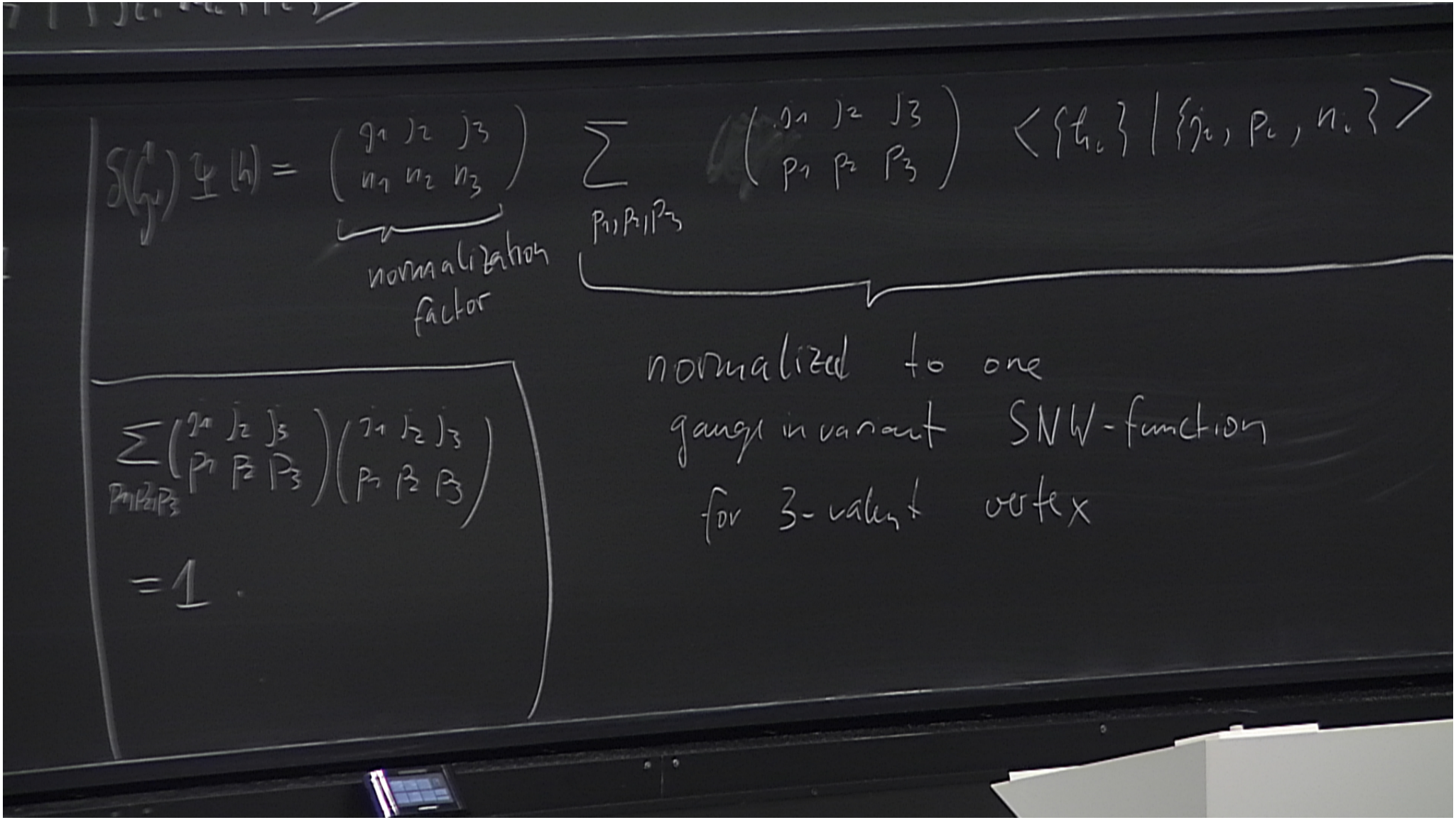

Spin network function

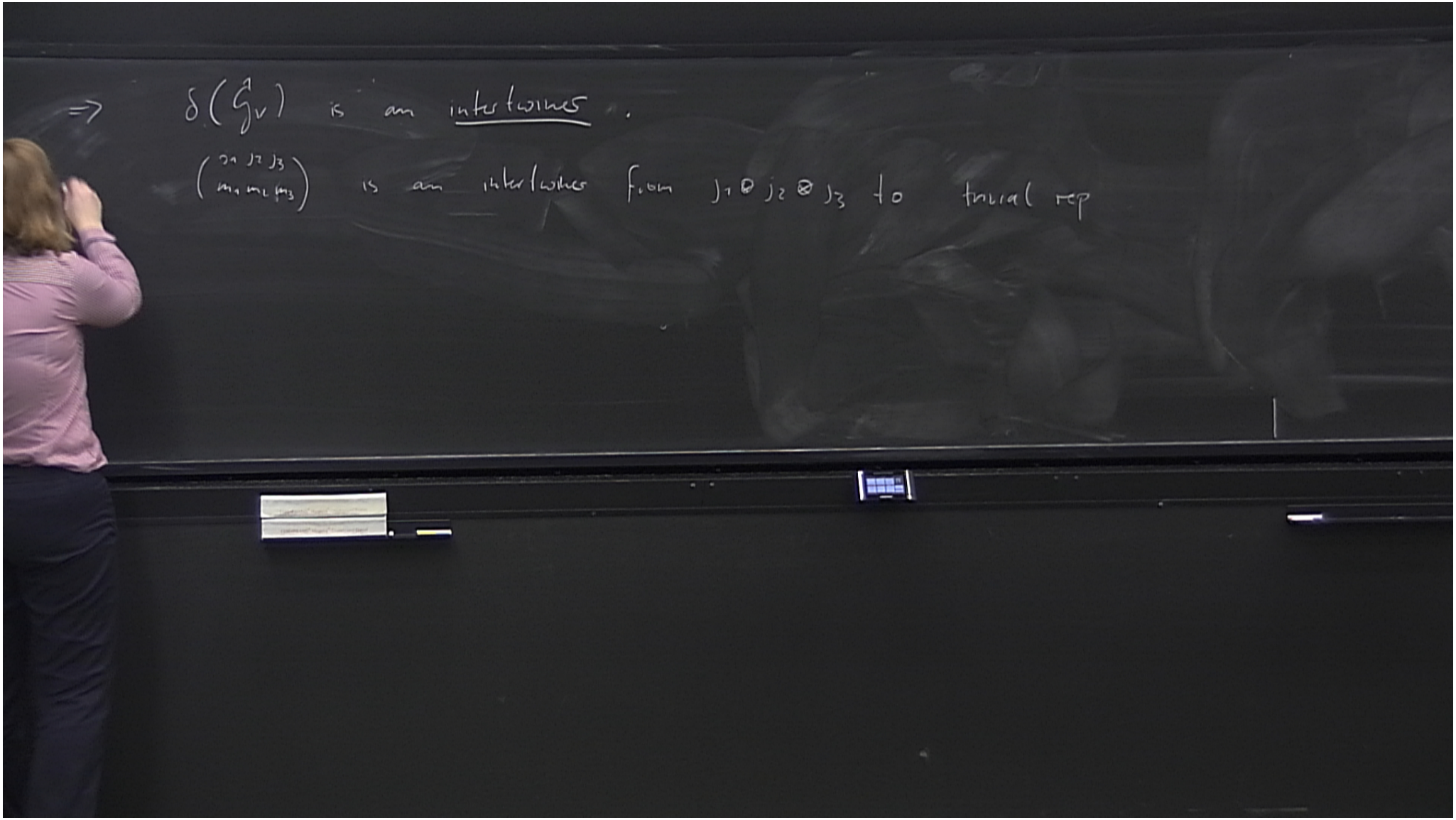

$\langle \{R_i\} | \{j_i, m_i, P_i\} \rangle$
$\int D^{j_1}_{P_1 n_1}(g)\, D^{j_2}_{P_2 n_2}(g)\, D^{j_3}_{P_3 n_3}(g)\, dg$
$= \begin{pmatrix} j_1 & j_2 & j_3 \\ P_1 & P_2 & P_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$
Wigner 3j symbol


$\langle \{R_i\} | \{j_i, m_i, P_i\} \rangle$
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Wigner 3j symbol
normalization factor
$\sum_{p_1 p_2 p_3}$

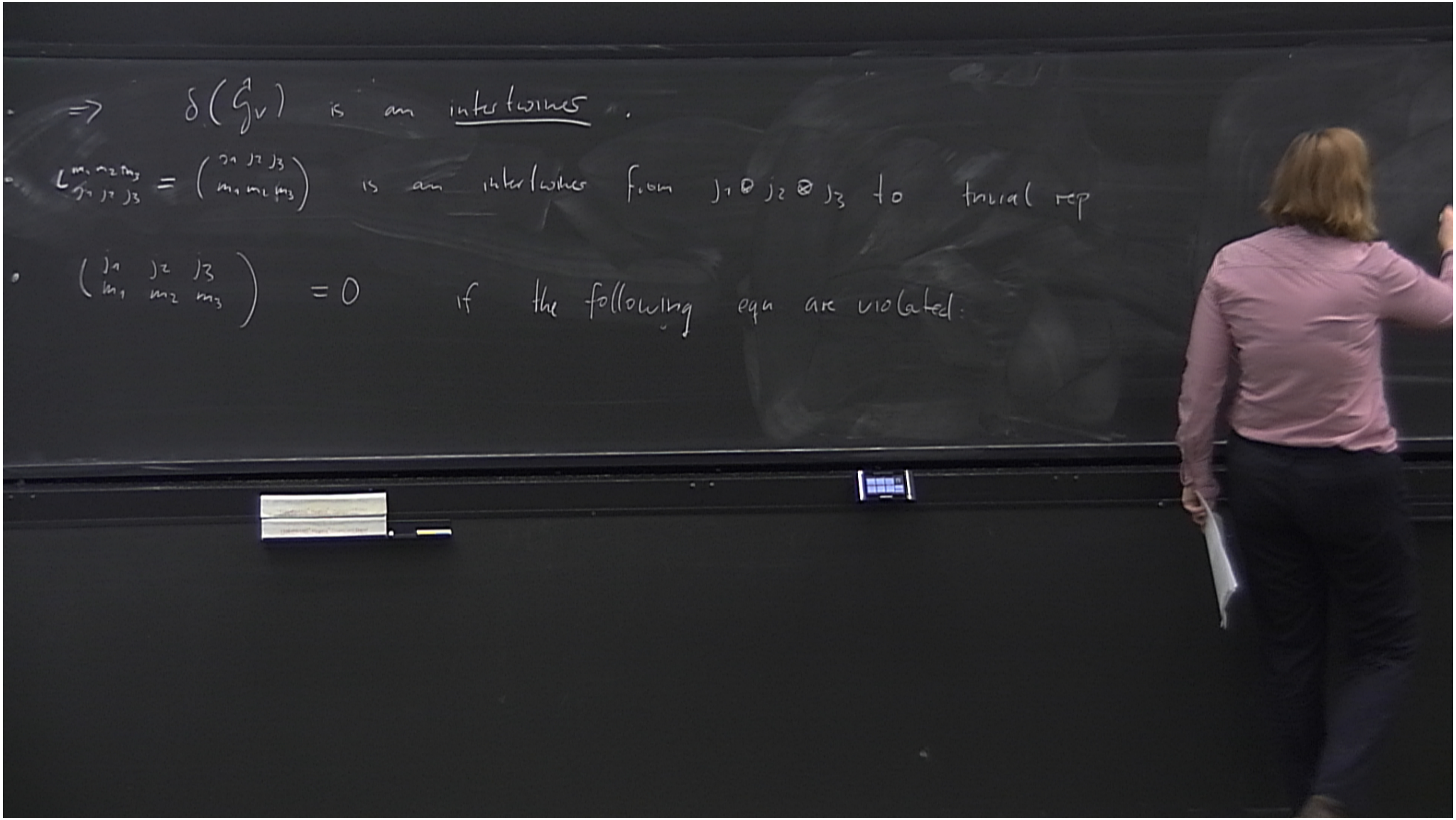

normalization factor
normalized to one
gauge invariant SNW-function
for 3-valent vertex

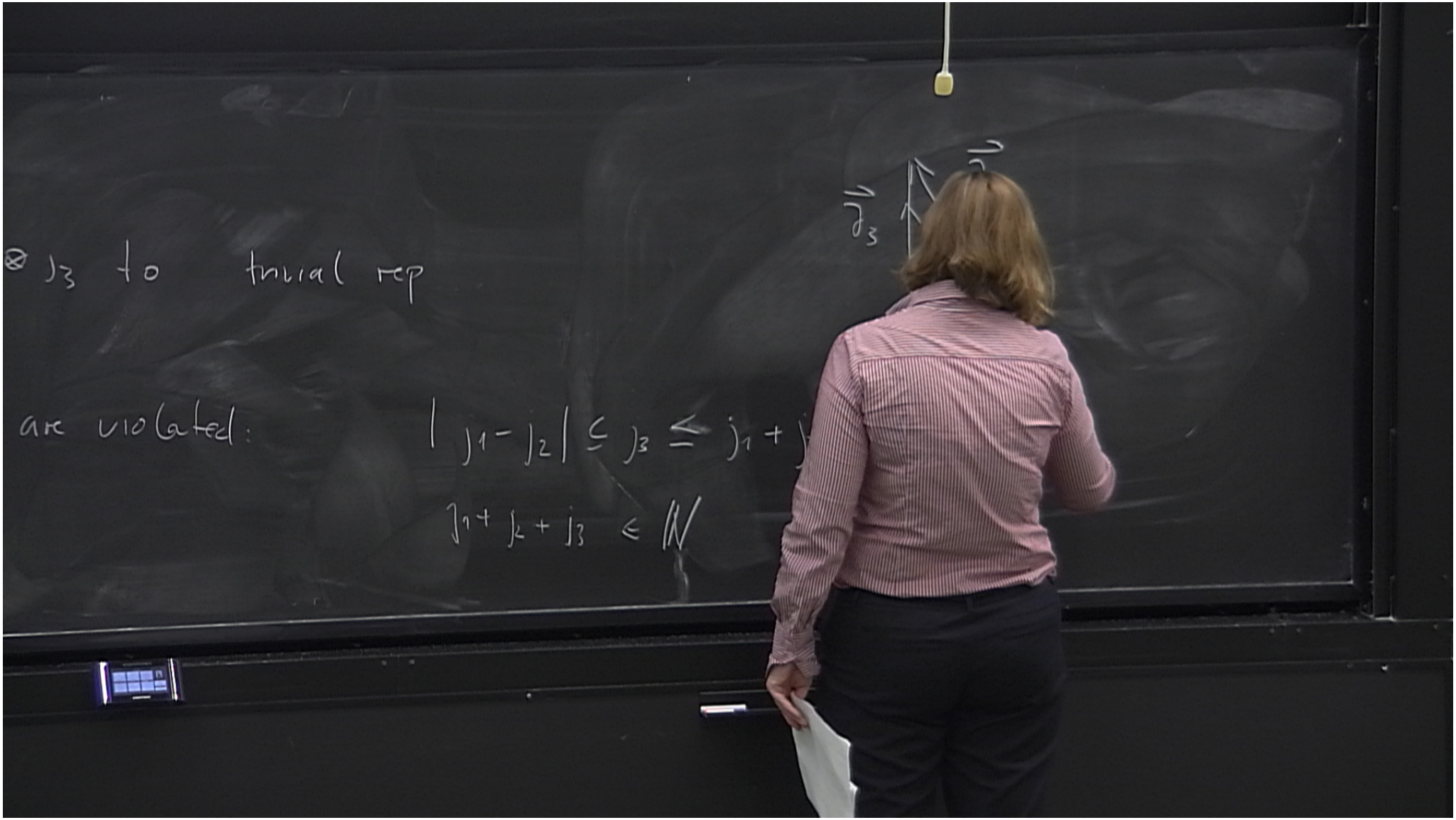

normalization factor
normalized to one
gauge invariant SNW-function
for 3-valent vertex
=1.

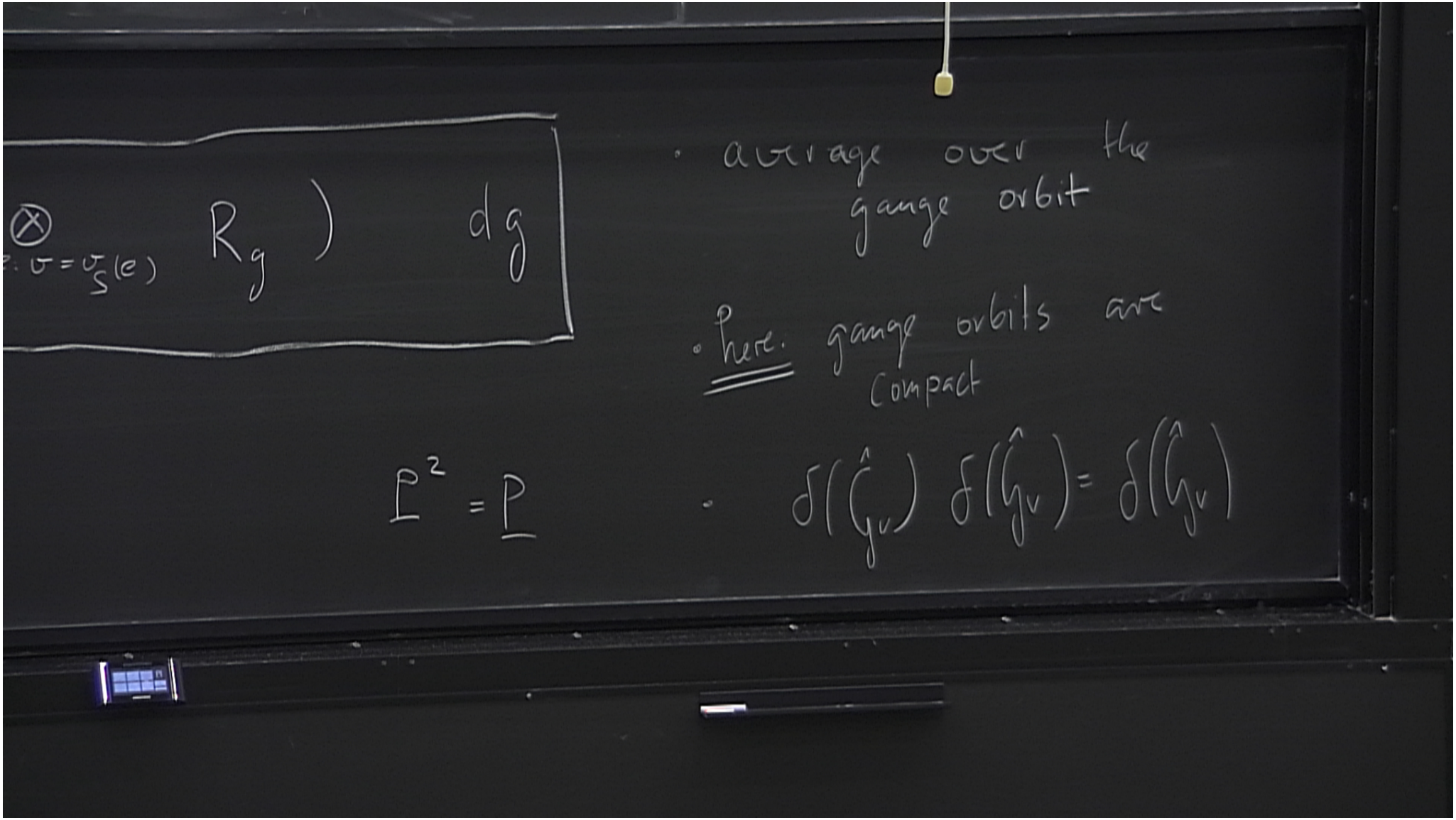

$\Rightarrow$ $\delta(\vec{\zeta}_v)$ is an intertwiner.
$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is an intertwiner from $j_1 \otimes j_2 \otimes j_3$ to trivial rep

$\Rightarrow$ $\delta(\hat{g}_v)$ is an intertwiner.

$L^{m_1 m_2 m_3}_{j_1 j_2 j_3} = \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is an intertwiner from $j_1 \otimes j_2 \otimes j_3$ to trivial rep


$\Rightarrow \quad \delta(\vec{\zeta}_v)$ is an intertwiner.
$L^{m_1 m_2 m_3}_{j_1 j_2 j_3} = \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is an intertwiner from $j_1 \otimes j_2 \otimes j_3$ to trivial rep
$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = 0$ if the following eqn are violated:


$\otimes j_3$ to trivial rep
are violated:
$|j_1 - j_2| \leq j_3 \leq j_1 +$
$j_1 + j_2 + j_3 \in \mathbb{N}$


$\otimes$
$v = v_S(e)$
R_g) dg
· average over the gauge orbit
· Here: gauge orbits are compact
$P^2 = P$
· $\delta(\hat{\zeta}_v)\,\delta(\hat{\zeta}_v) = \delta(\hat{\zeta}_v)$

$j_1 + j_2 + j_3 \in 2\mathbb{N}$

- Gauge-trafo's for vertex w/ outgoing edges

→ right translation

→ infinitesimal version: left invariant vector field

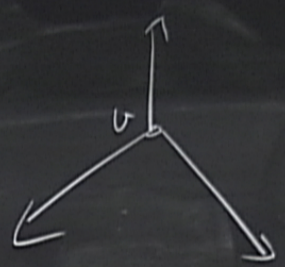
→ $\hat{E}^k_e = -i\hbar L^k$



$g_v =$

edges

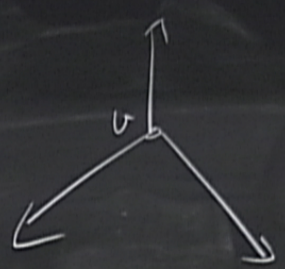
ant vector field



$$(\vec{g}_v)^k = (\vec{E}_{e_1})^k + (\vec{E}_{e_2})^k + (\vec{E}_{e_3})^k$$

edges

ant vector field

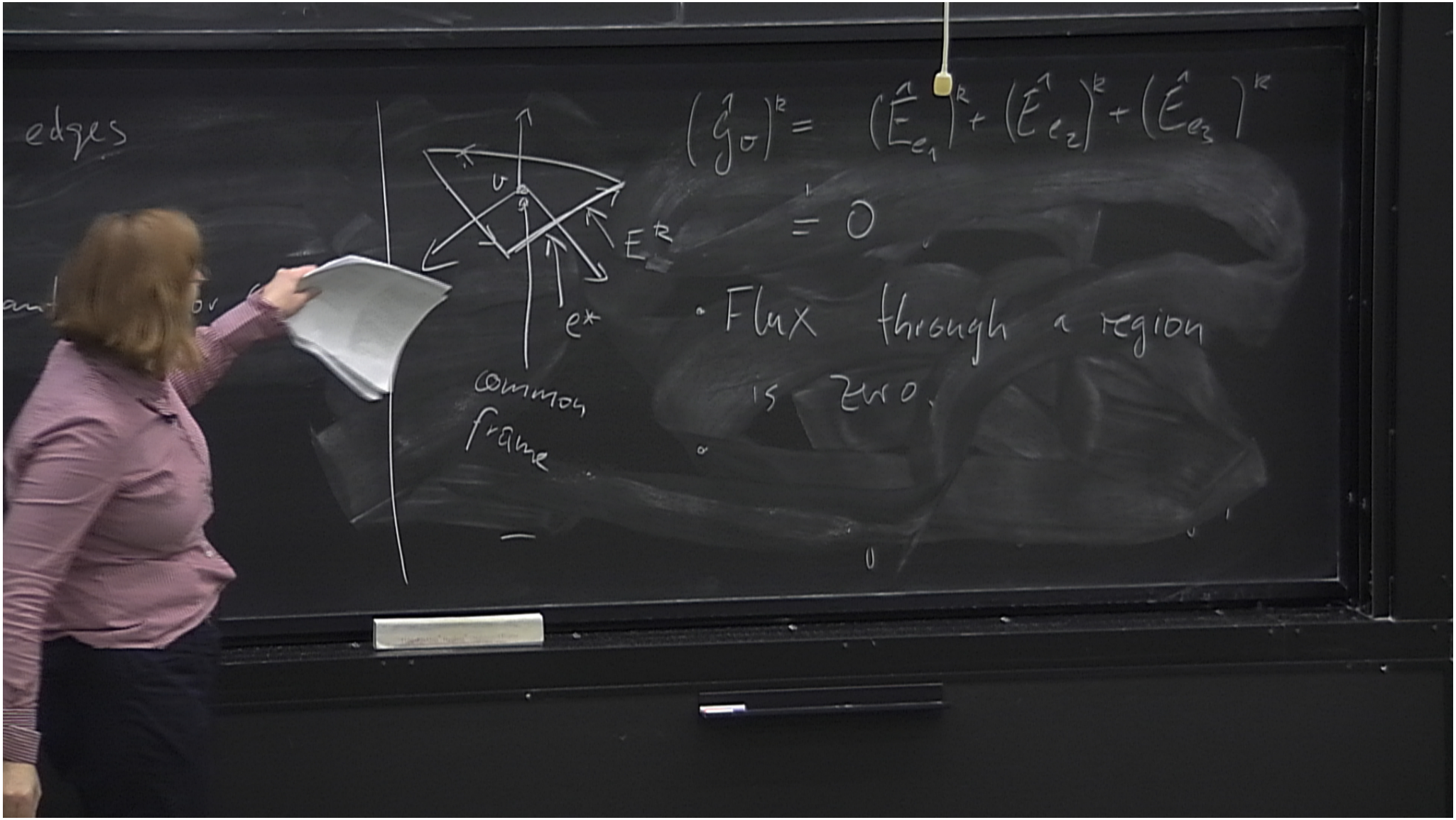


$$(\vec{g}_v)^2 = (\vec{E}_{e_1})^2 + (\vec{E}_{e_2})^2 + (\vec{E}_{e_3})^2$$

$$= 0$$

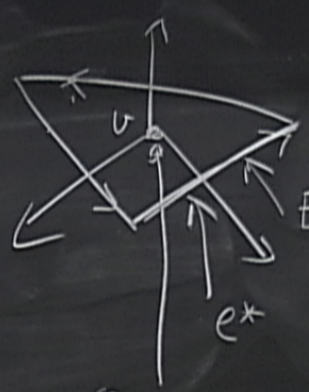
Flux through a region is zero.


edges
ant vector field
v
$(\vec{g}_v)^2 = (\vec{E}_{e_1})^2 + (\vec{E}_{e_2})^2 + (\vec{E}_{e_3})^2$
$= 0$
Flux through a region
is zero.


edges
v
g
E^*
e^*
common
frame
$(\hat{g}_v)^* = (\hat{E}_{e_1})^* + (\hat{E}_{e_2})^* + (\hat{E}_{e_3})^*$
$= 0$
• Flux through a region
is zero.

edges

ant vector field



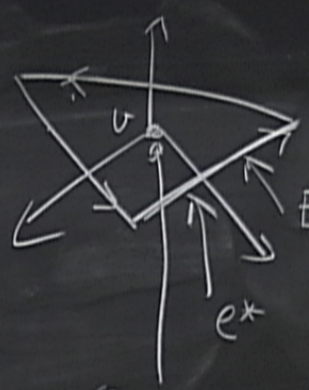
$$(\hat{g}_v)^k = (\hat{E}_{e_1})^k + (\hat{E}_{e_2})^k + (\hat{E}_{e_3})^k$$

$$= 0$$

- Flux through
- is zero.
- triangle is

edges

ant vector field



$$(\vec{g}_v)^k = (\vec{E}_{e_1})^k + (\vec{E}_{e_2})^k + (\vec{E}_{e_3})^k$$

$$= 0$$

- Flux through a region is zero.
- triangle is "closed".