

Title: 12/13 PSI - Quantum Gravity Review Lecture 13

Date: Feb 13, 2013 10:15 AM

URL: <http://pirsa.org/13020056>

Abstract:

Hilbert space for one edge

$$\{E_e^j, h_e\}$$



## Hilbert space for one edge

$$\{E_e^j, (h_e)_{mn}\} = (h_e T^j)_{mn}$$

$$\{E_e^j, E_e^k\} = \epsilon^{jkl} E_e^l$$

$$\{(h_e)_{mn}, (h_e)_{m'n'}\} = 0$$

$$\cdot \mathcal{H}_e = L_2(\underbrace{G}_{\text{SU}(2)}, dg)$$

• Peter-Weyl:  
 (semi-simple  
 compact  
 Lie-groups)

basis: matrix elements  
 of irreducible unitary  
 representations of  $G = \text{SU}(2)$



- $\mathcal{H}_e = L_2(\underbrace{G}_{\text{SU}(2)}, dg)$

- Peter-Weyl: (semi-simple compact Lie-groups)
- basis: matrix elements of irreducible unitary representations of  $G = \text{SU}(2)$
- Spin  $j$ -rep.

- $\mathcal{H}_e = L_2(\underbrace{G}_{\text{SU}(2)}, dg)$

- Peter-Weyl: (semi-simple compact Lie-groups)
- basis: matrix elements of irreducible unitary representations of  $G = \text{SU}(2)$
- Spin  $j$ -rep.







$$= \int_{SU(2)} \mathcal{L}_2(G, dg)$$

gl: - basis matrix elements  
of irreducible unitary  
representations of  $G = SU(2)$   
- spin  $j$ -rep.

$$\int D_{mn}^j(g) D_{m'n'}^{j'}(g) dg$$

$$= \frac{1}{d_j} \delta^{jj'} \delta_{mm'} \delta_{nn'}$$

$$\langle g | j, m, n \rangle = \sqrt{d_j} D_{mn}^j(g)$$

$$\sum_{j \in \frac{N}{2}} \sum_{m=-j}^{+j} \sum_{n=-j}^{+j} |j, m, n\rangle \langle j, m, n| = \mathbb{1}$$



## Right and Left translation

$$[R_h \psi(g)] = \psi(gh) \quad \leftarrow \quad \psi'(g) = \psi(gh)$$

$$R_{h_1} \circ R_{h_2} = R_{h_1 h_2}$$

$$R_{h_1} \psi'(h) = \psi'(gh_1) = \psi(gh_1 h)$$



## Right and Left translation

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$$R_h D_{mn}^j(g)$$



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$$R_h D_{mn}^j(g) = D_{mn}^j(gh) \stackrel{\text{representative property}}{=} \sum_p D_{mp}^j(g) D_{pn}^j(h)$$



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$$R_h D_{mn}^j(g) = D_{mn}^j(gh) \stackrel{\text{representative property}}{=} \sum_{p=1}^j D_{mp}^j(g) D_{pn}^j(h)$$



$$R_h |j, m, h\rangle = \sum_p |j, m, p\rangle D_{pm}^j(h)$$



$|g, h\rangle$   
 $|g, h, h\rangle$   
 $n(h)$

$$R_h |j, m, h\rangle = \sum_p |j, m, p\rangle D_{pm}^j(h)$$

$|j, m, \cdot\rangle$



$$R_h |j, m, h\rangle = \sum_p |j, m, p\rangle D_{pm}^j(h)$$

$\Rightarrow \{|j, m, \cdot\rangle\}$  basis of  $j$ -rep of  $SU(2)$

$|gh\rangle$   
 $g|h\rangle$   
 $n(h)$

$$R_h |j, m, h\rangle = \sum_p |j, m, \textcircled{p}\rangle D_{pm}^j(h)$$

$\Rightarrow \{ |j, m, \cdot\rangle \}$  basis of  $j$ -rep of  $SU(2)$   
 $\uparrow \quad \uparrow \quad \uparrow$   
 $\text{fixed, fixed} \quad n = -j, \dots, +j$



$$R_h |j, m, h\rangle = \sum_p |j, m, \textcircled{p}\rangle D_{pm}^j(h)$$

$\Rightarrow \{ |j, m, \cdot\rangle \}$  basis of  $V_{j, \pi p}$  of  $SU(2)$   
 $\uparrow \quad \uparrow \quad \uparrow$   
 $\text{fixed, fixed} \quad n = -j, \dots, +j$

for  $1 \leq i, j \leq n$   $S = 0$

Spin  $j$ -rep.

$$L_h D_{mn}^j(g) = D_{mn}^j(h^{-1}g)$$

$$L_{h_1} L_{h_2} = L_{h_1 h_2}$$



spin 1/2 rep.  $S = 0$

spin  $j$ -rep.

$$L_h D_{mn}^j(g) = D_{mn}^j(h^{-1}g) = \sum_p D_{mp}^j(h^{-1}) D_{pn}^j(g)$$

$$L_{h_1} L_{h_2} = L_{h_1 h_2}$$

for  $l \in \mathbb{N}$   $S = 0$

Spin  $j = n/2$

$$L_h D_{mn}^j(g) = D_{mn}^j(h^{-1}g) = \sum_p D_{mp}^j(h^{-1}) D_{pn}^j(g)$$

$$L_{h_1} L_{h_2} = L_{h_1 h_2}$$

$$L_g |j, m, n\rangle = \sum_{p=-j}^j D_{mp}^j(h^{-1}) |j, p, n\rangle$$



$$V_j \rightarrow D_{mR}^j G^R = G_m^R$$

translation

$$\psi'(g) = \psi(g_h)$$

$$R_{h_n} \psi'(h) = \psi'(g) \\ = \psi(g)$$

$$= \sum_{p=-j}^j D_{mp}^j(g)$$

representational property

$$R_h |j, m, h\rangle = \sum_p |j, m, p\rangle D_{pm}^j(h)$$

$$\Rightarrow \{ |j, m\rangle \} \text{ basis of } V_j \text{-rep of } SU(2)$$

$\uparrow \quad \uparrow \quad \uparrow$   
 fixed, fixed  $m = -j, \dots, +j$



$$V_j^R \rightarrow D_{mR}^j G^R = G_m^R$$

translation

$$\psi'(g) = \psi(g_h)$$

$$R_{h_n} \psi'(h) = \psi'(g h_n) \\ = \psi(g h_n h)$$

$$= \sum_{p=j}^i D_{mp}^j(g) D_{pn}^j(h)$$

representational  
property

$$R_h |j, m, h\rangle = \sum_p |j, m, p\rangle D_{pm}^j(h)$$

$$\Rightarrow \{ |j, m, \cdot\rangle \} \text{ basis of } V_j \text{-rep of } SU(2)$$

$\uparrow \quad \uparrow \quad \uparrow$   
 fixed, fixed  $m = -j, \dots, +j$



$$L_g D_{mn}^j(g) = D_{mn}^j(g^{-1}g) = \sum_p D_{mp}^j(g^{-1}) D_{pn}^j(g)$$

$$L_{g_1} L_{g_2} = L_{g_1 g_2}$$

$$L_g |j, m, n\rangle = \sum_{p=-j}^j D_{np}^j(g) |j, p, n\rangle$$

$\Rightarrow$  contragredient rep. / dual

$$D_{nm}^j(g^{-1})$$

$$D_{mn}^j(g^{-1}) = \sum_p D_{mp}^j(g^{-1}) D_{pn}^j(g)$$

$$L_g |j, m, n\rangle = \sum_{p=-j}^j D_{mp}^j(g^{-1})$$

$$\Rightarrow \text{contragredient rep. / dual rep.} \quad D_{mn}^j(g) = D_{nm}^j(g^{-1}) \stackrel{!}{=}$$

$\overline{D_{mn}^j(g)}$   
unitary



$$D_{mn}^j(g^{-1}) = \sum_p D_{mp}^j(g^{-1}) D_{pn}^j(g)$$

$$|j, m, n\rangle = \sum_{p=-j}^j D_{mp}^j(g^{-1}) |j, p, n\rangle$$

= a predict rep. / dual rep.  $(D^j)^*_{mn}(g) = D_{nm}^j(g^{-1}) \stackrel{!}{=} \overline{D_{mn}^j(g)}$

$$\overline{D_{mn}^j(g)} = D_{nm}^j(g^{-1})$$

unitary

$$D_{mn}^j(g^{-1}) = \sum_p D_{mp}^j(g^{-1}) D_{pn}^j(g)$$

$$L_g |j\rangle = \sum_{p=1}^j D_{mp}^j(g^{-1}) |j, p, n\rangle$$

$$\Rightarrow \text{cont. + rep. / dual rep. } (D^j)^*_{mn}(g) = D_{nm}^j(g^{-1}) \stackrel{!}{=} \overline{D_{mn}^j(g)}$$

$$\overline{D_{mn}^j(g)} = D_{nm}^j(g^{-1})$$

unitary



$$D_{mn}^j(g^{-1}) = \sum_p D_{mp}^j(g^{-1}) D_{pn}^j(g)$$

$$L_g |j, m, n\rangle = \sum_{p=-j}^j D_{mp}^j(g^{-1}) |j, p, n\rangle$$

$$\Rightarrow \text{contragredient rep. / dual rep. } j^*: (D^j)^*_{mn}(g) = D_{nm}^j(g^{-1}) \stackrel{!}{=} \overline{D_{mn}^j(g)}$$

$j^*$  is unitarily equivalent to  $j$ .

$$\overline{D_{mn}^j(g)} = D_{nm}^j(g^{-1})$$

unitary

$$D_{mn}^j(g^{-1}) = \sum_p D_{mp}^j(g^{-1}) D_{pn}^j(g)$$

$$L_g |j, m, n\rangle = \sum_{p=-j}^j D_{mp}^j(g^{-1}) |j, p, n\rangle$$

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unitary

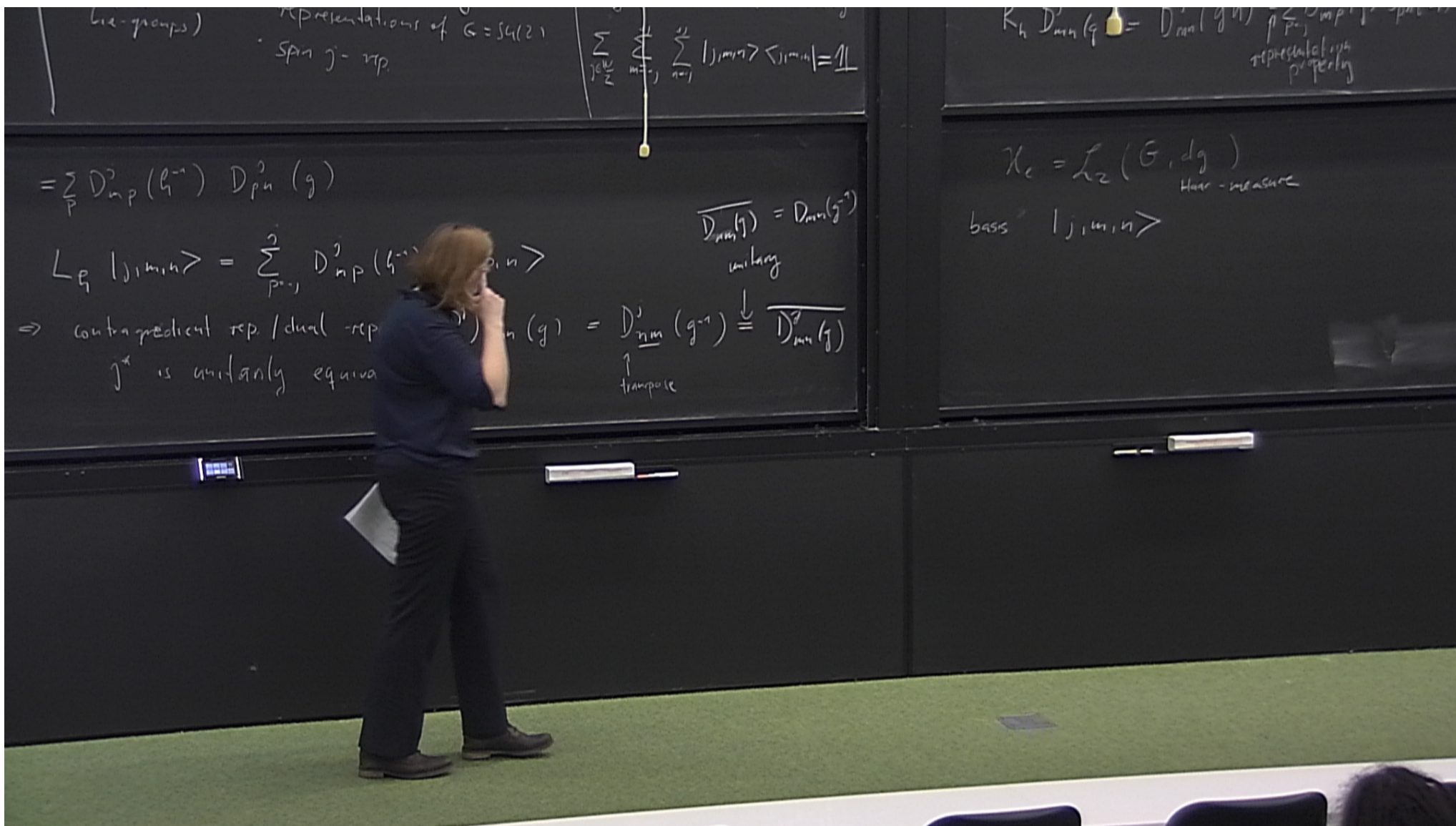


representation  
property

$$\mathcal{H}_e = L_2(G, dg)$$

Haar - measure

basis  $|j, m, n\rangle$





representation  
property

$$\mathcal{H}_e = L_2(G, dg)$$

Haar - measure

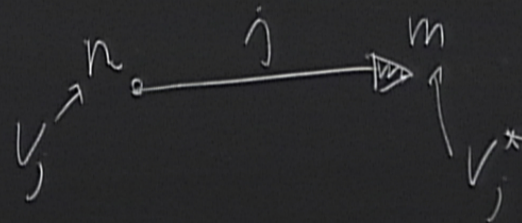
basis  $|j, m, n\rangle$   
left transl.  $\nearrow$   $\nwarrow$  right transl.

representation  
property

$$\mathcal{H}_c = L_2(G, dg) \quad \text{Haar-measure}$$

$$= \bigoplus_{j \in \mathbb{N}} V_j \otimes V_j^*$$

basis  $|j, m, n\rangle$   
 left transl.  $\nearrow$   
 right transl.  $\nwarrow$





representation  
property

$$\mathcal{H}_e = L_2(G, dg)$$

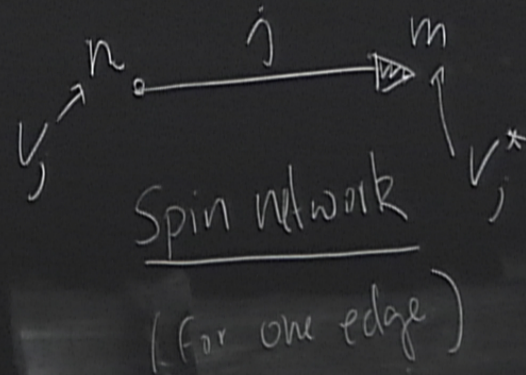
Haar - measure

$$= \bigoplus_{j \in \frac{1}{2}\mathbb{N}} V_j \otimes V_j^*$$

basis  $|j, m, n\rangle$

left transl.  $\nearrow$

right transl.  $\nwarrow$

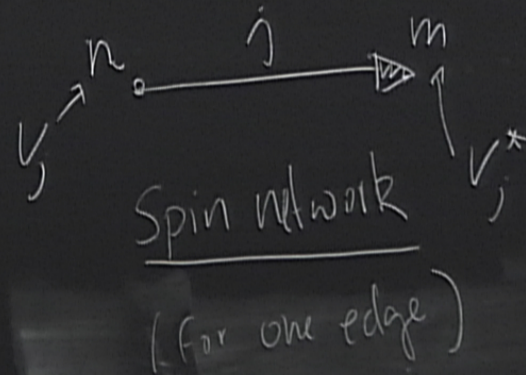


representation  
property

$$\mathcal{H}_e = L_2(G, dg) \quad \text{Haar-measure}$$

$$= \bigoplus_{j \in \frac{1}{2}\mathbb{N}} V_j \otimes V_j^*$$

basis  $|j, m, n\rangle$   
 left transl.  $\nearrow$   
 right transl.  $\nwarrow$





$$R_{\alpha} |j, m, n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

Action of fluxes

$$R_n |j, m, n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$[R_n, \hat{L}_z] = 0$$

$$\hat{L}_z |j, m, n\rangle = m |j, m, n\rangle$$



Action of fluxes

$$R_{\alpha} |j, m, n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$[R_{\alpha}, \chi(g)] = \chi(g h)$$

$$\chi(g) = \frac{d}{ds}$$

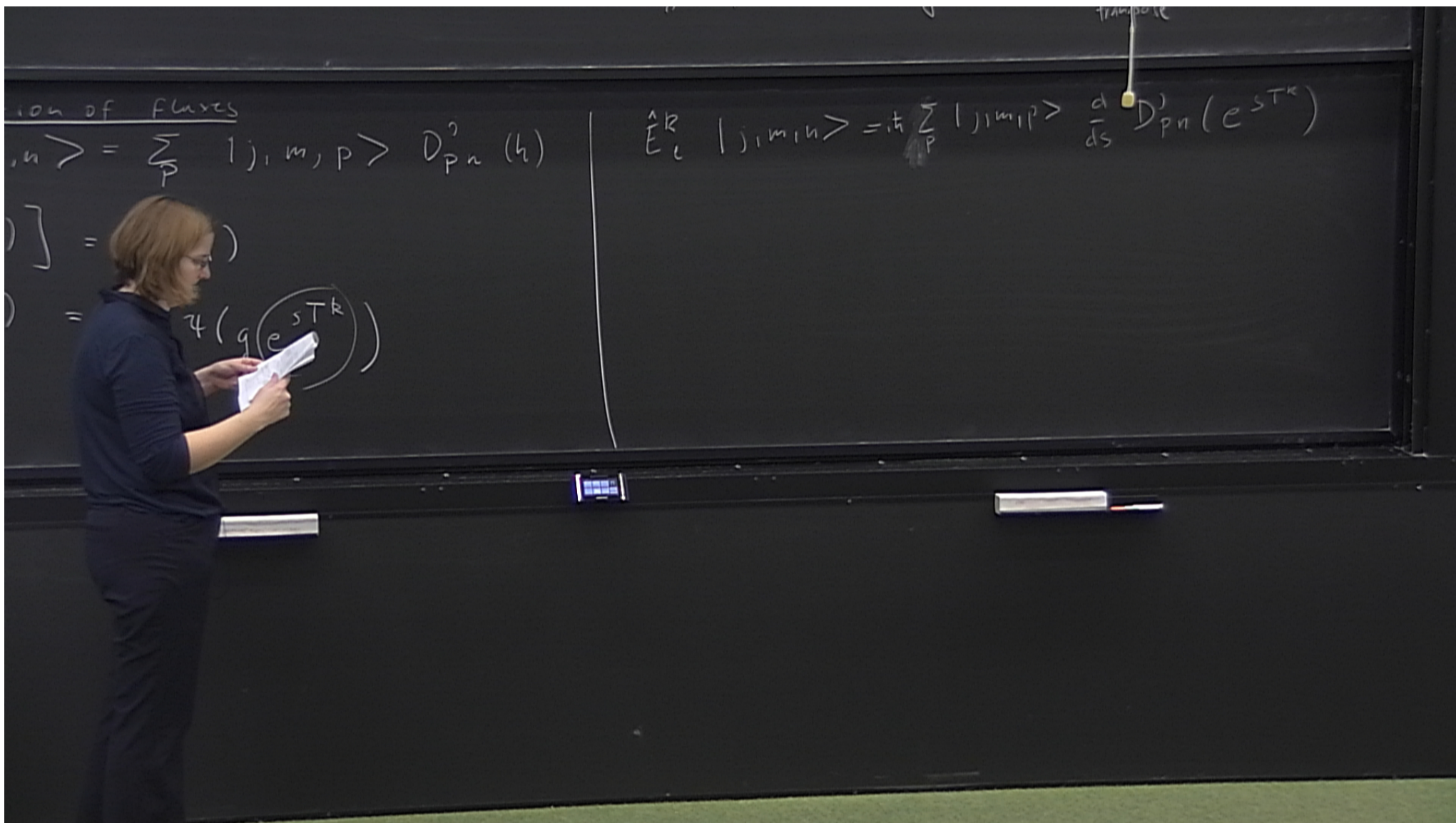
Action of fluxes

$$R_n |j, m, n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$[R_n \psi(g)] = \psi(gh)$$

$$\hat{L}_c^k \psi(g) = i\hbar \frac{d}{ds} \psi(g(e^{sT^k}))$$





ion of fluxes

$$|n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$] = 4(gh)$$

$$= i\hbar \frac{d}{ds} 4(g e^{sT^k})$$

$$\hat{E}_c^k |j, m, n\rangle = i\hbar \sum_p |j, m, p\rangle \underbrace{\frac{d}{ds} D_{pn}^j(e^{sT^k})}_{= D_{pn}^j(T^k)}$$



ion of fluxes

$$|n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$] = 4(gh)$$

$$= i\hbar \frac{d}{ds} 4(g(e$$

$$\hat{E}_L^R |j, m, n\rangle = i\hbar \sum_p |j, m, p\rangle \underbrace{\frac{d}{ds} D_{pn}^j(e^{sT^k})}_{= -D_{pn}^j(T^k)}$$

$$T^k = -\frac{i}{2}(\sigma_R)$$

Representation matrix of  
Lie algebra rep.  $j$   
(evaluated on Lie algebra  
element  $T^k$ )



ion of fluxes

$$|n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$] = 4(gh)$$

$$= i\hbar \frac{d}{ds} 4(g(e^{sT^k}))$$

$$\hat{E}_L^R |j, m, n\rangle = i\hbar \sum_p |j, m, p\rangle \frac{d}{ds} D_{pn}^j(e^{sT^k})$$

$$T^k = -\frac{i}{2}(\sigma_R)$$

$$= D_{pn}^j(T^k)$$

Representation matrix of  
Lie algebra rep.  $j$   
(evaluated on Lie algebra  
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ion of fluxes

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$$T^k = -\frac{i}{2}(\sigma_R)$$

$$= D_{pn}^j(T^k)$$

Representation matrix of  
Lie algebra rep.  $j$   
(evaluated on Lie algebra  
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ion of fluxes

$$|n\rangle = \sum_p |j, m, p\rangle D_{pn}^j(h)$$

$$] = 4(gh)$$

$$= i\hbar \frac{d}{ds} 4(g(e^{sT^k}))$$

$$\hat{E}_L^R |j, m, n\rangle = i\hbar \sum_p |j, m, p\rangle \frac{d}{ds} D_{pn}^j(e^{sT^k})$$

$$T^k = -\frac{i}{2}(\sigma_R)$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$D_{mn}^j(T^3)$  is diagonal

$$= -D_{pn}^j(T^k)$$

Representation matrix of  
Lie algebra rep.  $j$   
(evaluated on Lie algebra  
element  $T^k$ )



representation  
property

$$D^{1/2}(T^k) = (T^{1/2})_{mn}$$

$$= \begin{pmatrix} \overset{m}{1/2} & \overset{n}{1/2} & \overset{m}{1/2} & \overset{n}{-1/2} \\ -1/2 & 1/2 & -1/2 & -1/2 \end{pmatrix}$$

representation  
property

$$D^{1/2}_{mn}(T^k) = (T^k)_{mn}$$

$$= -\frac{i}{2} (\sigma^k)_{mn}$$

index  
positions

$$\begin{pmatrix} m & n \\ 1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 & -1/2 \end{pmatrix}$$



$j$  is unitarily equivalent to  $j$

↑  
transpose

for one edge

$$= (h_e T^j)_{mn}$$

$$= e^{jkl} E_e^l$$

$$e)_{lm} \} = 0$$

$$\sum_R \hat{E}_e^R \hat{E}_e^R |j, m, n\rangle = i\hbar \sum_P \hat{E}_e^R |j, m, P\rangle D_{pn}^j(T^R) \\ = (-\hbar^2) \sum_R \sum_{P, P'} |j, m, P'\rangle D_{P'P}^j(T^R) D_{pn}^j(T^R)$$

$j$  is unitarily equivalent to  $j$

↑ transpose

ce for one edge

$$= (h_c T^j)$$

$$\} = e^{j k l F}$$

$$e)_{k, m'} \} = 0$$

$$\begin{aligned} \sum_R \hat{E}_e^R \hat{E}_e^R |j, m, n\rangle &= i \hbar \sum_P \hat{E}_e^R \left( \sum_P |j, m, P\rangle D_{pn}^j(T^R) \right) \\ &= (-\hbar^2) \left( \sum_R \right) \left( \sum_{P, P'} |j, m, P'\rangle D_{P'P}^j(T^R) D_{pn}^j(T^R) \right) \\ C_{pn}^j &= \sum_R \sum_P D_{P'P}^j(T^R) D_{pn}^j(T^R) \end{aligned}$$



$J$  is unitarily equivalent to  $J$

↑ transpose

ce for one edge

$$= (-i)_{mn}$$

$$\{ = \hat{E}_e^L$$

$$\left[ \sum_R \hat{E}_e^R \hat{E}_e^{R*} \right] |j, m, n\rangle = i \hbar \sum_P \hat{E}_e^R \left( \sum_P |j, m, P\rangle D_{pn}^j(T^R) \right)$$

$$= (-\hbar^2) \left( \sum_R \right) \left( \sum_{P, P'} |j, m, P'\rangle D_{P'P}^j(T^R) D_{pn}^j(T^R) \right)$$

CASIMIR

$$C_{pn}^j = \sum_R \sum_P D_{P'P}^j(T^R) D_{pn}^j(T^R)$$

$$\sum_n C_{pn}^j D_{nm}^j(g) = \sum_n D_{pn}^j(g) C_{nm}^j \quad \leftarrow \text{use commutation relations of } SU(2)$$

inter twining property



$$\sum_n C_{pn}^j D_{nm}^j(g) = \sum_n D_{pn}^j(g) C_{nm}^j \quad \leftarrow \text{use commutation relations of } SU(2)$$

(a) Interwining property

Lemma:

$C$  is an intertwiner acting on an irreducible rep.  $\Rightarrow C = \lambda \mathbb{1}$



$$\sum_n C_{pn}^j D_{nm}^j(g) = \sum_n D_{pn}^j(g) C_{nm}^j \quad \leftarrow \text{use commutation relations of } SU(2)$$

Casimir. intertwining property

Schur's Lemma:  $C$  is an intertwiner acting on an irreducible rep.  $\Rightarrow C = \lambda \mathbb{1}$ ,  $\boxed{\lambda = j(j+1)}$



$$\sum_k \hat{E}_e^k \hat{t}_e^k$$

← spectrum  $j(j+1) (-\hbar)^2$

$$D_{nm}^j(g) = \sum_n D_{pn}(g) \quad C_{nm}^j \quad \leftarrow \text{use commutation relations of } SU(2)$$

winning property

req:  $C$  is an intertwiner acting on an irreducible rep.  $\Rightarrow C = \lambda \mathbb{1}$ ,  $\boxed{\lambda = -j(j+1)}$  discrete



$$\sum_k \hat{E}_e^k \hat{t}_e^k$$

← spectrum  $j(j+1) (\hbar)^2$

$$\int_{e^*} h_{e^*}^k(s) e_a^k \gamma^a(s) T_R h_{e^*}^{-1}(s) ds$$

$$\sum_k e_a^k e_b^k = g_{ab}$$

$$\sum_R \hat{E}_e^k \hat{t}_e^k$$

← spectrum  $j(j+1) (\hbar)^2$

$$\int_{\mathbb{R}^n} h(s) e_a^k \dot{\gamma}^a(s) T_R h_{e^k}(s) ds$$

$$e^* \sum_R e_a^k e_b^k = g_{ab} \leftarrow \text{Z-metric } (\sum \text{Zdim})$$

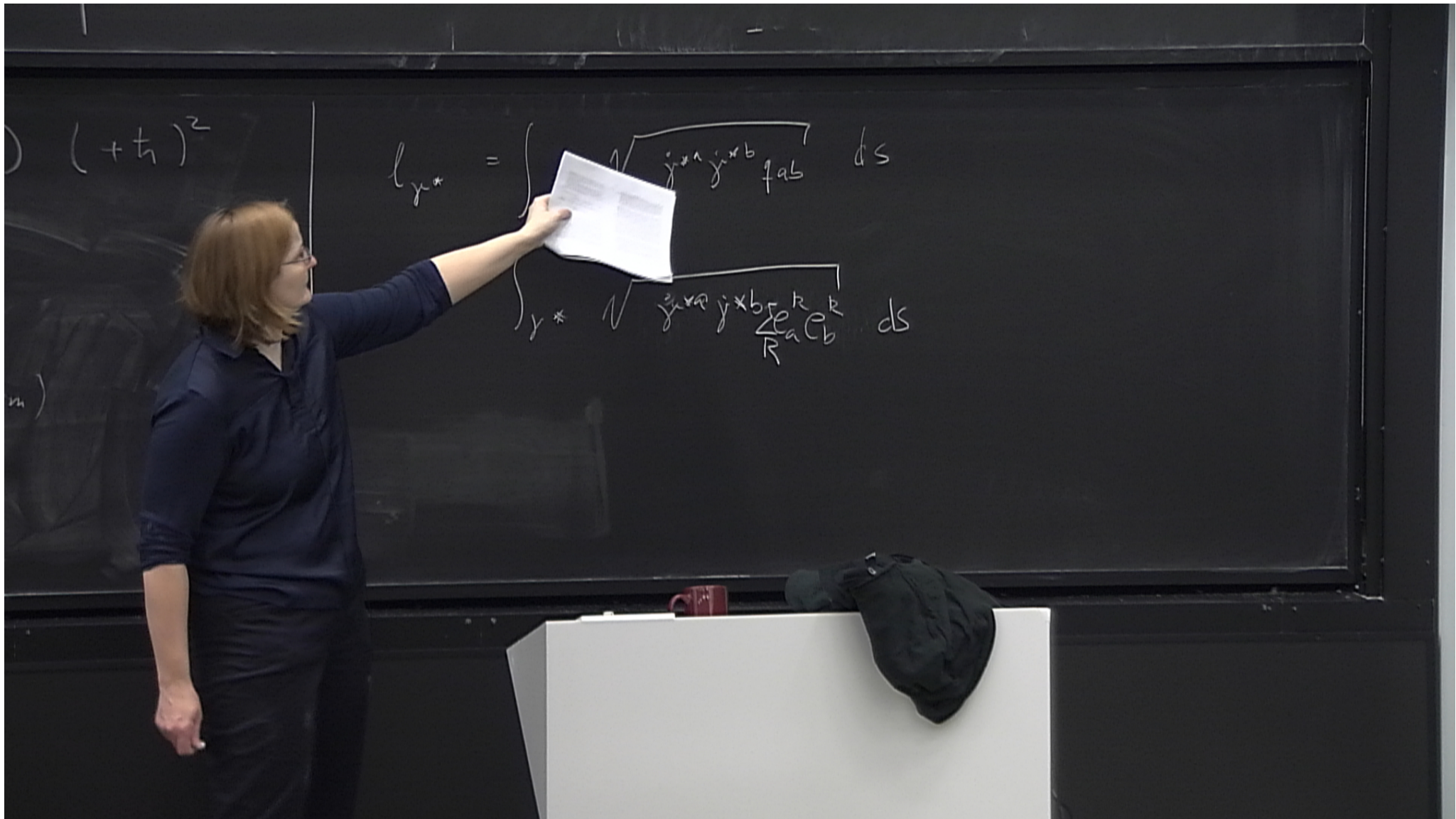


$$\sum_R \hat{E}_e^k \hat{t}_e^k$$

← spectrum  $j(j+1) (\hbar)^2$

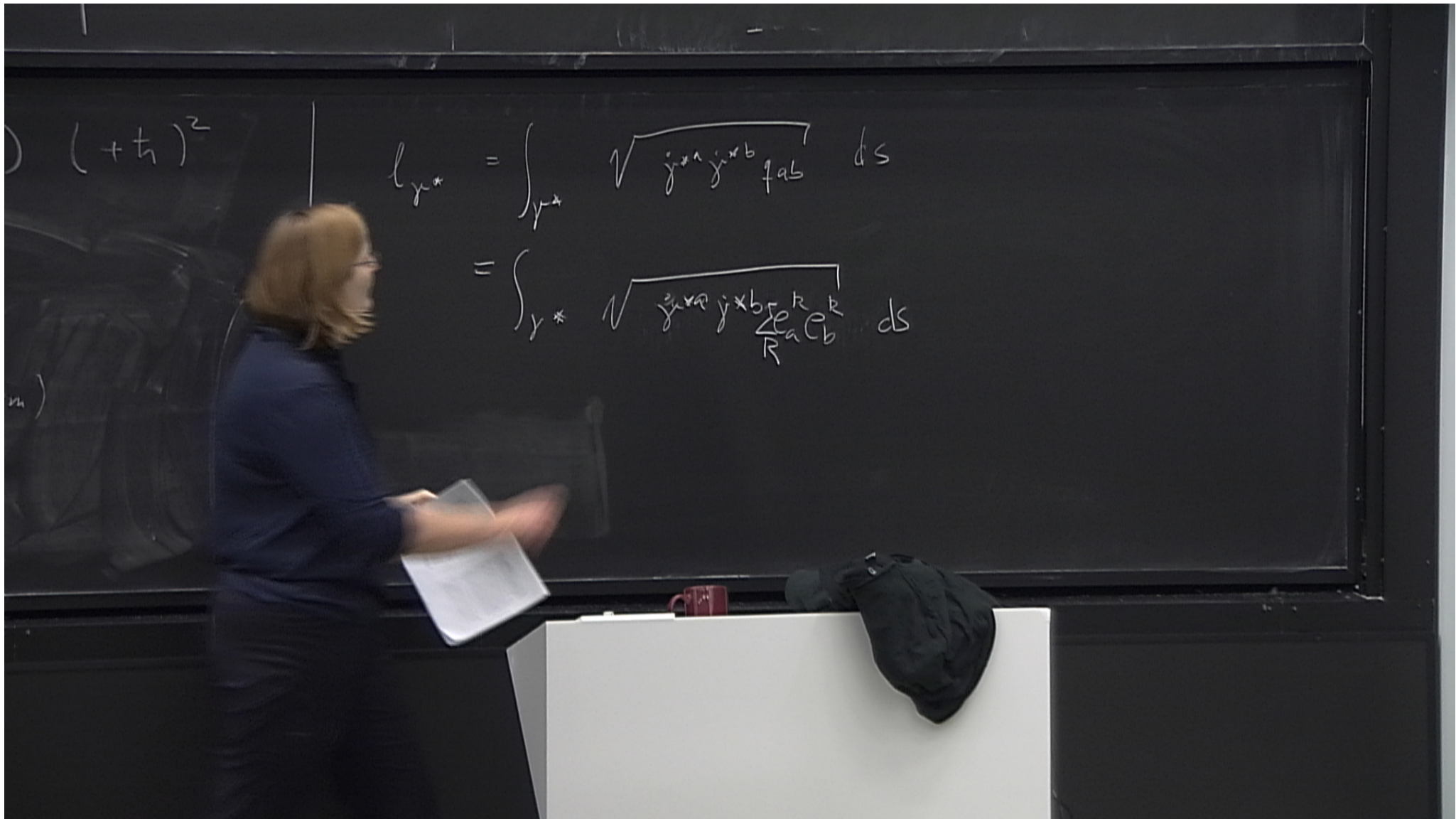
$$\int h_{e^a}^k(s) e_a^k \gamma^a(s) T_R h_{e^a}^{-1}(s) ds$$

$$e^* \sum_R e_a^k e_b^k = g_{ab} \leftarrow \text{Z-metric } (\sum 2\text{dim})$$



$$l_{\gamma^*} = \int \sqrt{ \gamma^{*a} \gamma^{*b} f_{ab} } ds$$
$$\gamma^* \sqrt{ \gamma^{*a} \gamma^{*b} \sum_R e_a^k e_b^k } ds$$





$$) (+t)^2$$

$$l_{\gamma^*} = \int_{\gamma^*} \sqrt{g^{*a} g^{*b} g_{ab}} \, ds$$

$$= \int_{\gamma^*} \sqrt{g^{*a} g^{*b} \sum_R e_a^k e_b^k} \, ds$$

$$\approx \sqrt{\sum_R \left( \int_{\gamma^*} e_a^k g^{*a} \, ds \right)^2}$$



$$) (+t)^2$$

$$l_{\gamma^*} = \int_{\gamma^*} \sqrt{g^{*a} g^{*b} f_{ab}} ds$$

$$= \int_{\gamma^*} \sqrt{g^{*a} g^{*b} \sum_R e_a^k e_b^k} ds$$

$$\gamma^*_{\text{small}} \approx \sqrt{\sum_R \left( \int_{\gamma^*} e_a^k g^{*a} ds \right)^2}$$



$$R_h D_{mn}^j(g) = D_{mn}^j(gh) = \sum_{p=1}^j D_{mp}^j(g) D_{pn}^j(h)$$

representation property

$$\sum_k \hat{E}_e^k \hat{E}_e^k \approx \rho_{e^*}^2$$

$$\sum_k \hat{E}_e^k \hat{E}_e^k \leftarrow \text{spectrum } j(j+1) (\hbar)^2$$

$$\int_{e^*} h(s) e_a^k \dot{y}^a(s) T_R h_{e^*}^{-1}(s) ds$$

$$\sum_k e_a^k e_b^k = g_{ab} \leftarrow \text{Z-metric } (\sum \text{Zdim})$$

$$l_{y^*} = \left( \begin{array}{c} j^* \otimes j^* \\ j^* \otimes j^* \\ j^* \otimes j^* \end{array} \right)$$

$$v^* \xrightarrow{\text{small}} \dots$$



$$R_h D_{mn}^j(g) = D_{mn}^j(gh) = \sum_{p,q} D_{mp}^j(g) D_{pn}^j(h)$$

representation property

$$\sum_k \hat{E}_e^k \hat{E}_e^k \approx \hat{\rho}_{e^*}^2$$

$$\sum_k \hat{E}_e^k \hat{E}_e^k \leftarrow \text{spectrum } j(j+1) (\hbar)^2$$

$$\int_{e^*} h(s) e_a^k \dot{y}^a(s) T_R h_{e^*}^{-1}(s) ds$$

$$\sum_k e_a^k e_b^k = g_{ab}$$

$$l_{y^*} = \int_{y^*} \sqrt{\dot{y}^{*\mu} \dot{y}^{*\mu}}$$

$$= \int_{y^*} \sqrt{\dot{y}^{*\mu} \dot{y}^{*\mu}}$$

$$y^{* \text{ small}} \approx \left| \sum_k \right| \int_{y^*} e_a^k \dot{y}^{*a}$$



$$\rho(g, h) = \sum_{p=1}^j D_{mp}^j(g) D_{pn}^j(h)$$

representation property

$$\sum_R \hat{E}_e^1 \hat{E}_e^R \approx \hat{L}_e^2 \approx \frac{(l_{\text{Planck}}^{30})^2 j(j+1)}{\text{spectrum}}$$

← spectrum  $j(j+1) (\hbar)^2$

$$h_{e^a}^{-1}(s) ds$$

$a, b$  ← Z-metric ( $\sum 2 \dim$ )

$$l_{\gamma^*} = \int_{\gamma^*} \sqrt{\gamma^{*a} \gamma^{*b} g_{ab}} ds$$

$$= \int_{\gamma^*} \sqrt{\gamma^{*a} \gamma^{*b} \sum_R e_a^R e_b^R} ds$$

$$\gamma_{\text{small}}^* \approx \sqrt{\sum_R \left( \int_{\gamma^*} e_a^R \gamma^{*a} ds \right)^2}$$