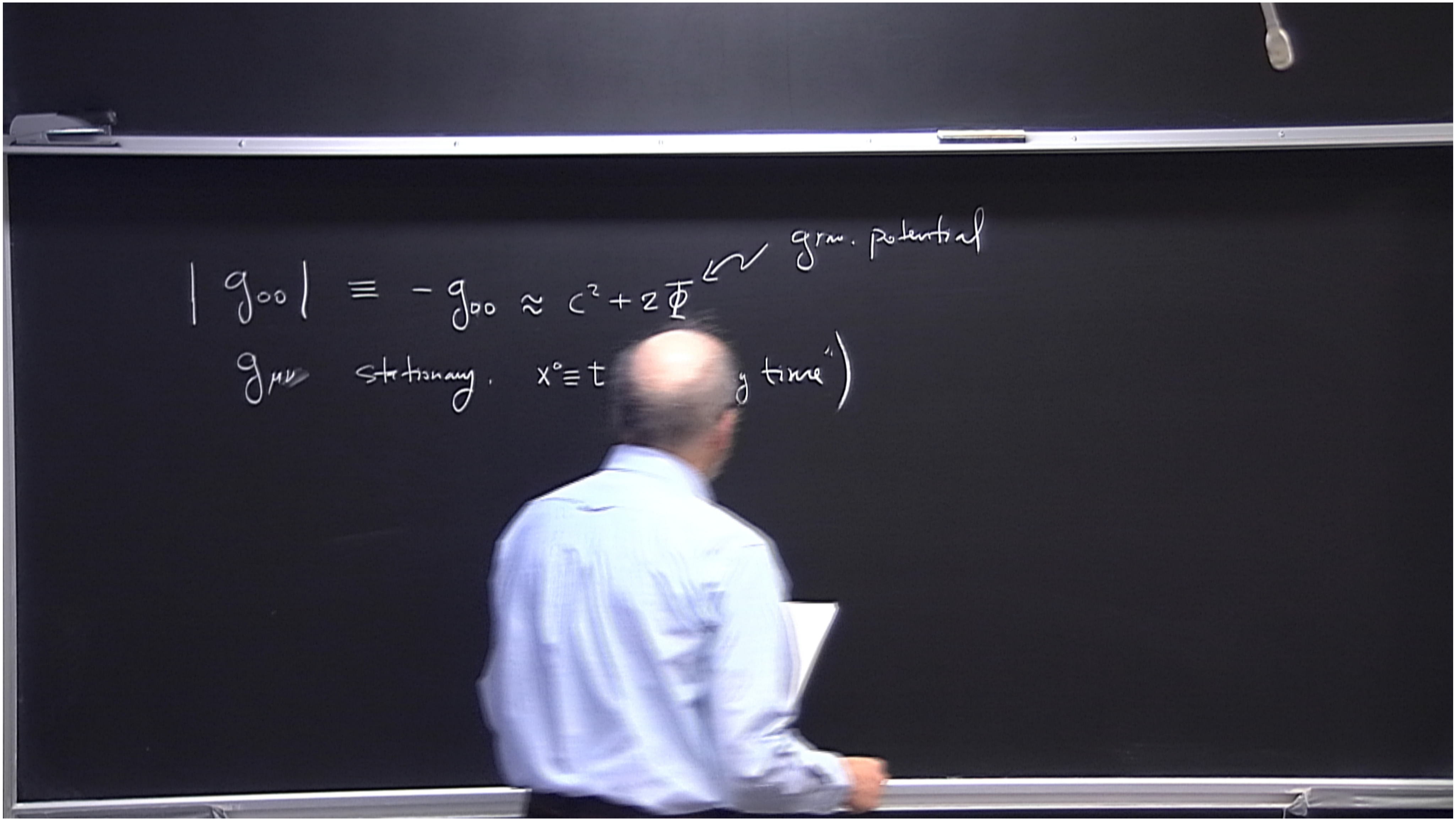
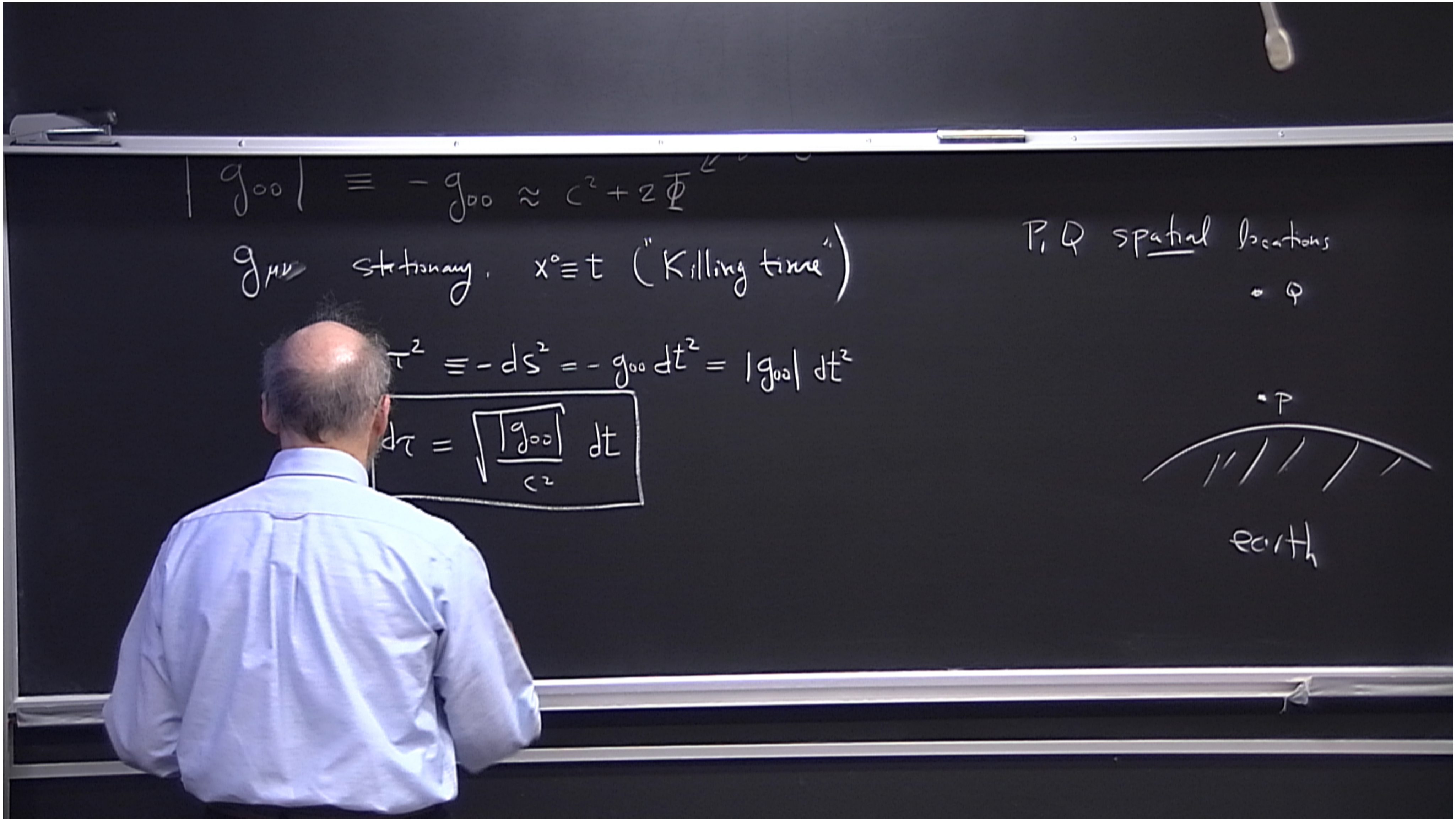
Title: Advanced General Relativity - Lecture 10

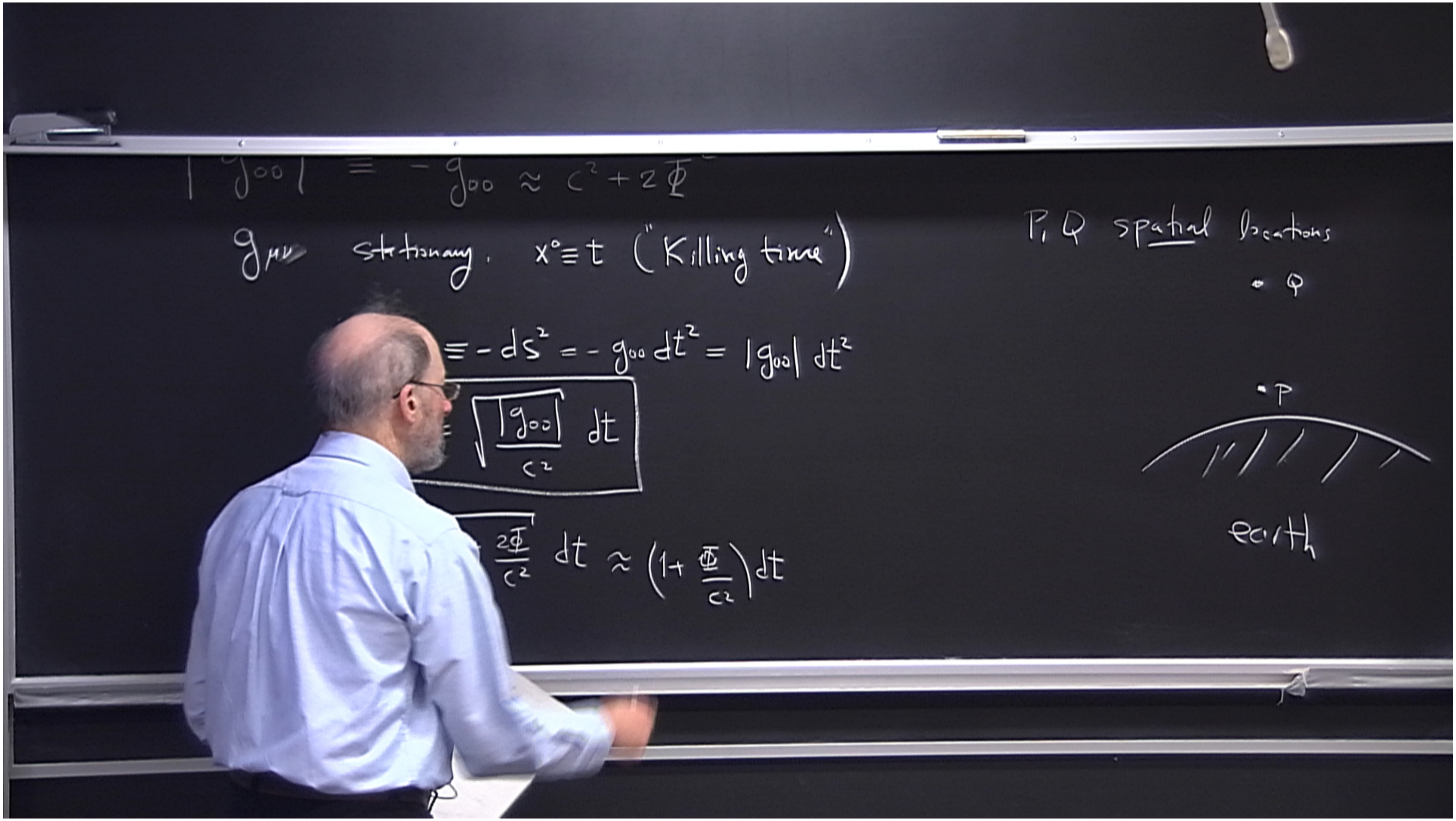
Date: Feb 21, 2013 05:00 PM

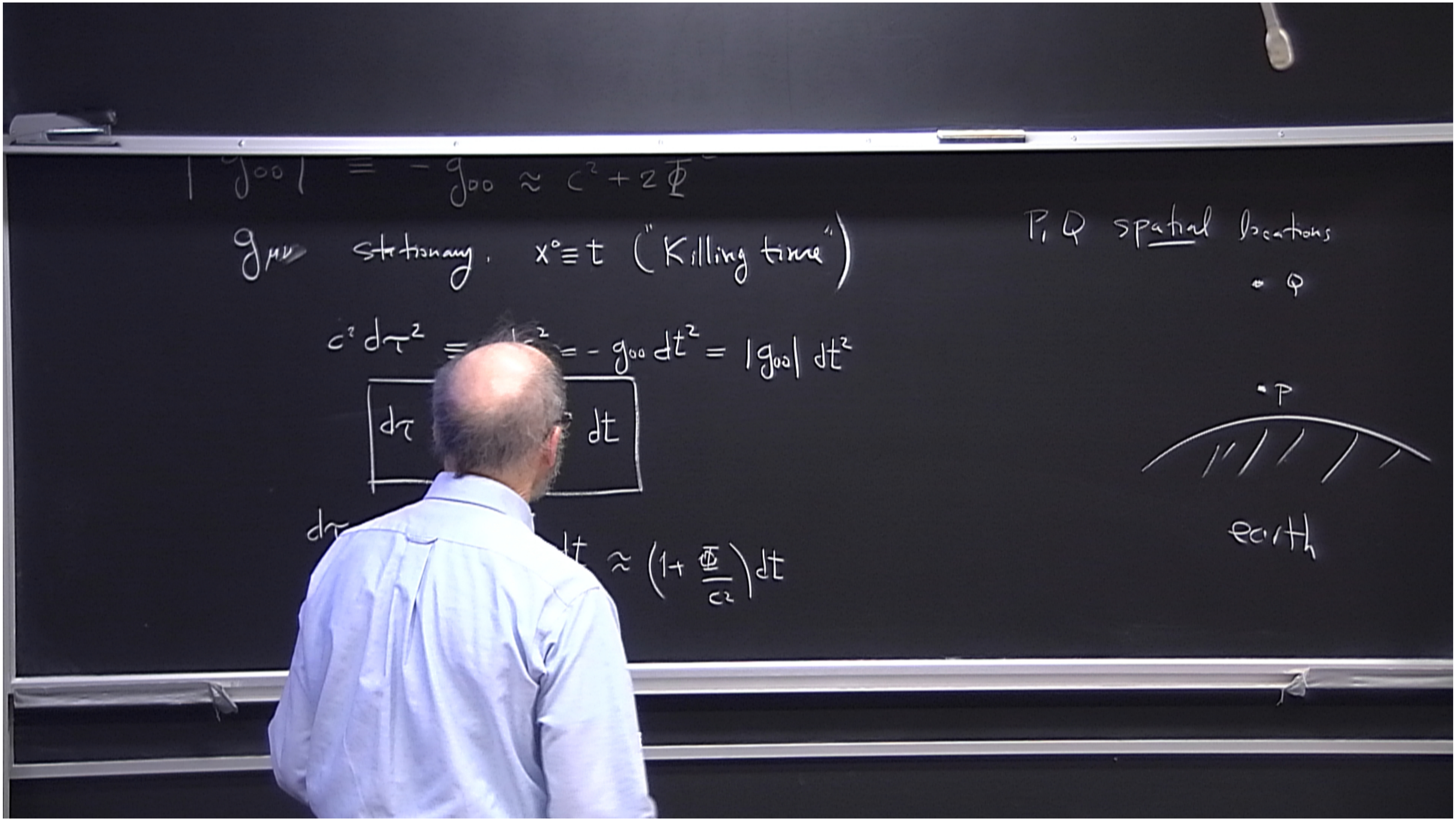
URL: http://pirsa.org/13020021

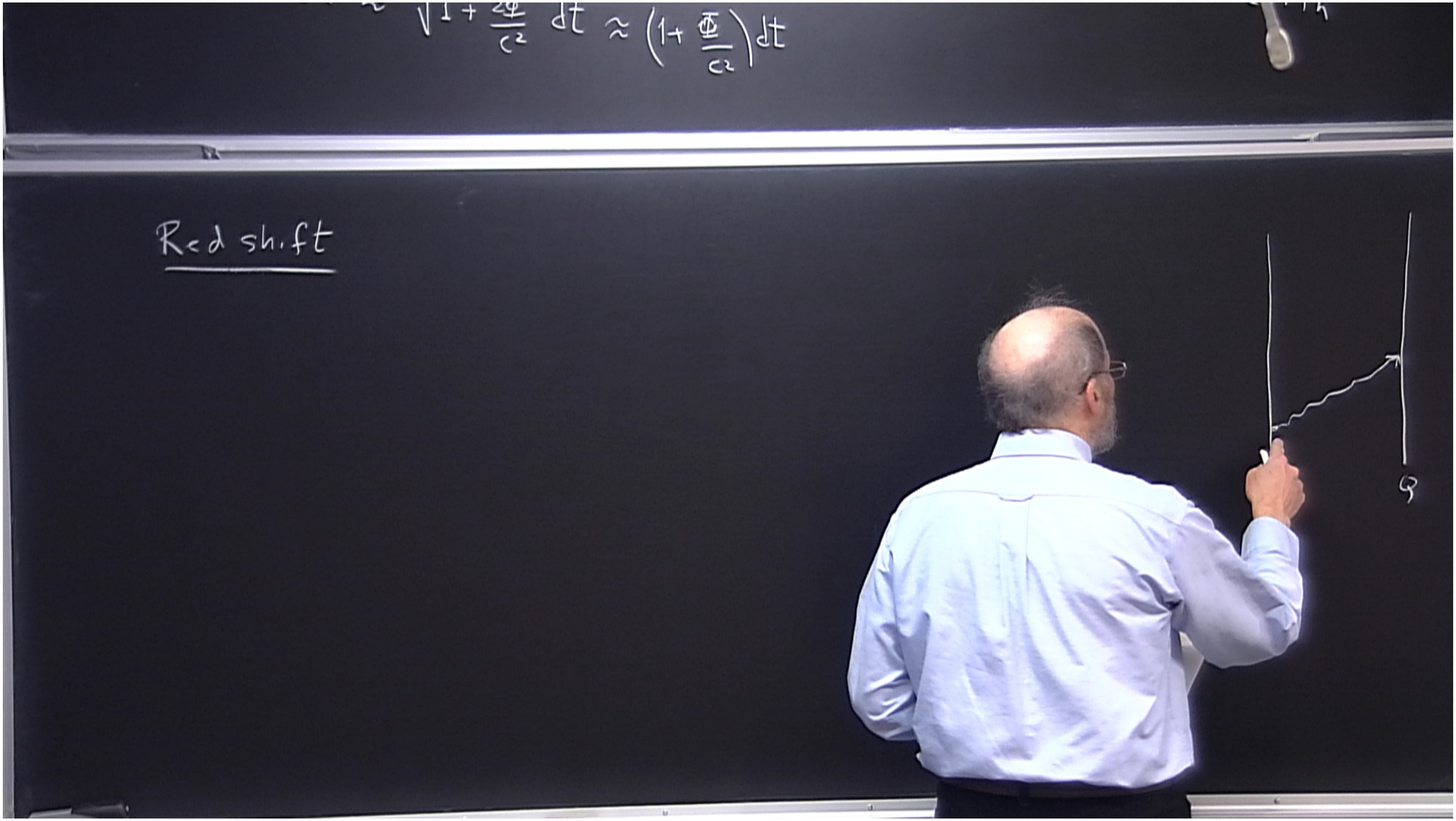
Abstract:

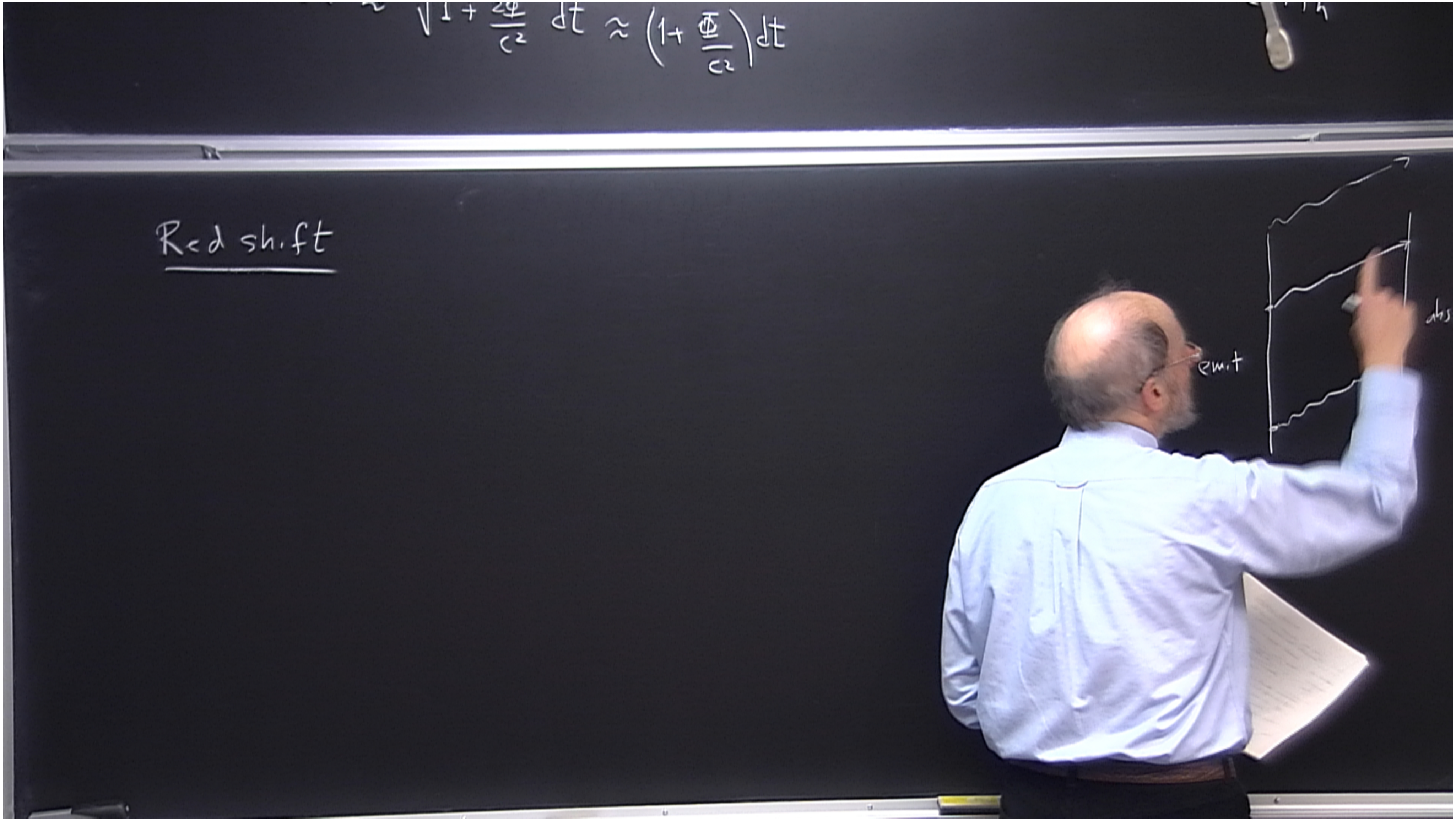

$|g_{00}| \equiv -g_{00} \approx c^2 + 2\Phi$
grav. potential
$g_{\mu\nu}$ stationary. $x^0 \equiv t$
time")

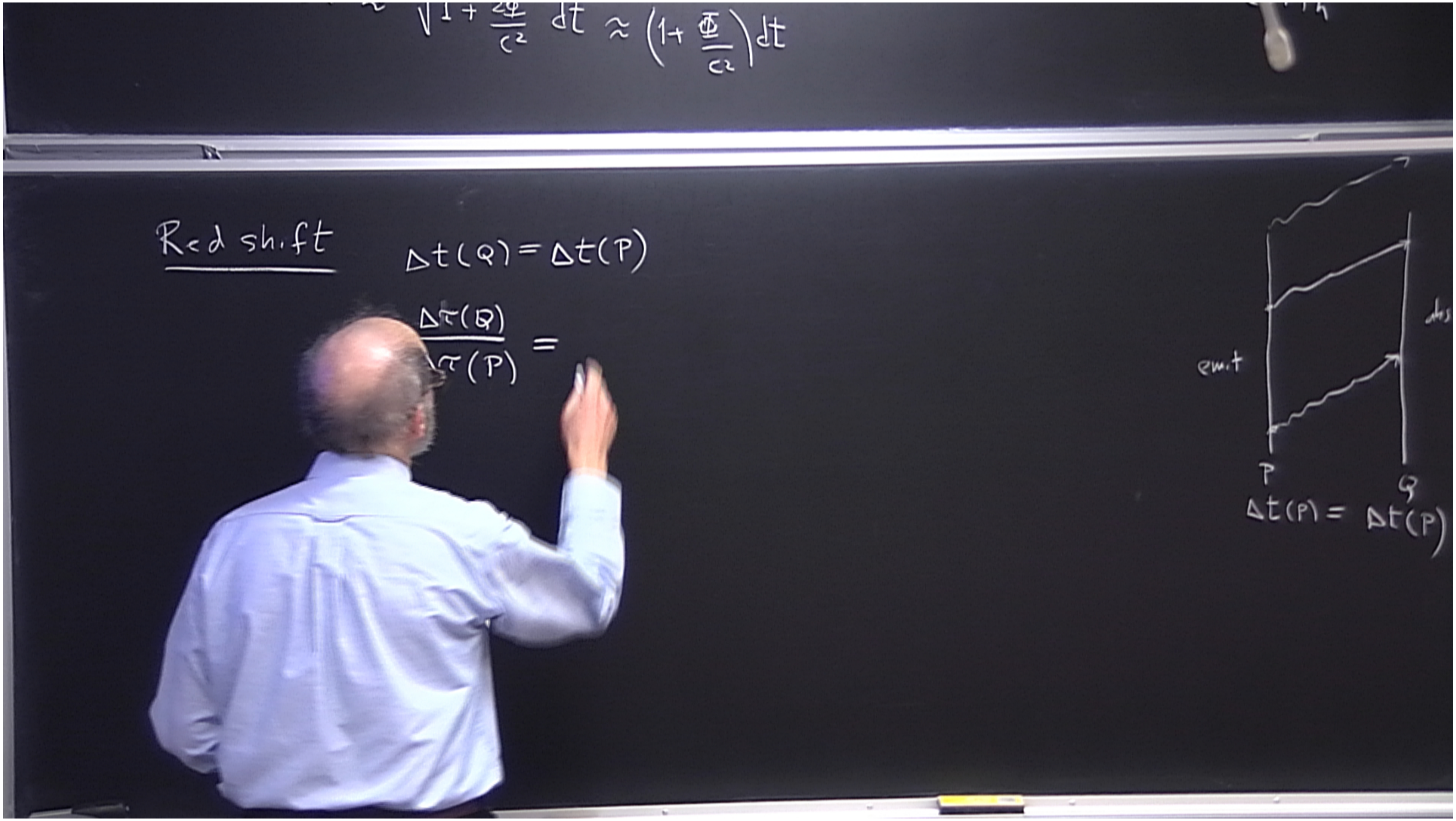

$|g_{00}| \equiv -g_{00} \approx c^2 + 2\Phi$
$g_{\mu\nu}$ stationary. $x^0 \equiv t$ ("Killing time")
$\tau^2 \equiv -ds^2 = -g_{00}\,dt^2 = |g_{00}|\,dt^2$
$d\tau = \sqrt{\frac{|g_{00}|}{c^2}}\,dt$
P, Q spatial locations
Q
P
earth

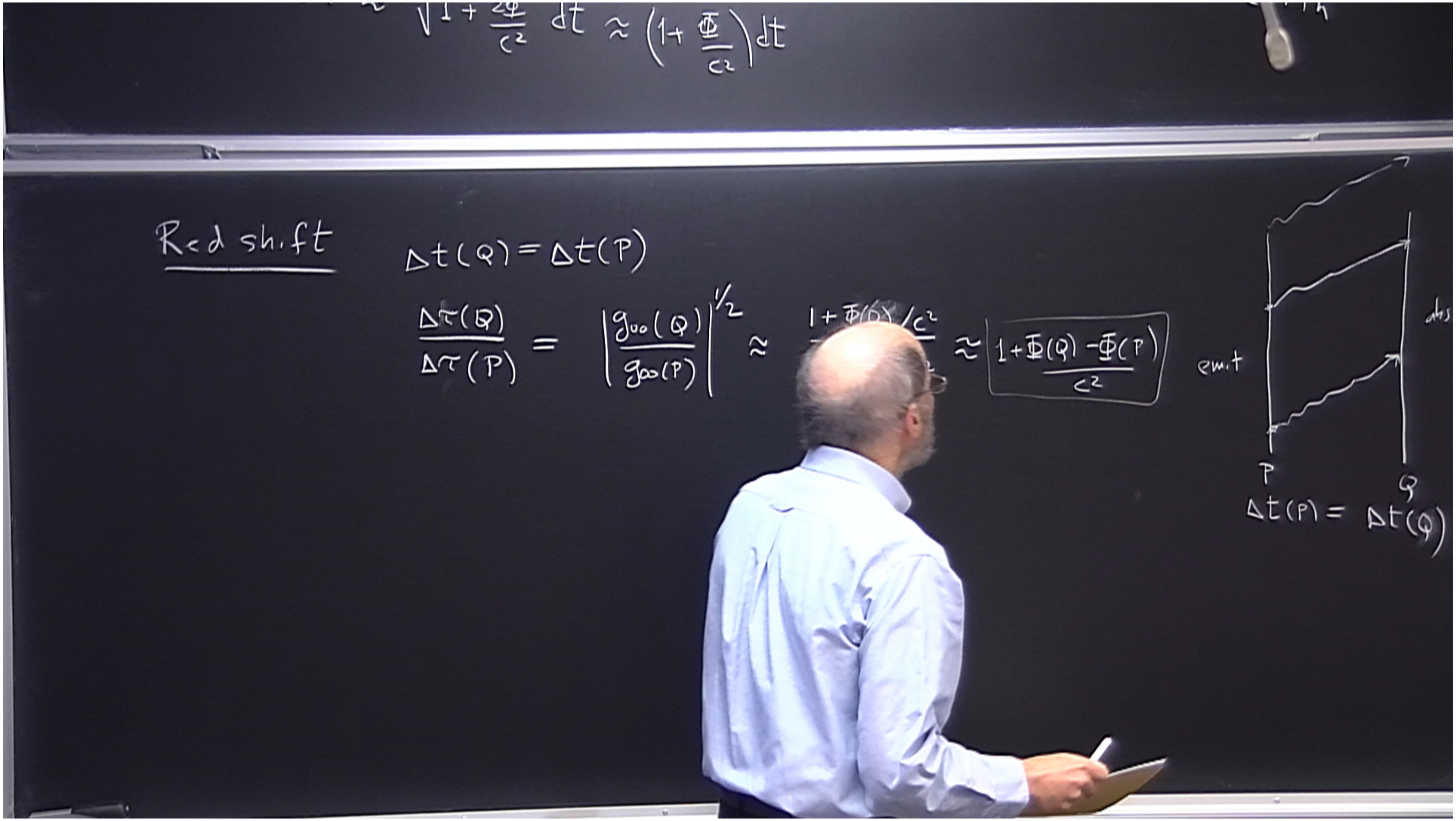

$|g_{00}| \equiv -g_{00} \approx c^2 + 2\Phi$
$g_{\mu\nu}$ stationary. $x^0 \equiv t$ ("Killing time")
$\equiv -ds^2 = -g_{00}\, dt^2 = |g_{00}|\, dt^2$
$\sqrt{\frac{|g_{00}|}{c^2}}\, dt$
$\frac{2\Phi}{c^2}\, dt \approx \left(1 + \frac{\Phi}{c^2}\right) dt$
P, Q spatial locations
Q
P
earth

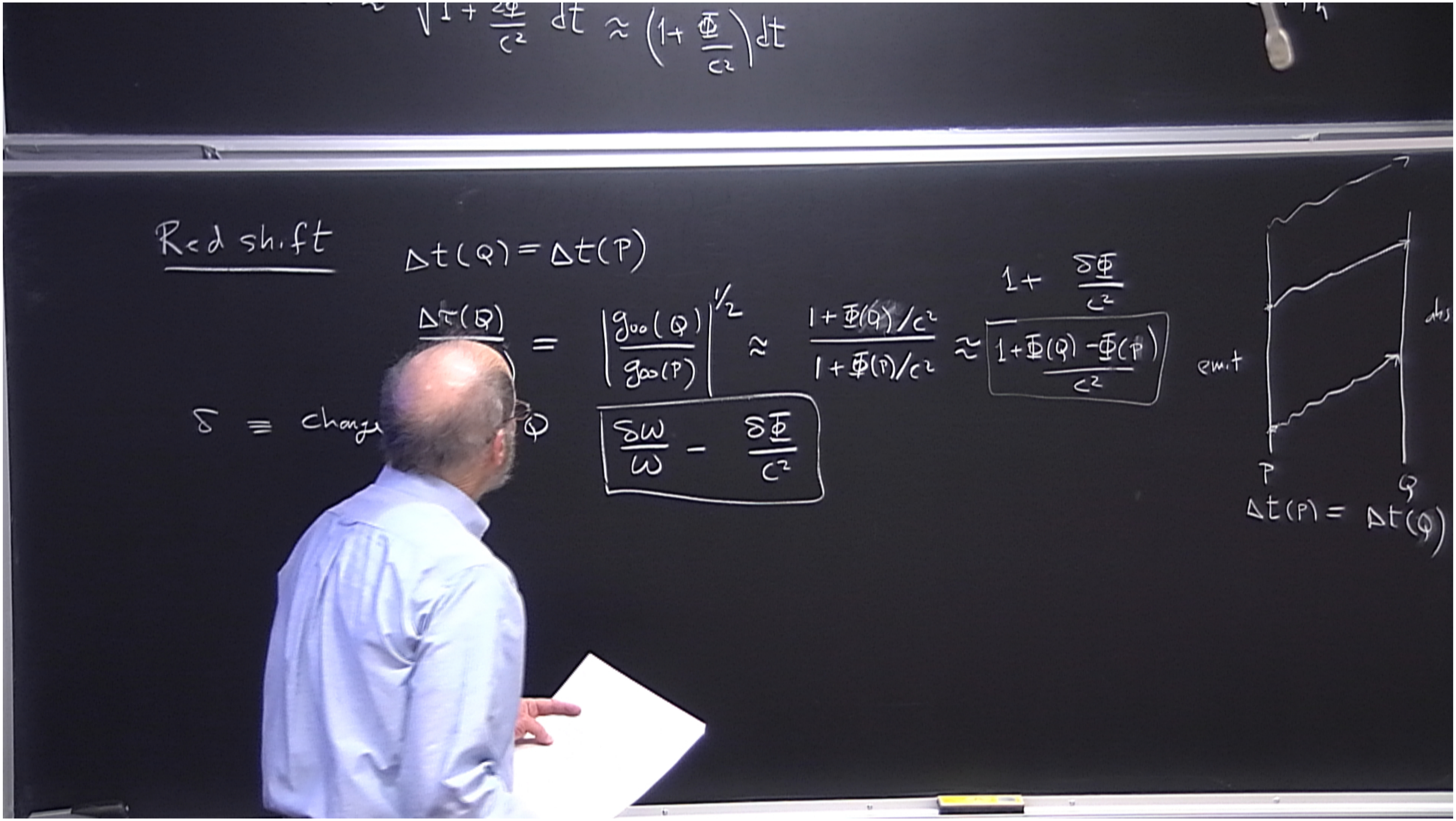

$|g_{00}| = -g_{00} \approx c^2 + 2\Phi$
$g_{\mu\nu}$ stationary. $x^0 \equiv t$ ("Killing time")
$c^2 d\tau^2 = \ldots = -g_{00}\, dt^2 = |g_{00}|\, dt^2$
$d\tau$
dt
$\approx \left(1 + \frac{\Phi}{c^2}\right) dt$
P, Q spatial locations
Q
P
earth

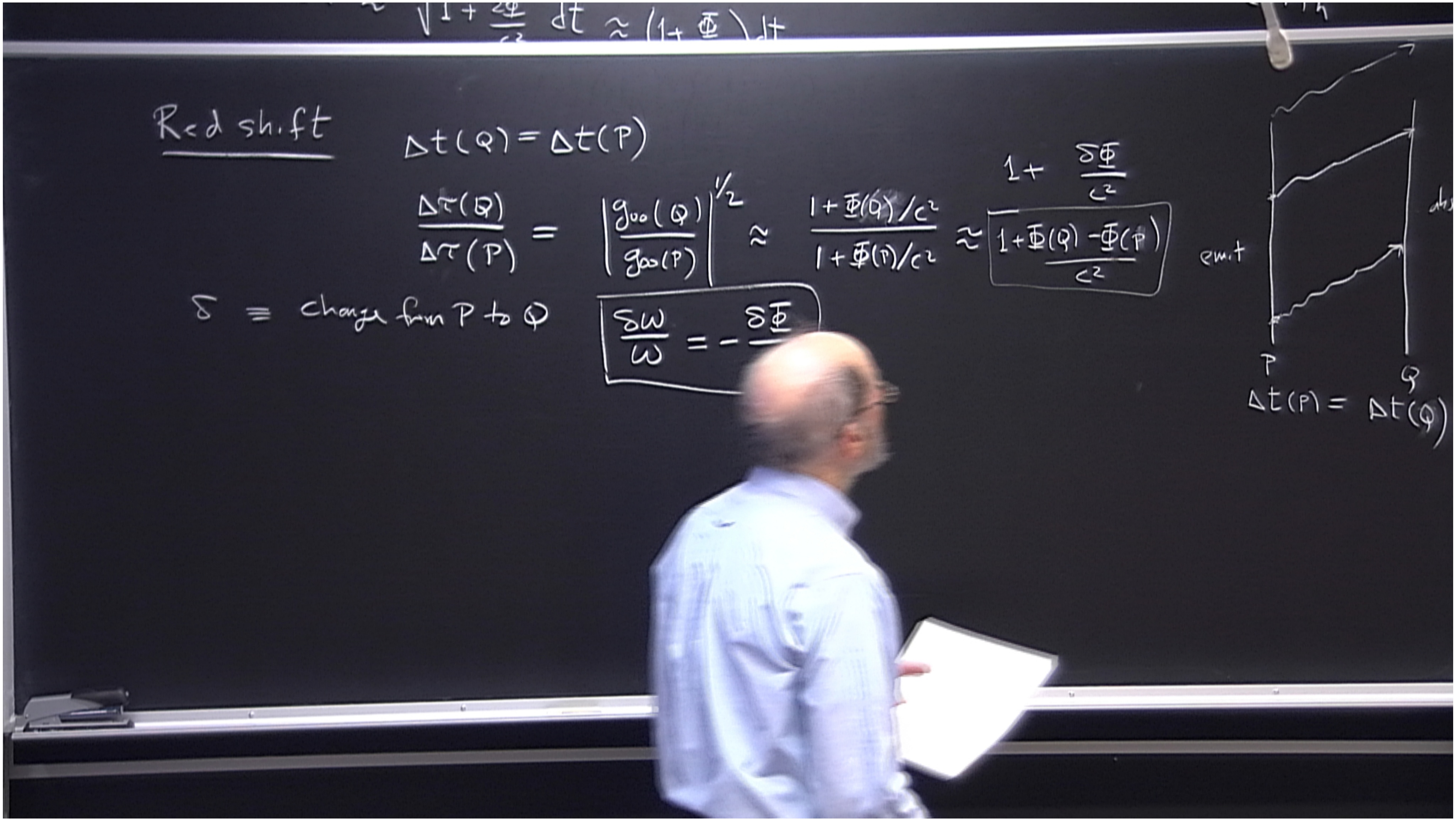

$\approx \left(1+\frac{\Phi}{c^2}\right)dt$
Red shift

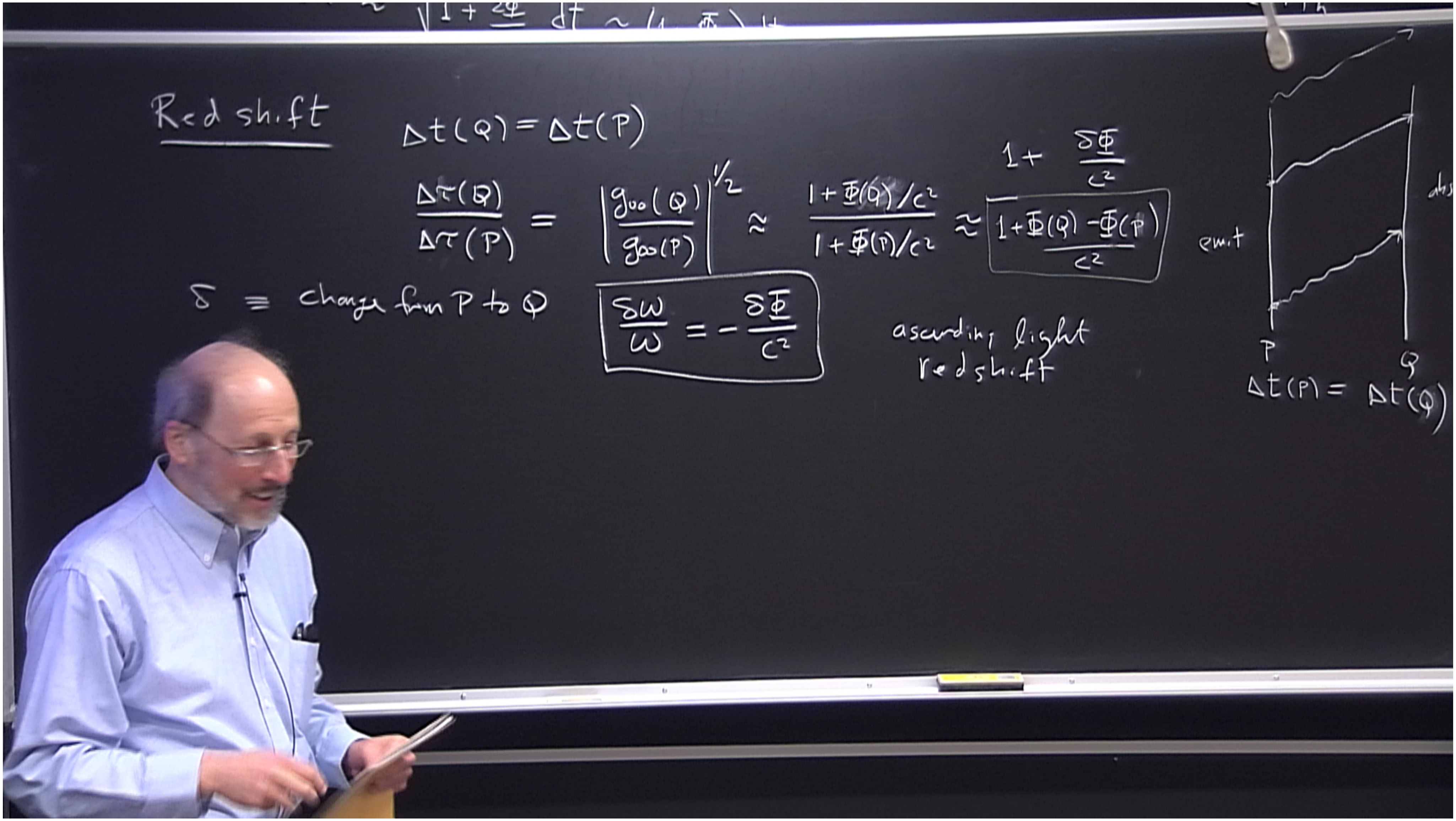

$\sqrt{1+\frac{2\Phi}{c^2}}\,dt \approx \left(1+\frac{\Phi}{c^2}\right)dt$
Red shift
emit
abs

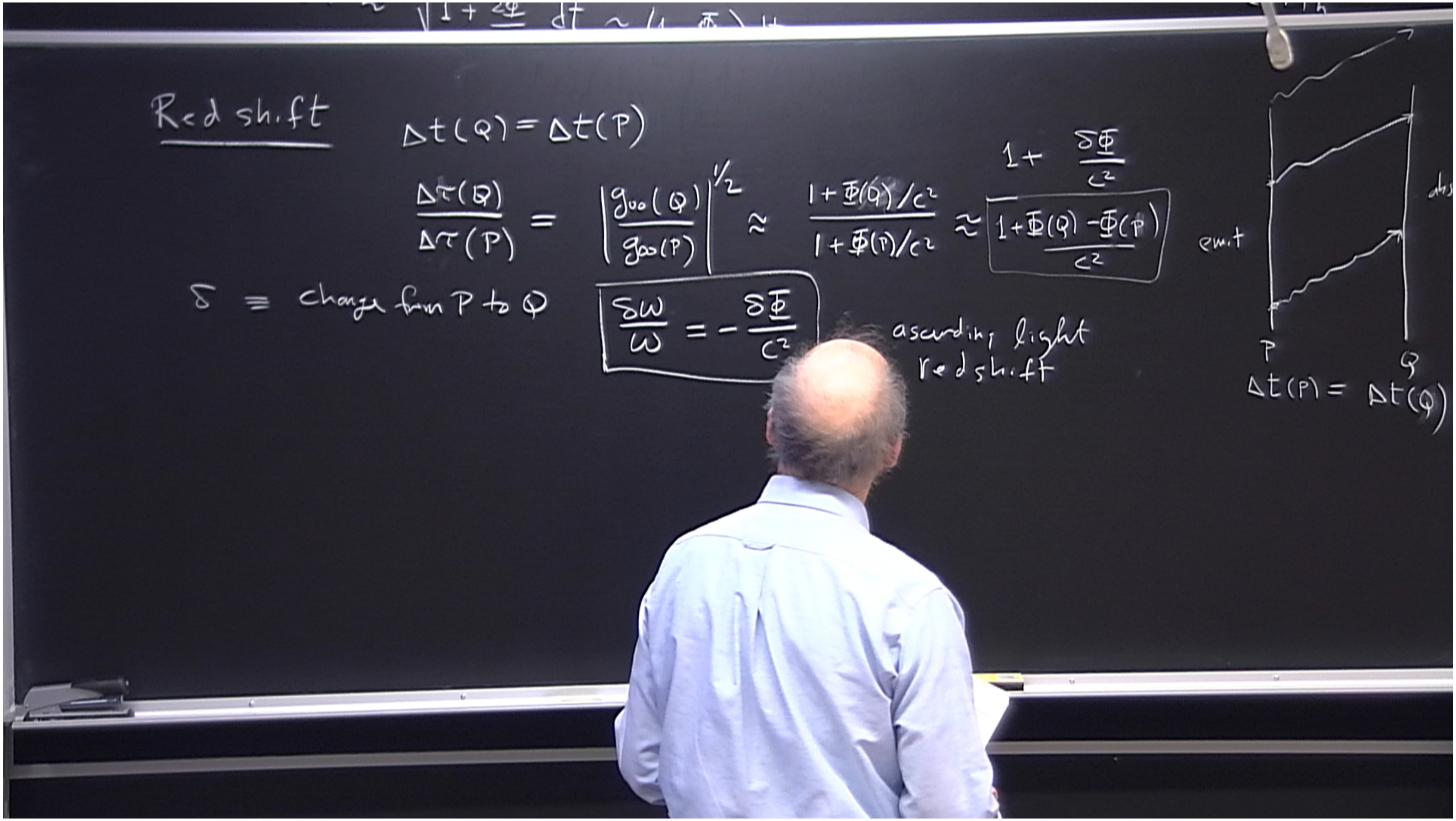

≈ (1+ Φ/c²) dt
Red shift
Δt(Q) = Δt(P)
Δτ(Q) / Δτ(P) =
emit
P
Q
Δt(P) = Δt(P)

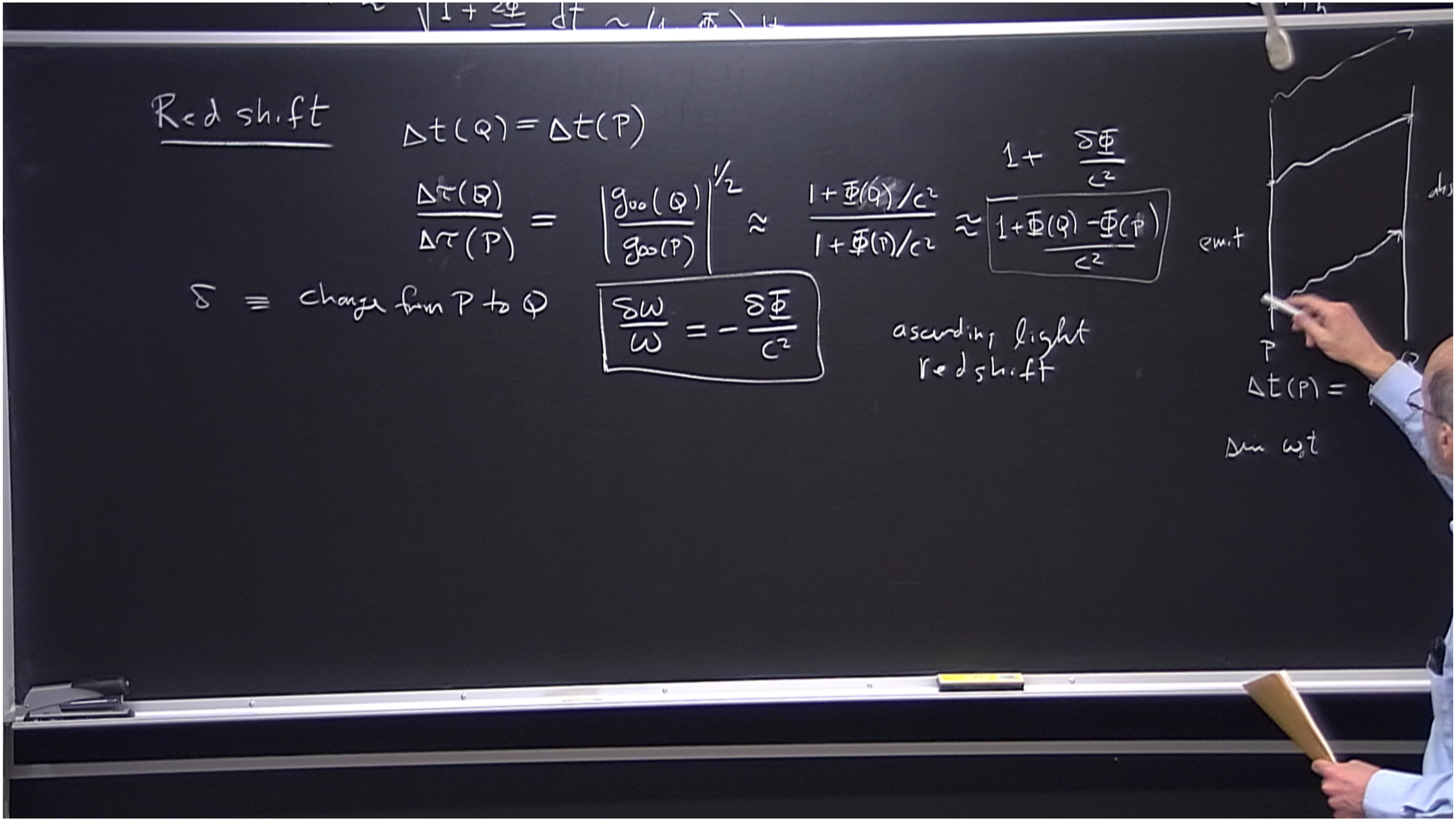

$\approx \left(1 + \frac{\Phi}{c^2}\right) dt$
Red shift
$\Delta t(Q) = \Delta t(P)$
$\frac{\Delta\tau(Q)}{\Delta\tau(P)} = \left|\frac{g_{00}(Q)}{g_{00}(P)}\right|^{1/2} \approx$
$\approx \boxed{1 + \frac{\Phi(Q) - \Phi(P)}{c^2}}$
emit
abs
P
Q
$\Delta t(P) = \Delta t(Q)$


Red shift
Δt(Q) = Δt(P)
Δτ(Q)
= |g₀₀(Q)/g₀₀(P)|^½ ≈ (1 + Φ(Q)/c²)/(1 + Φ(P)/c²) ≈ 1 + (Φ(Q) − Φ(P))/c²
1 + δΦ/c²
δ ≡ change
δω/ω − δΦ/c²
emit
abs
P
Q
Δt(P) = Δt(Q)
≈ (1 + Φ/c²) dt


Redshift
Δt(Q) = Δt(P)
Δτ(Q)/Δτ(P) = |g00(Q)/g00(P)|^1/2 ≈ (1+Φ(Q)/c²)/(1+Φ(P)/c²) ≈ 1 + (Φ(Q) − Φ(P))/c²
1 + δΦ/c²
δ ≡ change from P to Q
δω/ω = − δΦ
emit
P
Q
Δt(P) = Δt(Q)


Redshift
Δt(Q) = Δt(P)
Δτ(Q)/Δτ(P) = |g00(Q)/g00(P)|^1/2 ≈ (1 + Φ(Q)/c²)/(1 + Φ(P)/c²) ≈ 1 + (Φ(Q) − Φ(P))/c²
1 + δΦ/c²
δ ≡ change from P to Q
δω/ω = − δΦ/c²
ascending light
Redshift
emit
P
Q
Δt(P) = Δt(Q)


Redshift
Δt(Q) = Δt(P)
Δτ(Q)/Δτ(P) = |g₀₀(Q)/g₀₀(P)|^½ ≈ (1 + Φ(Q)/c²)/(1 + Φ(P)/c²) ≈ 1 + (Φ(Q) − Φ(P))/c²
1 + δΦ/c²
δ ≡ change from P to Q
δω/ω = − δΦ/c²
ascending light
redshift
emit
P
Q
Δt(P) = Δt(Q)


Redshift
$\Delta t(Q) = \Delta t(P)$
$\frac{\Delta\tau(Q)}{\Delta\tau(P)} = \left|\frac{g_{00}(Q)}{g_{00}(P)}\right|^{1/2} \approx \frac{1+\Phi(Q)/c^2}{1+\Phi(P)/c^2} \approx \boxed{1+\frac{\Phi(Q)-\Phi(P)}{c^2}}$
$1 + \frac{\delta\Phi}{c^2}$
$\delta \equiv$ change from P to Q
$\boxed{\frac{\delta\omega}{\omega} = -\frac{\delta\Phi}{c^2}}$
ascending light
redshift
emit
P
$\Delta t(P) =$
$\sin \omega_0 t$

Redshift

$$\Delta t(Q) = \Delta t(P)$$

$$\frac{\Delta\tau(Q)}{\Delta\tau(P)} = \left|\frac{g_{00}(Q)}{g_{00}(P)}\right|^{1/2} \approx \frac{1+\Phi(Q)/c^2}{1+\Phi(P)/c^2} \approx \boxed{1+\frac{\Phi(Q)-\Phi(P)}{c^2}}$$

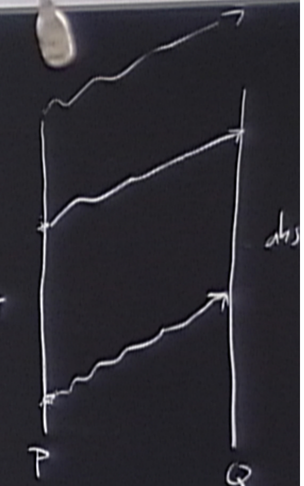
$$1 + \frac{\delta\Phi}{c^2}$$

$\delta \equiv$ change from P to Q

$$\boxed{\frac{\delta\omega}{\omega} = -\frac{\delta\Phi}{c^2}}$$

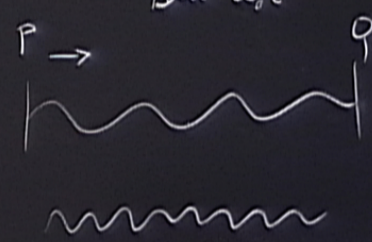
ascending light
redshift

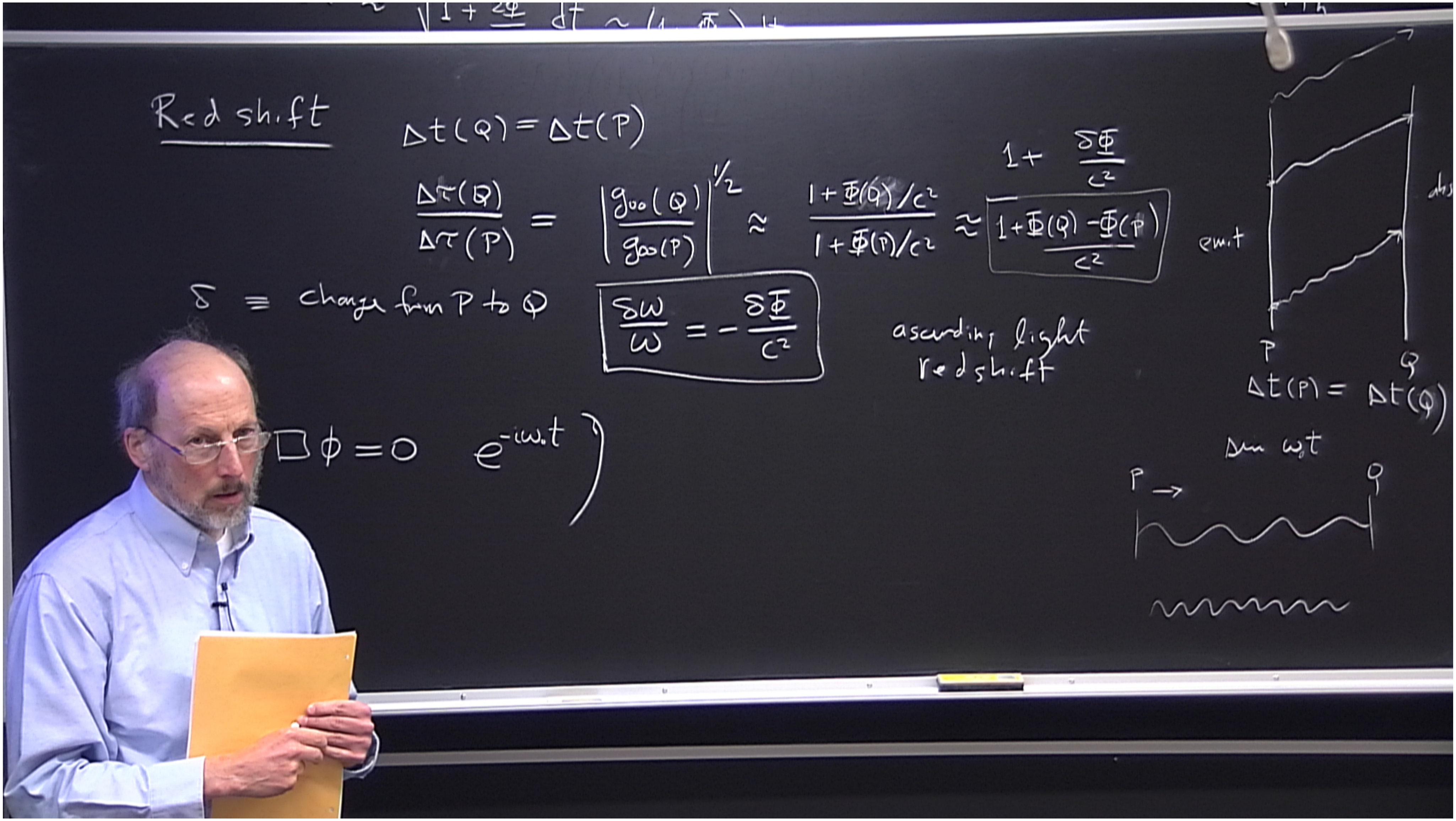
emit

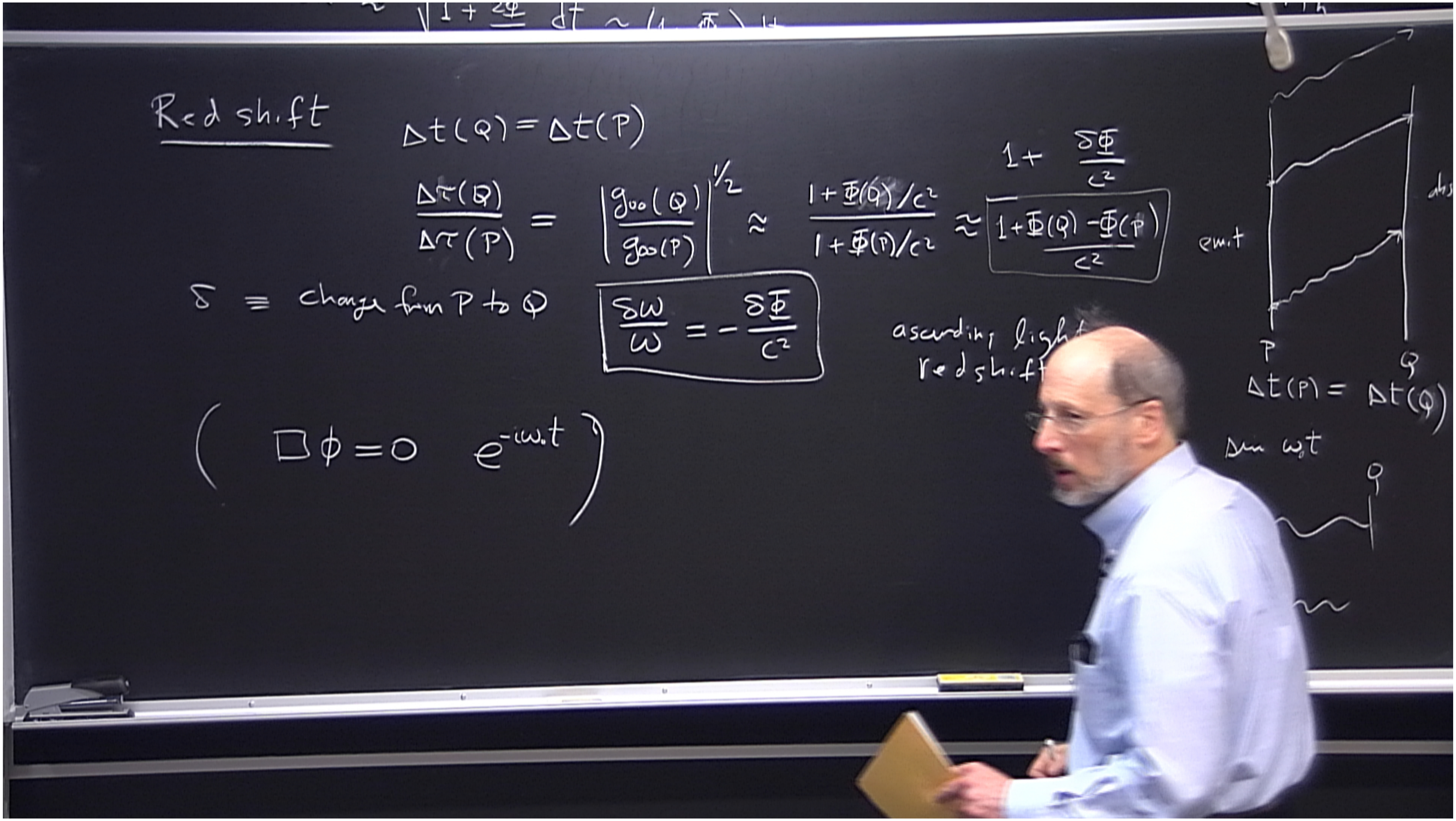


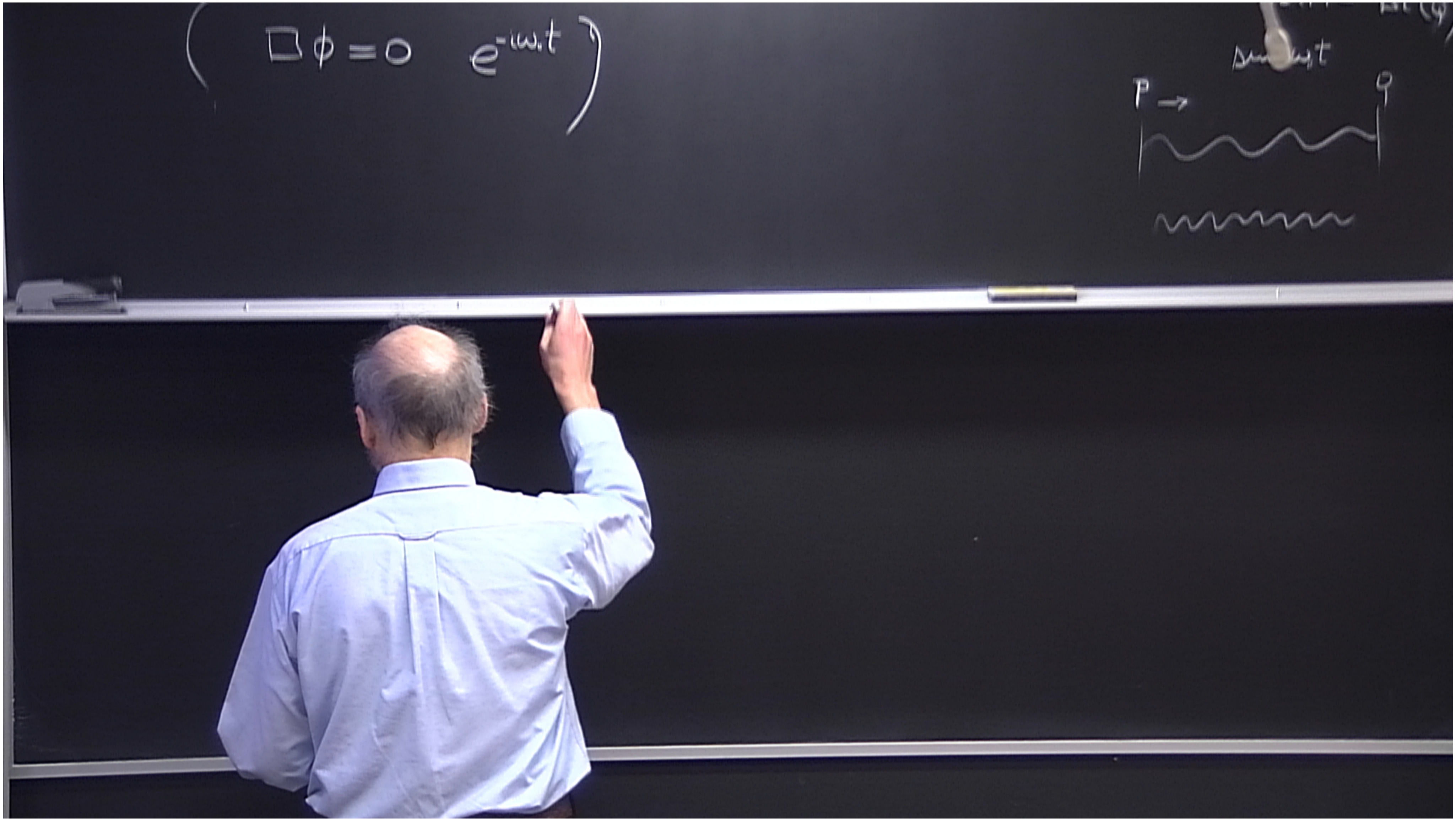
$$\Delta t(P) = \Delta t(Q)$$

$\sin \omega_0 t$

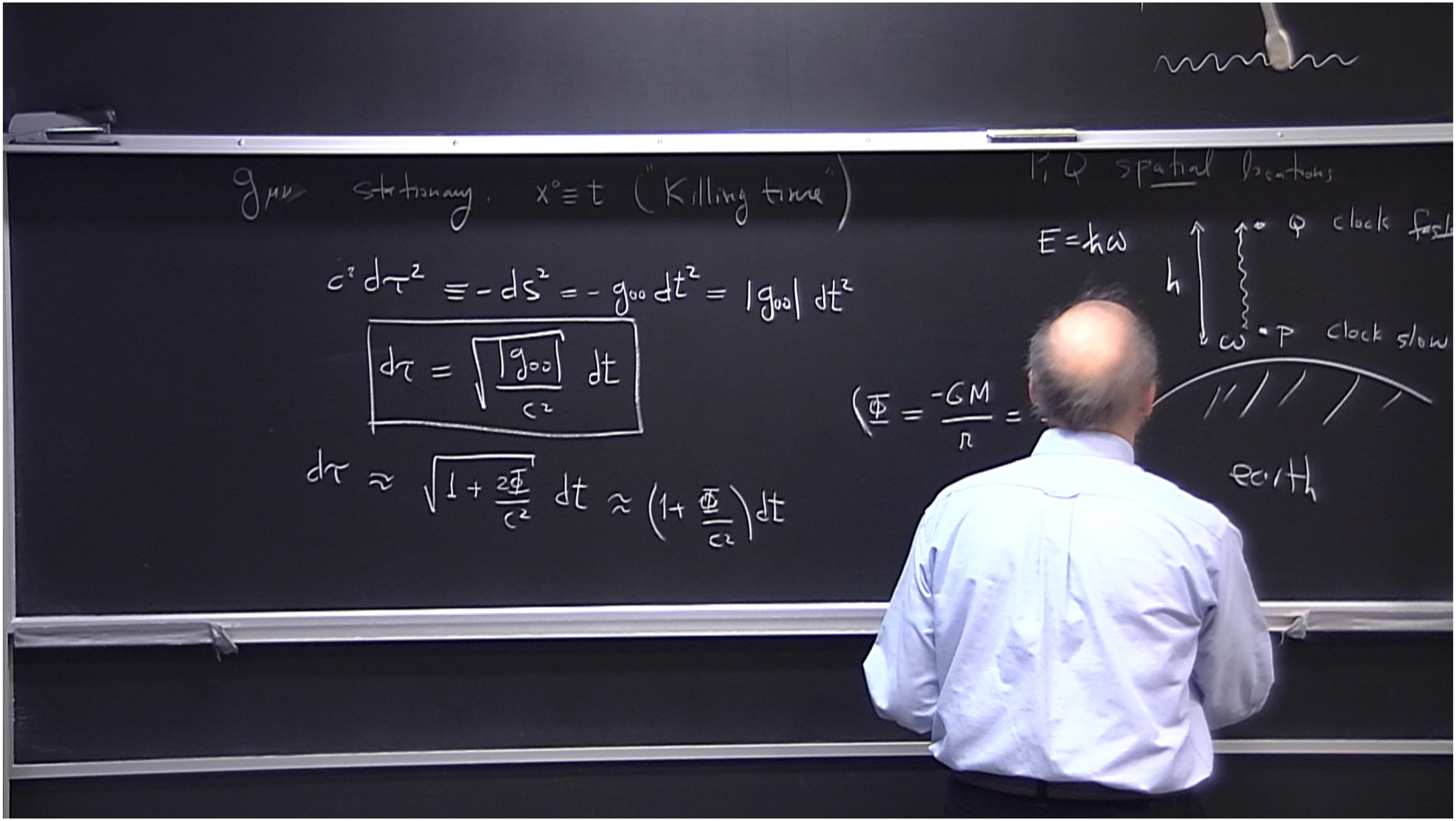


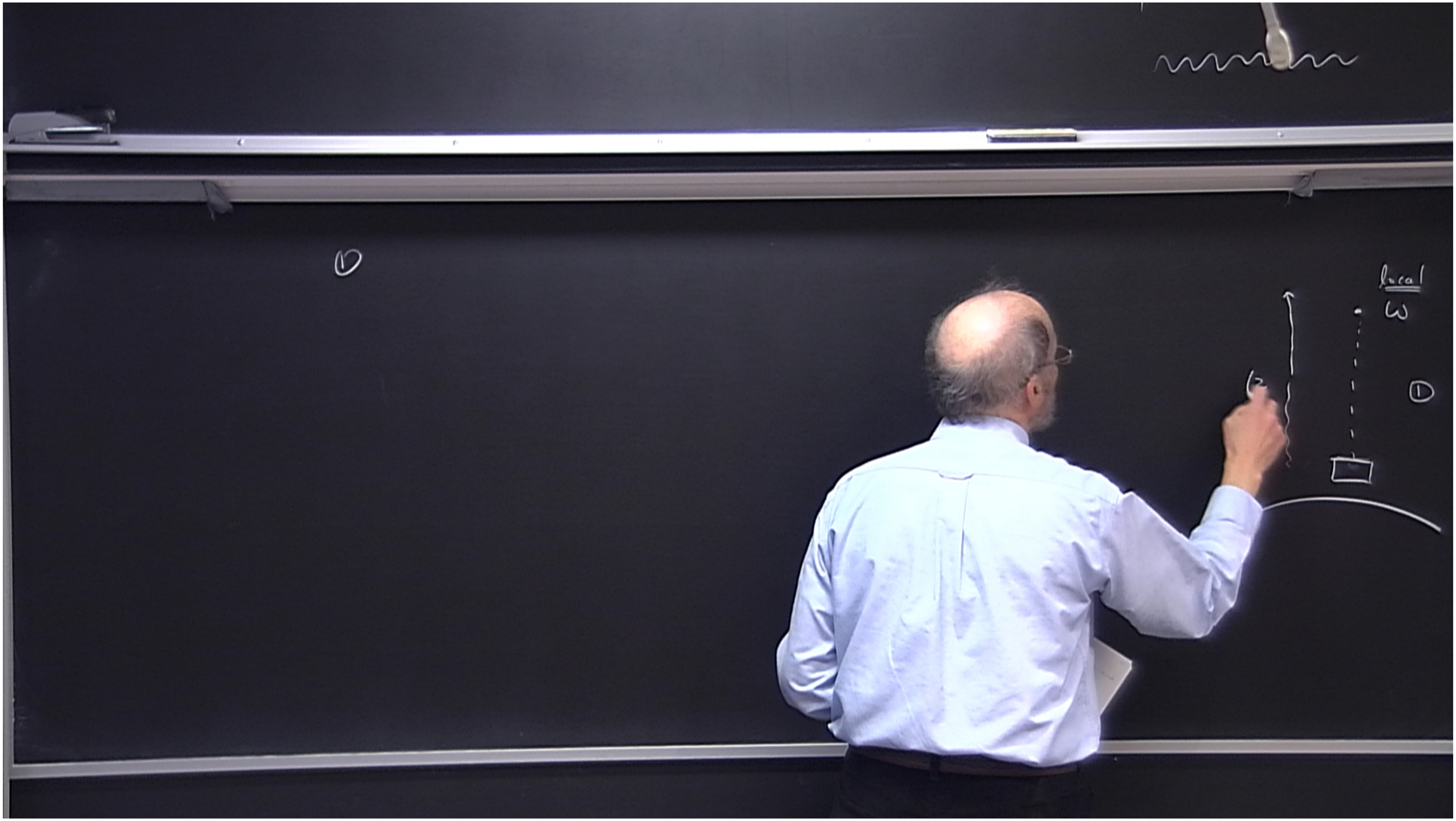

Redshift
Δt(Q) = Δt(P)
Δτ(Q) / Δτ(P) = |g₀₀(Q) / g₀₀(P)|^½ ≈ (1 + Φ(Q)/c²) / (1 + Φ(P)/c²) ≈ 1 + (Φ(Q) − Φ(P)) / c²
1 + δΦ / c²
δ ≡ change from P to Q
δω / ω = − δΦ / c²
ascending light
redshift
emit
P
Q
Δt(P) = Δt(Q)
□φ = 0   e^(−iω₀t)
sin ω₀t

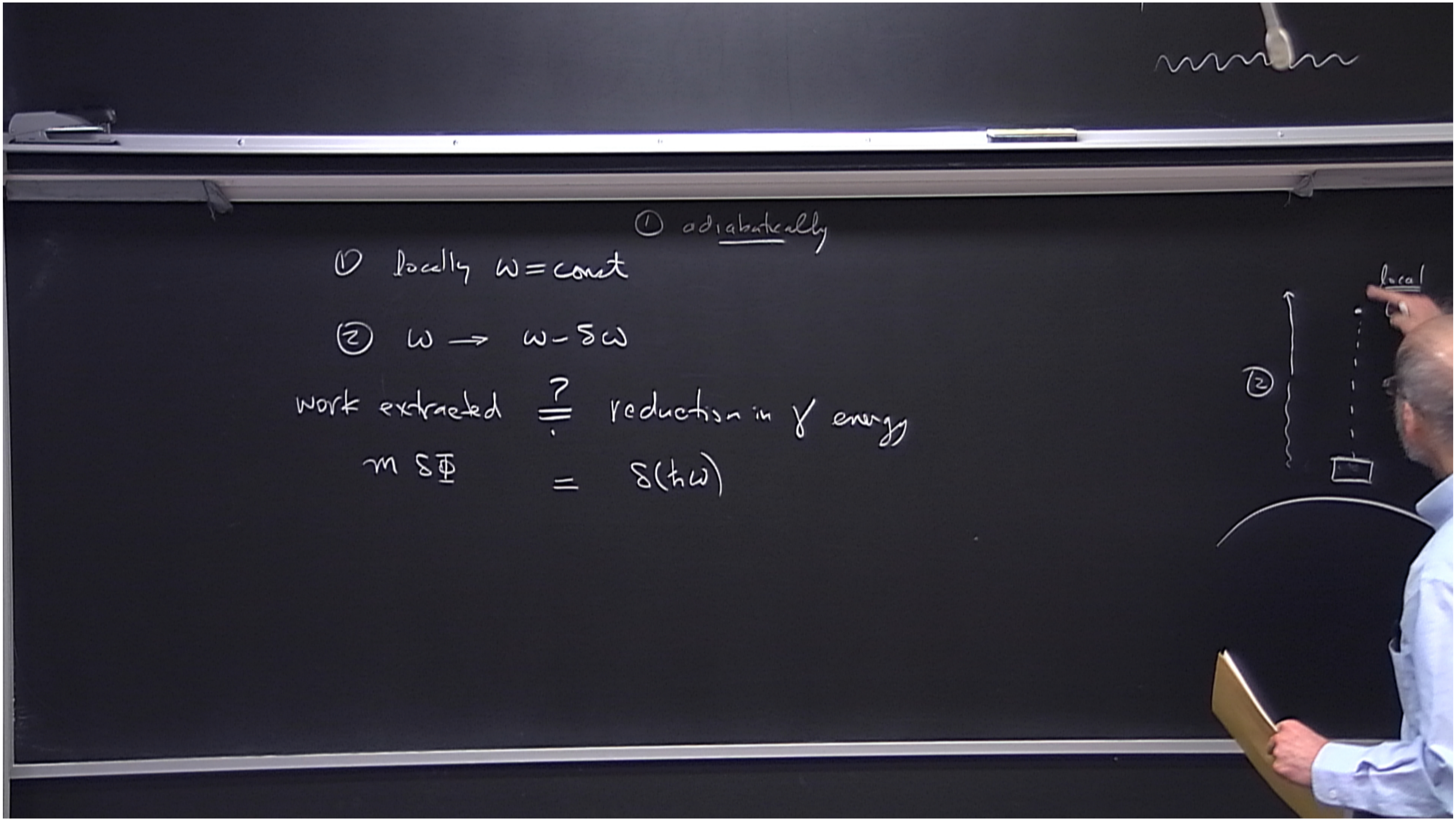

Red shift
Δt(Q) = Δt(P)
Δτ(Q)/Δτ(P) = |g₀₀(Q)/g₀₀(P)|^½ ≈ (1+Φ(Q)/c²)/(1+Φ(P)/c²) ≈ 1 + (Φ(Q) − Φ(P))/c²
1 + δΦ/c²
δ ≡ change from P to Q
δω/ω = − δΦ/c²
ascending light
Redshift
(□φ = 0   e^{−iω₀t})
emit
P
Q
Δt(P) = Δt(Q)
sin ω₀t



$\Box\phi = 0 \quad e^{-i\omega_i t}$
$p \rightarrow$
q


$g_{\mu\nu}$ stationary. $x^0 \equiv t$ ("Killing time")
$c^2 d\tau^2 \equiv -ds^2 = -g_{00}\, dt^2 = |g_{00}|\, dt^2$
$\boxed{d\tau = \sqrt{\frac{|g_{00}|}{c^2}}\, dt}$
$d\tau \approx \sqrt{1 + \frac{2\Phi}{c^2}}\, dt \approx \left(1 + \frac{\Phi}{c^2}\right) dt$
$(\Phi = \frac{-GM}{r} =$
P, Q spatial locations
$E = \hbar\omega$
h
Q clock fast
ω P clock slow
earth


local
ω


① adiabatically
① locally $\omega = \text{const}$
② $\omega \to \omega - \delta\omega$
work extracted $\stackrel{?}{=}$ reduction in γ energy
$m\,\delta\Phi = \delta(\hbar\omega)$
local
②

(1) adiabatically

(1) locally $\omega = \text{const}$

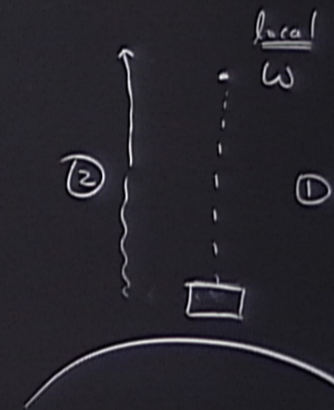
(2) $\omega \rightarrow \omega - \delta\omega$

work extracted $\overset{?}{=}$ reduction in γ energy

$$-m\,\delta\Phi \overset{?}{=} \delta(\hbar\omega)$$

$$-\frac{\cancel{\hbar}\omega}{c^2}\,\delta\Phi \approx \cancel{\hbar}\,\delta\omega$$

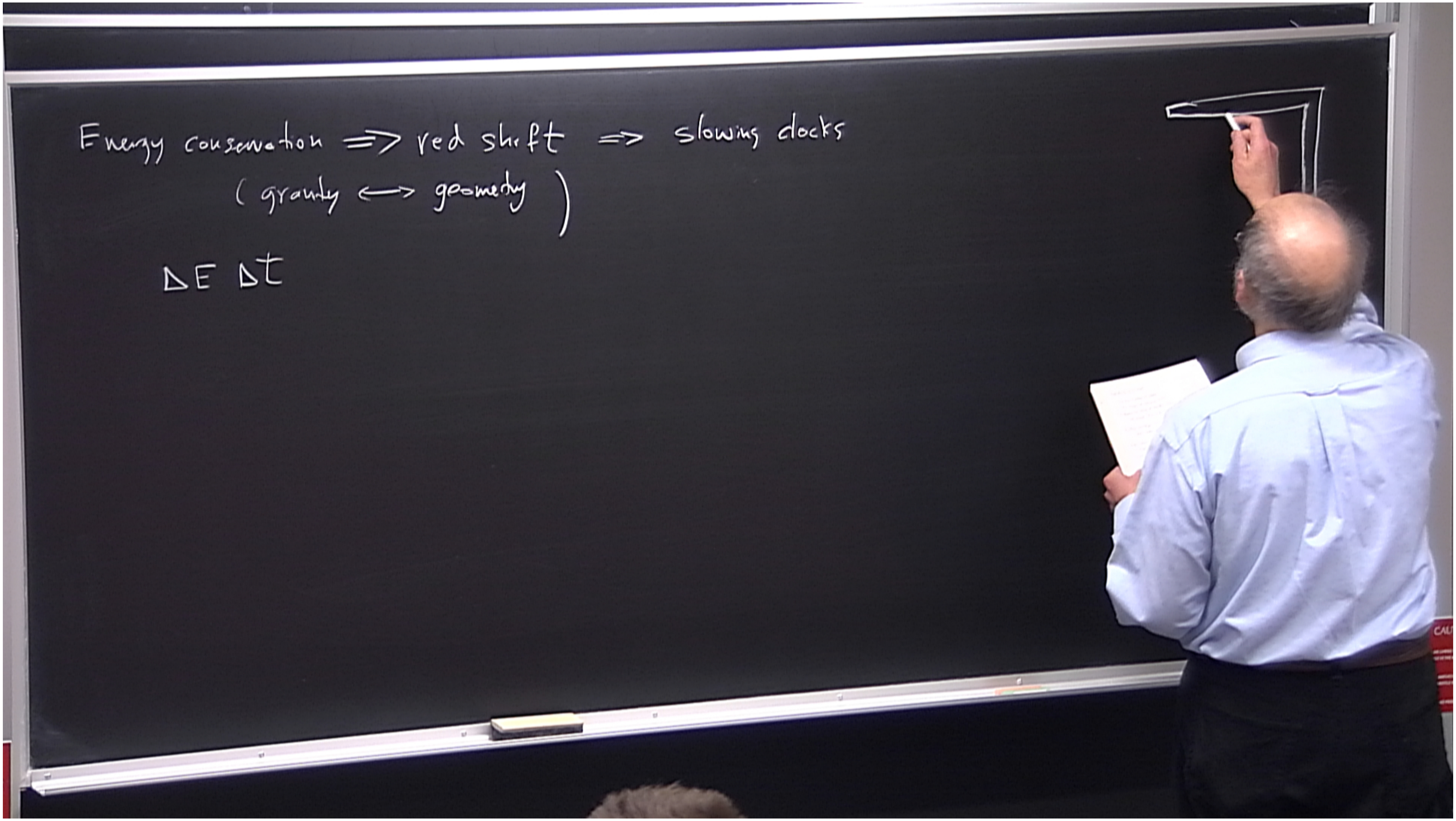
$$-\frac{\delta\Phi}{c^2} \approx \frac{\delta\omega}{\omega}$$

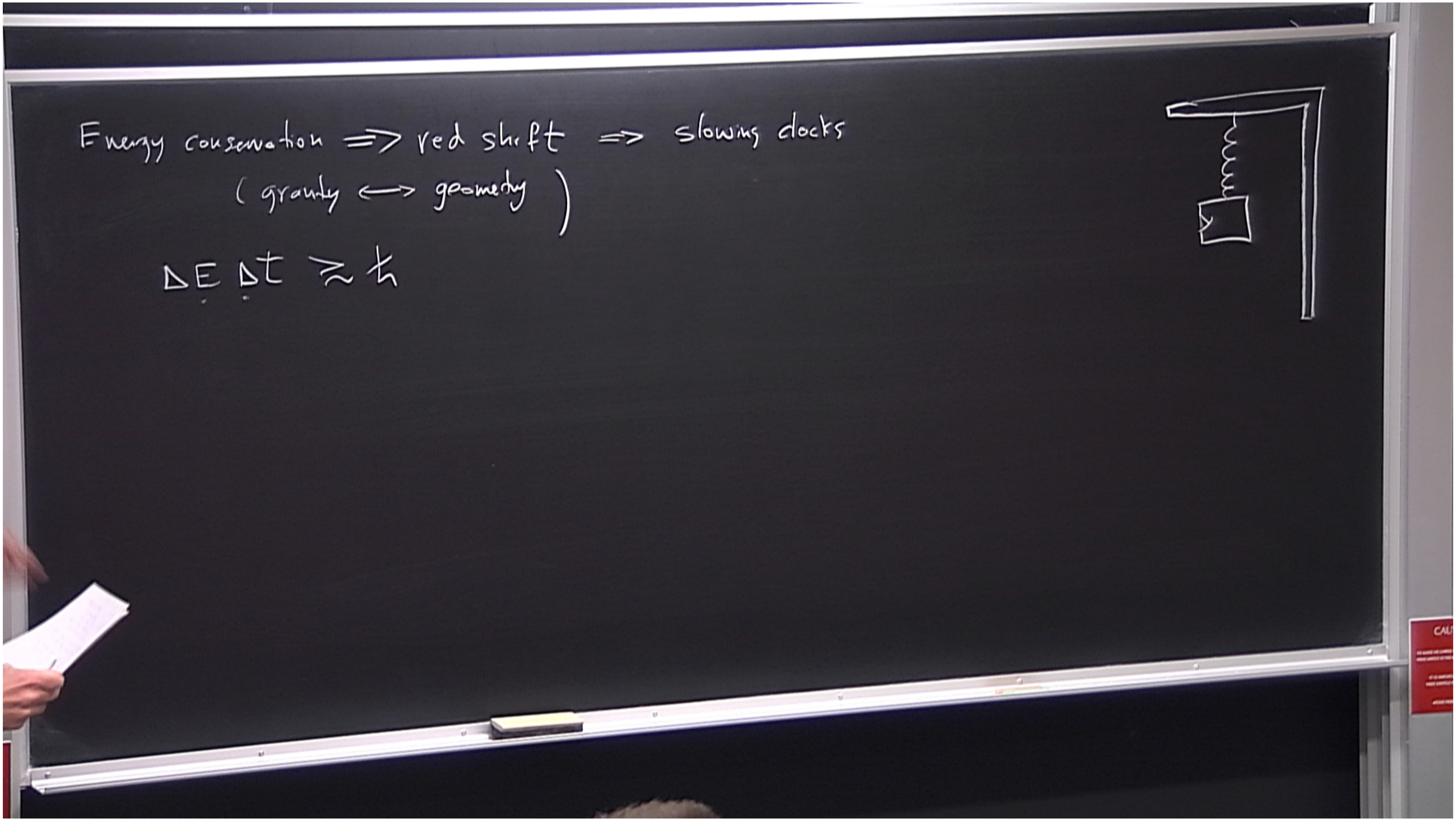


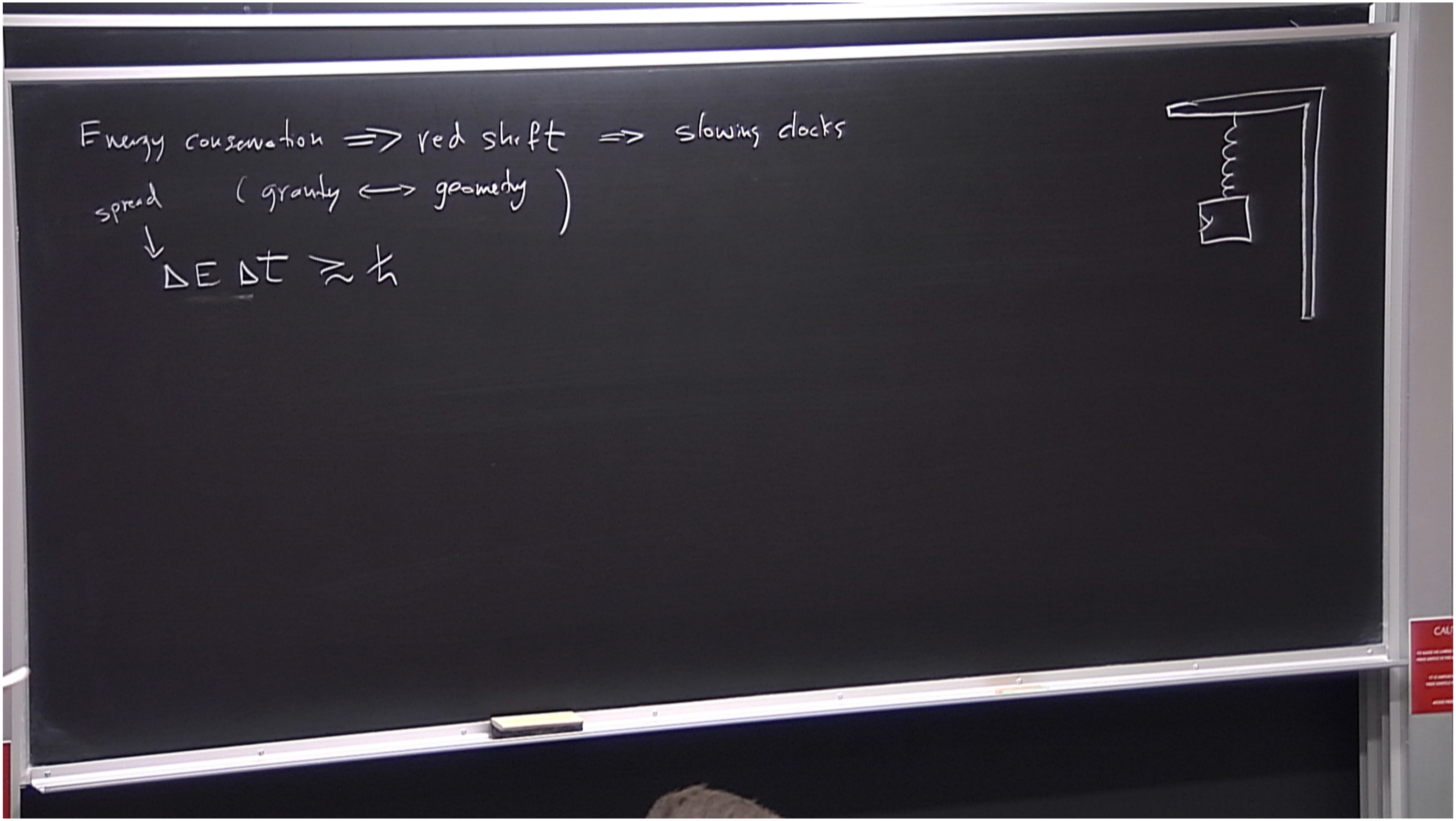
Energy conservation $\Rightarrow$ red shift $\Rightarrow$ slowing clocks

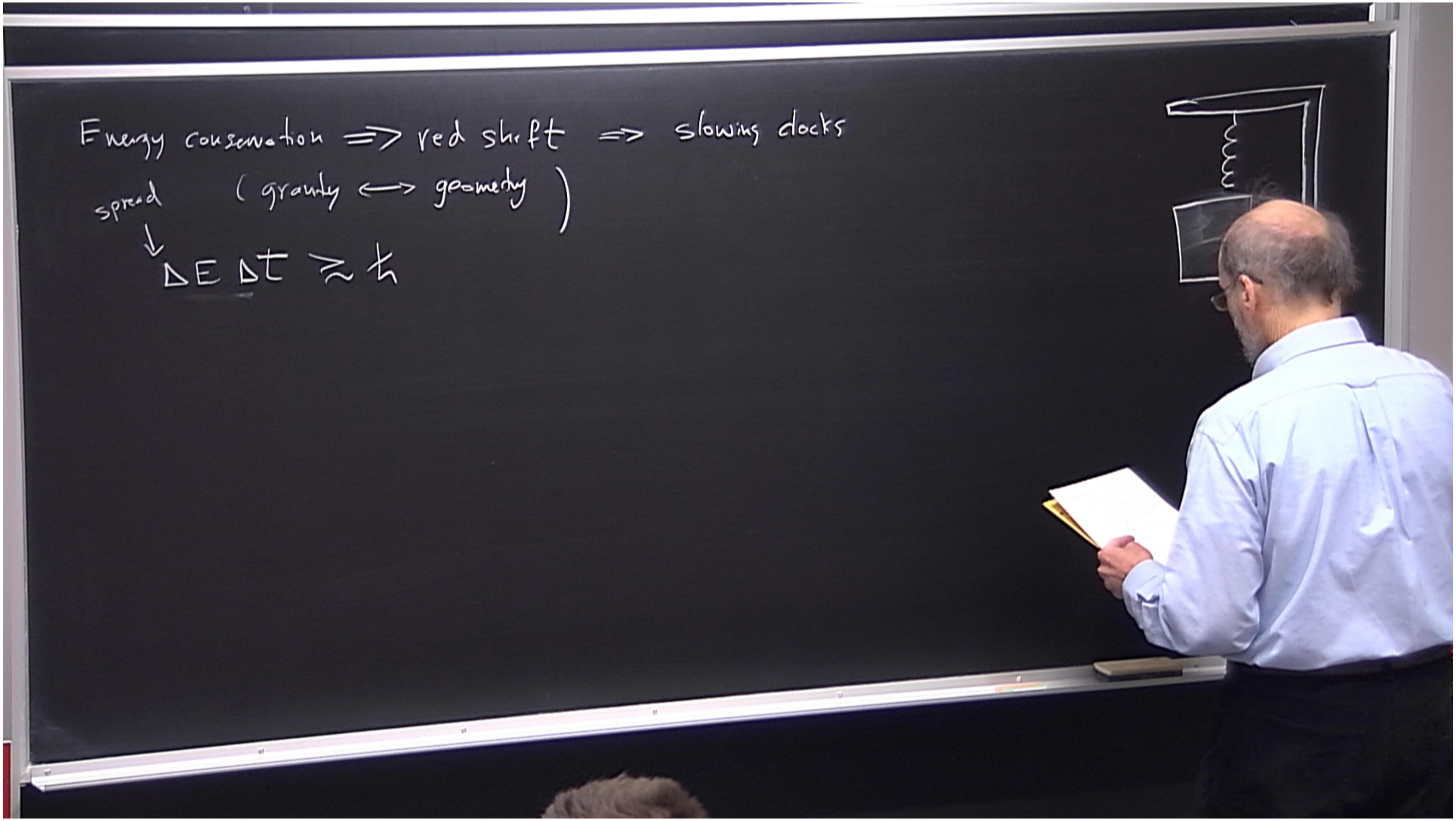
Energy conservation $\Rightarrow$ red shift $\Rightarrow$ slowing clocks

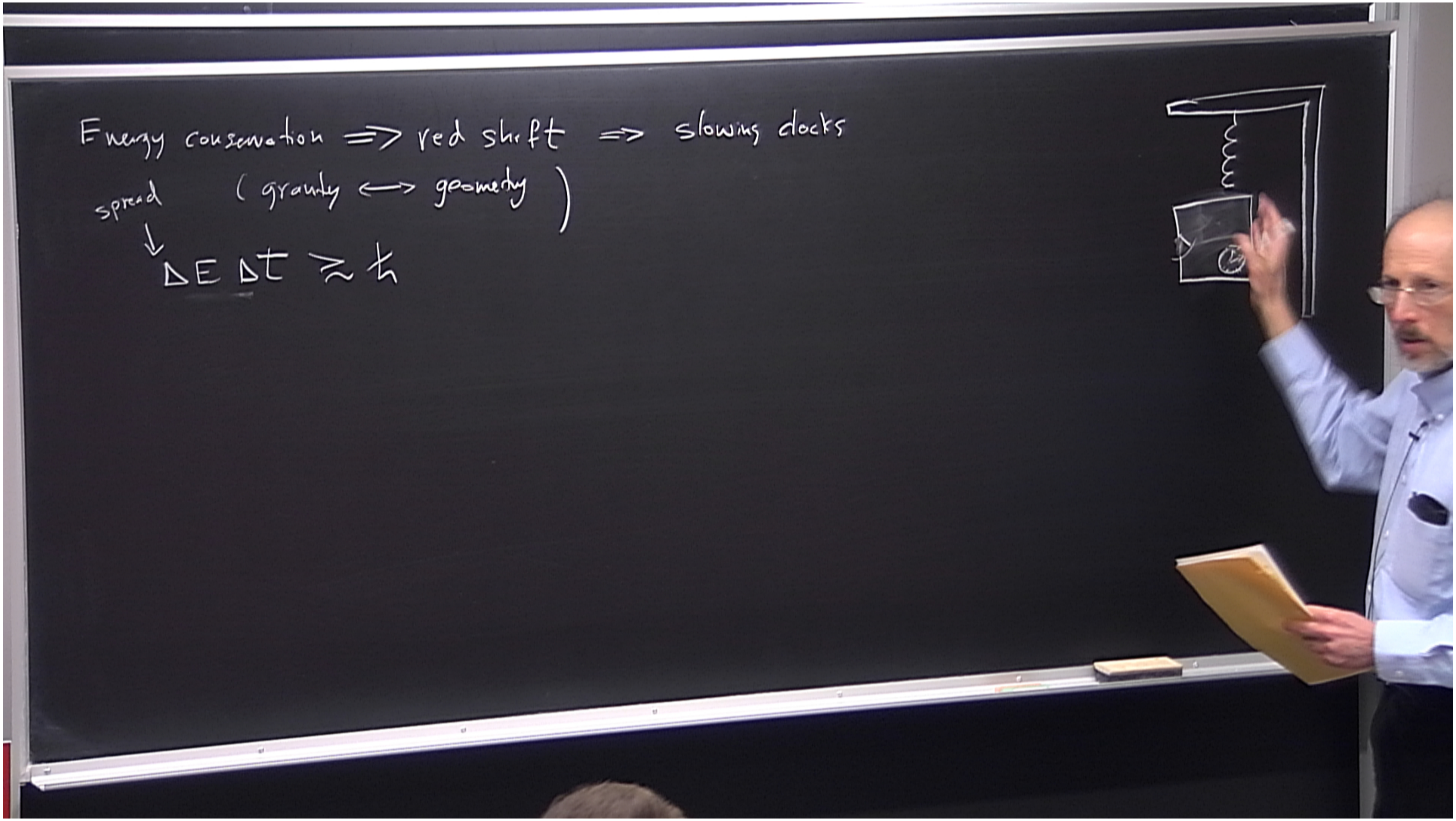
(gravity $\longleftrightarrow$ geometry)

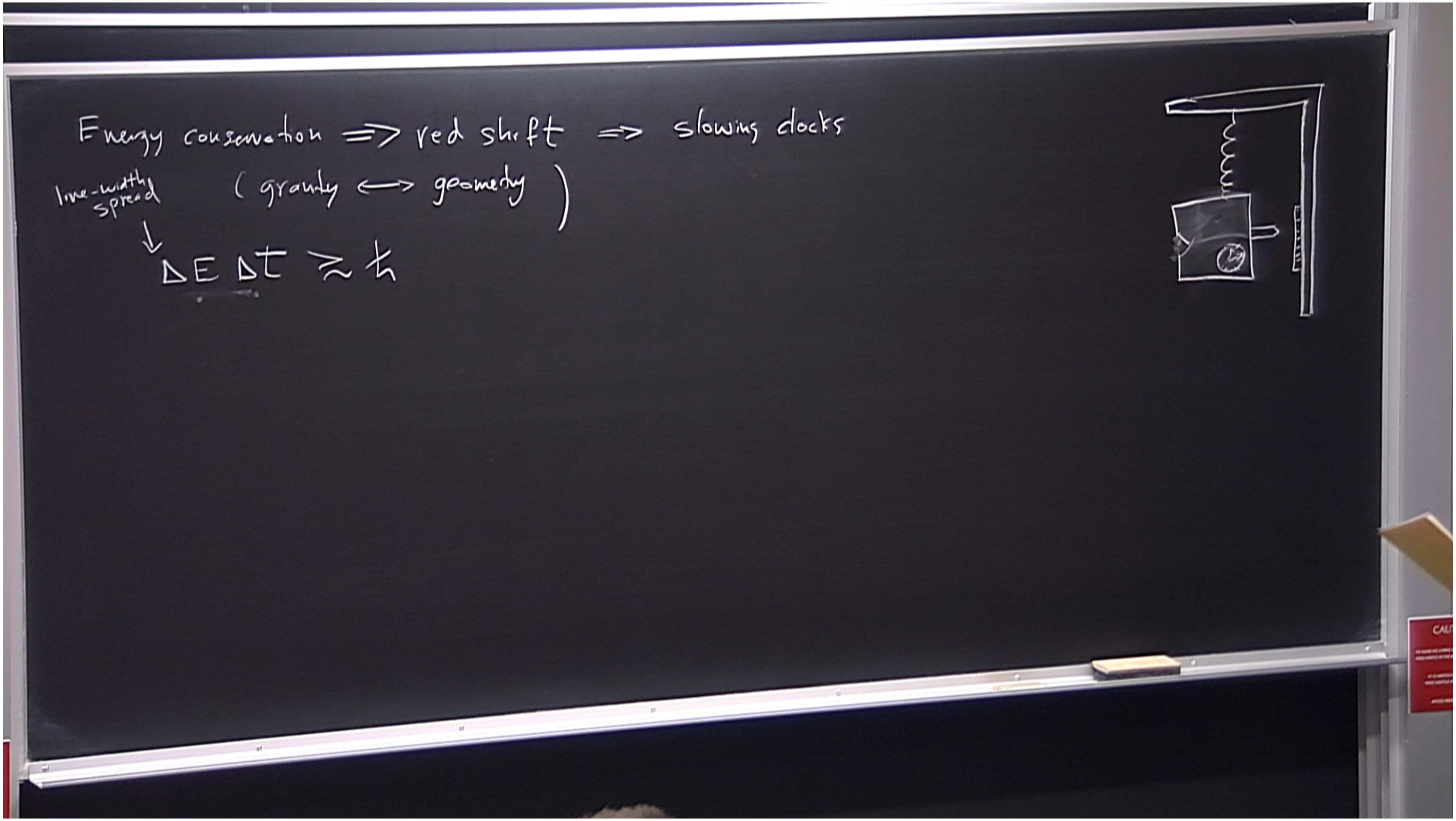

Energy conservation ⇒ red shift ⇒ slowing clocks
(gravity ⟷ geometry)
ΔE Δt


Energy conservation ⇒ red shift ⇒ slowing clocks
(gravity ⟷ geometry)
$\Delta E \, \Delta t \gtrsim \hbar$

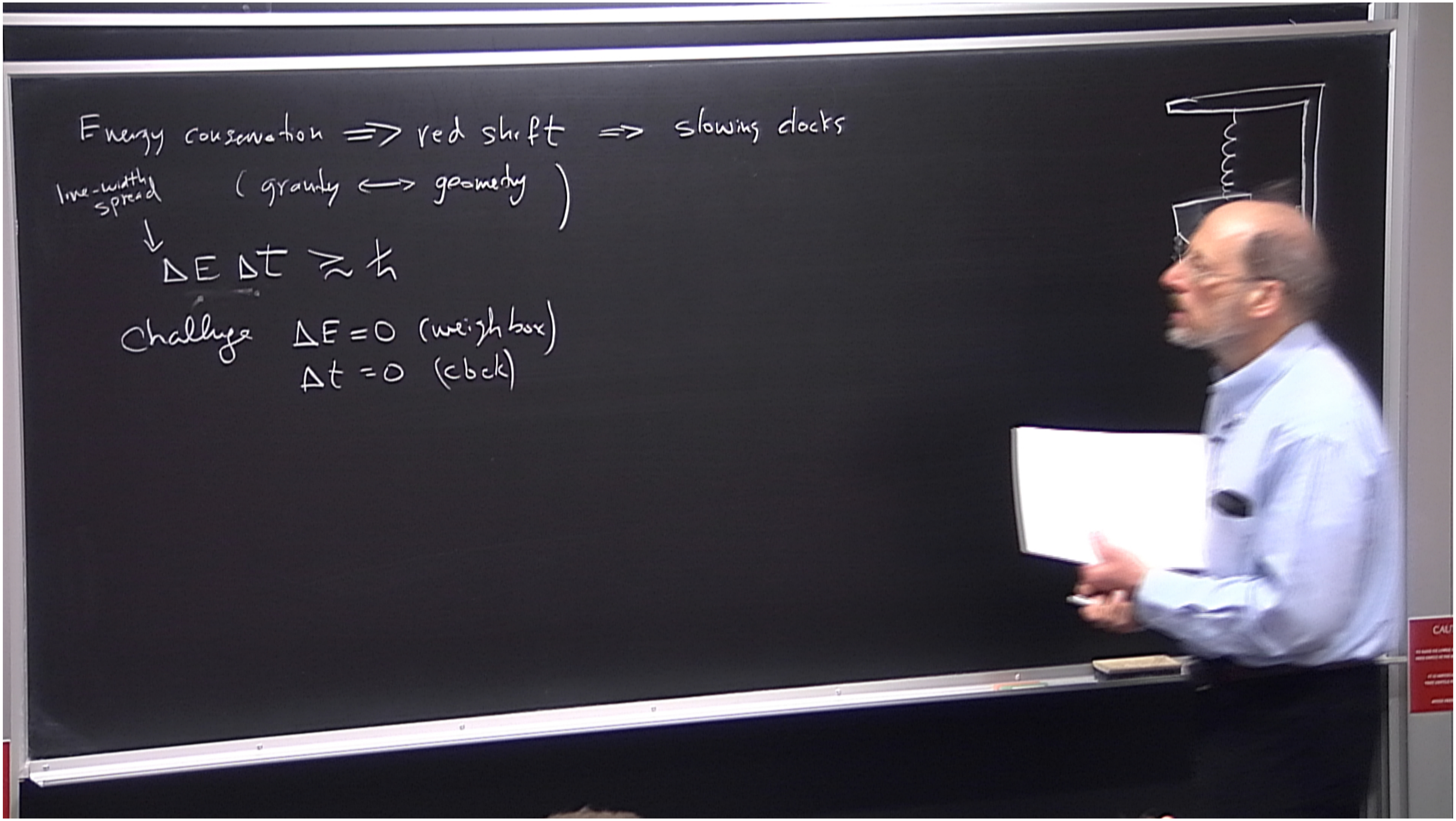

Energy conservation $\Rightarrow$ red shift $\Rightarrow$ slowing clocks
(gravity $\longleftrightarrow$ geometry)
spread
$\Delta E \, \Delta t \gtrsim \hbar$

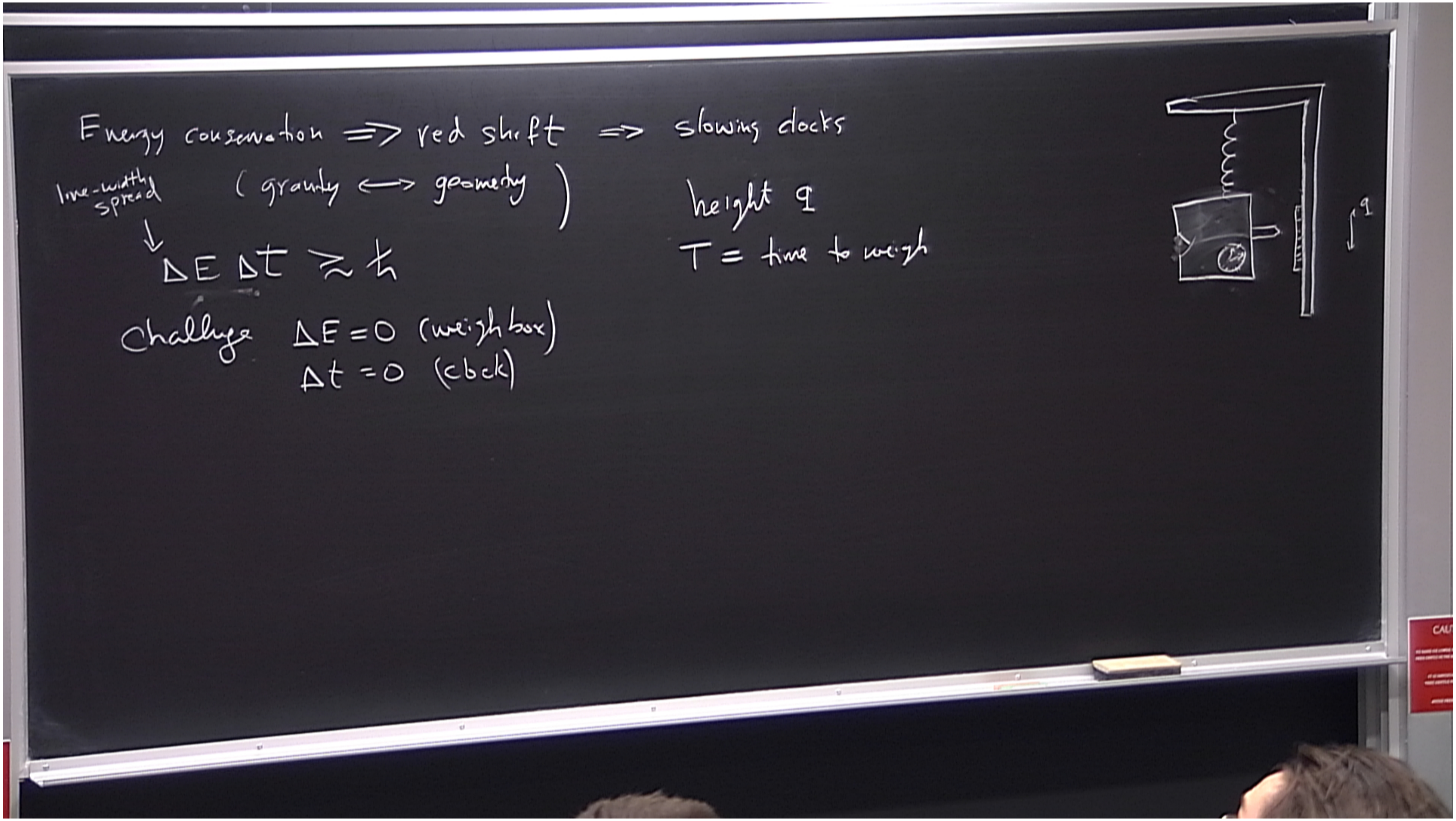

Energy conservation ⇒ red shift ⇒ slowing clocks
spread
(gravity ⟷ geometry)
$\Delta E \, \Delta t \gtrsim \hbar$

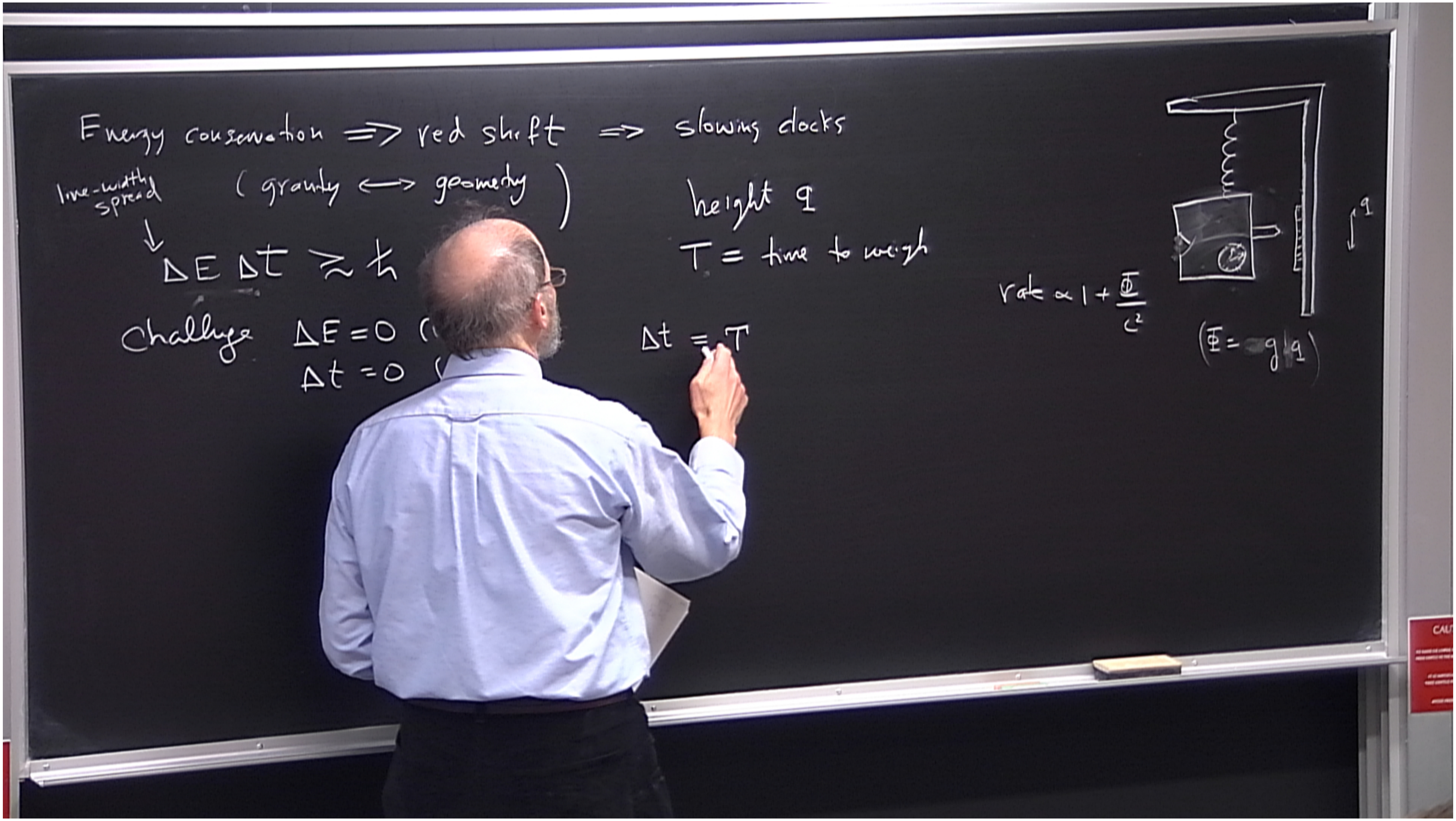

Energy conservation ⇒ red shift ⇒ slowing clocks
(gravity ⟷ geometry)
spread
$\Delta E \, \Delta t \gtrsim \hbar$

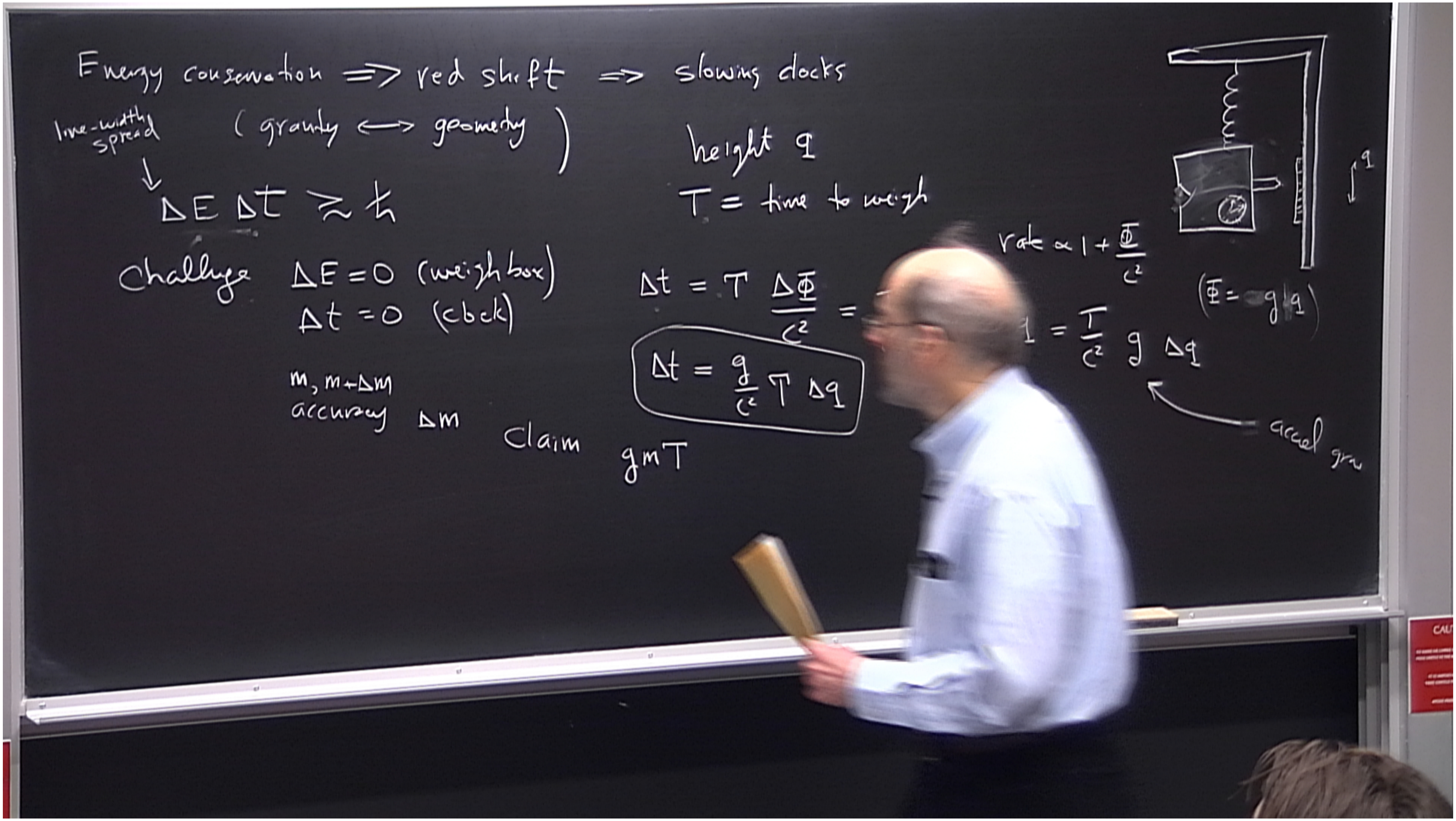


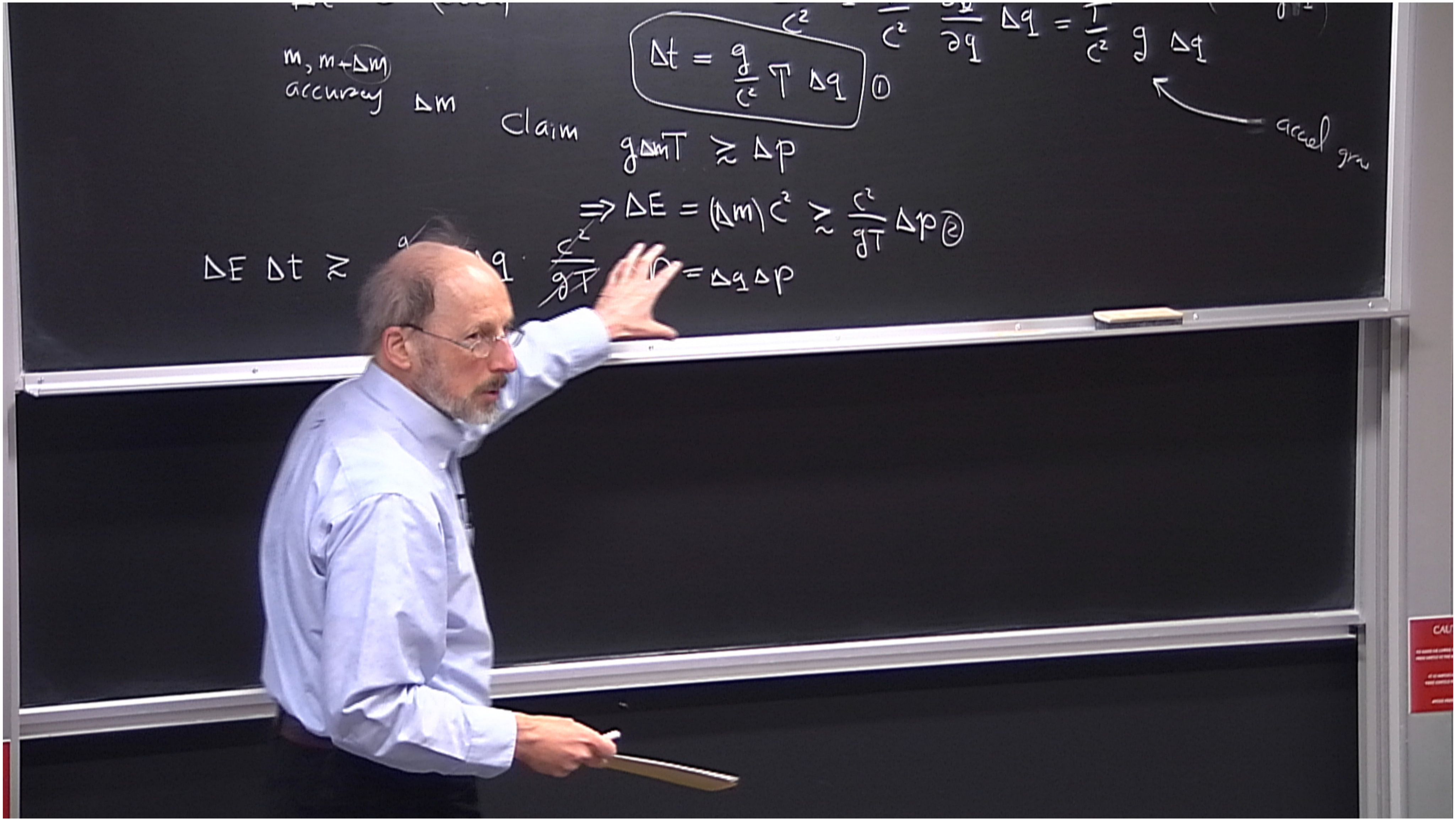
Energy conservation ⇒ red shift ⇒ slowing clocks
line-width spread
(gravity ⟷ geometry)
ΔE Δt ≳ ħ


Energy conservation => red shift => slowing clocks
line-width spread
(gravity <-> geometry)
ΔE Δt ≳ ħ
Challenge ΔE = 0 (weigh box)
Δt = 0 (clock)


Energy conservation ⇒ red shift ⇒ slowing clocks
line-width spread
(gravity ⟷ geometry)
height q
ΔE Δt ≳ ħ
T = time to weigh
Challenge ΔE = 0 (weigh box)
Δt = 0 (clock)
q


Energy conservation => red shift => slowing clocks
line-width spread
(gravity <-> geometry)
height
T = time to weigh
ΔE Δt ≳ ħ
Challenge ΔE = 0
Δt = 0
Δt = T
rate ∝ 1 + Φ/c²


Energy conservation => red shift => slowing clocks
line-width spread
(gravity <-> geometry)
ΔE Δt ≳ ħ
height q
T = time to weigh
rate ∝ 1 + Φ/c²
Challenge ΔE = 0 (weigh box)
Δt = 0 (clock)
Δt = T ΔΦ/c² =
(Φ = g q)
= T/c² g Δq
Δt = g/c² T Δq
m, m+Δm
accuracy Δm
Claim
gmT
accel grav


m, m+Δm
accuracy Δm
Claim
gΔmT ≳ Δp
Δt = g/c² T Δq ①
accel grav
⇒ ΔE = (Δm)c² ≳ c²/gT Δp ②
ΔE Δt ≳
= ΔqΔp

line-width spread

$$\Delta E\,\Delta t \gtrsim \hbar$$

(gravity $\longleftrightarrow$ geometry)

Challenge $\Delta E = 0$ (weigh box)

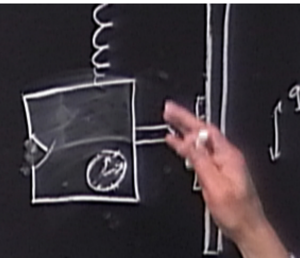
$\Delta t = 0$ (clock)

$m,\ m+\Delta m$

accuracy Δm

height q

T = time to weigh



rate $\propto 1 + \dfrac{\Phi}{c^2}$

$(\Phi = g\,q)$

$$\Delta t = T\,\frac{\Delta\Phi}{c^2} = \frac{T}{c^2}\,\frac{\partial\Phi}{\partial q}\,\Delta q = \frac{T}{c^2}\,g\,\Delta q$$

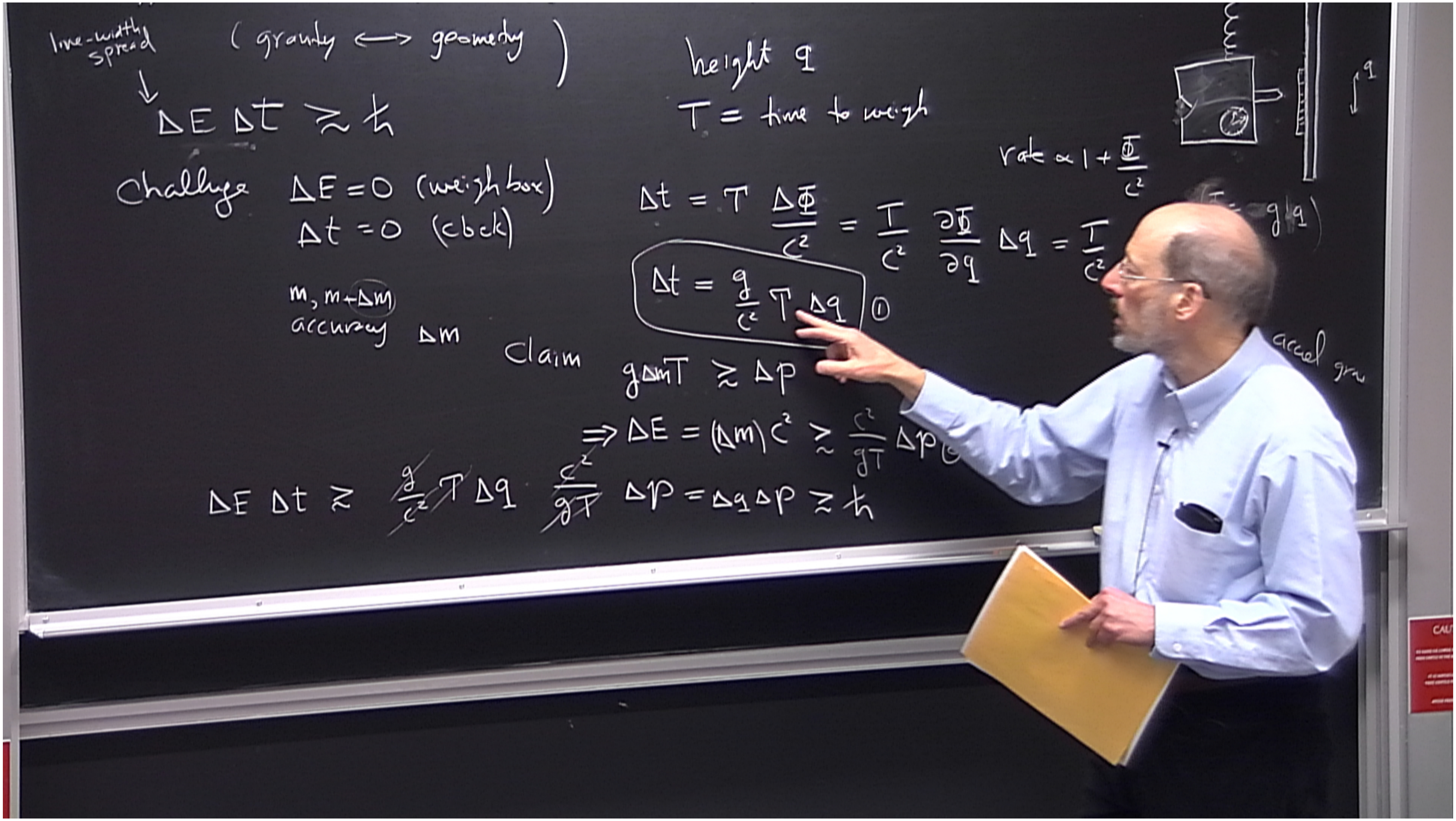
$$\boxed{\Delta t = \frac{g}{c^2}\,T\,\Delta q} \quad ①$$

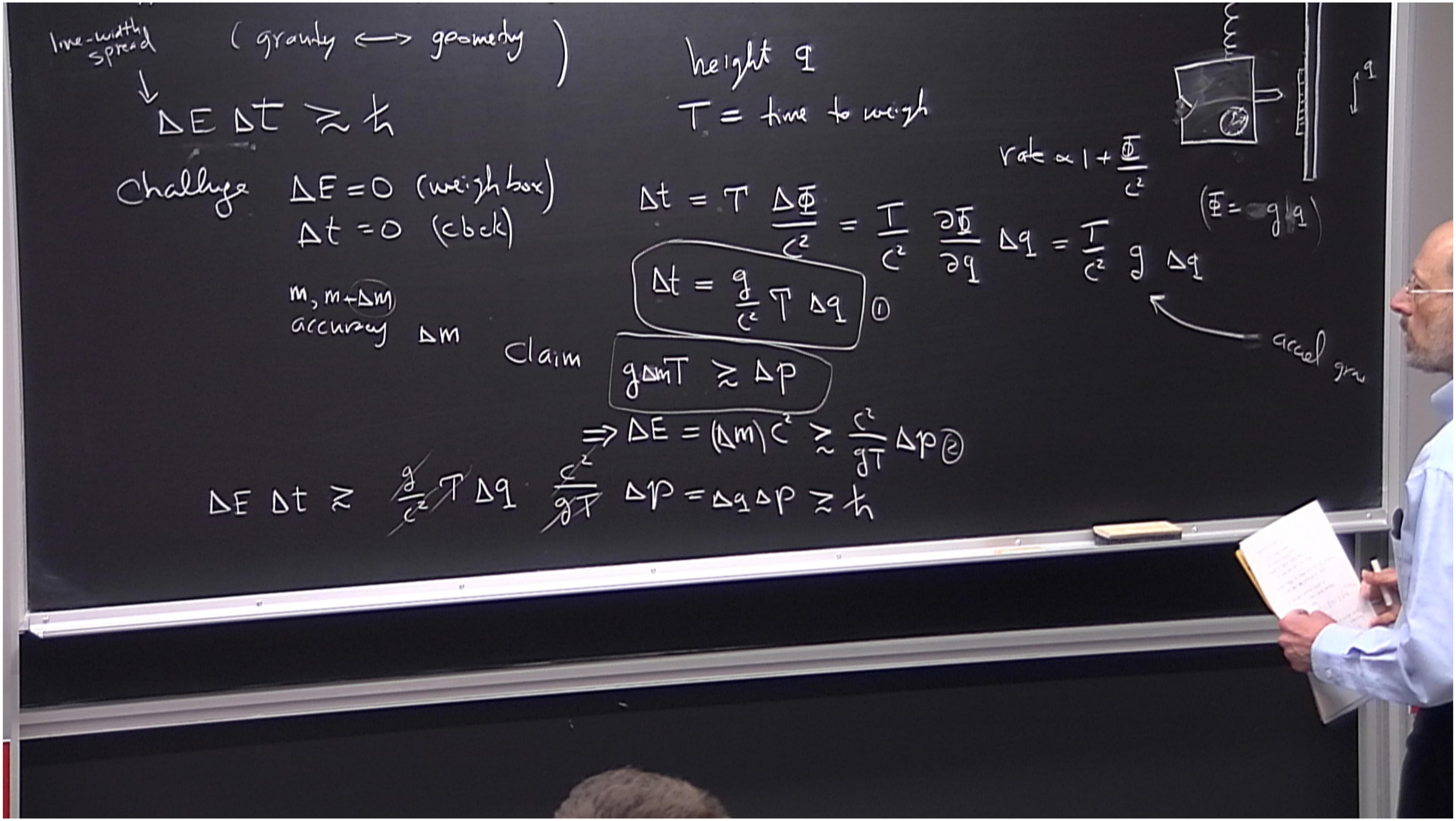
accel grav

Claim $g\Delta m T \gtrsim \Delta p$

$$\Rightarrow \Delta E = (\Delta m)c^2 \gtrsim \frac{c^2}{gT}\,\Delta p \quad ②$$

$$\Delta E\,\Delta t \gtrsim \cancel{\frac{g}{c^2}}\,\cancel{T}\,\Delta q \cdot \frac{\cancel{c^2}}{\cancel{gT}}\,\Delta p = \Delta q\,\Delta p \gtrsim \hbar$$


line-width spread
(gravity ⟷ geometry)
ΔE Δt ≳ ħ
height q
T = time to weigh
rate ∝ 1 + Φ/c²
Challenge ΔE = 0 (weigh box)
Δt = 0 (clock)
Δt = T ΔΦ/c² = T/c² ∂Φ/∂q Δq = T/c²
Δt = g/c² T Δq ①
m, m+Δm
accuracy Δm
Claim gΔmT ≳ Δp
⇒ ΔE = (Δm)c² ≳ c²/gT Δp
ΔE Δt ≳ g/c² T Δq · c²/gT Δp = Δq Δp ≳ ħ
accel grav


line-width spread
(gravity $\longleftrightarrow$ geometry)
$\Delta E\,\Delta t \gtrsim \hbar$
height q
T = time to weigh
rate $\propto 1 + \frac{\Phi}{c^2}$
$(\Phi = g\,q)$
q
Challenge $\Delta E = 0$ (weigh box)
$\Delta t = 0$ (clock)
$\Delta t = T\,\frac{\Delta\Phi}{c^2} = \frac{T}{c^2}\,\frac{\partial\Phi}{\partial q}\,\Delta q = \frac{T}{c^2}\,g\,\Delta q$
accel grav
$m,\ m+\Delta m$
accuracy Δm
$\boxed{\Delta t = \frac{g}{c^2}\,T\,\Delta q}$ ①
claim $\boxed{g\Delta m T \gtrsim \Delta p}$
$\Rightarrow \Delta E = (\Delta m)\,c^2 \gtrsim \frac{c^2}{gT}\,\Delta p$ ②
$\Delta E\,\Delta t \gtrsim \cancel{\frac{g}{c^2}}\,\cancel{T}\,\Delta q\ \cancel{\frac{c^2}{gT}}\,\Delta p = \Delta q\,\Delta p \gtrsim \hbar$

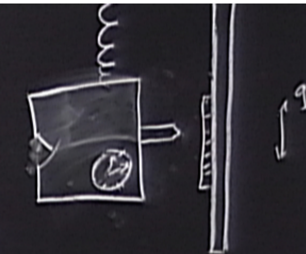
line-width spread

$$\Delta E\,\Delta t \gtrsim \hbar$$

(gravity $\longleftrightarrow$ geometry)

height q

T = time to weigh



rate $\propto 1 + \frac{\Phi}{c^2}$

Challenge $\Delta E = 0$ (weigh box)

$\Delta t = 0$ (clock)

$$\Delta t = T\,\frac{\Delta\Phi}{c^2} = \frac{T}{c^2}\,\frac{\partial\Phi}{\partial q}\,\Delta q = \frac{T}{c^2}\,g\,\Delta q$$

($\Phi = g\,q$)

accel grav

$$\boxed{\Delta t = \frac{g}{c^2}\,T\,\Delta q} \quad ①$$

$m,\ m+\Delta m$

accuracy Δm

Claim $\boxed{g\,\Delta m\,T \gtrsim \Delta p}$

$$\Rightarrow \Delta E = (\Delta m)c^2 \gtrsim \frac{c^2}{gT}\,\Delta p \quad ②$$

$$\Delta E\,\Delta t \gtrsim \cancel{\frac{g}{c^2}}\,\cancel{T}\,\Delta q\;\cancel{\frac{c^2}{gT}}\,\Delta p = \Delta q\,\Delta p \gtrsim \hbar$$

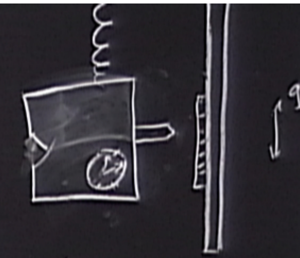
line-width spread

(gravity $\longleftrightarrow$ geometry)

$$\Delta E\,\Delta t \gtrsim \hbar$$

height q

T = time to weigh



rate $\propto 1 + \frac{\Phi}{c^2}$

Challenge $\Delta E = 0$ (weigh box)

$\Delta t = 0$ (clock)

$(\Phi = g q)$

$$\Delta t = T \frac{\Delta \Phi}{c^2} = \frac{T}{c^2} \frac{\partial \Phi}{\partial q} \Delta q = \frac{T}{c^2} g \Delta q$$

$$\boxed{\Delta t = \frac{g}{c^2} T \Delta q} \quad (1)$$

accel grav

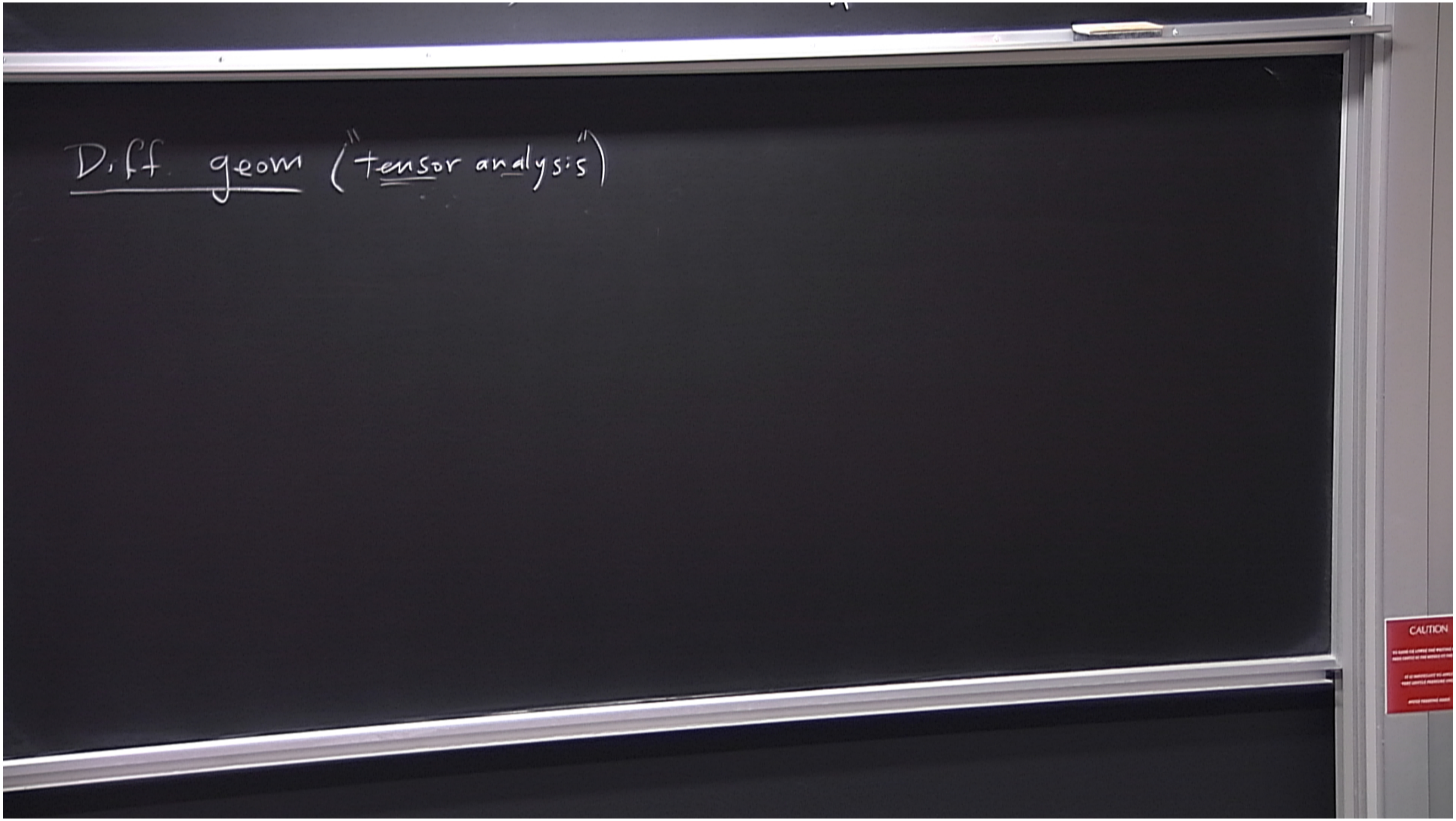
$m,\ m+\Delta m$

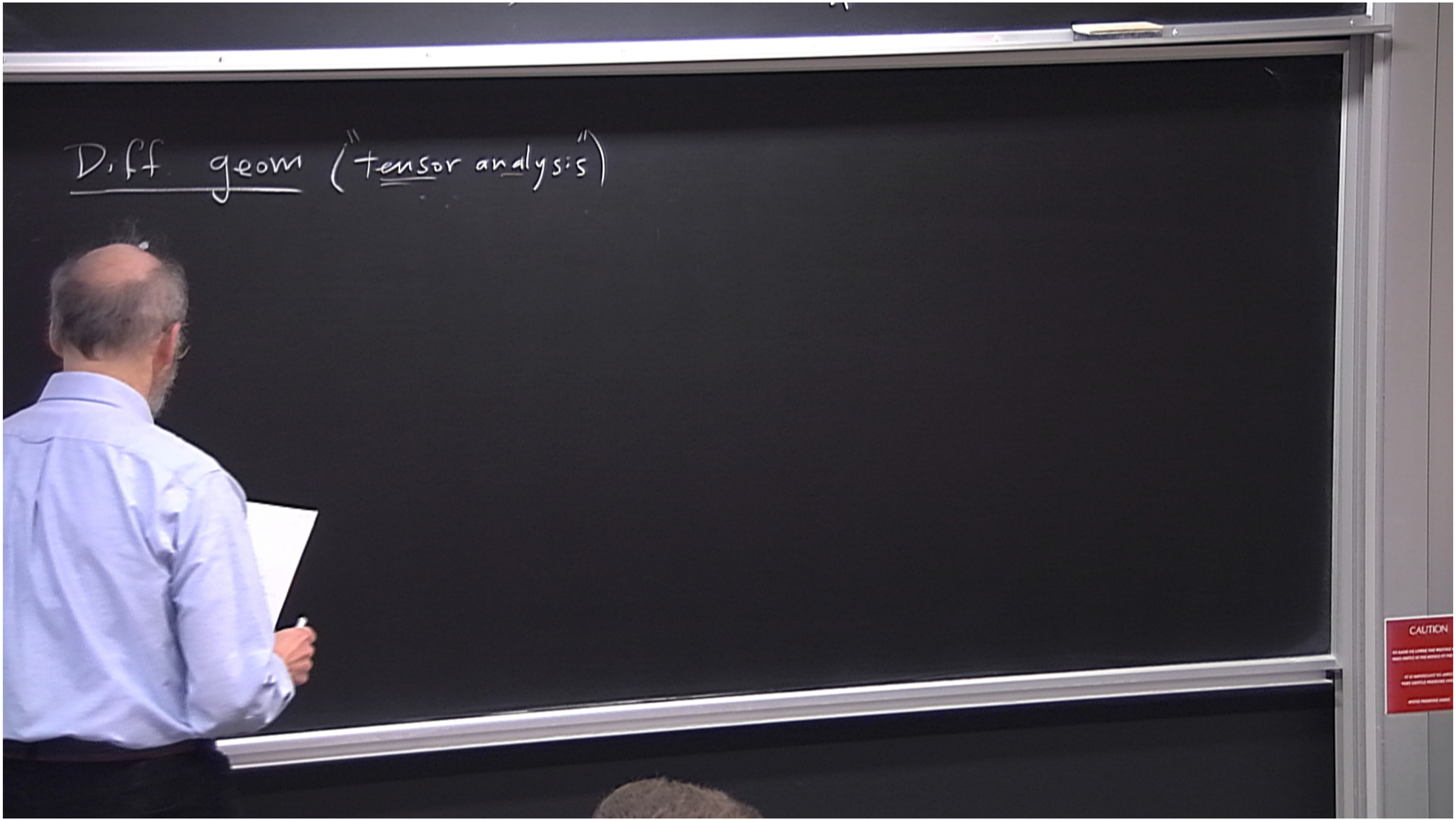
accuracy Δm

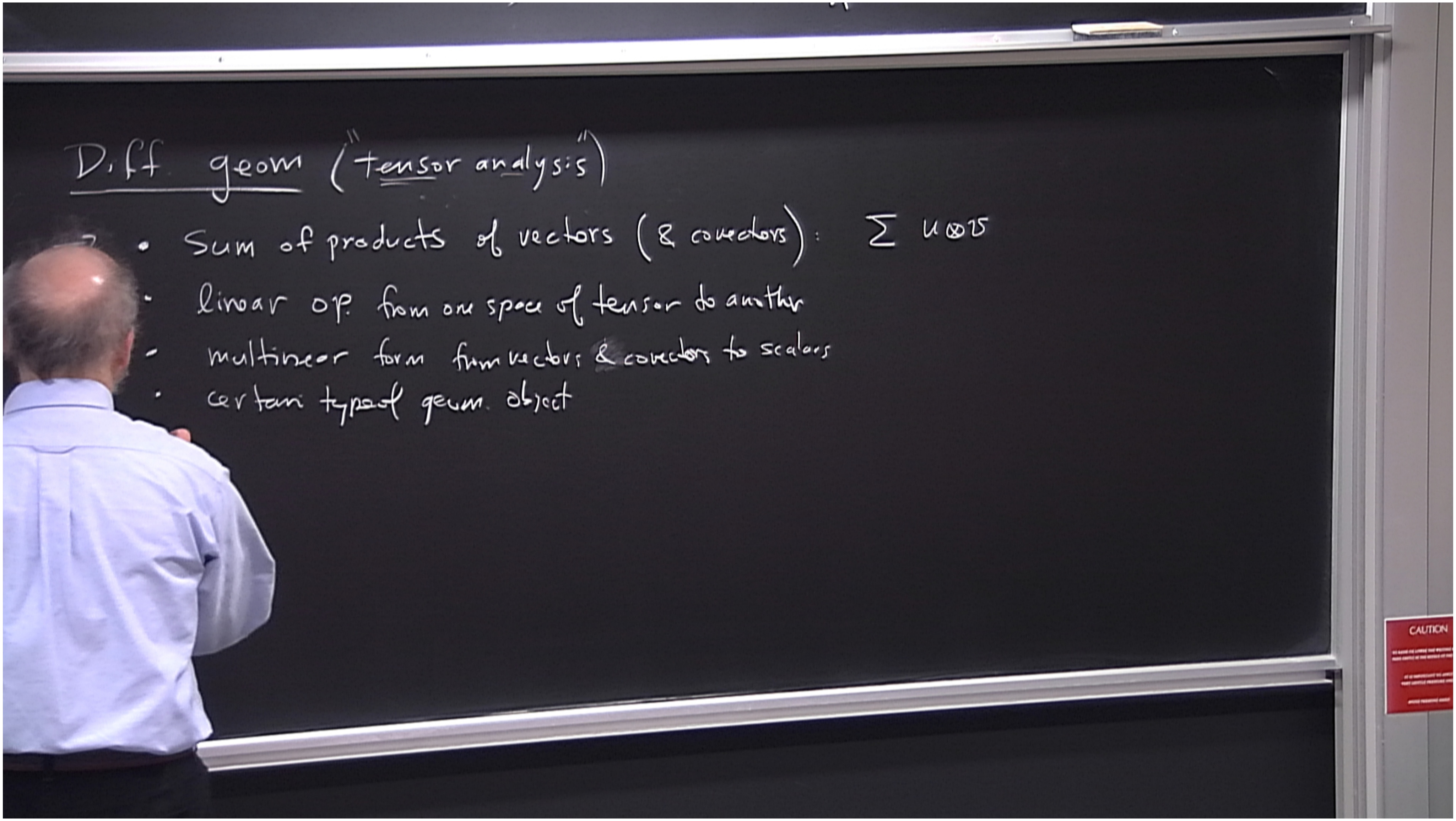
Claim $\boxed{g \Delta m T \gtrsim \Delta p}$

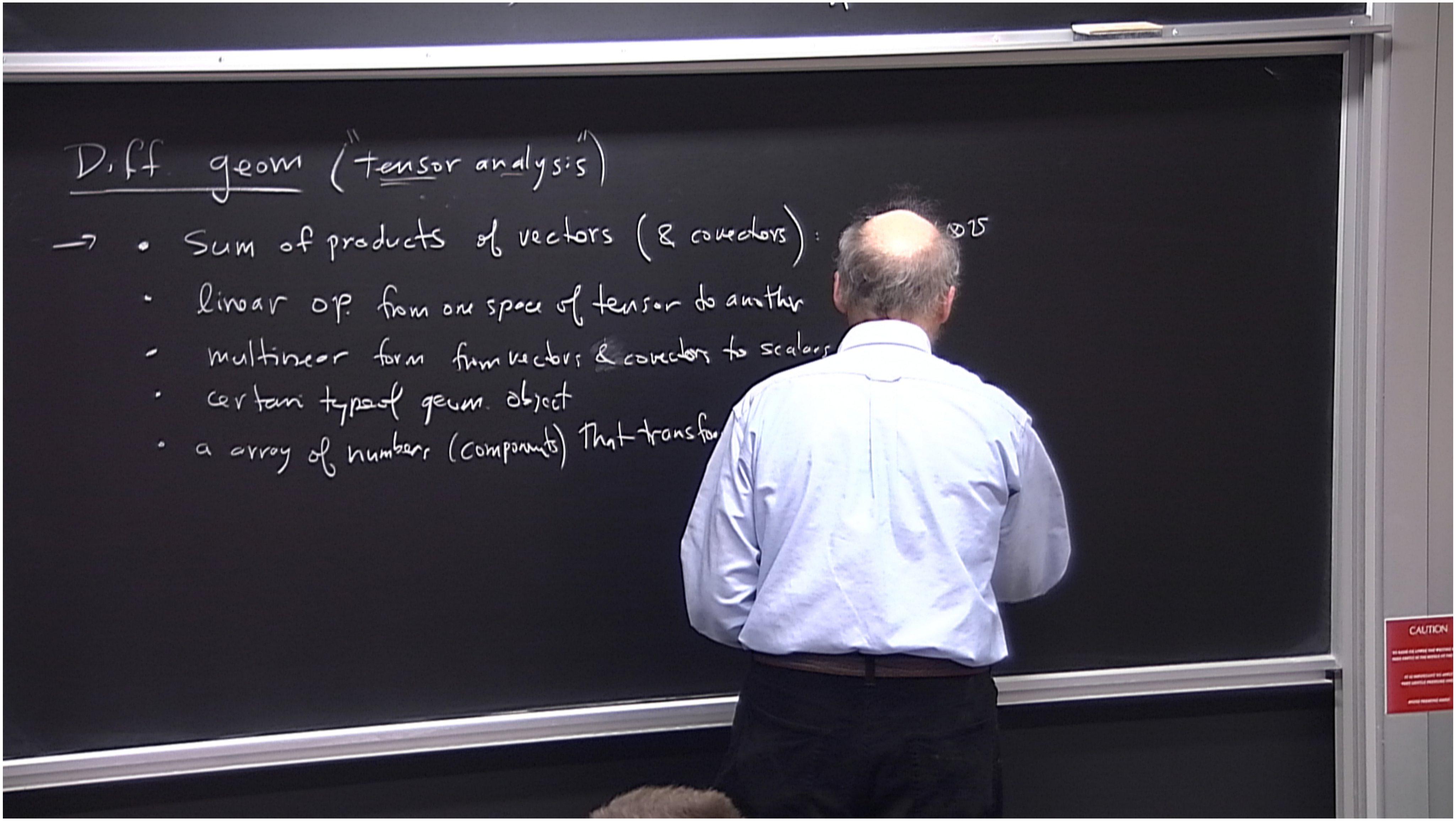
$$\Rightarrow \Delta E = (\Delta m) c^2 \gtrsim \frac{c^2}{gT} \Delta p \quad (2)$$

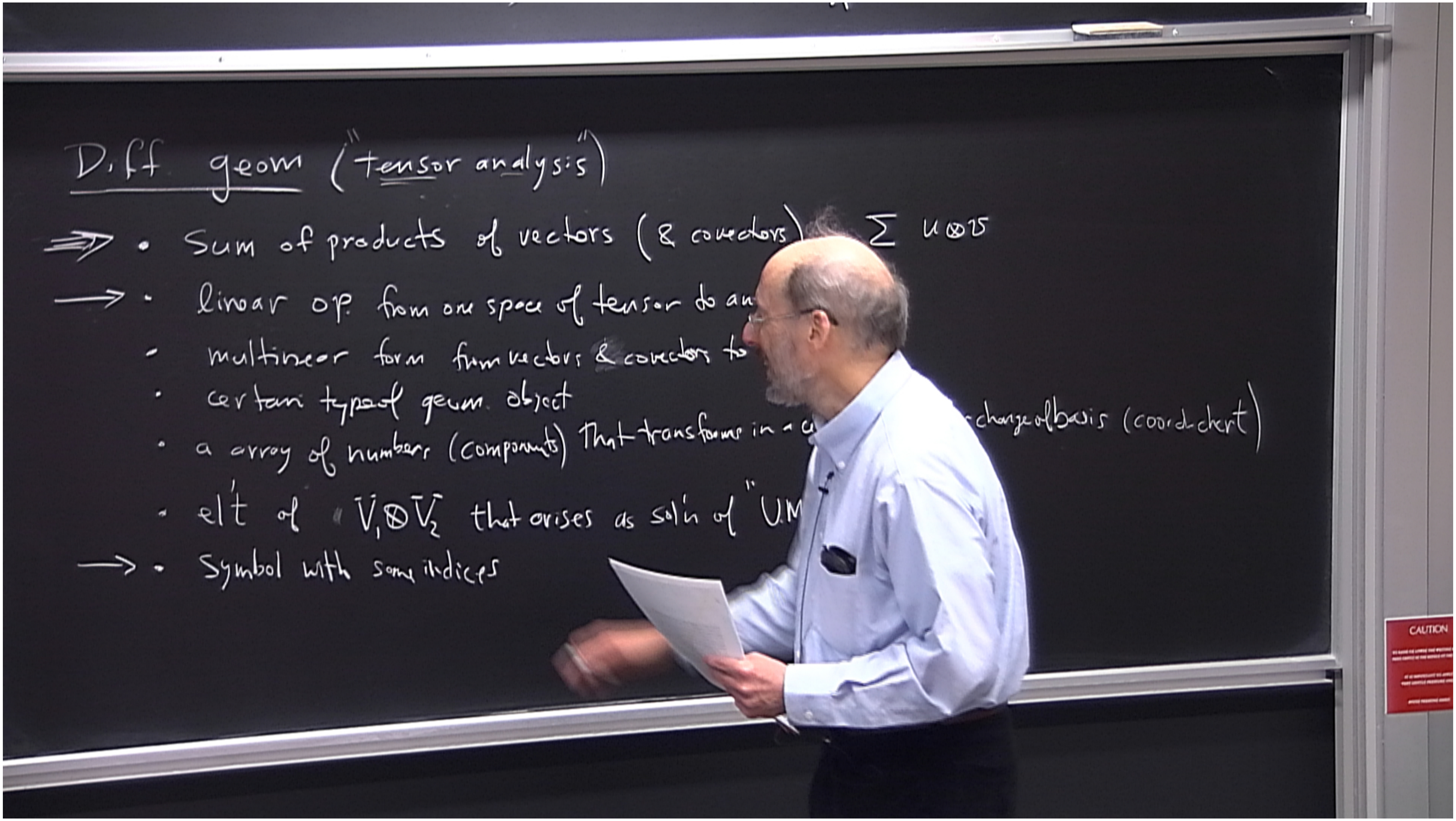
$$\Delta E\,\Delta t \gtrsim \cancel{\frac{g}{c^2}}\cancel{T} \Delta q \, \cancel{\frac{c^2}{gT}} \Delta p = \Delta q \Delta p \gtrsim \hbar$$

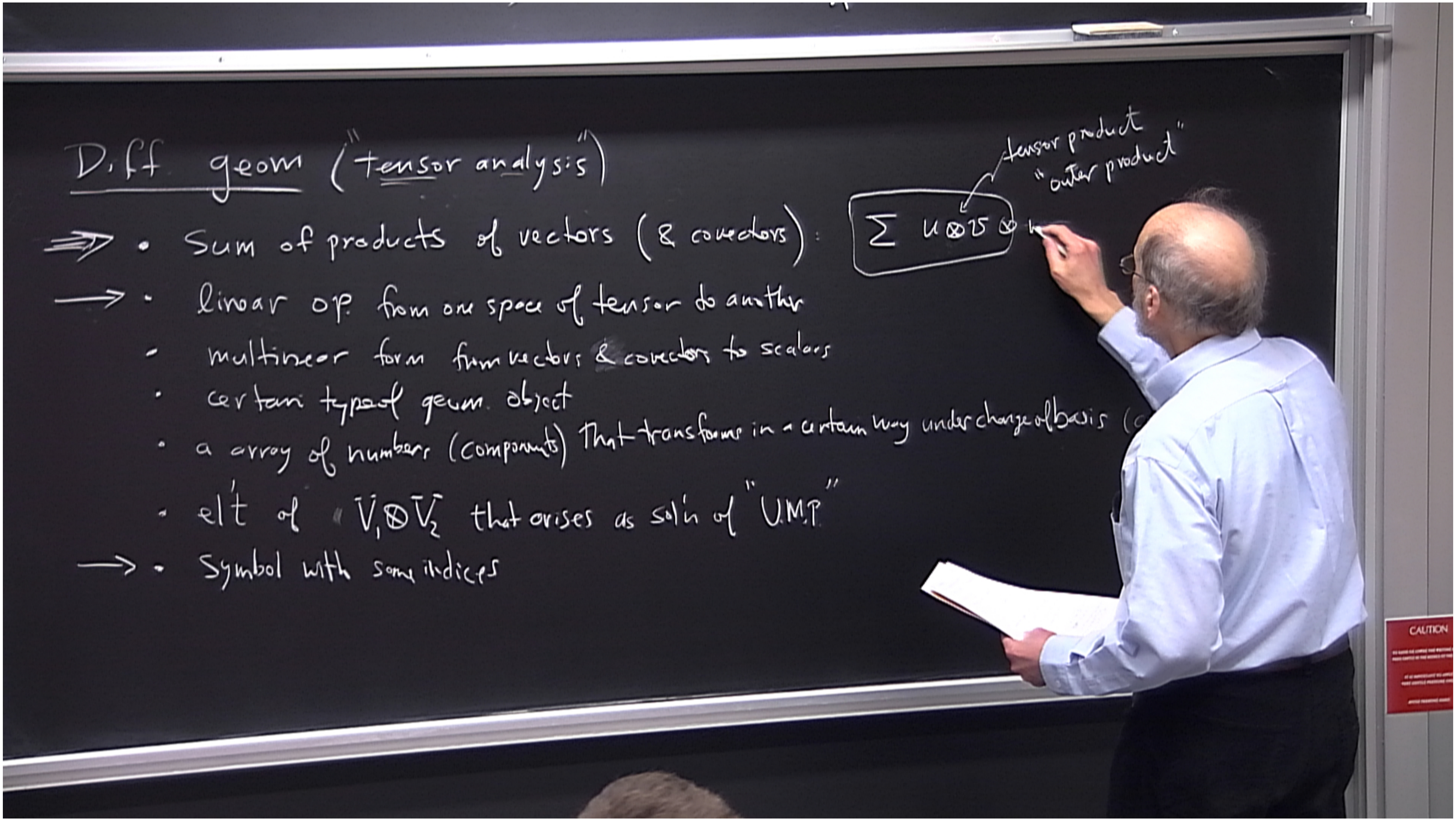

Diff. geom ("tensor analysis")

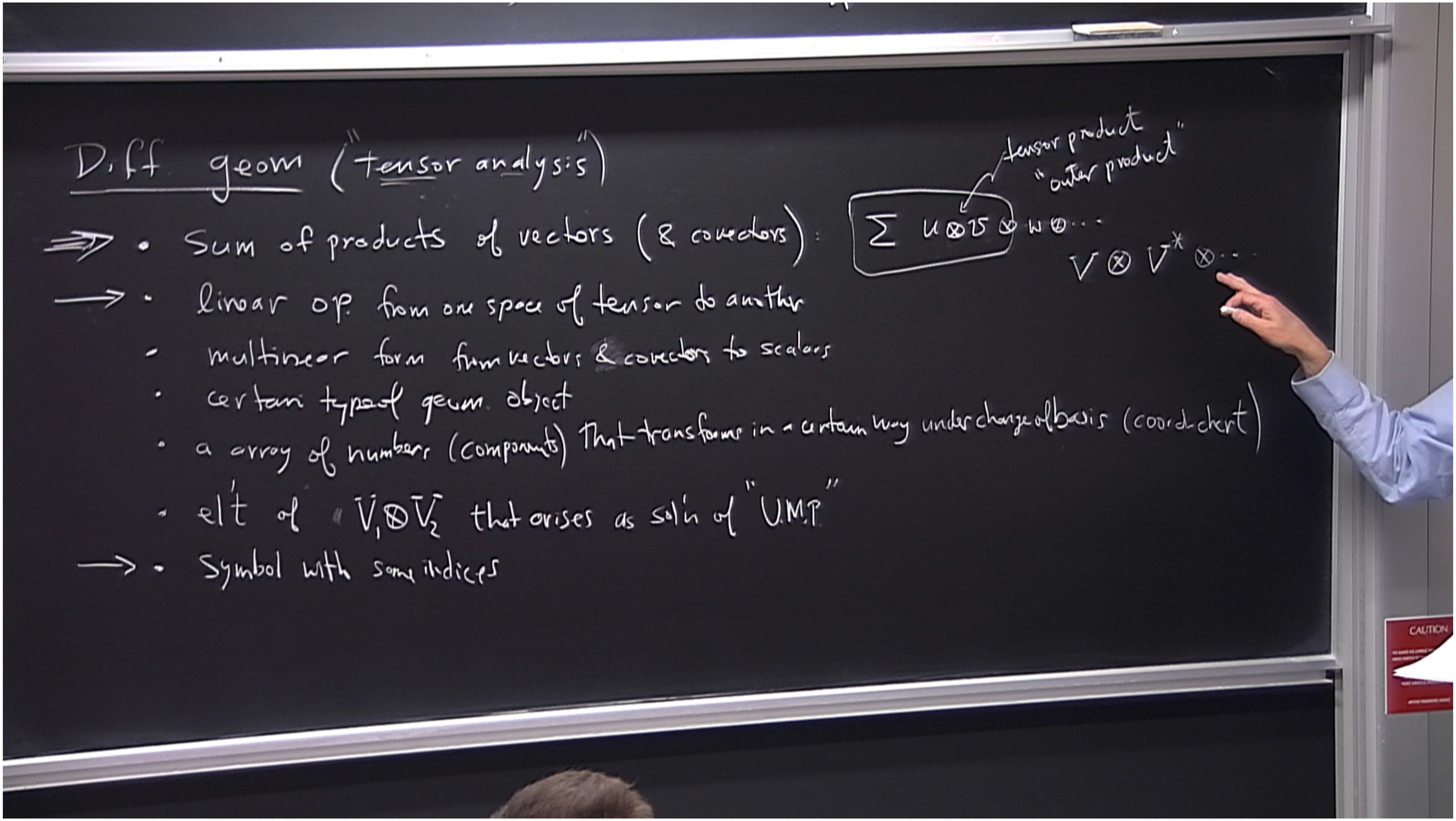

Diff geom ("tensor analysis")


Diff geom ("tensor analysis")
• Sum of products of vectors (& covectors): $\sum u \otimes v$
• linear op. from one space of tensor to another
• multilinear form from vectors & covectors to scalars
• certain type of geom. object
CAUTION

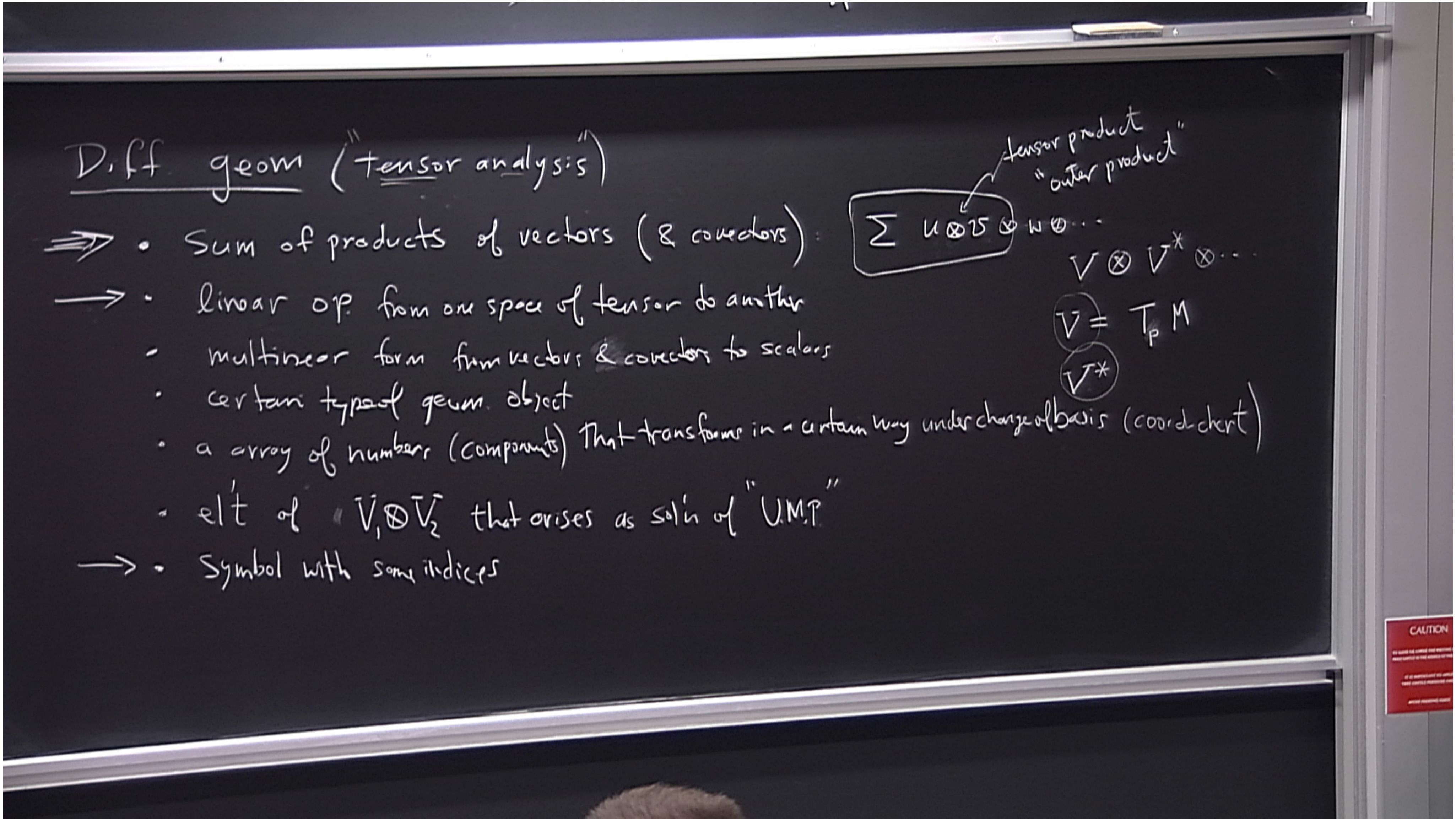

Diff geom ("tensor analysis")
→ • Sum of products of vectors (& covectors):
• linear op. from one space of tensor to another
• multilinear form from vectors & covectors to scalars
• certain type of geom. object
• a array of numbers (components) That transfo

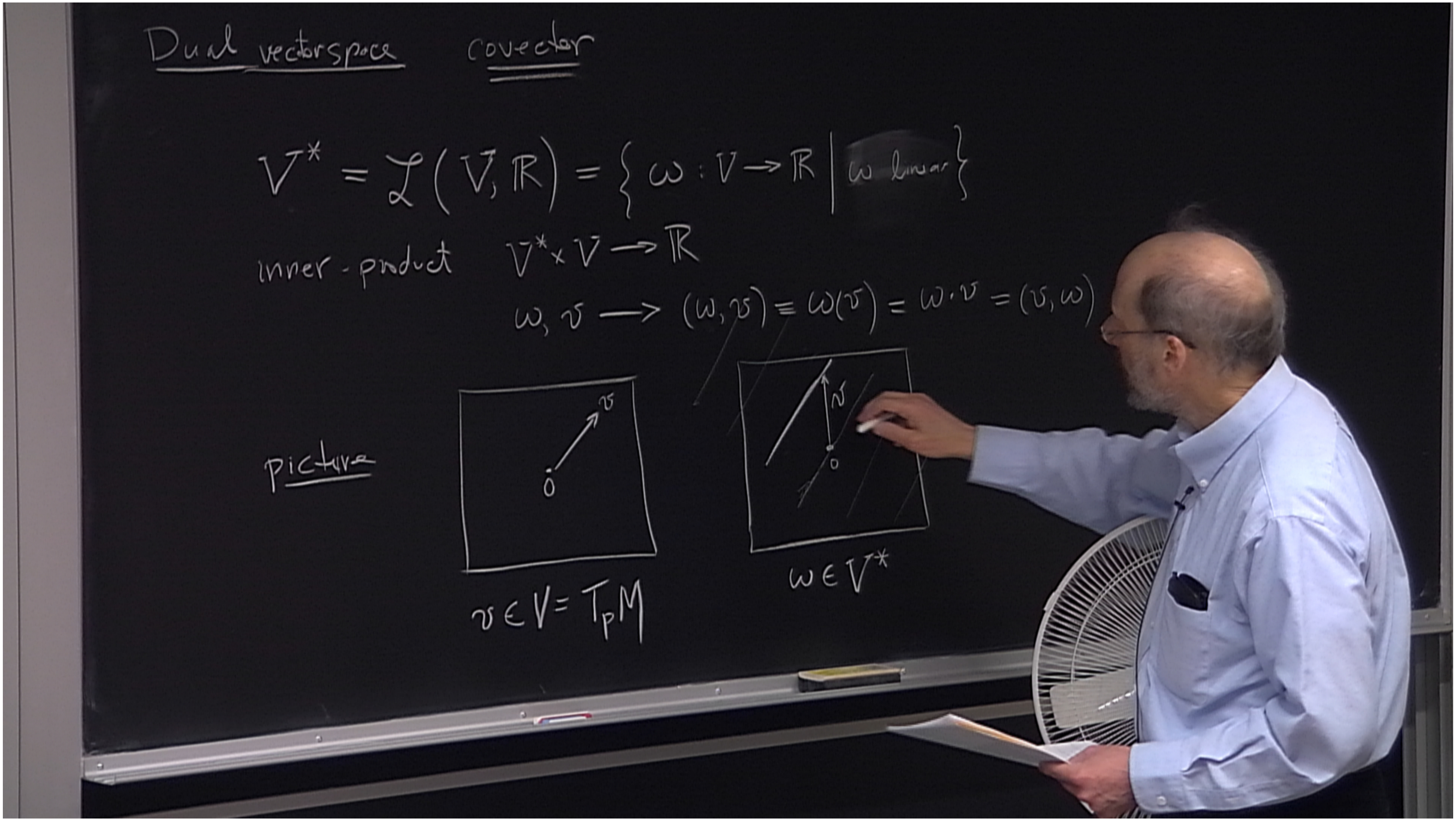

Diff geom ("tensor analysis")
• Sum of products of vectors (& covectors) Σ u⊗v
• linear op. from one space of tensor to an
• multilinear form from vectors & covectors to
• certain type of geom. object
• a array of numbers (components) that transforms in a
change of basis (coord. chart)
• el't of V₁⊗V₂ that arises as sol'n of "U.M
• Symbol with some indices
CAUTION

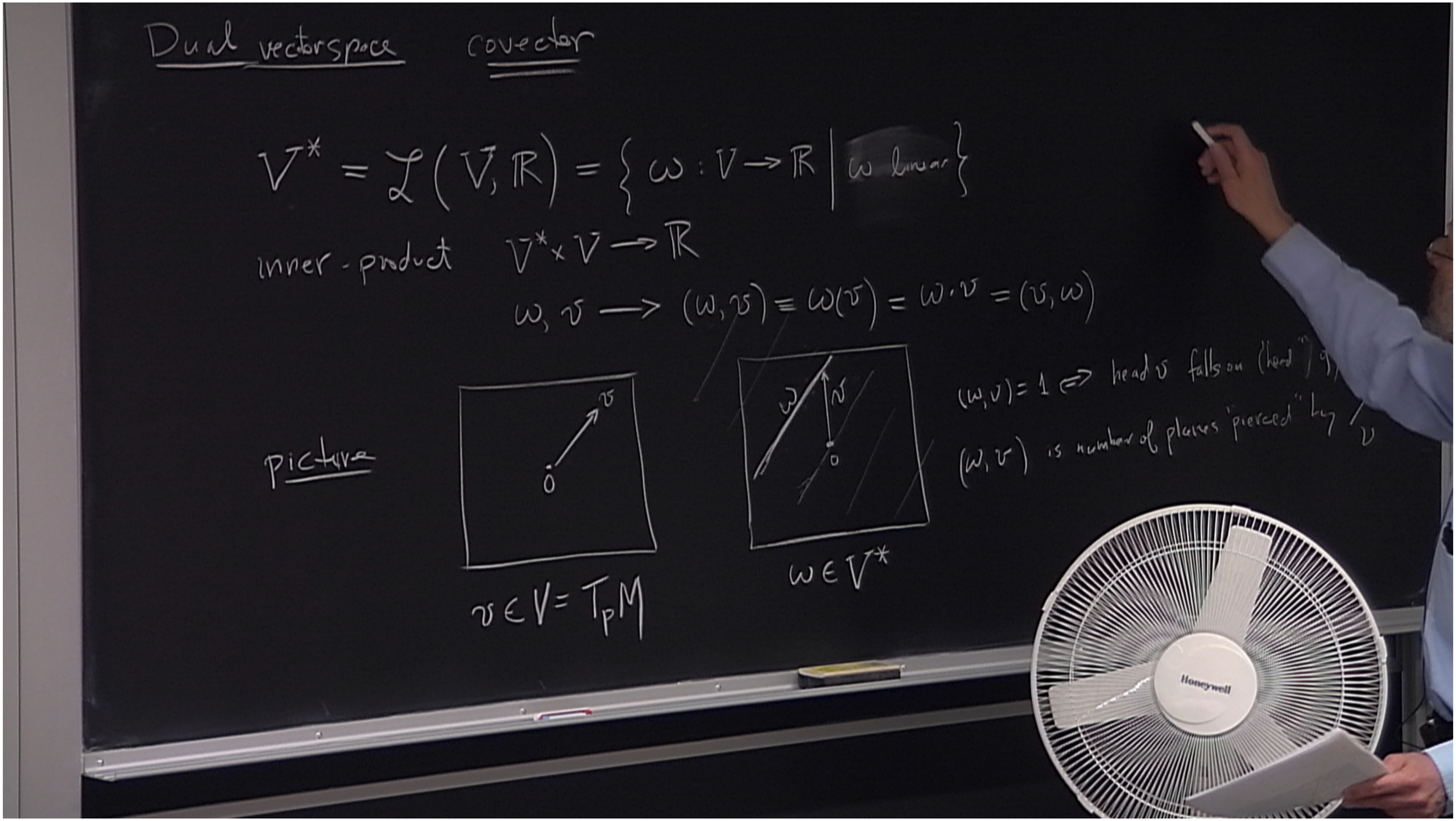

Diff geom ("tensor analysis")
tensor product
"outer product"
• Sum of products of vectors (& covectors): Σ u⊗v⊗
• linear op. from one space of tensor to another
• multilinear form from vectors & covectors to scalars
• certain type of geom. object
• a array of numbers (components) that transforms in a certain way under change of basis
• el't of V1⊗V2 that arises as sol'n of "U.M.P"
• Symbol with some indices
CAUTION

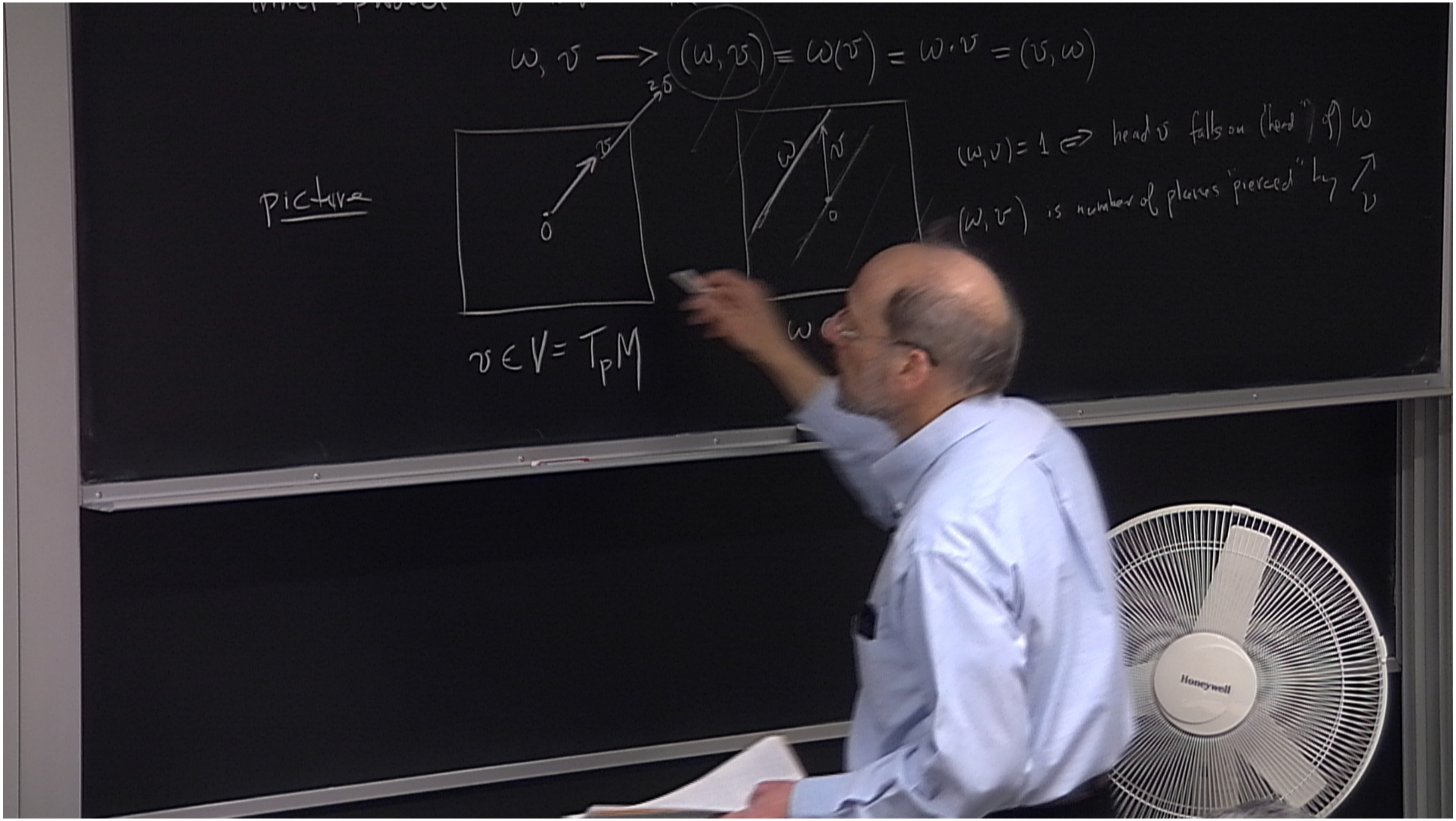

Diff geom ("tensor analysis")
tensor product
"outer product"
• Sum of products of vectors (& covectors): Σ u⊗v⊗w⊗…
V⊗V*⊗…
• linear op. from one space of tensor to another
• multilinear form from vectors & covectors to scalars
• certain type of geom. object
• a array of numbers (components) that transforms in a certain way under change of basis (coord. chart)
• el't of V1⊗V2 that arises as sol'n of "U.M.P"
• Symbol with some indices

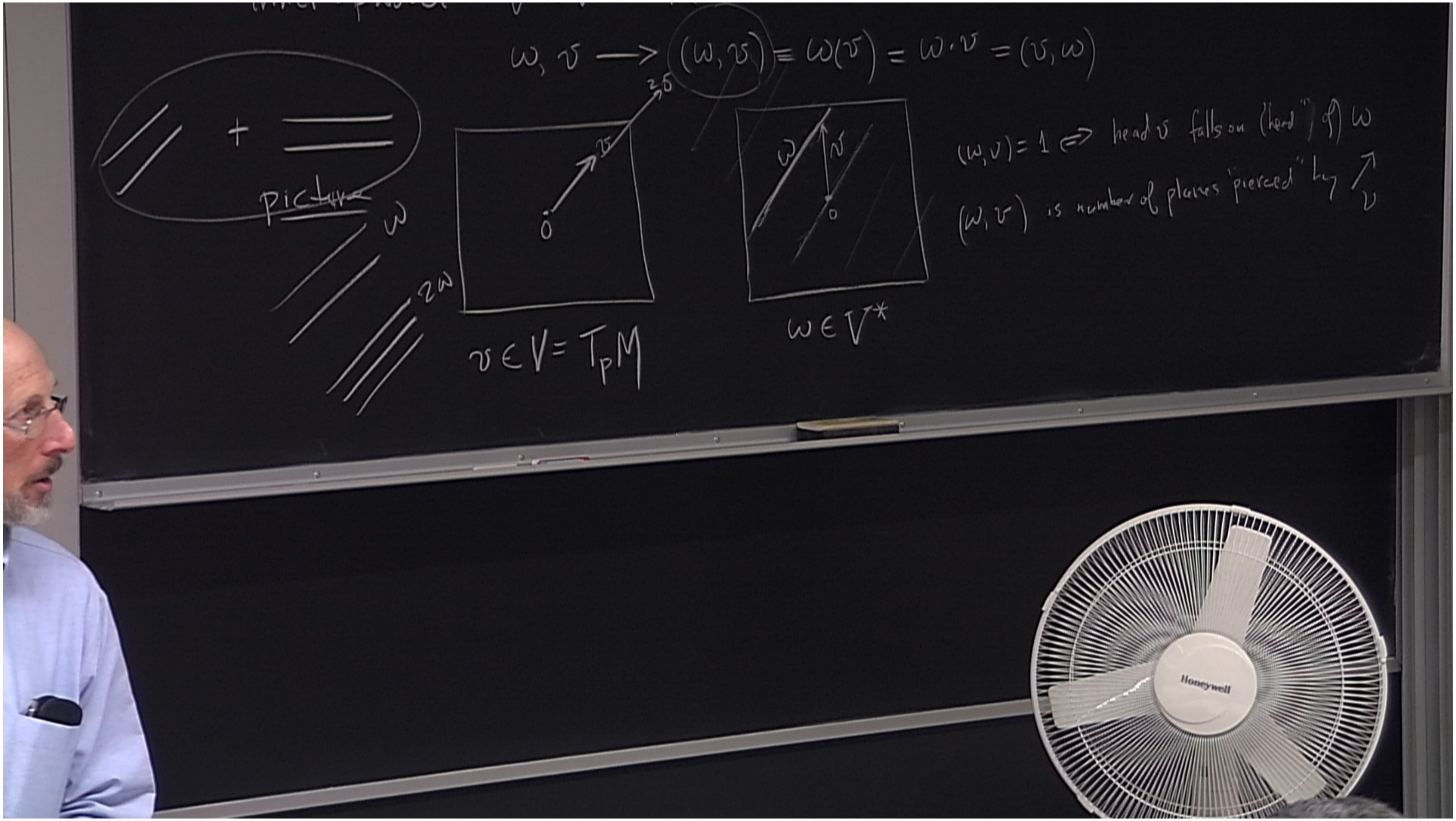

Dual vectorspace


Diff. geom ("tensor analysis")
tensor product
"outer product"
• Sum of products of vectors (& covectors): $\sum u \otimes v \otimes w \otimes \cdots$
$V \otimes V^* \otimes \cdots$
• linear op. from one space of tensor to another
$V = T_p M$
• multilinear form from vectors & covectors to scalars
V^*
• certain type of geom. object
• a array of numbers (components) that transforms in a certain way under change of basis (coord. chart)
• elt of $V_1 \otimes V_2$ that arises as sol'n of "U.M.P"
• Symbol with some indices
CAUTION


Dual vectorspace
covector
V* = L(V, ℝ) = {ω : V → ℝ | ω linear}
inner-product V* × V → ℝ
ω, v ⟶ (ω, v) ≡ ω(v) = ω·v = (v, ω)
picture
v
0
v ∈ V = TpM
ω ∈ V*


Dual vectorspace
covector
V* = L(V, R) = {ω : V → R | ω linear}
inner-product V* × V → R
ω, v → (ω, v) ≡ ω(v) = ω·v = (v, ω)
(ω, v) = 1 ⇔ head v falls on
(ω, v) is number of planes "pierced" by v
picture
v
0
v ∈ V = TpM
ω
v
0
ω ∈ V*


ω, v ⟶ (ω, v) = ω(v) = ω·v = (v, ω)
picture
2v
v
0
ω
v
0
v ∈ V = T_p M
(ω, v) = 1 ⟺ head v falls on "head" of ω
(ω, v) is number of planes "pierced" by v


ω, v ⟶ (ω, v) ≡ ω(v) = ω·v = (v, ω)
picture
ω
2ω
2v
v
0
v ∈ V = T_p M
ω
v
0
ω ∈ V*
(ω, v) = 1 ⟺ head v falls on "head" of ω
(ω, v) is number of planes "pierced" by v
Honeywell