Title: Holographic description of cosmological singularity

Date: Jan 22, 2013 11:00 AM

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Abstract: In my talk I will discuss our recent paper hep-th/1211.1322 where we construct a 3D conformal field theory dual to asymptotically AdS cosmology in four dimensions. Due to the scale invariance this dual theory allows an infinite family of instantons each of which breaks the conformal group O(3,2) down to O(3,1). These instantons are dual to bulk instantons responsible for nucleating an O(3,1) invariant cosmological bubble. Presumably they indicate an

infinite instability rate, however we are able to sum over all of them completely. The resulting theory is manifestly stable, unitary and finite.

Outline

1. Introduction
2. The CFT dual
   - Old lore
   - Instantons
   - spontaneous breaking of scale invariance
3. Generating functional
   - sum over the instantons
   - two-point function
4. Conclusions

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Introduction

- AdS/CFT is a unique theoretical laboratory to address various fundamental conundrums, e.g. big bang singularity, canonical measure problem, the conformal factor problem and etc.

- Cosmologies in AdS space are not realistic! They possess a negative cosmological constant and negative space curvature. The dual theory is only tractable at weak gauge coupling, i.e. stringy regime.

- Nevertheless, the methods developed within AdS/CFT constitute a significant step towards tackling the above fundamental puzzles.

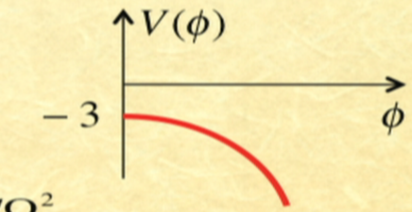
Introduction

(T.Hertog, K.Maeda 04') (T.Hertog,G.T.Horowitz 04',05') (B.Craps,T.Hertog,N.Turok 09')

- *The "simplest" and best-defined theoretical frameworks for holographic 4d cosmology is M-theory in asymptotically AdS which can be consistently truncated to* (M.J.Duff, J.T.Liu 99')

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}(\nabla\phi)^2+2+\cosh(\sqrt{2}\phi)\right]+\dots$$

$$m_\phi^2=-2\ >\ m_{BF}^2=-\frac{d^2}{4}=-\frac{9}{4}\quad\Rightarrow$$ *AdS is pert. stable!*



Coleman – De Luccia bubble $ds^2=d\rho^2+b^2(\rho)d\Omega_3^2$

$$\phi\sim\alpha\, e^{-\rho}+\beta\, e^{-2\rho}\quad,\quad b(\rho)\sim e^{\rho}$$

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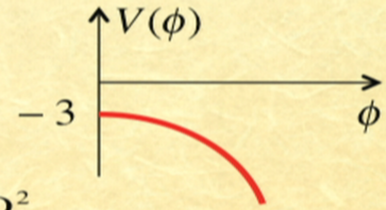

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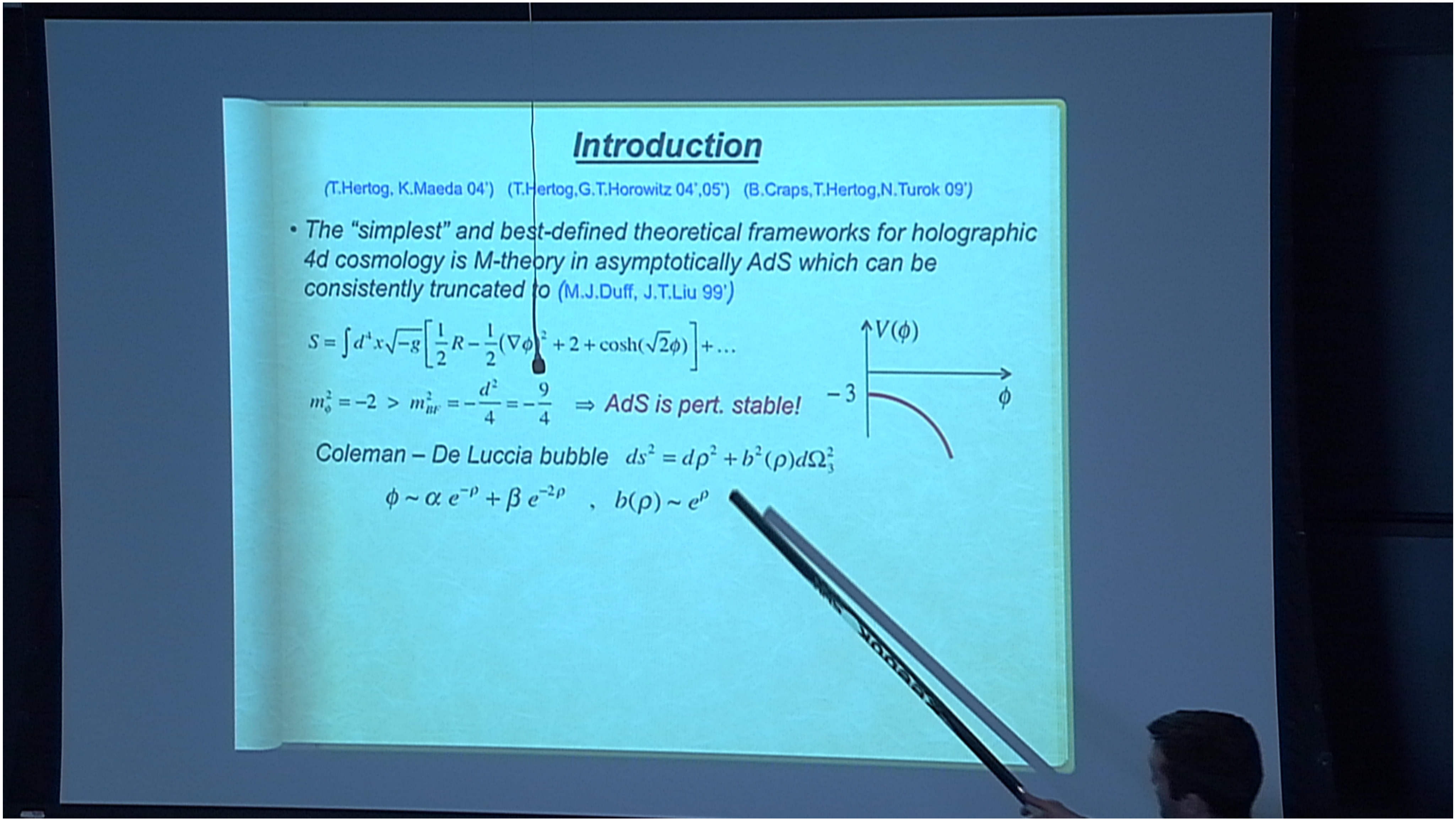
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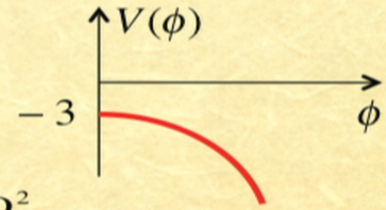

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- *Non-supersymmetric but still asymptotically AdS boundary conditions allow for interesting big crunch cosmologies in the bulk.*

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- *Holographic description.*

ϕ *is dual to* O *with* $\Delta_O = 1$ *and*

$$\alpha = \langle O \rangle \ , \ \beta \sim \alpha^2 \ \Leftrightarrow \ S = S_{CFT} + \frac{f}{3}\int O^3$$

$$O \sim Tr(Y^{\dagger}Y) \sim \vec{\phi}^2$$

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- Construction of the dual CFT begins with the O(N) vector model on S^3

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- Introducing auxiliary fields, yields

$$S_E = \int_{S^3} \left[\frac{1}{2}(\partial\vec{\phi})^2 + \frac{R}{16}\vec{\phi}^2 + \frac{1}{2}s(\vec{\phi}^2 - N\rho) + N\frac{g_6}{6}\rho^3 \right]$$

- Classical equivalence

$$\delta\phi : (-\nabla^2 + \frac{R}{8} + s)\vec{\phi} = 0$$

$$\delta s : \vec{\phi}^2 = N\rho$$

$$\delta\rho : s = g_6\rho^2$$

$$\Rightarrow \left[-\nabla^2 + \frac{R}{8} + \frac{g_6}{N^2}(\vec{\phi}^2)^2 \right]\vec{\phi} = 0$$

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$$I(\phi) \equiv \int d\rho \int ds\, e^{s(\phi-\rho)+\rho^3}$$

$$\frac{dI}{d\phi} = -\int d\rho \int ds\, e^{\rho^3} \frac{d}{d\rho} e^{s(\phi-\rho)}$$

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- *Integrate out* $\vec{\phi}$

$$Z\left(\vec{J}\right) = \int DsD\rho\; e^{-NS_{eff}(s,\rho)+\frac{1}{2}\iint \vec{J}(-\nabla^2+\frac{R}{8}+s)^{-1}\vec{J}}$$

$$S_{eff}(s,\rho) = \frac{1}{2}\int\left(\frac{g_6}{3}\rho^3 - s\rho\right) + \frac{1}{2}Tr\ln(-\nabla^2 + \frac{R}{8} + s)$$

The CFT dual

- Previous studies of the model in flat space

(D.Amit, T.Appelquist, M.Bander, W.Bardeen, U.Heinz, M.Moshe, R.Pisarski, E.Rabinovici 75'-85')

$$\bar{\rho} = \langle x | (-\partial^2 + \bar{s})^{-1} | x \rangle = \int_\Lambda \frac{d^3 k_E}{(2\pi)^3} \frac{1}{k_E^2 + \bar{s}} = \frac{\Lambda}{2\pi^2} - \frac{\sqrt{\bar{s}}}{4\pi}$$

$$\Rightarrow \quad \bar{s} = g_6 \bar{\rho}^2 = g_6 \frac{\bar{s}}{16\pi^2} \quad \Rightarrow \quad \bar{s} = 0,\ \forall g_6 \qquad \text{or} \qquad g_6 = 16\pi^2,\ \forall \bar{s}$$

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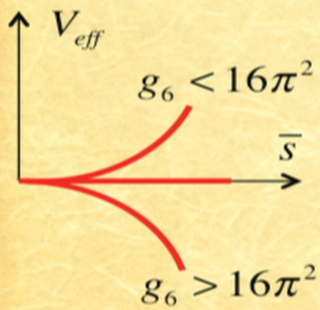

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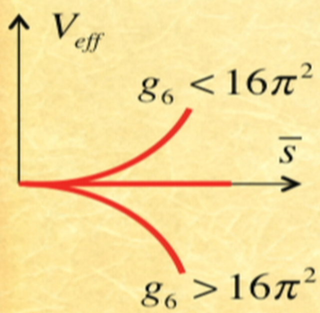

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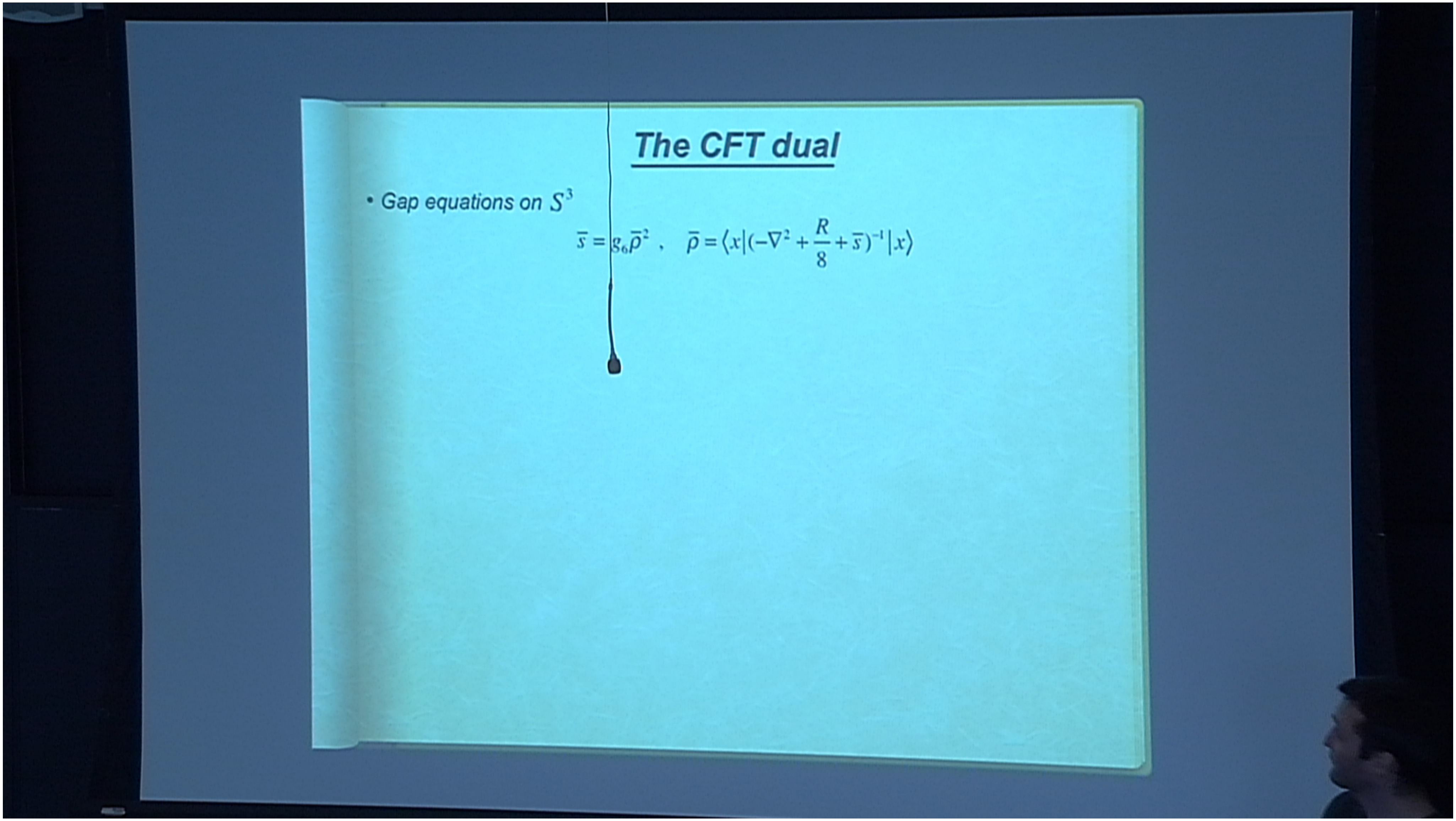
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$$\bar{s}(\hat{\eta};a,\hat{u}) = \frac{4\bar{s}_a}{[1+\hat{\eta}\cdot\hat{u}+\frac{r_0^2}{a^2}(1-\hat{\eta}\cdot\hat{u})]^2} \quad , \hat{\eta}\in S^3, \hat{u} \text{ is centre of } \bar{s}$$

- *Weyl invariance guarantees that all these instantons possess the same Euclidean action action S_{inst}, therefore we need to sum over all of them!*
- $\bar{s}(\hat{\eta};a,\hat{u}) = \bar{s}(\hat{\eta};\frac{r_0^2}{a},-\hat{u})$ *therefore to avoid double counting we restrict summation over a to $r_0 \leq a < \infty$*
- *For all $a \neq r_0$ each instanton, if individually continued to Lorentzian time, i.e., to dS_3, become singular.*
- *Each instanton spontaneously breaks conformal symmetry O(4,1) to O(4)*

$$\bar{s}|_{\mathbb{R}^3} = \bar{s}_a\left(\frac{2a^2}{a^2+r^2}\right)^2$$

broken generators: P_μ, D, K_μ

unbroken generators: $L_{\mu\nu}, L_{\mu 4} = K_\mu + a^2 P_\mu$

- *Instanton is dual to the bubble in the bulk which breaks O(4,1) of H_4 to O(4)*
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- *Instanton is dual to the bubble in the bulk which breaks O(4,1) of* H_4 *to O(4)*
- *Infinite number of instantons indicate Infinite instability rate?*

Generating functional

- For each instanton we work in the Weyl frame in which it is homogeneous
- In this frame quadratic fluctuations around a given instanton have one negative mode – the homogeneous mode, $\overline{s}$, and four zero modes – the $l=1$ harmonics associated with 3 translations and 1 dilation in $\mathbb{R}^3$.
- Partition function to leading order

$$Z_0 = \int DsD\rho\, e^{-NS_{eff}(s,\rho)} \sim e^{-S_{inst}} \sum_{a,\hat{u}} \int d\,\overline{s} \prod_{l=2}^{\infty} ds_l e^{-S_{fluc}(a,\hat{u};\overline{s},s_l)}$$

$$S_{eff}(s,\rho) = \frac{1}{2}\int \left(\frac{g_6}{3}\rho^3 - s\rho \right) + \frac{1}{2} Tr \ln(-\nabla^2 + \frac{R}{8} + s)$$

- No dependence on ρ since it has trivial dynamics (appears algebraically)
- To ensure convergence $\overline{s} \longrightarrow i\overline{s}$!!! Imaginary partition function ???

  No! Because $\rho^3 = 3\overline{\rho}\,\delta\rho^2 < 0 \Rightarrow \delta\rho \to -i\delta\rho$

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- Finally,

$$\sum_{a,\hat{u}} \equiv \int_{r_0}^{\infty} \frac{da}{a} \int_{S^3} C_d C_t^3\;,\quad C_d \sim \sqrt{\frac{r_0^3 a(a^2 - ar_0 + r_0^2)}{(a+r_0)^4}}\;,\quad C_t \sim \left|\frac{a-r_0}{a+r_0}\right|$$

Generating functional

- Since higher harmonics occur at least quadratically in the action, they may consistently be set to zero to determine the integration contour for the homogenious mode.

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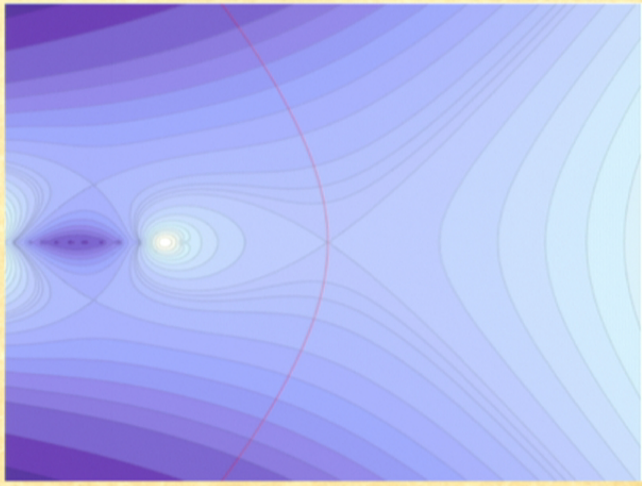
$$\Rightarrow Z_0 \sim -i\int d\,\overline{s}\; Ai(\alpha\overline{s})\, e^{-\frac{N}{2}Tr\ln(-\nabla^2 + \frac{R}{8} + \overline{s})} \;,\; \alpha = (V_{S^3}N/2)\,g_6^{-\frac{1}{3}}$$

- In the large N, we have

$$Ai(\alpha\overline{s}) \sim (\alpha\overline{s})^{-\frac{1}{4}} \exp\left(-\frac{V_{S^3}N}{3\sqrt{g_6}}\;\overline{s}^{\frac{3}{2}}\right)$$



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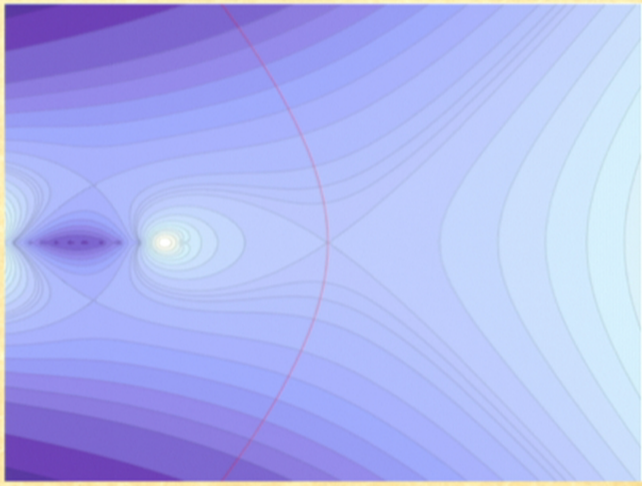

Generating functional

- Through a straightforward computation one can evaluate the Gaussian integral over fluctuations

$$\int d\bar{s}\prod_{l=2}^{\infty} ds_l e^{-S_{fluc}(a,\hat{u};\bar{s},s_l)} = \frac{(a\Lambda)^\kappa}{a^4} + \ldots, \quad \kappa = -\frac{(M\coth(\pi M))^3}{2(4\pi)^3} g_6^2\left(1-\frac{g_6}{192}\right)$$

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- Finally,

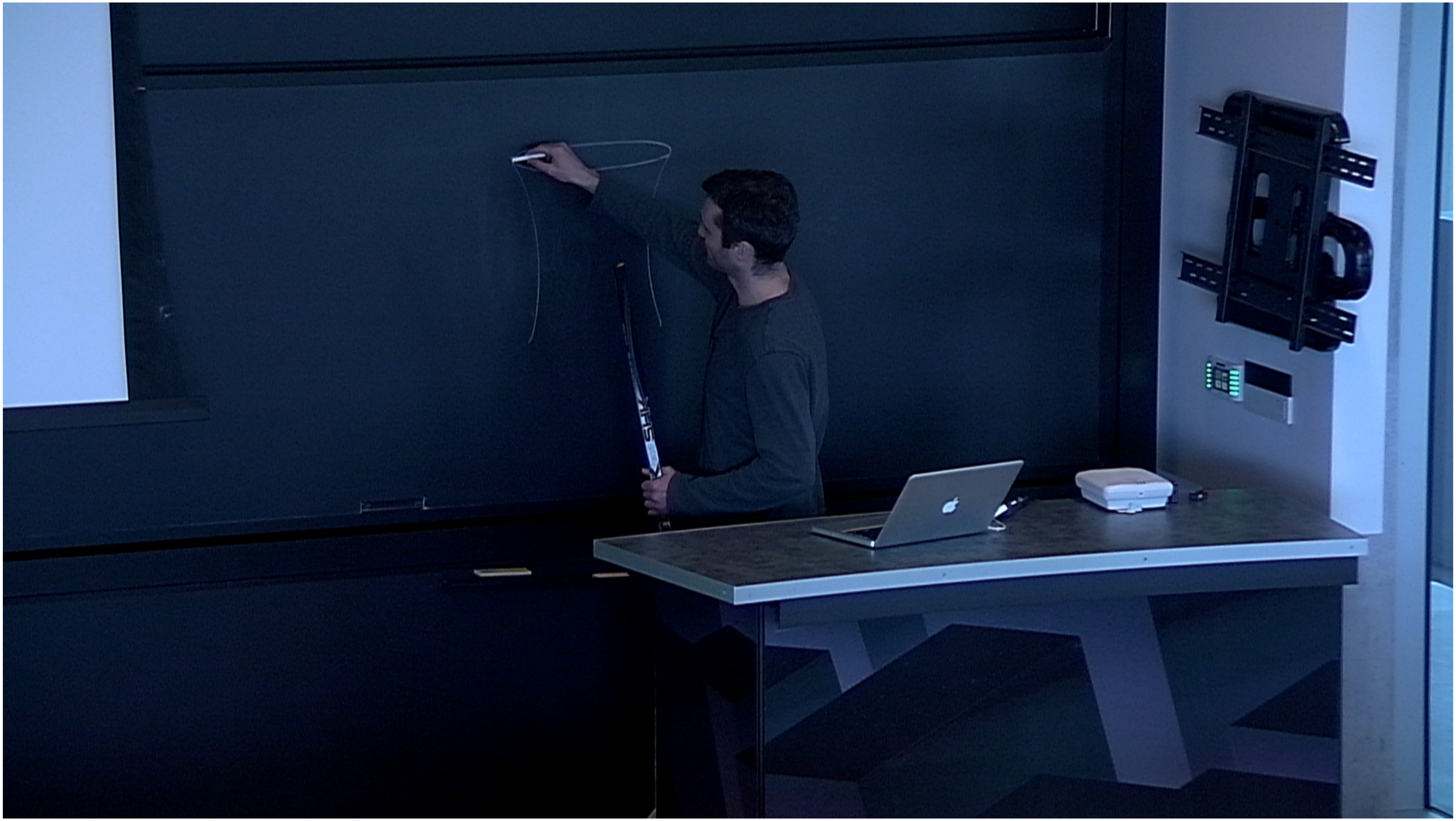
$$Z_0 \sim (r_0\Lambda)^{\kappa} e^{-S_{inst}} \quad \Rightarrow \quad \frac{dZ_0}{d\Lambda} = 0 \Leftrightarrow \beta_g = \frac{3}{2\pi^2 N} g_6^2\left(1 - \frac{g_6}{192}\right)$$

Conclusions

- The dual to a 4d M-theory cosmology is constructed for weak gauge coupling, i.e., when the bulk is in the stringy regime.

To be done:

- The behavior of the holographic dual at strong gauge coupling through understanding the bulk behavior in the Einstein gravity regime, including perturbations.
- Study the double holographic dual through the asymptotics on the dS_3 , i.e., 2d CFT defined on the past and future S^2 boundaries of dS_3.
- Propagate information across the cosmological singularity in a unique manner.
- Comparison to other boundary conditions in the bulk.



The End

Thank You!