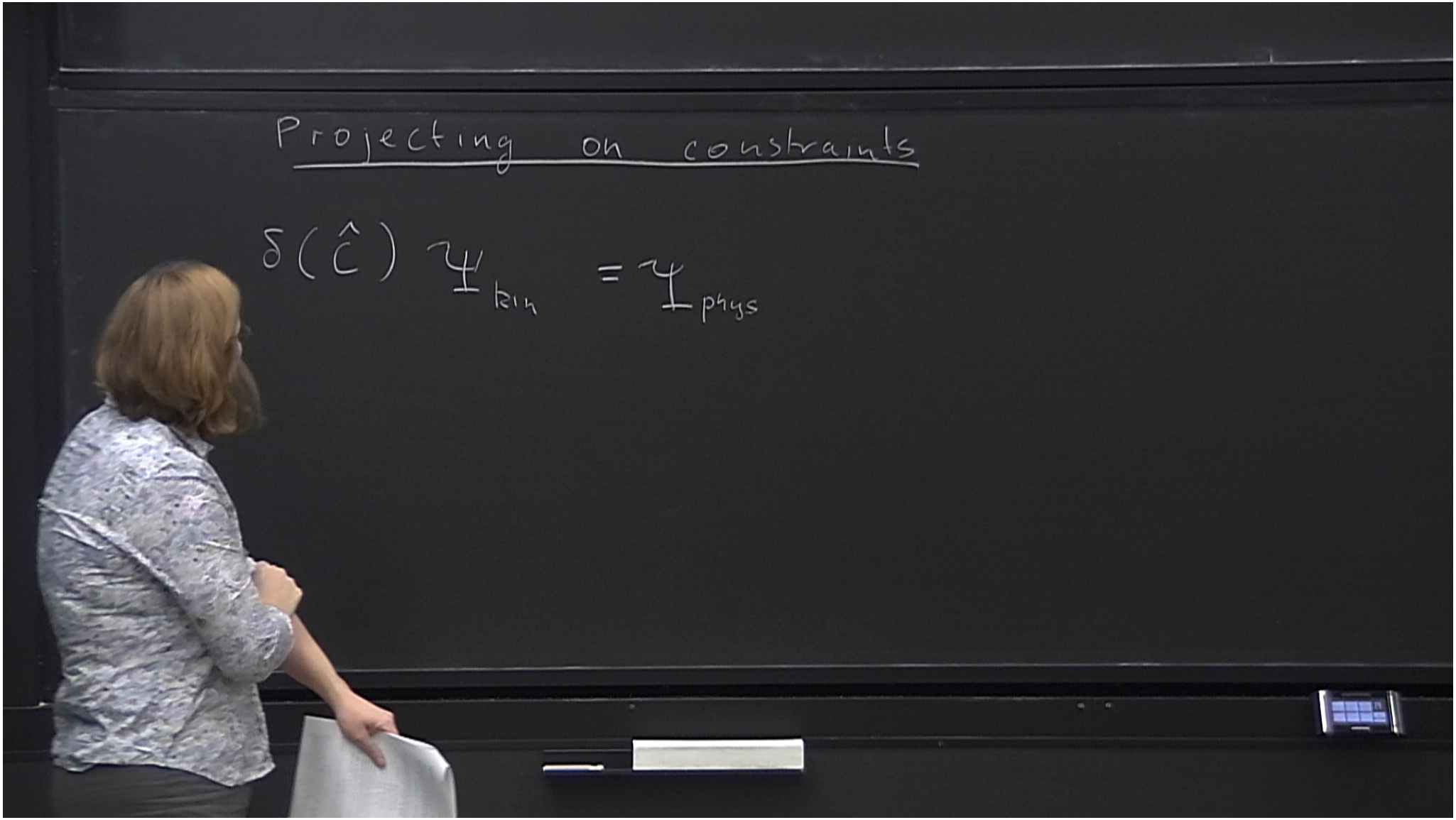
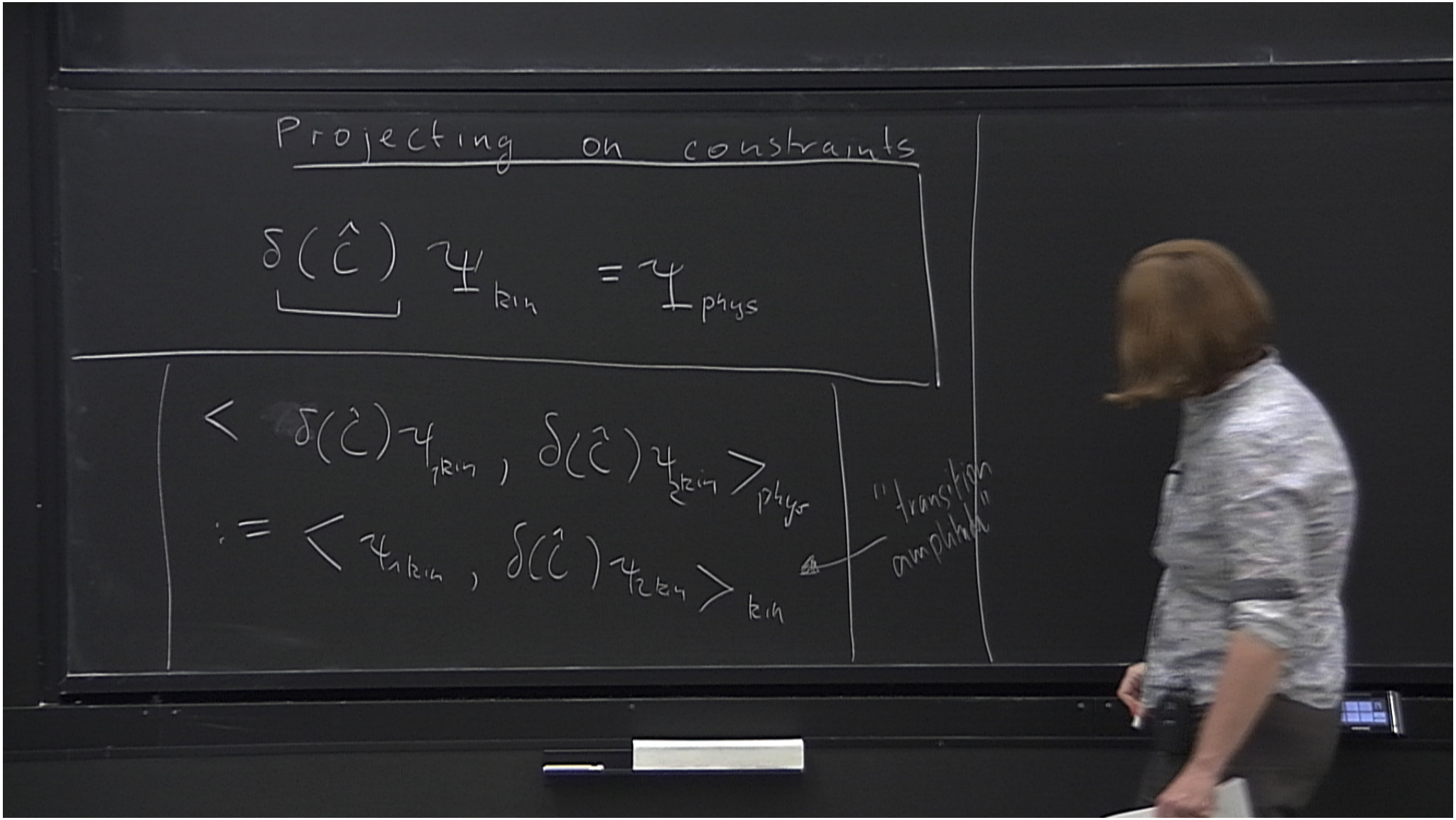
Title: 12/13 PSI - Quantum Gravity Review Lecture 8

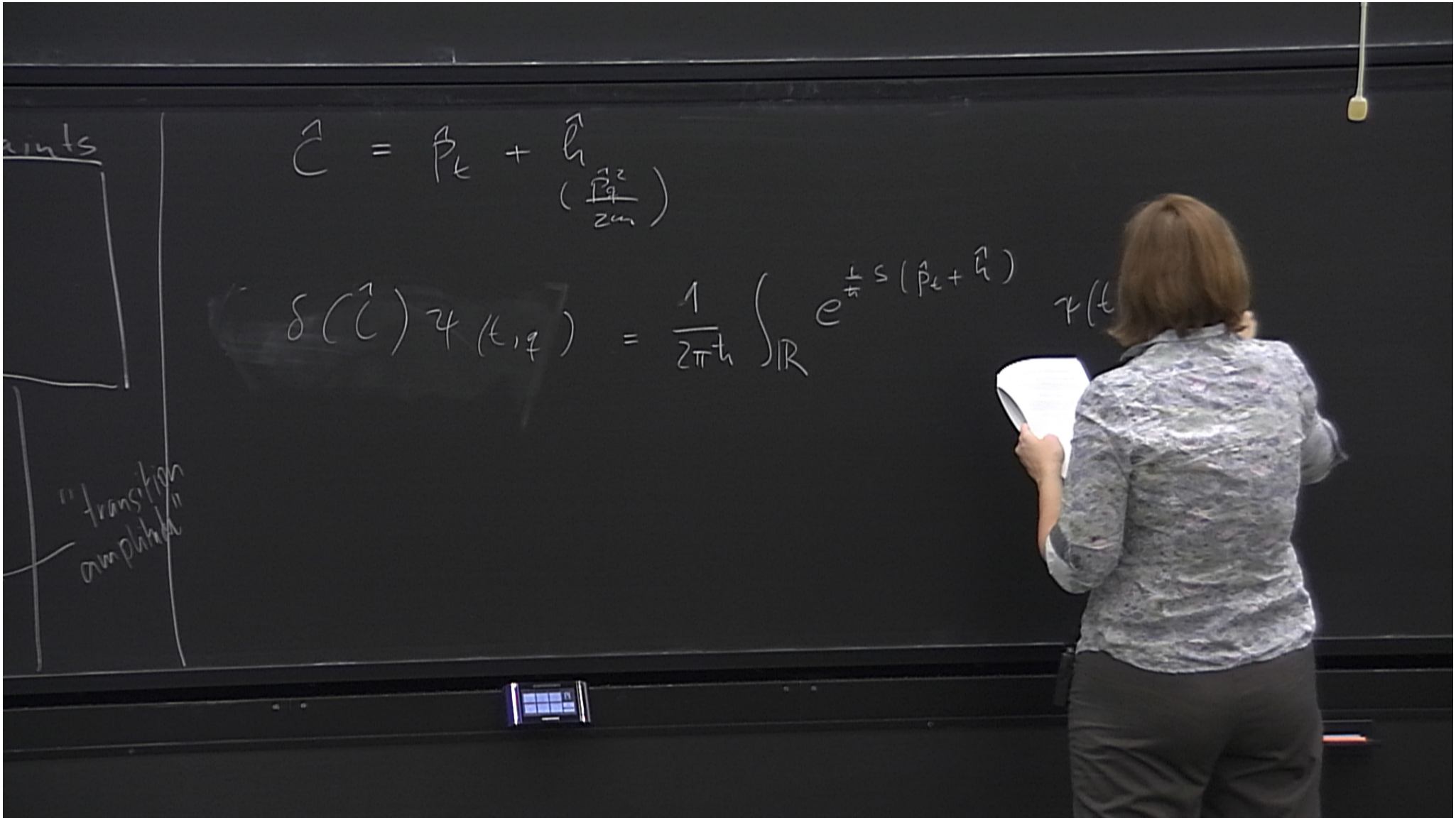
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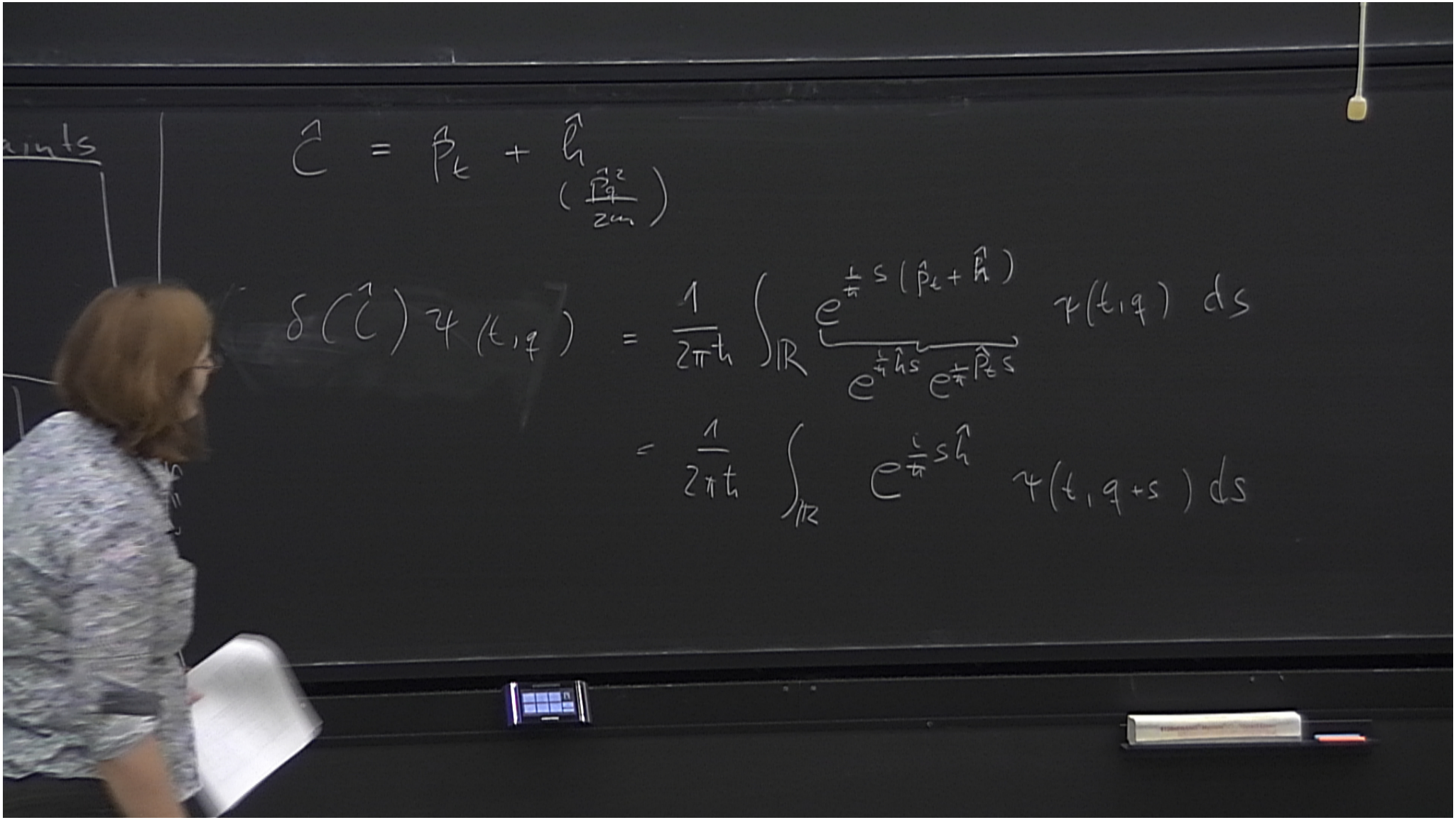
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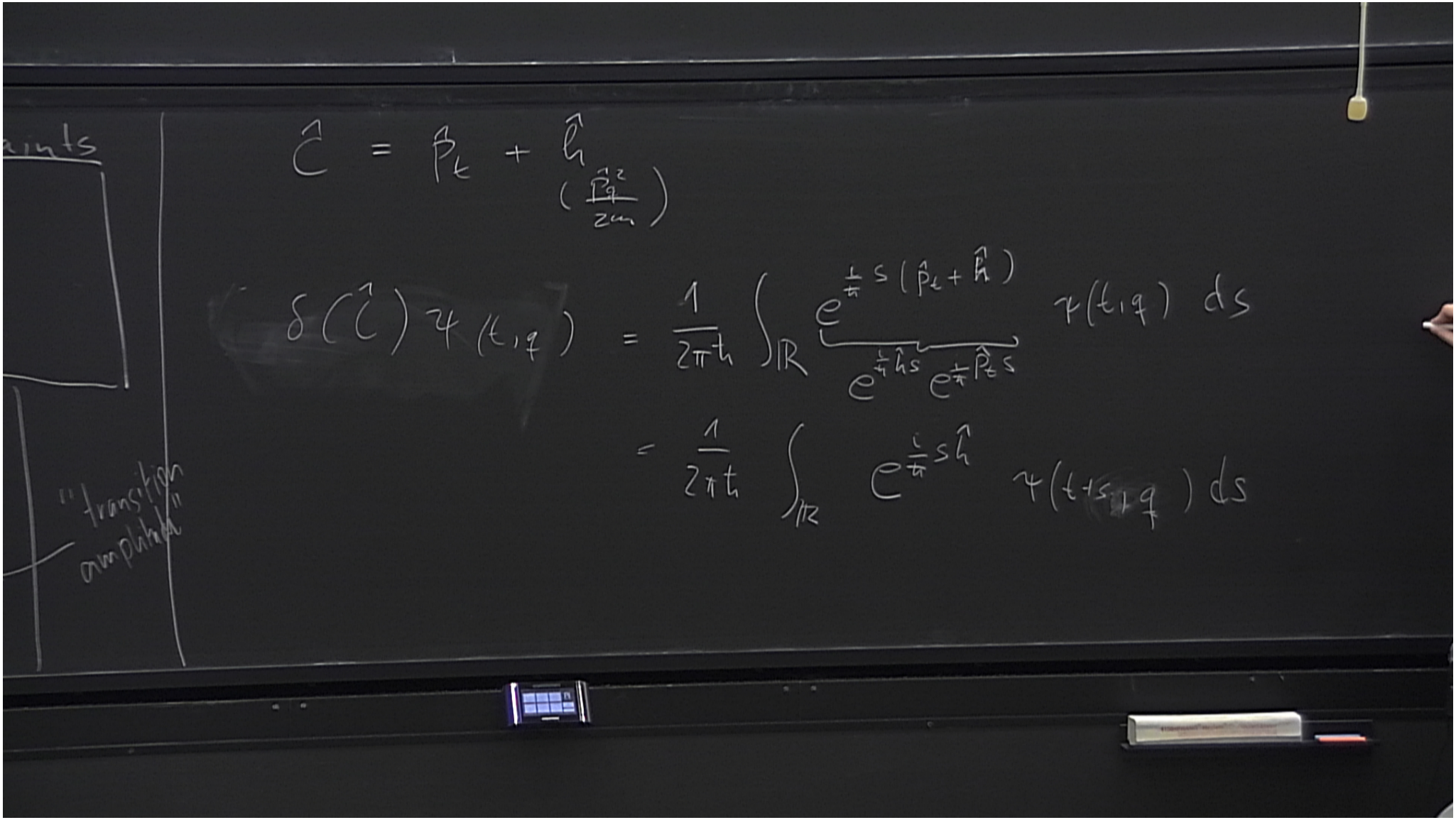
Abstract:


Projecting on constraints
$\delta(\hat{C})\,\Psi_{kin} = \Psi_{phys}$


Projecting on constraints
$\delta(\hat{C})\,\psi_{kin} = \psi_{phys}$
$< \delta(\hat{C})\psi_{1kin}, \delta(\hat{C})\psi_{2kin} >_{phys}$
$:= < \psi_{1kin}, \delta(\hat{C})\psi_{2kin} >_{kin}$
"transition amplitude"


$\vec{C} = \vec{p}_t + \vec{h}$
$\left(\frac{\vec{p}^2}{2m}\right)$
$\delta(\vec{C})\,\psi(t,q) = \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{\frac{1}{i\hbar}S(\hat{p}_t+\vec{h})}$
"transition amplitude"


$\vec{C} = \hat{p}_t + \hat{h}_{\left(\frac{\vec{p}^2}{2m}\right)}$
$\delta(\vec{C})\,\psi(t,q) = \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{\frac{1}{i\hbar}s(\hat{p}_t+\hat{h})}\,\psi(t,q)\,ds$
$e^{\frac{1}{i\hbar}\hat{h}s}\,e^{\frac{1}{i\hbar}\hat{p}_t s}$
$= \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{\frac{i}{\hbar}s\hat{h}}\,\psi(t,q+s)\,ds$


$\vec{C} = \hat{P}_t + \vec{h}$
$\left(\frac{\hat{P}_q^2}{2m}\right)$
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$e^{\frac{i}{\hbar}\hat{h}s}\,e^{\frac{i}{\hbar}\hat{P}_t s}$
$= \frac{1}{2\pi\hbar}\int_{\mathbb{R}} e^{\frac{i}{\hbar}s\hat{h}}\,\psi(t+s,q)\,ds$
"transition amplitude"

$\Psi_+(t,q)$ is a solution

w/ $\Psi_+(r,q) = \psi(r,q)$.

$\Psi_+(t,q)$ is a solution

w/ $\Psi_+(r,q) = \psi(r,q)$.

$$\langle \psi_1, \delta(\hat{C})\psi_2 \rangle_{RIG} = \frac{\hat{\cdot}}{2\pi\hbar} \times$$

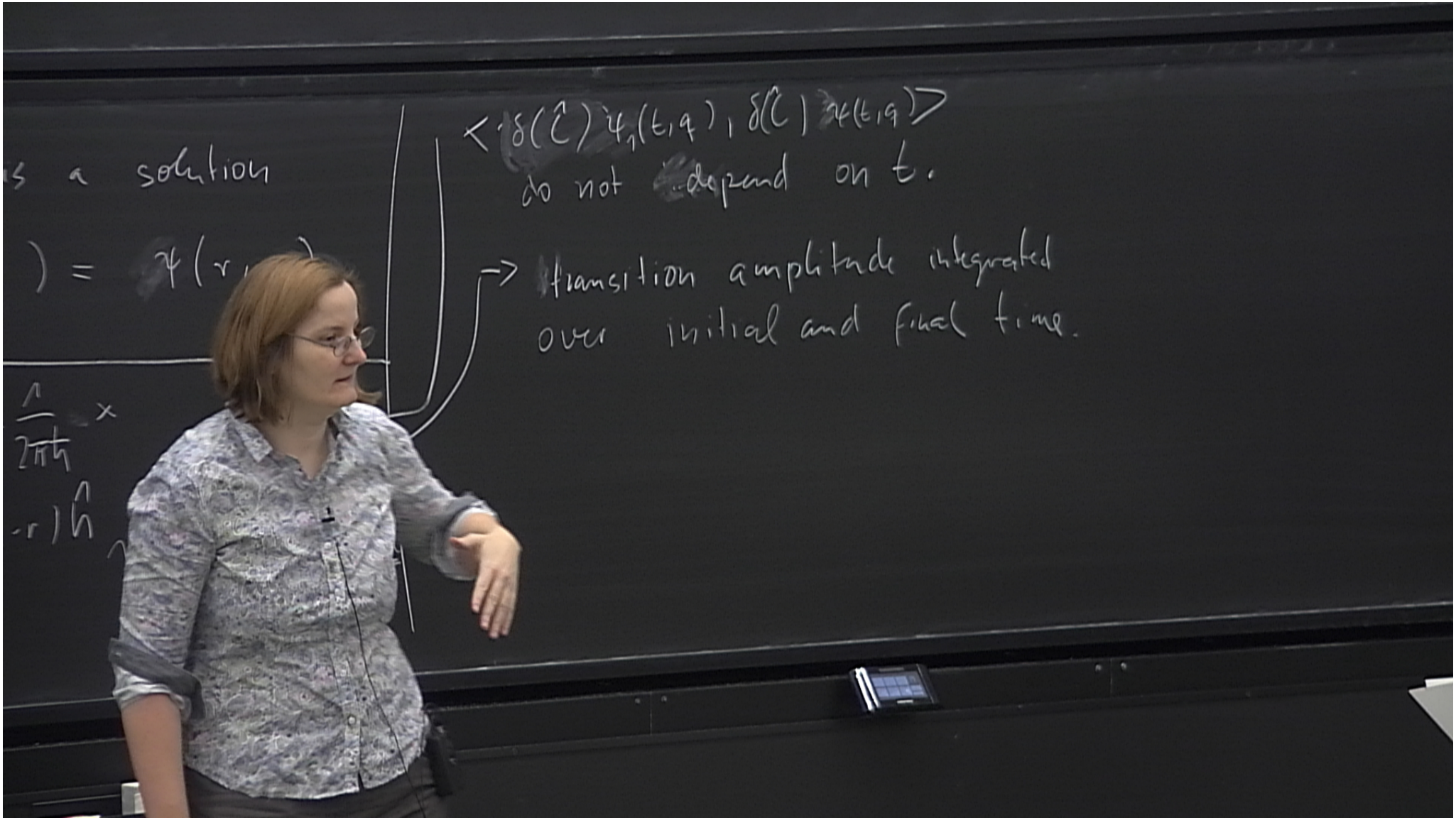
$$\iiint \overline{\psi_1}(q,t)\, e^{-\frac{i}{\hbar}(t-r)\hat{h}}\, \psi_2(r,q)\, dr\, dq\, dt$$

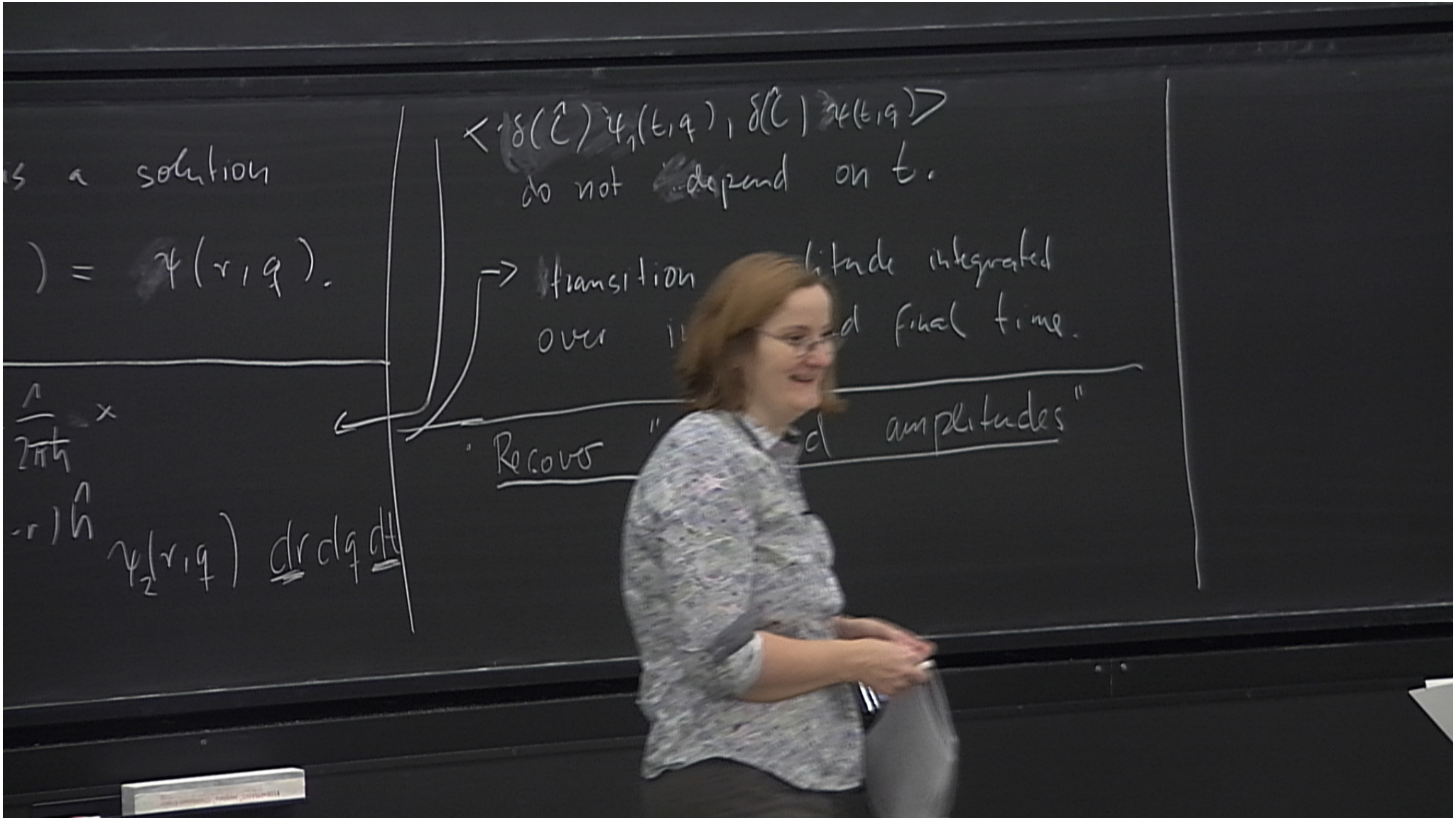
$\Psi_+(t,q)$ is a solution

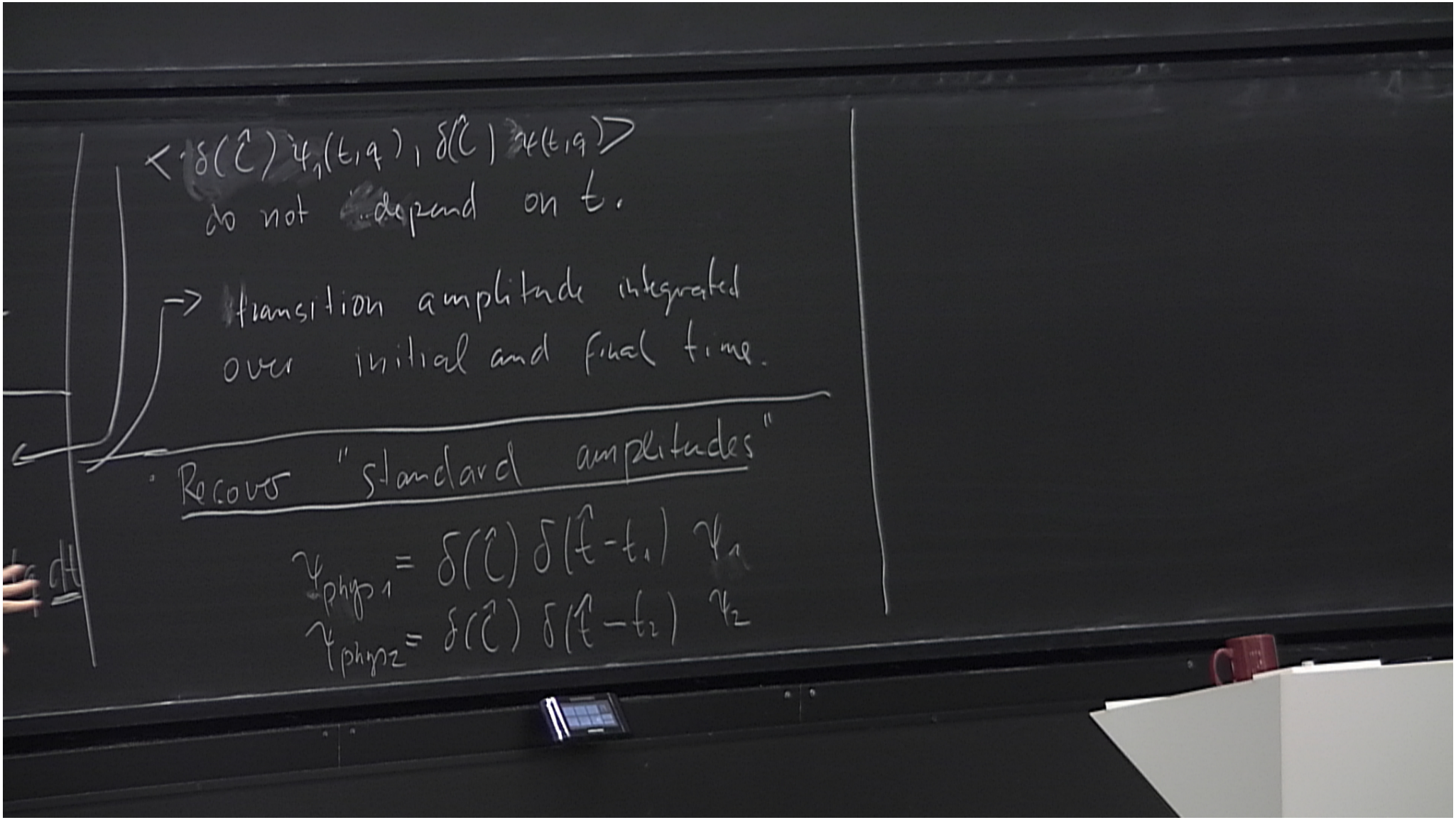
w/ $\Psi_+(r,q) = \psi(r,q)$.

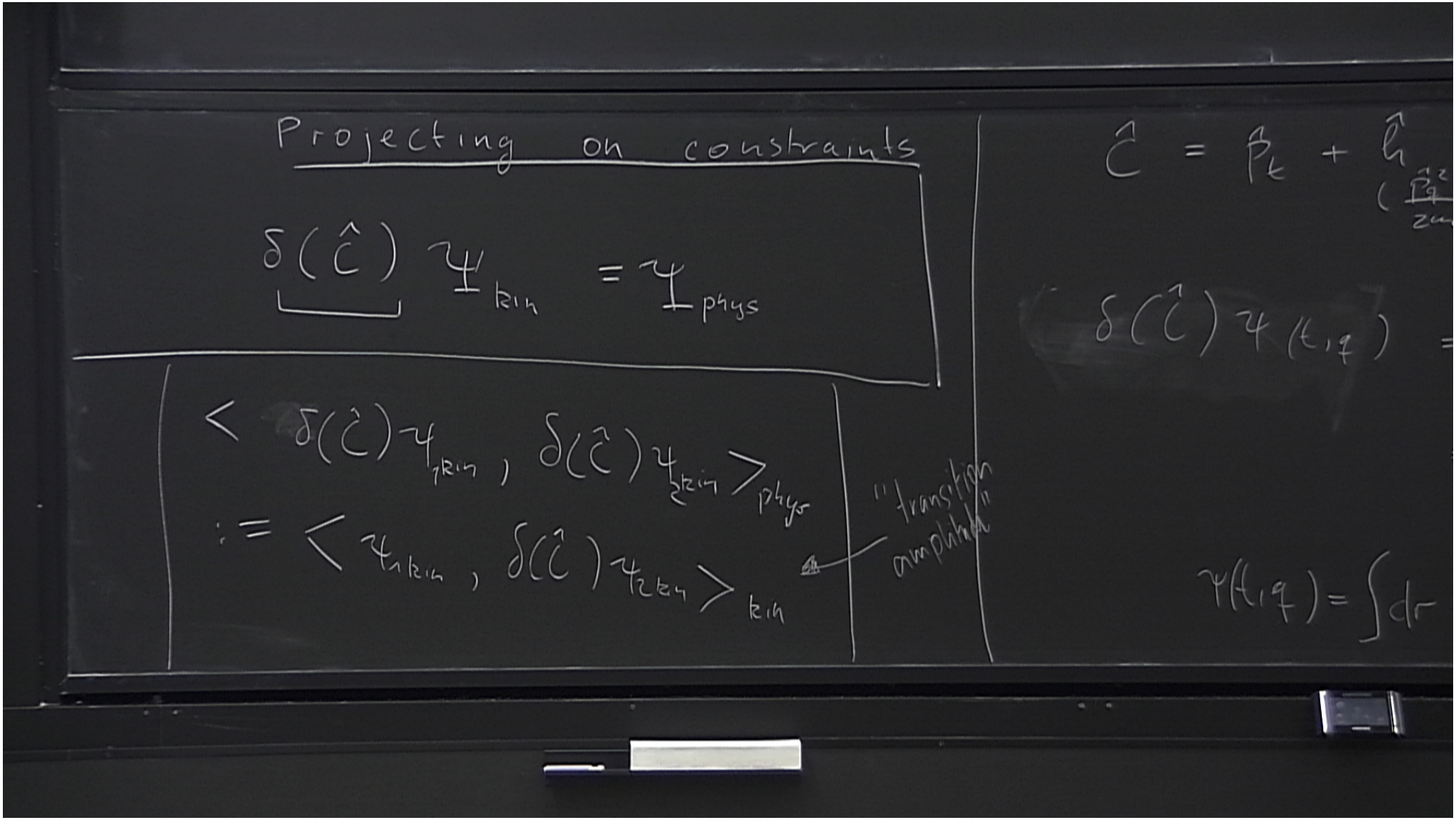
$$\langle \psi_1, \delta(\hat{C})\psi_2 \rangle_{RIG} = \frac{\hat{}}{2\pi\hbar} \times$$

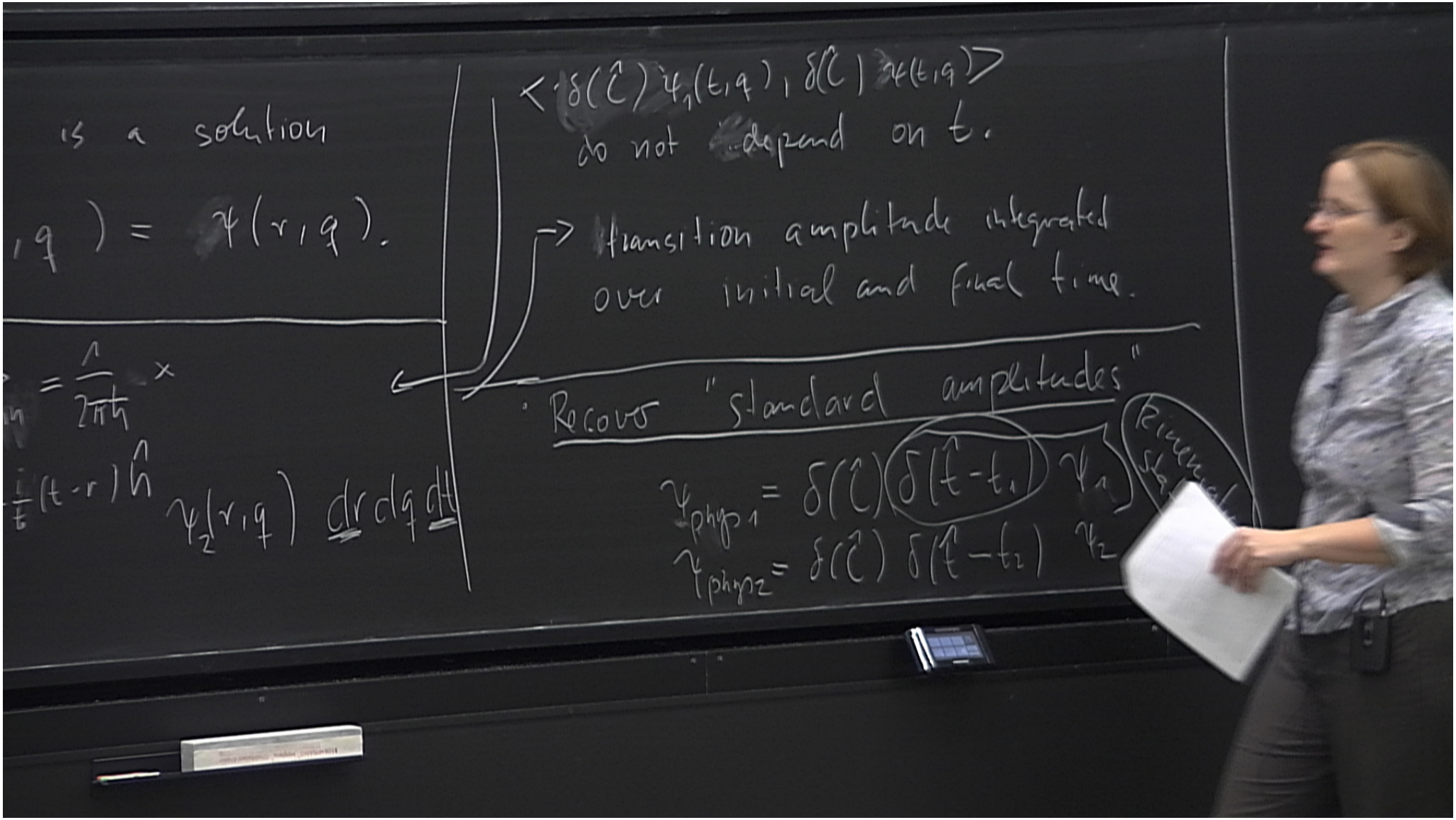
$$\iiint \overline{\psi_1}(q,t)\, e^{-\frac{i}{\hbar}(t-r)\hat{h}}\, \psi_2(r,q)\, dr\, dq\, dt$$

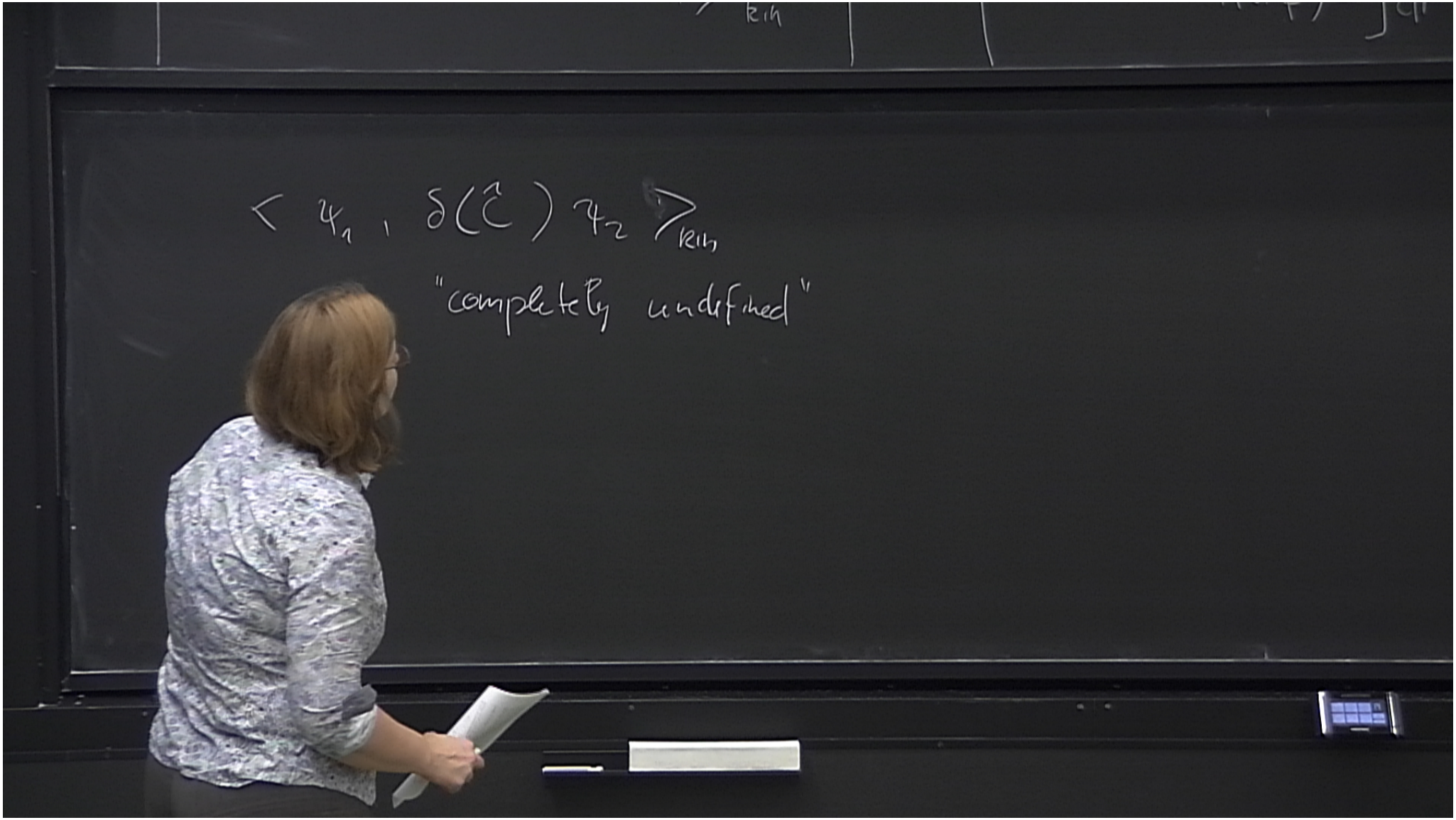

s a solution
) =
$\langle \delta(\hat{C}) \psi_1(t,q), \delta(\hat{C}) \psi(t,q) \rangle$
do not depend on t.
-> transition amplitude integrated
over initial and final time.


s a solution
) = ψ(r, q).
⟨δ(Ĉ) ψ1(t, q), δ(Ĉ) ψ(t, q)⟩
do not depend on t.
→ transition
over
final time.
Recover
amplitudes"
ψ2(r, q) dr dq dt


$\langle \delta(\hat{C})\,\psi_1(t,q),\ \delta(\hat{C})\,\psi_2(t,q)\rangle$
do not depend on t.
→ transition amplitude integrated over initial and final time.
Recover "standard amplitudes"
$\psi_{phys\,1} = \delta(\hat{C})\,\delta(\hat{t}-t_1)\,\psi_1$
$\psi_{phys\,2} = \delta(\hat{C})\,\delta(\hat{t}-t_2)\,\psi_2$


Projecting on constraints
$\delta(\hat{C})\,\psi_{kin} = \psi_{phys}$
$\langle \delta(\hat{C})\psi_{1kin}, \delta(\hat{C})\psi_{2kin} \rangle_{phys}$
$:= \langle \psi_{1kin}, \delta(\hat{C})\psi_{2kin} \rangle_{kin}$
"transition amplitude"
$\hat{C} = \hat{p}_t + \hat{h}$
$\delta(\hat{C})\psi(t,q) =$
$\psi(t,q) = \int dr$

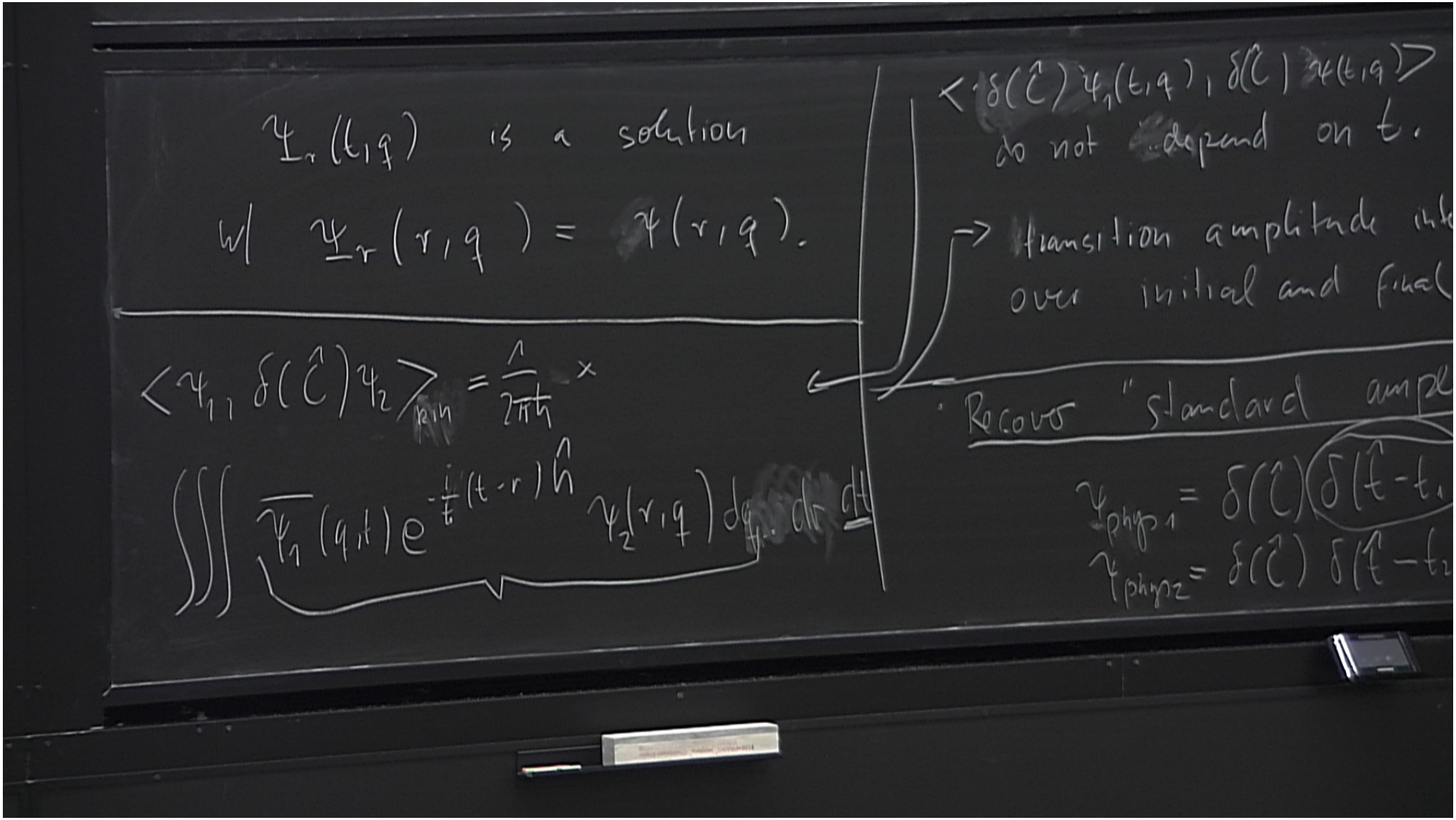

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$\psi(r,q) = \psi(r,q)$.
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$\frac{i}{\hbar}(t-r)\hat{h}\,\psi_2(r,q)\,dr\,dq\,dt$

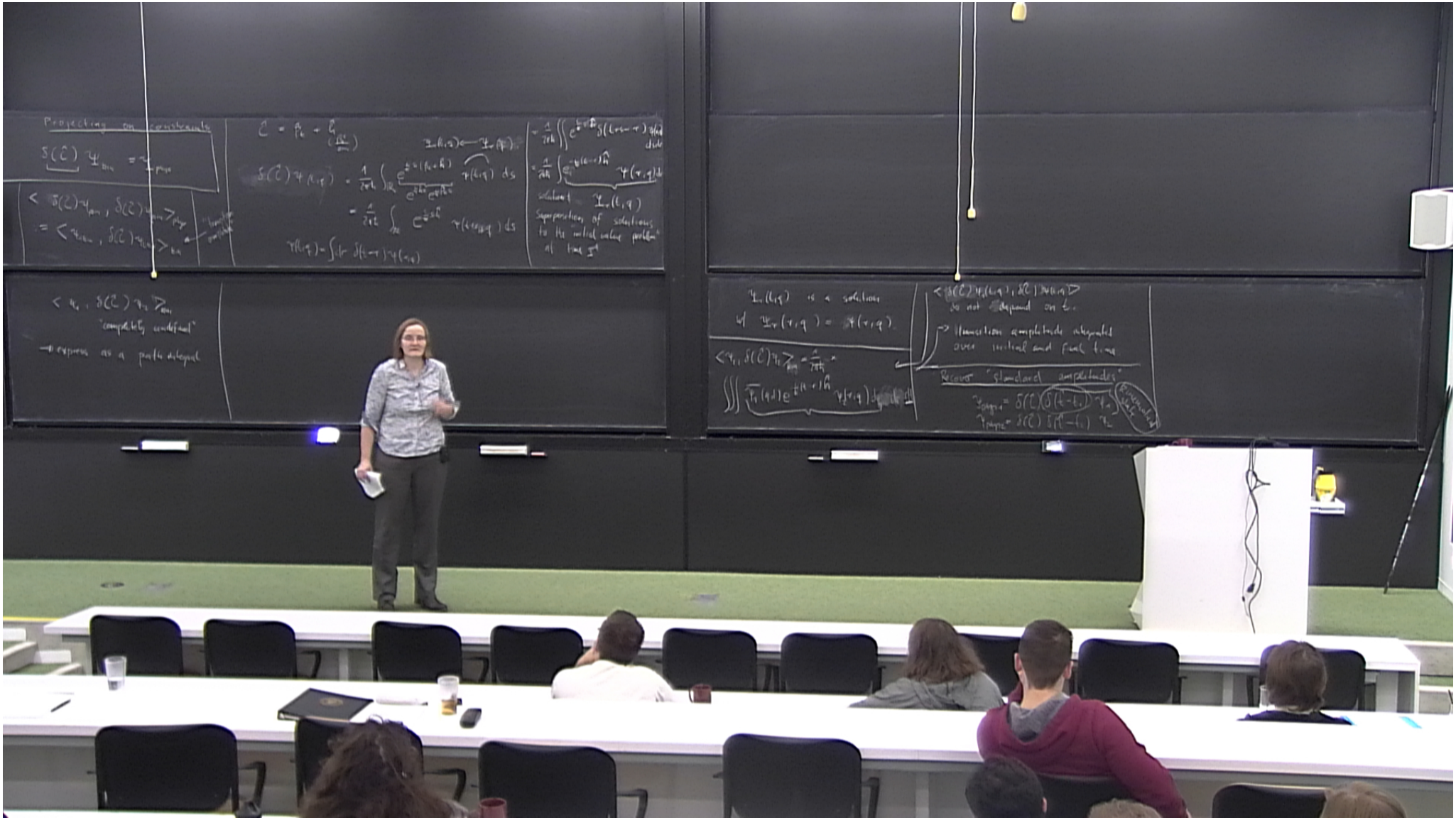

$\langle \psi_1, \delta(\hat{C})\psi_2 \rangle_{Kin}$
"completely undefined"

$\langle \psi_1 , \delta(\hat{C}) \psi_2 \rangle_{Kin}$

"completely undefined"

→ express as a path integral


$\Psi_r(t,q)$ is a solution
w/ $\Psi_r(r,q) = \psi(r,q)$.
$\langle \psi_1, \delta(\hat{C})\psi_2 \rangle_{Kin} = \frac{\hat{}}{2\pi\hbar}$
$\iiint \overline{\psi_1}(q,t) e^{-\frac{i}{\hbar}(t-r)\hat{h}} \psi_2(r,q)\, dq\, dr\, dt$
$\langle \delta(\hat{C})\psi_1(t,q), \delta(\hat{C})\psi(t,q) \rangle$
do not depend on t.
→ transition amplitude
over initial and
Recovers "standard
$\psi_{phys1} = \delta(\hat{C})\delta(\hat{t}-t_1$
$\psi_{phys2} = \delta(\hat{C})\,\delta(\hat{t}-t_2$

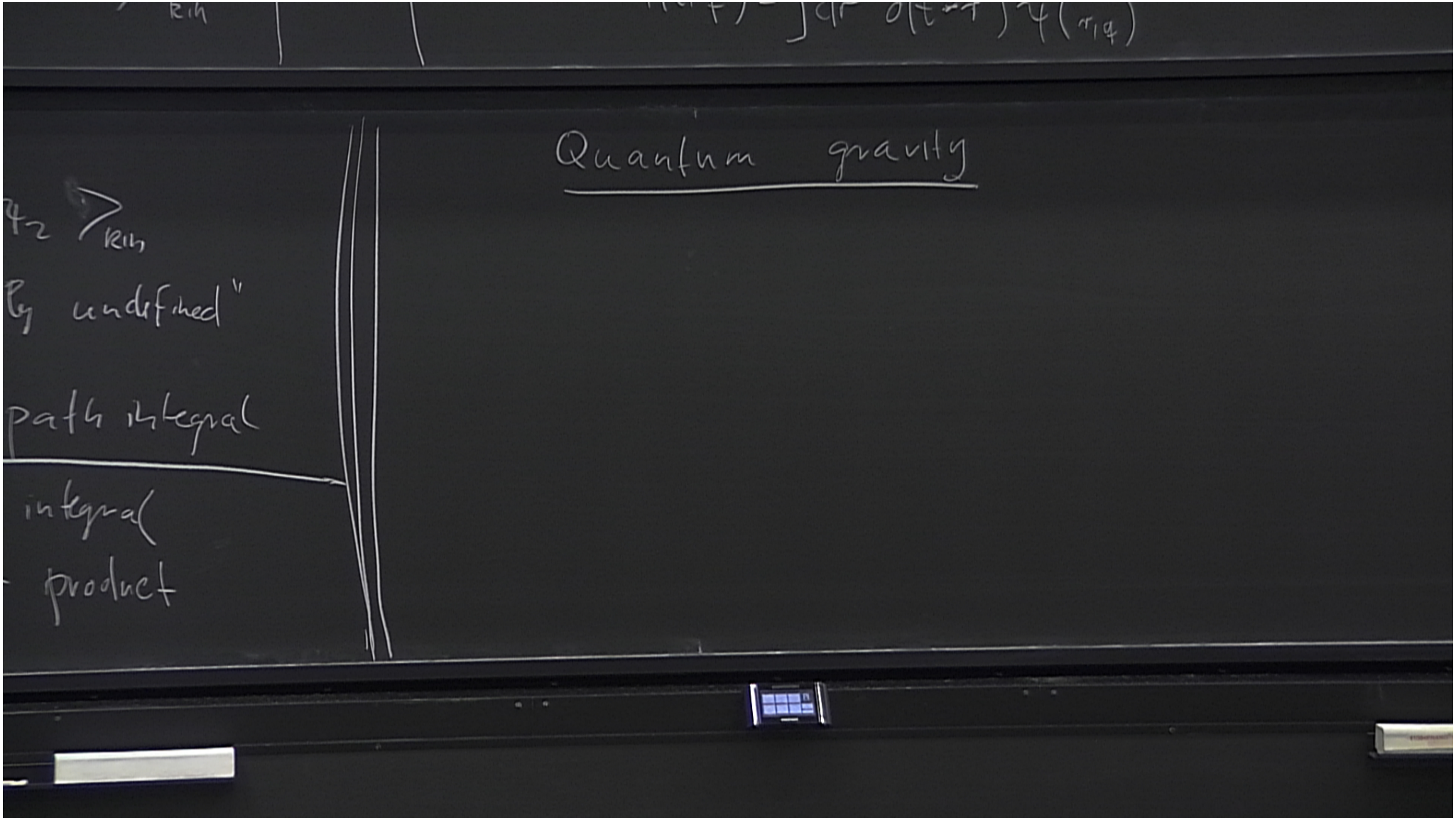

Projecting on eigenstates
"completely undefined"
Recover "standard amplitudes"

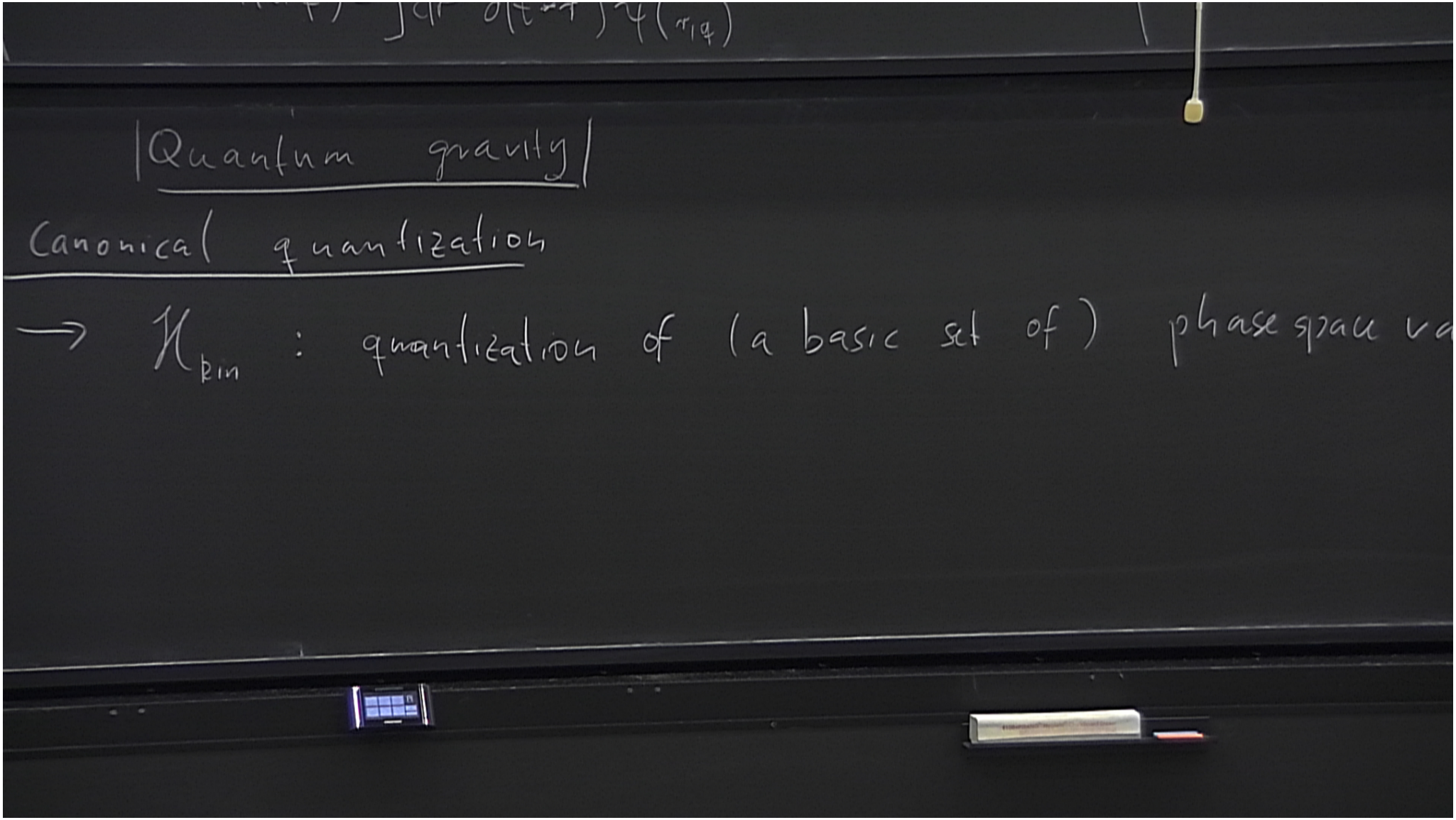
$\langle \psi_1 , \delta(\hat{C}) \psi_2 \rangle_{Kin}$

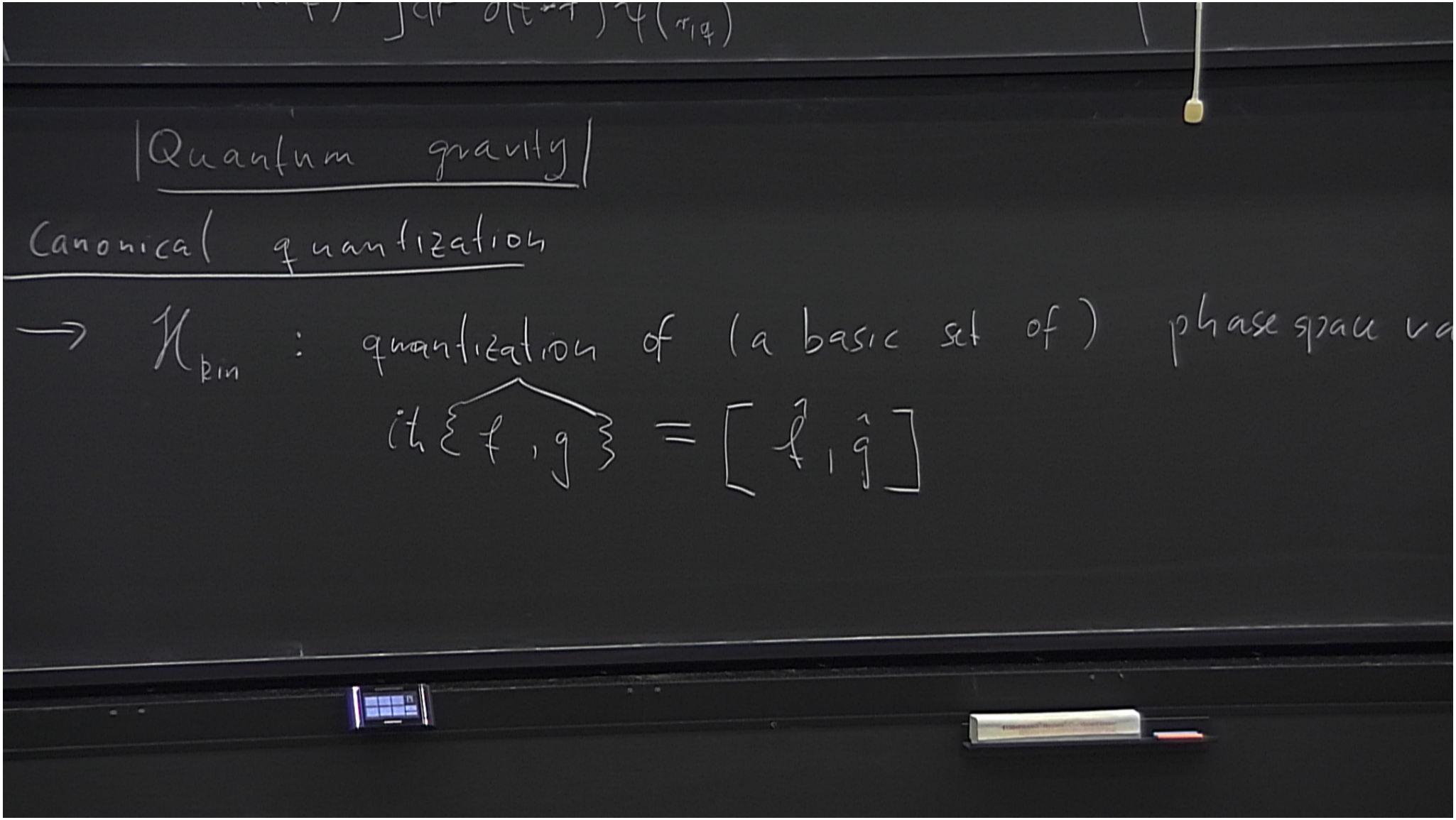
"completely undefined"

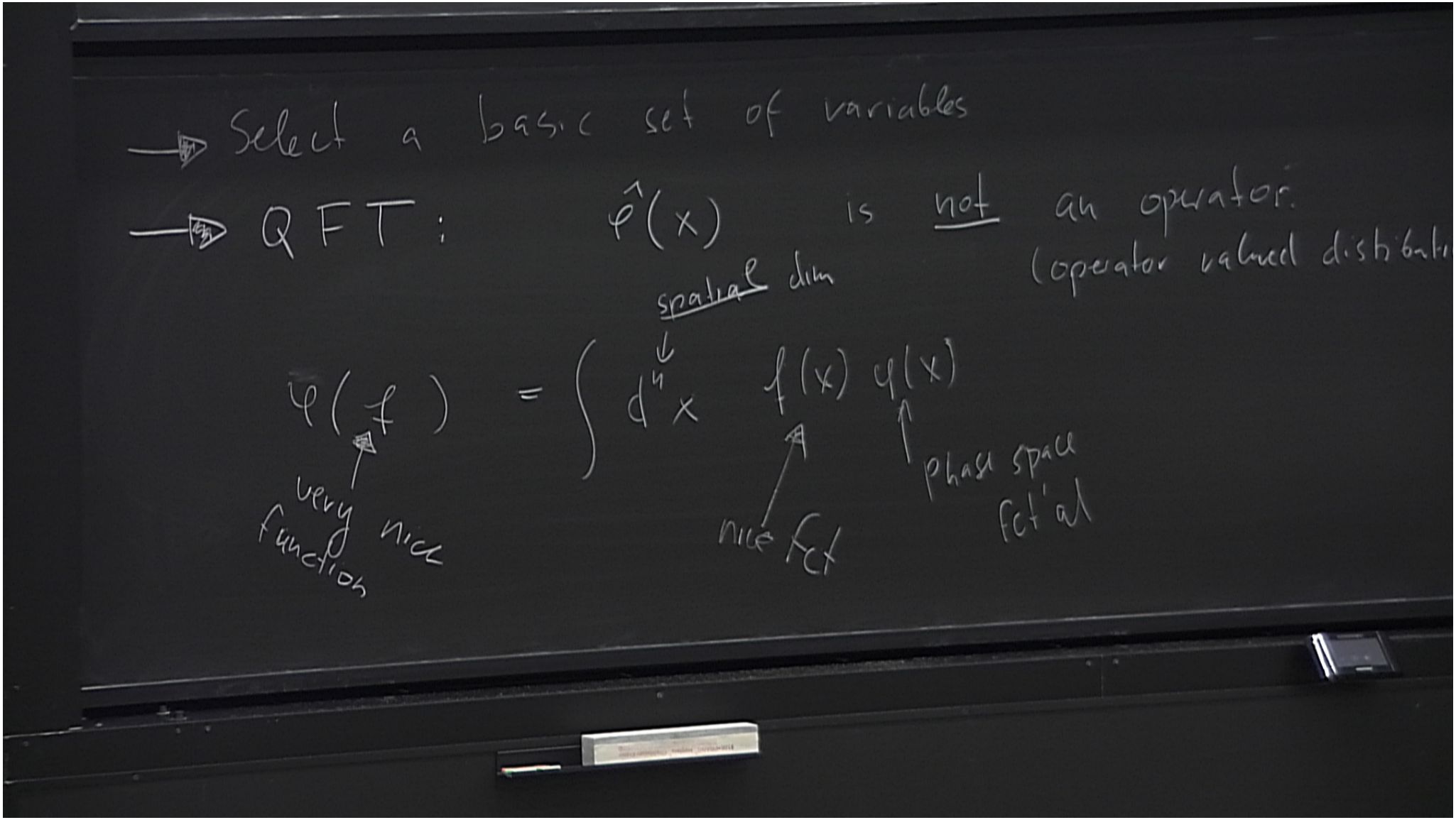
→ express as a path integral

- start w/ path integral
  → physical inner product


Quantum gravity
path integral
integral
product


Quantum gravity
Canonical quantization
→ $\mathcal{H}_{kin}$: quantization of (a basic set of) phase space va


Quantum gravity
Canonical quantization
→ $\mathcal{H}_{kin}$: quantization of (a basic set of) phase space va
$i\hbar\{f,g\} = [\hat{f},\hat{g}]$


Select a basic set of variables
QFT: $\hat{\varphi}(x)$ is not an operator.
(operator valued distribu
spatial dim
$\varphi(f) = \int d^n x \; f(x) \varphi(x)$
very nice function
nice fct
phase space fct'al

→ Select a basic set of variables

→ QFT: $\hat{\varphi}(x)$ is not an operator.

(operator valued distribution)

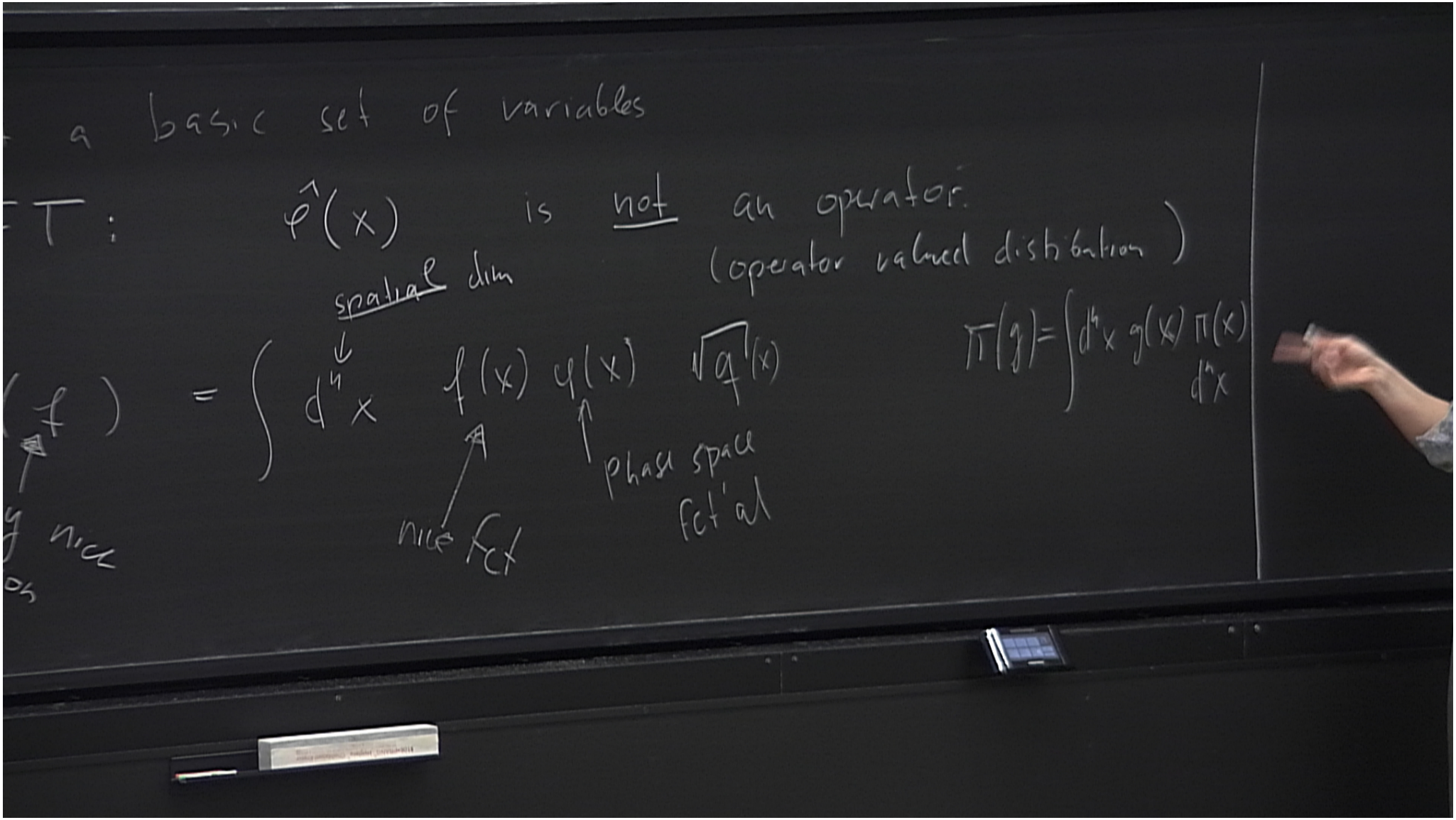
$$\varphi(f) = \int d^{n}x \; f(x)\, \varphi(x)\, \sqrt{q}(x)$$

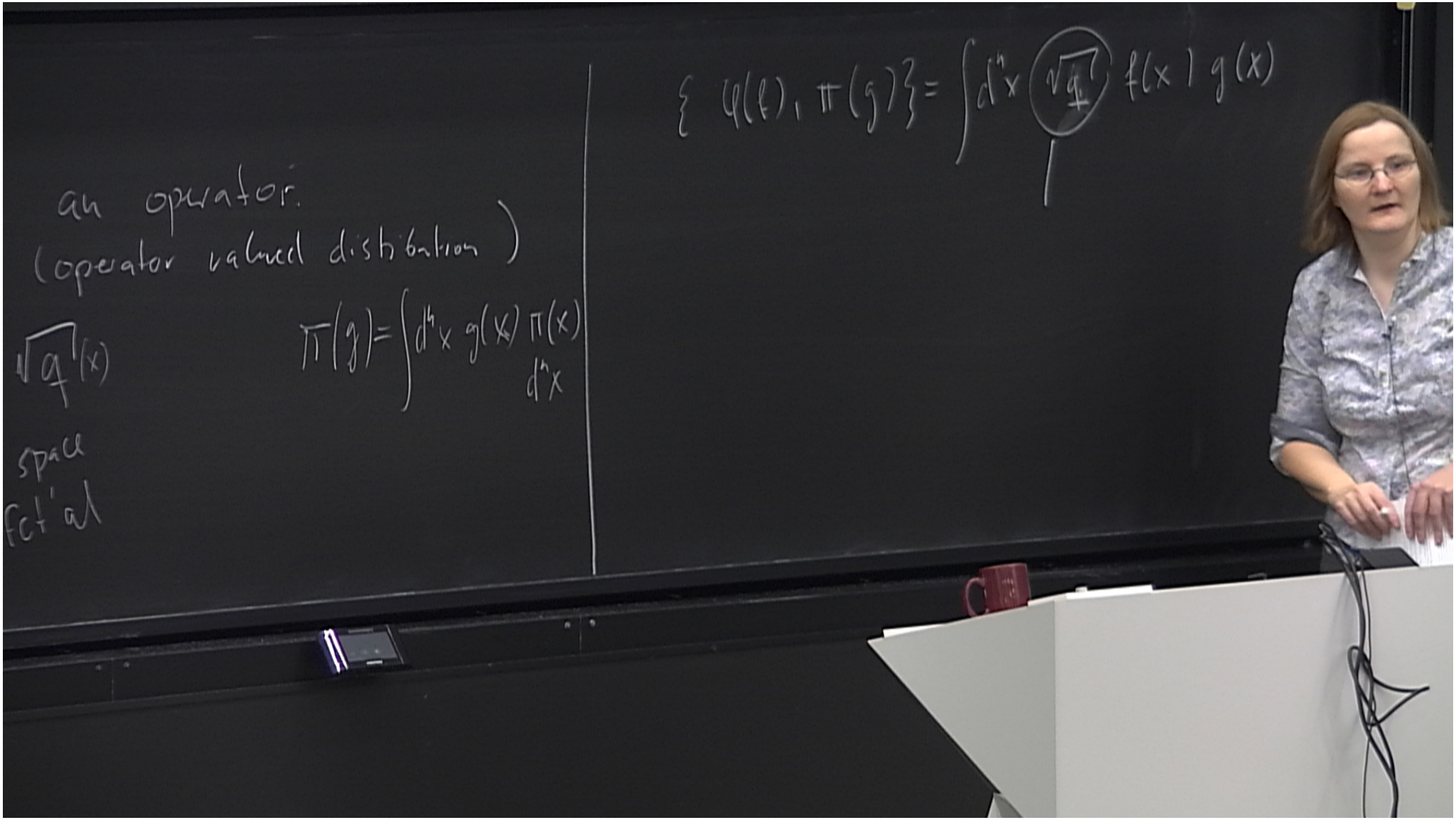
spatial dim

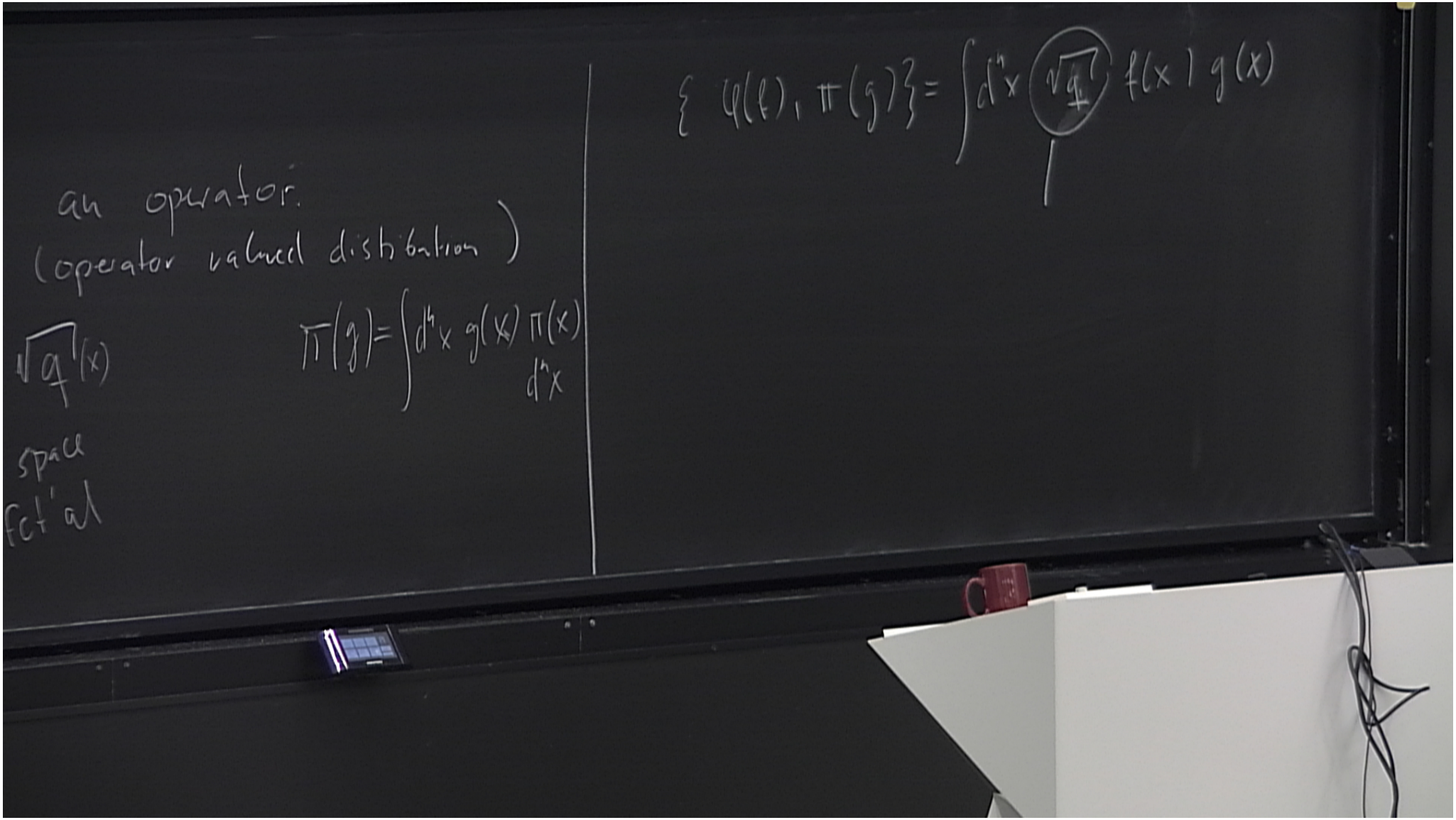
very nice function

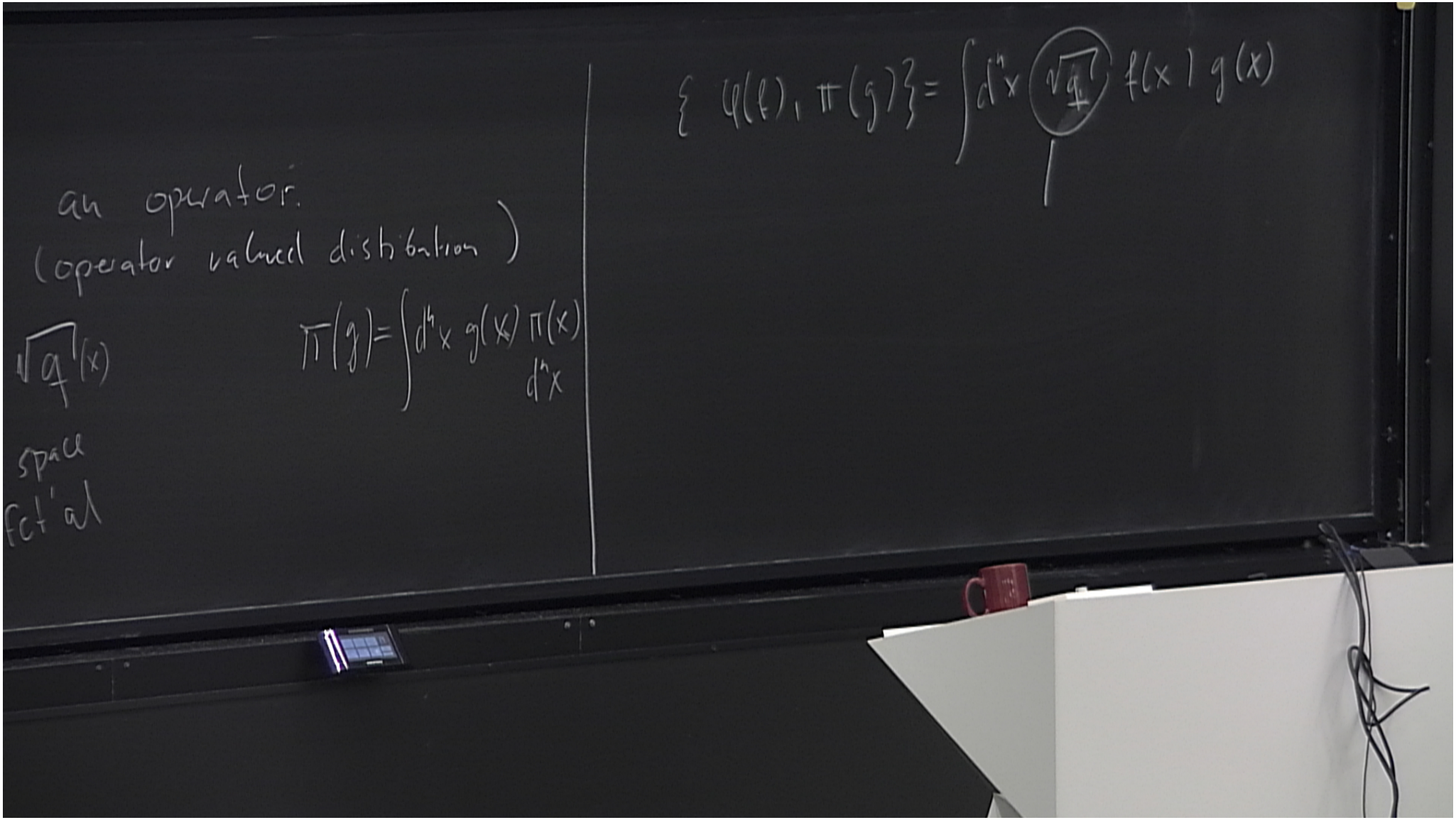
nice fct

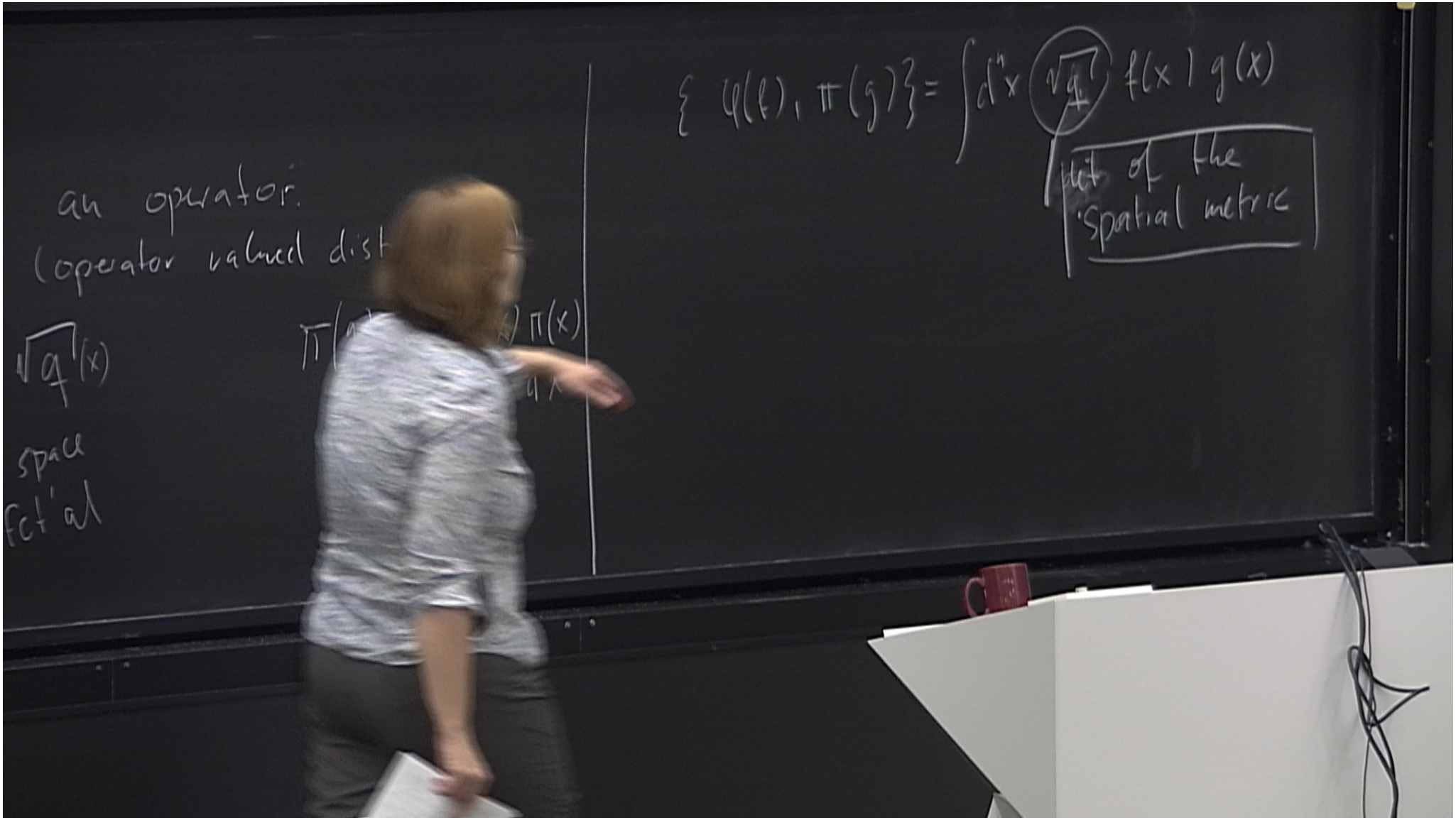
phase space fct'al

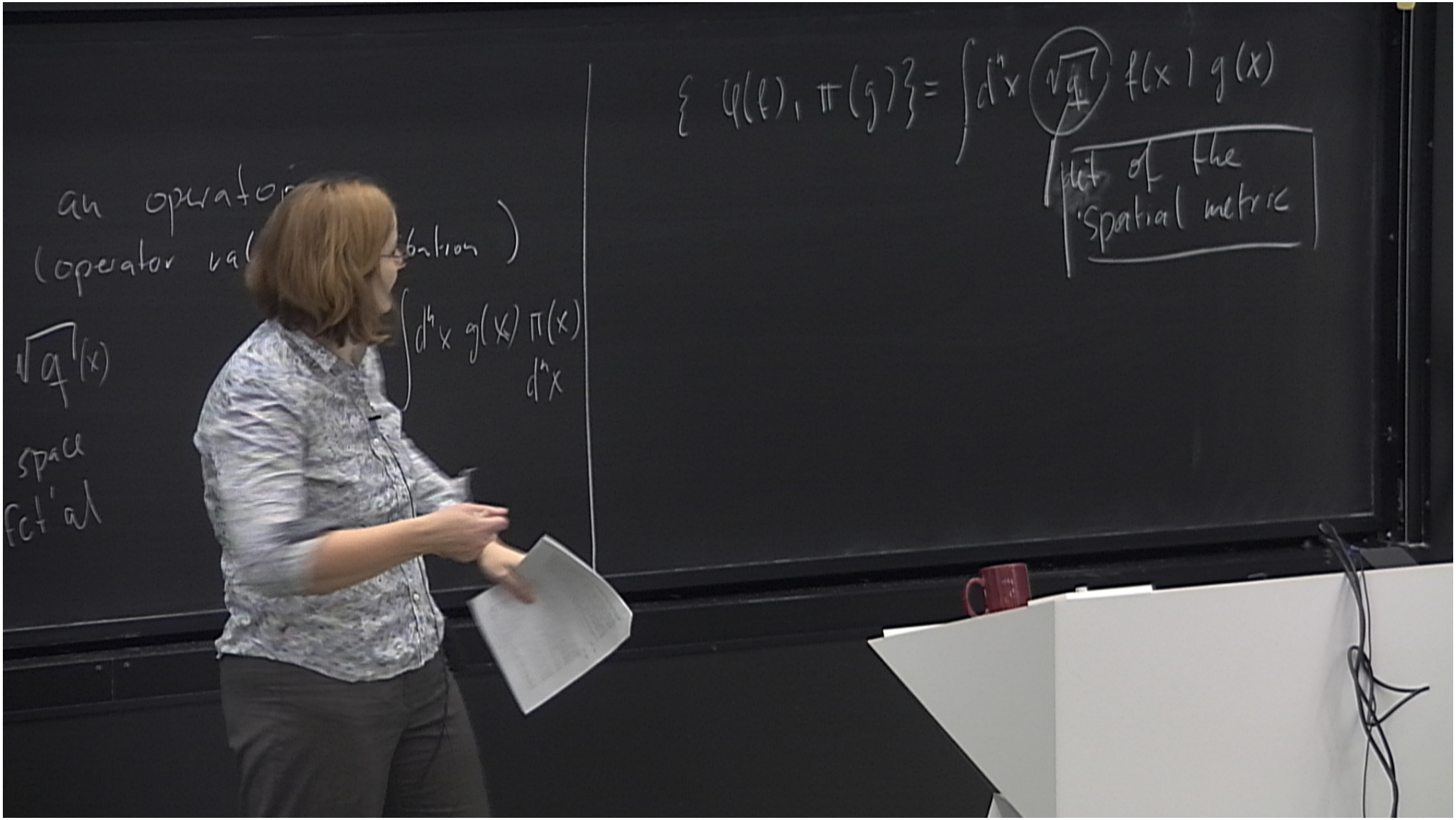

a basic set of variables
$\vec{\varphi}(x)$ is not an operator
(operator valued distribution)
spatial dim
$\pi(g) = \int d^n x \, g(x) \, \pi(x)$
$d^n x$
$f(x)$ $q(x)$ $\sqrt{q}(x)$
nice fct
phase space fct'al

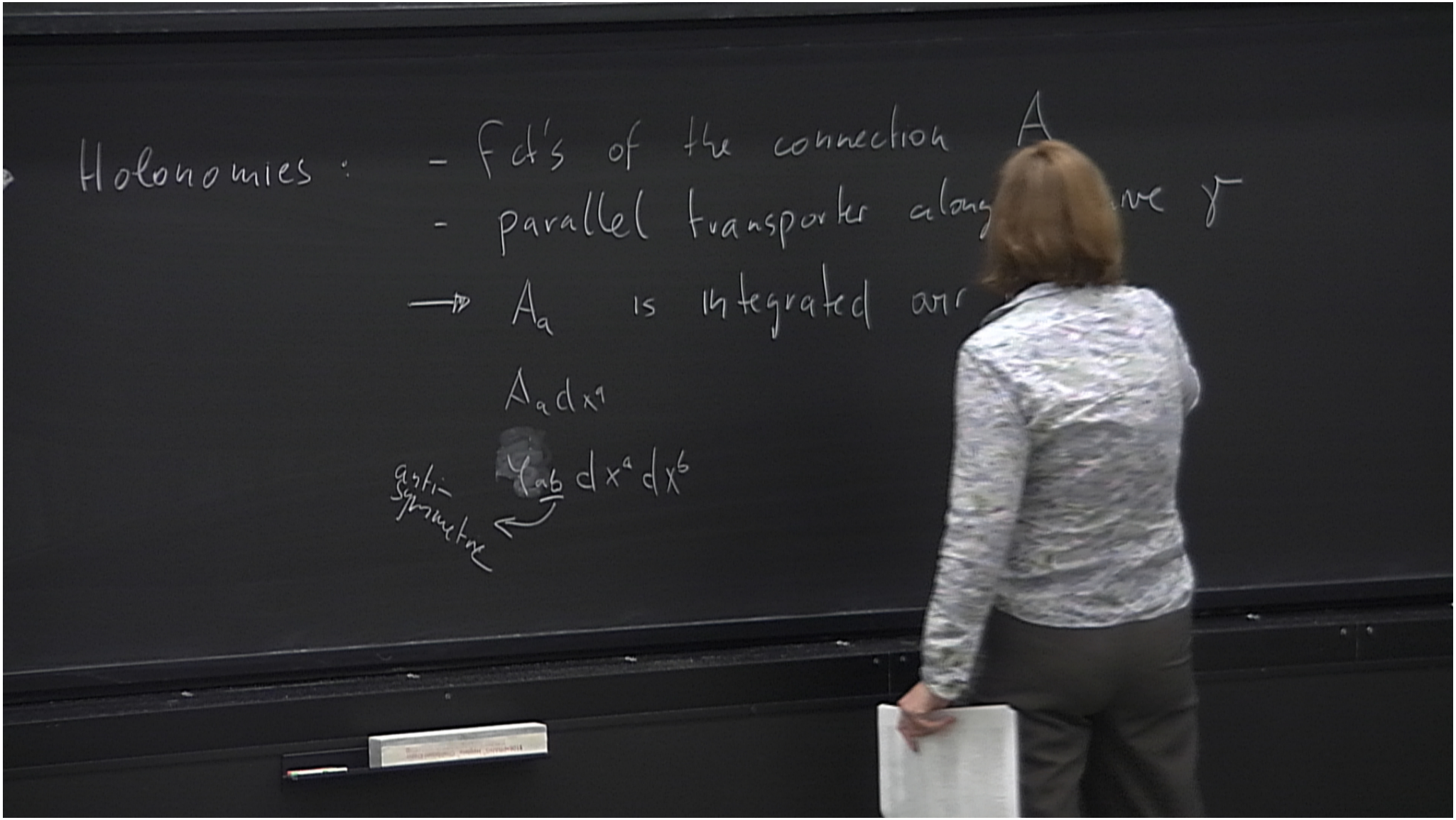

an operator
(operator valued distribution)
$\sqrt{q}(x)$
$\pi(g) = \int d^n x\, g(x)\, \pi(x)$
$d^n x$
space
fct'al
$\{\varphi(f), \pi(g)\} = \int d^n x \sqrt{q}\, f(x)\, g(x)$

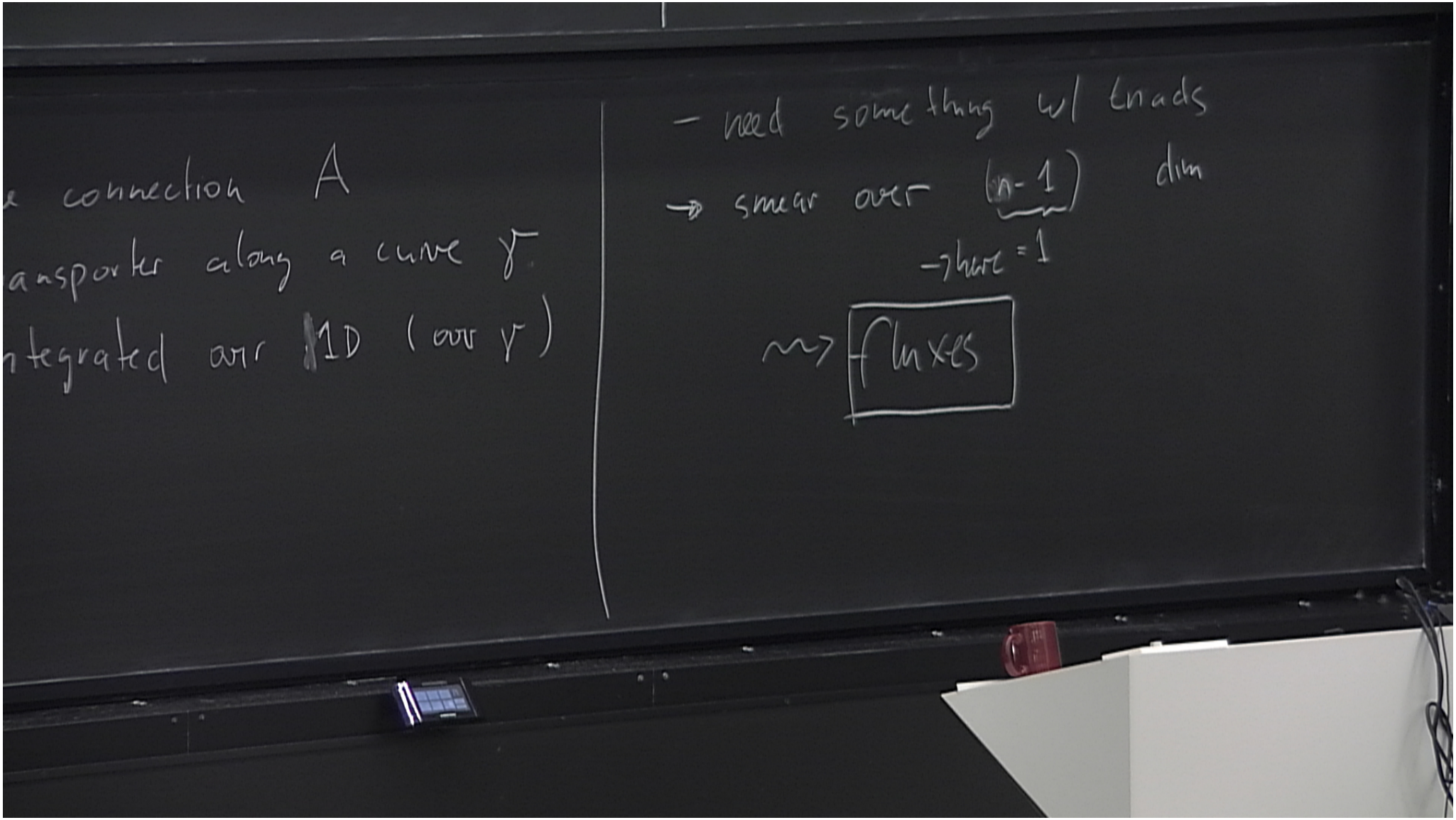

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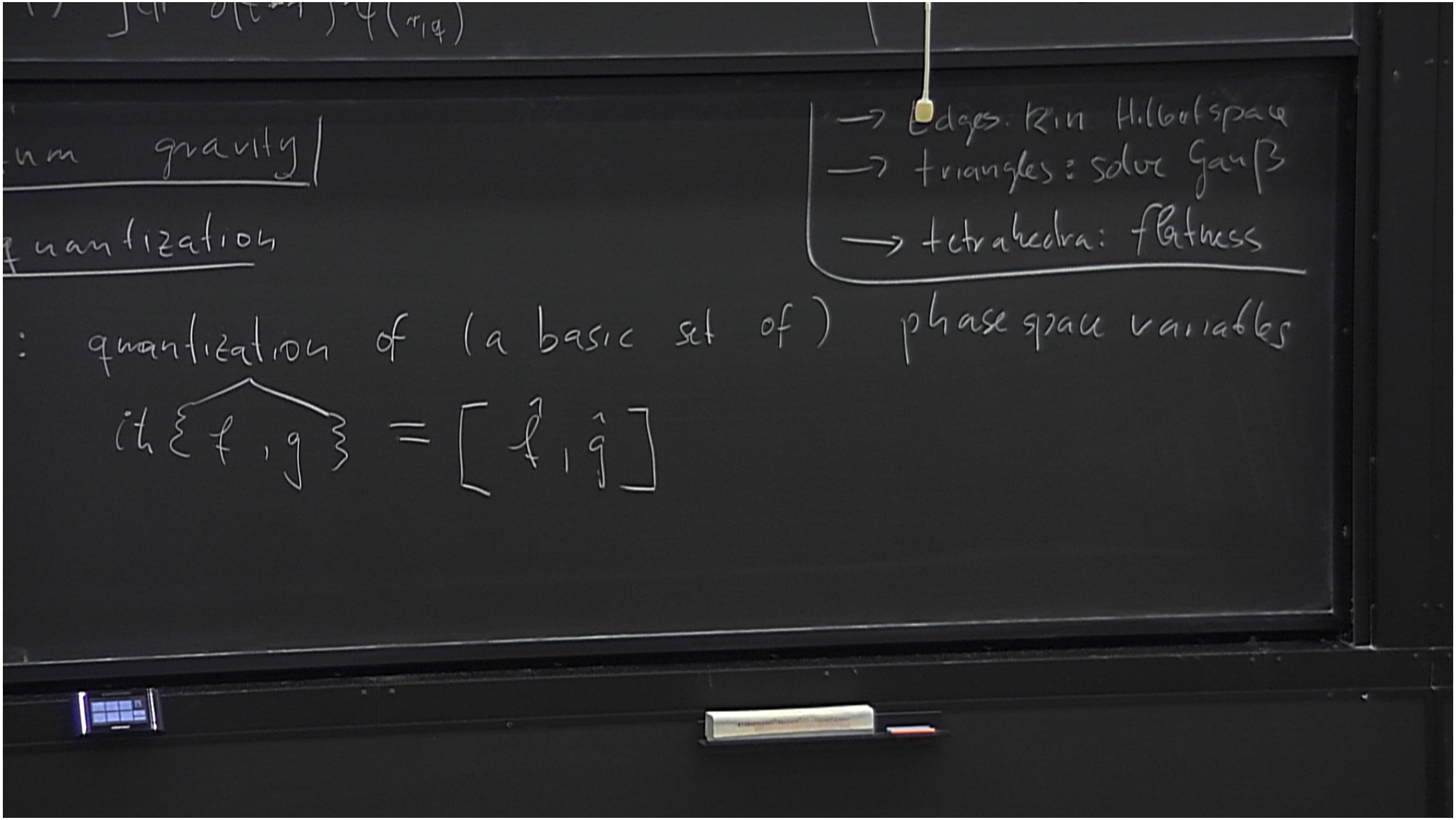

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an operator
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det of the
spatial metric


$\{\varphi(f), \pi(g)\} = \int d^n x \sqrt{q}\, f(x)\, g(x)$
det of the spatial metric
$\int d^n x\, g(x)\, \pi(x)$
$d^n x$
$\sqrt{q}(x)$


Holonomies :
- fct's of the connection A
- parallel transporter along
→ A_a is integrated
$A_a dx^a$
$A_{ab} dx^a dx^b$
anti-
symmetric


connection A
ansporter along a curve γ
ntegrated over 1D (over γ)
- need something w/ knots
→ smear over (n-1) dim
-> here = 1
~> fluxes


→ Edges: Rin Hilbertspace
→ triangles: solve Gauß
→ tetrahedra: flatness
: quantization of (a basic set of) phase space variables
$i\hbar\{f,g\} = [\hat{f},\hat{g}]$