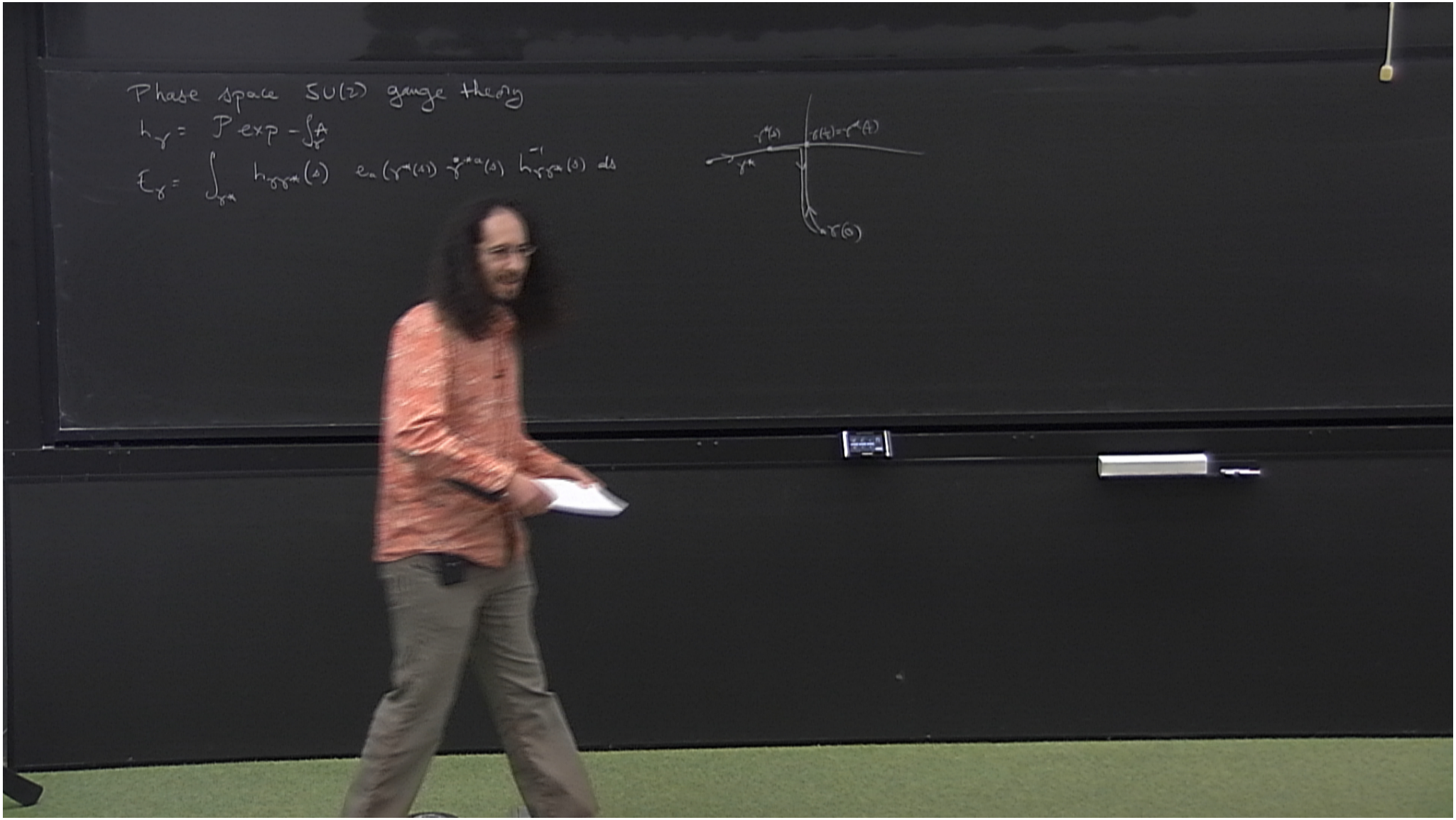
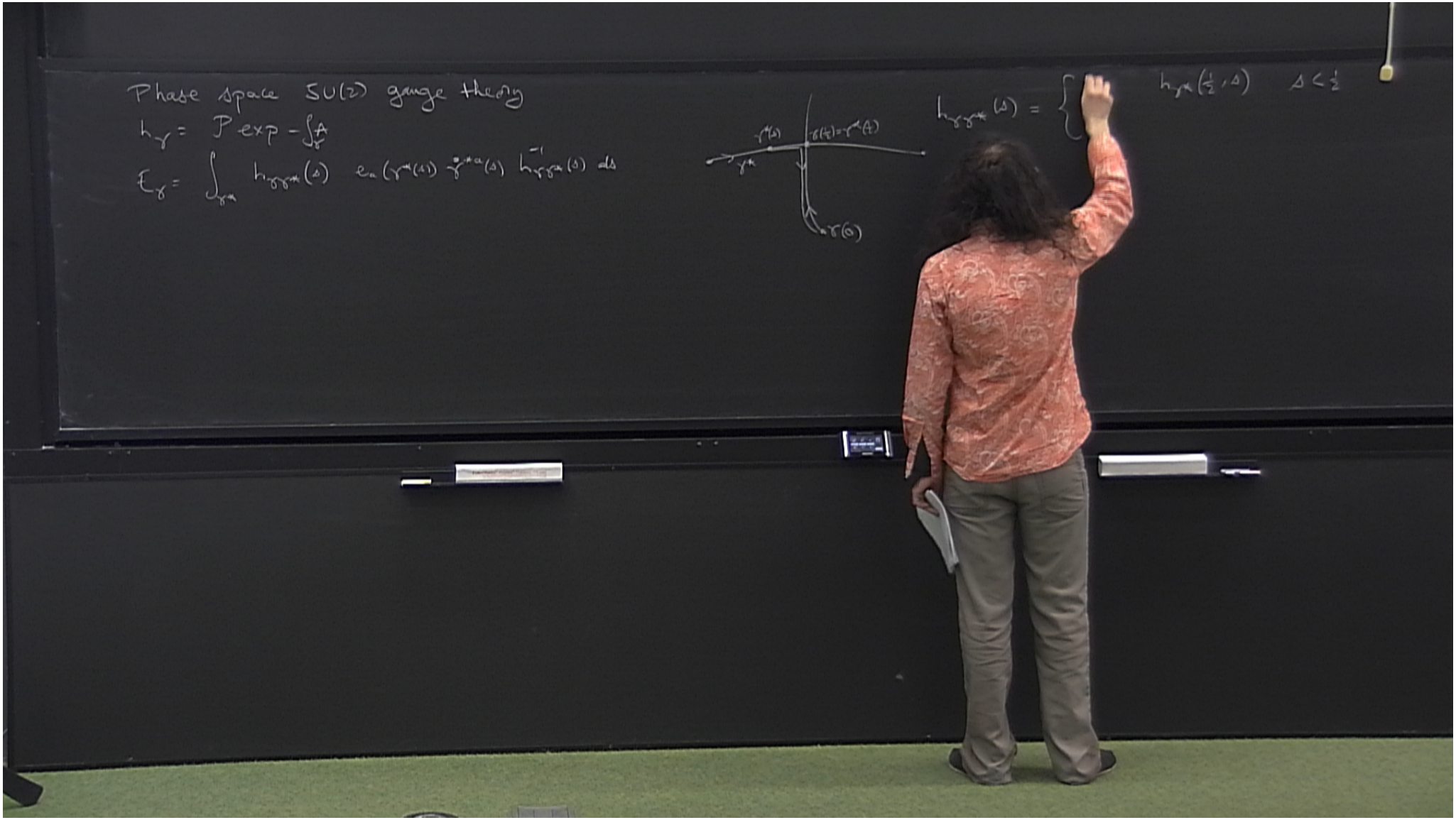
Title: 12/13 PSI - Quantum Gravity Review Lecture 12

Date: Feb 12, 2013 10:15 AM

URL: http://pirsa.org/13020055

Abstract:

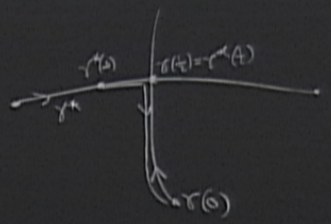

Phase space SU(2) gauge theory
$h_\gamma = \mathcal{P}\exp -\int_\gamma A$
$E_\gamma = \int_{\gamma\cap\sigma} h_{\gamma\gamma\cap\sigma}(s)\ e_a(\gamma\cap\sigma(s))\ \dot\gamma^{\cap\sigma\,a}(s)\ h^{-1}_{\gamma\gamma\cap\sigma}(s)\ ds$
$\gamma(s)$
$\sigma(t)=\gamma(s)$
$\gamma^{\cap\sigma}$
$\sigma(0)$


Phase space SU(2) gauge theory
$h_\gamma = \mathcal{P} \exp - \int_\gamma A$
$E_\gamma = \int_{\gamma * \gamma} h_{\gamma s \gamma * (s)} \; e_a(\gamma^*(s)) \; \dot{\gamma}^{*a}(s) \; h^{-1}_{\gamma s \gamma * (s)} \; ds$
$h_{\gamma s \gamma *}(s) = \begin{cases} h_{\gamma * (\frac{1}{2}, s)} & s < \frac{1}{2} \end{cases}$
$\gamma^*(s)$
$\gamma(t) = \gamma^*(t)$
γ^*
$\gamma(0)$

Phase space SU(2) gauge theory

$h_\gamma = \mathcal{P}\exp -\int_\gamma A$

$E_\gamma = \int_{\gamma\cap\sigma} h_{\gamma\gamma\cap\sigma}(s)\; e_a(\gamma\cap\sigma(s))\; \dot\gamma^{\cap\sigma a}(s)\; h^{-1}_{\gamma\gamma\cap\sigma}(s)\; ds$



$$h_{\gamma\gamma\cap\sigma}(s) = \begin{cases} h_\gamma^{-1}(\tfrac{1}{2},0)\, h_{\gamma\cap\sigma}(\tfrac{1}{2},s) & s<\tfrac{1}{2} \\ h_\gamma^{-1}(\tfrac{1}{2},0)\, h^{-1}_{\gamma\cap\sigma}(s,\tfrac{1}{2}) & s>\tfrac{1}{2} \end{cases}$$

Phase space SU(2) gauge theory

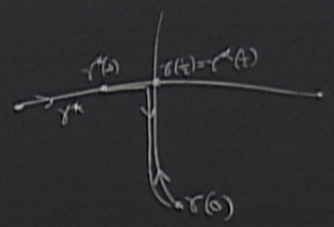
$h_\gamma = \mathcal{P} \exp -\int_\gamma A$

$$\int_{\gamma^*} h_{\gamma\gamma^*}(s)\; e_a(\gamma^*(s))\; \dot{\gamma}^{*a}(s)\; h^{-1}_{\gamma\gamma^*}(s)\; ds$$

$$\{(h_\gamma)_{AB}, (h_\gamma)_{CD}\} = 0$$

$$\{E^i_\gamma, (h_\gamma)_{AB}\} = (h_\gamma \tau^i)_{AB}$$

$$\{E^i_\gamma, E^j_\gamma\} = \epsilon^{ij}{}_k E^k_\gamma$$

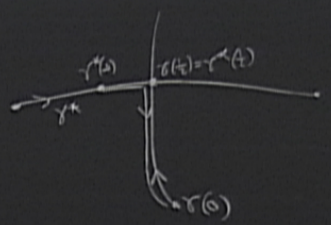


$$h_{\gamma\gamma^*}(s) = \begin{cases} h^{-1}_\gamma(t,0)\, h_{\gamma^*}(t,s) & s < t \\ h^{-1}_\gamma(t,0)\, h^{-1}_{\gamma^*}(s,t) & s > t \end{cases}$$

Phase space SU(2) gauge theory

$h_\gamma = \mathcal{P} \exp - \int_\gamma A$

$E_\gamma = \int h_{\gamma\gamma^{*a}(s)} \; e_a(\gamma^{*a}(s)) \; \dot{\gamma}^{*a}(s) \; h^{-1}_{\gamma\gamma^{*a}(s)} \; ds$



$h_{\gamma\gamma^*}(s) = \begin{cases} h_\gamma^{-1}(\frac{1}{2},0)\, h_{\gamma^*}(\frac{1}{2},s) & s < \frac{1}{2} \\ h_\gamma^{-1}(\frac{1}{2},0)\, h^{-1}_{\gamma^*}(s,\frac{1}{2}) & s > \frac{1}{2} \end{cases}$

Algebra

$\{(h_\gamma)_{AB}, (h_\gamma)_{CD}\} = 0$

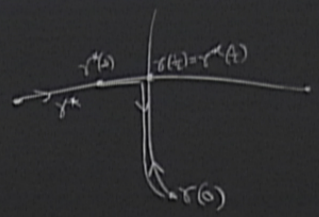
$\{E^i_\gamma, (h_\gamma)_{AB}\} = (h_\gamma T^i)_{AB} = \frac{d}{dt} \left(h_\gamma e^{tT^i}\right)_{|t=0}$

$\{E^i_\gamma, E^j_\gamma\} = \epsilon^{ij}{}_k E^k_\gamma \rightarrow$ SU(2) structure

Phase space SU(2) gauge theory

$h_\gamma = \mathcal{P}\exp -\int_\gamma A$

$E_\gamma = \int_{\gamma*} h_{\gamma\gamma*}(s)\ e_a(\gamma*(s))\ \dot\gamma*^a(s)\ h^{-1}_{\gamma\gamma*}(s)\ ds$

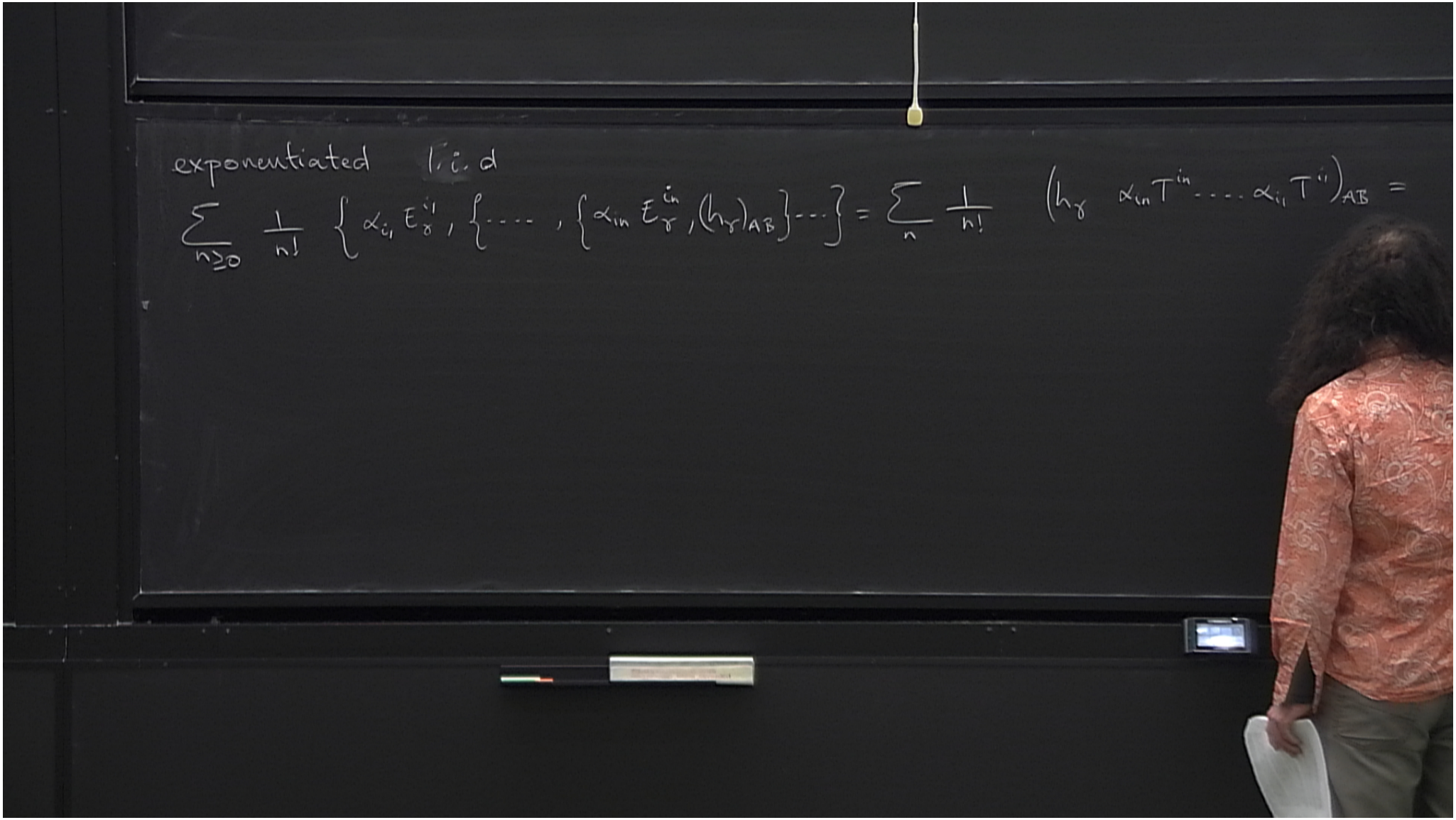


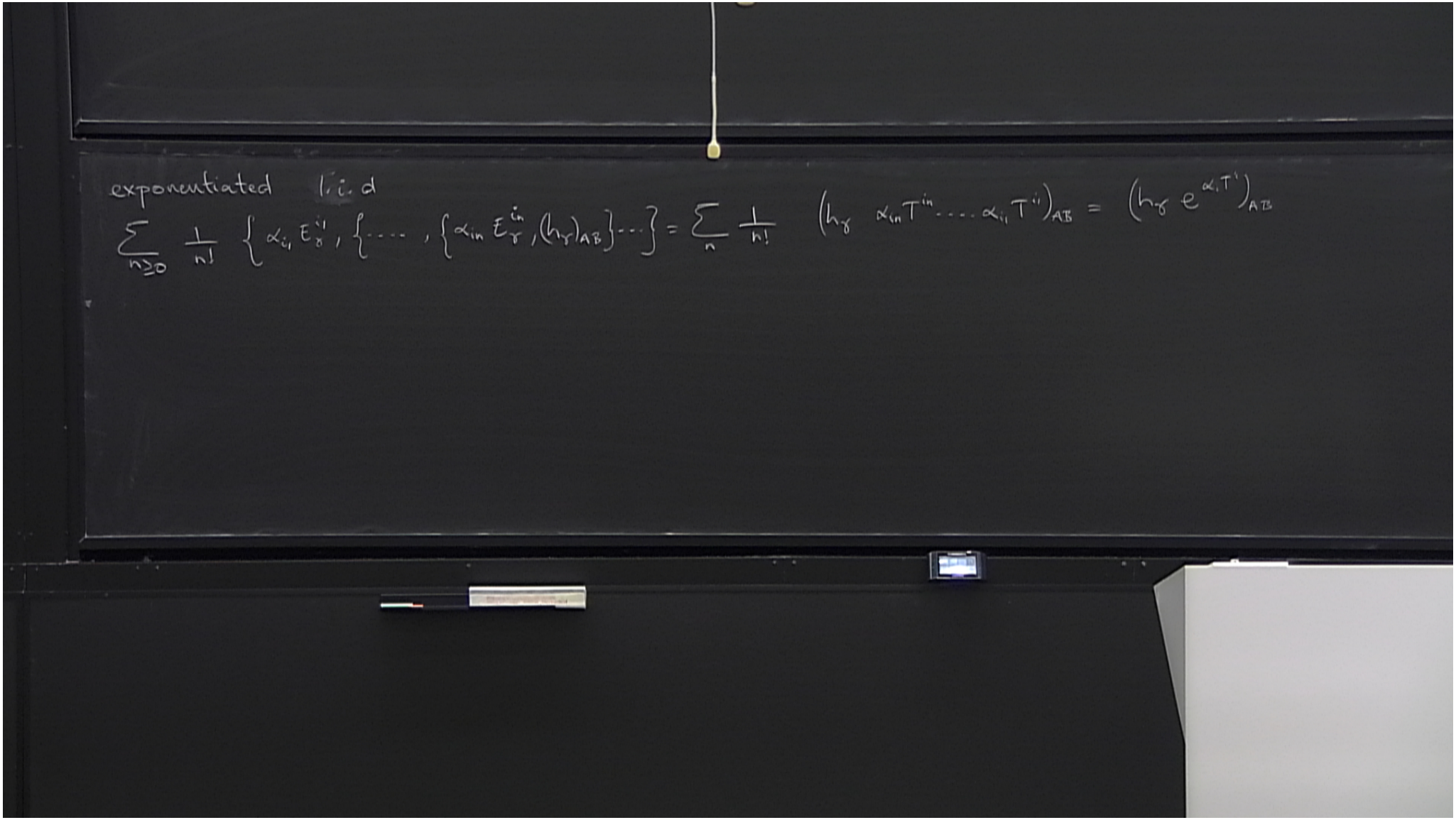
$h_{\gamma\gamma*}(s) = \begin{cases} h_\gamma^{-1}(\frac{1}{2},0)\, h_{\gamma*}(\frac{1}{2},s) & s<\frac{1}{2} \\ h_\gamma^{-1}(\frac{1}{2},0)\, h^{-1}_{\gamma*}(s,\frac{1}{2}) & s>\frac{1}{2} \end{cases}$

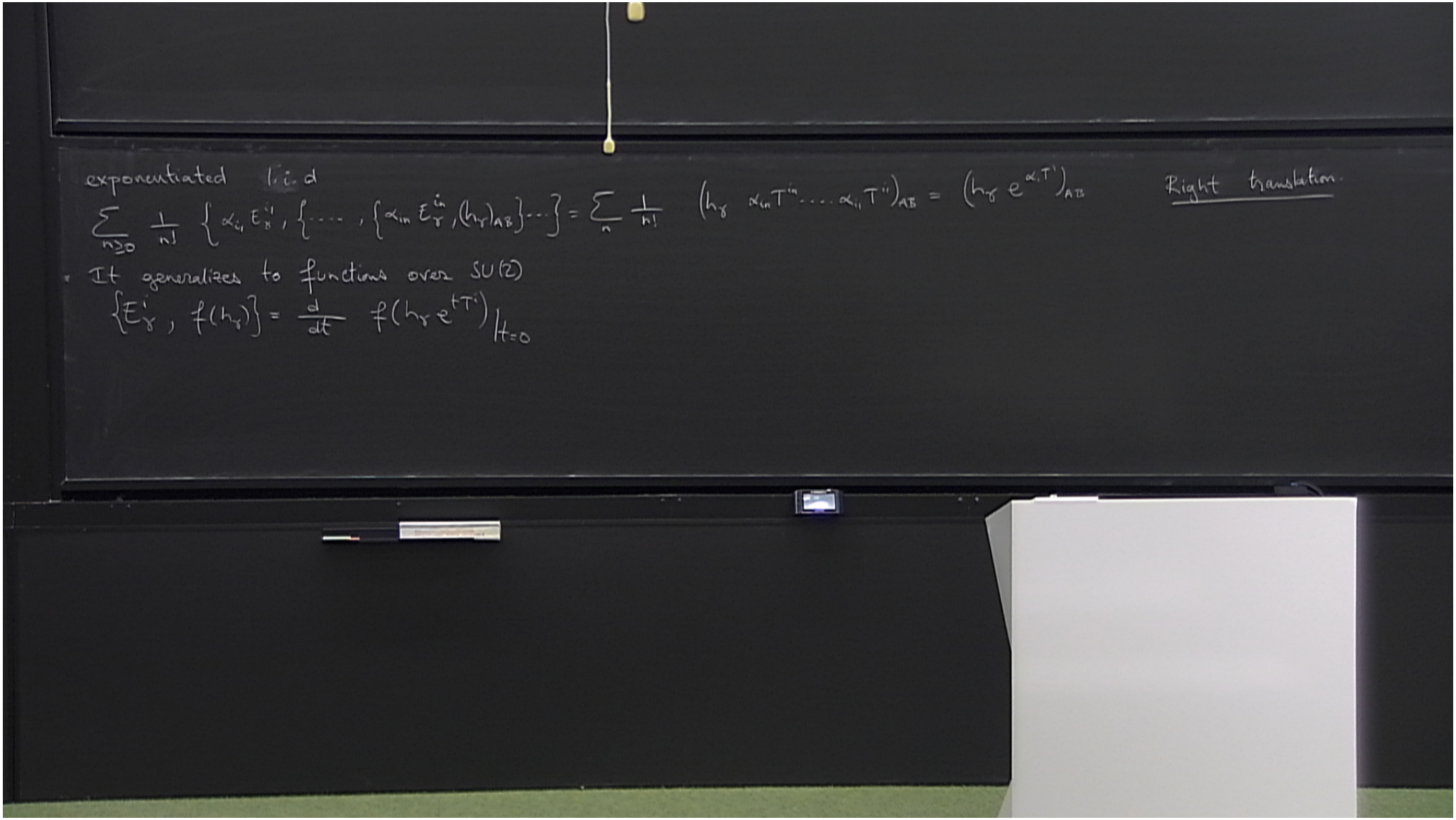
Algebra: $\{(h_\gamma)_{CD}\} = 0$

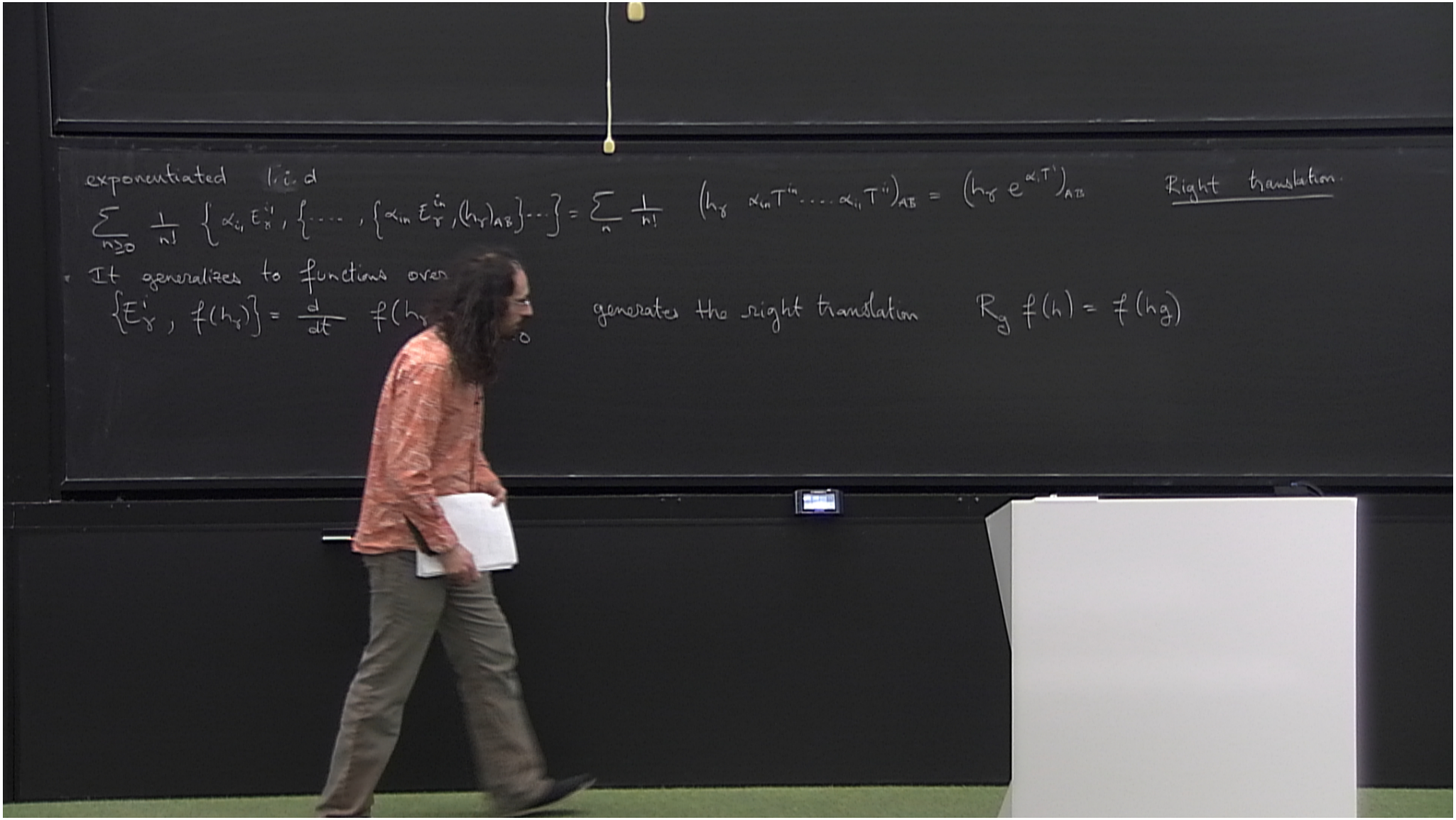
$\{\ \ _{AB}\} = (h_\gamma T^i)_{AB} = \frac{d}{dt}\left(h_\gamma e^{tT^i}\right)_{|t=0}$ left invariant derivative (commutes with operators acting on the left)

$\{E^j_\gamma\} = \epsilon^{ij}{}_k E^k_\gamma$ → SU(2) structure


exponentiated i.i.d
$\sum_{n\geq 0} \frac{1}{n!} \{\alpha_{i_1} E^{i_1}_{\gamma}, \{\cdots, \{\alpha_{i_n} E^{i_n}_{\gamma}, (h_\gamma)_{AB}\}\cdots\}\} = \sum_n \frac{1}{n!} \left(h_\gamma \alpha_{i_n} T^{i_n} \cdots \alpha_{i_1} T^{i_1}\right)_{AB} =$


exponentiated l.i.d
$\sum_{n\geq 0} \frac{1}{n!} \{\alpha_{i_1} E^{i_1}_\gamma, \{\cdots, \{\alpha_{i_n} E^{i_n}_\gamma, (h_\gamma)_{AB}\}\cdots\}\} = \sum_n \frac{1}{n!} (h_\gamma \alpha_{i_n} T^{i_n} \cdots \alpha_{i_1} T^{i_1})_{AB} = (h_\gamma e^{\alpha_i T^i})_{AB}$


exponentiated l.i.d
$\sum_{n\geq 0} \frac{1}{n!} \{\alpha_{i_1} E^{i_1}_{\gamma}, \{\cdots, \{\alpha_{i_n} E^{i_n}_{\gamma}, (h_\gamma)_{AB}\}\cdots\}\} = \sum_n \frac{1}{n!} (h_\gamma \alpha_{i_n} T^{i_n} \cdots \alpha_{i_1} T^{i_1})_{AB} = (h_\gamma e^{\alpha_i T^i})_{AB}$
Right translation.
It generalizes to functions over SU(2)
$\{E^i_\gamma, f(h_\gamma)\} = \frac{d}{dt} f(h_\gamma e^{tT^i})|_{t=0}$


exponentiated Lie algebra
Right translation.
It generalizes to functions over
generates the right translation

exponentiated l.i.d

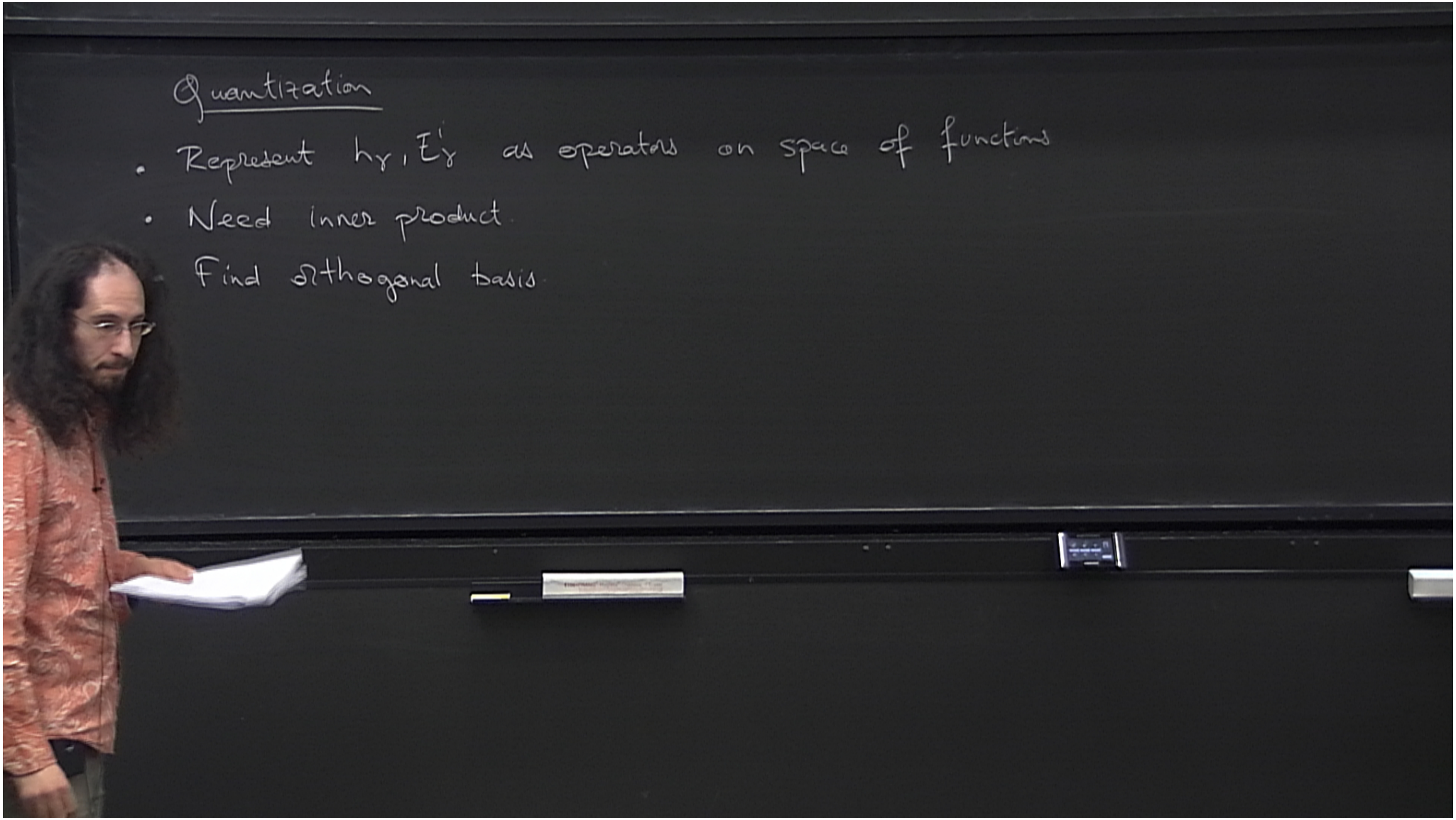
$$\sum_{n\geq 0} \frac{1}{n!} \left\{ \alpha_{i_1} E^{i_1}_{\gamma}, \left\{ \cdots, \left\{ \alpha_{i_n} E^{i_n}_{\gamma}, (h_\gamma)_{AB} \right\} \cdots \right\} \right\} = \sum_n \frac{1}{n!} \left(h_\gamma \, \alpha_{i_n} T^{i_n} \cdots \alpha_{i_1} T^{i_1} \right)_{AB} = \left(h_\gamma \, e^{\alpha_i T^i} \right)_{AB}$$

Right translation.

- It generalizes to functions over SU(2)

$$\left\{ E^i_\gamma, f(h_\delta) \right\} = \frac{d}{dt} f(h_\gamma e^{tT^i})\Big|_{t=0}$$

generates the right translation $R_g f(h) = f(hg)$

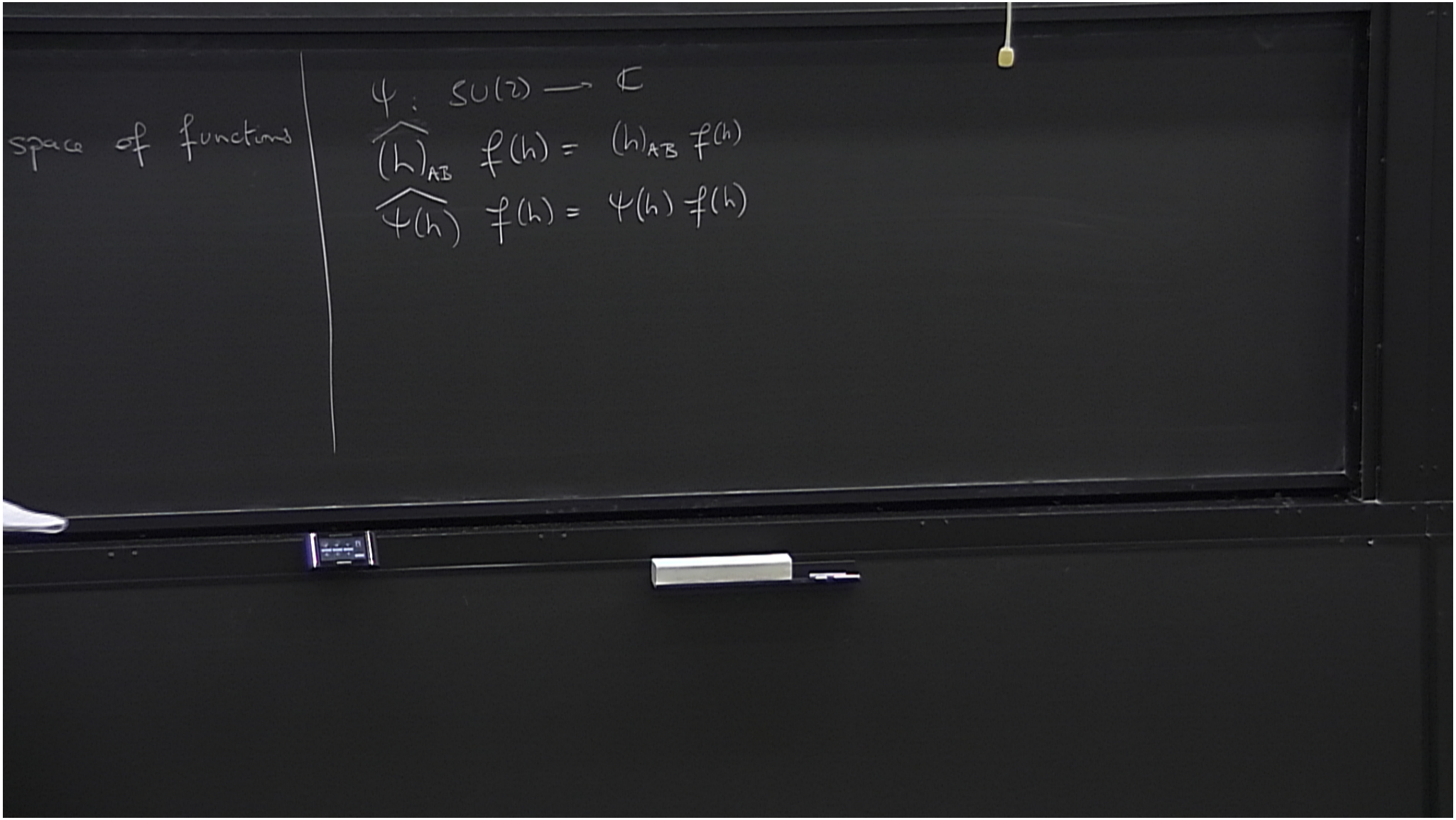

Quantization
• Represent h_γ, E'_γ as operators on space of functions
• Need inner product
Find orthogonal basis

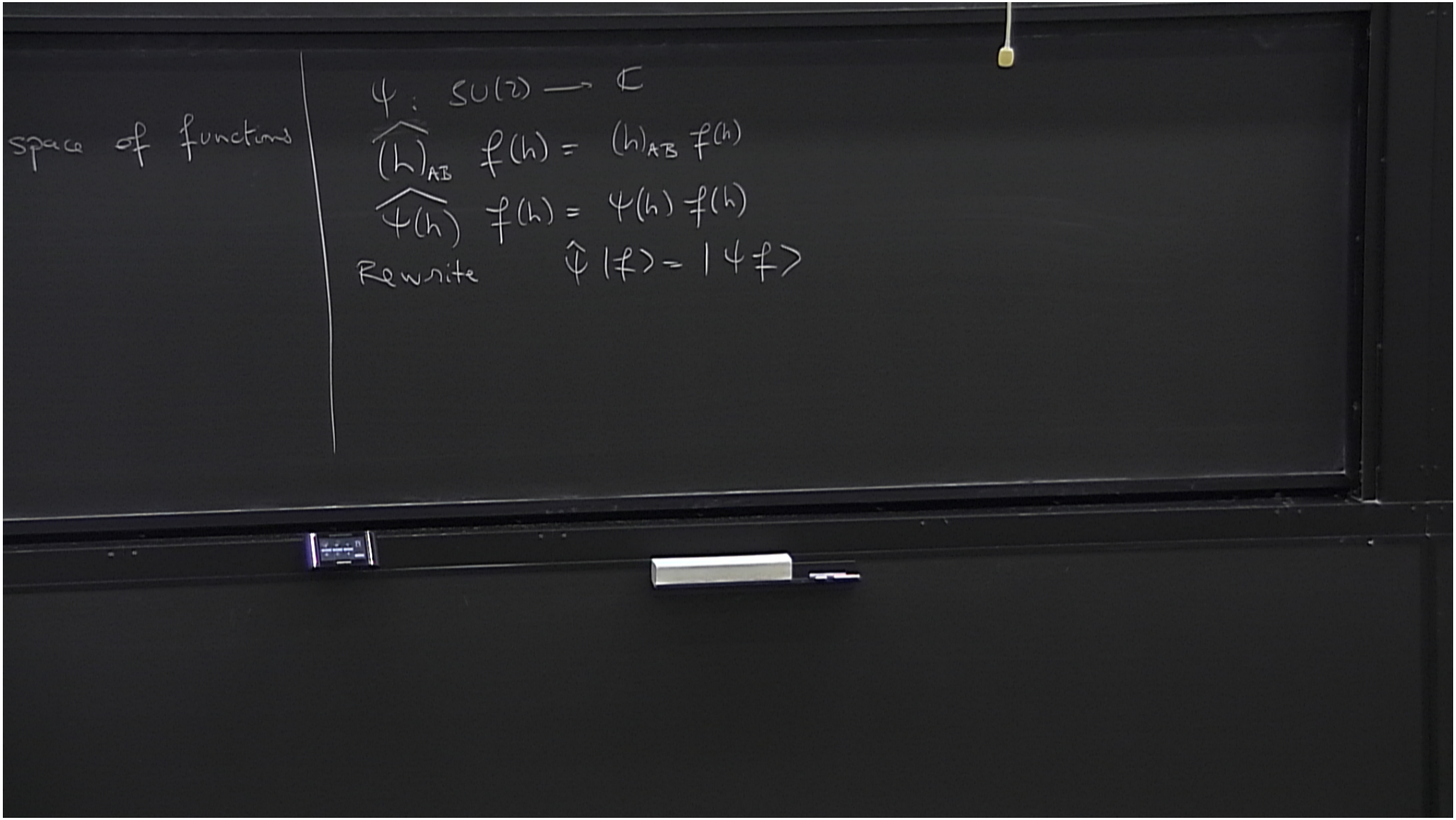
space of functions

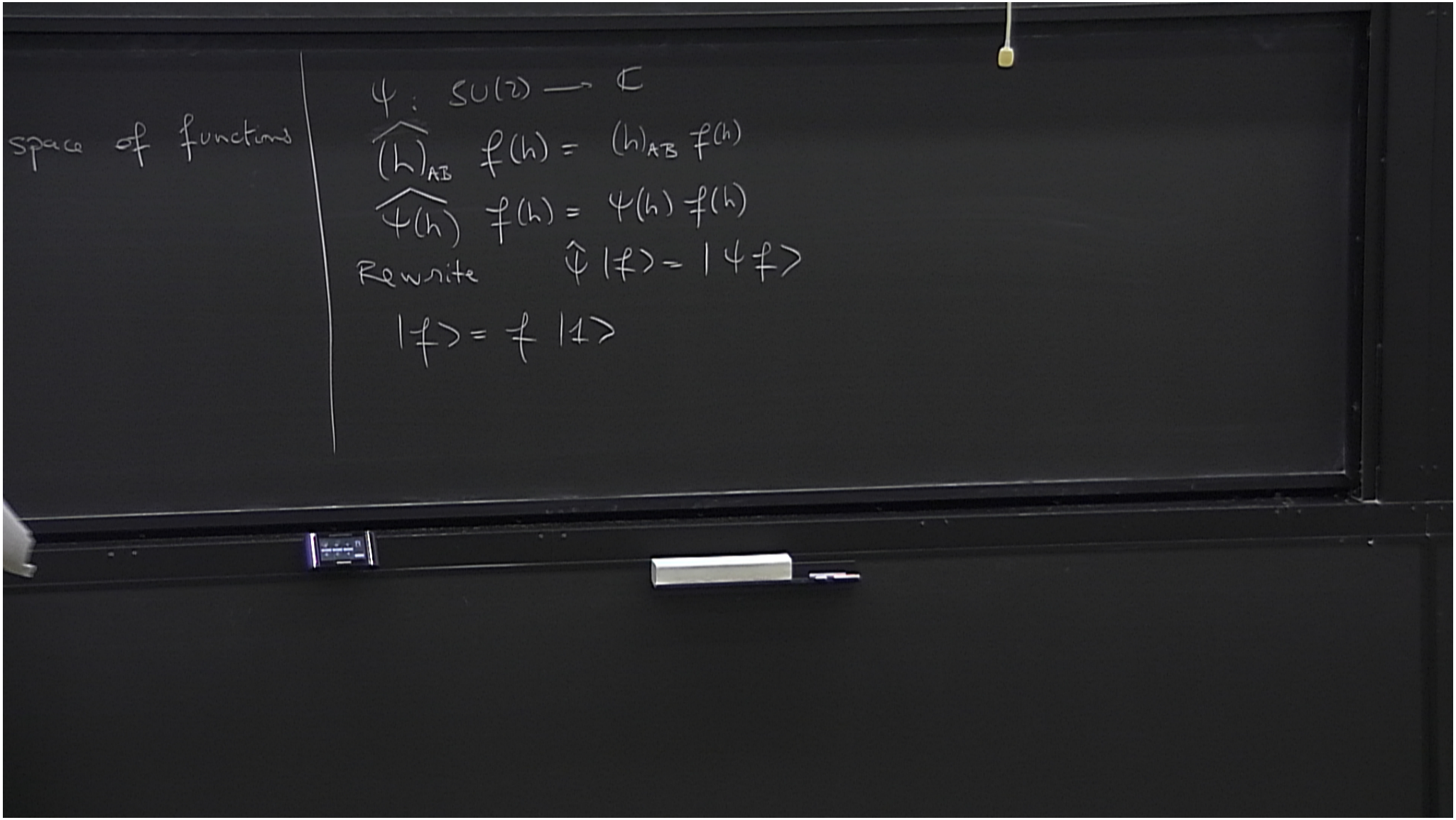
$\psi : SU(2) \longrightarrow \mathbb{C}$

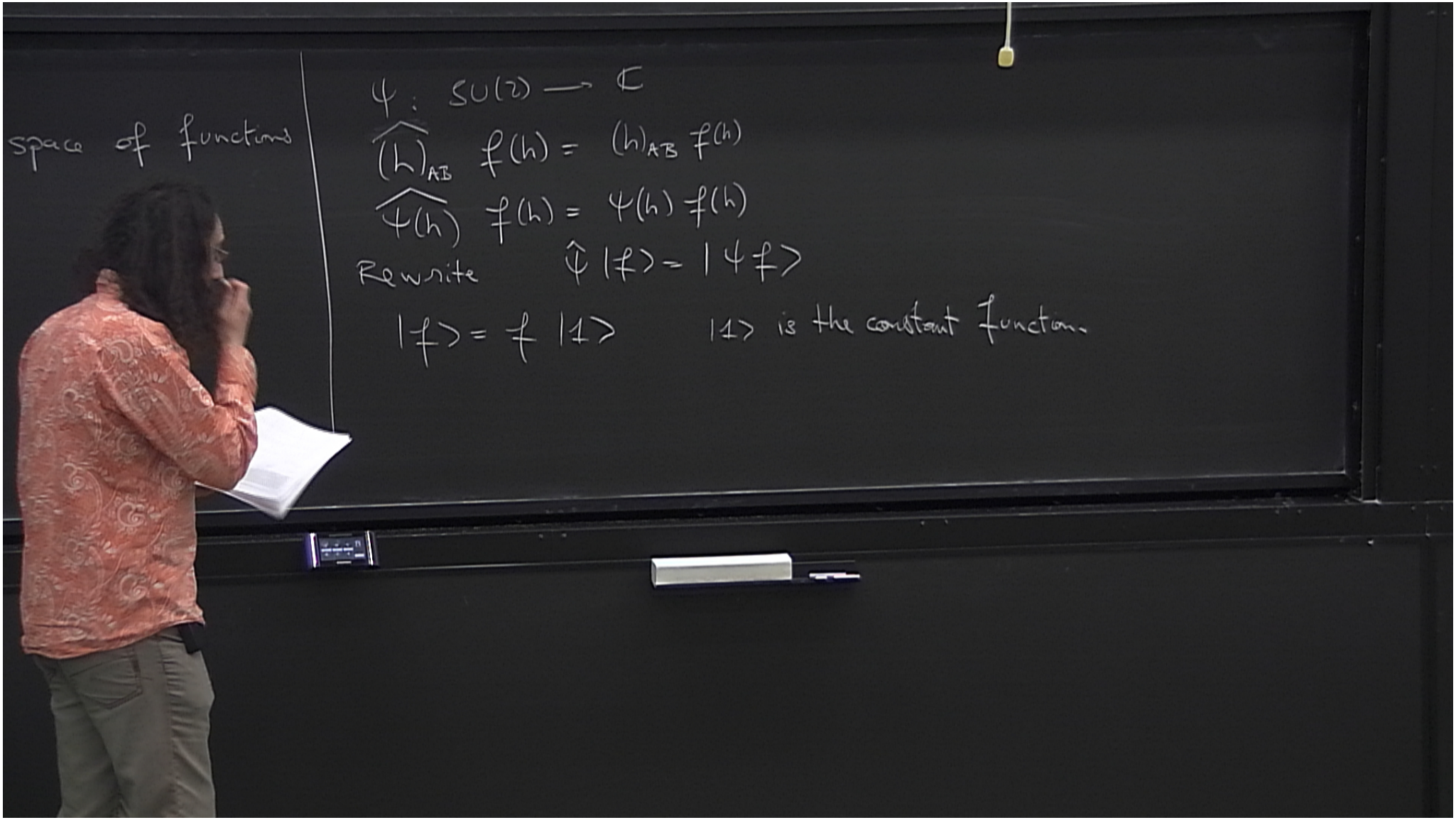
$\widehat{(h)_{AB}}\, f(h) = (h)_{AB}\, f(h)$

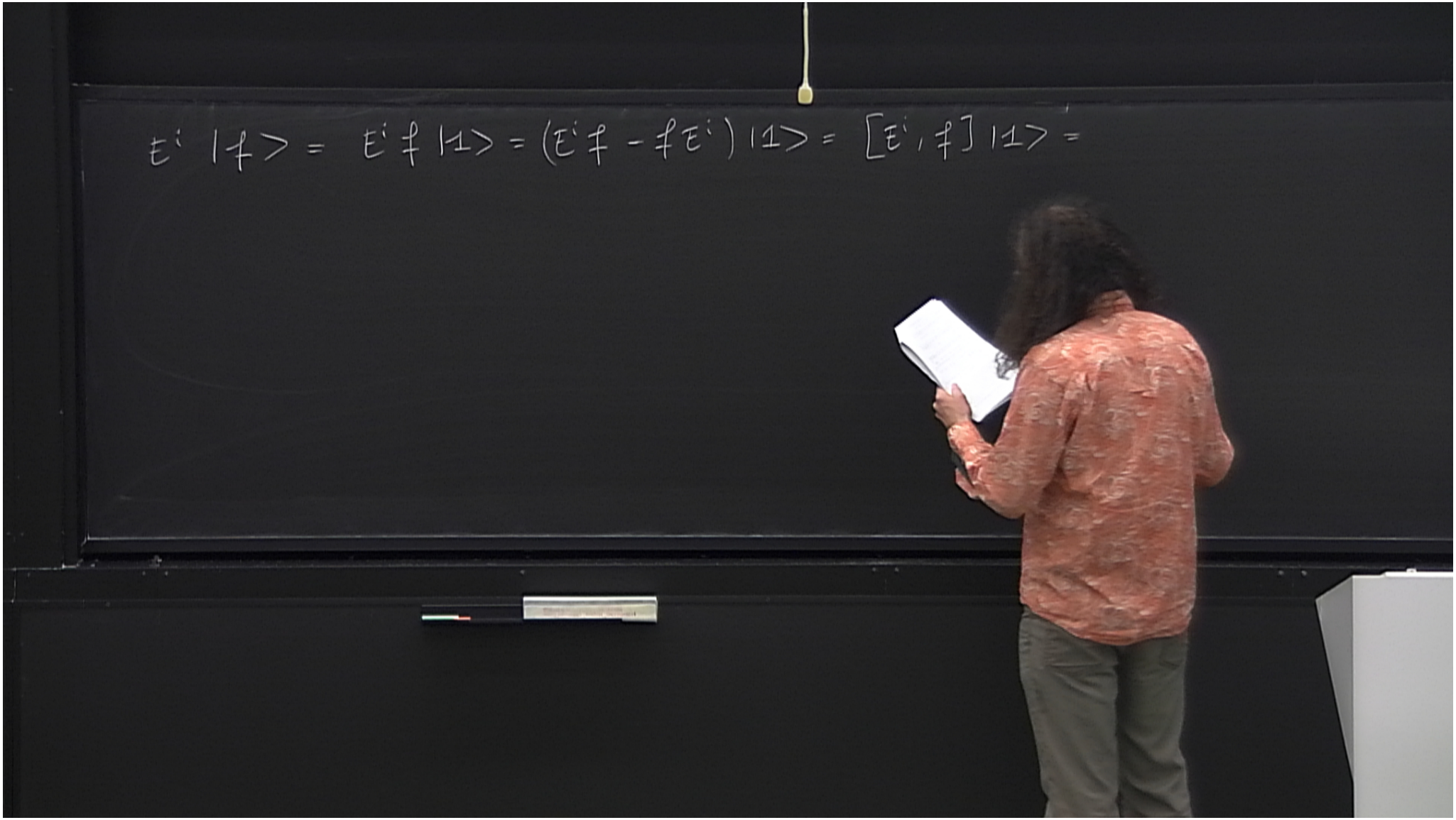
$\widehat{\psi(h)}\, f(h) = \psi(h)\, f(h)$

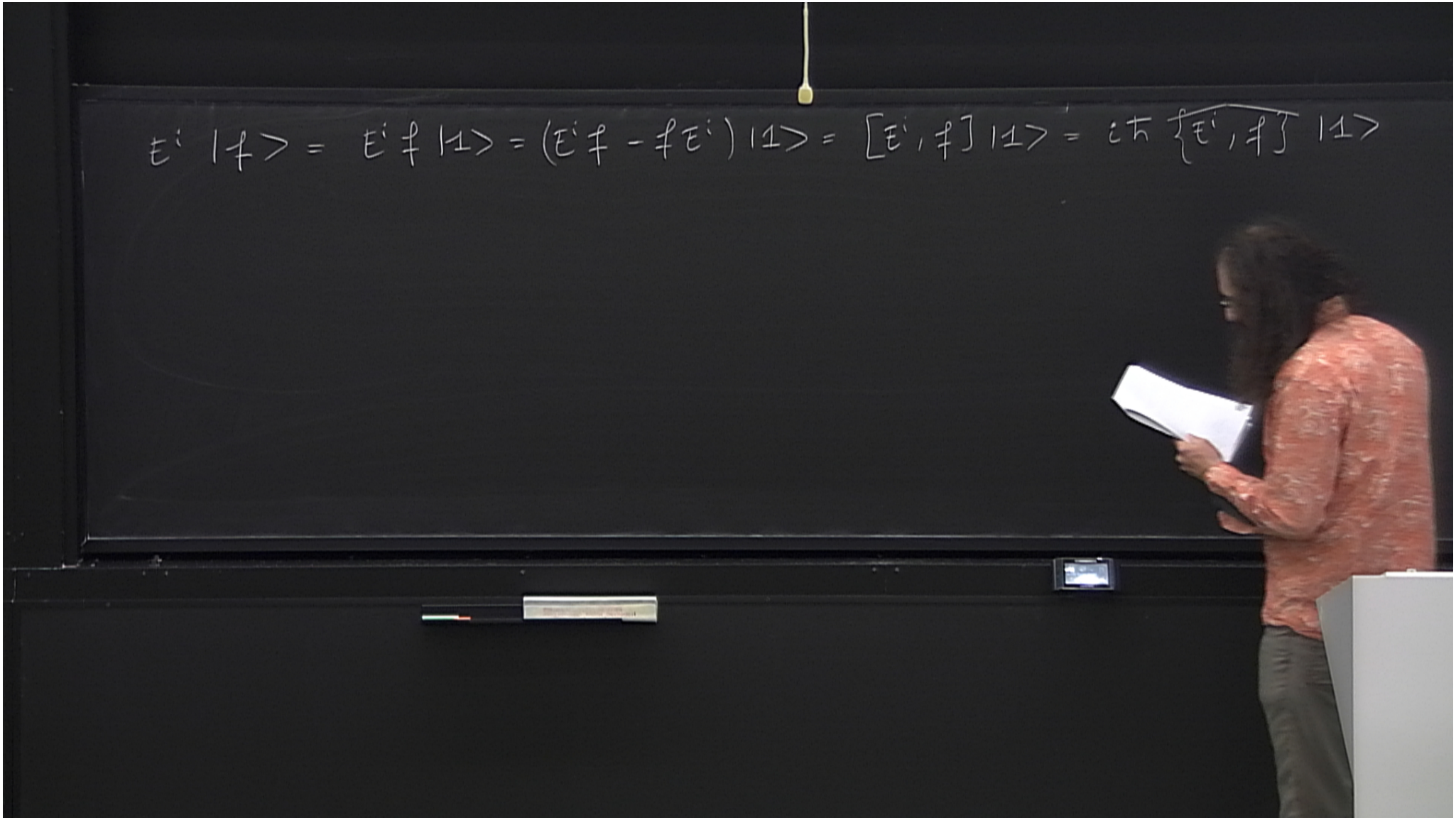


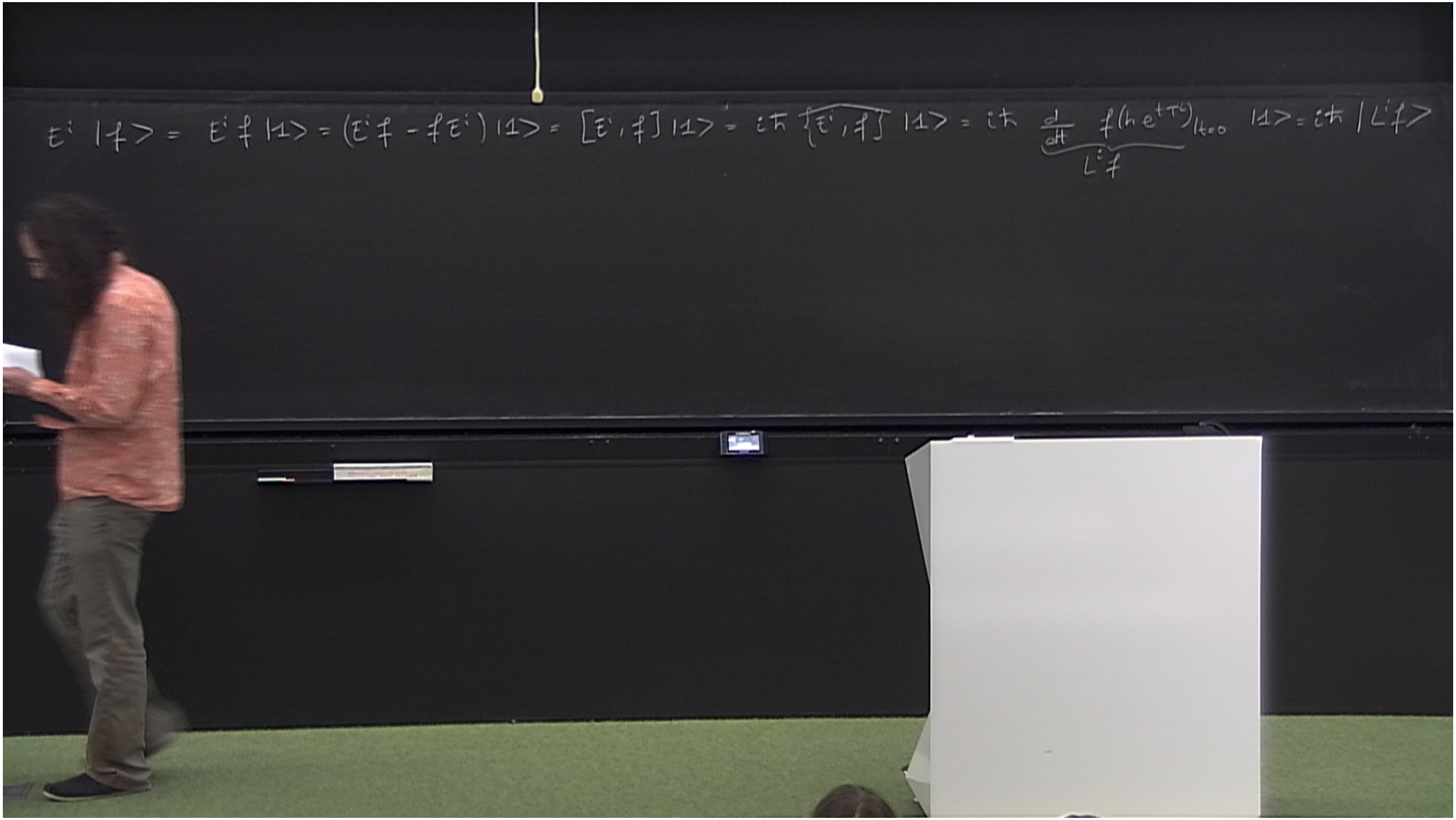

space of functions
$\psi : SU(2) \longrightarrow \mathbb{C}$
$\widehat{(h)_{AB}}\, f(h) = (h)_{AB}\, f(h)$
$\widehat{\psi(h)}\, f(h) = \psi(h)\, f(h)$
Rewrite
$\hat{\psi}\,|f\rangle = |\psi f\rangle$


space of functions
$\psi : SU(2) \longrightarrow \mathbb{C}$
$\widehat{(h)_{AB}}\, f(h) = (h)_{AB}\, f(h)$
$\widehat{\psi(h)}\, f(h) = \psi(h)\, f(h)$
Rewrite
$\hat{\psi}\,|f\rangle = |\psi f\rangle$
$|f\rangle = f\,|1\rangle$


space of functions
$\psi : SU(2) \longrightarrow \mathbb{C}$
$\widehat{(h)}_{AB}\, f(h) = (h)_{AB}\, f(h)$
$\widehat{\psi(h)}\, f(h) = \psi(h)\, f(h)$
Rewrite $\hat{\psi}\,|f\rangle = |\psi f\rangle$
$|f\rangle = f\,|1\rangle$ $|1\rangle$ is the constant function.


$\hat{E}^i |f\rangle = \hat{E}^i \hat{f} |1\rangle = (\hat{E}^i \hat{f} - \hat{f}\hat{E}^i) |1\rangle = [\hat{E}^i, \hat{f}] |1\rangle =$

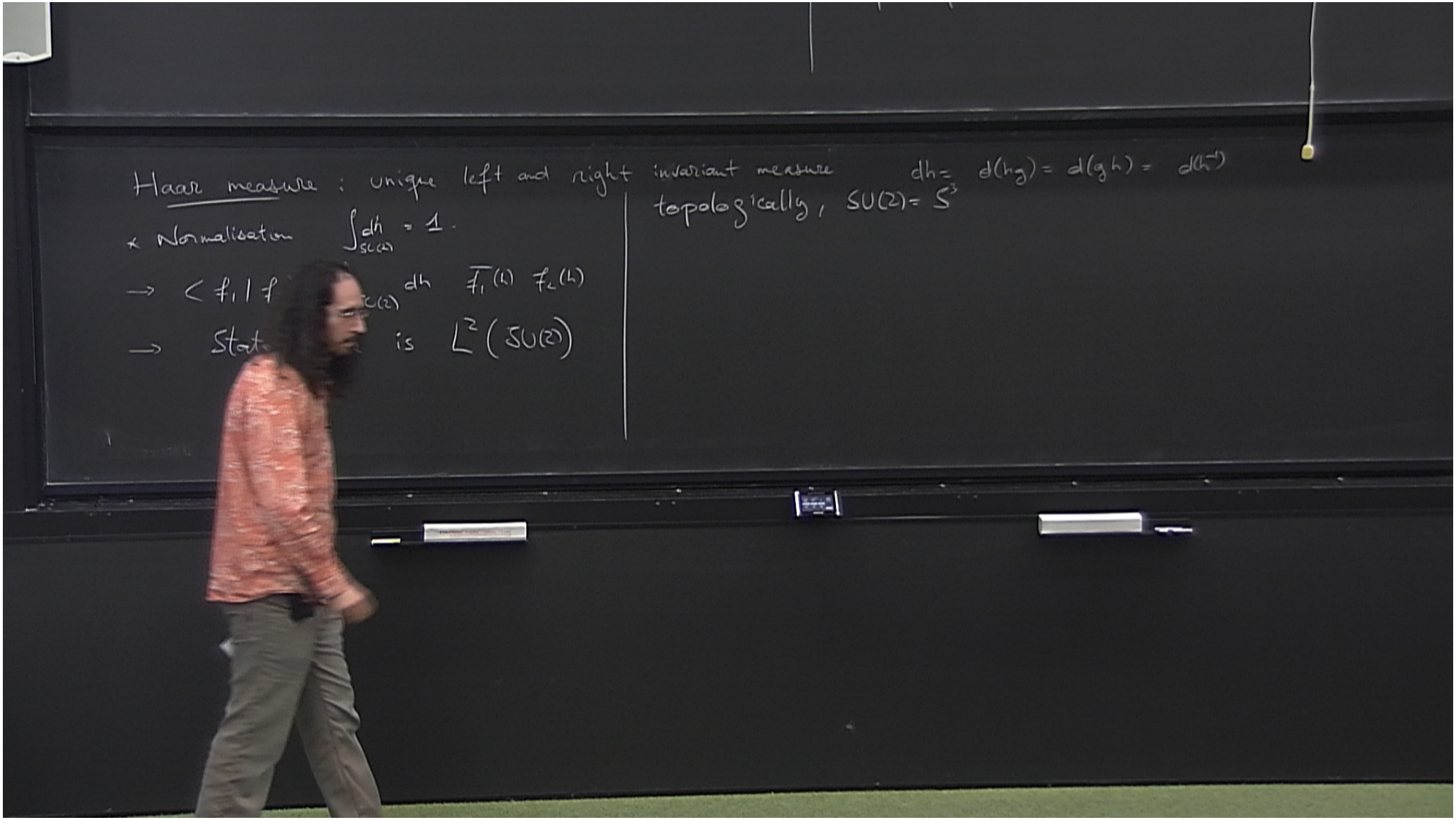

$E^i |f\rangle = E^i f |1\rangle = (E^i f - f E^i) |1\rangle = [E^i, f] |1\rangle = i\hbar \{E^i, f\} |1\rangle$

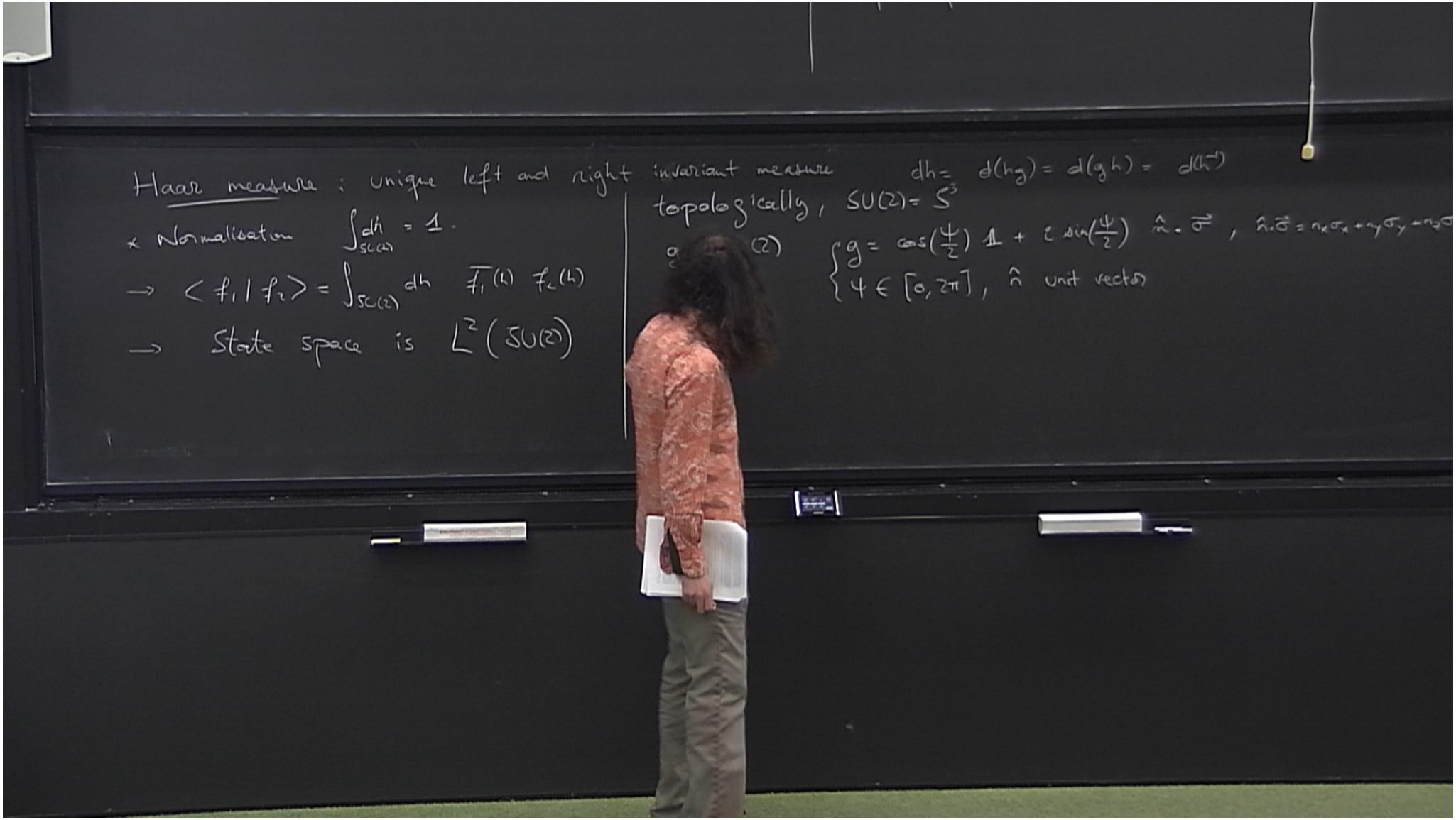

$L^i |f\rangle = L^i f |1\rangle = (L^i f - f L^i) |1\rangle = [L^i, f] |1\rangle = i\hbar \{L^i, f\} |1\rangle = i\hbar \underbrace{\frac{d}{dt} f(h e^{tY})_{|t=0}}_{L^i f} |1\rangle = i\hbar |L^i f\rangle$

$$\hat{E}^i |f\rangle = \hat{E}^i f |1\rangle \underset{(\hat{E}^i|1\rangle = 0)}{=} (\hat{E}^i f - f \hat{E}^i)|1\rangle = [\hat{E}^i, f]|1\rangle = i\hbar \{\hat{E}^i, f\}|1\rangle = i\hbar \underbrace{\frac{d}{dt} f(h e^{t\tau^i})_{|t=0}}_{L^i f} |1\rangle = i\hbar |L^i f\rangle$$

* Exponentiated fluxes should act as unitary operators
  ↳ generate right translations.

$$\langle f_1 | f_2 \rangle = \int_{SU(2)} dh \; \overline{f_1(h)} \; f_2(h) = \int_{SU(2)} dh \; \overline{f_1(hg)} \; f_2(hg) \quad \forall g$$


Haar measure : unique left and right invariant measure dh = d(hg) = d(gh) = d(h⁻¹)
topologically, SU(2) = S³
× Normalisation ∫ dh = 1
→ < f₁ | f
dh f̄₁(h) f₂(h)
→ Stat
is L²(SU(2))


Haar measure : unique left and right invariant measure dh = d(hg) = d(gh) = d(h⁻¹)
* Normalisation ∫ dh = 1
→ ⟨f₁ | f₂⟩ = ∫ dh f̄₁(h) f₂(h)
→ State space is L²(SU(2))
topologically, SU(2) = S³
g ∈ SU(2)
g = cos(ψ/2) 𝟙 + i sin(ψ/2) n̂ · σ⃗ , n̂ · σ⃗ = n_x σ_x + n_y σ_y + n_z σ
ψ ∈ [0, 2π], n̂ unit vector

Haar measure : unique left and right invariant measure $dh = d(hg) = d(gh) = d(h^{-1})$

topologically, $SU(2) = S^3$

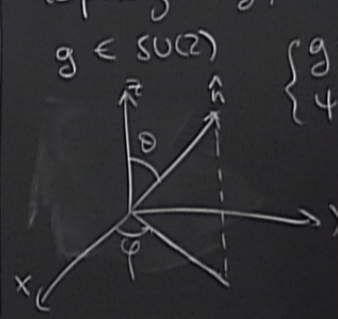
* Normalisation $\int_{SU(2)} dh = 1$.

→ $\langle f_1 | f_2 \rangle = \int_{SU(2)} dh \; \overline{f_1}(h) \, f_2(h)$

→ State space is $L^2(SU(2))$

$g \in SU(2)$ $\quad \begin{cases} g = \cos(\frac{\psi}{2}) + i \sin(\frac{\psi}{2}) \; \hat{n} \cdot \vec{\sigma} \;, \quad \hat{n} \cdot \vec{\sigma} = n_x \sigma_x + n_y \sigma_y + n_z \sigma \\ \psi \in [\quad \hat{n} \text{ unit vector} \quad \hat{n} = (\theta, \varphi) \end{cases}$

$\frac{1}{4\pi^2} (\sin \frac{\psi}{2})^2 \, d\psi \, \sin\theta \, d\theta \, d\varphi$

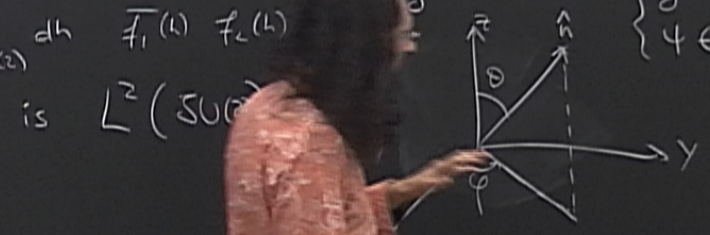


Haar measure : unique left and right invariant measure $dh = d(hg) = d(gh) = d(h^{-1})$

topologically, $SU(2) = S^3$

* Normalisation $\int_{SU(2)} dh = 1$.

$g \in SU(2)$ $\begin{cases} g = \cos\left(\frac{\psi}{2}\right) \mathbb{1} + i \sin\left(\frac{\psi}{2}\right) \hat{n} \cdot \vec{\sigma}, \quad \hat{n}\cdot\vec{\sigma} = n_x\sigma_x + n_y\sigma_y + n_z\sigma \\ \psi \in [0, 2\pi], \ \hat{n} \text{ unit vector} \quad \hat{n} = (\theta, \varphi) \end{cases}$



$\rightarrow \langle f_1 | f_2 \rangle = \int_{SU(2)} dh \ \overline{f_1}(h) \ f_2(h)$

$\rightarrow$ State space is $L^2(SU($

$dg = \frac{1}{4\pi^2} \left(\sin\frac{\psi}{2}\right)^2 d\psi \sin\theta \, d\theta \, d\varphi$

Haar measure : unique left and right invariant measure $dh = d(hg) = d(gh) = d(h^{-1})$

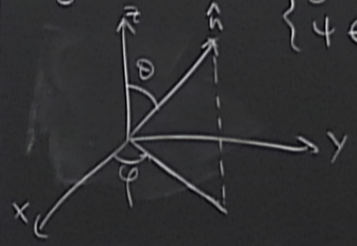
* Normalisation $\int_{SU(2)} dh = 1$.

→ $\langle f_1 | f_2 \rangle = \int_{SU(2)} dh \; \overline{f_1}(h) \, f_2(h)$

→ State space is $L^2(SU(2))$

topologically, $SU(2) = S^3$

$g \in SU(2)$ $\begin{cases} g = \cos\left(\frac{\psi}{2}\right) \mathbb{1} + i \sin\left(\frac{\psi}{2}\right) \hat{n} \cdot \vec{\sigma} \;, \quad \hat{n} \cdot \vec{\sigma} = n_x \sigma_x + n_y \sigma_y + n_z \sigma \\ \psi \in [0, 2\pi] \;, \quad \hat{n} \text{ unit vector} \end{cases}$



$dg = \frac{1}{4\pi^2} (\sin \frac{\psi}{2}$ … $d\varphi$

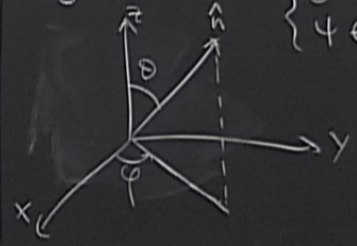
$= \delta(x_1^2 +$ … dx

Haar measure : unique left and right invariant measure $dh = d(hg) = d(gh) = d(h^{-1})$

topologically, $SU(2) = S^3$

* Normalisation $\int_{SU(2)} dh = 1$.

$g \in SU(2)$ $\begin{cases} g = \cos\left(\frac{\psi}{2}\right) \mathbb{1} + i \sin\left(\frac{\psi}{2}\right) \hat{n} \cdot \vec{\sigma} \,, \quad \hat{n}\cdot\vec{\sigma} = n_x\sigma_x + n_y\sigma_y + n_z\sigma \\ \psi \in [0, 2\pi] \,, \ \hat{n} \text{ unit vector} \quad \hat{n} = (\theta, \varphi) \end{cases}$

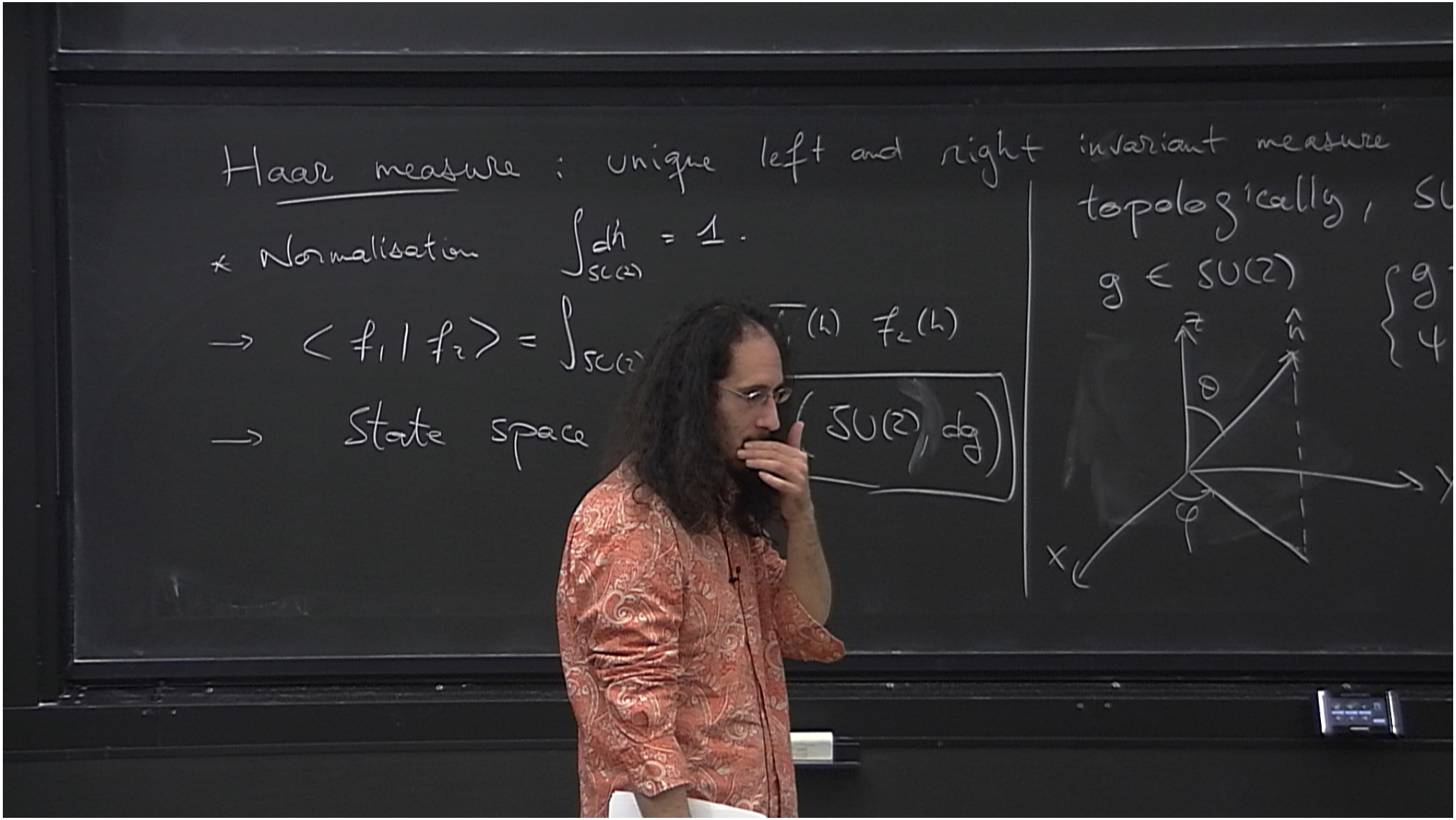


$\rightarrow \langle f_1 | f_2 \rangle = \int_{SU(2)} dh \ \overline{f_1}(h) \ f_2(h)$

$\rightarrow$ State space is $\boxed{L^2(SU(2), dg)}$

$$\boxed{dg = \frac{1}{4\pi^2} \left(\sin\frac{\psi}{2}\right)^2 d\psi \sin\theta \, d\theta \, d\varphi}$$

$$= \delta\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right) dx_1 dx_2 dx_3 dx_4$$


Haar measure : unique left and right invariant measure
* Normalisation $\int_{SU(2)} dh = 1$.
$\rightarrow \langle f_1 | f_2 \rangle = \int_{SU(2)}$ $\bar{f}_1(h)\ f_2(h)$
$\rightarrow$ State space $(SU(2), dg)$
topologically, SU
$g \in SU(2)$
z
n
θ
φ
x

Haar measure : unique left and right invariant measure

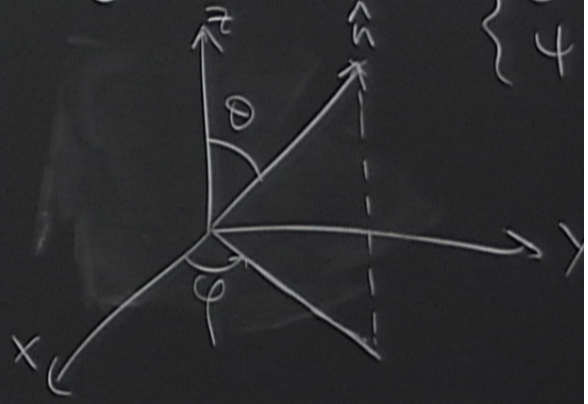
* Normalisation $\int_{SU(2)} dh = 1$.

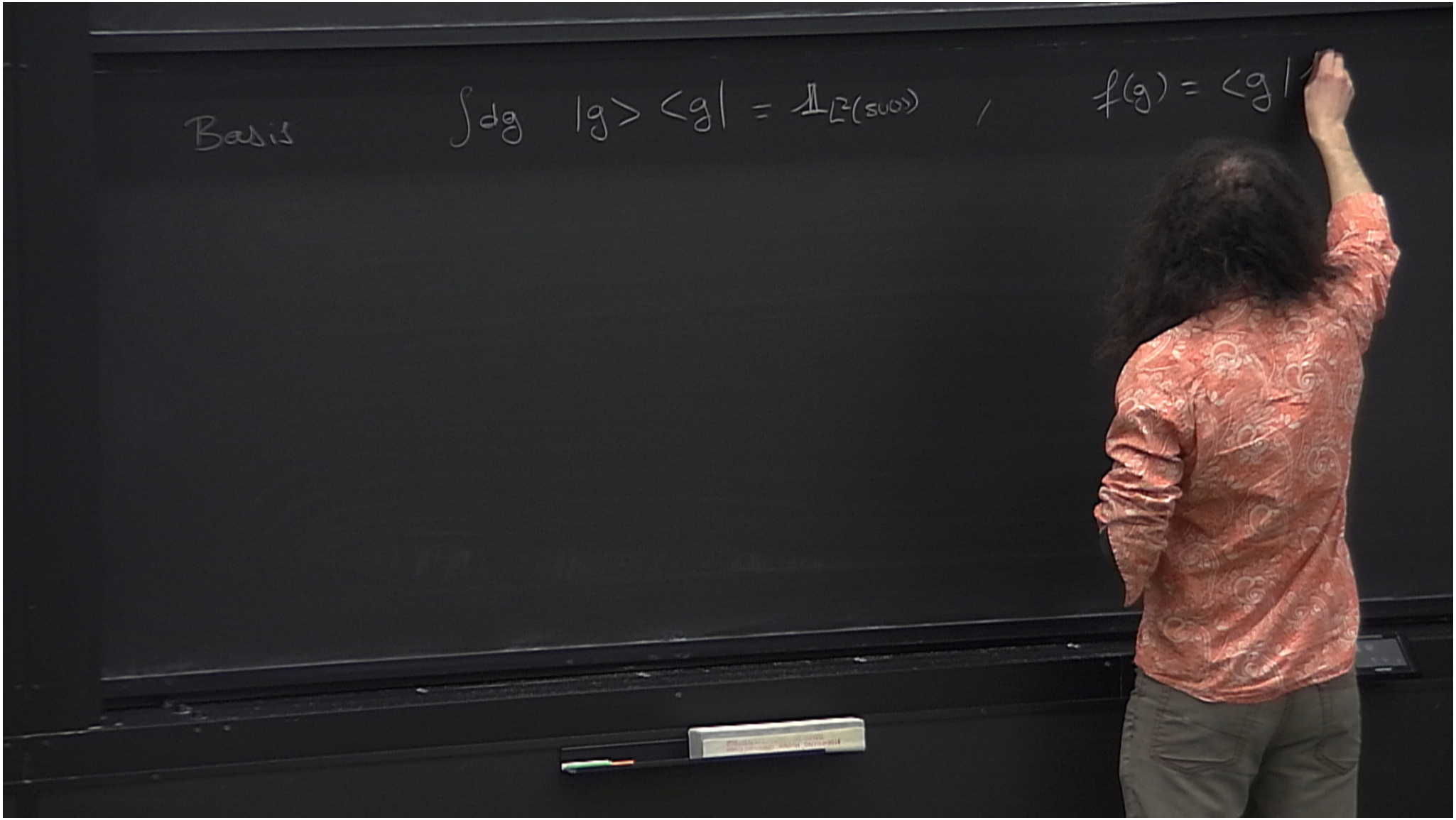
$\rightarrow \langle f_1 | f_2 \rangle = \int_{SU(2)} dh \; \overline{f_1}(h) \; f_2(h)$

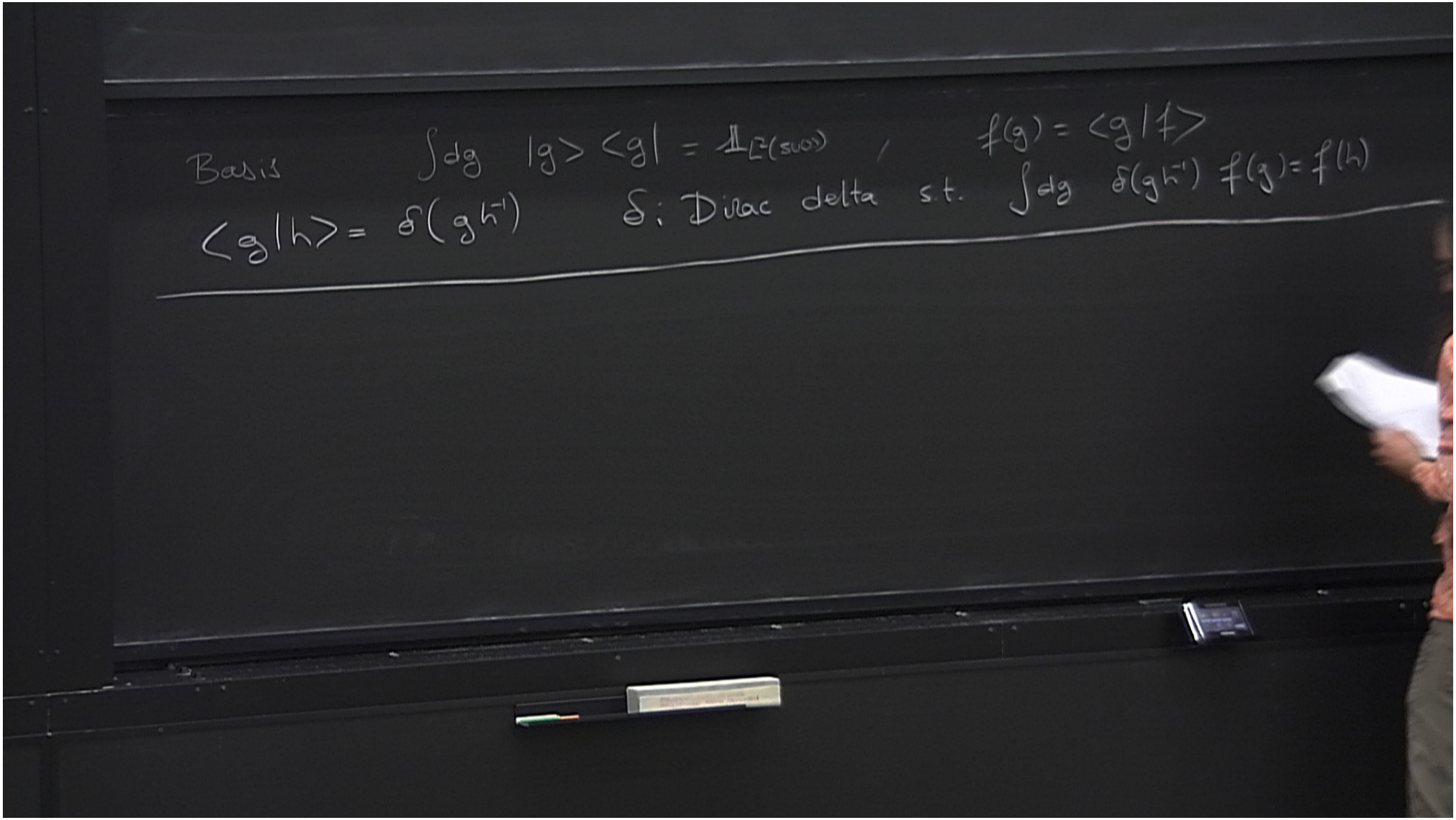
$\rightarrow$ State space is $\boxed{L^2(SU(2), dg)}$

topologically, SU

$g \in SU(2)$




Basis
$\int dg \; |g\rangle\langle g| = \mathbb{1}_{L^2(SU(2))}$
$f(g) = \langle g|$


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$\langle g|h\rangle = \delta(gh^{-1})$ δ: Dirac delta s.t. $\int dg\ \delta(gh^{-1}) f(g) = f(h)$

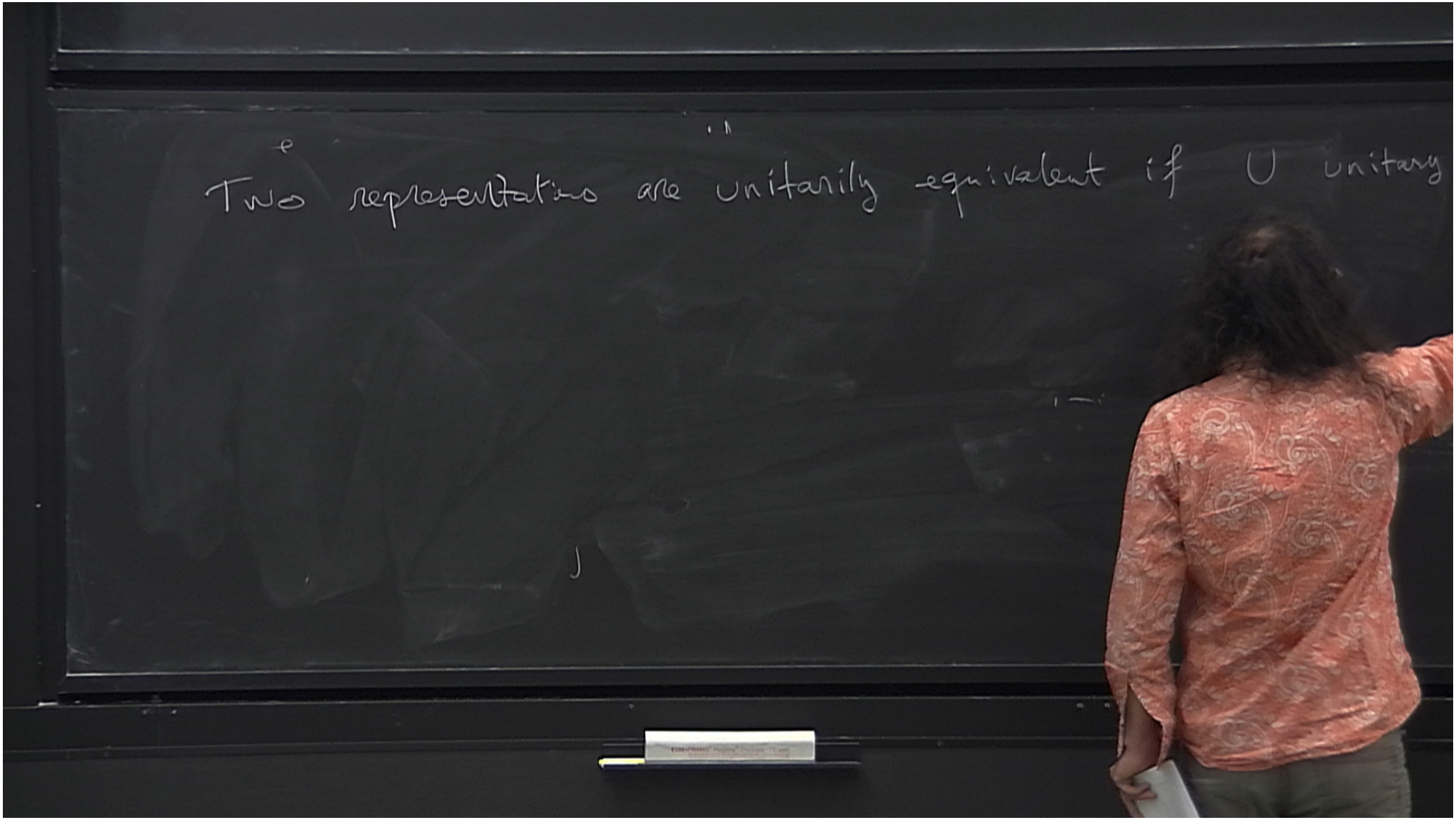
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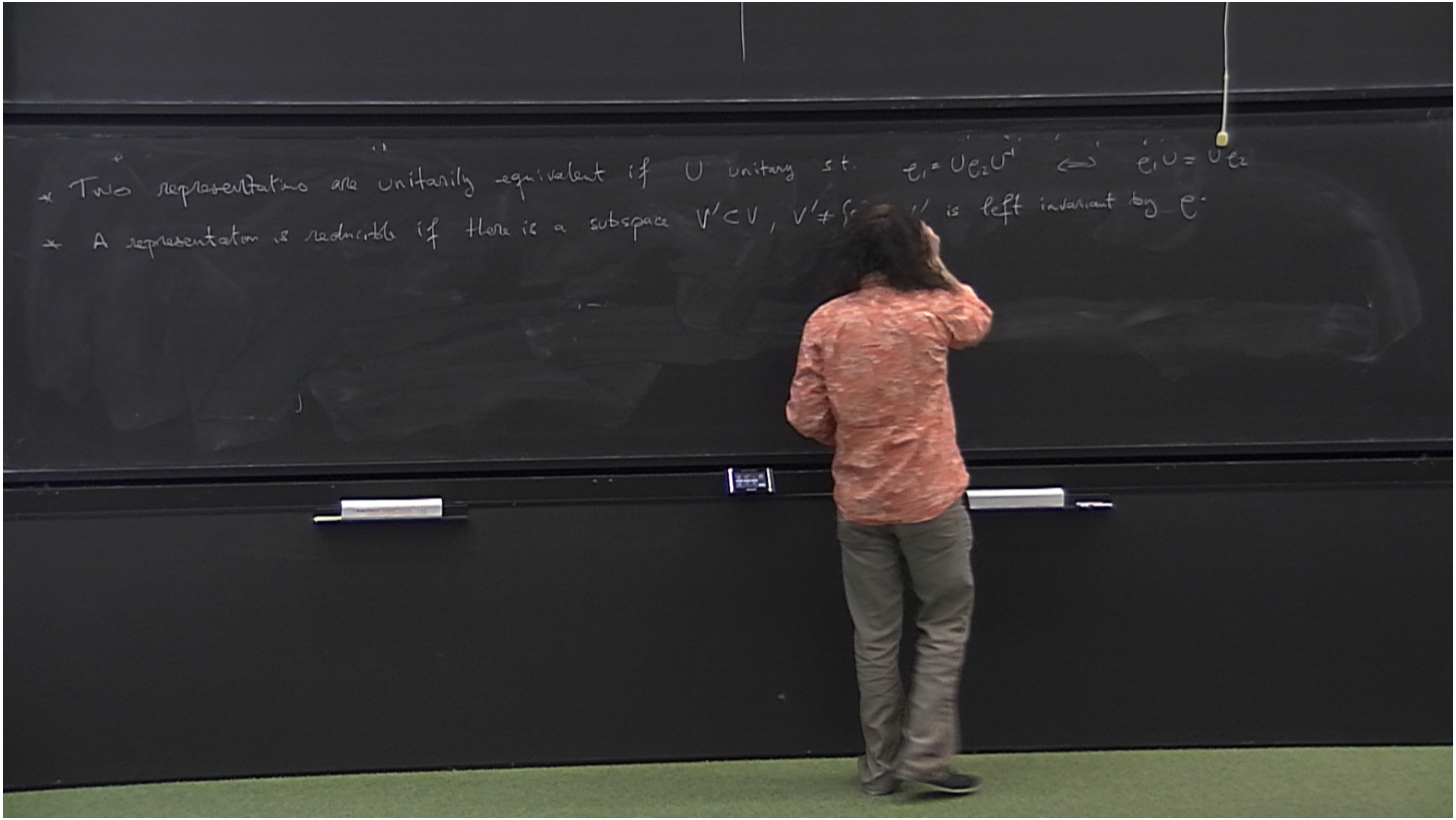
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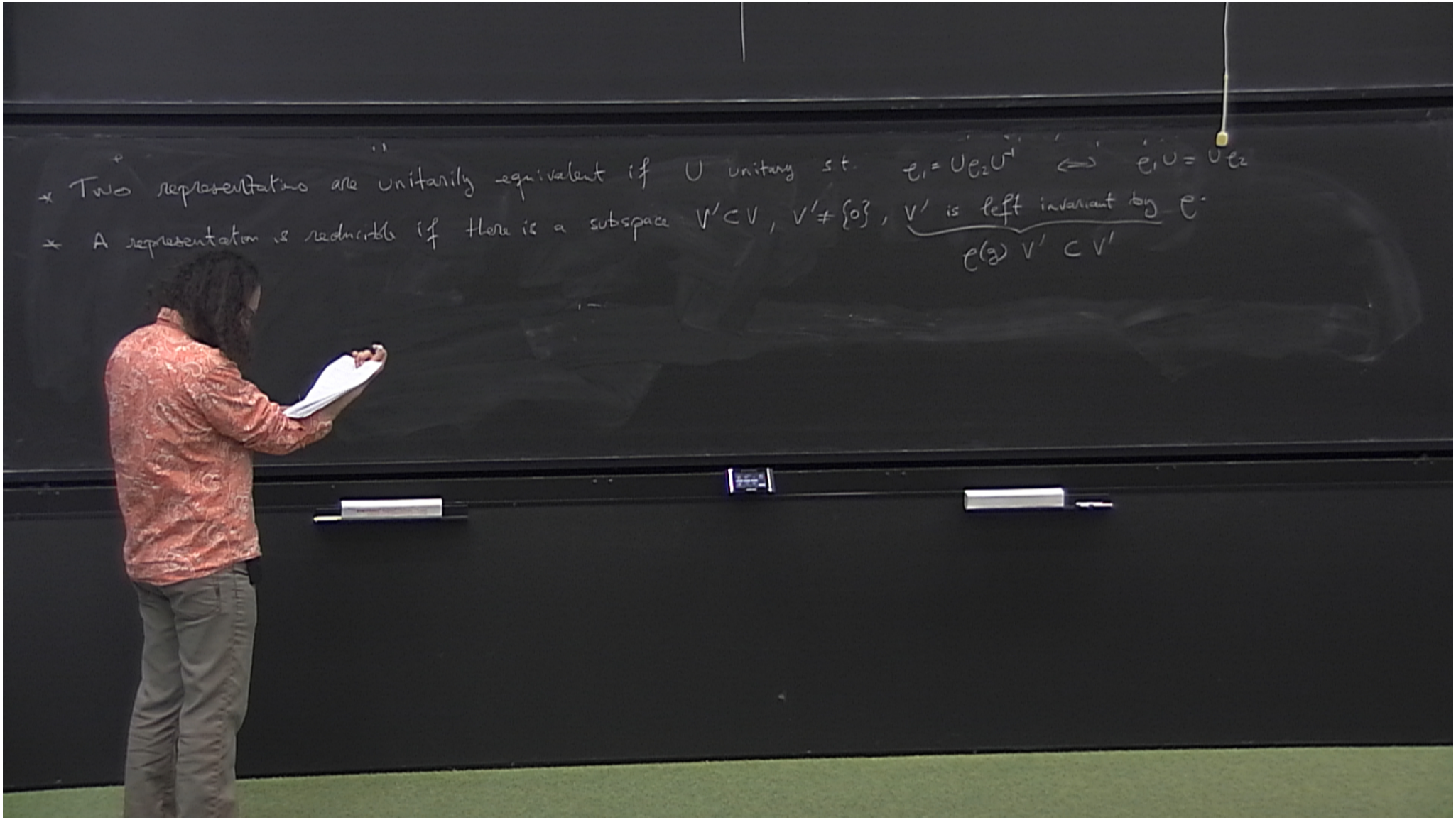
Representation theory $\rho: G \longrightarrow GL(V)$ V: Hilbert space.

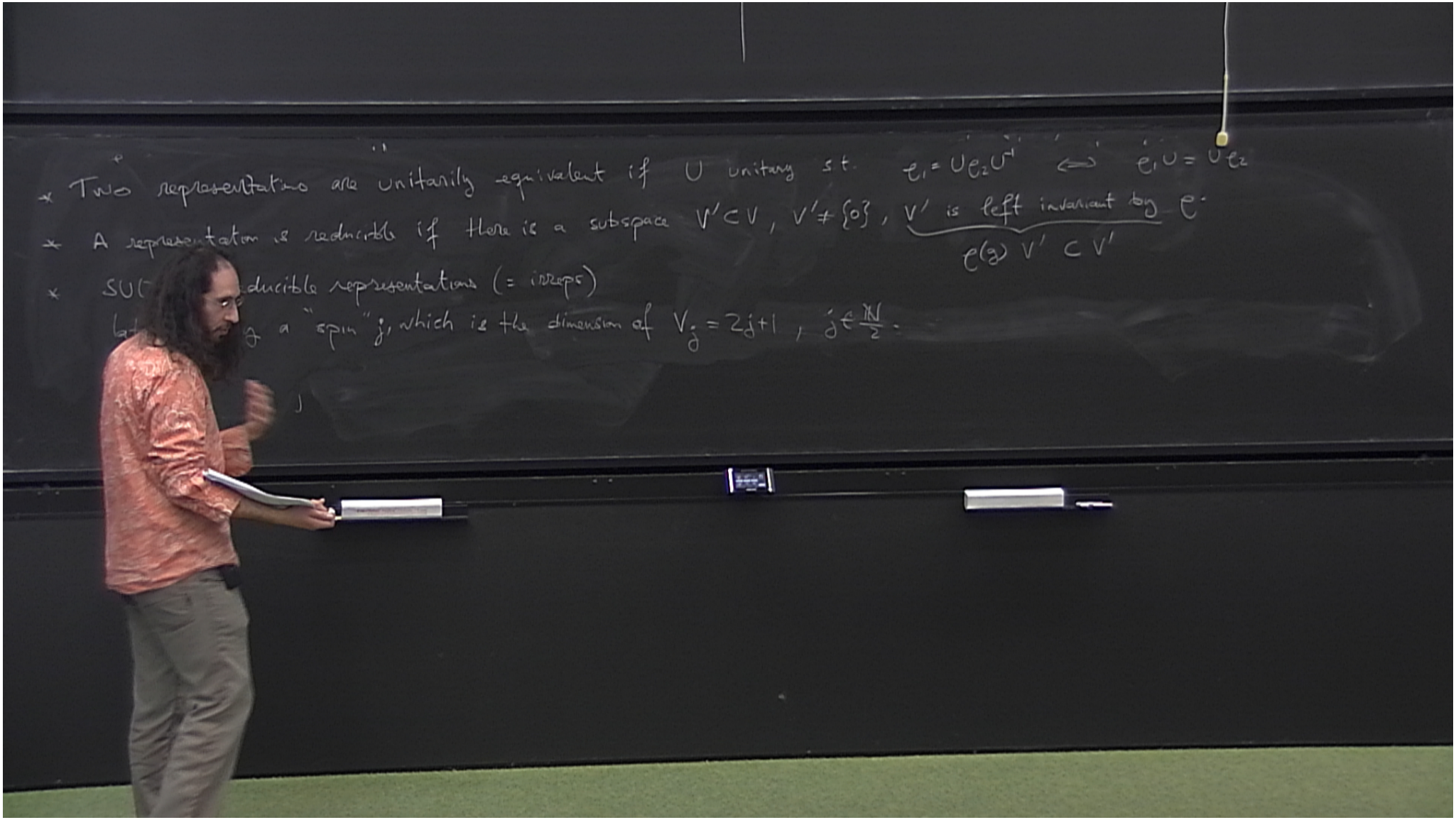
$g \longmapsto \rho(g)$ operator on V.

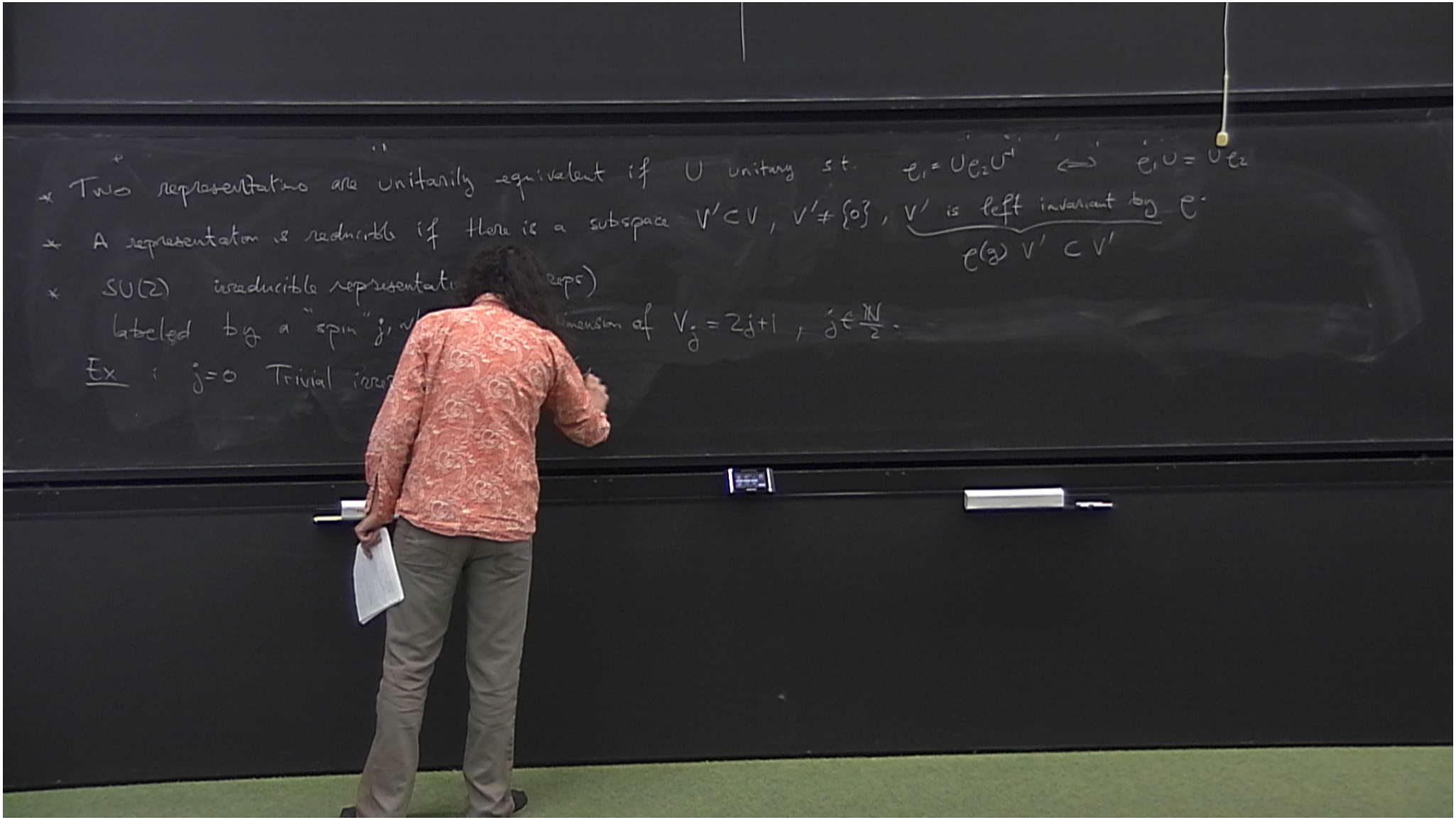
* Unitary representation: $\rho(g)$ is unitary $\forall g \in G$.


Two representations are unitarily equivalent if U unitary


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$\rho(g)\, V' \subset V'$


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* SU(ducible representations (= irreps)
a "spin" j, which is the dimension of $V_j = 2j+1$, $j \in \frac{\mathbb{N}}{2}$.

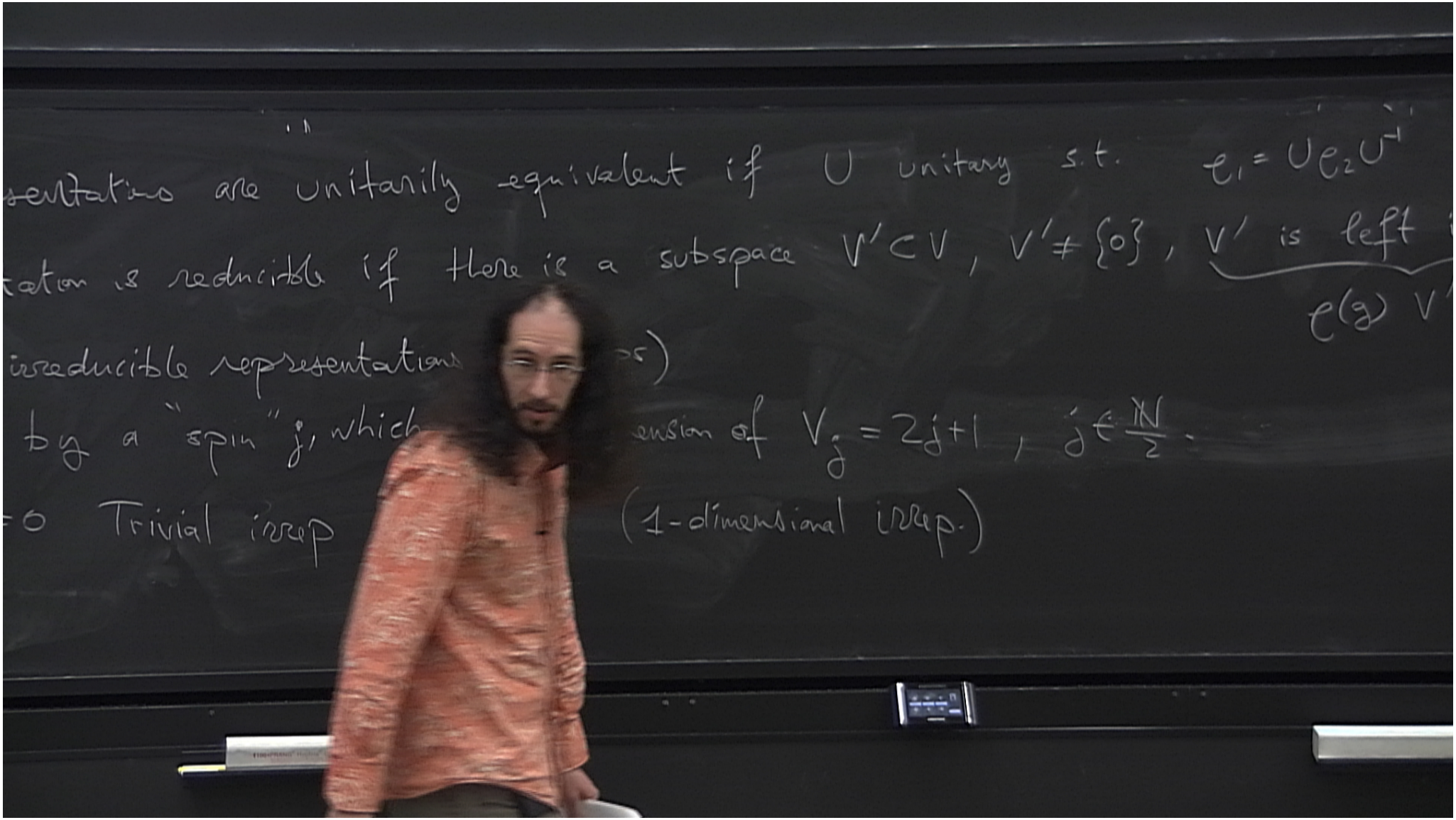

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* SU(2) irreducible representations (reps)
labeled by a "spin" j, ... dimension of $V_j = 2j+1$, $j \in \frac{\mathbb{N}}{2}$.
Ex : $j=0$ Trivial

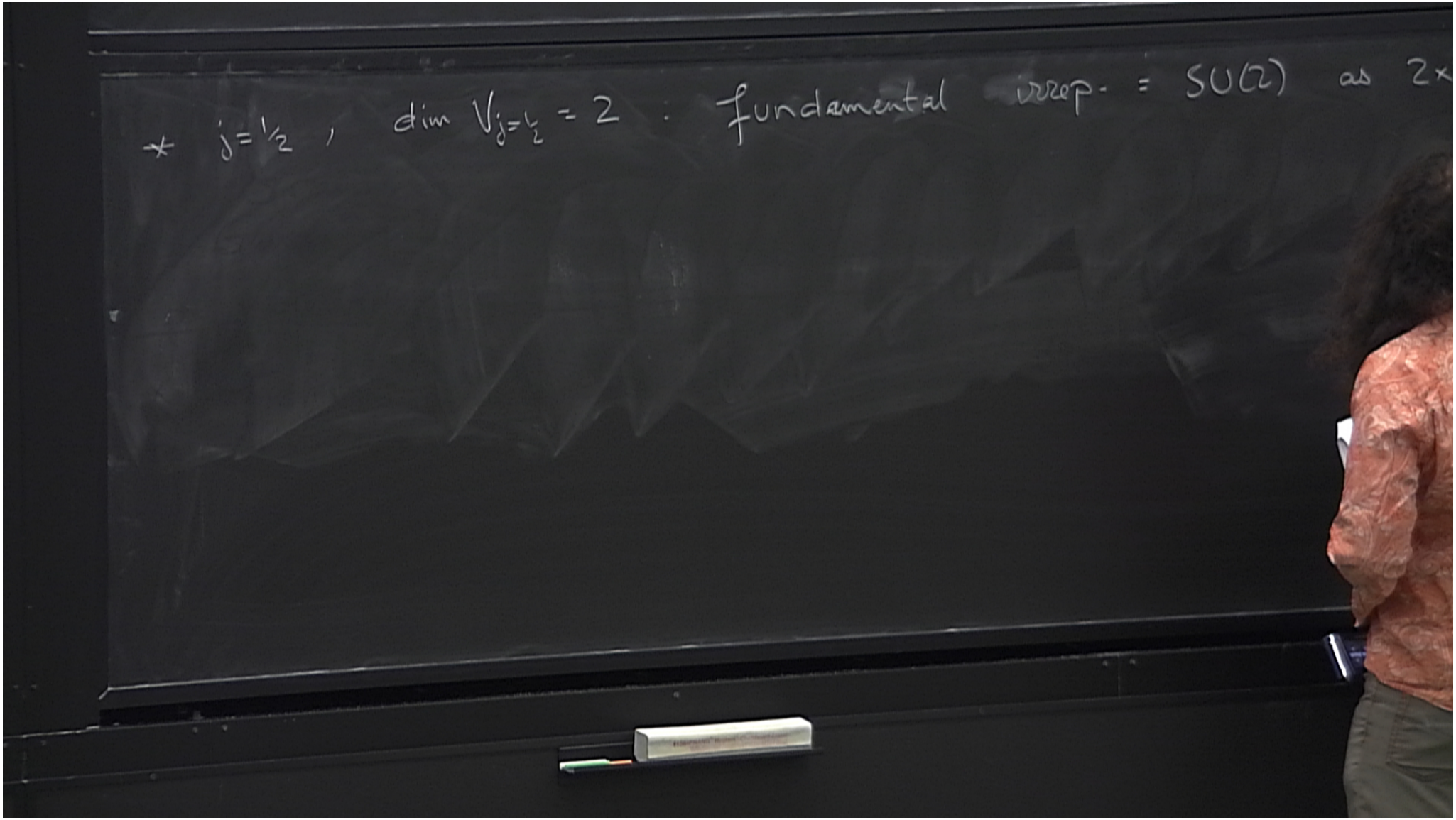
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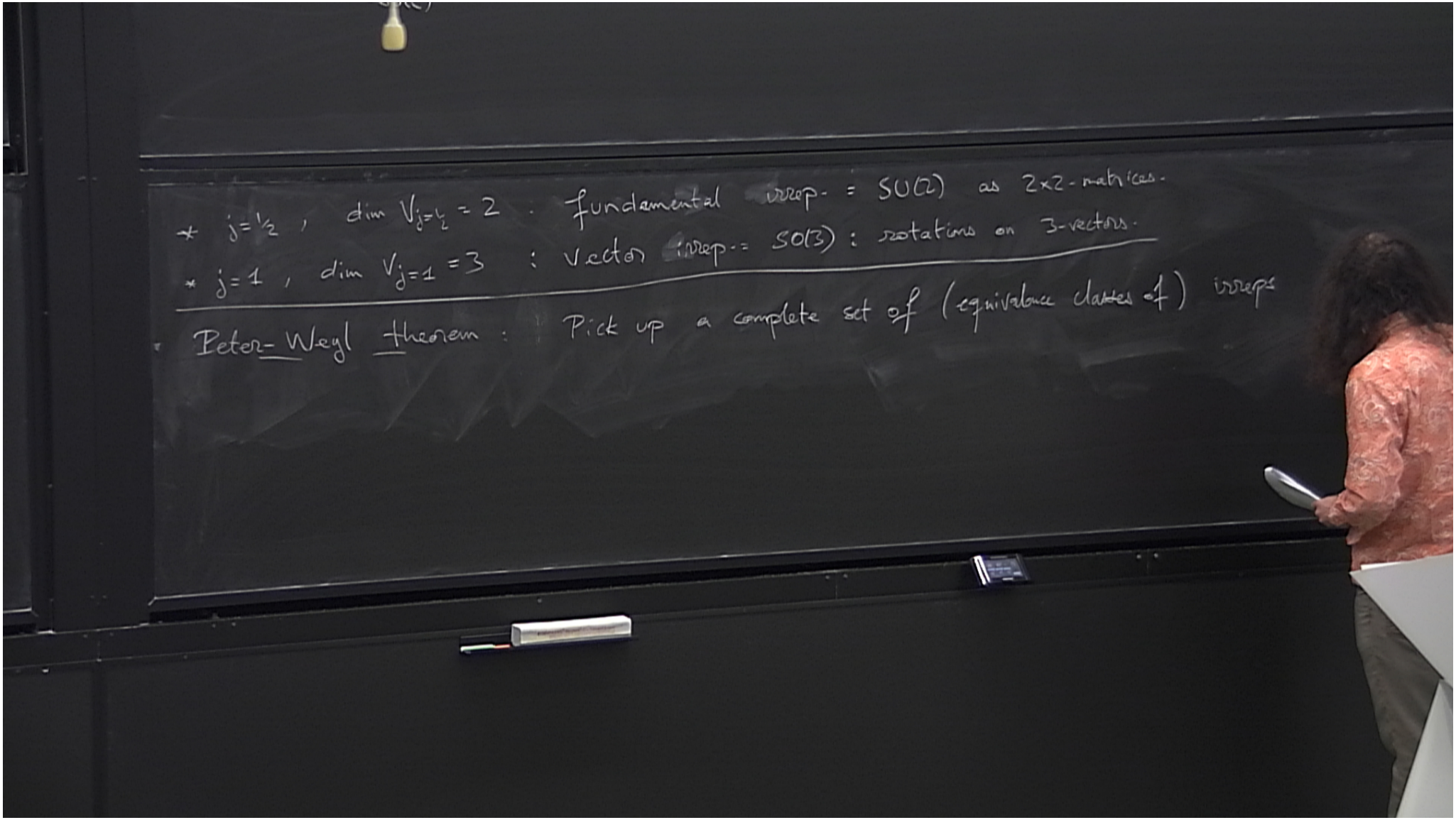
* A representation is reducible if there is a subspace $V' \subset V$, $V' \neq \{0\}$, V' is left invariant by ρ. ($\rho(g)\, V' \subset V'$)

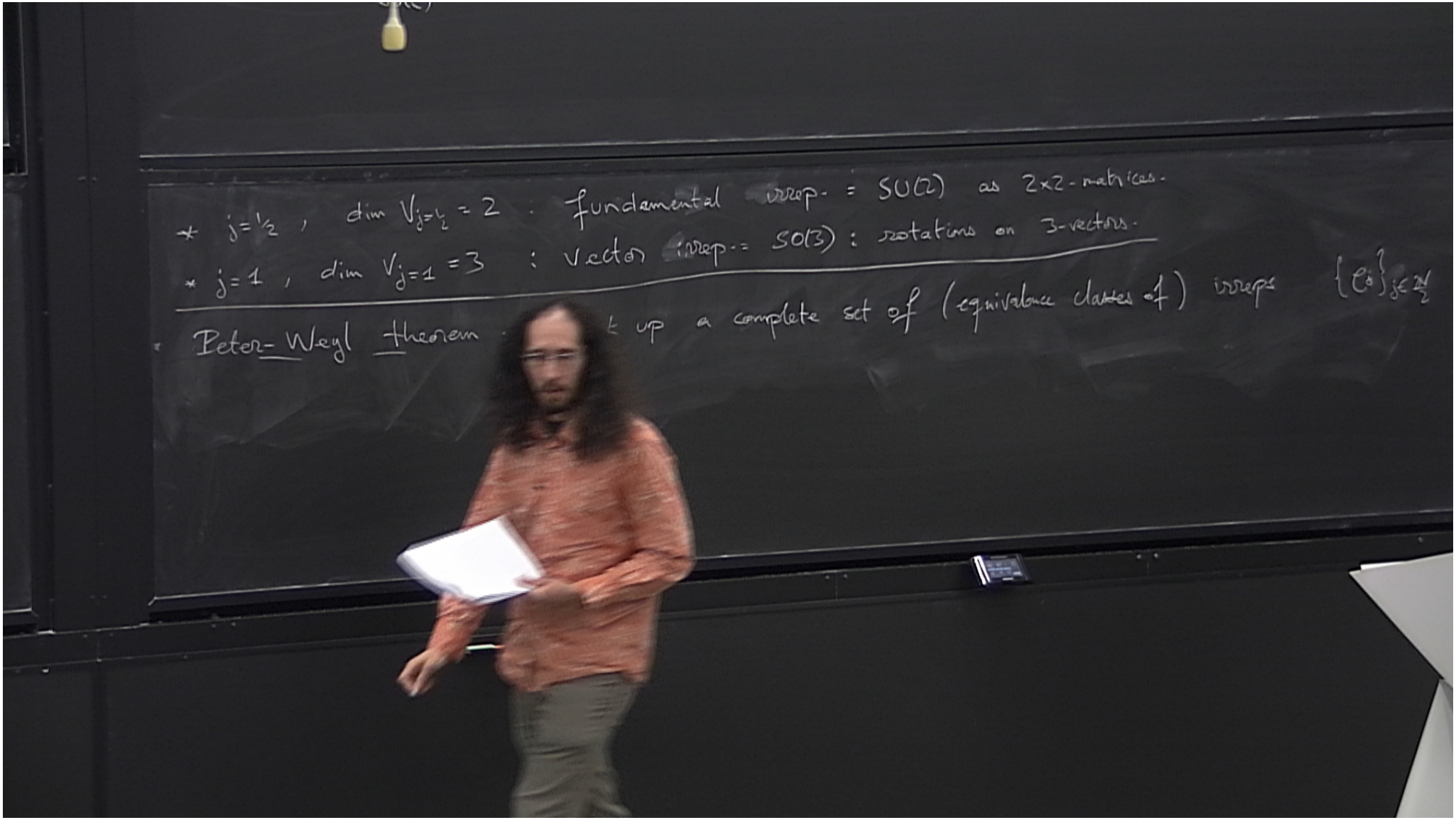
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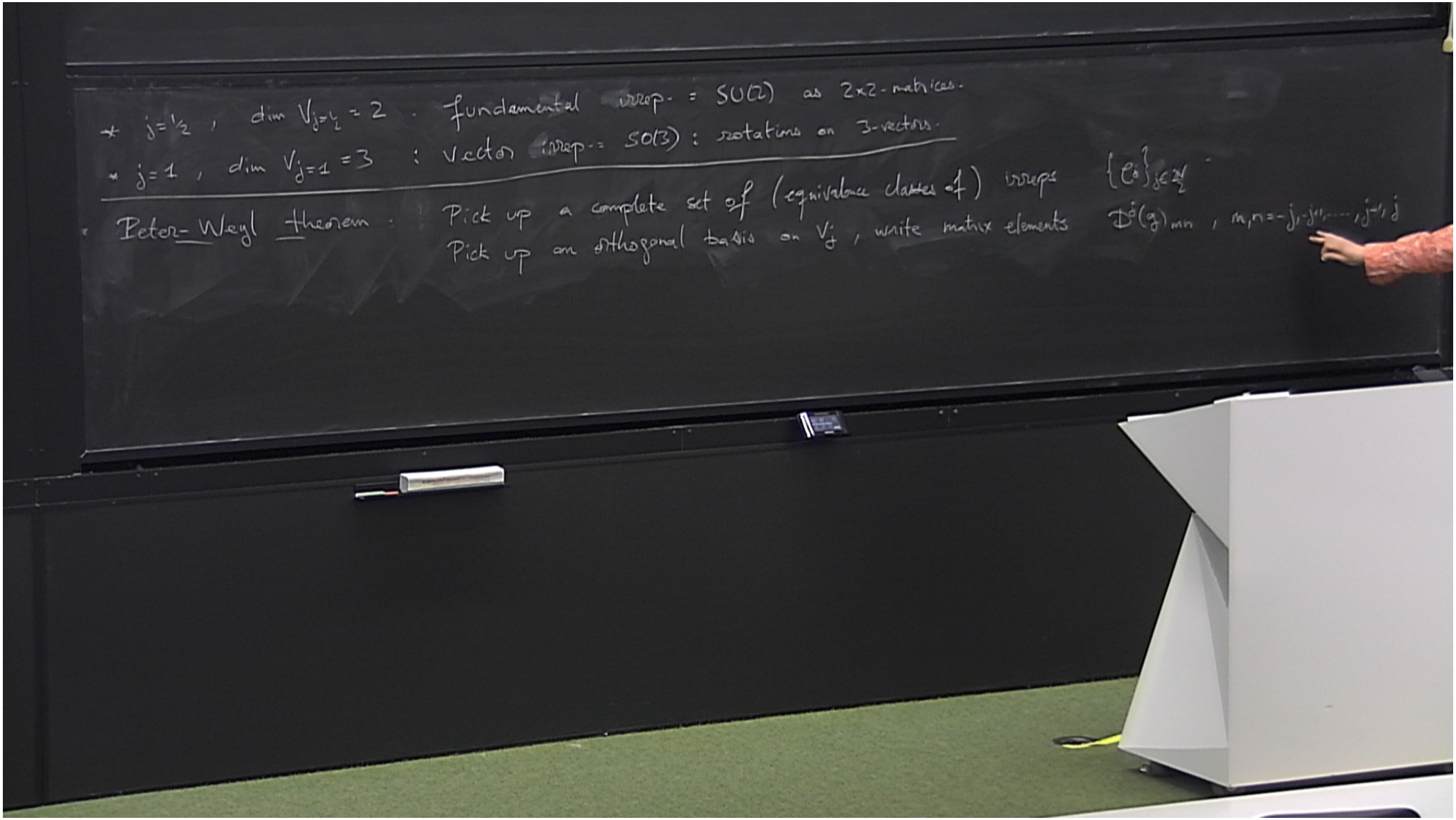
Ex : $j=0$ Trivial irrep $\rho(g) = \mathbb{1}$ (1-dimensional irrep.)

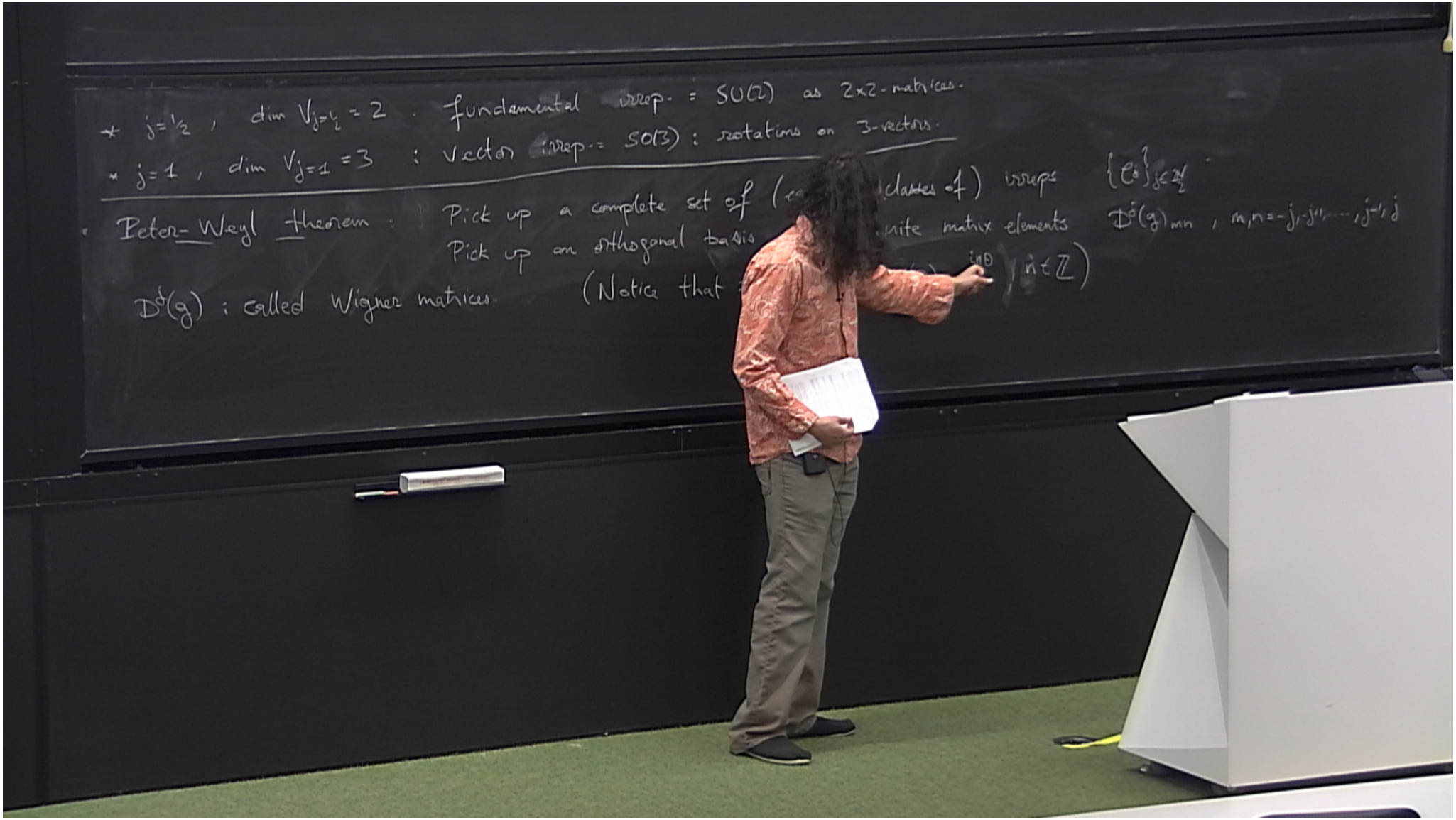

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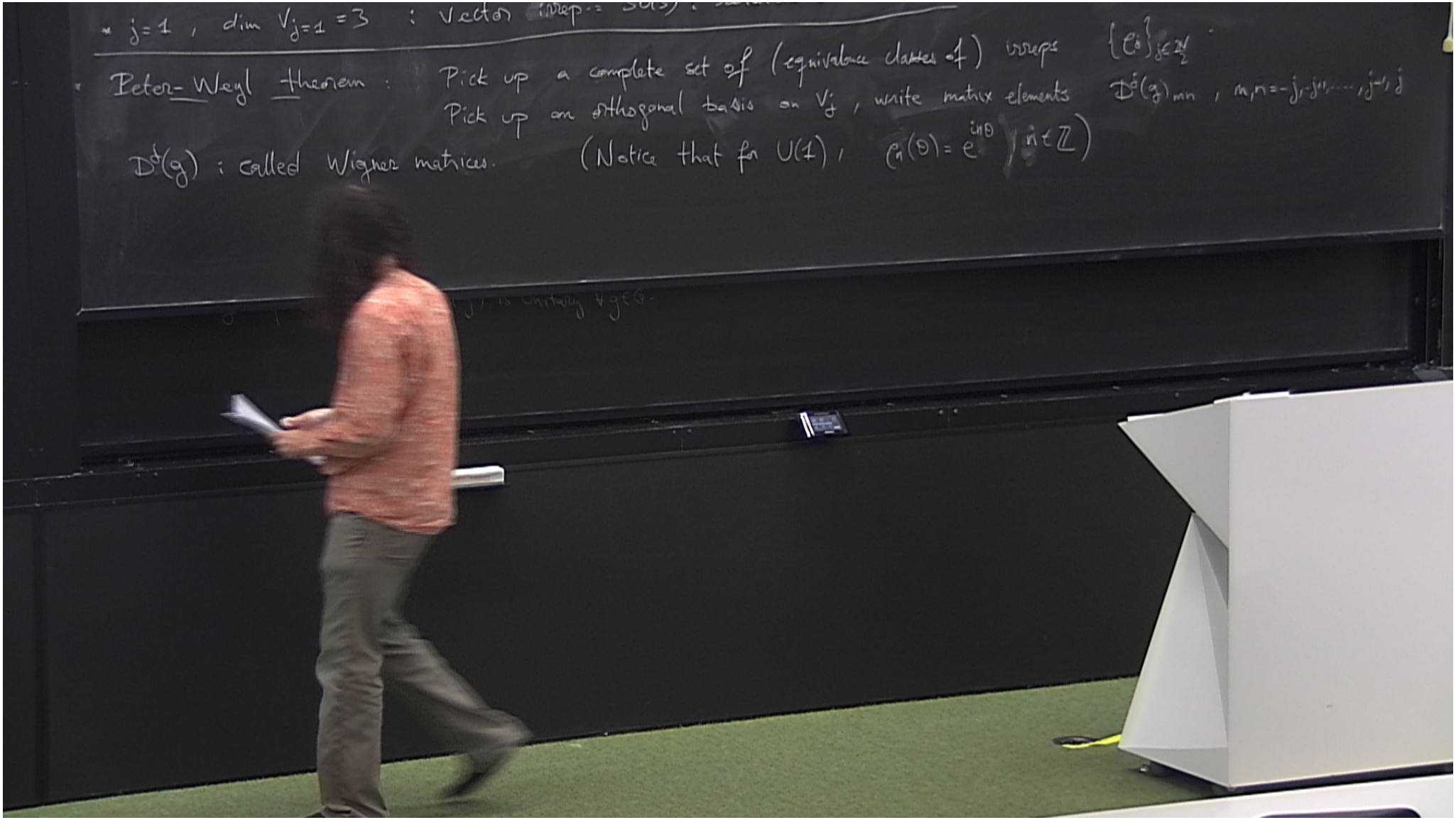

* $j=1/2$, $\dim V_{j=\frac{1}{2}} = 2$: fundamental irrep. = SU(2) as 2×

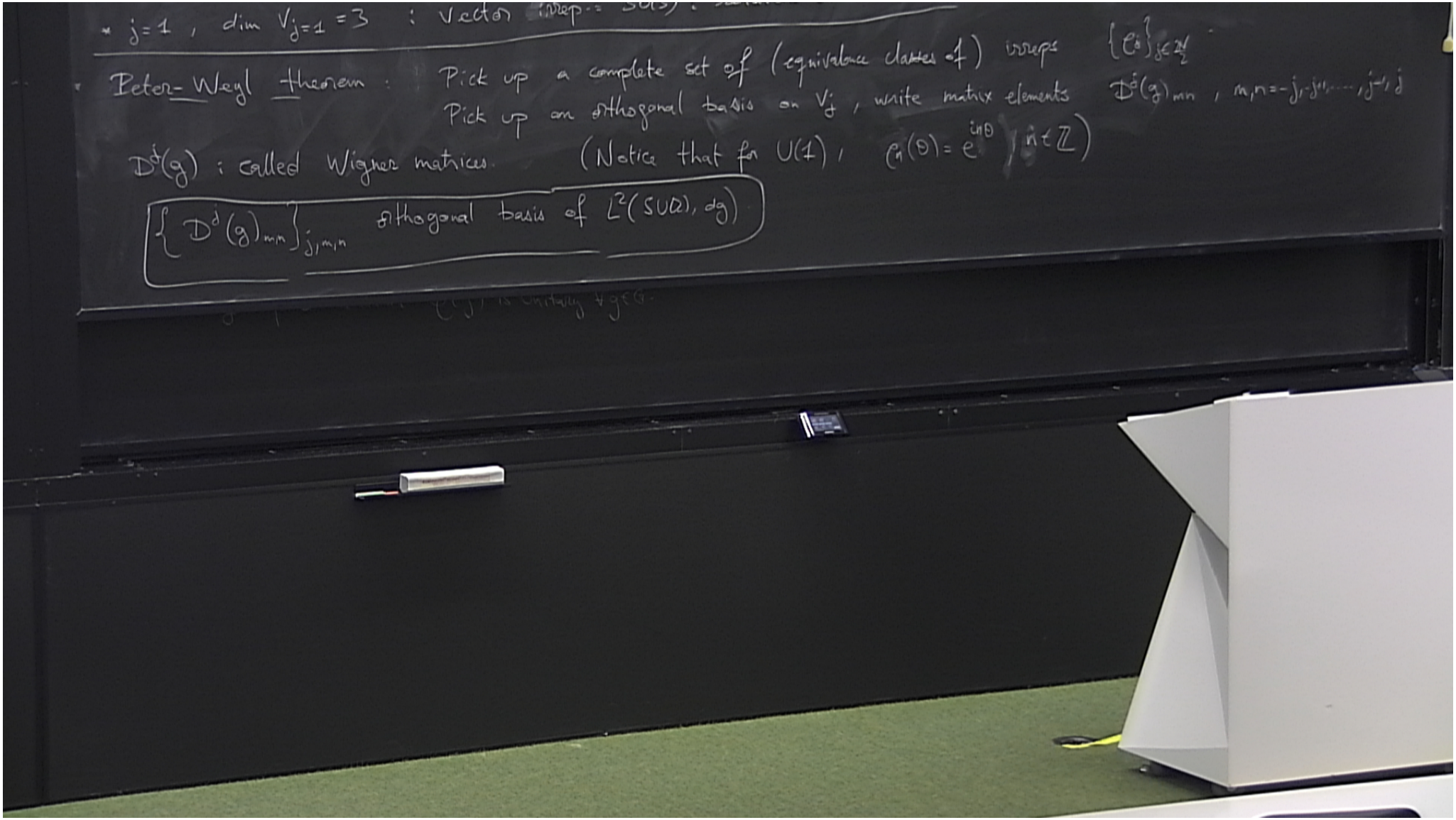

* $j=\frac{1}{2}$, $\dim V_{j=\frac{1}{2}} = 2$. fundamental irrep. = SU(2) as 2×2-matrices.
* $j=1$, $\dim V_{j=1} = 3$: vector irrep. = SO(3) : rotations on 3-vectors.
Peter-Weyl theorem : Pick up a complete set of (equivalence classes of) irreps

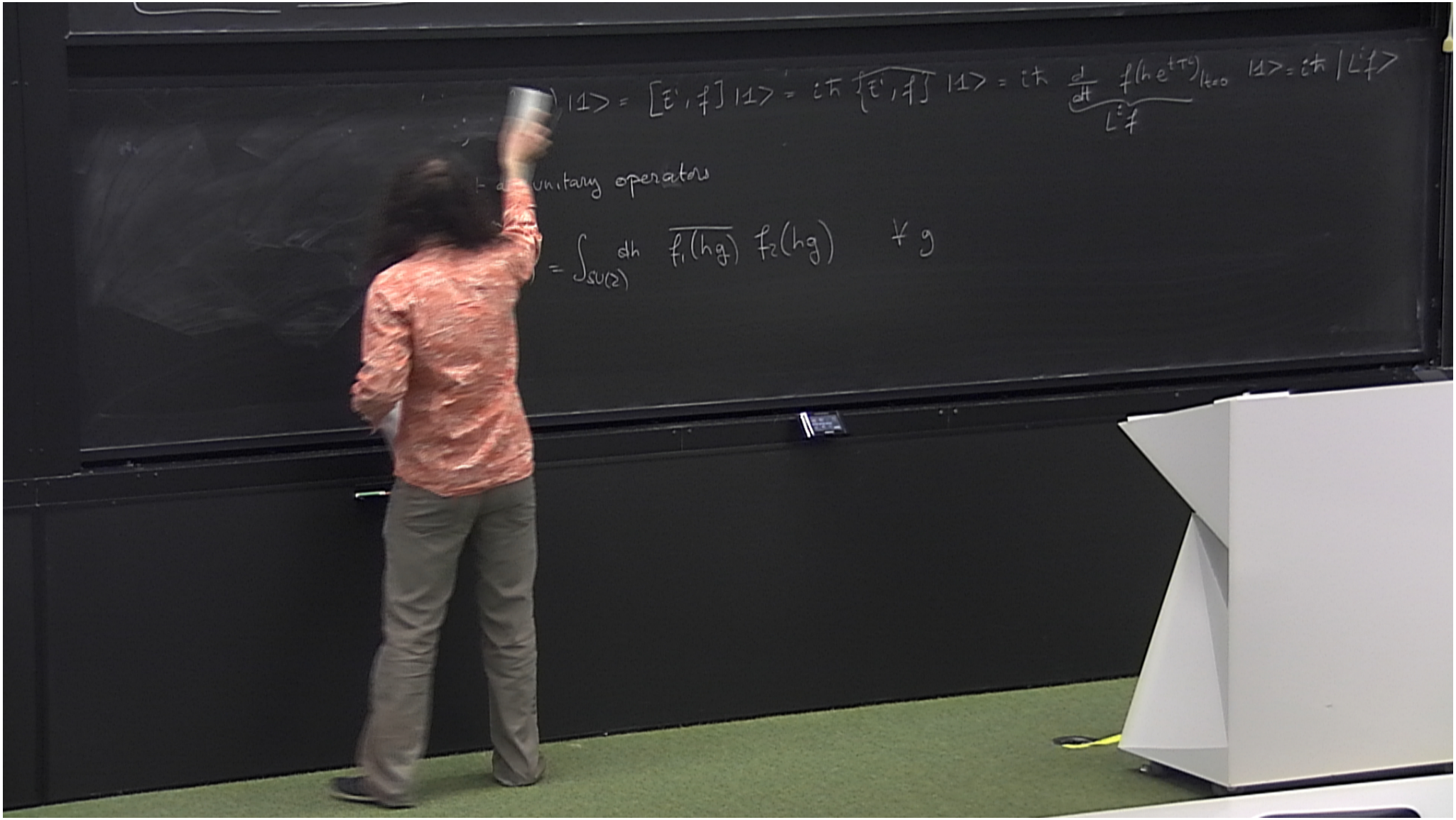

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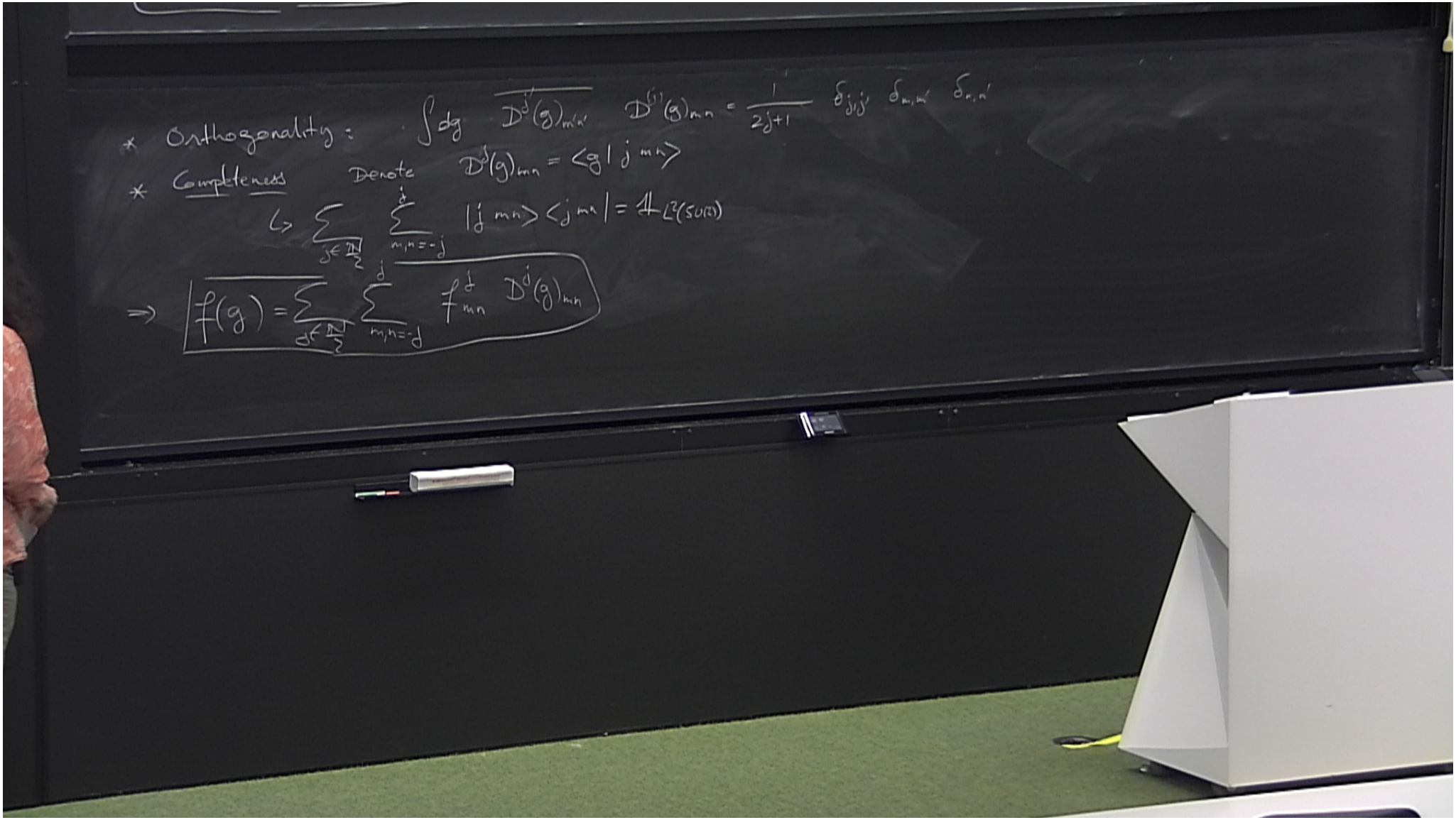

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Peter-Weyl theorem : Pick up a complete set of (equivalence classes of) irreps $\{\mathcal{C}_j\}_{j\in\frac{\mathbb{N}}{2}}$
Pick up an orthogonal basis on V_j, write matrix elements $D^j(g)_{mn}$, $m,n=-j,-j+1,\dots,j-1,j$

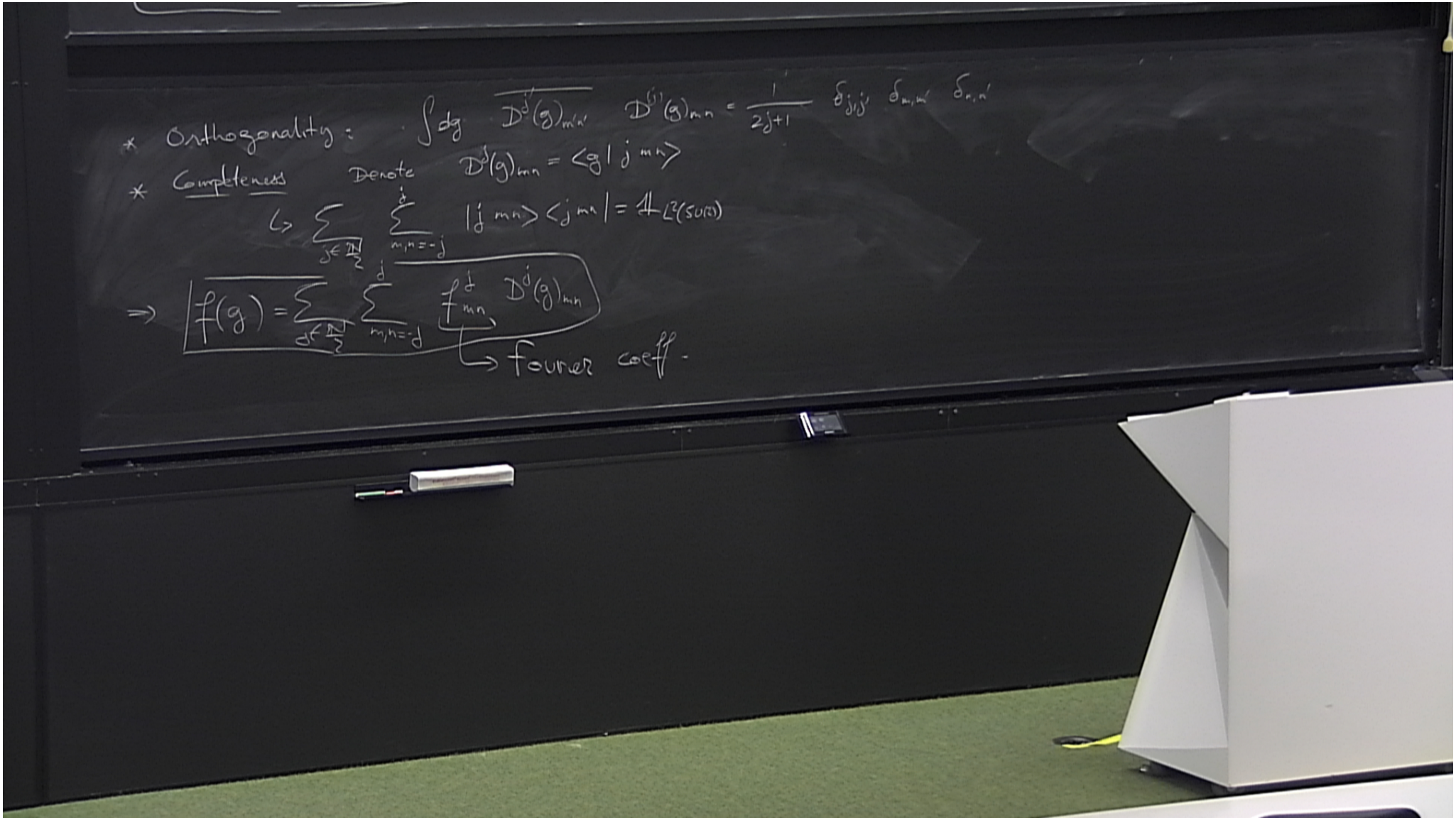

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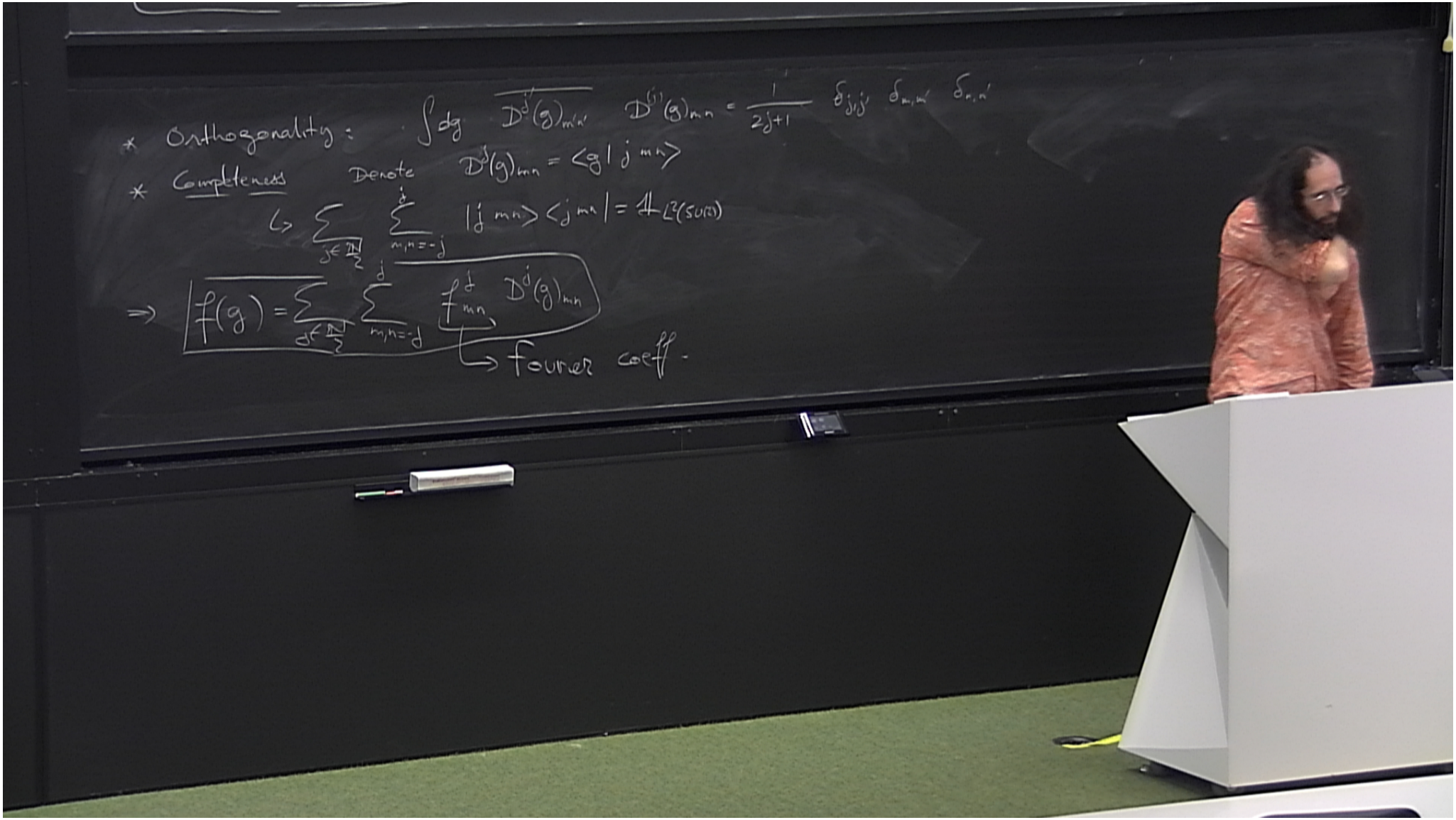

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$\{D^j(g)_{mn}\}_{j,m,n}$ orthogonal basis of $L^2(SU(2),dg)$


|1> = [E', f] |1> = iħ {E', f} |1> = iħ d/dt f(h e^{tY})|t=0 |1> = iħ |L'f>
L'f
unitary operators
= ∫_SU(2) dh f1(hg) f2(hg) ∀ g


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* Completeness Denote $D^j(g)_{mn} = \langle g | j\ mn \rangle$
$\hookrightarrow \sum_{j \in \mathbb{N}/2} \sum_{m,n=-j}^{j} |j\ mn\rangle\langle j\ mn| = \mathbb{1}_{L^2(SU(2))}$
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