

Title: 12/13 PSI - Quantum Gravity Review Lecture 5

Date: Feb 01, 2013 10:15 AM

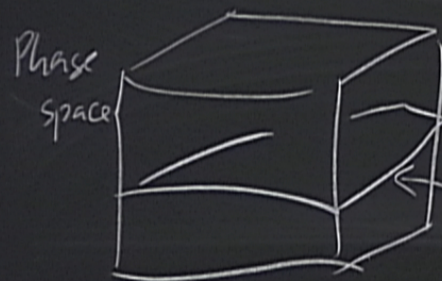
URL: <http://pirsa.org/13020044>

Abstract:

Canonical Analysis

constraints: some EDM expressed in
phase space variables,
appear w/out time
derivatives

Canonical Analysis



constraints: some EOM expressed in
phase space variables,
appear w/out time
derivatives

$$\begin{aligned} G_i &= 0 \\ J_i &= 0 \end{aligned}$$

$$\mathcal{H} = -e_0^i F_i + A_0^i G_i \quad \text{totally constrained.}$$

$$\mathcal{H} = -e_0^i F_i = A_0^i G_i \quad \text{totally constrained.}$$

- $\mathcal{H} = 0$ for solutions
- nevertheless $\mathcal{H}$ leads to a non-trivial flow $\Rightarrow$ gauge transformation

$$\mathcal{H} = \int [e_0^i F_i + A_0^j G_j] d^2x \quad \text{totally constrained.}$$

- $\mathcal{H} = 0$ for solutions
- nevertheless $\mathcal{H}$ leads to a non-trivial flow $\Rightarrow$ gauge transformation

$$g[\vec{L}] = \int G_i \vec{L}^i d^2x = \int \vec{L}^i [\partial_a E_j^a + \epsilon_{jmn} A_a^m E^{an}] d^2x$$

$\vec{L}^i$

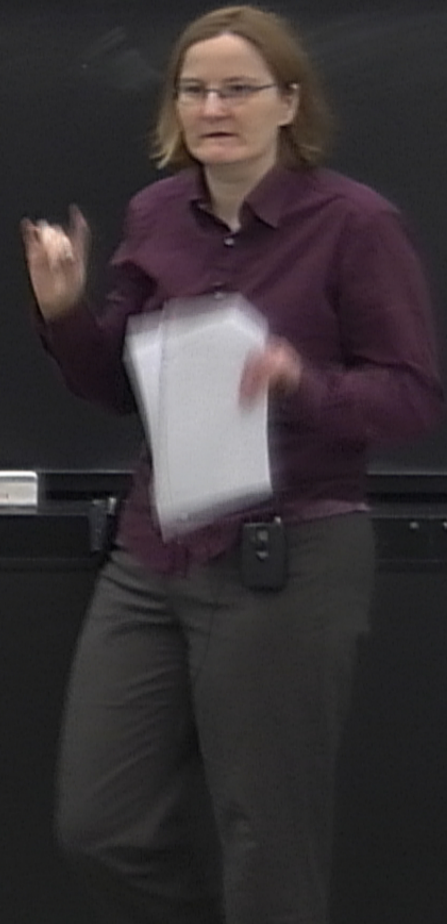
$$\mathcal{H} = \int [e^i_0 F_i - A^i_0 G_i] d^2x \quad \underline{\text{totally constrained.}}$$

- $\mathcal{H} = 0$ for solutions
- nevertheless $\mathcal{H}$ leads to a non-trivial flow $\Rightarrow$ gauge transformation

$$g[\underset{\substack{\uparrow \\ L}}{L}] = \int G_i L^i d^2x = \int L^i [\partial_a E^a_i + \epsilon_{jml} A^l_a E^{aj}] d^2x$$

$$\delta_{\perp} E_R^b(x) = \{ E_R^b(x), G[\mathcal{L}] \} = \left(\{ E_R^b(x), \mathcal{L}^i(y) \in_{m\ell} A \right.$$

$$\delta_{\perp} E_R^b(x) = \{E_R^b(x), G[\perp]\} = \int \{E_R^b(x), \perp^j(y) \epsilon_{,ml} A_{a(y)}^m E^{al}(y)\} d^2 y$$



$$\delta_{\perp} E_R^b(x) = \{E_R^b(x), G[\mathcal{L}]\} = \int \{E_R^b(x), \mathcal{L}^i(y) \epsilon_{,m} A_{a(y)}^m E_{(y)}^{a\epsilon}\} d^2 y$$

$$\{E, A\} = -\delta\delta\delta \rightarrow \int \mathcal{L}^i(y) \delta(x,y)$$

$$\delta_{\perp} E_R^b(x) = \{E_R^b(x), G[L]\} = \int \{E_R^b(x), L^i(y) \epsilon_{,ml} A_{a(y)}^m E^{al}(y)\} d^2 y$$

$$\stackrel{\{E, A\} = -\delta\delta\delta}{=} \int -L^i(y) \delta(x,y) \epsilon_{,kl} E^{bl}(y) d^2 y$$

$$= -L^i \epsilon_{,kl} E^{bl}(x)$$

$$\delta_{\perp} E_k^b(x) = \{E_k^b(x), G[L]\} = \int \{E_k^b(x), L^i(y) \epsilon_{,ml} A_{a(y)}^m E^{al}(y)\} d^2 y$$

$$\stackrel{\{E, A\} = -\delta\delta\delta}{=} \int -L^i(y) \delta(x,y) \epsilon_{,kl} E^{bl}(y) d^2 y$$

$$\boxed{\delta_{\perp} E_k^b(x) = -L^i \epsilon_{,kl} E^{bl}(x)}$$

$V_{\text{eff}}(r)$

$$\delta_{\perp} E_k^b(x) = \{ E_k^b(x), G[L] \} = \int \{ E_k^b(x), L^i(y) \epsilon_{,kl} A_a^m(y) E^{al}(y) \} d^2 y$$

$$\{E, A\} = -\delta\delta\delta \quad \rightarrow \quad \int -L^i(y) \delta(x,y) \epsilon_{,kl} E^{bl}(y) d^2 y$$

$$\boxed{\delta_{\perp} E_k^b(x) = -L^i \epsilon_{,kl} E^{bl}(x)}$$

agree w/ rotation
(symmetries of action)
(for e_n^b)

leaves t, y, ϕ but $\partial_t \partial_\phi$

$V_{\text{eff}}(r)$

$$\delta_{\perp} E_k^b(x) = \{E_k^b(x), G[\mathcal{L}]\} = \int \{E_k^b(x), \mathcal{L}^i(y) \epsilon_{,ml} A_a^m(y) E^{al}(y)\} d^2 y$$

$$\{E, A\} = -\delta\delta\delta \quad \rightarrow \quad \int -\mathcal{L}^i(y) \delta(x,y) \epsilon_{,kl} E^{bl}(y) d^2 y$$

$$\boxed{\delta_{\perp} E_k^b(x) = -\mathcal{L}^i \epsilon_{,kl} E^{bl}(x)}$$

agree w/ rotation
(symmetries of action)
(for e_n^k)

$$\delta_\Lambda A'_a(x) = \{ A'_a(x), g[\mathcal{L}] \}$$

$$= \{ A'_a(x), \{$$

$$E = g_{tt} \dot{t} = \left(1 - \frac{2GM}{r}\right) \dot{t} \quad \left. \vphantom{\frac{2GM}{r}} \right\} \text{con}$$

$$h = g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi}$$

$$\text{plus } V \dot{t}^2 - \frac{\dot{r}^2}{V} - r^2 \dot{\phi}^2 = \eta \quad \left. \vphantom{\frac{\dot{r}^2}{V}} \right\} \begin{array}{l} 1 \ m^2 > 0. \\ 0 \ m^2 = 0. \end{array}$$

or

$$\frac{1}{2} E^2 = \frac{1}{2} \dot{r}^2 + \underbrace{\left(\frac{h^2}{2r^2} + \frac{q}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

$$\delta_A A_b^k(x) = \{ A_a^j(x), g[L] \}$$

$$= \{ A_b^k(x), \int (\lambda \partial_a E_j^a + \epsilon_{mnc} A_a^m \partial_b \phi^c) dy d\phi \}$$

$$E = g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \quad \left. \vphantom{\frac{2GM}{r}} \right\} \text{const}$$

$$h = g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi}$$

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$$\delta_\Lambda A_b^k(x) = \{ A_a^j(x), g[L] \}$$

$$= \{ A_b^k(x), \underbrace{\left(\Lambda \partial_a E_j^a + \epsilon_{mnc} A_a^m E^{ac} \right)}_{-(2\partial_a L)} \}$$

(for e_m^a)

$$\left. \begin{aligned} E &= g_{tt} \dot{t} = \left(1 - \frac{2GM}{r}\right) \dot{t} \\ h &= g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi} \end{aligned} \right\} \text{ cons.}$$

plus $V \dot{t}^2 - \frac{\dot{r}^2}{V} - r^2 \dot{\phi}^2 = \eta$ 1 $m^2 > 0$,
0 $m^2 = 0$.

or

$$\frac{1}{2} E^2 = \frac{1}{2} \dot{r}^2 + \underbrace{\left(\frac{h^2}{2r^2} + \frac{q}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

$$\delta_\Lambda A_b^k(x) = \{ A_a^j(x), g[L] \}$$

$$= \{ A_b^k(x), \underbrace{\int (\Lambda \partial_a E_j^a + \Lambda^j \epsilon_{jme} A_a^m E^{ae}) (y) d^2 y}_{-(2\Lambda^j)} \}$$

(for e_m^a)

$$\left. \begin{aligned} E &= g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \\ h_{\phi\phi} \dot{\phi} &= r^2 \dot{\phi} \end{aligned} \right\} \text{con}$$

$$\text{ph} \quad -\dot{r}^2 - r^2 \dot{\phi}^2 = \eta \quad \begin{matrix} 1 & m^2 > 0 \\ 0 & m^2 = 0 \end{matrix}$$

or

$$\frac{1}{2} E + \underbrace{\left(\frac{h^2}{2r^2} + \frac{q}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

$$\delta_\Lambda A_b^k(x) = \{ A_a^j(x), g[L] \}$$

$$= \{ A_b^k(x), \underbrace{\int (\Lambda \partial_a E_j^a + \Lambda^j \epsilon_{jme} A_a^m E^{ae}) dy}_{-\int (2\Lambda^j) \eta E_j^a dy} \}$$

(for e_m^a)

$$\left. \begin{aligned} E &= g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \\ h &= g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi} \end{aligned} \right\} \text{ cons.}$$

plus $V \dot{t}^2 - \frac{\dot{r}^2}{V} - r^2 \dot{\phi}^2 = \eta$ 1 $m^2 > 0$
0 $m^2 = 0$

$$-2 = \frac{1}{2} \dot{r}^2 + \underbrace{\left(\frac{h^2}{2r^2} + \frac{\eta}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

$$\begin{aligned}\delta_{\Lambda} A_b^k(x) &= \{ A_a^j(x), g[L] \} \\ &= \{ A_b^k(x), \underbrace{\int (\Lambda \partial_a E_j^a + \Lambda^j \epsilon_{jmc} A_a^m E^{ac}) dy}_{-\int (\partial_a \Lambda^j) E_j^a dy} d^2 y \} \\ &= -\partial_b \Lambda^k\end{aligned}$$

(for e_m)

$$\begin{aligned}E &= g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \\ h &= g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi}\end{aligned} \quad \left. \vphantom{\begin{aligned}E &= g_{tt} \dot{t} \\ h &= g_{\phi\phi} \dot{\phi}\end{aligned}} \right\} \text{const.}$$

plus $V \dot{t}^2 - \frac{\dot{r}^2}{V} - r^2 \dot{\phi}^2 = \eta$ 1 $m^2 > 0$
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$$\frac{1}{2} E^2 = \frac{1}{2} \dot{r}^2 + \underbrace{\left(\frac{h^2}{2r^2} + \frac{q}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

$$\begin{aligned}
 \delta_{\Lambda} A_b^k(x) &= \{ A_a^j(x), g[L] \} \\
 &= \{ A_b^k(x), \underbrace{\int (\Lambda \partial_a E_j^a + \Lambda^j \epsilon_{jmc} A_a^m E^{ac}) (y) d^3y}_{-\int (\partial_a \Lambda^j) E_j^a (y) d^3y} \} \\
 &= -\partial_b \Lambda^k + \Lambda^j \epsilon_{jmk} A_b^m
 \end{aligned}$$

(for e_m^a)

$$\begin{aligned}
 E &= g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \\
 h &= g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi}
 \end{aligned}
 \left. \vphantom{\begin{aligned} E &= g_{tt} \dot{t} \\ h &= g_{\phi\phi} \dot{\phi} \end{aligned}} \right\} \text{const}$$

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$$\delta_{\Lambda} A_b^k(x) = \{ A_a^j(x), g[L] \}$$

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$$\boxed{\delta_{\Lambda} A_b^k = -\partial_b \Lambda^k + \Lambda^j \epsilon_{jmc} A_b^m}$$

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$$\boxed{\delta_\Lambda A_b^k = -\partial_b \Lambda^k + \Lambda^j \epsilon_{jmk} A_b^m}$$

infinitesimal
rotation in
internal index

$$E = g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \quad \left. \vphantom{\frac{2GM}{r}} \right\} \text{const}$$

$$1 - g_{\phi\phi} \dot{\phi}^2 = r^2 \dot{\phi}^2$$

$$V \dot{t}^2 - \frac{\dot{r}^2}{V} - r^2 \dot{\phi}^2 = \eta \quad \left. \vphantom{\frac{\dot{r}^2}{V}} \right\} \begin{array}{l} 1 \quad m^2 > 0 \\ 0 \quad m^2 = 0 \end{array}$$

or

$$\frac{1}{2} \dot{r}^2 + \underbrace{\left(\frac{\hbar^2}{2r^2} + \frac{q}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

$$\delta_{\Lambda} A_b^k(x) = \{ A_a^j(x), G[\Lambda] \}$$

$$= \{ A_b^k(x), \underbrace{\int (\Lambda^j \partial_a E_j^a + \Lambda^j \epsilon_{jmc} A_a^m E^{ac}) (y) d^3y}_{-\int (\partial_a \Lambda^j) E_j^a (y) d^3y} \}$$

$$\delta_{\Lambda} A_b^k = -\partial_b \Lambda^k + \Lambda^j \epsilon_{jmk} A_b^m$$

$\Rightarrow$ Gauss constraints generate rotations.

$$E = g_{tt} \dot{t} = (1 - \frac{2GM}{r}) \dot{t} \quad \left. \vphantom{\frac{2GM}{r}} \right\} \text{con}$$

$$h = g_{\phi\phi} \dot{\phi} = r^2 \dot{\phi}$$

$$\text{plus } V \dot{t}^2 - \frac{\dot{r}^2}{v} - r^2 \dot{\phi}^2 = \eta \quad \left. \vphantom{\frac{\dot{r}^2}{v}} \right\} \begin{array}{l} 1 \text{ } m^2 > 0. \\ 0 \text{ } m^2 = 0. \end{array}$$

or

$$\frac{1}{2} E^2 = \frac{1}{2} \dot{r}^2 + \underbrace{\left(\frac{h^2}{2r^2} + \frac{q}{2} \right) \left(1 - \frac{2GM}{r} \right)}_{V_{\text{eff}}(r)}$$

Flatness constraints

$$F[N] = \int N^i F_i d^2y$$

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$$\delta_N A_b^R(x) = \{A_b^R(x), F[N]\} = 0$$

$$\{A_a^i(x), E_b^j(y)\} = \delta_{ij}^k \delta_a^b \delta(x,y)$$

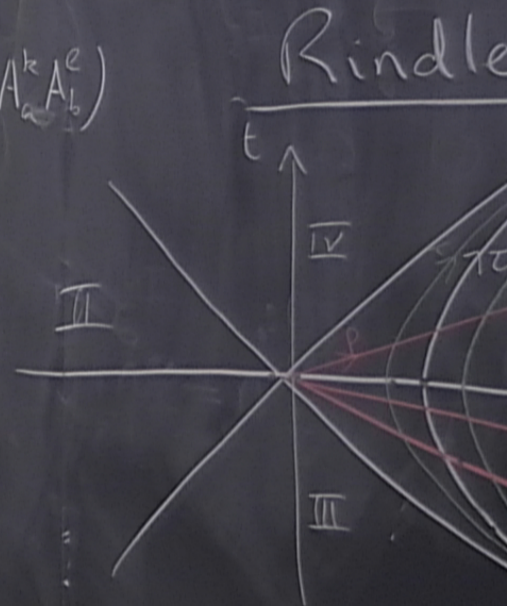
$$\{A_a^i(x), A_b^k(y)\} = 0 \quad \{E_b^i(x), E_j^a(y)\} = 0$$

Gauss constraint:

$$G_i = \frac{1}{2} T_{abj} \tilde{E}^{ab} = \partial_a E_j^a + \epsilon_{jcm} A_a^c F^{am}$$

Flatness-constraint:

$$F_j = \frac{1}{2} F_{ab} \tilde{E}^{ab} = \tilde{E}^{ab} \left(\partial_a A_b^j + \frac{1}{2} \epsilon^j_{kl} A_a^k A_b^l \right)$$



Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$\delta_N A_b^R(x) = \{A_b^R(x), F[N]\} = 0$$

agrees with
action of
translation symmetry

Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$\delta_N A_b^R(x) = \{A_b^R(x), F[N]\} = 0$$

$$\delta_N E_k^b(x) = \{E_k^b(x), \int \frac{1}{2} \tilde{E}^{ac}$$

agrees with
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translation symmetry

Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$\delta_N A_b^k(x) = \{A_b^k(x), F[N]\} = 0$$

$$\delta_N E_k^b(x) = \{A_b^k(x), \int \frac{1}{2} \tilde{e}^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon^j_{lm} A_a^l A_b^m) d^2 y\}$$

Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$\delta_N A_b^R \{ A_b^k(x), F[N] \} = 0 \quad \text{agrees with action of transl. symm.}$$

$$\delta_N E \{ E(x), \int d^2 y \epsilon^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon^j_{lm} A_a^l A_b^m) N^j d^2 y \}$$

Flatness constraints

$$F[N] = \int N^j \mathcal{F}_j d^2 y$$

$$A_b^R(x) = \{ A_b^k(x), F[N] \} = 0$$

agrees with
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$$E_b^R(x) = \{ E_b^k(x), \int \frac{1}{2} \tilde{e}^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon^j_{lm} A_a^l A_b^m) N^j d^2 y \}$$

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$$\begin{aligned} \delta_N E_k^b(x) &= \{E_k^b(x), \int \frac{1}{2} \tilde{E}^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon_{lm}^j A_a^l A_b^m) N^j d^2 y\} \\ &= \tilde{E}^{cb} (\partial_c N_k + \epsilon_{kml} A_c^l N^l)(x) \end{aligned}$$

Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$\delta_N A_b^R(x) = \{A_b^R(x), F[N]\} = 0 \quad \text{agrees with action of transl. symm.}$$

$$\delta_N E_k^b(x) = \{E_k^b(x), \left(\frac{1}{2} \tilde{e}^{ac} (\partial_a A_b^i + \frac{1}{2} \epsilon^j_{lm} A_a^l A_b^m) N^j d^2 y \right)\}$$

$$E_k^b = \tilde{e}^b_a e_a^k \quad \tilde{e}^{cb} (\partial_c N_k + \epsilon_{kml} A_c^m N^l)(x)$$

$e_a^k \mapsto \frac{\partial x^k}{\partial x^a}$

Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$A_b^R(x) = \{ A_b^R(x), F[N] \} = 0$$

agrees with
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$$E_b^R(x) = \{ E_b^R(x), \int \frac{1}{2} \tilde{e}^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon^j_{lm} A_a^l A_b^m) N^j d^2 y \}$$

$$\left[\begin{matrix} e_a^k \\ e_a^k + D_a N^k \end{matrix} \right] \xrightarrow{\tilde{e}^{ac}} \tilde{e}^{cb} (\partial_c N_k + \epsilon_{kml} A_c^m N^l)(x)$$

Flatness constraints

$$F[N] = \int N^i F_i d^2 y$$

$$\delta_N A_b^k(x) = \{A_b^k(x), F[N]\} = 0 \quad \text{agrees with action of transl. symm.}$$

$$\delta_N E_k^b(x) = \{E_k^b(x), \int \frac{1}{2} \tilde{E}^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon_{lm}^j A_a^l A_b^m) N^j d^2 y\}$$

$$\begin{aligned} & \left. \begin{array}{l} E_k^b = \tilde{E}^{ba} e_a^k \\ e_a^k \mapsto e_a^k + D_a N^k \end{array} \right\} \tilde{E}^{cb} (\partial_c N_k + \epsilon_{kml} A_c^m N^l)(x) \end{aligned}$$

Flatness constraints

$$F[N] = \int N^j \mathcal{F}_j d^2 y$$

$$\delta_N A_b^R(x) = \{A_b^R(x), F[N]\} = 0$$

agrees with
action of transl. symm.
translation symmetry

$$\delta_N E_k^b(x) = \{E_k^b(x), \int \frac{1}{2} \tilde{E}^{ac} (\partial_a A_b^j + \frac{1}{2} \epsilon^j_{lm} A_a^l A_b^m) N^j d^2 y\}$$

$$\underbrace{E_k^b = \tilde{E}^{ba} e_{ak}}_{\text{translation: } e_a^k \mapsto e_a^k + D_a N^k} = \tilde{E}^{cb} (\partial_c N_k + \epsilon_{kml} A_c^m N^l)(x)$$

Flatness constraints generate
translation symmetry.

agrees with
action of transl. symm.

$$\int d^2y \{ N^j \}$$

s with
n of transl. symm.
d²y }

Flatness constraints generate
translation symmetry.

⇒ $\mathcal{H}$ generates a gauge symmetry,
which is identified with
time evolution.

Flatness constraints generate translation symmetry.

$\Rightarrow H$ generates a gauge symmetry, which is identified with time evolution.

with
of transl. symm.

NEWTON: EINSTEIN

Accelerating observer

$$X^M = \left(\frac{1}{a} \sinh at, \frac{1}{a} \cosh at \right)$$

$$\dot{X}^M = (\cosh at, \sinh at)$$

$$\ddot{X}^M = (a \sinh at, a \cosh at)$$

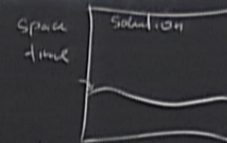
Flatness constraints

$$F[N] = \int N^i \mathcal{F}_i d^2 y$$

$$\delta_N A^R_b(x) = \{ A^R_b(x), F[N] \}$$

$$\delta_N E^b_k(x) = \{ E^b_k(x), \int \frac{1}{2} \tilde{E}^a_c (A^c_a A^M_b) N^j d^2 y \}$$

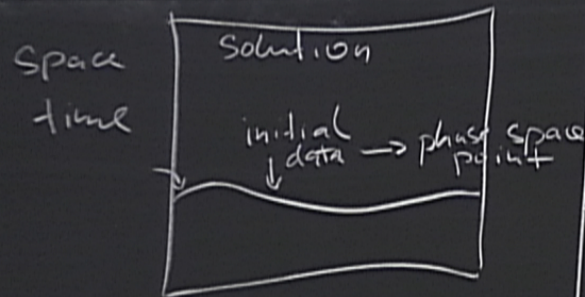
transformation: $E^a_k \mapsto \tilde{E}^a_k + D_k N^a$



agrees with action of transl symm.

Flatness constraints generate translation symmetry

$\Rightarrow H$ generates a gauge symmetry, which is identified with time evolution.



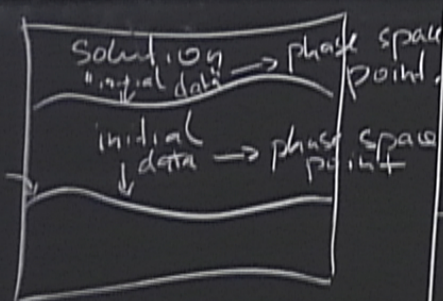
s with
n of transl. symm.

d^2y

Flatness constraints generate translation symmetry.

$\Rightarrow H$ generates a gauge symmetry, which is identified with time evolution.

space
time



s with
n of transl. symm.

d^2y

Flatness constraints generate
translation symmetry.

$\Rightarrow H$ generates a gauge symmetry,
which is identified with
time evolution.

ints

$$\int_j d^2 y$$

$t = x^0$ Fixed

$$\begin{cases} A^k(t, x^a) \\ E^k(t, x^a) \end{cases}$$

Solution

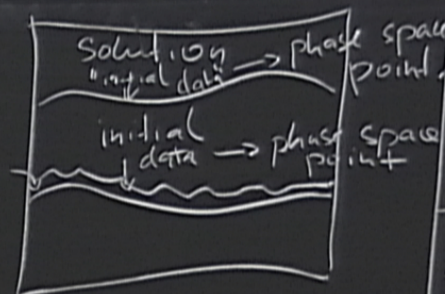
Space
time

$$\begin{cases} A^k(x^0, x^a) \\ E^k(x^0, x^a) \end{cases}$$

satisfying EOM

agrees with

action of transl. symm.



Flatness
translat

$\Rightarrow H$ general
which is
time ev

$$\{N\} = 0$$

$$\left\{ \frac{1}{2} \tilde{E}^{ac} \left(\partial_a A_b^j + \frac{1}{2} \epsilon_{lm} A_a^l A_b^m \right) N^j d^2 y \right\}$$

$$- \epsilon_{kml} A_c^m N^l(x)$$

ints

$$N^j \int d^2 y$$

$t = x^0$ Fixed

$$A_a^k(t, x^a)$$

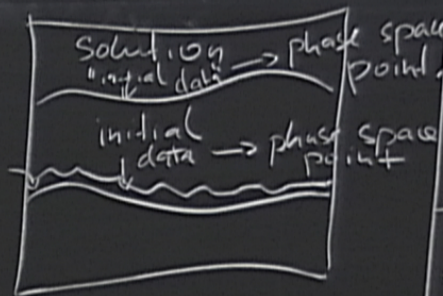
$$E_k^b(t, x^a)$$

Solution

$$\left\{ \begin{array}{l} A_\mu^k(x^0, x^a) \\ E_\mu^k(x^0, x^a) \end{array} \right.$$

satisfying EOM

Space
time



agrees with
action of transl. symm.

Flatness
translat

$\Rightarrow H$ general
which is
time ev

$$\{ \dots, F[N] \} = 0$$

$$\left\{ \int d^2 y \left(\partial_a A_b^j + \frac{1}{2} \epsilon^{jlm} A_a^l A_b^m \right) N^j \right\}$$

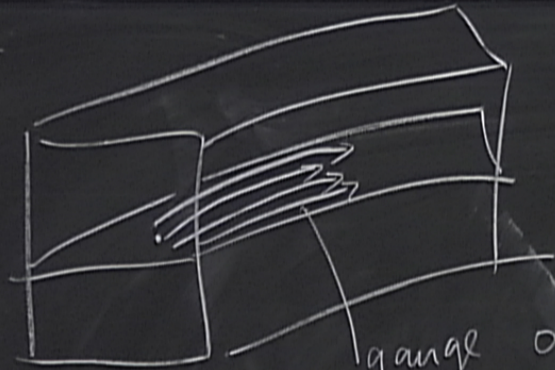
$$\epsilon_{km\ell} A_c^m N^\ell(x)$$

Flow generated by constraints leaves
the constraint hypersurface invariant.

$$\{F_i = 0, G_i = 0\}$$

$$\delta_\Lambda G[\Lambda'] = \{G[\Lambda'], G[\Lambda]\} = 0 \quad \text{on the constraint hypersurface}$$

$$= G[\epsilon_{jmn} \Lambda'^n]$$



gauge orbits
(for arbitrary L)

= {set of gauge equivalent phase space points to p } = gauge orbit through p

Flow generated by constraints leaves the constraint hypersurface invariant.

$$\{F_i = 0, G_i = 0\}$$

$$\delta_{\Lambda} G[\Lambda'] = \{G[\Lambda'], G[\Lambda]\} = 0 \quad \text{on the constraint hypersurface}$$

$$= G[\epsilon_{ijm} N^i \Lambda^j]$$

$$\delta_N F[N'] = 0$$

$$\delta_{\Lambda} F[N] = \{G[\Lambda], F[N]\} = -\delta_N G[\Lambda] =$$

Flow generated by constraints leaves
the constraint hypersurface invariant.

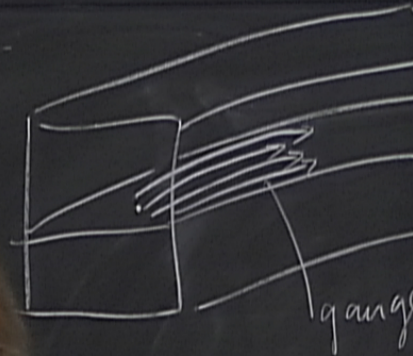
$$\{F_i = 0, G_j = 0\}$$

$$\delta_{\perp} G[\Lambda] = \{G[\Lambda], G[\Lambda]\} = 0$$

on the
constraint
hypersurface

$$\delta_N F[N] = 0$$

$$\delta_{\perp} F[N] = \{F[N], G[\Lambda]\} = -\delta_N G[\Lambda]$$



Λ^k