

Title: 12/13 PSI - Quantum Gravity Review Lecture 10

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Abstract:

Valentin Bonzon

3d Gravity

* Gauge field A_μ^i

* Cotriad e_μ^i

$$S = \int d^3x \epsilon^{\mu\nu\rho} e_\mu^i F_{\nu\rho}^i$$

Valentin Bonzom

3d Gravity

* Gauge field A_μ^i

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$$S = \int d^3x \epsilon^{\mu\nu\sigma} e_\mu^i F_{\nu\sigma}^i$$

Valentin Bonzon

3d Gravity
x Gauge field A_μ^i
- e_μ^i

$$S = \int d^3x \epsilon^{\mu\nu\sigma} e_\mu^i F_{\nu\sigma}^i$$

$$al : \{A_a^i(x), e_b^j(y)\} = \epsilon_{ab} \delta^{ij} \delta^{(3)}(x-y)$$

Valentin Bonzom

3d Gravity

* Gauge field A_μ^i

* Cotriad e_μ^i

* Canonical : $\{A_a^i(x), e_b^j(y)\} = \epsilon_{ab} \delta^{ij} \delta^{(3)}(x-y)$

$$E^{bi} = \epsilon^{ba} e_a^i$$

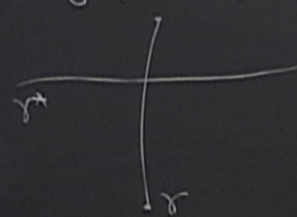
Electrodyn

* Gauge field A_μ^i } $S = \int d^3x \epsilon^{\mu\nu\sigma} e_\mu^i F_{\nu\sigma}^i$
 * Cotriad e_μ^i

* Canonical : $\{A_a^i(x), e_b^j(y)\} = \epsilon_{ab} \delta^{ij} \delta^{(3)}(x-y)$
 $E^{bi} = \epsilon^{ba} e_a^i$

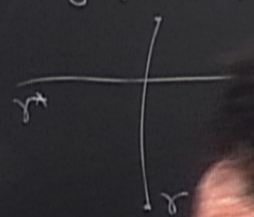
* Holonomies $h_\gamma(A) = \text{Pexp} - \int_\gamma A$

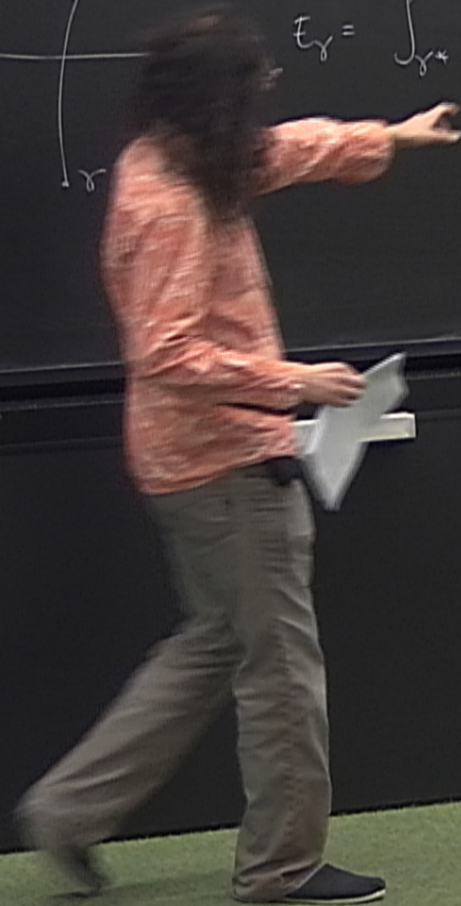
$$\left\{ \int_{\gamma} A, \int_{\gamma^*} e \right\} = \int_{\gamma} \int_{\gamma^*}$$

$$\left\{ \int_A, \int_{\gamma^*} e \right\} = \int_{\gamma} \int_{\gamma^*} \delta^{(2)}(x-y)$$


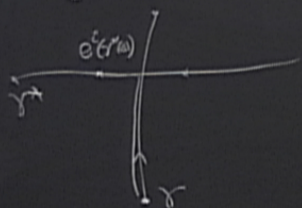
$$E_{\gamma} = \int_{\gamma^*} e(r^*)$$

$$\left\{ \int_{\gamma} A, \int_{\gamma^*} e \right\} = \int_{\gamma} \int_{\gamma^*} \delta^{(2)}(x-y)$$

$$E_{\gamma} = \int_{\gamma^*} e_a(r^*(\omega)) \dot{\gamma}^{*a}(\omega) d\omega$$




$$\left\{ \int_{\gamma} A, \int_{\gamma^*} e \right\} = \int_{\gamma} \int_{\gamma^*} \delta^{(2)}(x-y)$$



$$E_{\gamma} = \int_{\gamma^*} h_{\gamma\gamma^*}(s) e_a^i(\gamma^*(s)) \dot{\gamma}^{*a}(s) h_{\gamma\gamma^*}^{-1}(s) ds$$

$$E_{\gamma} \mapsto g(\gamma(0)) E_{\gamma} g^{-1}(\gamma(1))$$

Basis of 2×2 matrices $\{1, \sigma_1, \sigma_2, \sigma_3\}$

$$\text{tr } \sigma_i = 0$$

$$\text{tr}(\sigma_i \sigma_j) = 2 \delta_{ij}$$

$$M = M_0 1 + M_1 \sigma_1 + M_2 \sigma_2 + M_3 \sigma_3$$

$$\text{tr } M = 2M_0, \quad \text{tr}(M \sigma_i) = 2M_i$$

Basis of 2×2 matrices $\{1, \sigma_1, \sigma_2, \sigma_3\}$

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$$\sigma_i \sigma_j = \delta_{ij} 1 + i \epsilon_{ijk} \sigma_k$$

$$e_a = e_a^i$$

$$T_i =$$

Basis of 2×2 matrices $\{1, \sigma_1, \sigma_2, \sigma_3\}$

$$\text{tr } \sigma_i = 0$$

$$\text{tr}(\sigma_i \sigma_j) = 2 \delta_{ij}$$

$$T_i = -\frac{i}{2} \sigma_i$$

$$M = M_0 1 + M_1 \sigma_1 + M_2 \sigma_2 + M_3 \sigma_3$$

$$\text{tr } M = 2M_0, \quad \text{tr}(M \sigma_i) = 2M_i$$

$$\sigma_i \sigma_j = \delta_{ij} 1 + i \epsilon_{ijk} \sigma_k$$

$$e_a = e_a^i T_i$$

$$E_Y = E_Y^i T_i, \quad E_Y^i = -2 \operatorname{tr}(E_Y T_i)$$

$$E_Y^i = \int_{Y^*} -2 \operatorname{tr}$$

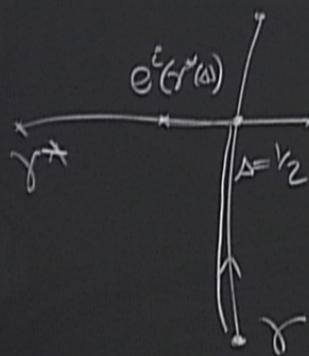
Calculate the

$$E_\gamma = E_\gamma^i T_i, \quad E_\gamma^i = -2 \operatorname{tr}(E_\gamma T_i)$$

$$E_\gamma^i = \int_{\gamma^*} \underbrace{-2 \operatorname{tr}(h_{\gamma\gamma^*}(\Delta) T_k h_{\gamma\gamma^*}^{-1}(\Delta) T_i)}_{f_{ik}(\Delta)} \quad \epsilon_{ab} E^{ab}(\gamma^*(\Delta)) \quad \gamma^* a(\Delta)$$

(calculate the)

$$= \left\{ \int_A, \int_{\gamma^*} e \right\} = \int_{\gamma} \int_{\gamma^*} \delta^{(2)}(x-y)$$



Def: $E_{\gamma} = \int_{\gamma^*} h_{\gamma\gamma^*}(s) e_a(r^*(s)) \gamma^{*a}(\Delta) h_{\gamma\gamma^*}^{-1}(\Delta) d\Delta$

Flux $E_{\gamma} \mapsto g(r(0)) E_{\gamma} g(r(1))^{-1}$

$$\epsilon_{ab} \gamma^a \gamma^{*b} > 0$$

$$\{E_\delta^i, h_\delta\} = -h_\gamma(1, \frac{1}{2} + \epsilon) \quad T^\delta h_\delta(\frac{1}{2} - \epsilon, 0)$$

$$\{E_\gamma^i, h_\gamma\} = -h_\gamma(1, \frac{1}{2} + \epsilon) \quad T^{\bar{\alpha}} h_\gamma(\frac{1}{2} - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ck}(s) \in_{ba} \bar{\gamma}^{*\alpha}(s) \bar{\gamma}^b(s')$$

$$\begin{aligned}
 \{E_{\gamma}^i, h_{\gamma}\} &= -h_{\gamma}(1, \frac{1}{2} + \epsilon) T^0 h_{\gamma}(\frac{1}{2} - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ik}(s) \epsilon_{ba} \tilde{\gamma}^{*a}(s) \tilde{\gamma}^c(s') \{E^{bk}(s) \\
 &= h_{\gamma}(1, \frac{1}{2} + \epsilon) T^0 h_{\gamma}(\frac{1}{2} - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ik}(s) \epsilon_{ba} \tilde{\gamma}^b(s) \tilde{\gamma}^{*a}(s') \tilde{\gamma}^{*c}(s), \gamma(s') \} ds'
 \end{aligned}$$

$$\begin{aligned}
\{E_g^i, h_g\} &= -h_g(1, k_2 + \epsilon) T^0 h_g(k_2 - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ik}(s) \epsilon_{ba} \tilde{\gamma}^{*a}(s) \tilde{\gamma}^b(s) \\
&= h_g(1, k_2 + \epsilon) T^0 h_g(k_2 - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ik}(s) \epsilon_{ba} \tilde{\gamma}^b(s) \tilde{\gamma}^{*a}(s) \underbrace{\delta^{(2)}(\gamma^* - \delta(s))}_{\delta(s)} \\
&\xrightarrow{\epsilon \rightarrow 0} h_g(1, k_2) T^0 h_g(k_2, 0) f_{ig}(k_2)
\end{aligned}$$

$$= h_g(1, k_2 + \epsilon) T_0 h_g(k_2 - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ik}(s) \epsilon_{ba} \gamma^a(s) \gamma^b(s) \delta(s - \frac{1}{2}) ds$$

$$\epsilon \rightarrow 0 \quad h_g(1, k_2) T_0 h_g(k_2, 0) f_{eg}(k_2)$$

$$f_{ij}(k_2) = -2 \operatorname{tr} \left(h_{g\gamma^*}(k_2) T_0 h_{\gamma\gamma^*}^{-1}(k_2) T_i \right)$$

Basis of 2×2 matrices $\{1, \sigma_1, \sigma_2, \sigma_3\}$

$$M = M_0 1 + M_1 \sigma_1 + M_2 \sigma_2 + M_3 \sigma_3$$

$$\text{tr } M = 2M_0, \quad \text{tr}(M\sigma_i) = 2M_i$$

$$\sigma_i \sigma_j = \delta_{ij} 1 + i \epsilon_{ijk} \sigma_k$$

$$e_a = e_a^i T_i$$

$$\text{tr } \sigma_i = 0$$

$$\text{tr}(\sigma_i \sigma_j) = 2\delta_{ij}$$

$$T_i = -\frac{i}{2} \sigma_i$$

$$\text{tr}(T_i T_j) = -\frac{1}{2} \delta_{ij}$$

$$= h_g(1, k_2 + \epsilon) T^0 h_g(k_2 - \epsilon, 0) \int_{\gamma^*} \int_{\frac{1}{2} - \epsilon}^{\frac{1}{2} + \epsilon} f_{ik}(s) \epsilon_{ba} \gamma^a(s) \gamma^b(s) \delta(s - \frac{1}{2})$$

$$\epsilon \rightarrow 0 \quad h_g(1, k_2) T^0 h_g(k_2, 0) f_{ij}(k_2)$$

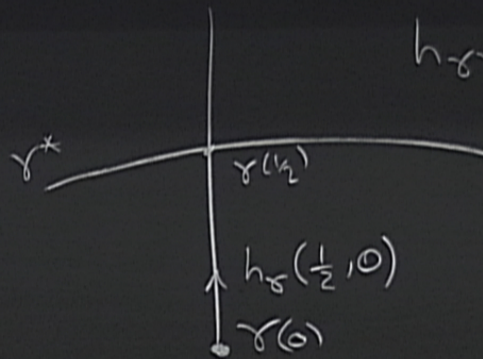
$$f_{ij}(k_2) = -2 \operatorname{tr} (h_{gg^*}(k_2) T_j h_{gg^*}^{-1}(k_2) T_i)$$

$$= -2 \operatorname{tr} (h_{gg^*}^{-1}(k_2) T_i h_{gg^*}(k_2) T_j)$$

$$f_{ij}(k_2) T^0 = -2 \operatorname{tr} (h_{gg^*}^{-1}(k_2) T_i h_{gg^*}(k_2) T_j) T^0 = h_{gg^*}^{-1}(k_2) T_i h_{gg^*}(k_2)$$

$$1 = h_{\gamma\gamma^*}(\frac{1}{2}) \quad ; \quad h_{\gamma\gamma^*}(\frac{1}{2})$$

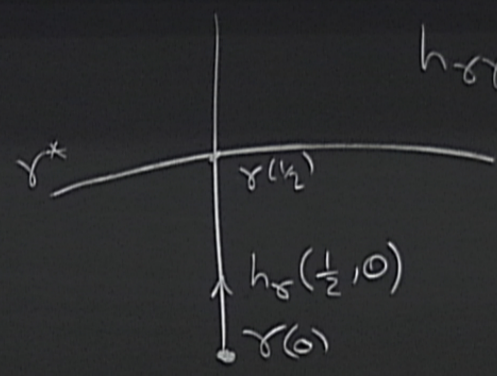
$$h_{\gamma^*}(\frac{1}{2}) \quad h_{\gamma}(\frac{1}{2}, 0)$$



$$h_{\gamma\gamma^*}(\frac{1}{2}) = h_{\gamma}^{-1}(\frac{1}{2}, 0)$$

$$1 = h_{\gamma\gamma^*}(\frac{1}{2}) \quad ; \quad h_{\gamma\gamma^*}(\frac{1}{2})$$

$$\underbrace{h_{\gamma^*}(\frac{1}{2}) \quad h_{\gamma}(\frac{1}{2}, 0)}_1$$



$$h_{\gamma\gamma^*}(\frac{1}{2}) = h_{\gamma}^{-1}(\frac{1}{2}, 0)$$

What is $\{E_{\gamma}^i, E_{\gamma}^j\} = ?$ $\rightarrow$ direct calculation.

What is $\{E_x^i, E_x^j\} = ?$ $\rightarrow$ direct calculation.

$\rightarrow$ Use Jacobi:

$$\left\{ \left\{ E_x^i, E_x^j \right\}, h_x \right\} + \left\{ \left\{ h_x, E_x^i \right\}, E_x^j \right\} + \left\{ \left\{ E_x^j, h_x \right\}, E_x^i \right\} = 0$$

What is $\{E_x^i, E_x^j\} = ?$ $\rightarrow$ direct calculation.
 $\rightarrow$ Use Jacobi:

$$\begin{aligned} & \{ \{E_x^i, E_x^j\}, h_x \} + \{ \{h_x, E_x^i\}, E_x^j \} + \{ \{E_x^j, h_x\}, E_x^i \} = 0 \\ & \{ \{E_x^i, E_x^j\}, h_x \} = \{ +h_x T^i, E_x^j \} - \{ h_x T^j, E_x^i \} \\ & = \{ h_x, E_x^j \} T^i - \{ h_x, E_x^i \} T^j = h_x (T^j T^i - T^i T^j) = \epsilon \end{aligned}$$