

Title: 12/13 PSI - Condensed Matter Review Lecture 3

Date: Jan 30, 2013 11:30 AM

URL: <http://pirsa.org/13010099>

Abstract:

Review of last lecture

Transfer Matrix method

$$H = -K \sum_i^M \sigma_i^z \sigma_{i+1}^z - h \sum_i^M \sigma_i^z$$

PBC

$$\mathcal{Z} = \sum_{\{\sigma_i^z\}} e^{-H} = \text{Tr}(T_1 T_2)^M \quad \left\{ \begin{array}{l} T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} \\ T_2 = \begin{pmatrix} e^h & 0 \\ 0 & e^{-h} \end{pmatrix} \end{array} \right.$$

Scaling limit

Define lattice constant a

$$a \rightarrow 0$$

$$\tau = a t$$

$$\hbar = \frac{h}{a}$$

Keeping these
constant

$$L = M a$$

$$= \begin{pmatrix} e^{iK} & e^{-iK} \\ e^{-iK} & e^{iK} \end{pmatrix}$$
$$= \begin{pmatrix} e^{i\hbar} & 0 \\ 0 & e^{-i\hbar} \end{pmatrix}$$

σ_i^z

$$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix}$$
$$T_2 = \begin{pmatrix} e^h & 0 \\ 0 & e^{-h} \end{pmatrix}$$

Scaling limit

Define lattice constant a

$$a \rightarrow 0$$

$$\tau = a\ell$$

$$\tilde{h} = \frac{h}{a}$$

Keeping these
constant

$$L = Ma$$

$$\sum^{-1} = \frac{1}{a} \ln \coth K$$

$$C(\tau) = \langle \sigma^z(\tau) \sigma^z(0) \rangle \sim e^{-\tau/\xi}$$

Scaling limit

Define lattice constant a

$$a \rightarrow 0$$

$$\tau = a \ell$$

$$\tilde{h} = \frac{h}{a}$$

Keeping these
constant

$$L = M a$$

$$\xi^{-1} = \frac{1}{a} \ln \coth K$$

$$C(\tau) = \langle \sigma^z(\tau) \sigma^z(0) \rangle \sim e^{-\tau/\xi}$$

$$\text{for large } K \quad \xi \approx \frac{1}{2} e^{2K} \gg 1$$

Classical - Quantum mapping

D -dim Classical SM model



$d+1$ Quantum model

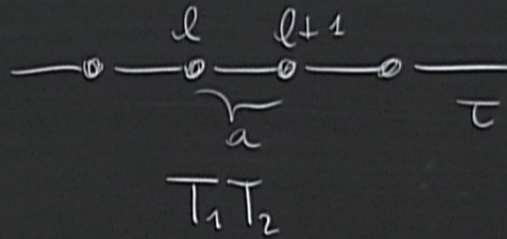
$$D = d+1$$

- The quantum theory is ~~THE~~ universal theory describing the whole univ. class
- Calculations are easier

quantum mapping

2 SM model

model

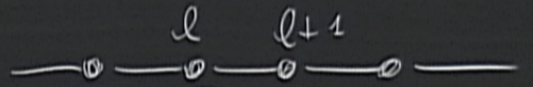


theory is THE universal theory
the whole univ. class
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quantum mapping

2 SM model

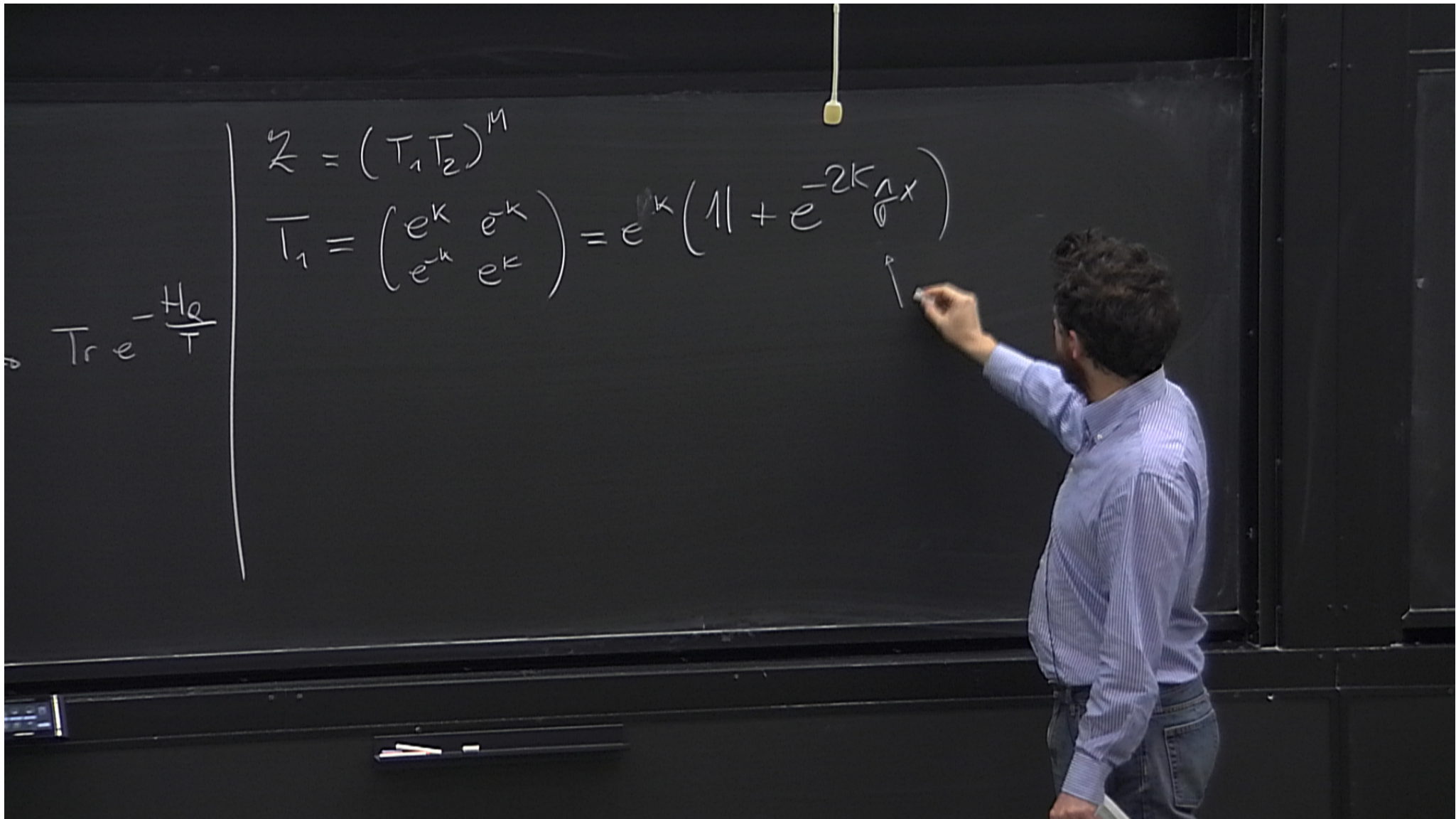
model



A horizontal line represents a 1D chain with four sites marked by small circles. The second and third sites from the left are labeled l and $l+1$ respectively. A bracket underneath the line connects the two sites between l and $l+1$, with the letter a centered below the bracket.

$$T_1 T_2 \approx e^{-a H_Q}$$

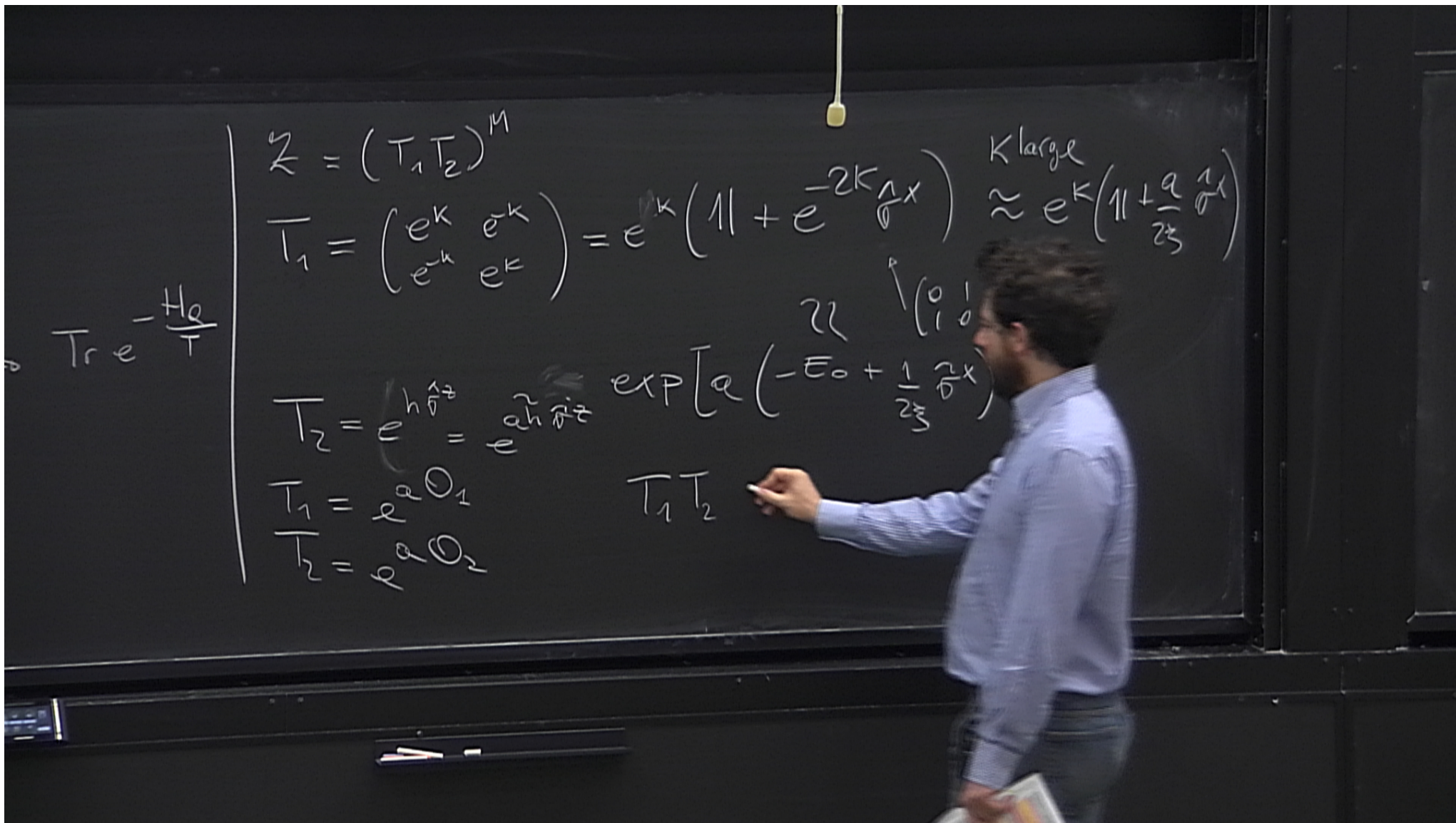
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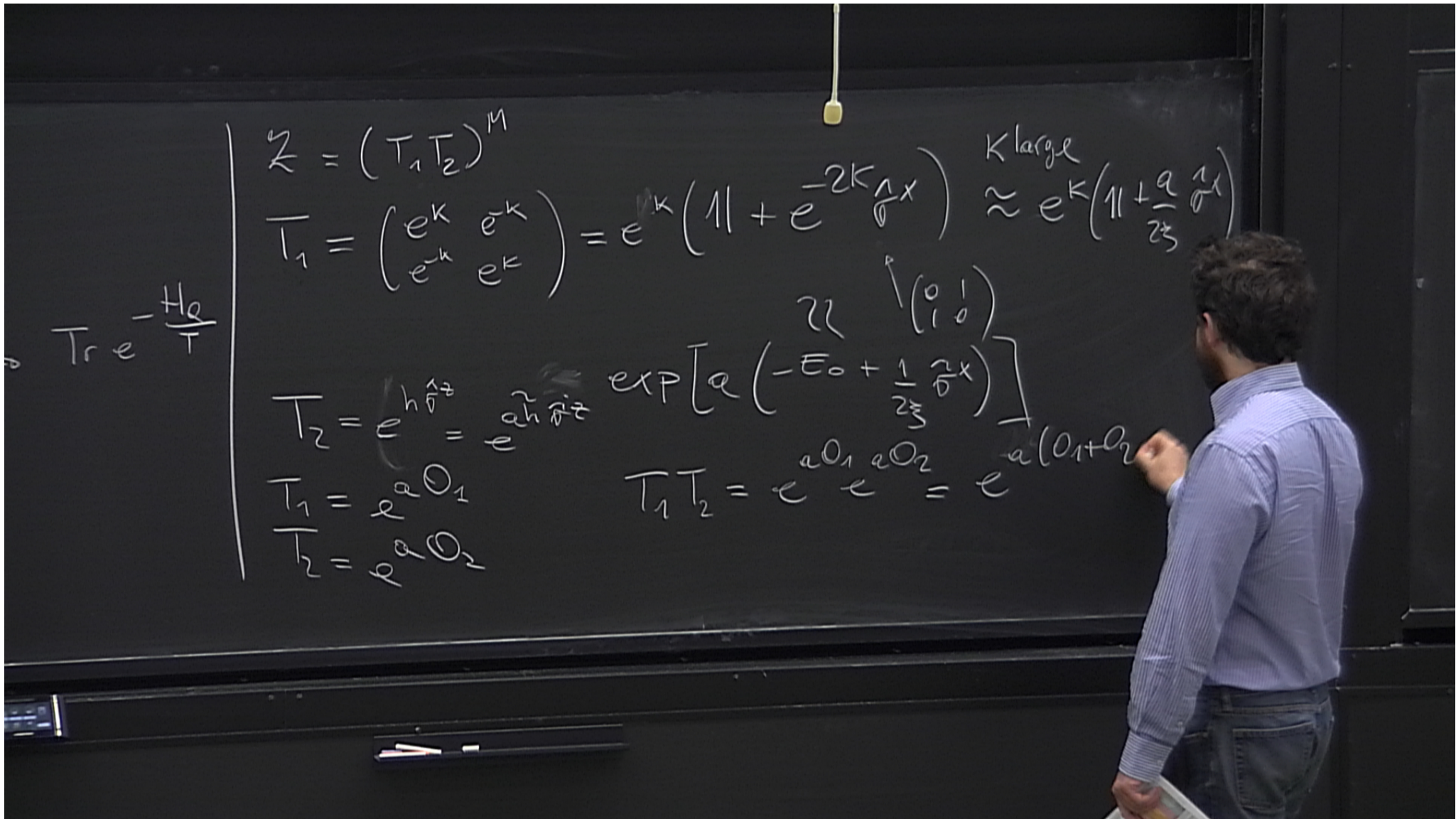


$$Z = (T_1 T_2)^M$$

$$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} = e^K \left(1 + e^{-2K} \frac{\sigma_x}{\sigma_y} \right) \stackrel{K \text{ large}}{\approx} e^K \left(1 + \frac{a}{2\epsilon} \frac{\sigma_x}{\sigma_y} \right)$$

$$T_2 = e^{h \hat{\sigma}_z} = e^{\tilde{a} h \hat{\sigma}_z} \exp \left[a \left(-E_0 + \frac{1}{2\epsilon} \frac{\sigma_x}{\sigma_y} \right) \right]$$





$$Z = (T_1 T_2)^M$$

$$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} = e^K \left(1 + e^{-2K} \frac{\sigma_x}{\sigma_z} \right) \stackrel{K \text{ large}}{\approx} e^K \left(1 + \frac{a}{2\epsilon} \frac{\sigma_x}{\sigma_z} \right)$$

$$T_2 = e^{h \hat{\sigma}_z} = e^{\frac{a}{2\epsilon} \hat{\sigma}_z} \exp \left[a \left(-E_0 + \frac{1}{2\epsilon} \frac{\sigma_x}{\sigma_z} \right) \right]$$

$$T_1 = e^{a \mathcal{O}_1} \quad T_1 T_2 = e^{a \mathcal{O}_1} e^{a \mathcal{O}_2} = e^{a(\mathcal{O}_1 + \mathcal{O}_2)}$$

$$T_2 = e^{a \mathcal{O}_2}$$

$$Z = (T_1 T_2)^M$$

$$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} = e^K \left(1 + e^{-2K} \frac{\sigma_x}{\sigma_z} \right) \stackrel{K \text{ large}}{\approx} e^K \left(1 + \frac{a}{2\epsilon} \frac{\sigma_x}{\sigma_z} \right)$$

$$T_2 = e^{h \hat{\sigma}_z} = e^{\frac{a}{2\epsilon} \hat{\sigma}_z} \exp \left[a \left(-E_0 + \frac{1}{2\epsilon} \frac{\sigma_x}{\sigma_z} \right) \right]$$

$$T_1 = e^{a Q_1}$$

$$T_2 = e^{a Q_2}$$

$$T_1 T_2 = e^{a Q_1} e^{a Q_2} = e^{a(Q_1 + Q_2)}$$

- calculations are easier

$$a \rightarrow 0 \quad T_1 T_2 \sim e^{a(O_1 + O_2)}$$

$$O_1 + O_2 = -E_0 + \frac{1}{\sigma} \frac{1}{\sigma^4} + \tilde{h} \frac{1}{\sigma^2} \equiv -H_Q$$

$$T_1 T_2 \approx e^{-a H_Q} \quad \frac{1}{\tilde{\xi}} = \Delta$$

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- calculations are easier

$$a \rightarrow 0 \quad T_1 T_2 \sim e^{a(O_1 + O_2)}$$

$$O_1 + O_2 = -E_0 + \frac{1}{\sigma} \frac{\partial^2}{\partial x^2} + \frac{\hbar}{\sigma} \frac{\partial^2}{\partial x^2} \equiv -H_Q$$

$$\boxed{T_1 T_2 \approx e^{-a H_Q}} \quad \swarrow \quad \frac{1}{\xi} = \Delta$$

$$\mathcal{Z} = (T_1 T_2)^M = \text{Tr} e^{-a M H_Q} =$$

- calculations are easier

$$a \rightarrow 0 \quad T_1 T_2 \sim e^{a(O_1 + O_2)}$$

$$O_1 + O_2 = -E_0 + \frac{1}{2\zeta} \frac{1}{\sigma^4} + \frac{\tilde{h}}{h} \frac{1}{\sigma^2} \equiv -H_Q$$

$$\boxed{T_1 T_2 \approx e^{-a H_Q}} \quad \swarrow \quad \frac{1}{\zeta} = \Delta$$

$$\mathcal{Z} = (T_1 T_2)^M = \text{Tr} e^{-a M H_Q} = \text{Tr} e^{-L H_Q}$$

- calculations are easier

$$\rightarrow 0 \quad T_1 T_2 \sim e^{a(\theta_1 + \theta_2)}$$

$$\theta_1 + \theta_2 = -E_0 + \frac{1}{\sigma} \frac{\hbar^2}{2} + \frac{\hbar^2}{2\sigma} \frac{\hbar^2}{2} \equiv -H_Q$$

$$\boxed{T_1 T_2 \approx e^{-a H_Q}} \quad \frac{1}{\sigma} = \Delta$$

$$Z = (T_1 T_2)^M = \text{Tr} e^{-a M H_Q} = \text{Tr} e^{-L H_Q} = \text{Tr} e^{-\frac{H_Q}{T}}$$

$L \equiv \frac{1}{T}$

$$E_{1,2} = E_0 \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \tilde{h}^2}$$

$$f = -T \ln Z = E_0 - T \ln \left[2 \cosh \frac{\sqrt{\left(\frac{\Delta}{2}\right)^2 + \tilde{h}^2}}{T} \right]$$

$$= \text{Tr} e^{-\frac{H_0}{T}}$$

$$E_{1,2} = E_0 \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \hbar^2}$$

$$f = -T \ln Z = E_0 - T \ln \left[2 \cosh \frac{\sqrt{\left(\frac{\Delta}{2}\right)^2 + \hbar^2}}{T} \right]$$

$$C_Q(\tau_1, \tau_2) \equiv \frac{1}{Z} \text{Tr} \left[e^{-H_Q/T} \hat{\sigma}^z(\tau_1) \hat{\sigma}^z(\tau_2) \right]$$

time ordered
 $\tau_1 > \tau_2$

$$= \text{Tr} e^{-\frac{H_Q}{T}}$$

$$E_{1,2} = E_0 \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \hbar^2}$$

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time ordered
 $\tau_1 > \tau_2$

$$= \text{Tr} e^{-\frac{H_Q}{T}}$$

$$\hat{\sigma}^z(\tau) \equiv e^{\tau H_Q} \hat{\sigma}^z e^{-\tau H_Q}$$

$$\boxed{\hat{\sigma}^z(\tau) = \hat{\sigma}^z \text{ at } \tau = 0}$$

$$\stackrel{\text{class}}{=} \lim_{a \rightarrow 0}$$

$$\langle \hat{\sigma}^z(\tau_1) \hat{\sigma}^z(\tau_2) \rangle_H \quad \text{classical average}$$

$$T \rightarrow 0$$

$$L \rightarrow \infty$$

$$C_Q(\tau_1, \tau_2) = \overline{\sum}_n |\langle 0 | \sigma^z | n \rangle|^2 e^{-(E_n - E_0) |\tau_1 - \tau_2|}$$

$$\overline{\sum}_n |n\rangle \langle n| = \mathbb{1}$$

$$T \rightarrow 0$$

$$L \rightarrow \infty$$

$$C_Q(\tau_1, \tau_2) = \overline{\sum}_n |\langle 0 | \sigma^z | n \rangle|^2 e^{-(E_n - E_0) |\tau_1 - \tau_2|}$$

$$\overline{\sum}_n |n\rangle \langle n| = \mathbb{1}$$

$$C_Q(\tau_1 - \tau_2) = \frac{4y_3^2}{z} + \frac{y_2^2}{z} \left(\frac{\cosh\left(\frac{\sqrt{z}}{2} (1 - 2y_1/2)\right)}{\cosh \frac{\sqrt{z}}{2}} \right) z = \frac{1}{E_1 - E_0} = \frac{1}{\Delta}$$

$$y_1 = \frac{\tau_1 - \tau_2}{L} \quad y_2 = \frac{\Delta}{T}$$

$$z \equiv y_2^2 + 4y_3^2 \equiv \Phi_\delta(y_1, y_2, y_3)$$

Mapping between

$d=1$ Quantum



$D=2$ Classical Ising

$\parallel$
 $1+1$

$$H_I = -J \sum_{i=1}^N \sigma_i^x - \sum_i \bar{J}_i \sigma_i^z \sigma_{i+1}^z$$

Mapping between

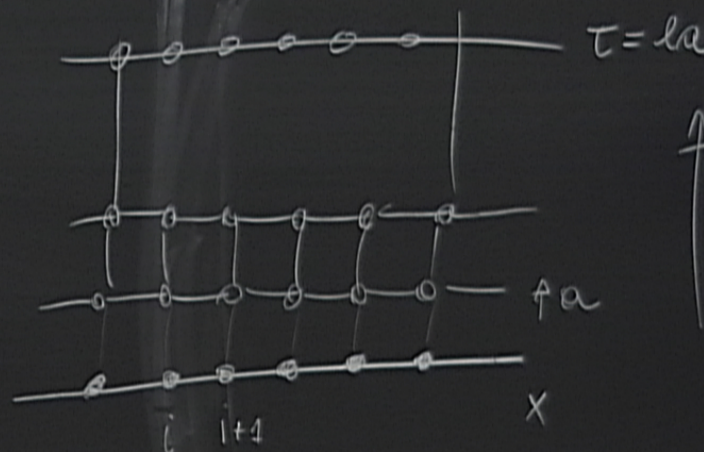
$d=1$ Quantum



$D=2$ Classical Ising

$\parallel$
 $1+1$

$$H_I = -J \sum_{i=1}^N \sigma_i^x - \bar{J} \sum_i \sigma_i^z \sigma_{i+1}^z$$

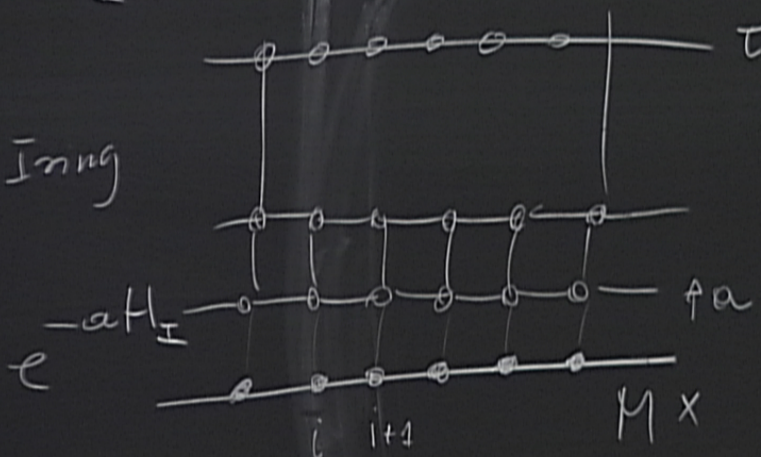


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$$H_I = -J \sum_{i=1}^M \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$



$$L = M \frac{a}{\tau} \quad \frac{1}{\tau} = L$$

$$\boxed{a \rightarrow 0 \quad \text{st} \quad \frac{1}{\tau} = L = M a = \text{const} \quad M \rightarrow \infty}$$

$$Z = \text{Tr} \exp \left(\frac{-H_I}{T} \right) =$$

$$T_1 = \exp \left(\beta a \sum_i^M \frac{\Delta x_i}{\sigma_i} \right)$$

$$T_2 = \exp \left(\beta a \sum_i^N \frac{\Delta z_i}{\sigma_i} \frac{\Delta z_{i+1}}{\sigma_{i+1}} \right)$$

$$\text{Tr} (T_1 T_2)^{M_c}$$

$$Z = (T_1 T_2)^{M_T} = \sum \langle \psi_{m_i(l)} | T_1 T_2 | \psi_{m_i(l+1)} \rangle$$

at basis

$(l+1) | \psi_{m_i} \rangle$ $\hat{J}$

$$Z = (T_1 T_2)^{M_T} = \sum_{\{m_i(\ell)\}} \prod_{\ell=1}^{M_T} \langle \{m_i(\ell)\} | T_1 T_2 | \{m_i(\ell+1)\} \rangle$$

$$\begin{matrix} \uparrow \\ 2^{M_1} \times 2^{M_2} \end{matrix}$$

at basis

$(\ell+1) | \{m_i\} \rangle$

$$Z = (T_1 T_2)^{M\tau} = \sum_{\{m_i(l)\}} \prod_{l=1}^{M\tau} \langle \{m_i(l)\} | T_1 T_2 | \{m_i(l+1)\} \rangle$$

1
2

$$\langle \{m_i(l)\} | T_1 | \{m_i(l+1)\} \rangle =$$

$$\langle \{m_i(l)\} | e^{\sum_{ja} J_{ja} \sigma_i^{ax}} | \{m_i(l+1)\} \rangle$$

$$= \sum_{\{e\}} B \prod_i m_i(l) m_i(l+1)$$

$$\mathcal{L} = \sum_{\{m_i(e)\}} \exp \left(\sum_{ie} J_a m_i(e) m_{i+1}(e) + B \right)$$

$$\exp \left(\sum_{i \in \mathcal{I}} J a m_i(l) m_{i+1}(l) + B m_i(l) m_{i+1}(l+1) \right)$$

$$e^{-2\beta} = \tanh(Jga)$$

$$\exp\left(\sum_{i\ell} J_a m_i(\ell) m_{i+1}(\ell) + B m_i(\ell) m_{i+1}(\ell+1)\right)$$

$$e^{-2\beta} = \tanh(Jg a)$$

$$\exp\left(\sum_{i \in \mathbb{Z}} J a m_i(l) m_{i+1}(l) + B m_i(l) m_{i+1}(l+1)\right)$$

$$e^{-2\beta} = \tanh(Jga)$$