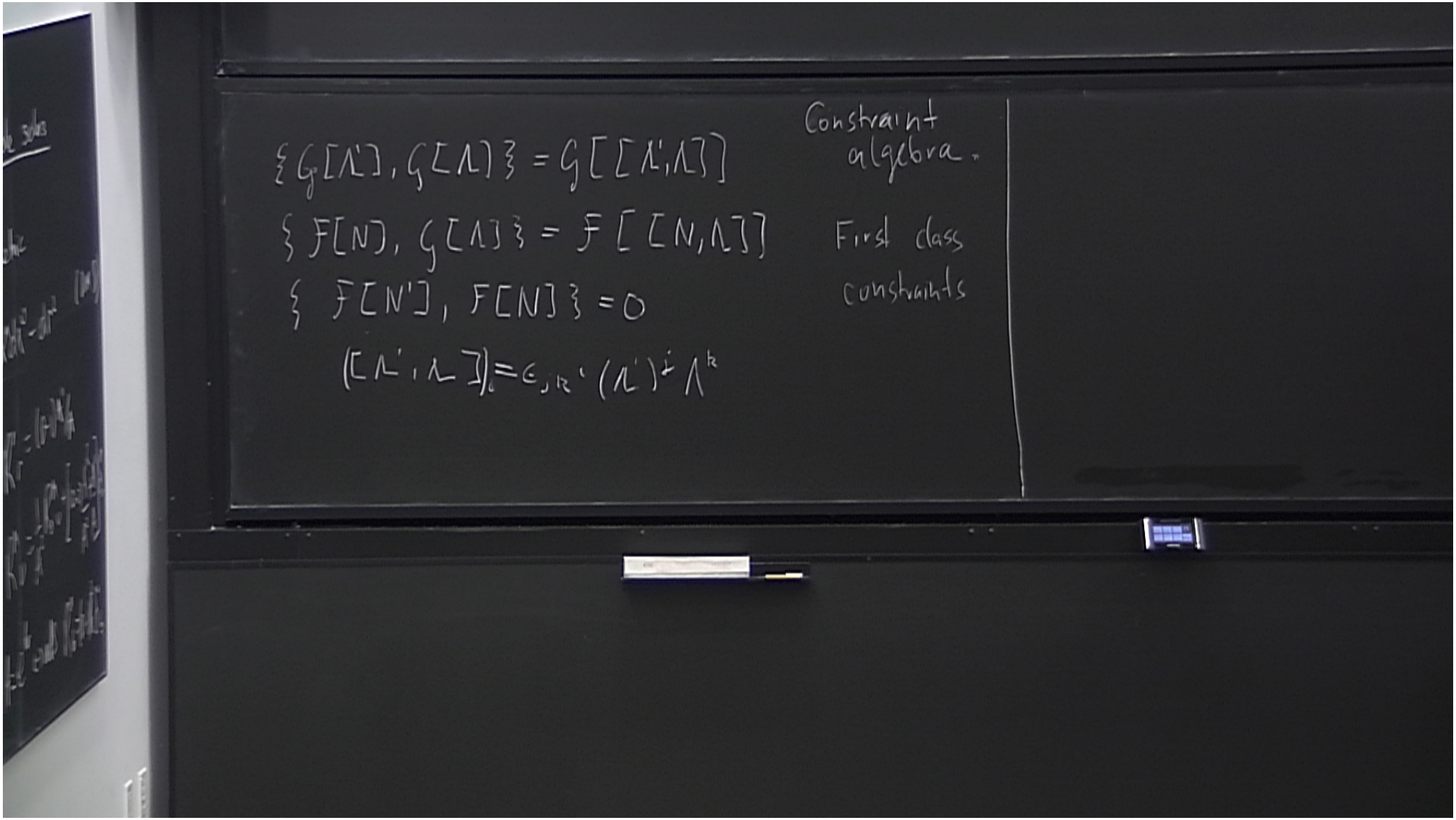
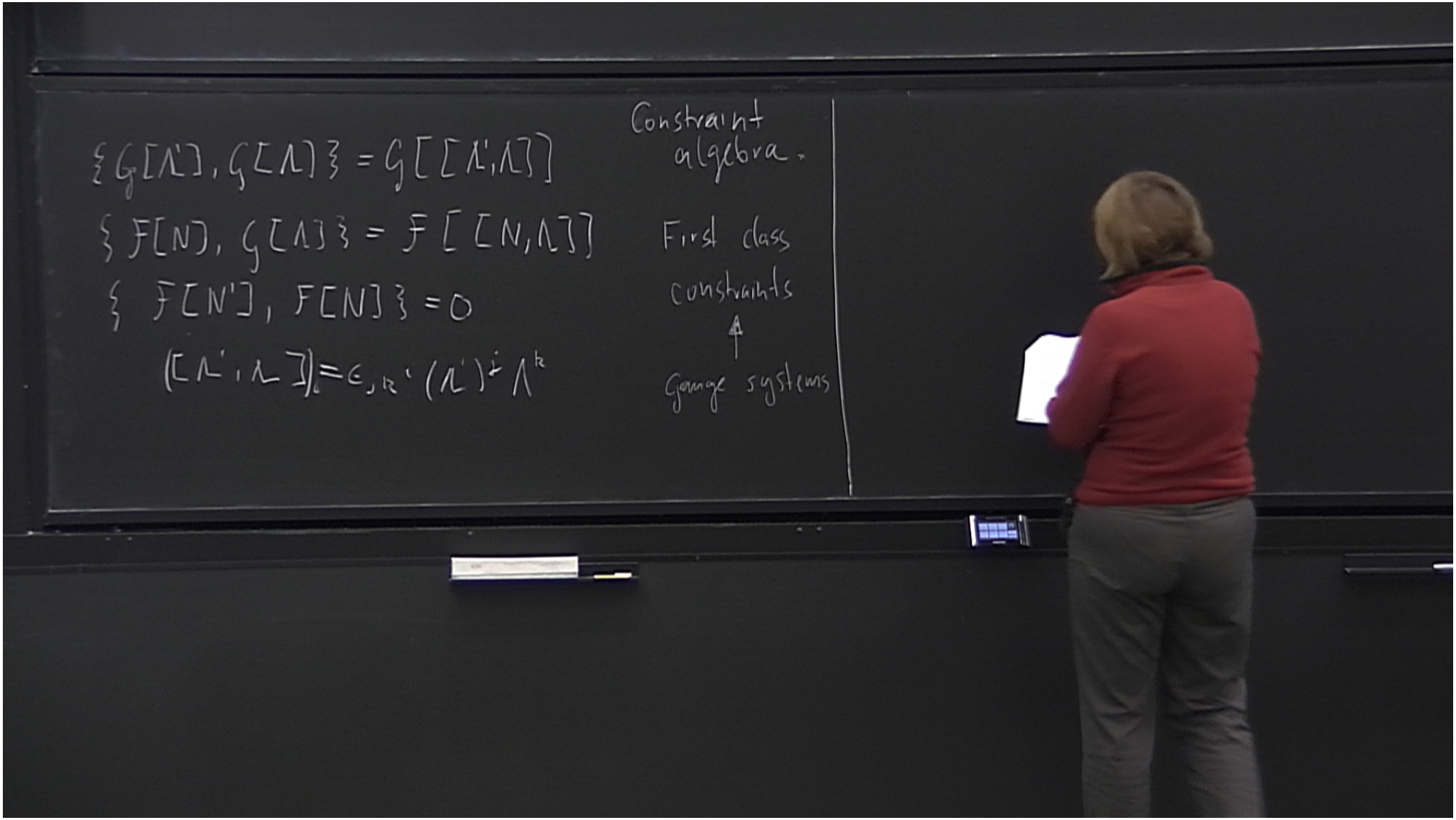
Title: 12/13 PSI - Quantum Gravity Review Lecture 6

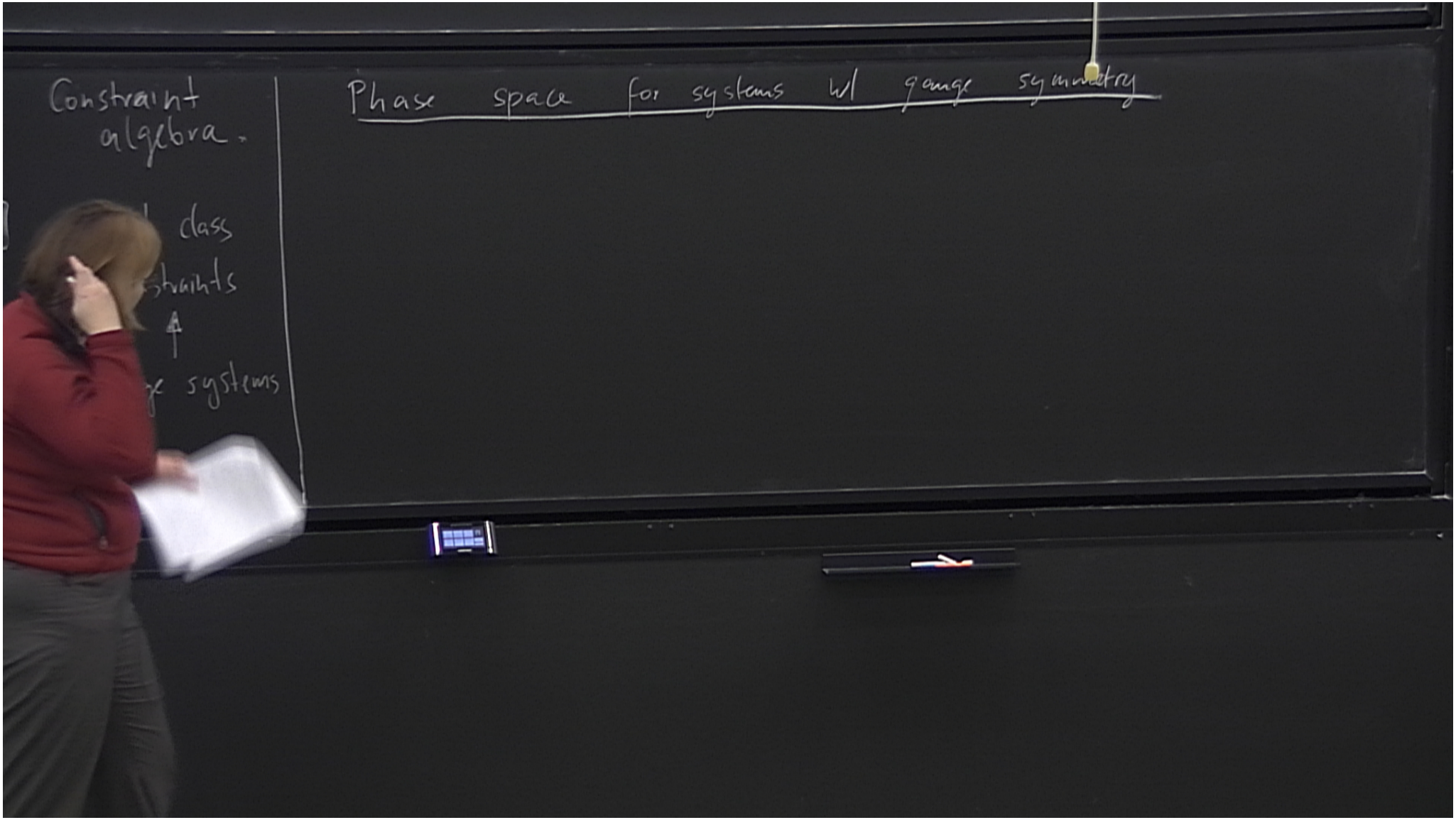
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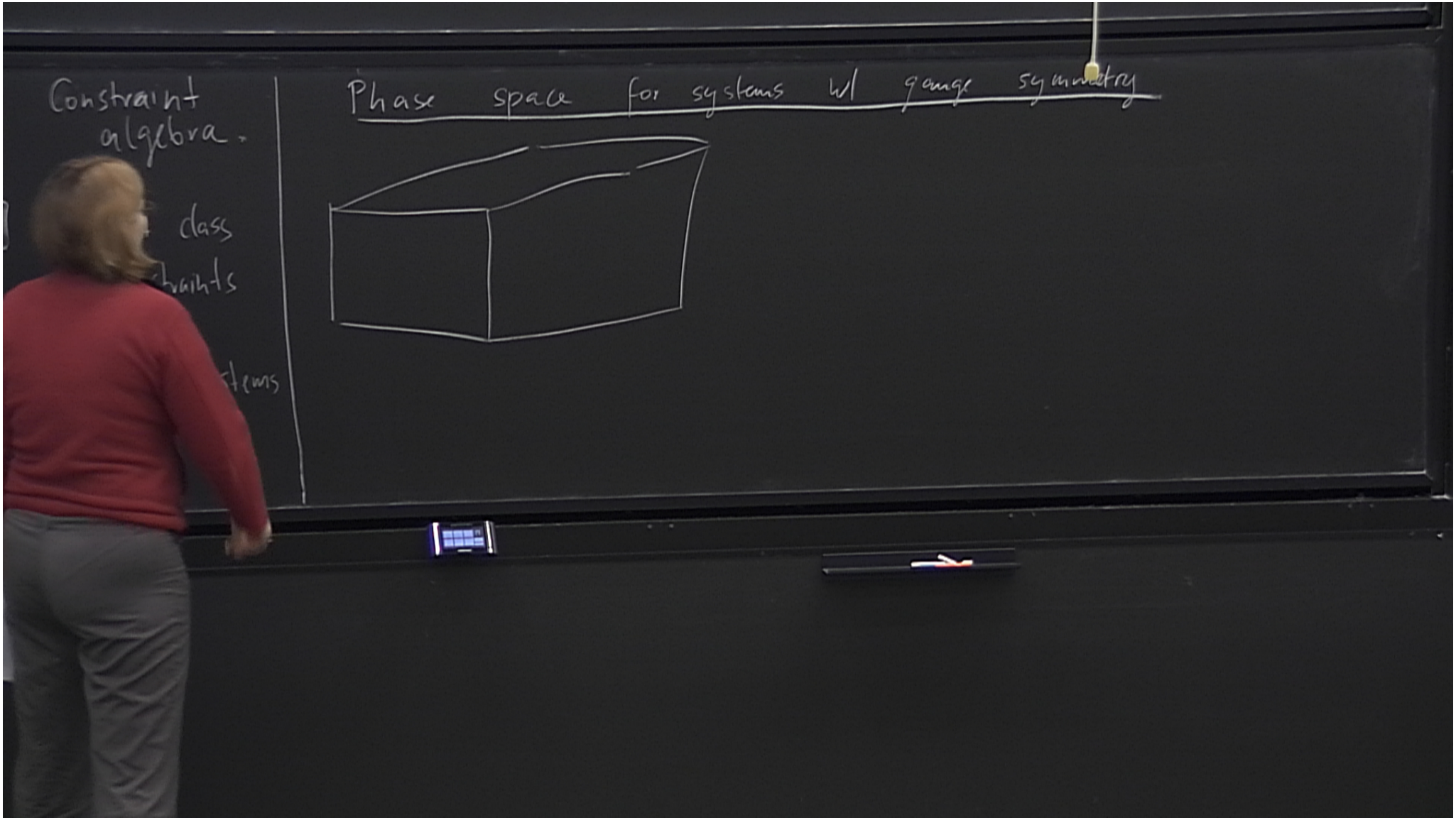
URL: http://pirsa.org/13020047

Abstract:


Constraint algebra.
First class constraints


{G[Λ'], G[Λ]} = G[[Λ', Λ]]
{F[N], G[Λ]} = F[[N, Λ]]
{F[N'], F[N]} = 0
[Λ', Λ]_i = C_{ik}^{j} (Λ') Λ^k
Constraint algebra.
First class constraints
Gauge systems


Constraint algebra.
class
straints
systems
Phase space for systems w/ gauge symmetry


Constraint algebra.
class
Phase space for systems w/ gauge symmetry

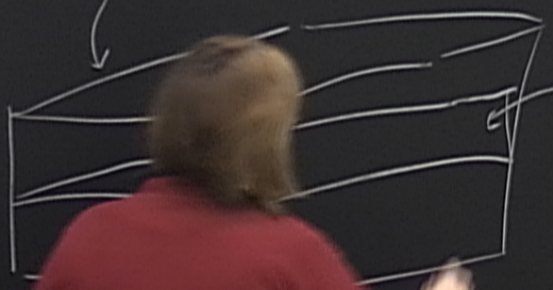
Constraint algebra.

First class constraints

↑

Gauge systems

Phase space for systems w/ gauge symmetry



constraint hypersurface. only these points lead to solutions

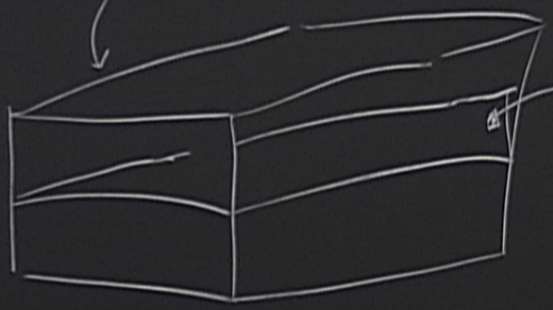
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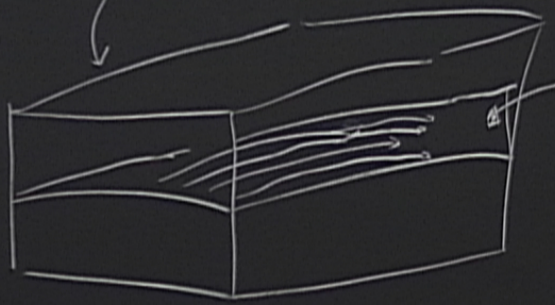
Constraint algebra.

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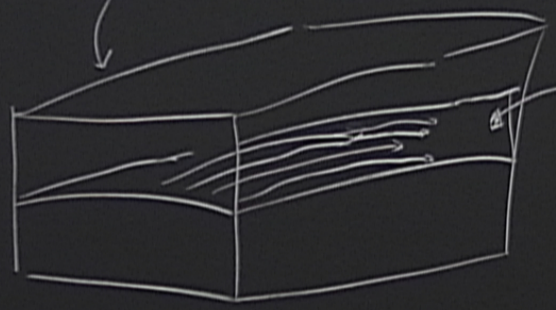
Constraint algebra.

First class constraints

↑

Gauge systems

Phase space for systems w/ gauge symmetry



constraint hypersurface. only these points lead to solutions

$\dot{E}^{aj}$, $\dot{A}^{j}_{a}$

$a = 1,2$

$j = 1,2,3$

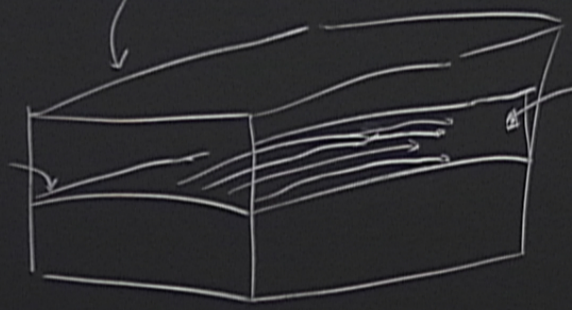
Constraint algebra.

First class constraints

↑

Gauge systems

Phase space for systems w/ gauge symmetry



constraint hypersurface. only these points lead to solutions

π^{q_i}, A_i

$q = 1,2$

$i = 1,2,3$

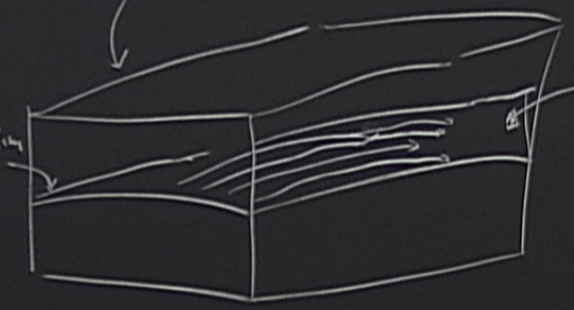
Constraint algebra.

First class constraints

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Gauge systems

Phase space for systems w/ gauge symmetry



constraint hypersurface. only these points lead to solutions

$\vec{E}^a$, $\vec{A}_a$

$a = 1,2$

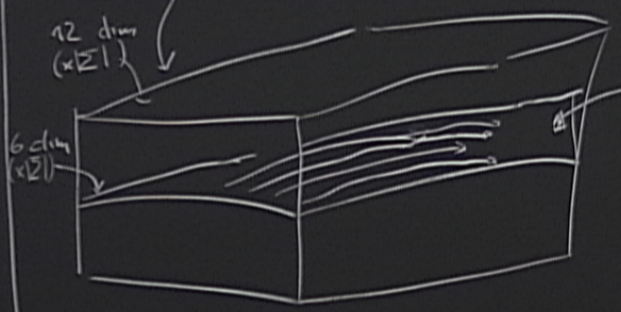
$i = 1,2,3$

Constraint algebra.

First class constraints

systems

Phase space for systems w/ gauge symmetry



constraint hypersurface. only these points lead to solutions

$\tilde{t}^{q}$, $\tilde{A}^{i}_{q}$

$q = 1,2$

$i = 1,2,3$

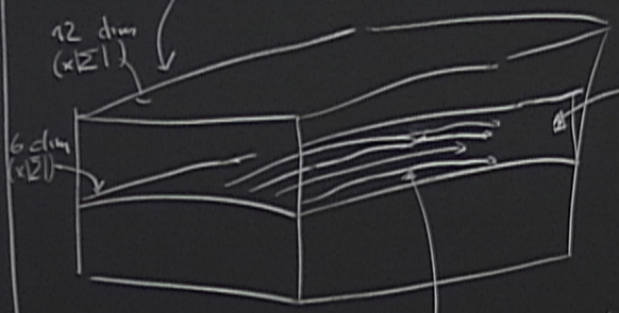
Constraint algebra.

First class constraints

↑

Gauge systems

Phase space for systems w/ gauge symmetry



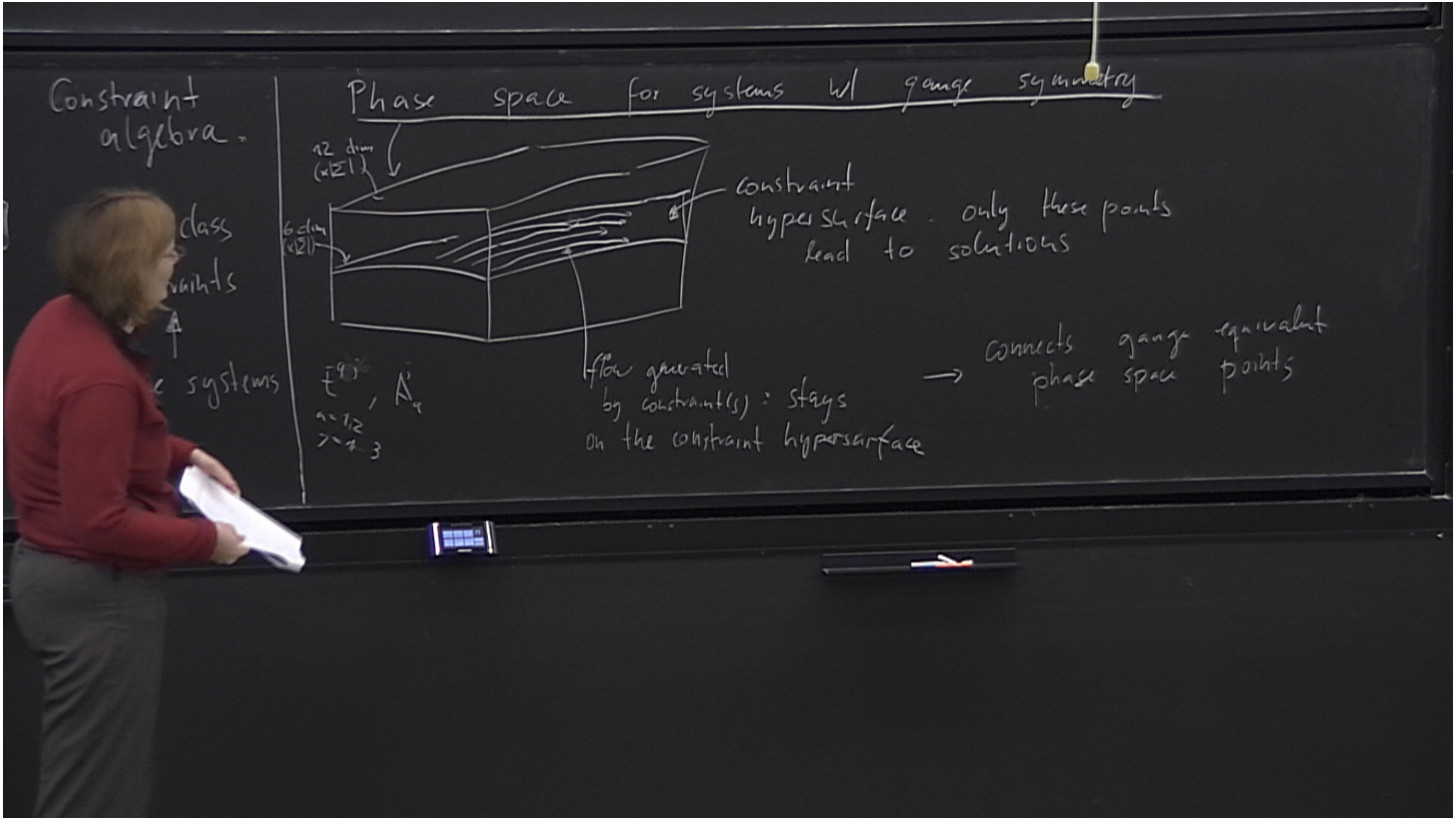
constraint hypersurface. only these points lead to solutions

$\bar{t}^{(q)}$, A^i_q

$q = 1,2$

$i = 1,2,3$

flow generated by constraint(s)


Constraint algebra.
class
systems
Phase space for systems w/ gauge symmetry
12 dim (x2)
6 dim (x2)
constraint hypersurface. only these points lead to solutions
flow generated by constraint(s): stays on the constraint hypersurface
→ connects gauge equivalent phase space points

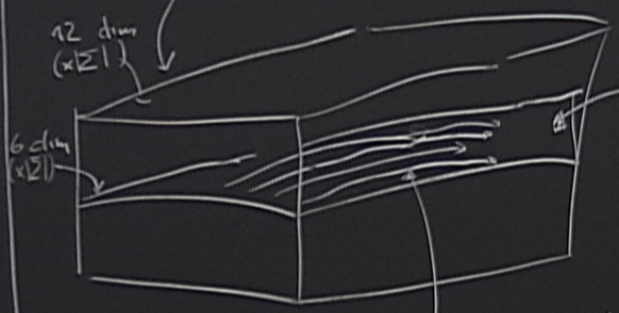
Constraint algebra.

First class constraints

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Gauge systems

Phase space for systems w/ gauge symmetry



constraint hypersurface. only these points lead to solutions

$\bar{t}^{(q)}$, A^i_q

$q = 1, 2$

$i = 1, 2, 3$

flow generated by constraint(s): stays on the constraint hypersurface → connects gauge equivalent phase space points

Gauge orbit (through p)
set of all points p' which
can be connected to p
by a flow generated by
constraints

Gauge orbit (through p)
set of all points p' which
can be connected to p
by a flow generated by
constraints

"physical states" → (solutions) ⟷ gauge orbits

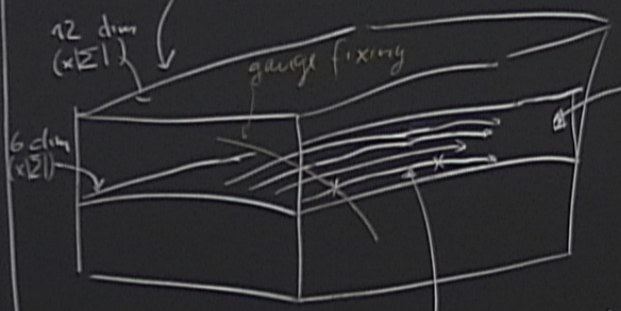
Constraint algebra.

First class constraints

↑

Gauge systems

Phase space for systems w/ gauge symmetry



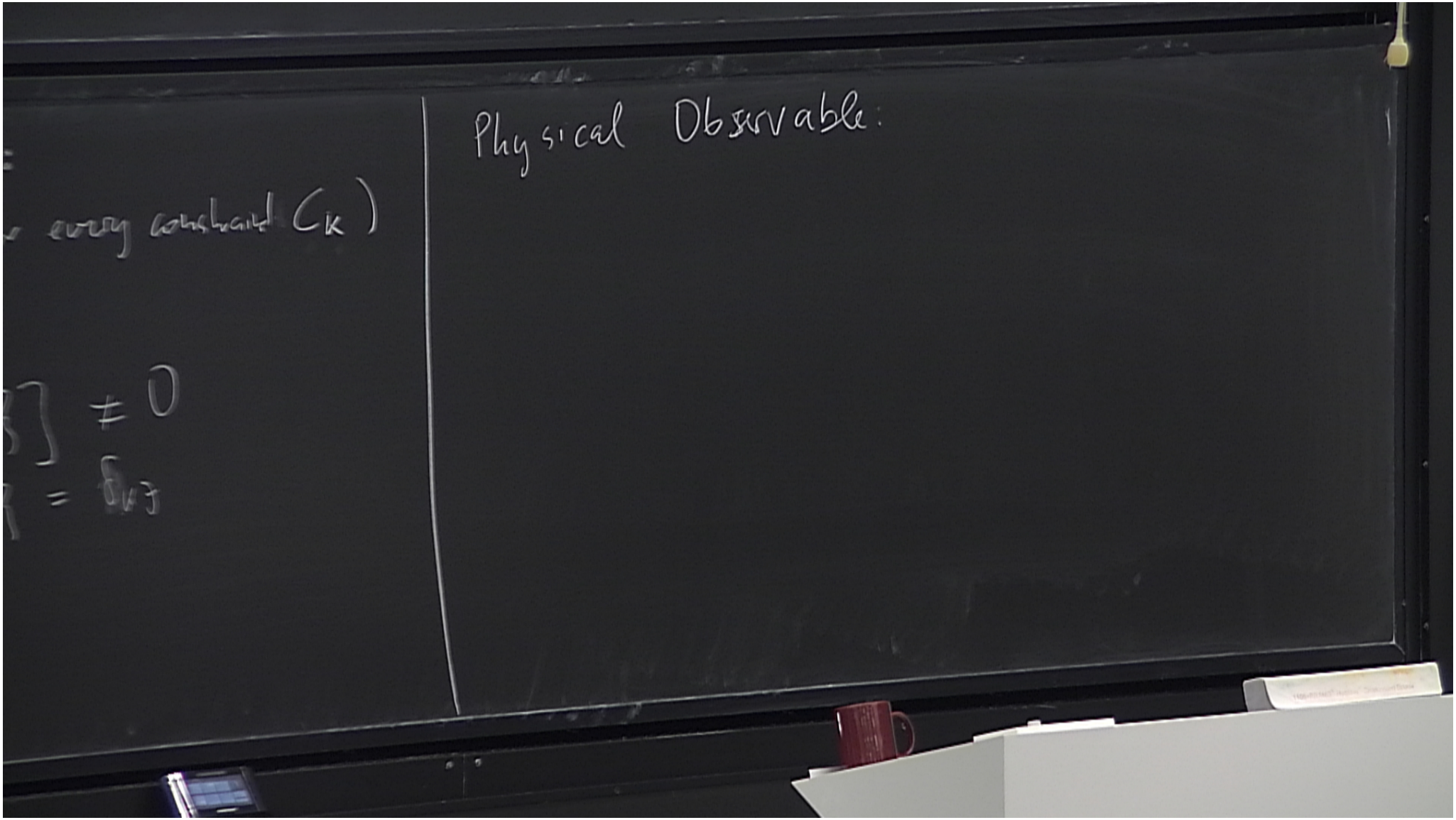
constraint hypersurface. only these points lead to solutions

$\vec{t}^{(q)}$, $\vec{A}_q$

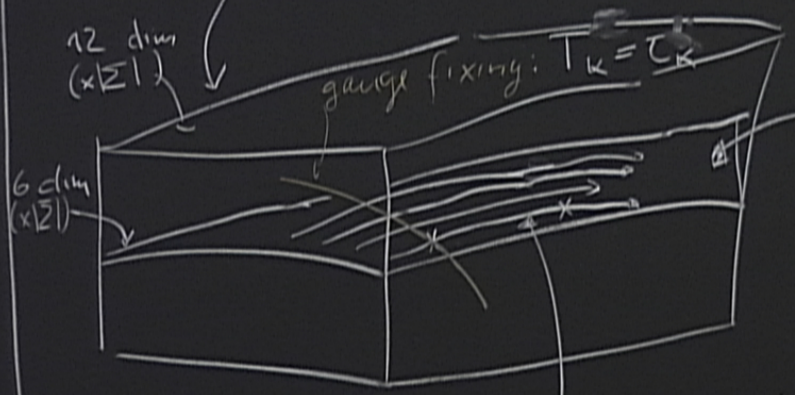
$q = 1,2$

$i = 1,2,3$

flow generated by constraint(s): stays on the constraint hypersurface → connects gauge equivalent phase space points


every constraint C_k)
$\neq 0$
Physical Observable:

Phase space for systems w/ gauge symmetry



$F_f(p) = f(p')$ where p' gauge equiv. p and T_k

constraint hypersurface: only these points lead to solutions

$\xi^{(q)}_j$, Λ_q

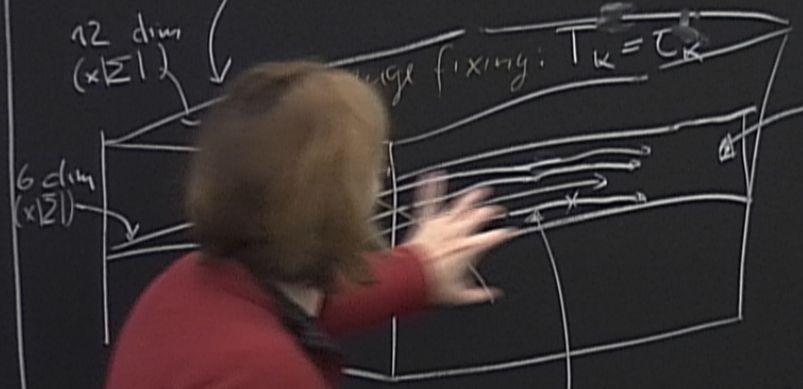
$q = 1,2$
$j = 1,2,3$

$\{T, C\} \neq 0$ flow generated by constraint(s): stays on the constraint hypersurface

flow generated by C

→ connects gau… phase spac…

Phase space for systems w/ gauge symmetry



$F_f(p)(\tau_k) = f(p')$ where p' gauge equiv p and $T_k(p') =$

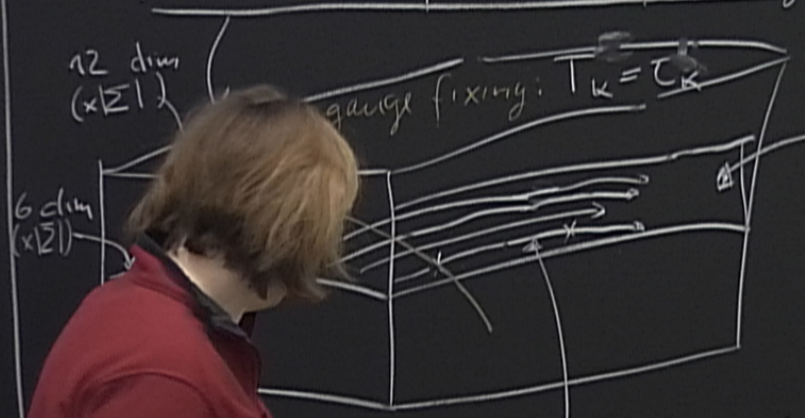
constraint hypersurface : only these points lead to solutions

$\{T, C\} \neq 0$ flow generated by constraint(s) : stays on the constraint hypersurface

flow generated by C

→ connects gauge equivalent phase space points

Phase space for systems w/ gauge symmetry



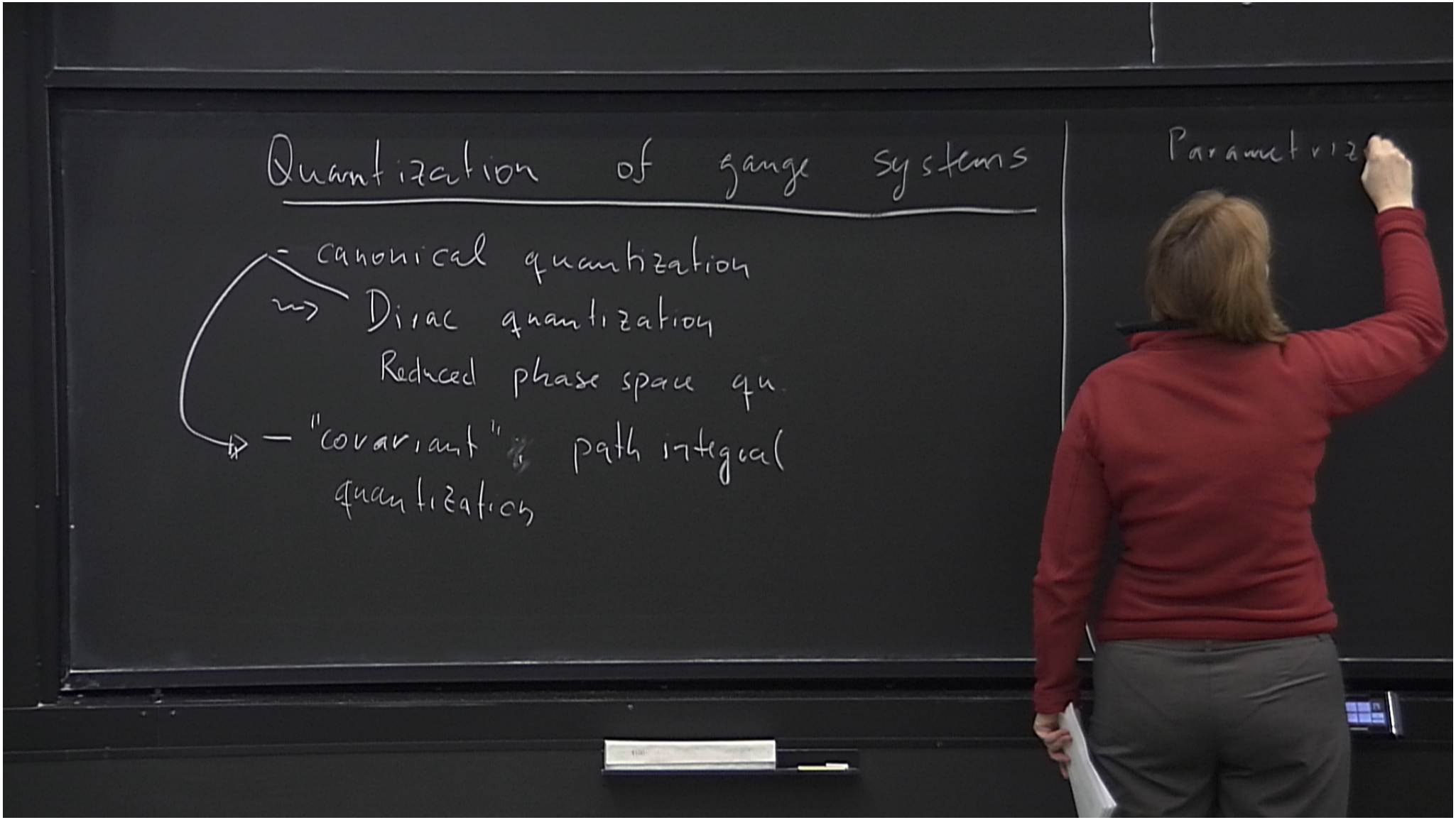
$F_f(p) = f(p')$ where p' gauge equiv p and $T_k(p') =$

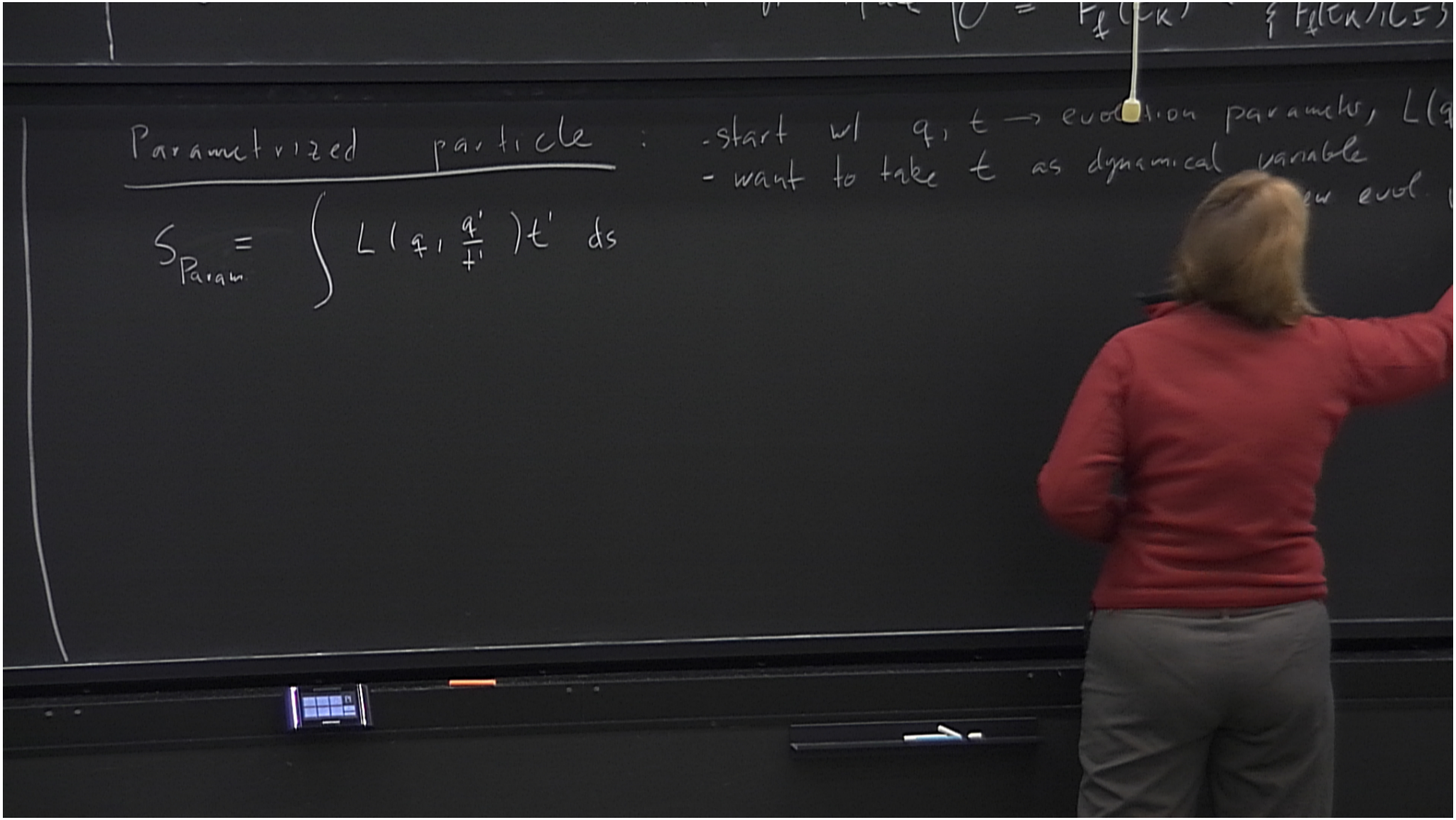
constraint hypersurface : only these points lead to solutions

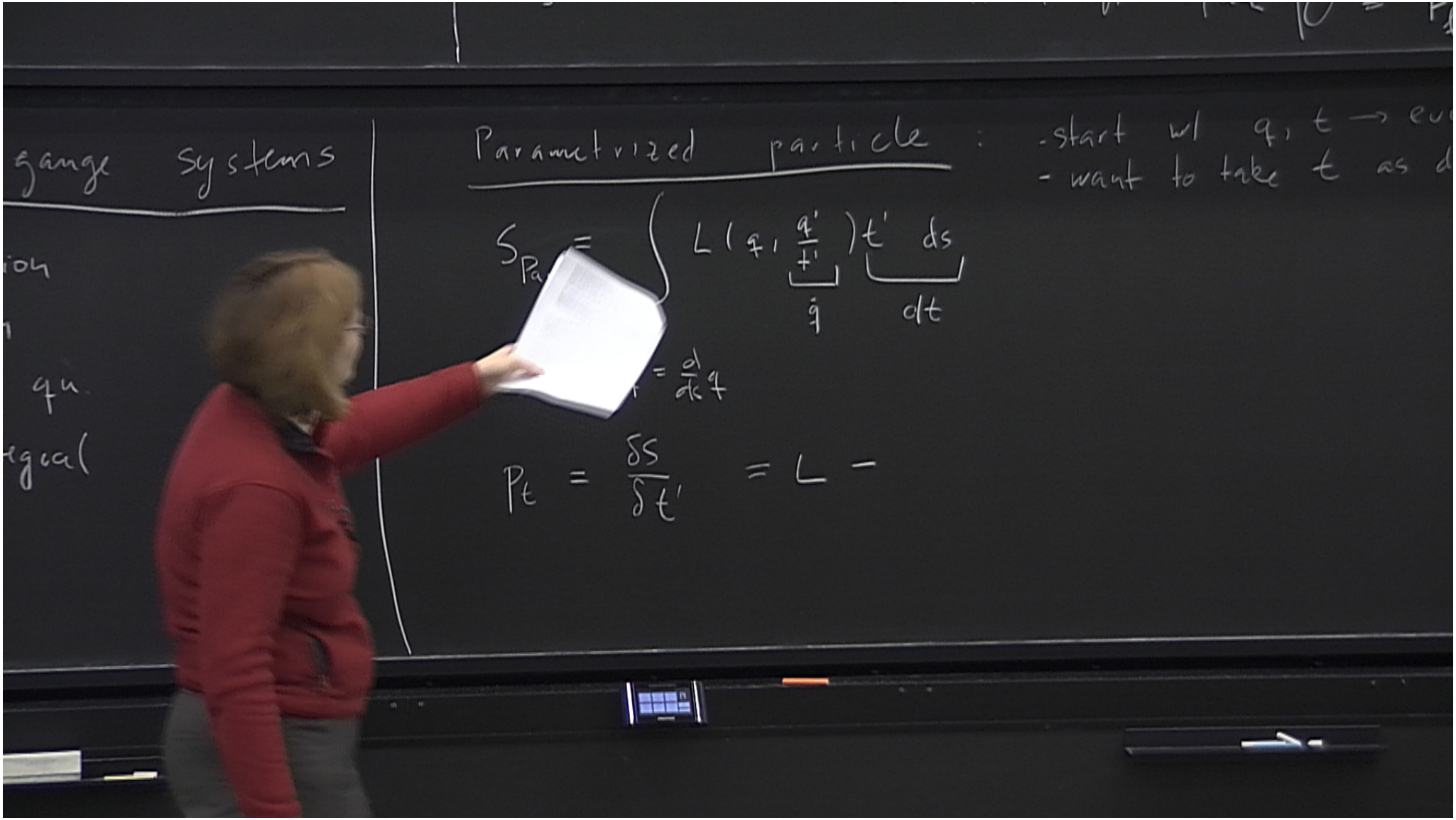
$\{T_i, C_j\} \neq 0$ flow generated by constraint(s) : stays on the constraint hypersurface

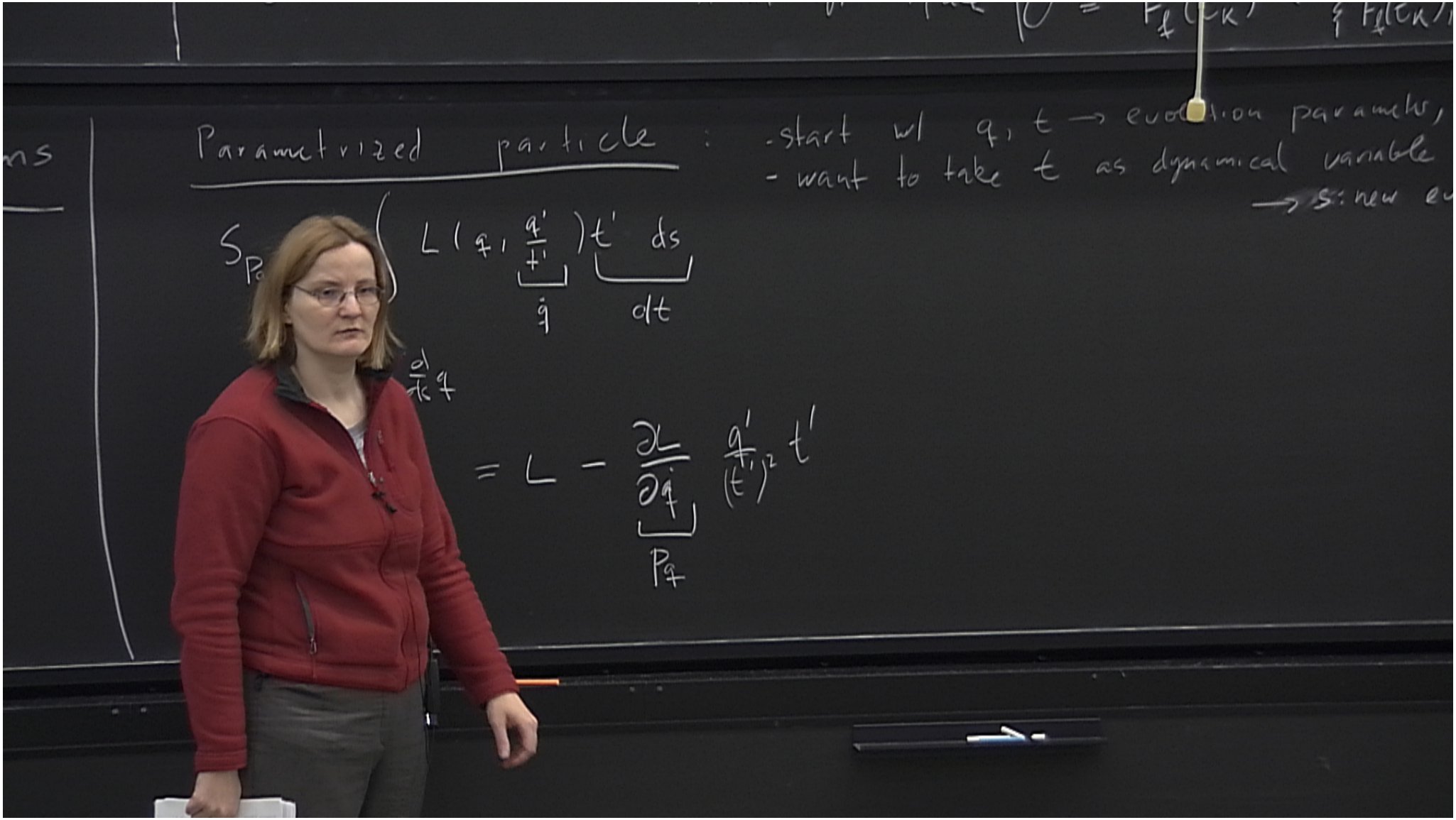
flow generated by C

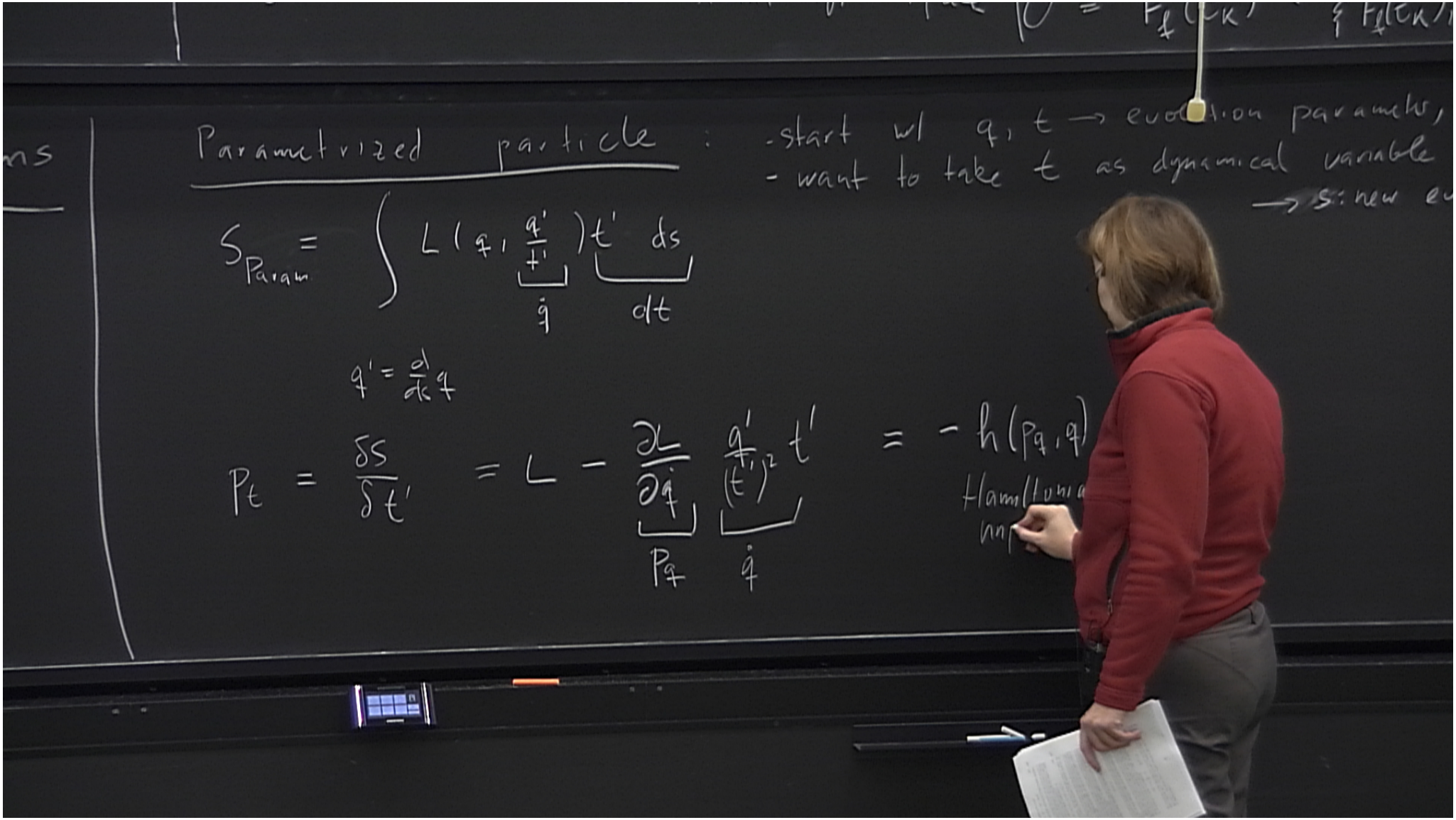
→ connects gauge equivalent phase space points


Quantization of gauge systems
- canonical quantization
⇝ Dirac quantization
Reduced phase space qu
- "covariant" path integral
quantization
Parametriz


Parametrized particle :
- start w/ q, t → evolution parameter, L(
- want to take t as dynamical variable
$S_{Param.} = \int L(q, \frac{q'}{t'}) t' \, ds$


gauge systems
Parametrized particle
- start w/ q, t → ev
- want to take t as
$S_{Pa} = \int L(q, \frac{q'}{t'}) t' \, ds$
$\dot{q}$
dt
$= \frac{d}{ds} q$
$P_t = \frac{\delta S}{\delta t'} = L -$


Parametrized particle : - start w/ q, t → evolution parameter,
- want to take t as dynamical variable
→ s: new ev
$S_P = \int L(q, \frac{q'}{t'})\, t'\, ds$
$\dot{q}$ dt
$\frac{\partial}{\partial \dot{q}} q$
$= L - \frac{\partial L}{\partial \dot{q}} \frac{q'}{(t')^2} t'$
p_q


Parametrized particle : - start w/ q, t → evolution parameter,
- want to take t as dynamical variable
→ S: new
$S_{Param} = \int L(q, \frac{q'}{t'}) t' \, ds$
$\dot{q}$ dt
$q' = \frac{d}{ds} q$
$p_t = \frac{\delta S}{\delta t'} = L - \frac{\partial L}{\partial \dot{q}} \frac{q'}{(t')^2} t' = -h(p_q, q)$
p_q $\dot{q}$
Hamiltonian

Parametrized particle

- start w/ $q, t \rightarrow$ evolution parameter, $L(q, \dot{q})$
- want to take t as dynamical variable
  $\rightarrow s$: new evol. para

$$S_{Param} = \int L\left(q, \underbrace{\frac{q'}{t'}}_{\dot{q}}\right) \underbrace{t'\, ds}_{dt}$$

$$q' = \frac{d}{ds} q$$

$$P_t = \frac{\delta S}{\delta t'} = L - \underbrace{\frac{\partial L}{\partial \dot{q}}}_{P_q} \underbrace{\frac{q'}{(t')^2}}_{\dot{q}} t' = -h(p_q, q) \Rightarrow$$

Hamiltonian for the unparametrized system

Parametrized particle

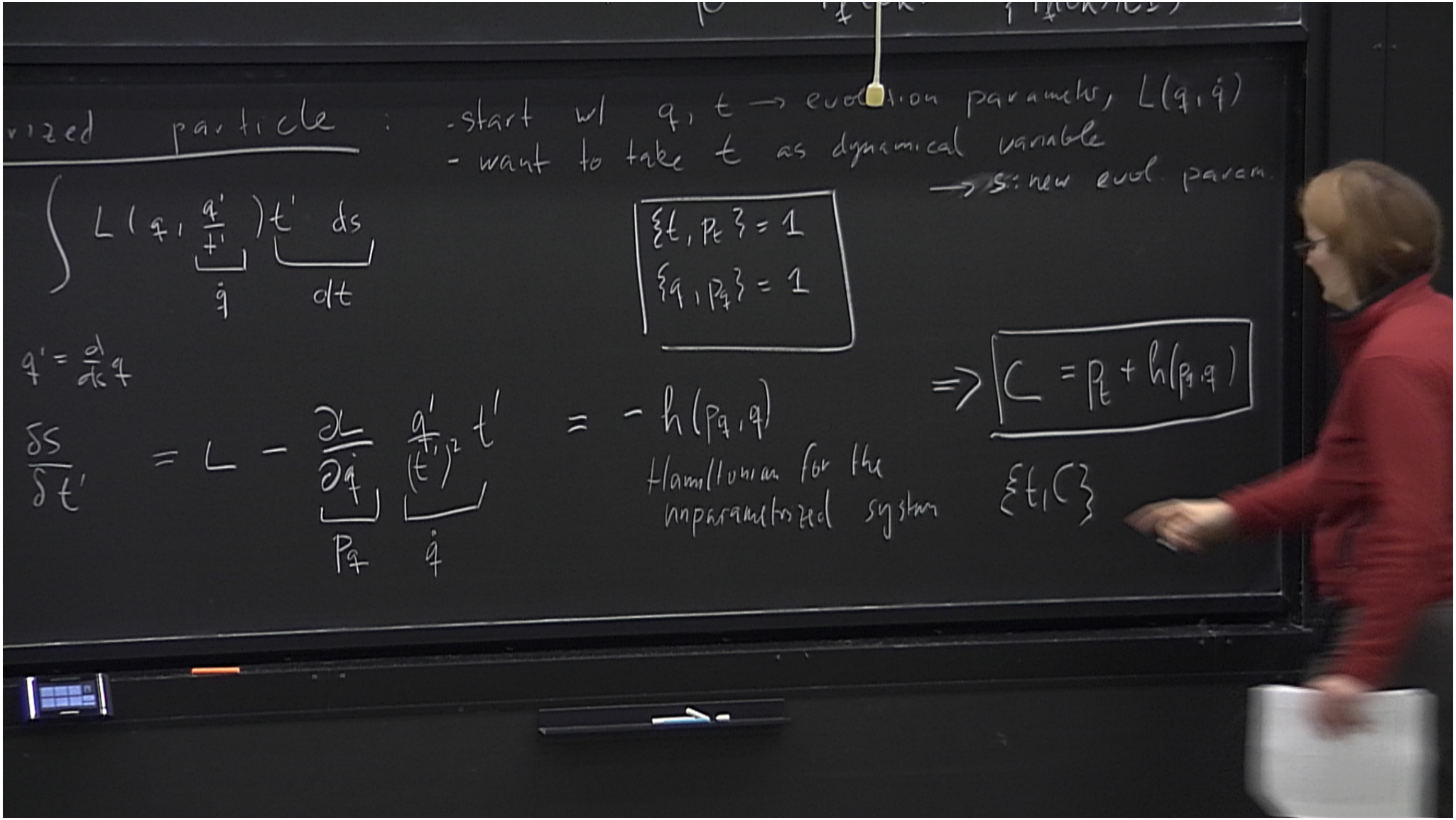
- start w/ $q, t \rightarrow$ evolution parameter, $L(q, \dot{q})$
- want to take t as dynamical variable
  $\rightarrow s$: new evol. param.

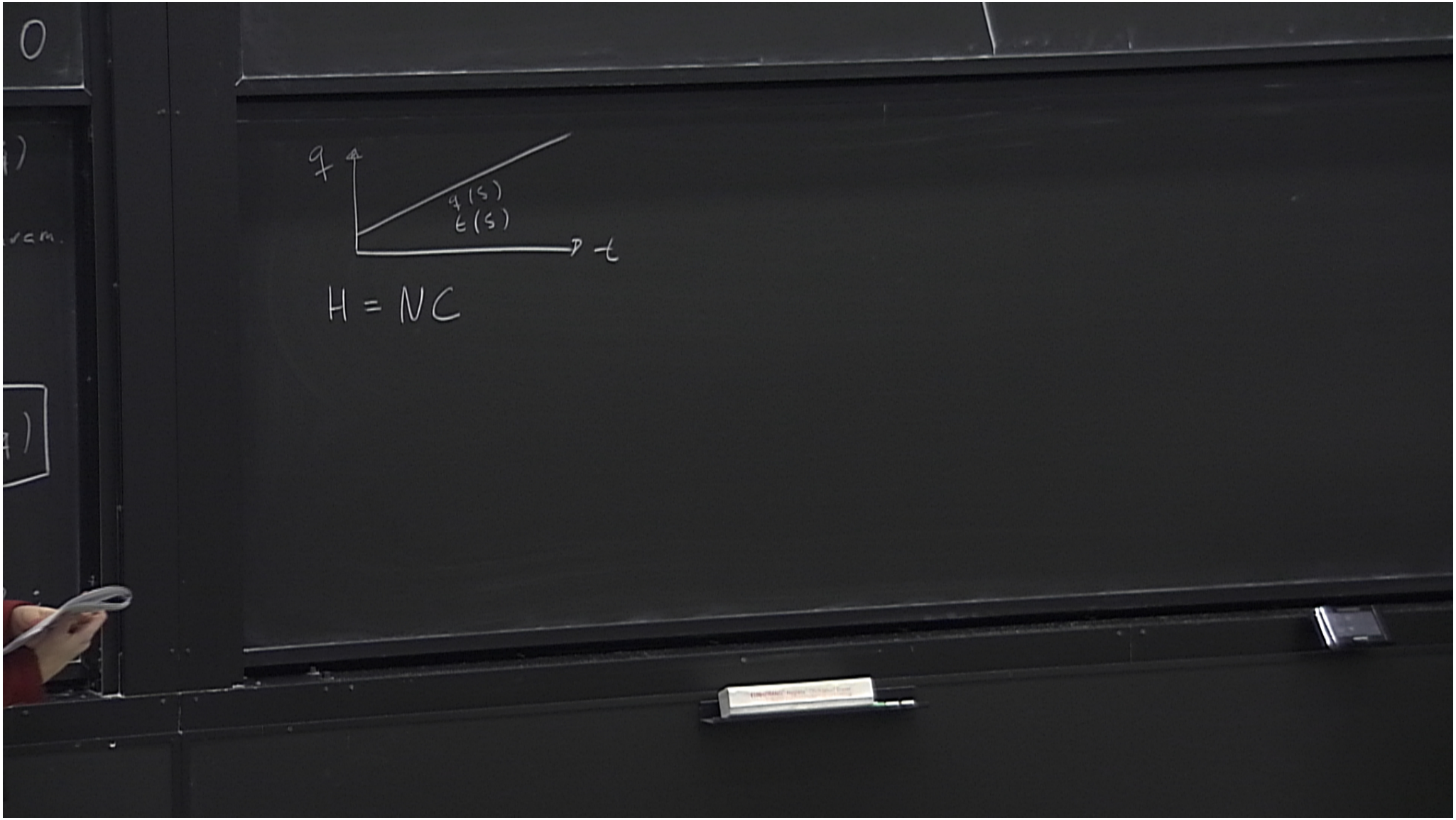
$$S_{Param} = \int L\left(q, \underbrace{\frac{q'}{t'}}_{\dot{q}}\right) \underbrace{t'\, ds}_{dt}$$

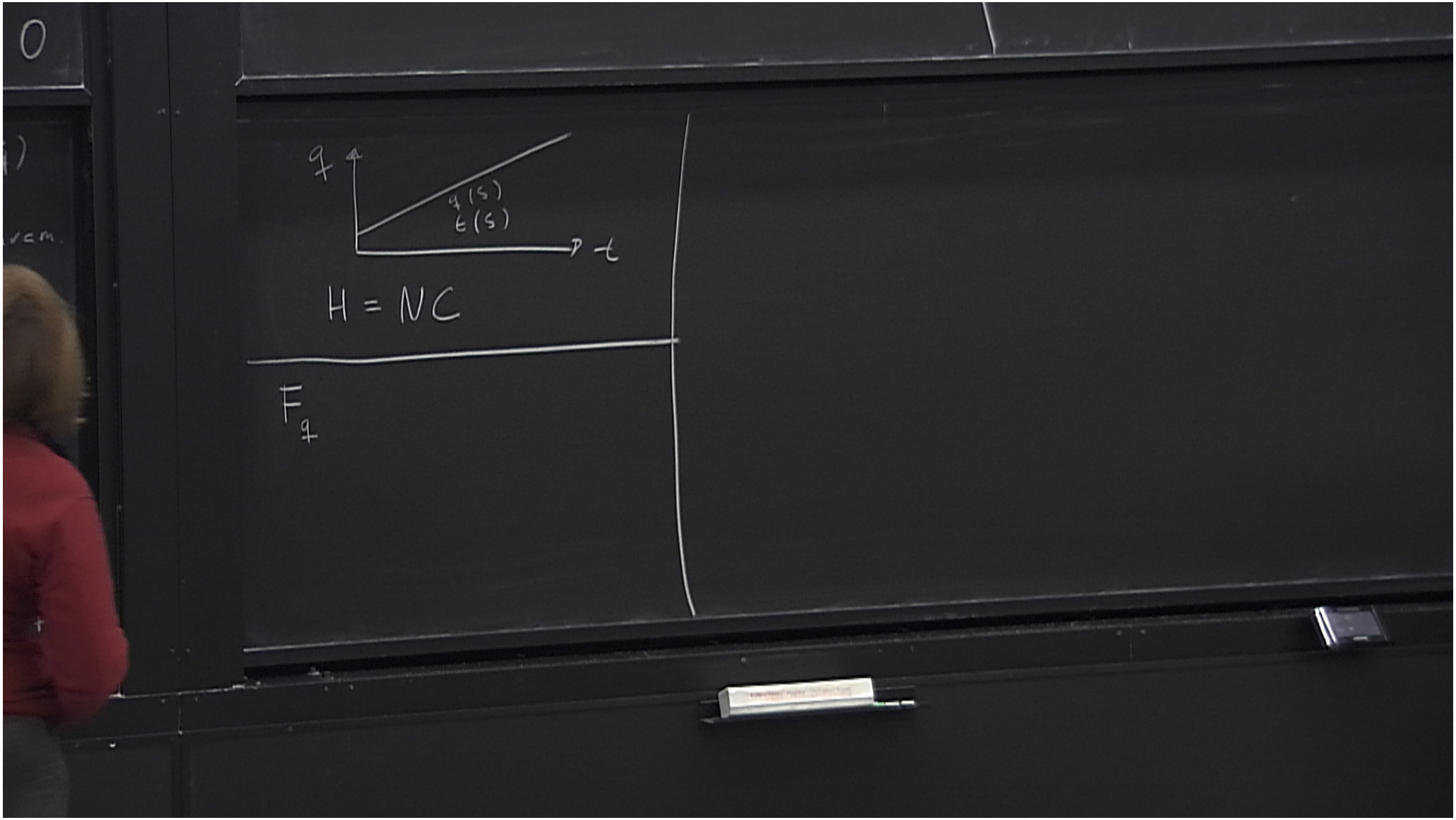
$$q' = \frac{dq}{ds}$$

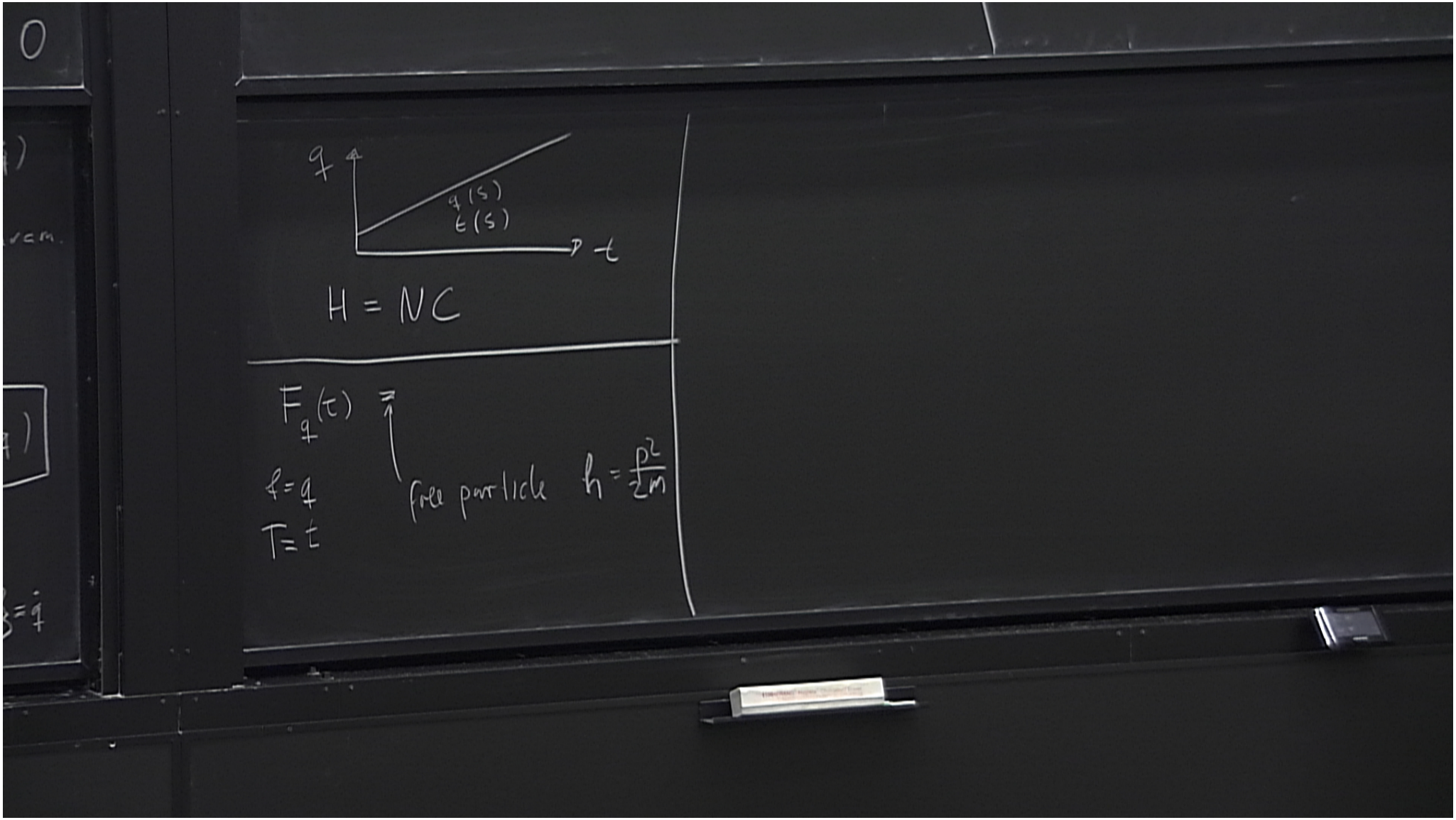
$$p_t = \frac{\delta S}{\delta t'} = L - \underbrace{\frac{\partial L}{\partial \dot{q}}}_{p_q} \underbrace{\frac{q'}{(t')^2}}_{\dot{q}} t' = -h(p_q, q) \Rightarrow L$$

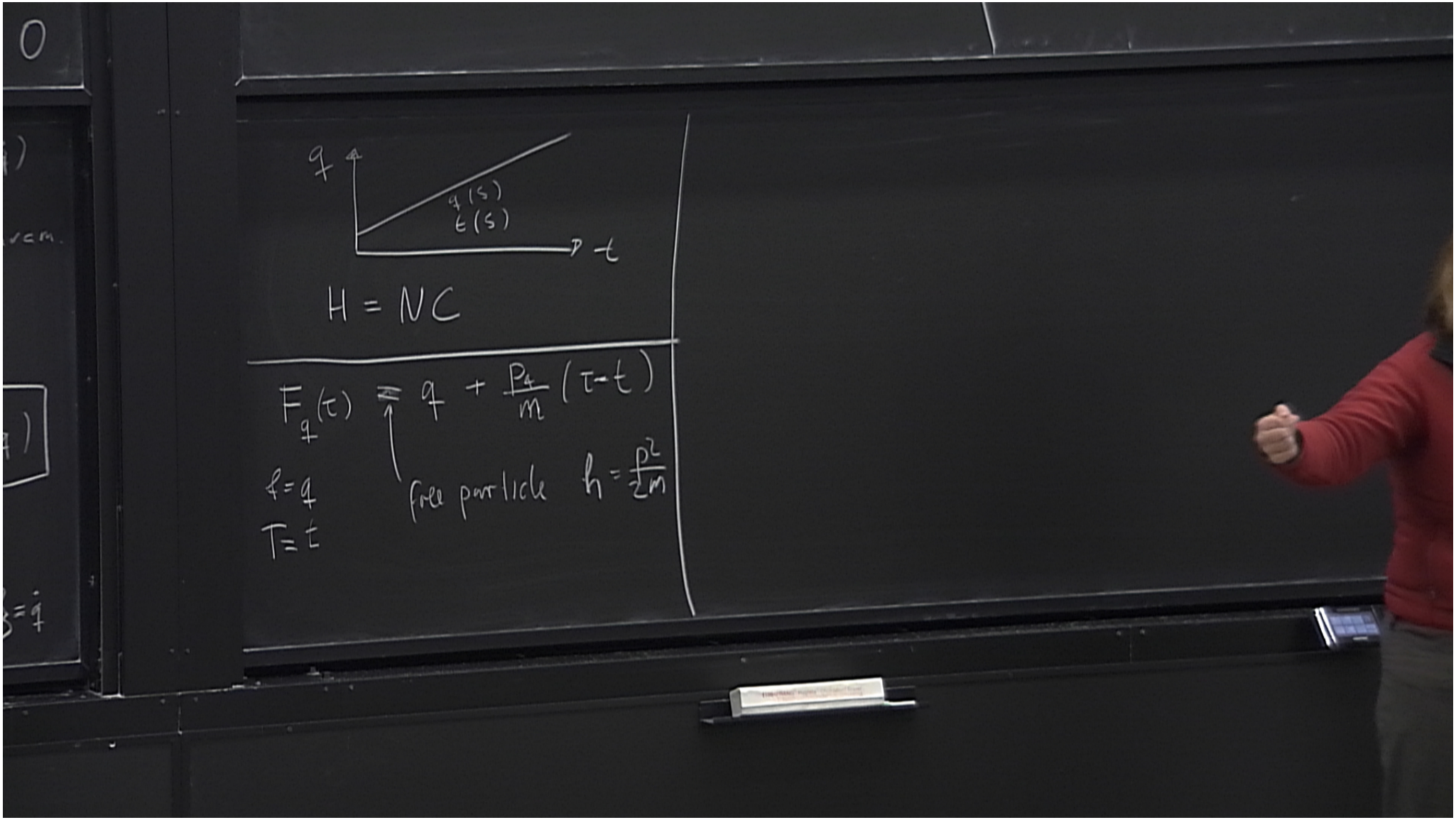
Hamiltonian for the unparametrized system

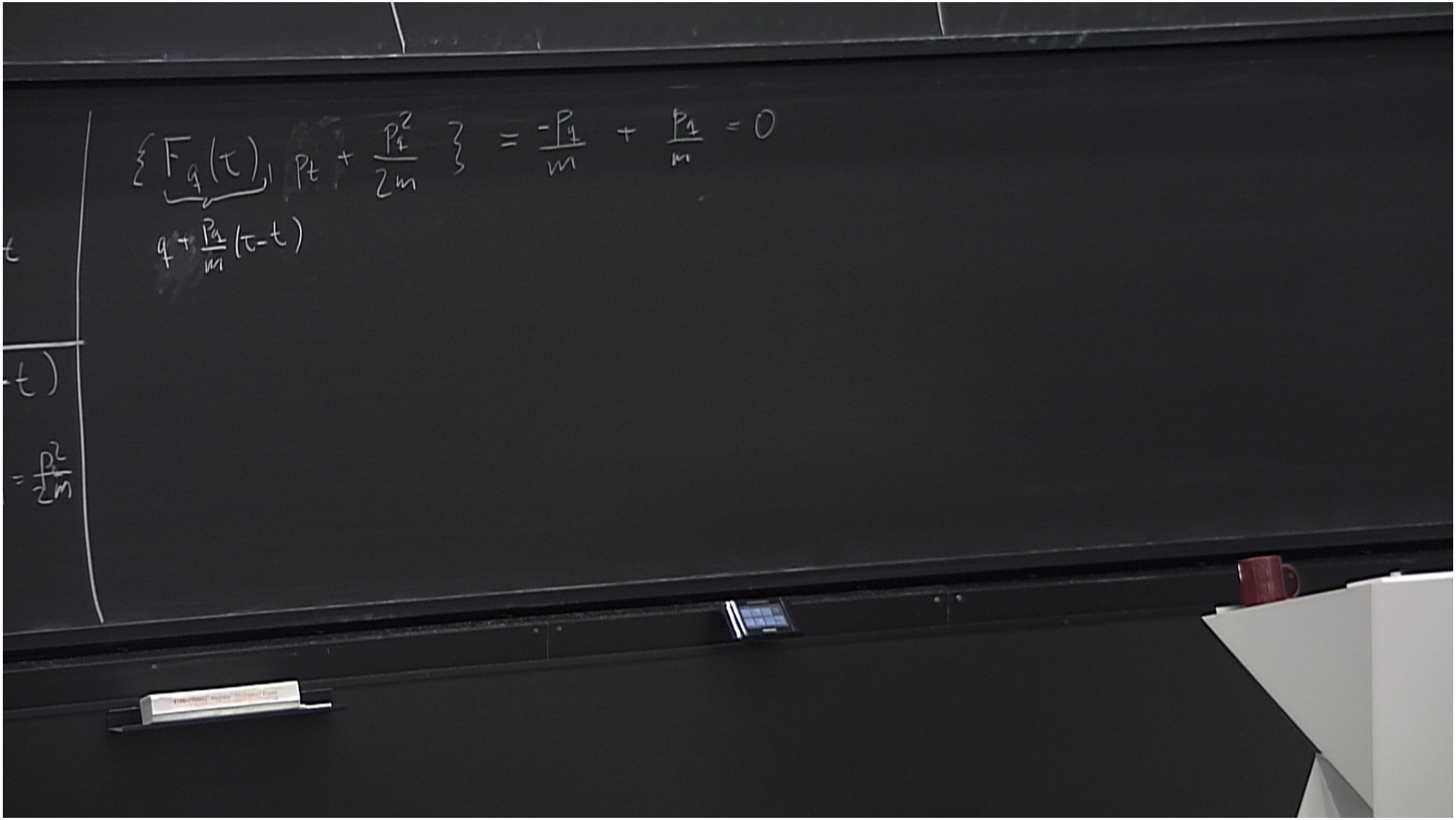

rized particle :
- start w/ q, t → evolution parameter, L(q, q̇)
- want to take t as dynamical variable
→ s: new evol param.
$\int L\left(q, \frac{q'}{t'}\right) t' \, ds$
$\dot{q}$ dt
$\{t, p_t\} = 1$
$\{q, p_q\} = 1$
$q' = \frac{d}{ds} q$
$\frac{\delta S}{\delta t'} = L - \frac{\partial L}{\partial \dot{q}} \frac{q'}{(t')^2} t' = -h(p_q, q)$
p_q $\dot{q}$
Hamiltonian for the unparametrized system
$\Rightarrow C = p_t + h(p_q, q)$
$\{t, C\}$

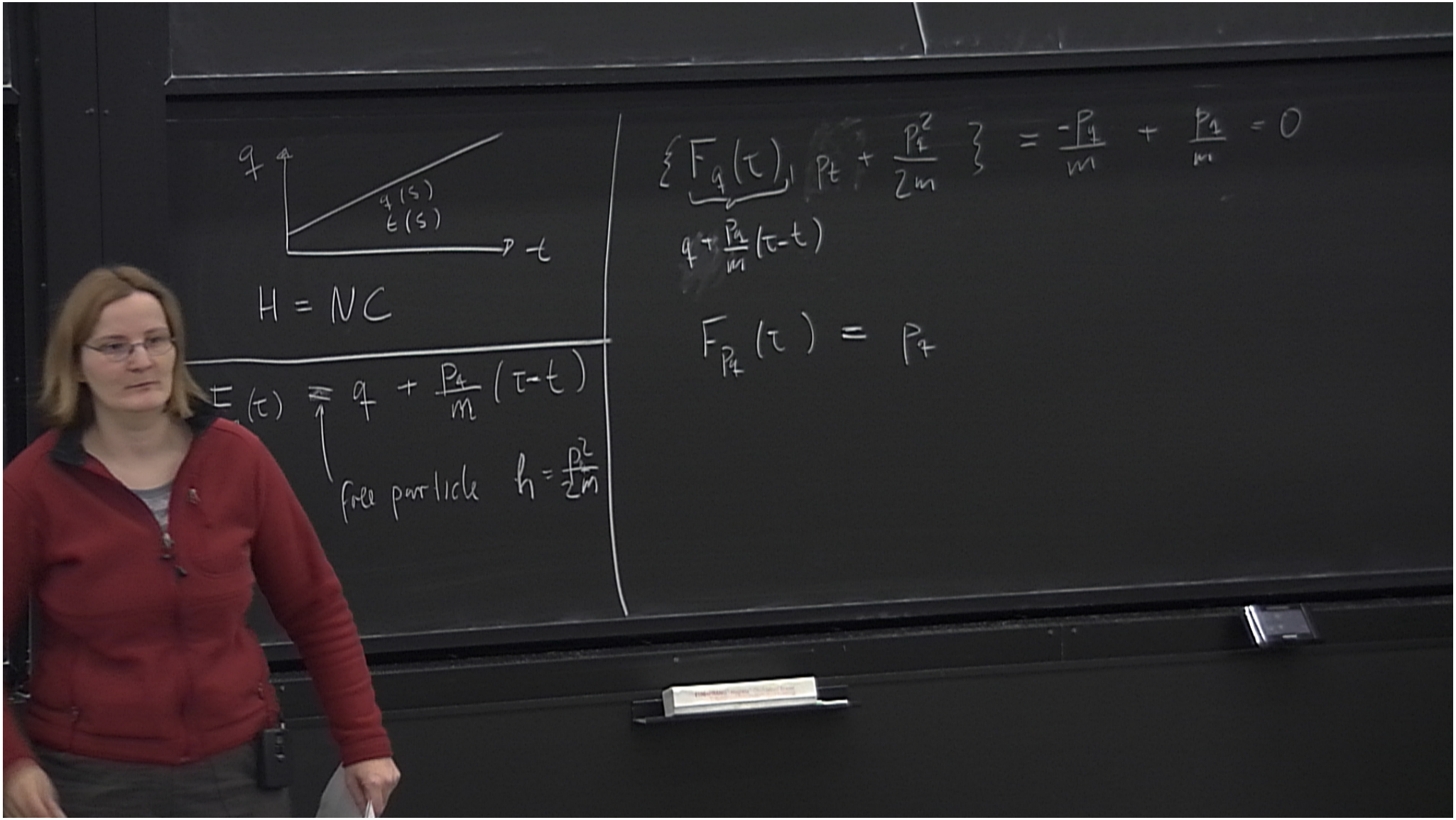

q
q(s)
ε(s)
t
H = NC

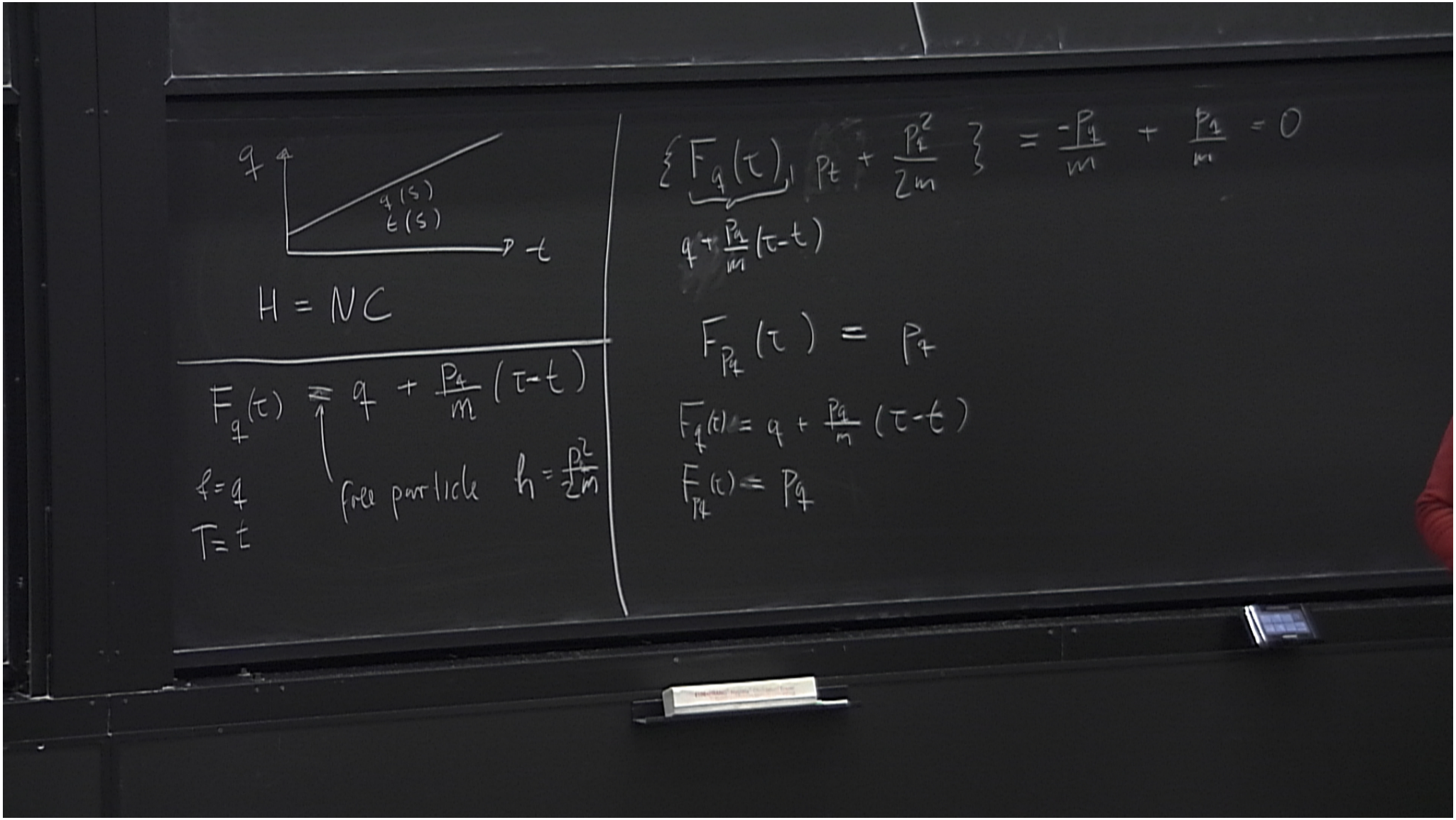

q
q(s)
ε(s)
t
H = NC
F_q

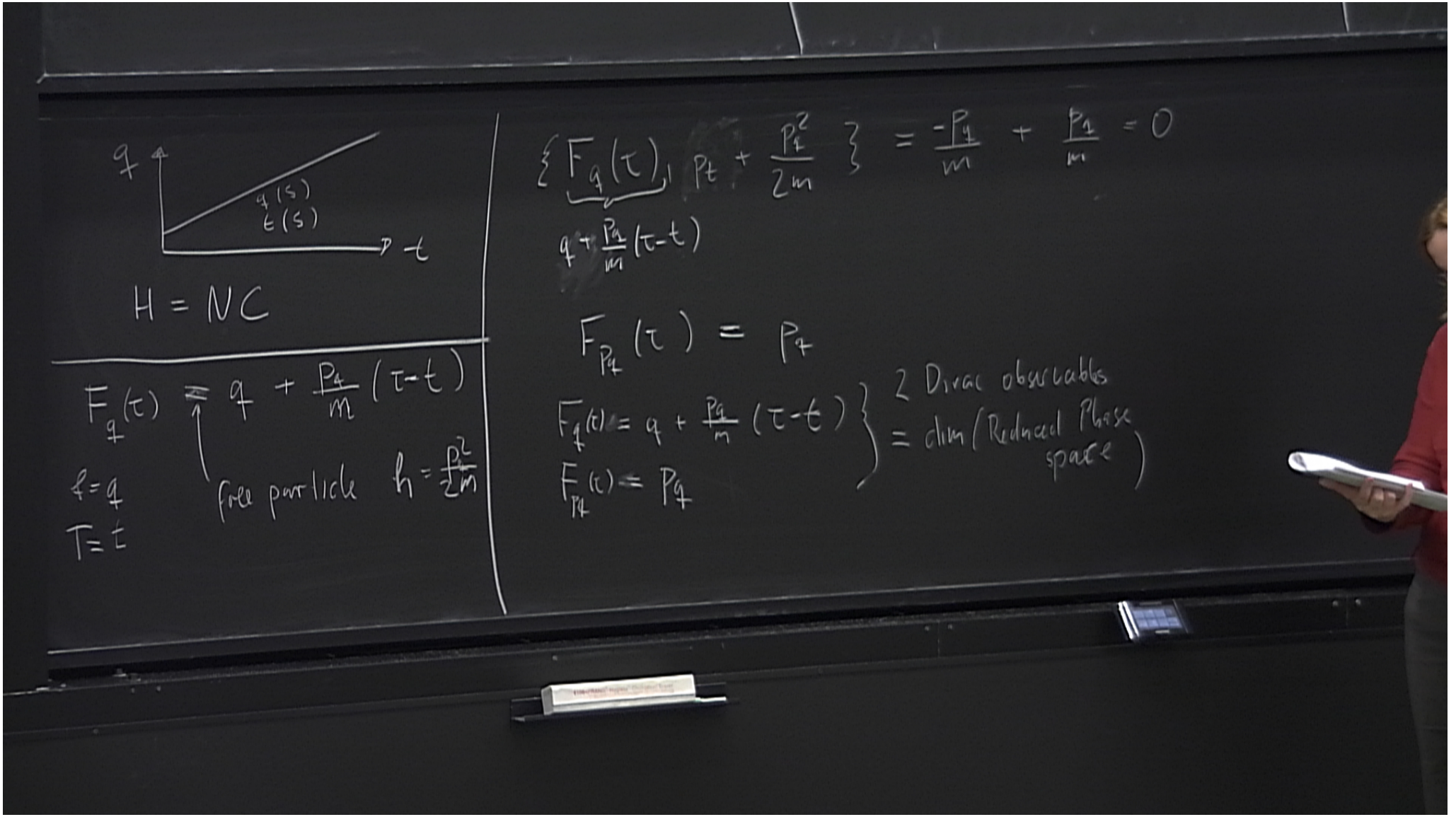

q
q(s)
ε(s)
t
H = NC
$F_q(\tau)$
$f = q$
$T = t$
free particle
$h = \frac{p^2}{2m}$

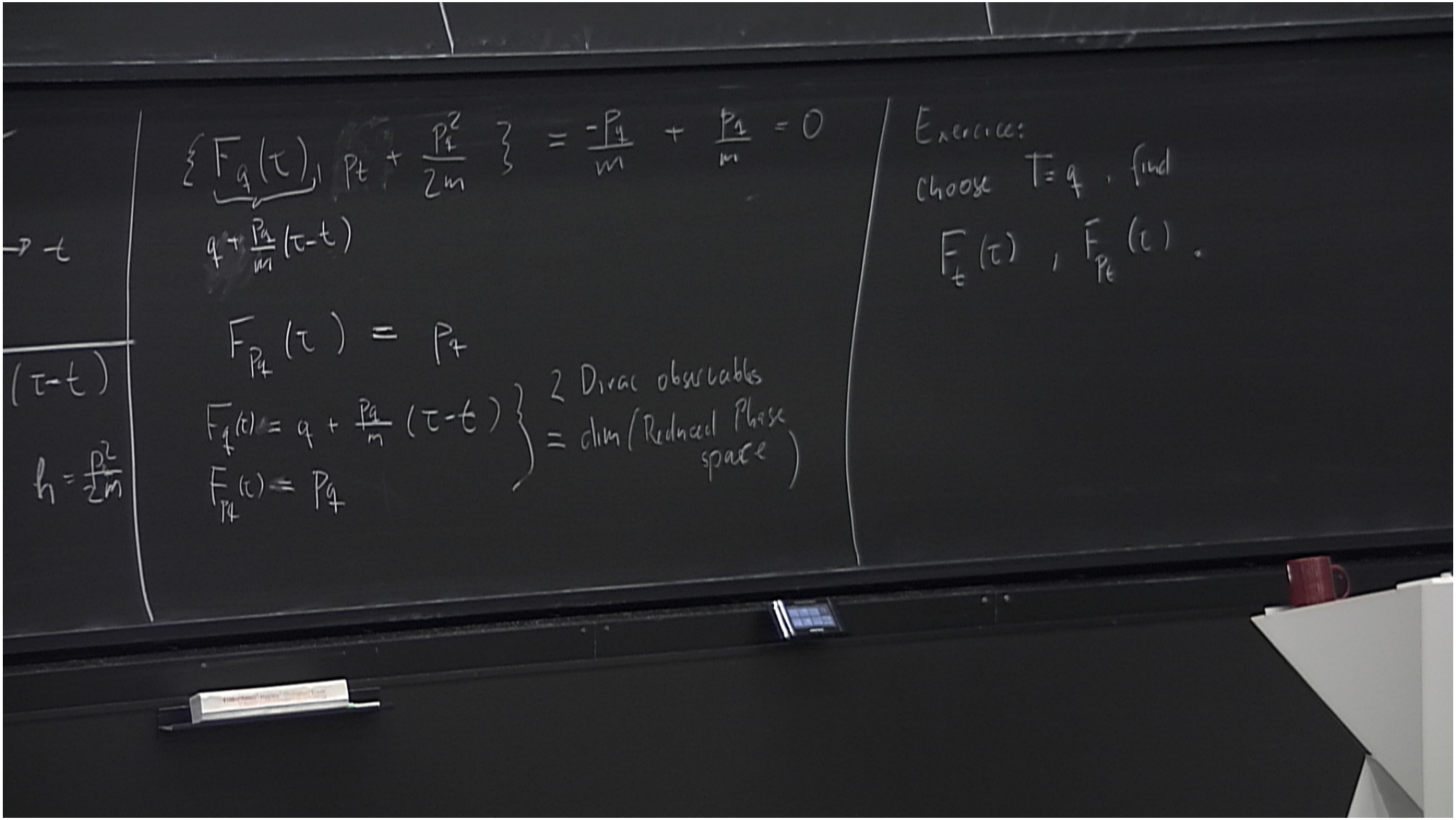

q
q(s)
ε(s)
t
H = NC
$F_q(\tau) = q + \frac{p_q}{m}(\tau - t)$
free particle
$h = \frac{p^2}{2m}$
f = q
T = t

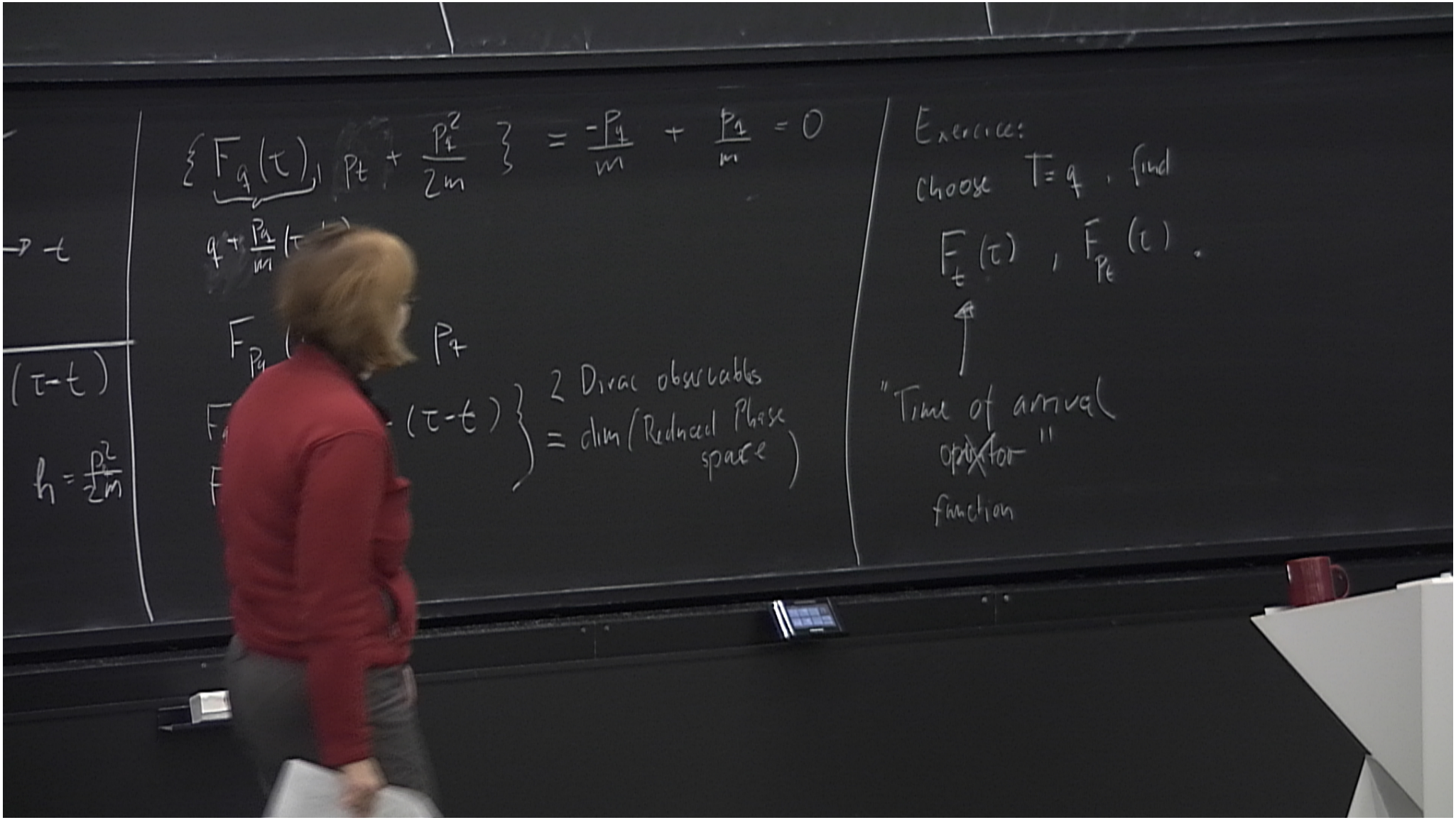

$\{F_q(t)\}\, p_t + \frac{p_t^2}{2m} \} = \frac{-p_1}{m} + \frac{p_1}{m} = 0$
$q + \frac{p_1}{m}(t-t)$

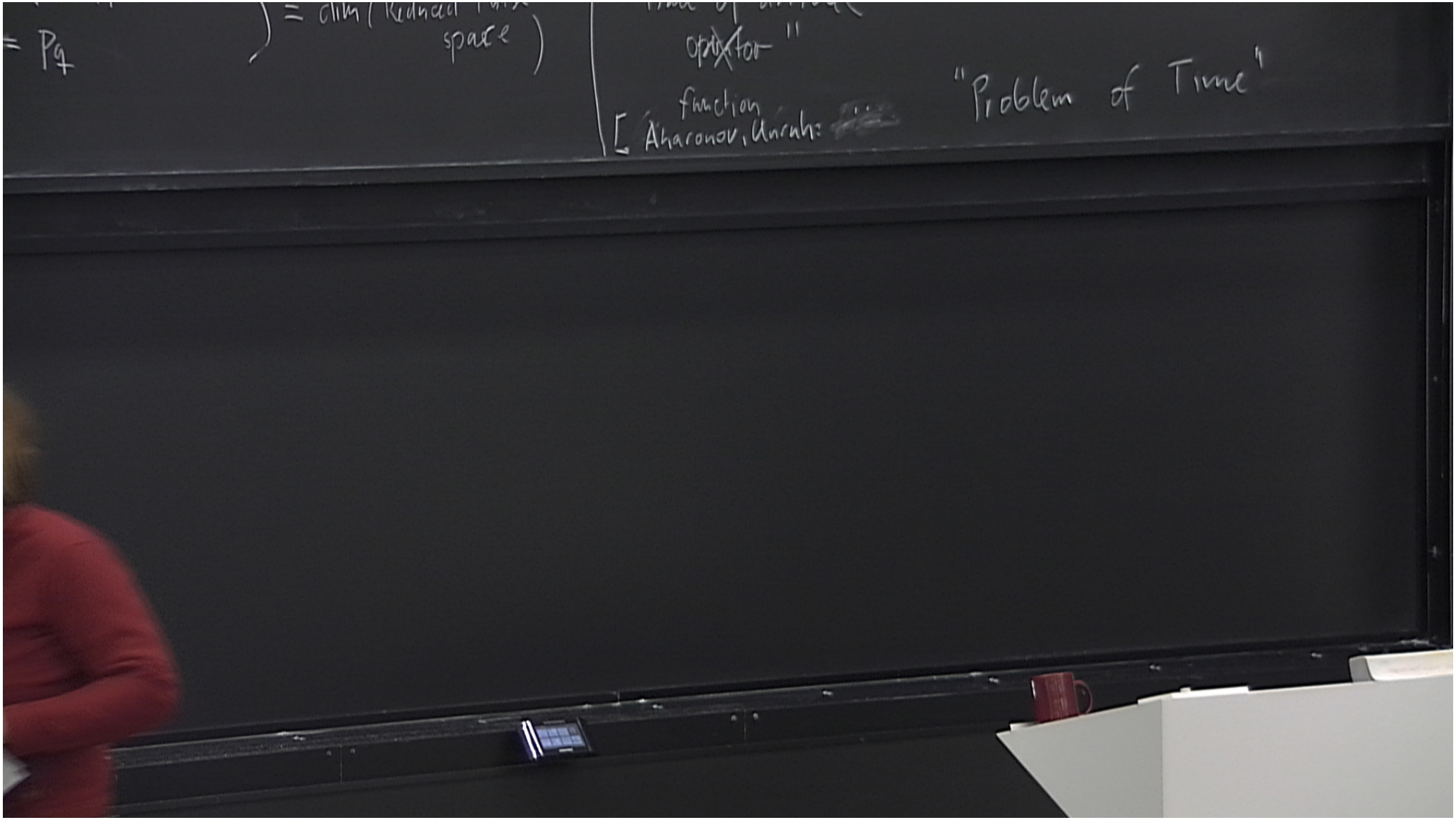

q
q(s)
ε(s)
t
H = NC
F_q(τ), P_t + P_t²/2m = -P_q/m + P_q/m = 0
q + P_q/m (τ-t)
F_{P_t}(τ) = P_t
q + P_q/m (τ-t)
free particle h = P²/2m


q
q(s)
ε(s)
t
H = NC
$F_q(\tau) = q + \frac{p_q}{m}(\tau - t)$
$f = q$
$T = t$
free particle
$h = \frac{p^2}{2m}$
$\{F_q(\tau), p_t + \frac{p_t^2}{2m}\} = -\frac{p_q}{m} + \frac{p_q}{m} = 0$
$q + \frac{p_q}{m}(\tau - t)$
$F_{p_t}(\tau) = p_t$
$F_q(t) = q + \frac{p_q}{m}(\tau - t)$
$F_{p_t}(t) = p_q$


q
q(s)
ε(s)
t
H = NC
$F_q(\tau) = q + \frac{p_q}{m}(\tau - t)$
$t = q$
$T = t$
free particle $h = \frac{p^2}{2m}$
$\{F_q(\tau), p_t + \frac{p_t^2}{2m}\} = \frac{-p_q}{m} + \frac{p_q}{m} = 0$
$q + \frac{p_q}{m}(\tau - t)$
$F_{p_t}(\tau) = p_t$
$F_q(t) = q + \frac{p_q}{m}(\tau - t)$
$F_{p_t}(t) = p_q$
2 Dirac observables
= dim (Reduced Phase space)


$\{F_q(\tau), p_t + \frac{p_t^2}{2m}\} = \frac{-p_q}{m} + \frac{p_q}{m} = 0$
$q + \frac{p_q}{m}(\tau - t)$
$F_{p_t}(\tau) = p_t$
$F_q(t) = q + \frac{p_q}{m}(\tau - t)$
$F_{p_q}(t) = p_q$
2 Dirac observables
= dim (Reduced Phase space)
Exercice:
Choose $T = q$. find
$F_t(\tau)$, $F_{p_t}(t)$.
$h = \frac{p^2}{2m}$


$\{F_q(\tau), P_t + \frac{P_t^2}{2m}\} = \frac{-P_q}{m} + \frac{P_q}{m} = 0$
$q + \frac{P_q}{m}(\tau - t)$
$F_{P_q}(\quad P_t$
$(\tau - t)\}$
2 Dirac observables
= dim (Reduced Phase space)
$h = \frac{p^2}{2m}$
Exercise:
Choose $T = q$. find
$F_t(\tau)$, $F_{P_t}(t)$.
"Time of arrival"
function


= Pq
space
for "
function
Aharonov, Unruh:
"Problem of Time"