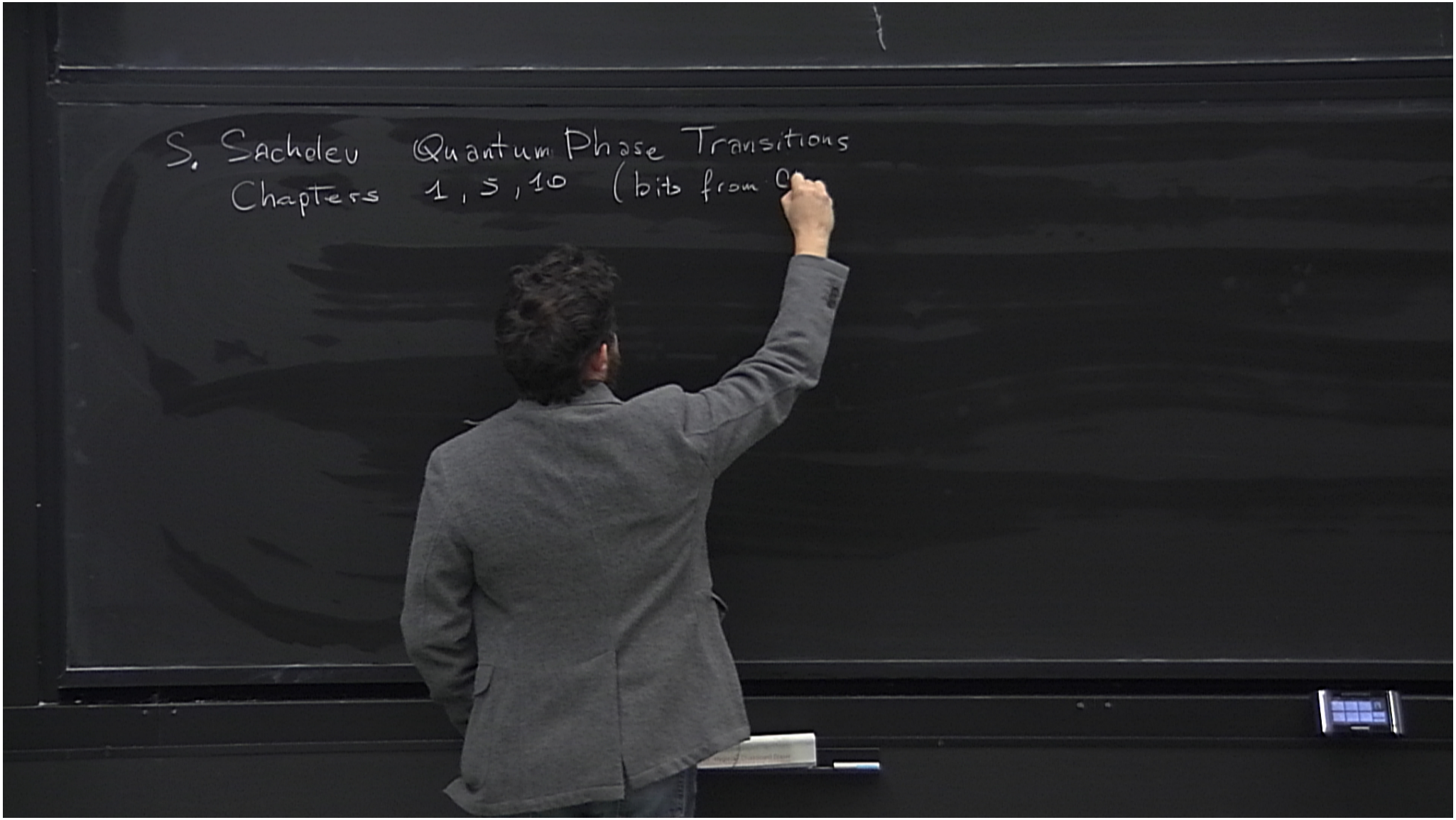
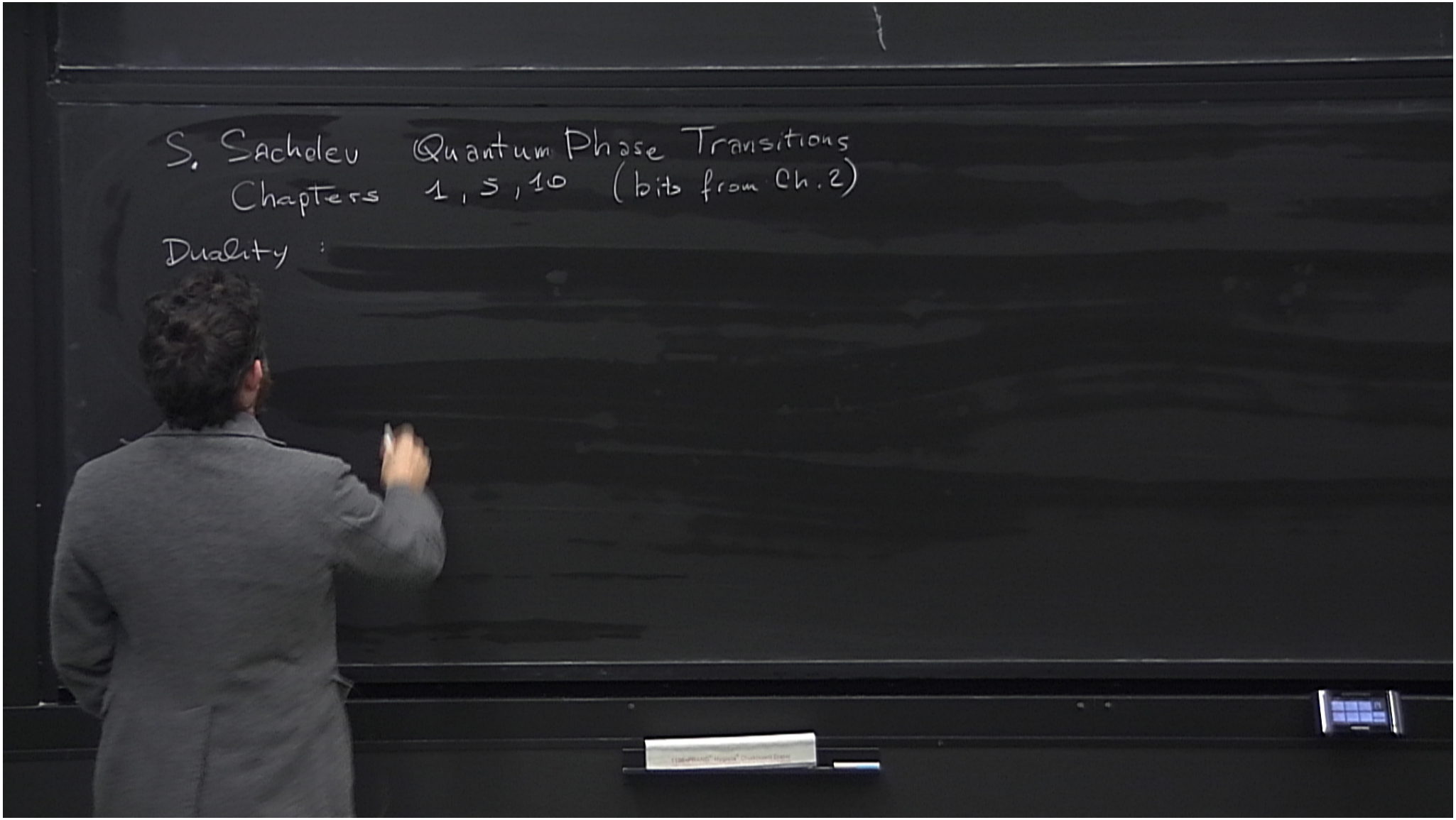
Title: 12/13 PSI - Condensed Matter Review Lecture 2

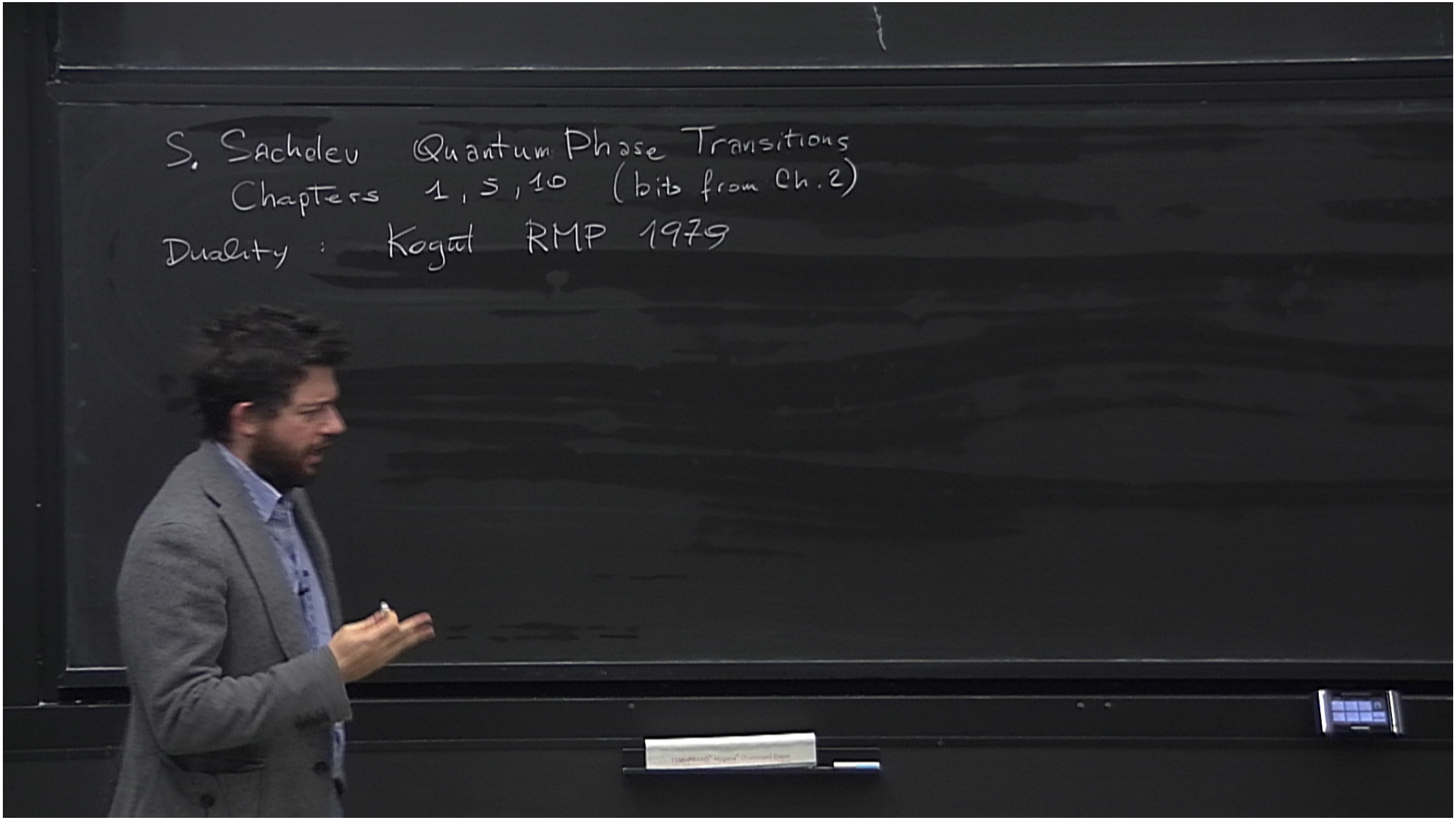
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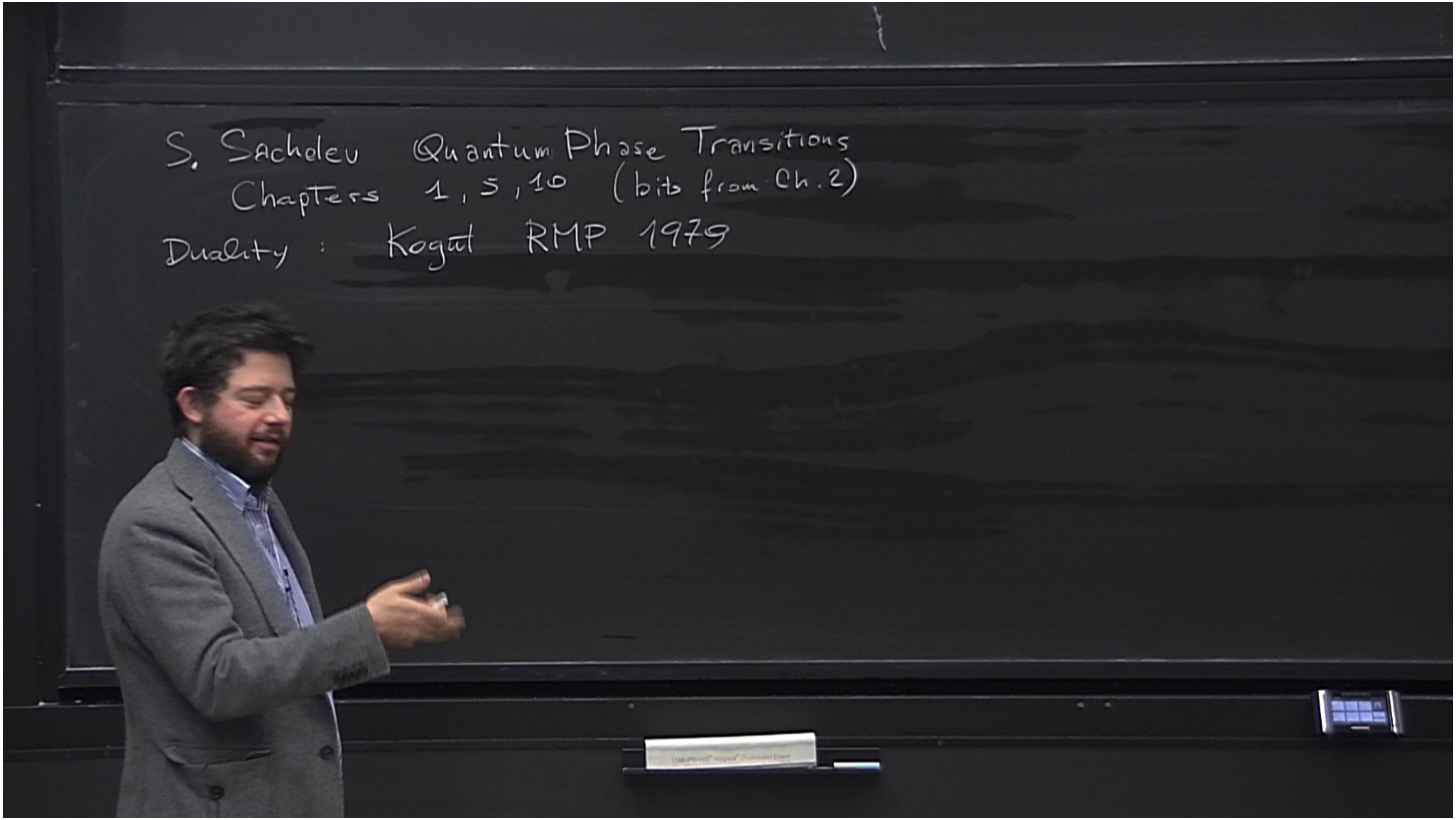
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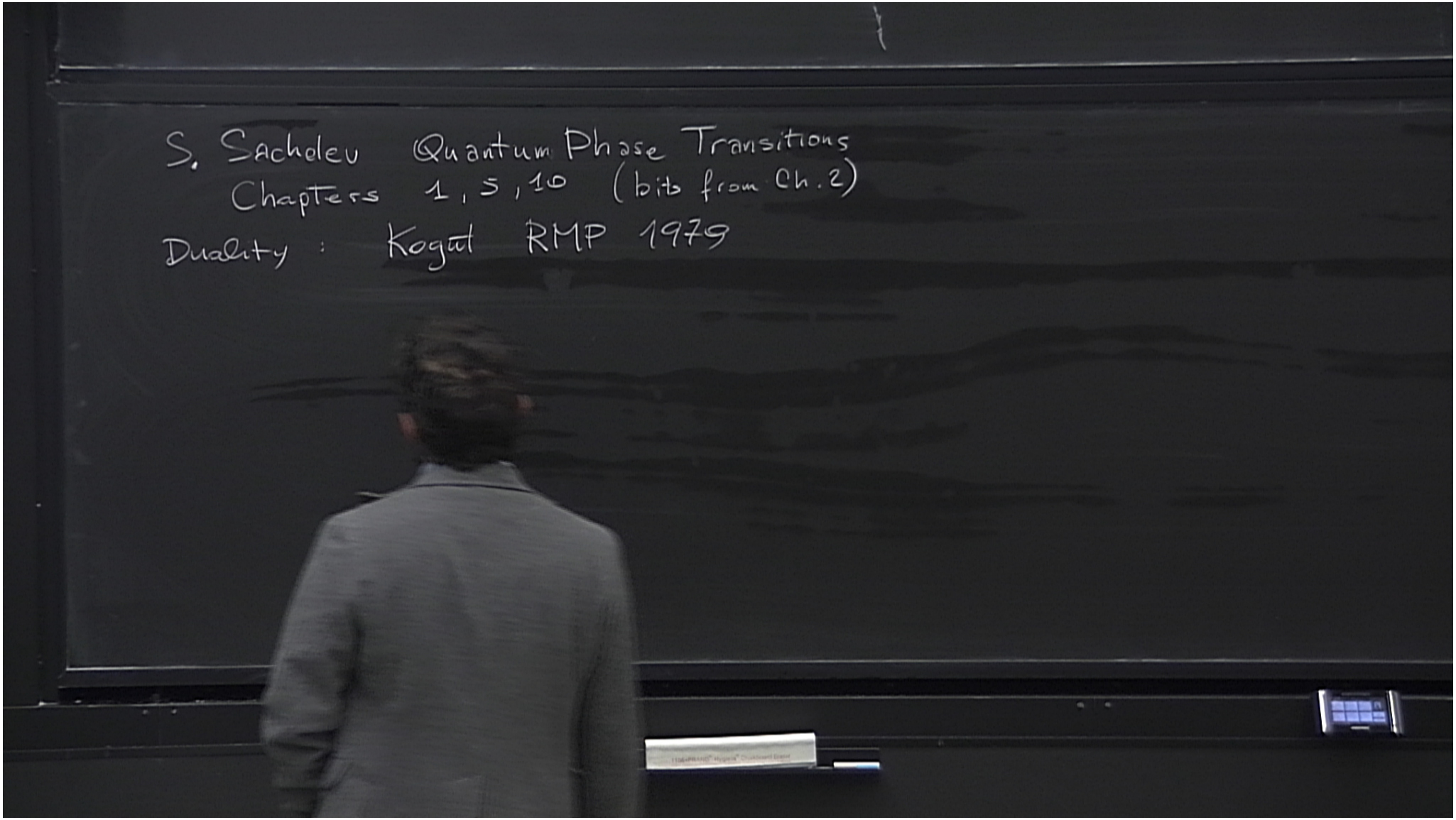
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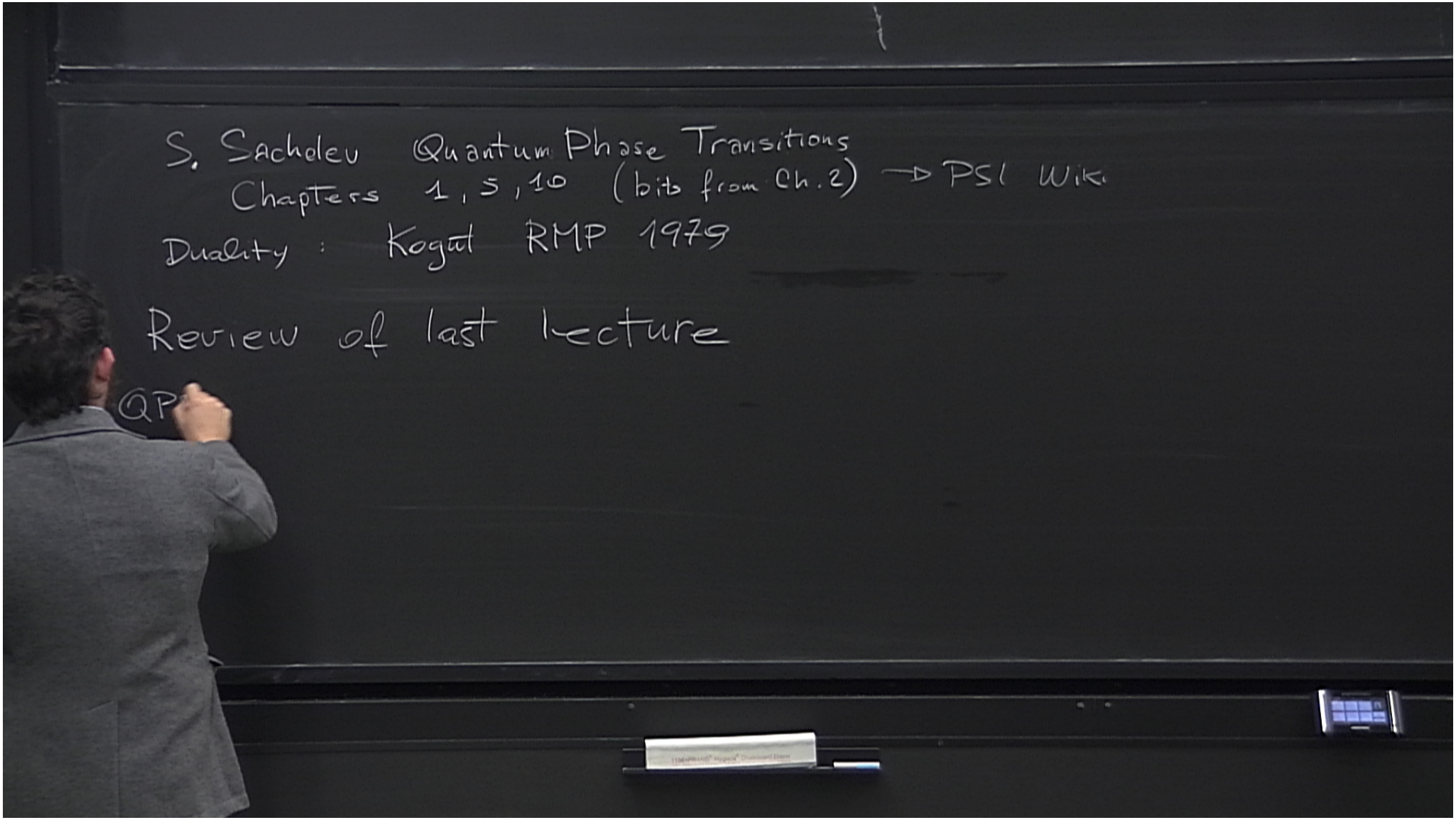

S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from

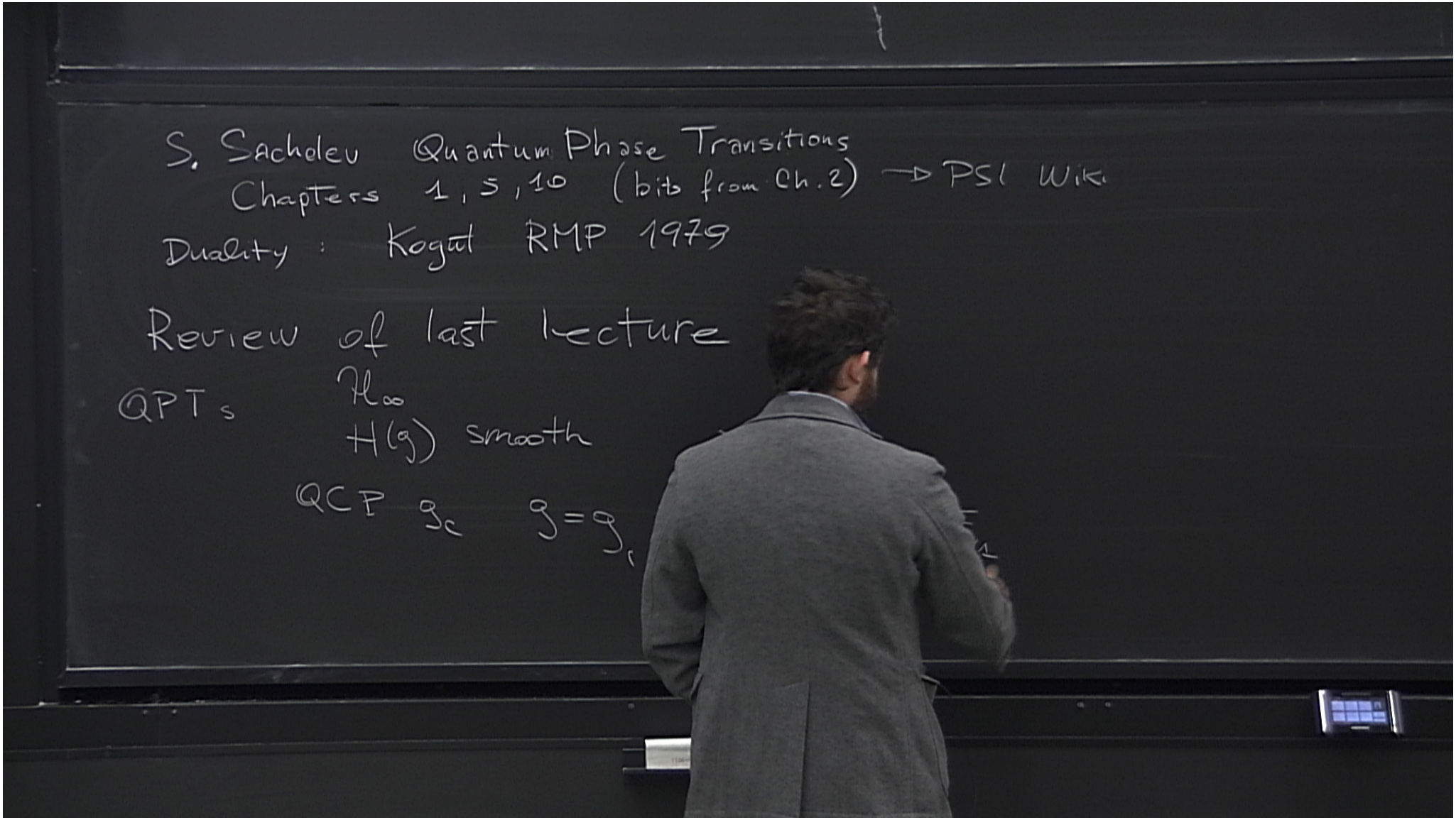

S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2)
Duality :


S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2)
Duality: Kogut RMP 1979


S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2)
Duality : Kogut RMP 1979


S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2)
Duality : Kogut RMP 1979


S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2) → PSI Wiki
Duality : Kogut RMP 1979
Review of last lecture
QP


S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2) → PSI Wiki
Duality : Kogut RMP 1979
Review of last lecture
QPTs
$\mathcal{H}_\infty$
$H(g)$ smooth
QCP g_c $g=g_c$

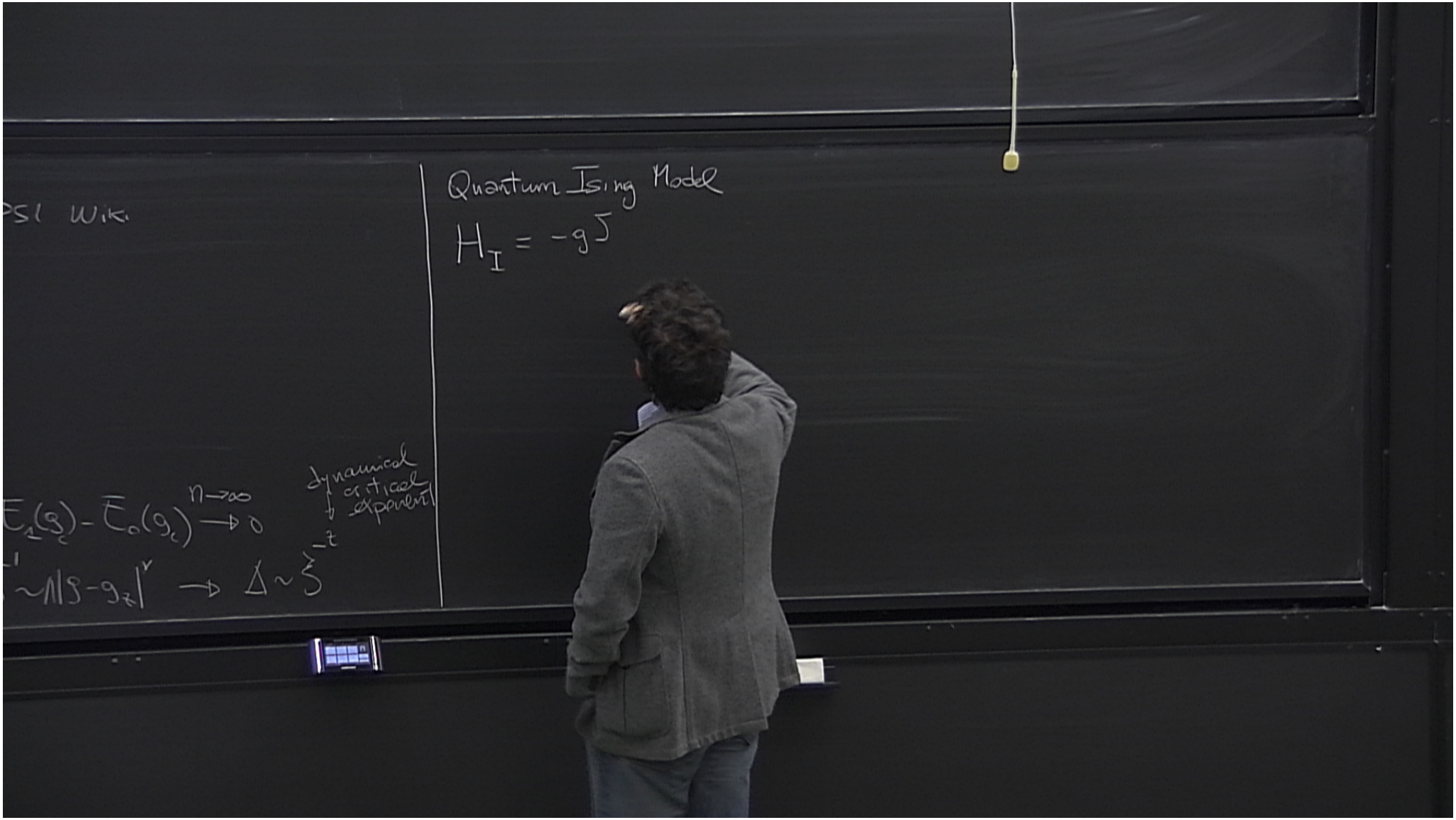
S. Sachdev Quantum Phase Transitions
Chapters 1, 5, 10 (bits from Ch. 2) → PSI Wiki
Duality: Kogut RMP 1979

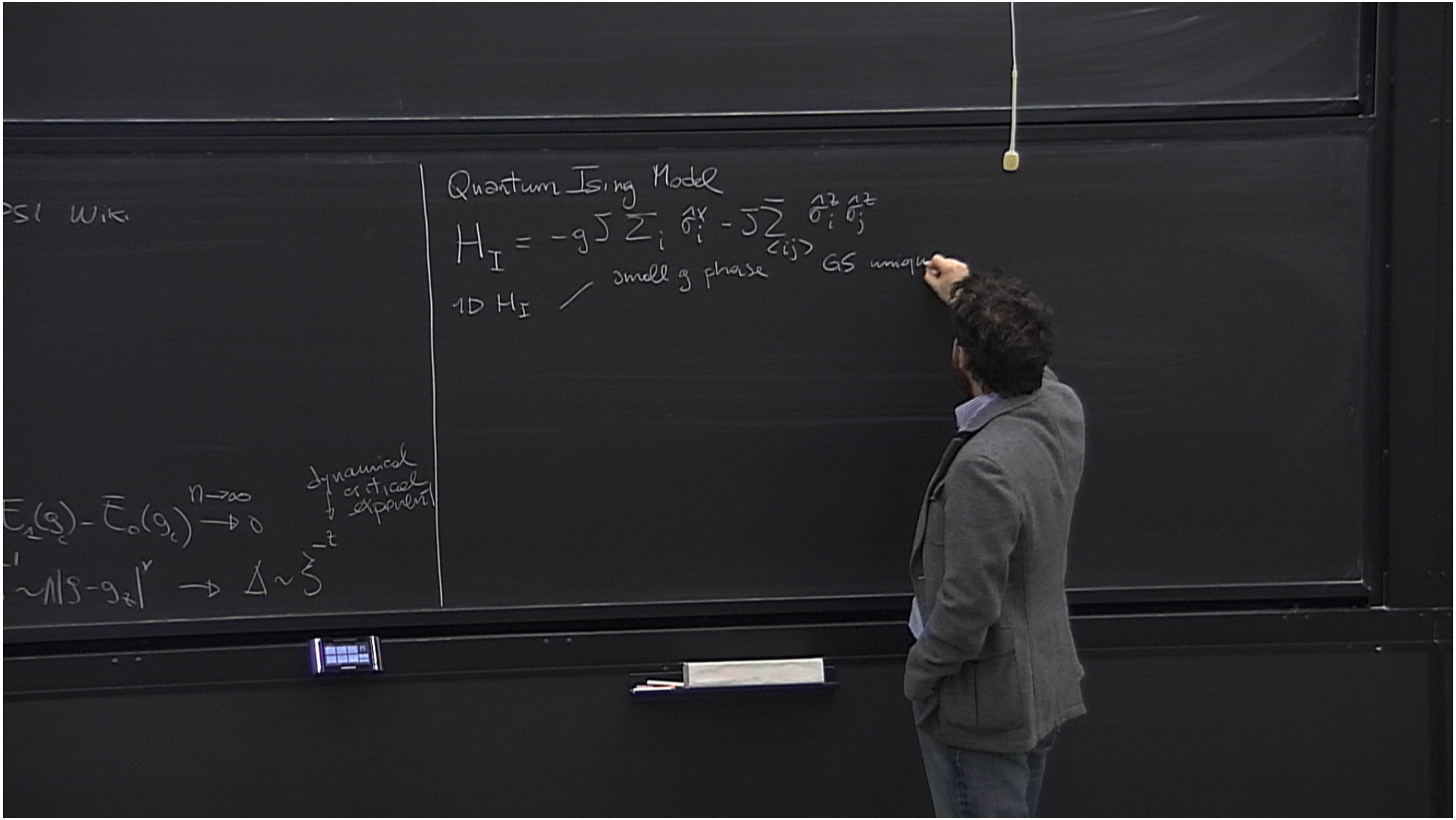
Review of last lecture

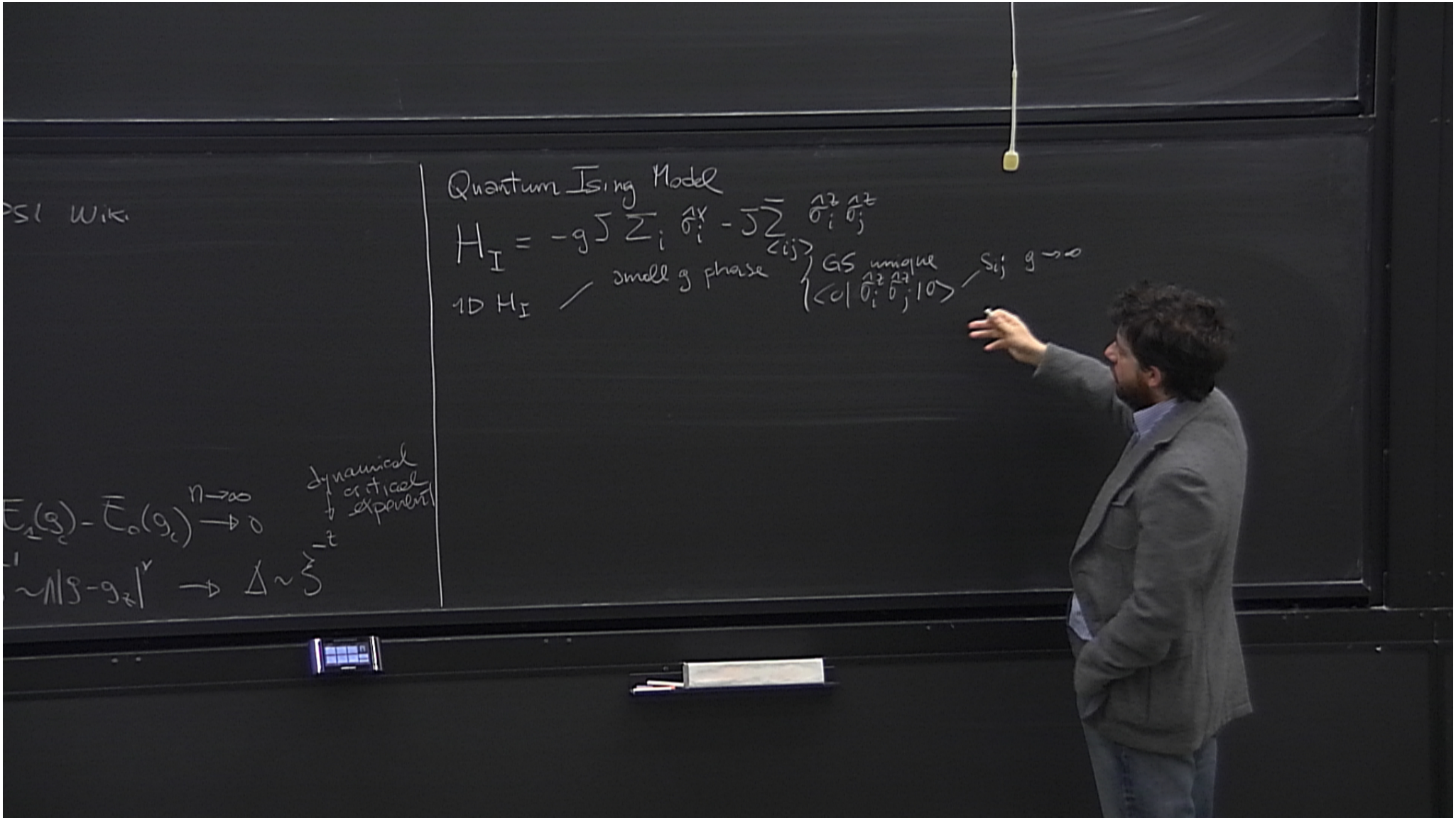
QPTs $\mathcal{H}_\infty$
$H(g)$ smooth

QCP g_c $g = g_c$ such that $\Delta|_{g_c} = E_1(g_c) - E_0(g_c) \xrightarrow{n\to\infty} 0$

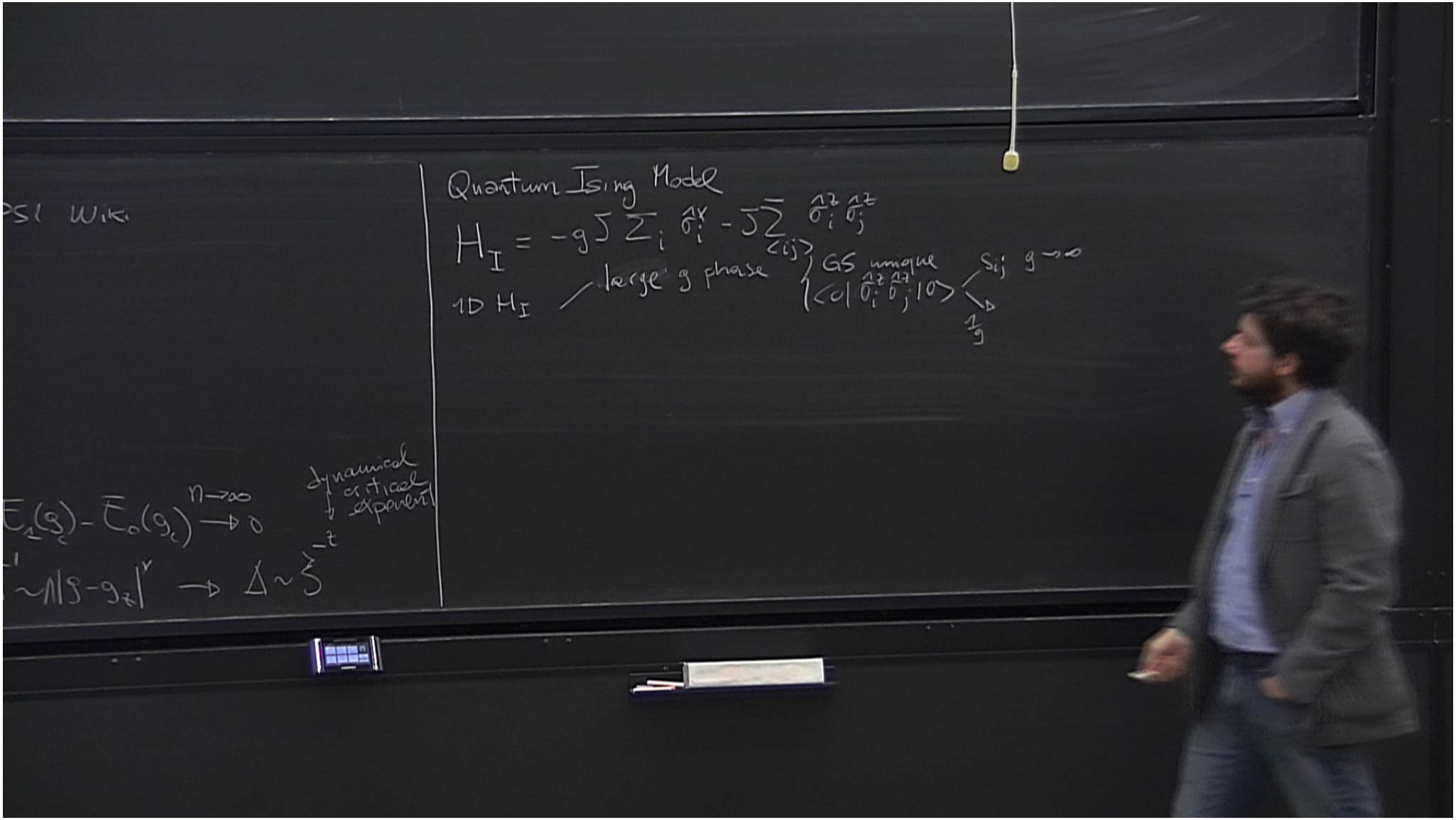
$g \sim g_c \sim J|g - g_c|^{z\nu}$ $\quad \xi^{-1} \sim \Lambda|g - g_c|^{\nu} \rightarrow \Delta \sim \xi^{-z}$

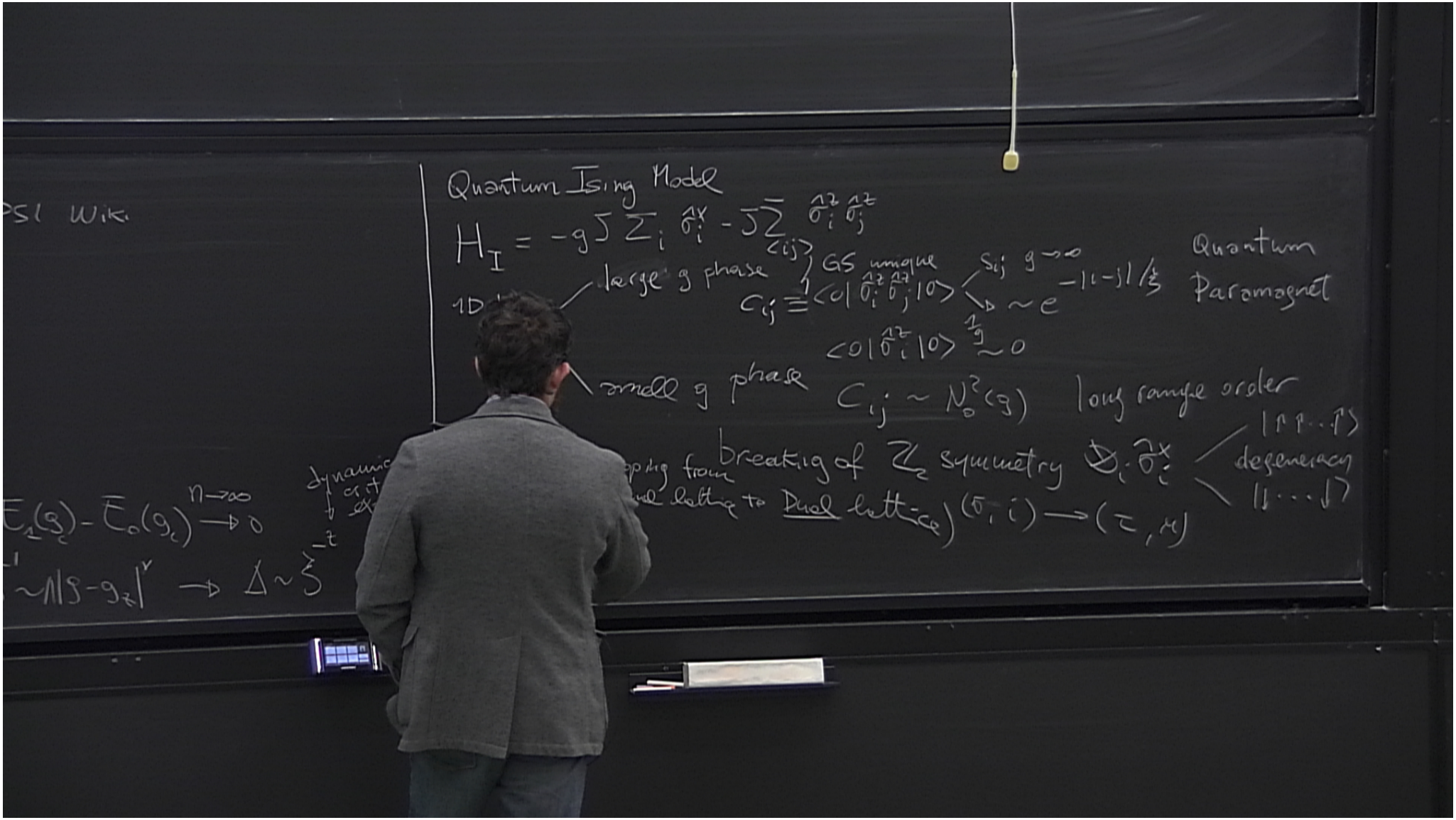

PSI Wiki
Quantum Ising Model
$H_I = -gJ$
$E_1(g_c) - E_0(g_c) \xrightarrow{n\to\infty} 0$
dynamical critical exponent
$\sim |g - g_c|^{\nu} \rightarrow \Delta \sim \xi^{-z}$


Quantum Ising Model
$H_I = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{<ij>} \hat{\sigma}_i^z \hat{\sigma}_j^z$
1D H_I
small g phase
GS unique
dynamical critical exponent
$n \to \infty$
$\to 0$
$\sim |g - g_c|^{\nu}$
$\Delta \sim \xi^{-z}$


Quantum Ising Model
$H_I = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{\langle ij\rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$
1D H_I — small g phase
GS unique
$\langle 0|\hat{\sigma}_i^z \hat{\sigma}_j^z|0\rangle$ — s_{ij} $g \to \infty$
$E_1(g_c) - E_0(g_c) \xrightarrow{n\to\infty} 0$
$\sim |g - g_c|^{\nu} \to \Delta \sim \xi^{-z}$
dynamical critical exponent


Review of last lecture


Quantum Ising Model
$H_I = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{\langle ij\rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$
1D H_I
large g phase
GS unique
$\langle 0|\hat{\sigma}_i^z \hat{\sigma}_j^z|0\rangle$
s_{ij} $g\to\infty$
$\frac{1}{g}$
$E_1(g_c) - E_0(g_c) \xrightarrow{n\to\infty} 0$
dynamical critical exponent
$\sim |g-g_c|^{\nu} \to \Delta \sim \xi^{-z}$


Quantum Ising Model
large g phase
GS unique
Quantum Paramagnet
small g phase
long range order
breaking of Z_2 symmetry
degeneracy
dual lattice

PSI Wiki

$E_1(g) - E_0(g_c) \xrightarrow{n\to\infty} 0$ dynamical critical exponent

$\Delta \sim |g - g_c|^{\nu} \to \Delta \sim \xi^{-z}$

Quantum Ising Model

$H_I = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$

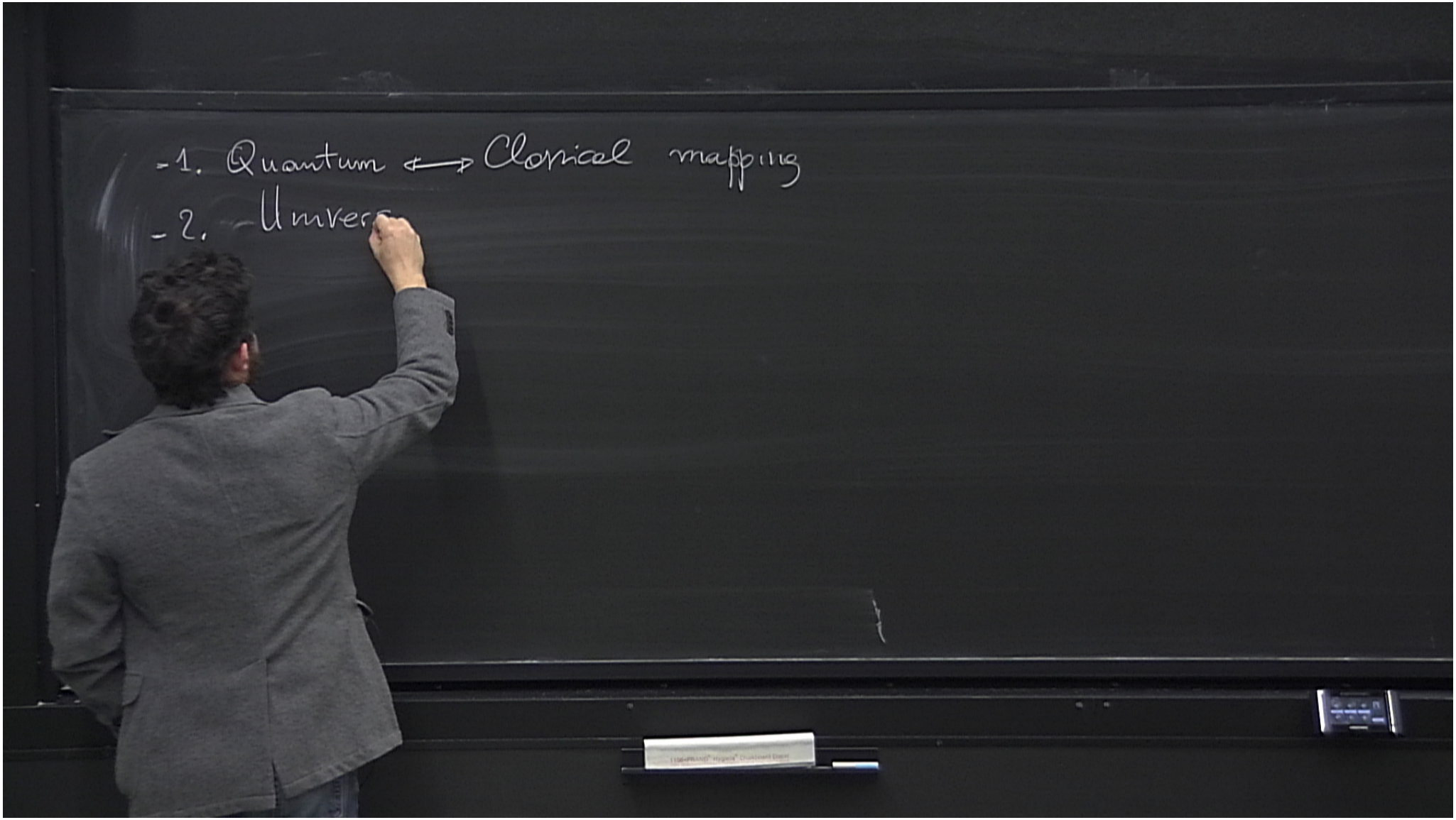
1D H_I — large g phase: GS unique; $C_{ij} \equiv \langle 0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | 0 \rangle$; S_{ij} $g \to \infty$ $\sim e^{-|i-j|/\xi}$ Quantum Paramagnet

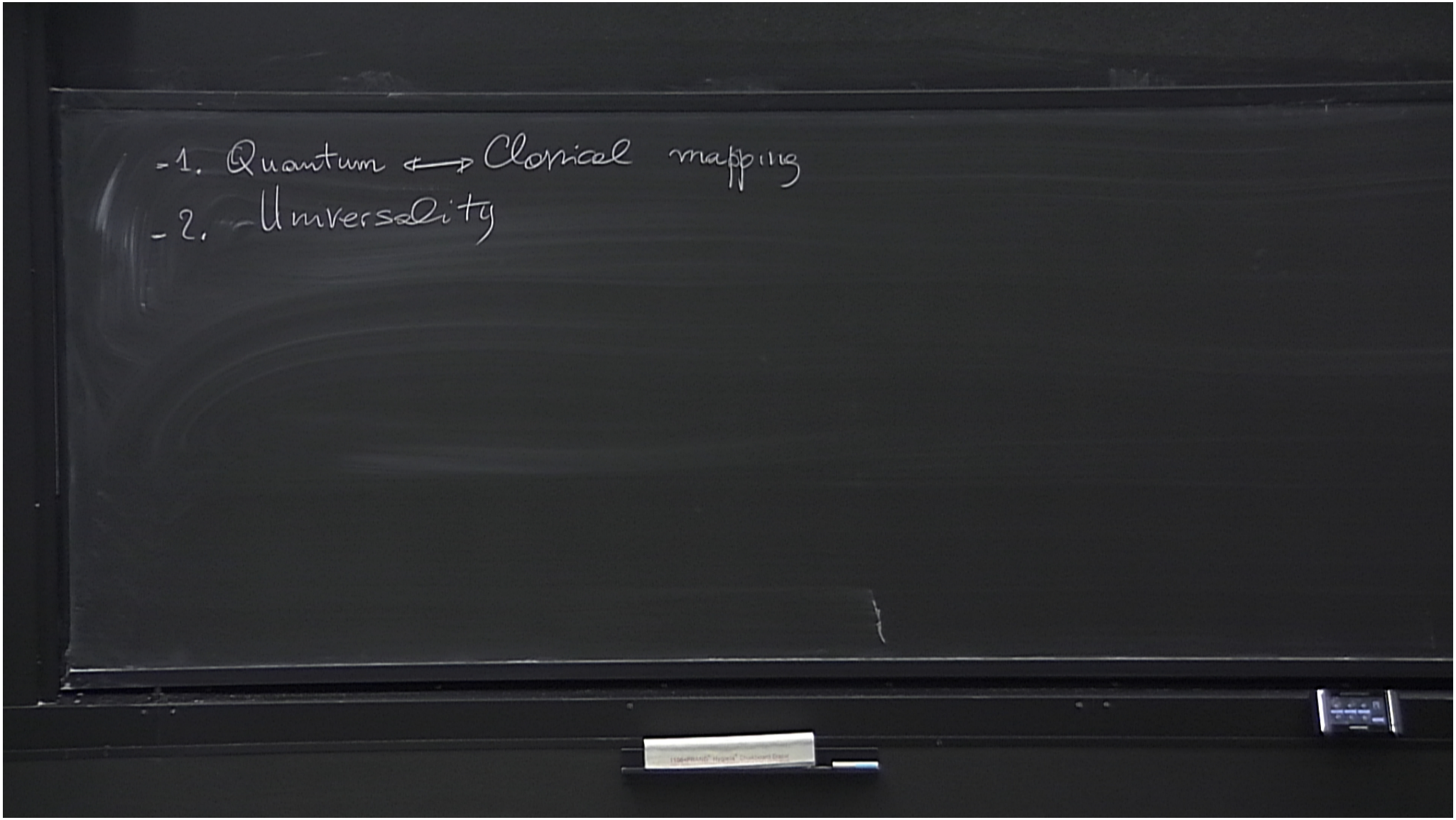
$\langle 0 | \hat{\sigma}_i^z | 0 \rangle \overset{1/g}{\sim} 0$

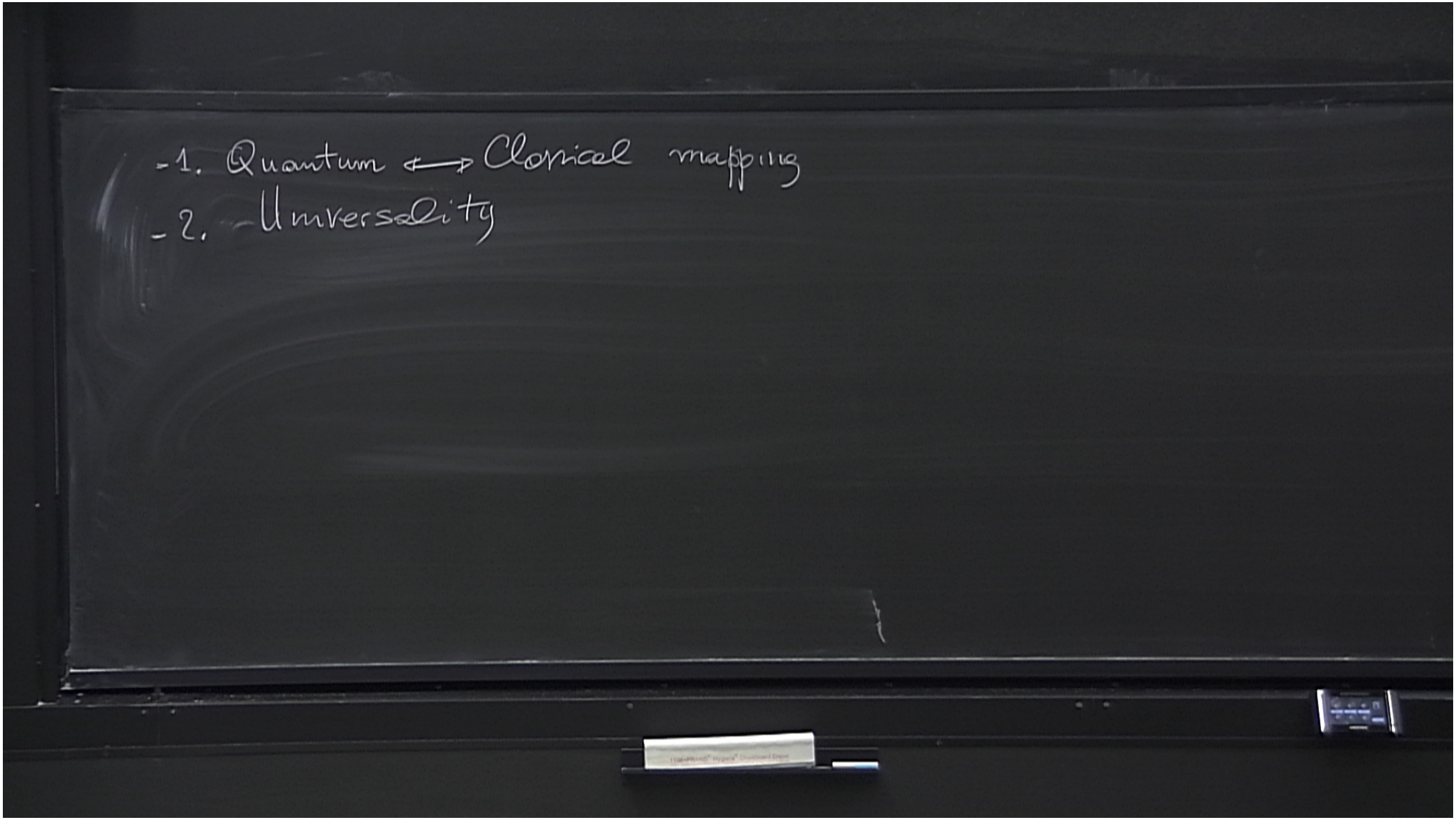
— small g phase: $C_{ij} \sim N_0^2(g)$ long range order; $|\uparrow\uparrow\ldots\uparrow\rangle$ $|\downarrow\ldots\downarrow\rangle$ degeneracy

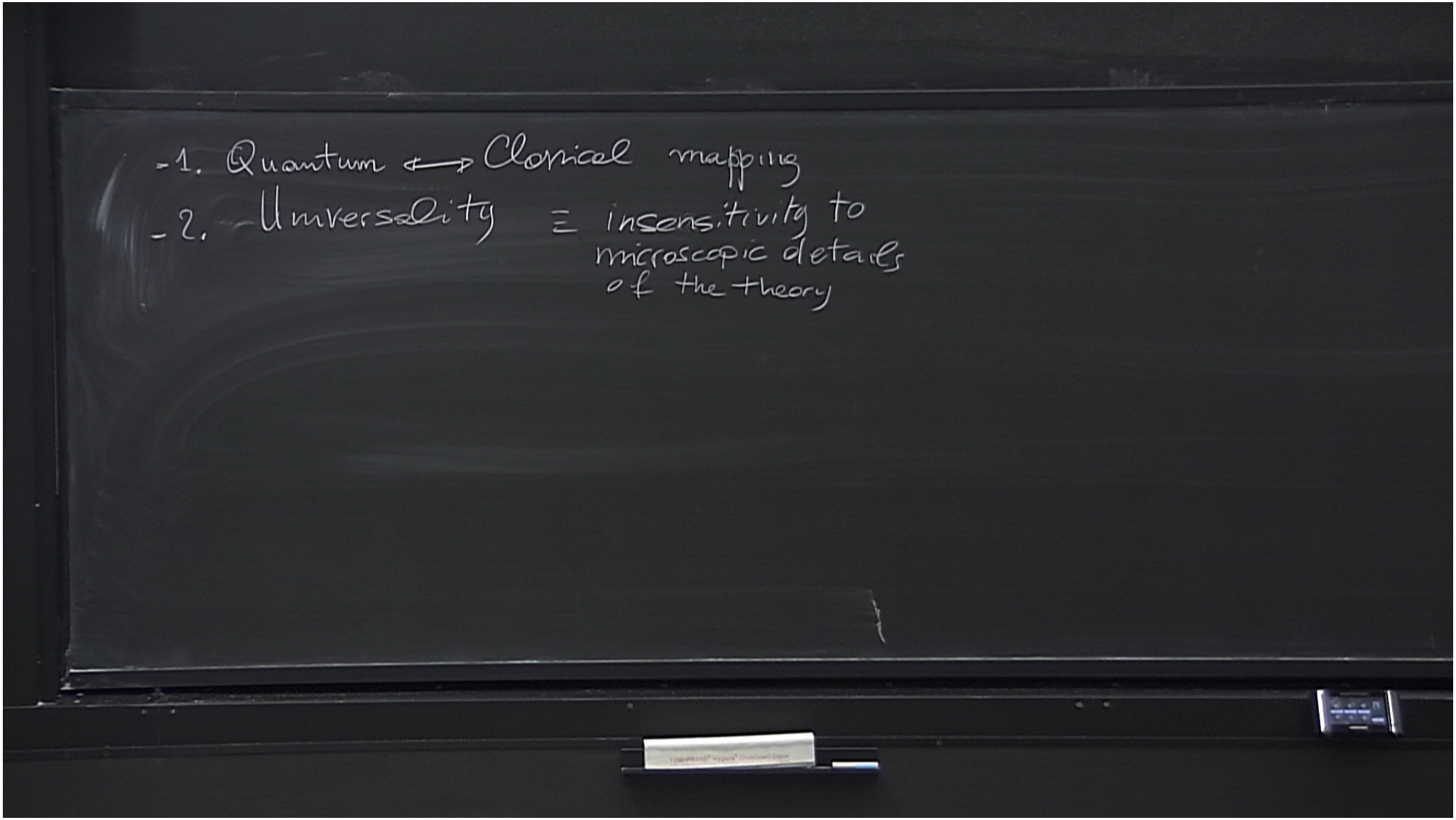
Duality (mapping from breaking of $\mathbb{Z}_2$ symmetry $\otimes_i \hat{\sigma}_i^x$ original lattice to Dual lattice) $(\sigma, i) \to (\tau, \mu)$

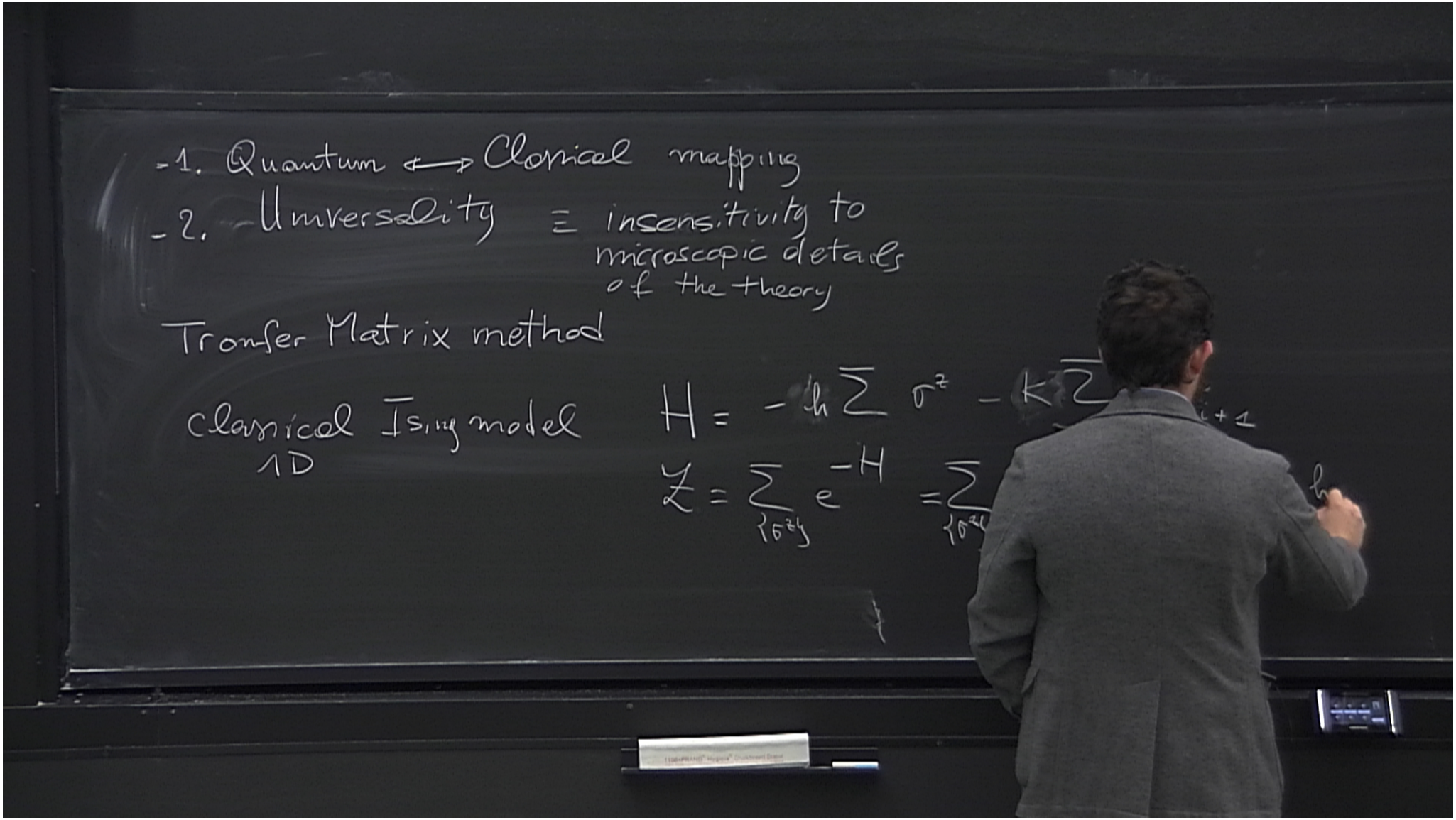
$H_I(\sigma, g) = g H_I(\tau, \frac{1}{g})$ self duality $\Rightarrow g_c = 1$

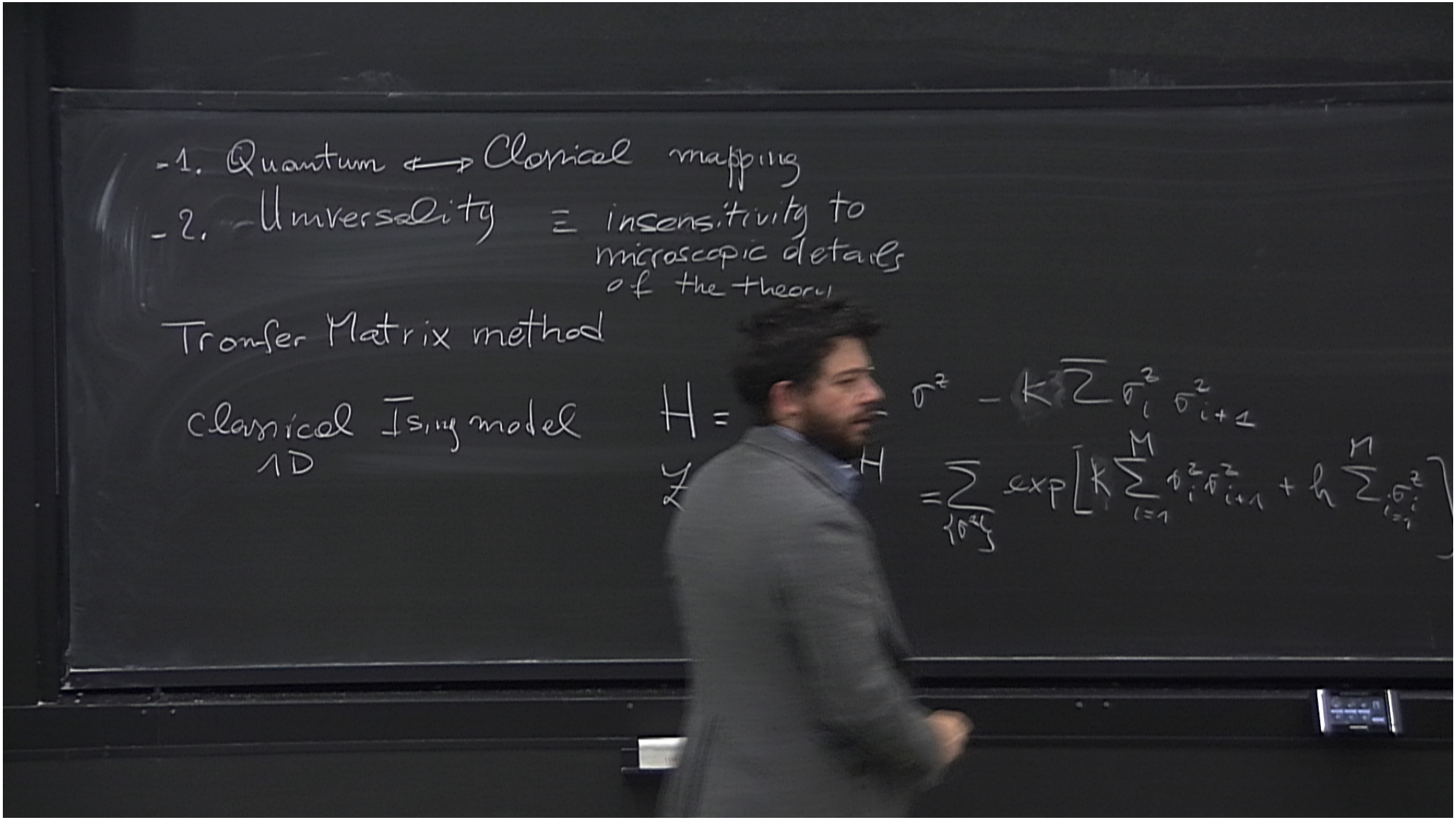

-1. Quantum ⟷ Classical mapping
-2. Univers


- 1. Quantum ⟷ Classical mapping
- 2. Universality


-1. Quantum ⟷ Classical mapping
-2. Universality


-1. Quantum ⟷ Classical mapping
-2. Universality ≡ insensitivity to microscopic details of the theory


-1. Quantum ⟷ Classical mapping
-2. Universality ≡ insensitivity to microscopic details of the theory
Transfer Matrix method
Classical Ising model
1D
$H = -h\sum \sigma^z - K\sum$
$Z = \sum_{\{\sigma^z\}} e^{-H} = \sum$


-1. Quantum ⟷ Classical mapping
-2. Universality ≡ insensitivity to microscopic details of the theory
Transfer Matrix method
Classical Ising model
1D
H =
Z
H
$-k\sum \sigma_i^z \sigma_{i+1}^z$
$= \sum_{\{\sigma_i\}} \exp\left[k \sum_{i=1}^{M} \sigma_i^z \sigma_{i+1}^z + h \sum_{i=1}^{M} \sigma_i^z\right]$

- 1. Quantum $\longleftrightarrow$ Classical mapping
- 2. Universality $\equiv$ insensitivity to microscopic details of the theory

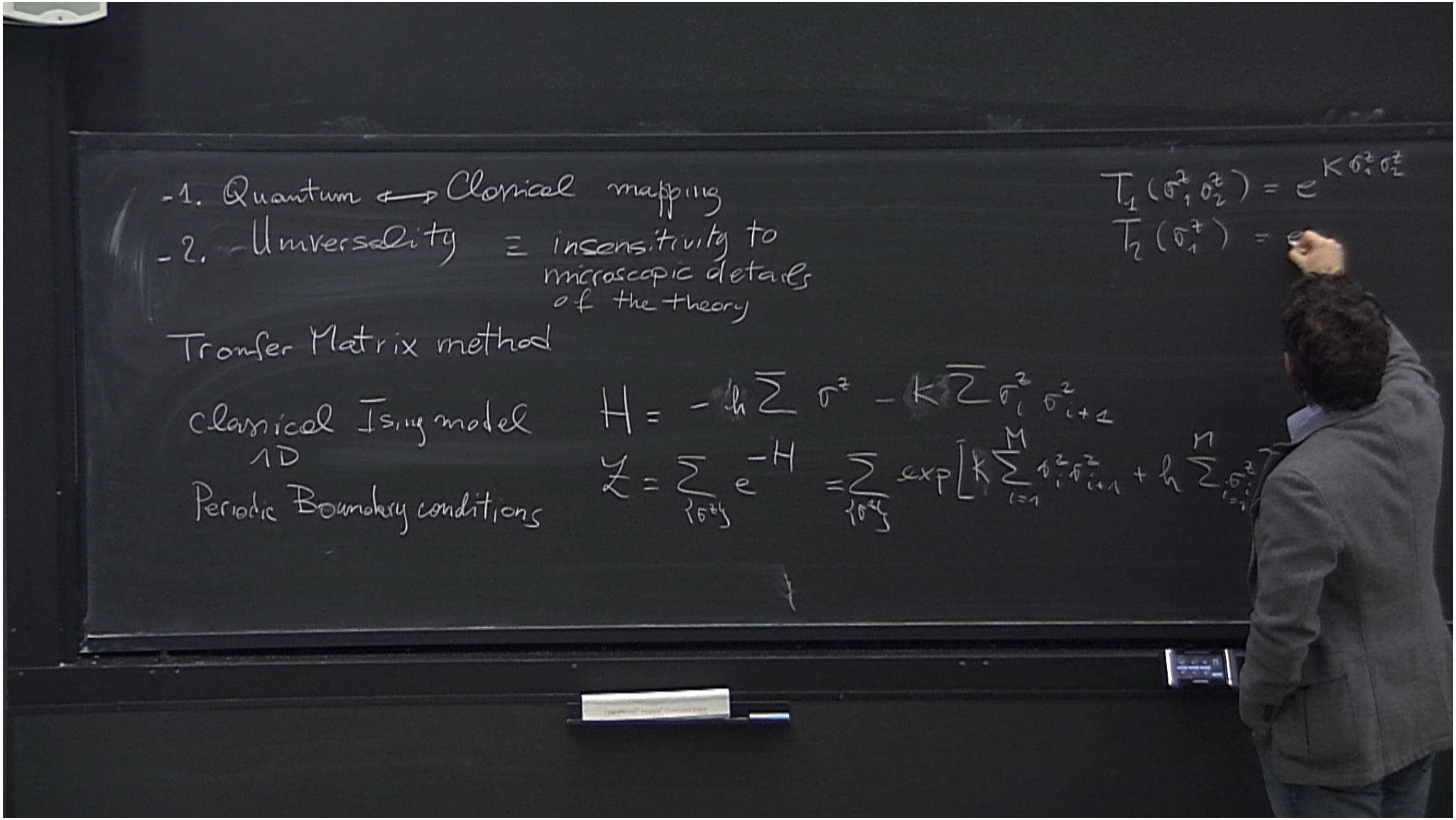
Transfer Matrix method

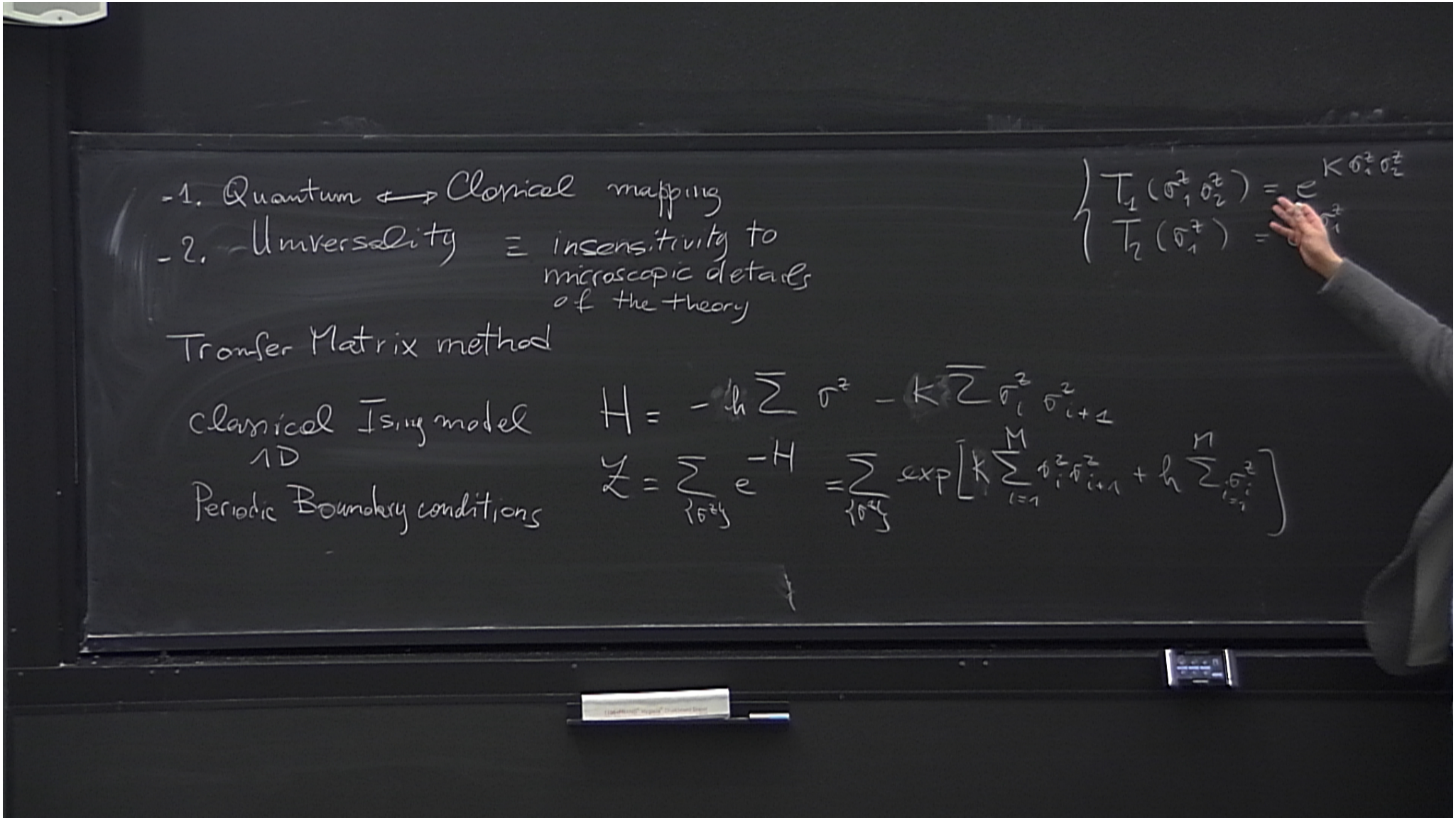
Classical Ising model 1D

Periodic Boundary conditions

$$H = -h\sum \sigma^z - K\sum \sigma_i^z \sigma_{i+1}^z$$

$$Z = \sum_{\{\sigma^z\}} e^{-H} = \sum_{\{\sigma^z\}} \exp\left[K\sum_{i=1}^{M} \sigma_i^z \sigma_{i+1}^z + h\sum_{i=1}^{M} \sigma_i^z\right]$$


-1. Quantum ⟷ Classical mapping
-2. Universality ≡ insensitivity to microscopic details of the theory
Transfer Matrix method
classical Ising model 1D
Periodic Boundary conditions
$T_1(\sigma_1^z, \sigma_2^z) = e^{K\sigma_1^z\sigma_2^z}$
$T_2(\sigma_1^z) =$
$H = -h\sum \sigma^z - K\sum \sigma_i^z \sigma_{i+1}^z$
$Z = \sum_{\{\sigma^z\}} e^{-H} = \sum_{\{\sigma^z\}} \exp\left[K\sum_{i=1}^{M} \sigma_i^z\sigma_{i+1}^z + h\sum_{i=1}^{M} \sigma_i^z\right.$


-1. Quantum ⟷ Classical mapping
-2. Universality ≡ insensitivity to microscopic details of the theory
Transfer Matrix method
Classical Ising model
1D
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$T_1(\sigma_1^z, \sigma_2^z) = e^{K\sigma_1^z \sigma_2^z}$
$T_2(\sigma_1^z) =$

- 1. Quantum $\longleftrightarrow$ Classical mapping
- 2. Universality $\equiv$ insensitivity to microscopic details of the theory

$$\begin{cases} T_1(\sigma_1^z, \sigma_2^z) = e^{K\sigma_1^z\sigma_2^z} \\ T_2(\sigma_1^z) = e^{h\sigma_1^z} \end{cases}$$

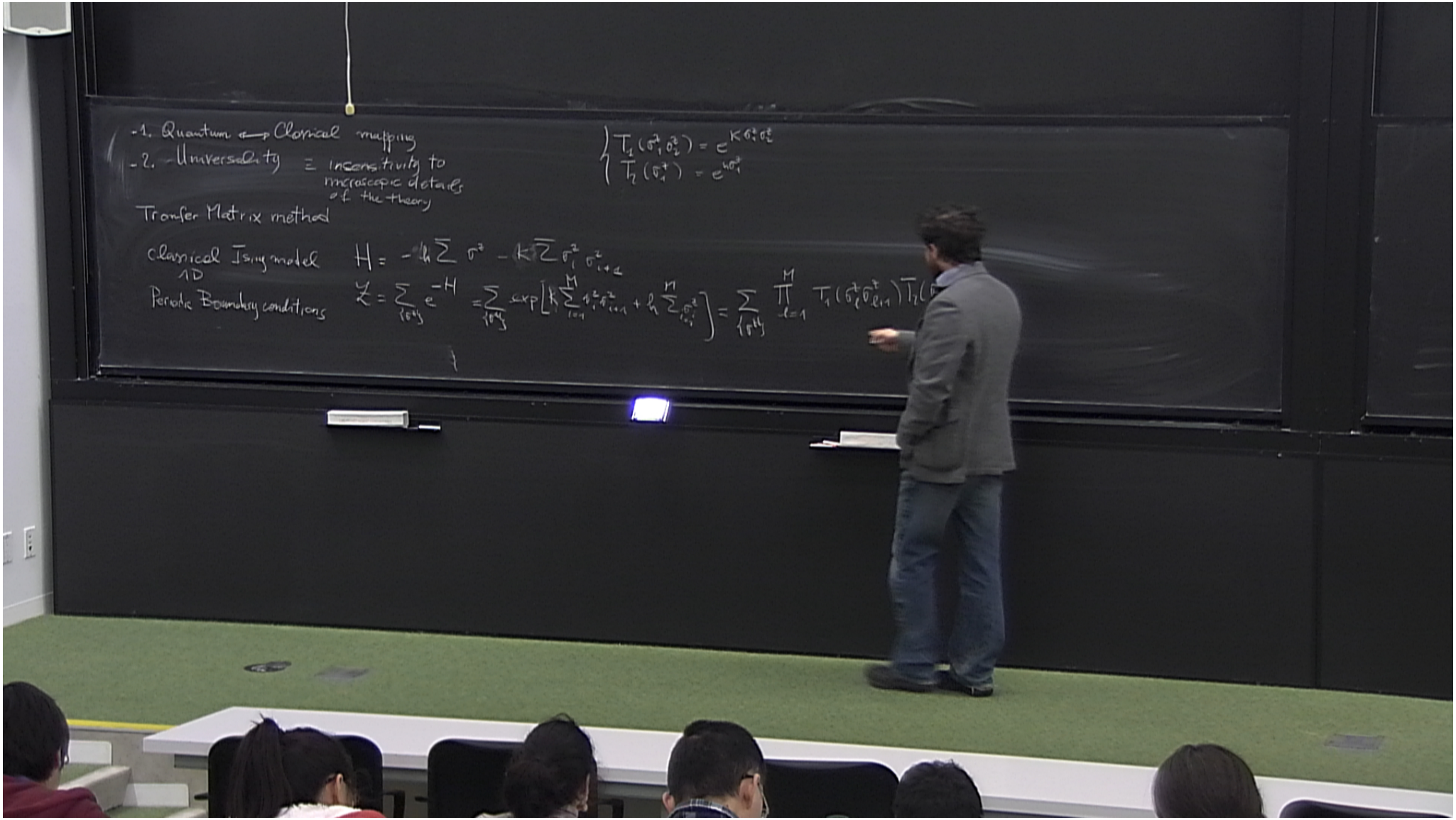
Transfer Matrix method

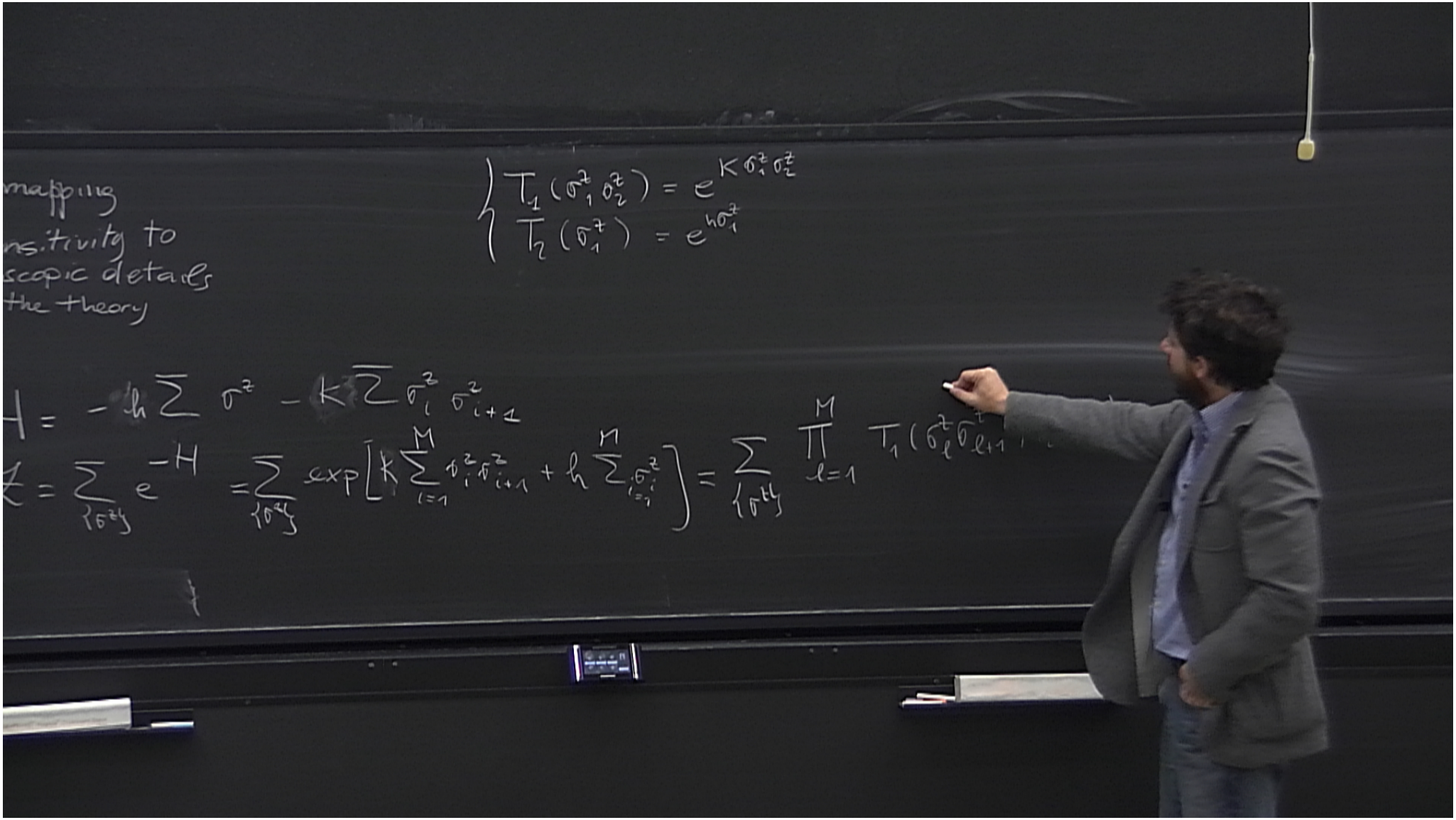
Classical Ising model 1D

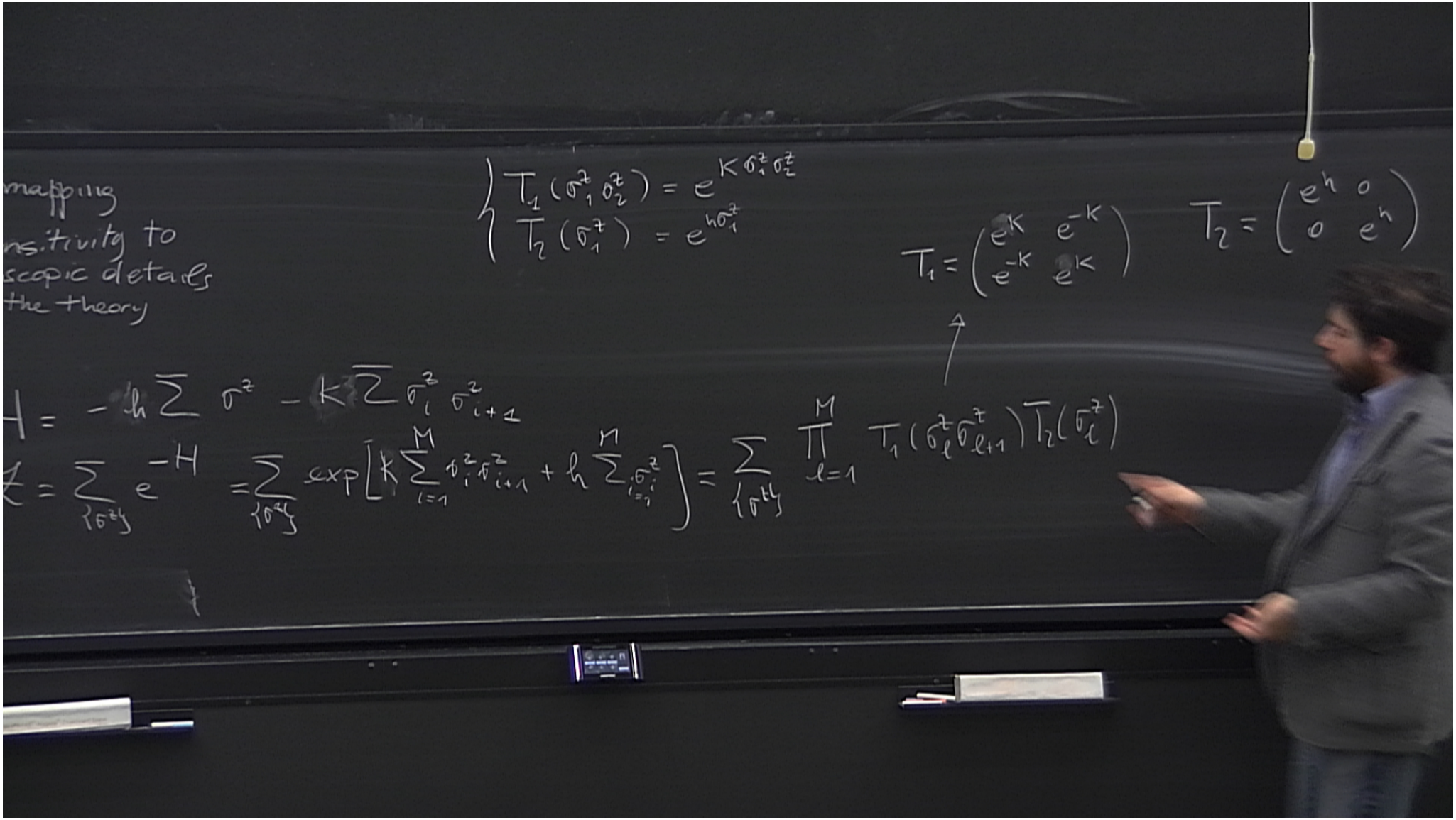
Periodic Boundary conditions

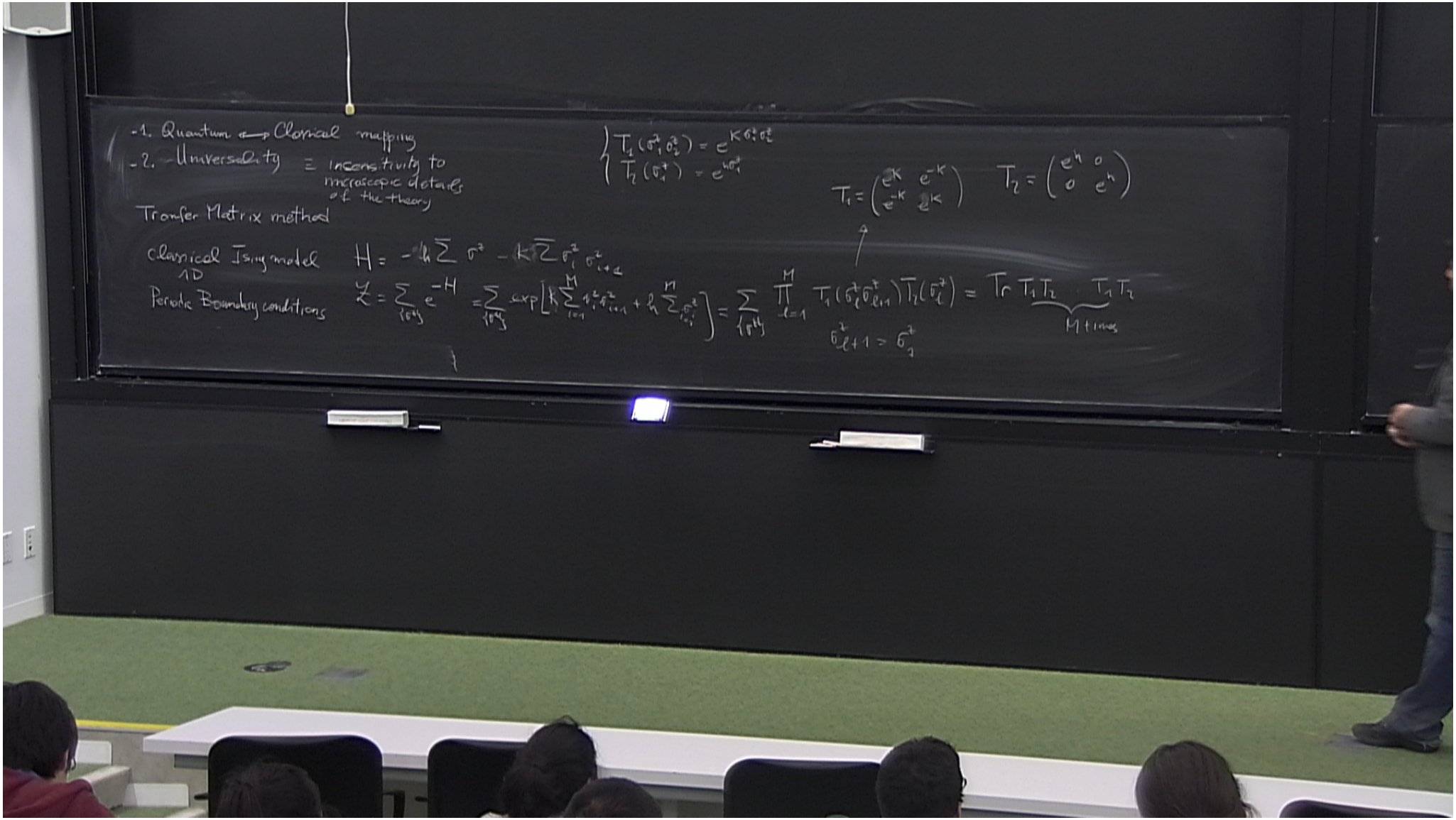
$$H = -h\sum \sigma^z - K\sum \sigma_i^z \sigma_{i+1}^z$$

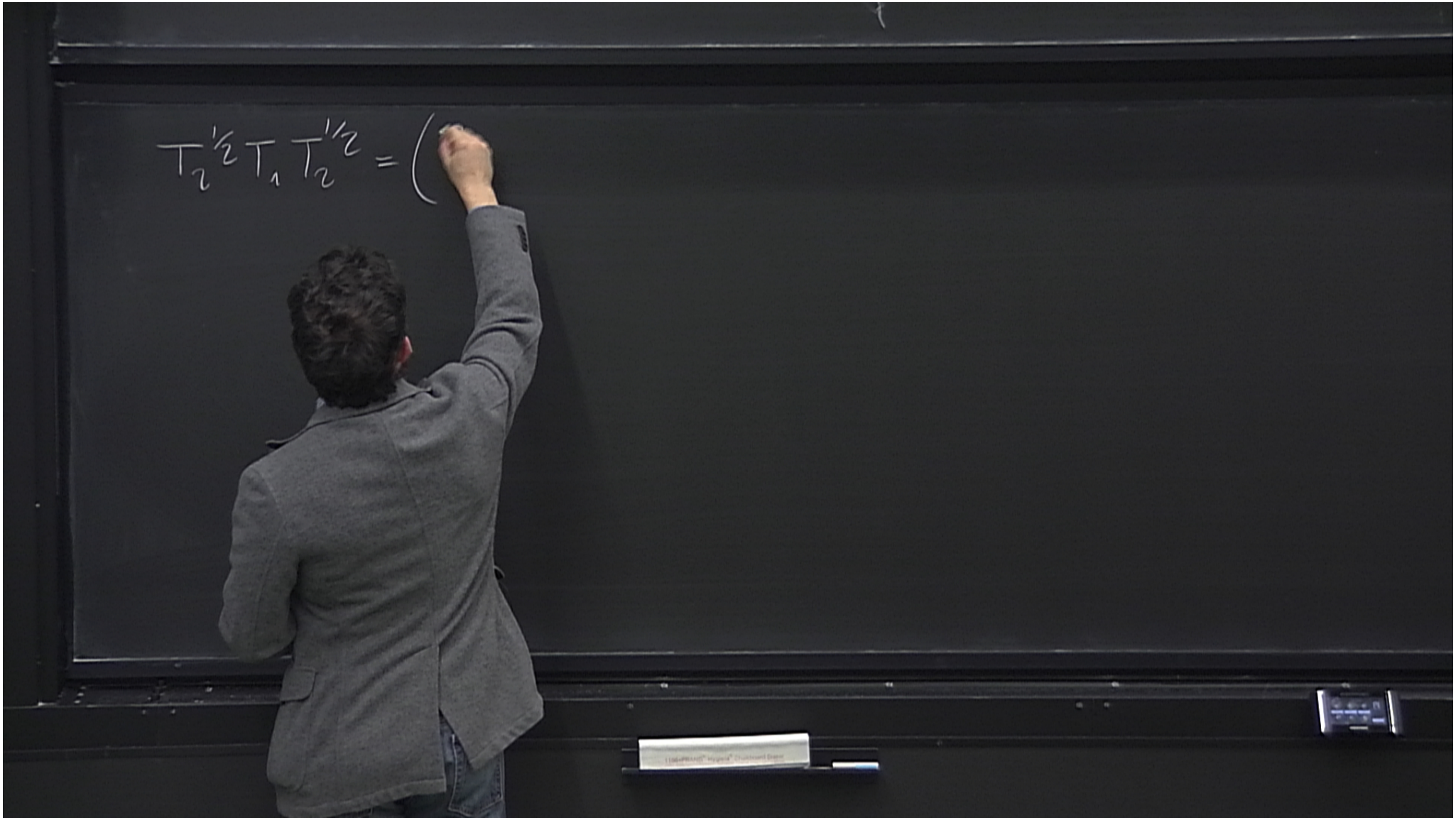
$$Z = \sum_{\{\sigma^z\}} e^{-H} = \sum_{\{\sigma^z\}} \exp\left[K\sum_{i=1}^{M} \sigma_i^z\sigma_{i+1}^z + h\sum_{i=1}^{M}\sigma_i^z\right] = \sum_{\{\sigma^z\}}$$

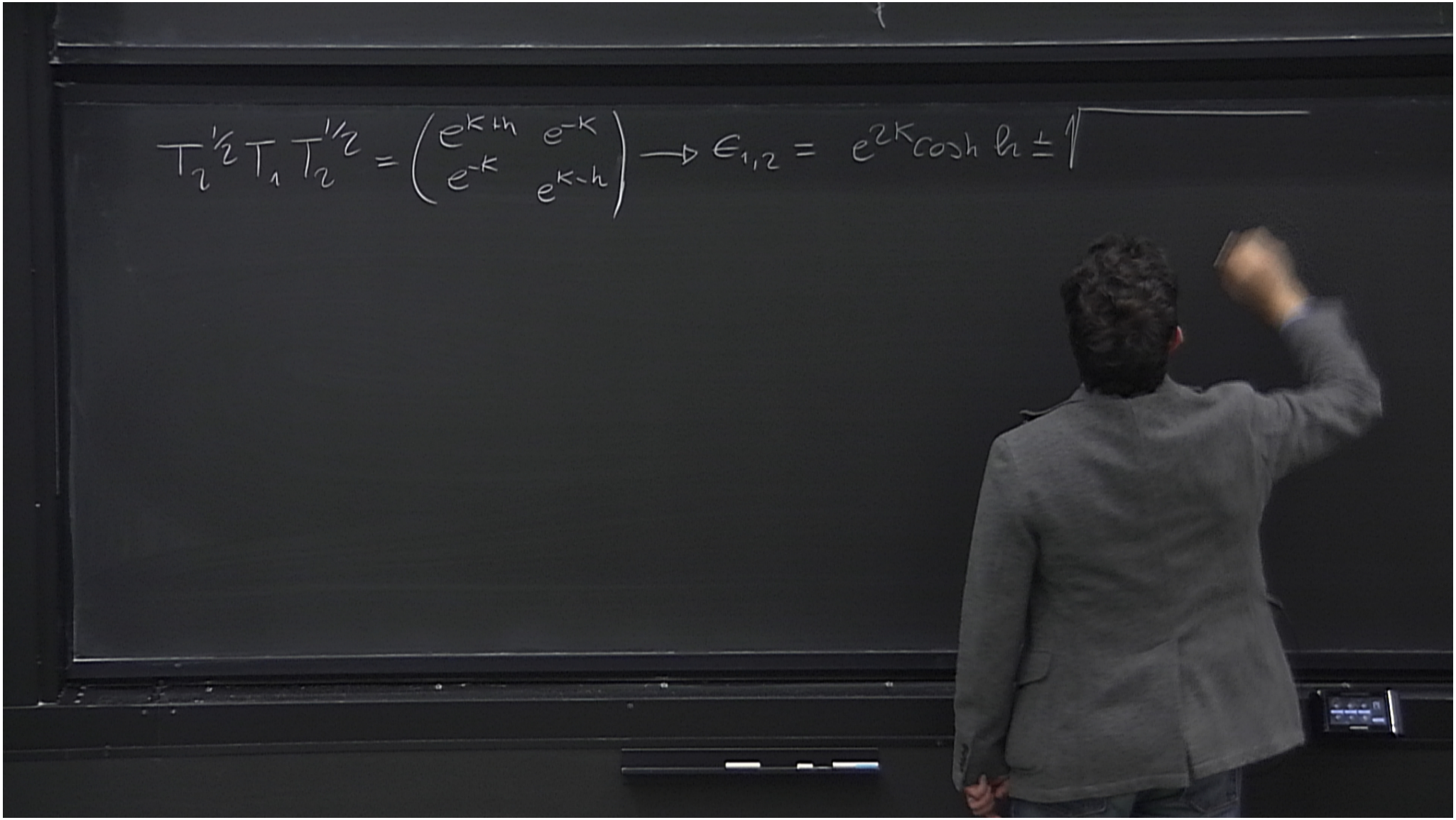

-1. Quantum ⟷ Clonical mapping
-2. Universality ≡ insensitivity to microscopic details of the theory
Transfer Matrix method
Clonical Ising model 1D
Periodic Boundary conditions

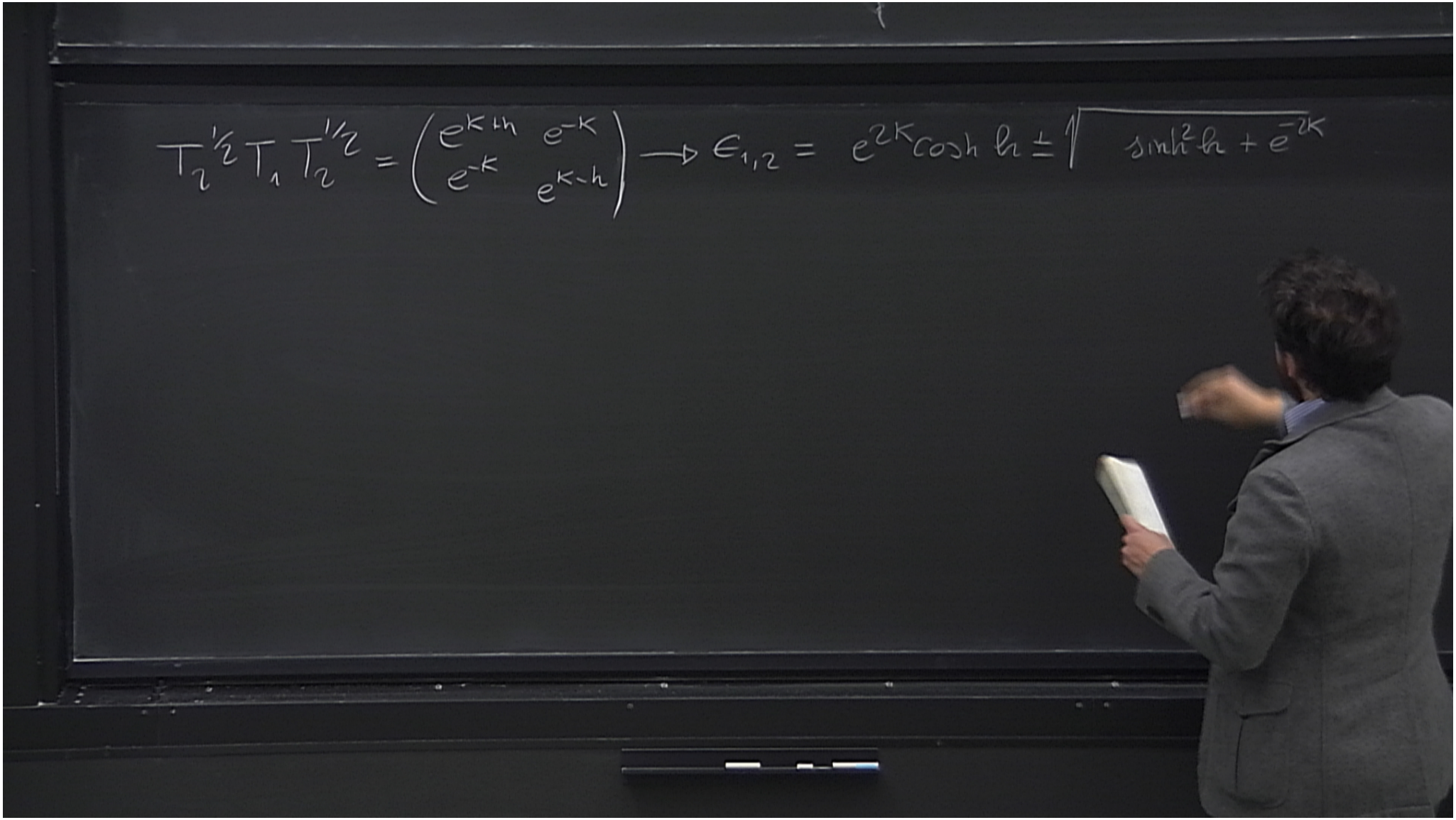

mapping
nsitivity to
scopic details
the theory
$\begin{cases} T_1(\sigma_1^z, \sigma_2^z) = e^{K\sigma_1^z\sigma_2^z} \\ T_2(\sigma_1^z) = e^{h\sigma_1^z} \end{cases}$
$H = -h\sum \sigma^z - K\sum \sigma_i^z \sigma_{i+1}^z$
$Z = \sum_{\{\sigma^z\}} e^{-H} = \sum_{\{\sigma^z\}} \exp\left[K\sum_{i=1}^{M} \sigma_i^z\sigma_{i+1}^z + h\sum_{i=1}^{M}\sigma_i^z\right] = \sum_{\{\sigma^z\}} \prod_{l=1}^{M} T_1(\sigma_l^z\sigma_{l+1}^z$

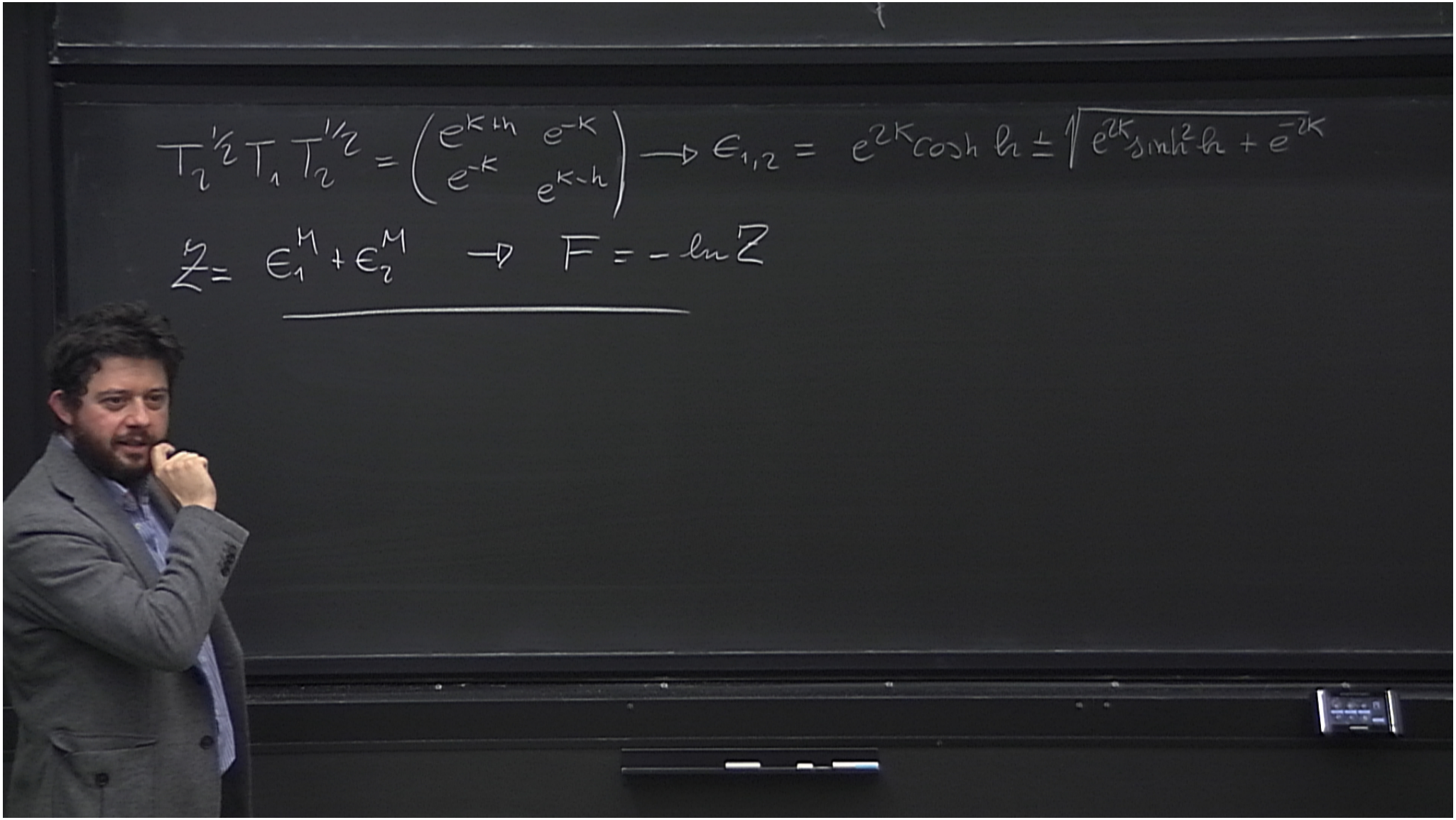

mapping
nsitivity to
scopic details
the theory
$\begin{cases} T_1(\sigma_1^z \sigma_2^z) = e^{K \sigma_1^z \sigma_2^z} \\ T_2(\sigma_1^z) = e^{h \sigma_1^z} \end{cases}$
$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix}$
$T_2 = \begin{pmatrix} e^h & 0 \\ 0 & e^h \end{pmatrix}$
$H = -h \sum \sigma^z - K \sum \sigma_i^z \sigma_{i+1}^z$
$Z = \sum_{\{\sigma^z\}} e^{-H} = \sum_{\{\sigma^z\}} \exp\left[K \sum_{i=1}^{M} \sigma_i^z \sigma_{i+1}^z + h \sum_{i=1}^{M} \sigma_i^z\right] = \sum_{\{\sigma^z\}} \prod_{l=1}^{M} T_1(\sigma_l^z \sigma_{l+1}^z) T_2(\sigma_l^z)$


-1. Quantum ⟷ Clonical mapping
-2. Universality = insensitivity to microscopic details of the theory
Transfer Matrix method
Classical Ising model 1D
Periodic Boundary conditions
Tr T1 T2 ... T1 T2
M times


$T_2^{1/2} T_1 T_2^{1/2} = ($


$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K} \cosh h \pm \sqrt{}$

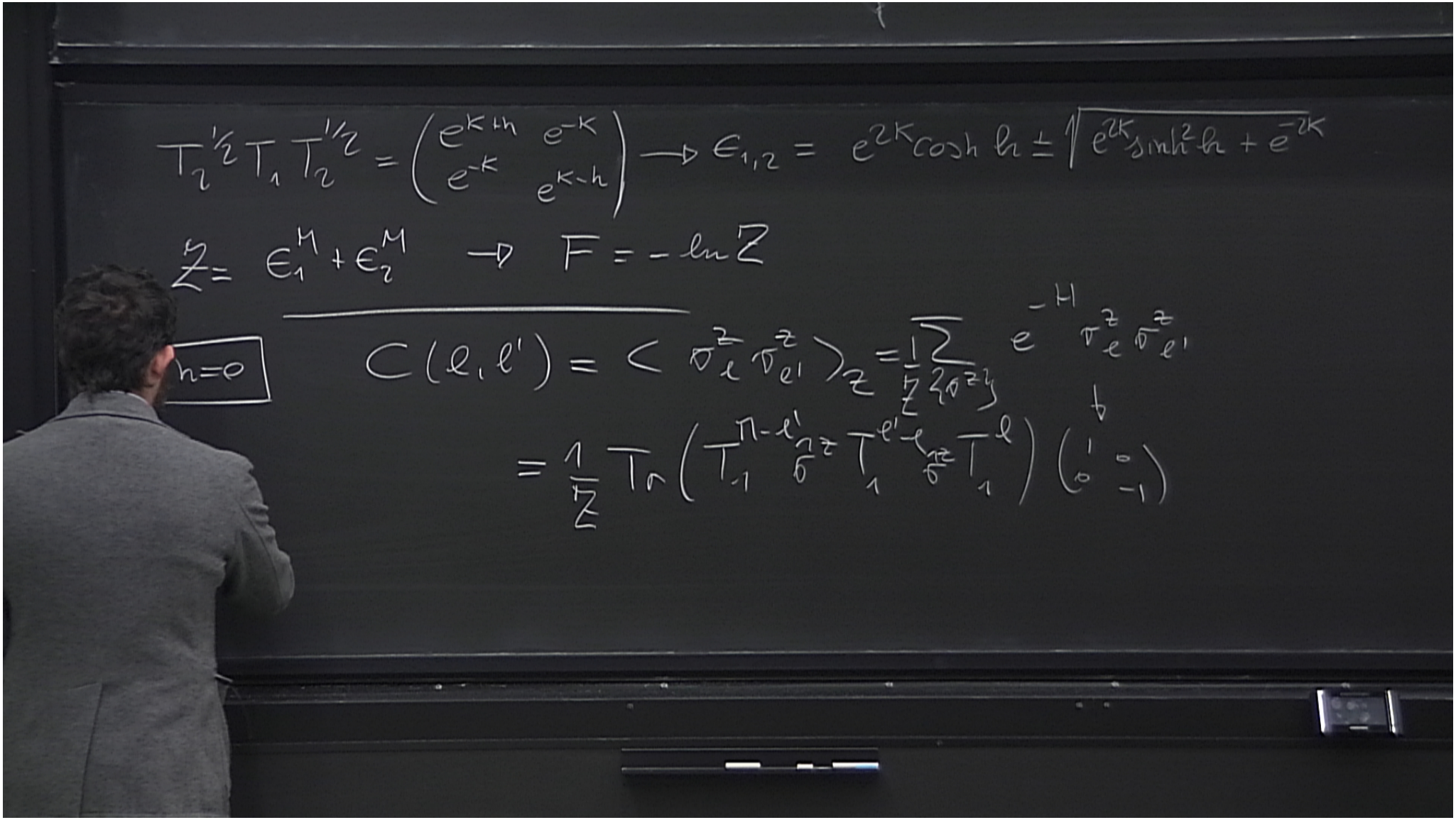

$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K} \cosh h \pm \sqrt{\sinh^2 h + e^{-2K}}$

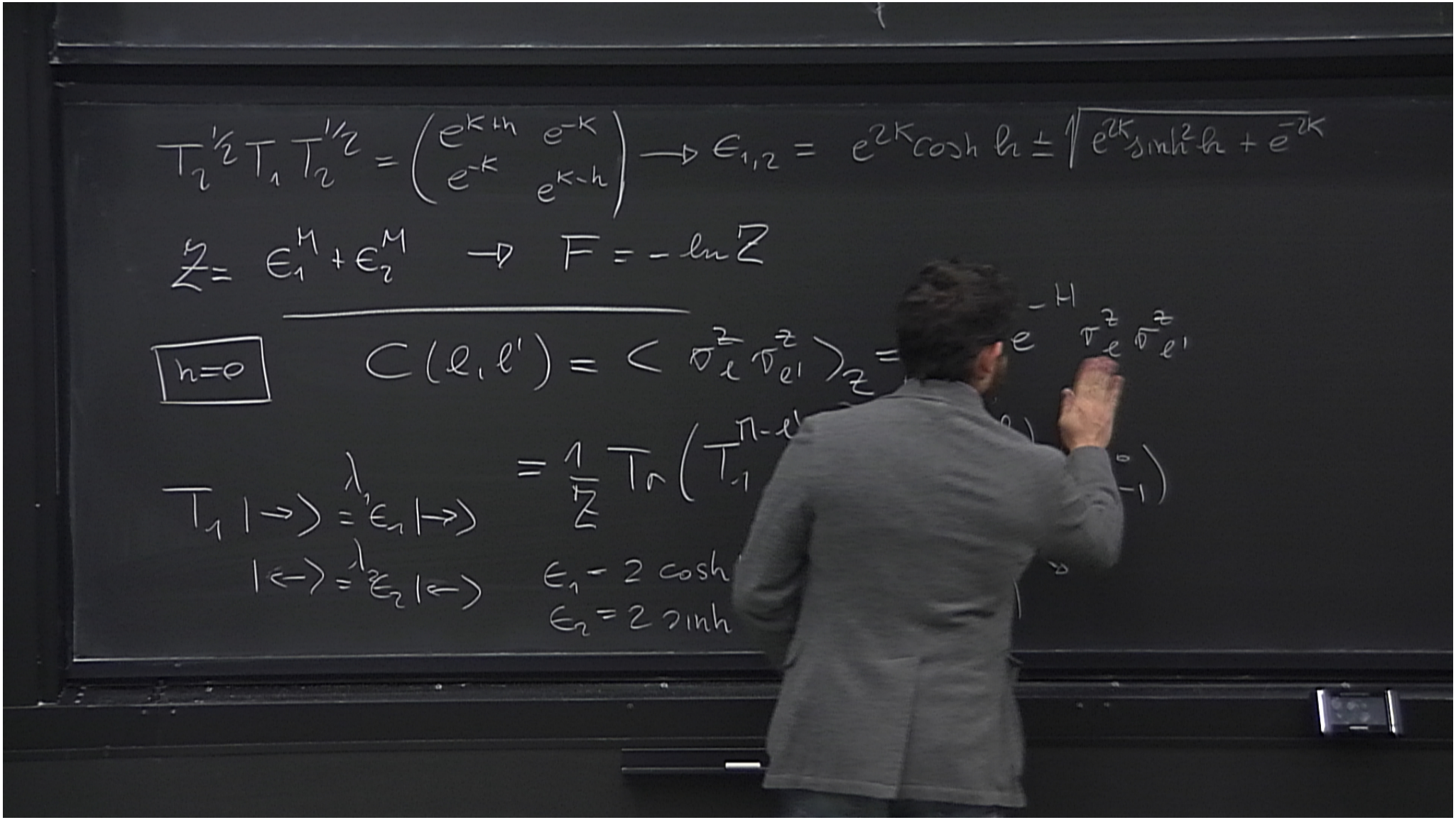

$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$
$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$

$$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \longrightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$$

$$Z = \epsilon_1^M + \epsilon_2^M \longrightarrow F = -\ln Z$$

$\boxed{h=0}$


$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$
$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$
$h=0$
$C(l,l') = \langle \sigma_l^z \sigma_{l'}^z \rangle_Z = \frac{1}{Z}\sum e^{-H} \sigma_l^z \sigma_{l'}^z$
$= \frac{1}{Z} \mathrm{Tr}\left(T_{-1}^{n-l'} \sigma^z T_1^{l'-l} \sigma^z T_1^{l} \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$


$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$
$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$
$h=0$
$C(\ell,\ell') = \langle \sigma_\ell^z \sigma_{\ell'}^z \rangle_Z =$
e^{-H}
$\sigma_\ell^z \sigma_{\ell'}^z$
$= \frac{1}{Z} \mathrm{Tr}\left(T_1^{\ell'-\ell} \right)$
$T_1 |\rightarrow\rangle = \lambda_1 \epsilon_1 |\rightarrow\rangle$
$|\leftarrow\rangle = \lambda_2 \epsilon_2 |\leftarrow\rangle$
$\epsilon_1 = 2\cosh$
$\epsilon_2 = 2\sinh$

$$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \longrightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{4K}\sinh^2 h + e^{-2K}}$$

$$Z = \epsilon_1^M + \epsilon_2^M \longrightarrow F = -\ln Z$$

$h=0$ $\qquad C(\ell,\ell') = \langle \sigma_\ell^z \sigma_{\ell'}^z \rangle_Z = \frac{1}{Z}\sum_{\{\sigma^z\}} e^{-H} \sigma_\ell^z \sigma_{\ell'}^z$

$$= \frac{1}{Z} \mathrm{Tr}\left(T_1^{n-\ell'} \sigma^z T_1^{\ell'-\ell} \sigma^z T_1^{\ell} \right)$$

$T_1 |\rightarrow\rangle = \lambda_1 |\rightarrow\rangle = \epsilon_1 |\rightarrow\rangle$

$|\leftarrow\rangle = \lambda_2 |\leftarrow\rangle = \epsilon_2 |\leftarrow\rangle$

$\epsilon_1 = 2\cosh K$

$\epsilon_2 = 2\sinh K$

$$\begin{pmatrix} \epsilon_1^{n-\ell'} & \\ & \epsilon_2^{n-\ell'} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$$

$$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$$

$h=0$

$$C(l,l') = \langle \sigma_l^z \sigma_{l'}^z \rangle_Z = \frac{1}{Z}\sum e^{-H} \sigma_l^z \sigma_{l'}^z$$

$$= \frac{1}{Z} \mathrm{Tr}\left(T_1^{n-l'} \sigma^z T_1^{l'-l} \sigma^z T_1^{l} \right) =$$

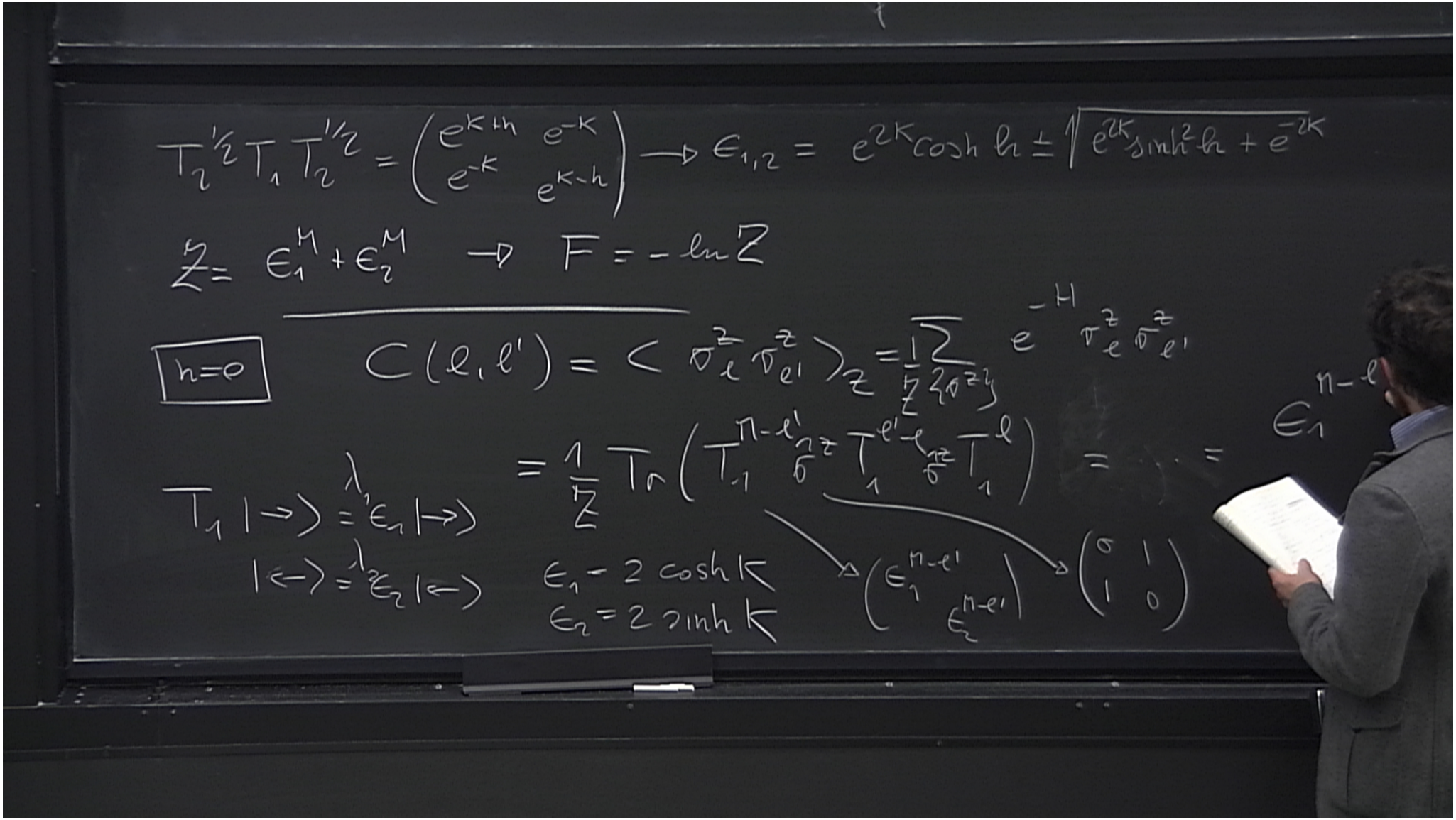
$$T_1 |\rightarrow\rangle = \lambda_1 |\rightarrow\rangle \qquad \lambda_1 = \epsilon_1$$

$$|\leftarrow\rangle = \lambda_2 |\leftarrow\rangle \qquad \lambda_2 = \epsilon_2$$

$$\epsilon_1 = 2\cosh K$$

$$\epsilon_2 = 2\sinh K$$

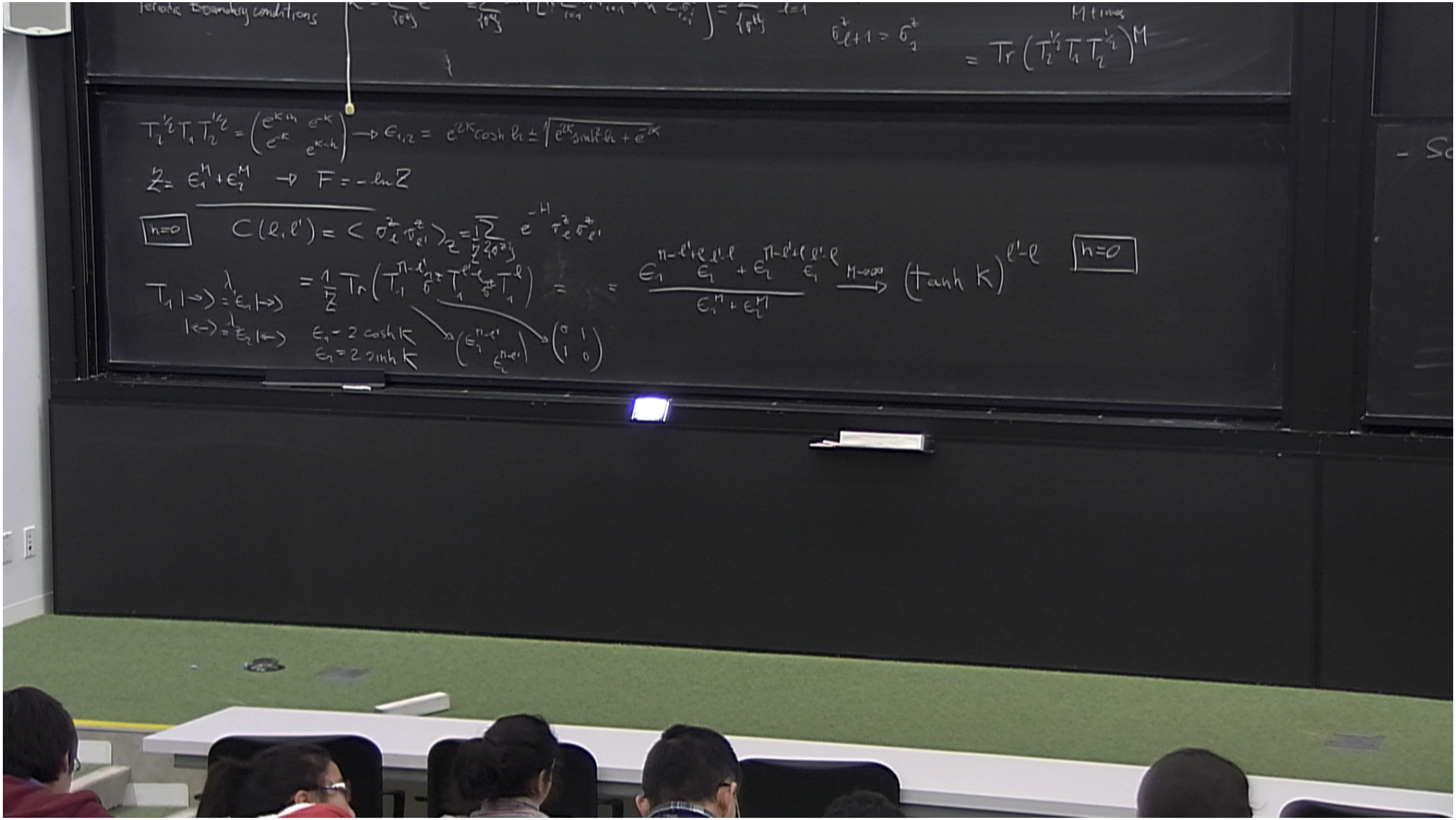
$$\begin{pmatrix} \epsilon_1^{n-l'} & \\ & \epsilon_2^{n-l'} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$


$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$
$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$
$h=0$
$C(l,l') = \langle \sigma_l^z \sigma_{l'}^z \rangle_Z = \frac{1}{Z}\sum e^{-H} \sigma_l^z \sigma_{l'}^z$
$= \frac{1}{Z} \mathrm{Tr}\left(T_1^{n-l'} \sigma^z T_1^{l'-l} \sigma^z T_1^{l} \right) = \ldots = \epsilon_1^{n-l}$
$T_1 |\rightarrow\rangle = \lambda_1 |\rightarrow\rangle = \epsilon_1 |\rightarrow\rangle$
$|\leftarrow\rangle = \lambda_2 |\leftarrow\rangle = \epsilon_2 |\leftarrow\rangle$
$\epsilon_1 = 2\cosh K$
$\epsilon_2 = 2\sinh K$
$\begin{pmatrix} \epsilon_1^{n-l'} & \\ & \epsilon_2^{n-l'} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$+ e^{-2K}$

$$\frac{\epsilon_1^{n-\ell'+\ell}\,\epsilon_2^{\ell'-\ell} + \epsilon_2^{n-\ell'+\ell}\,\epsilon_1^{\ell'-\ell}}{\epsilon_1^M + \epsilon_2^M} \xrightarrow{M\to\infty} (\tanh K)^{\ell'-\ell}$$

$h=0$


Periodic Boundary conditions
M times
$\sigma^z_{l+1} = \sigma^z_1$
$= \mathrm{Tr}\left(T_2^{1/2} T_1 T_2^{1/2}\right)^M$
$T_1^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{4K}\sinh^2 h + e^{-2K}}$
$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$
h=0
$C(l,l') = \langle \sigma^z_l \sigma^z_{l'} \rangle_Z = \frac{1}{Z}\sum e^{-H} \sigma^z_l \sigma^z_{l'}$
$T_1 |\rightarrow\rangle = \lambda_1 |\rightarrow\rangle$
$|\leftarrow\rangle = \lambda_2 |\leftarrow\rangle$
$\epsilon_1 = 2\cosh K$
$\epsilon_2 = 2\sinh K$
$= \frac{1}{Z}\mathrm{Tr}\left(T_1^{\Pi-l'} \sigma^z T_1^{l'-l} \sigma^z T_1^{l}\right) = \begin{pmatrix} \epsilon_1^{\Pi-l'} & \\ & \epsilon_2^{\Pi-l'} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$= \frac{\epsilon_1^{\Pi-l'+l} \epsilon_1^{l'-l} + \epsilon_2^{\Pi-l'+l} \epsilon_1^{l'-l}}{\epsilon_1^{\Pi} + \epsilon_2^{\Pi}} \xrightarrow{M\to\infty} (\tanh K)^{l'-l}$
h=0

$$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{2K}\sinh^2 h + e^{-2K}}$$

$$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$$

$h=0$ $\quad C(\ell,\ell') = \langle \sigma_\ell^z \sigma_{\ell'}^z \rangle_Z = \frac{1}{Z}\sum_{\{\sigma^z\}} e^{-H} \sigma_\ell^z \sigma_{\ell'}^z$

$$= \frac{1}{Z} \mathrm{Tr}\left(T_1^{n-\ell'} \sigma^z T_1^{\ell'-\ell} \sigma^z T_1^{\ell} \right) = \ldots = \frac{\epsilon_1^{n-\ell'+\ell} \epsilon_2^{\ell'}}{\epsilon_1^n}$$

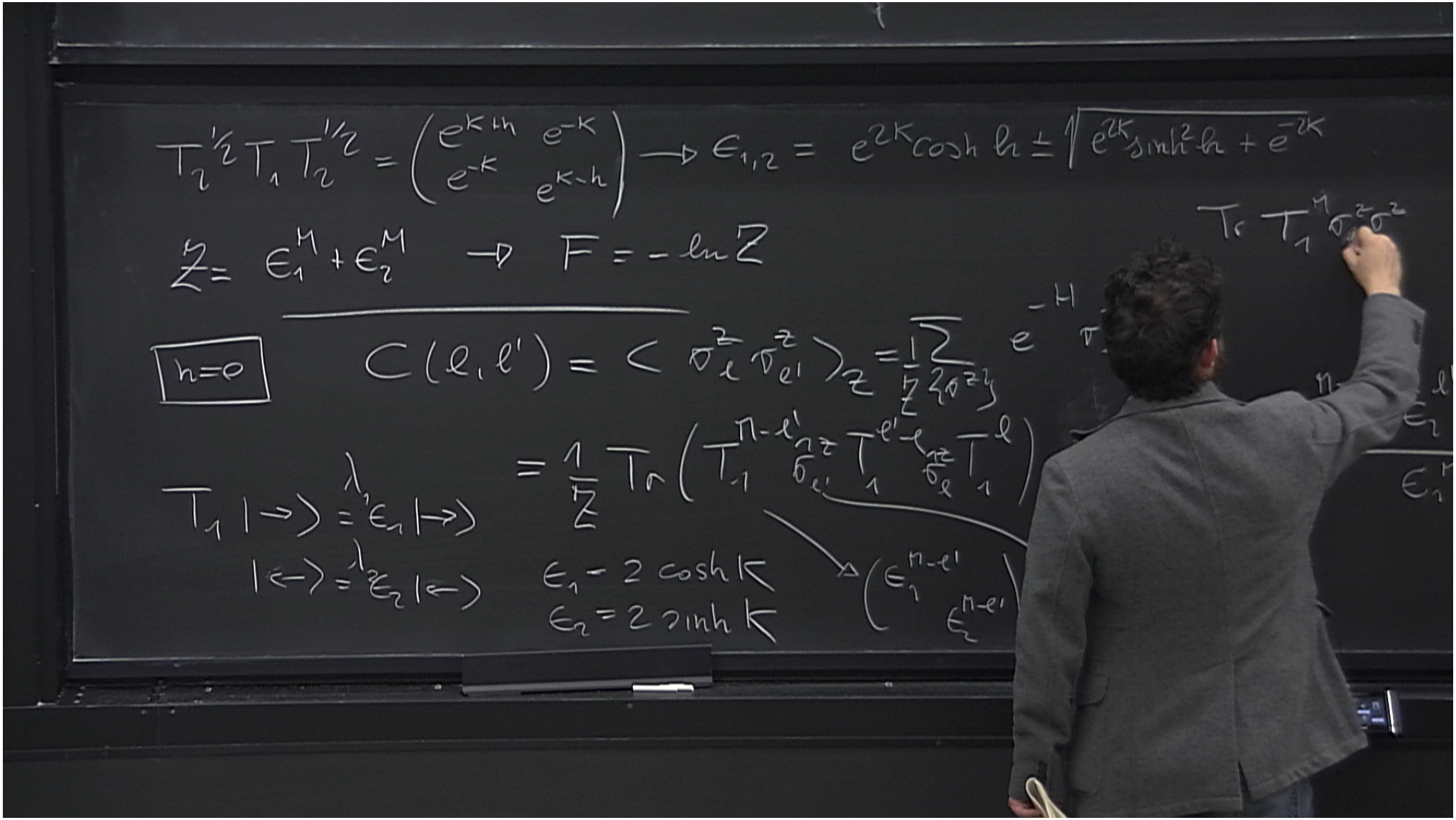
$T_1 |\rightarrow\rangle = \lambda_1 |\rightarrow\rangle = \epsilon_1 |\rightarrow\rangle$

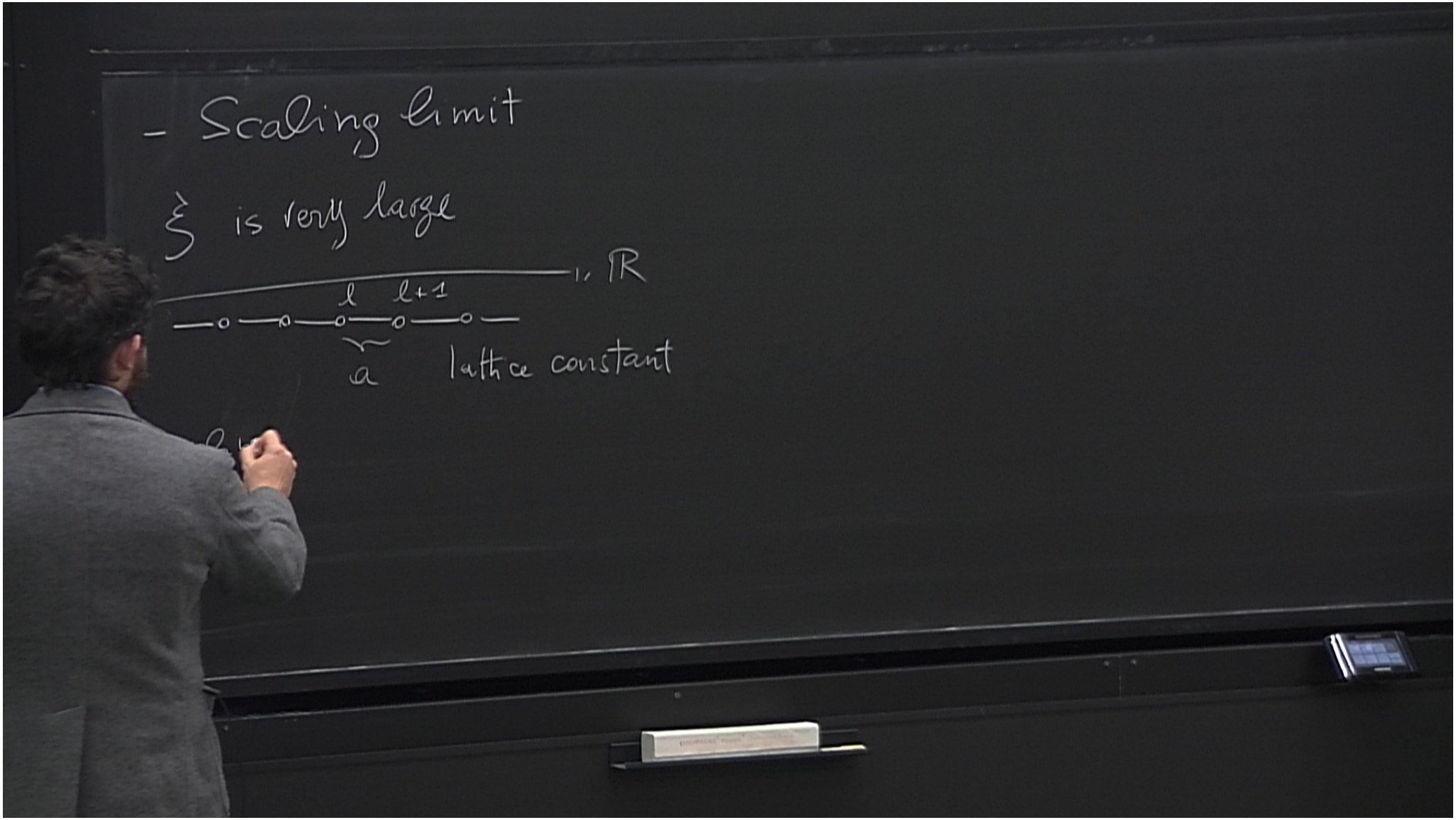
$|\leftarrow\rangle = \lambda_2 |\leftarrow\rangle = \epsilon_2 |\leftarrow\rangle$

$\epsilon_1 = 2\cosh K$

$\epsilon_2 = 2\sinh K$

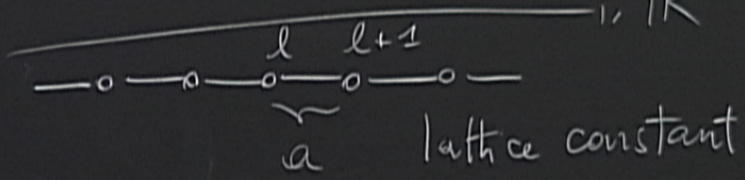
$$\begin{pmatrix} \epsilon_1^{n-\ell'} & \\ & \epsilon_2^{n-\ell'} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$


$T_2^{1/2} T_1 T_2^{1/2} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix} \rightarrow \epsilon_{1,2} = e^{2K}\cosh h \pm \sqrt{e^{4K}\sinh^2 h + e^{-2K}}$
$Z = \epsilon_1^M + \epsilon_2^M \rightarrow F = -\ln Z$
$h=0$
$C(\ell,\ell') = \langle \sigma_\ell^z \sigma_{\ell'}^z \rangle_Z = \frac{1}{Z}\sum e^{-H}$
$= \frac{1}{Z} \mathrm{Tr}\left(T_1^{n-\ell'} \sigma_\ell^z T_1^{\ell'-\ell} \sigma_{\ell'}^z T_1^{\ell} \right)$
$T_1 |\rightarrow\rangle = \lambda_1 |\rightarrow\rangle$
$|\leftarrow\rangle = \lambda_2 |\leftarrow\rangle$
$\epsilon_1 = 2\cosh K$
$\epsilon_2 = 2\sinh K$
$\begin{pmatrix} \epsilon_1^{n-\ell'} & \\ & \epsilon_2^{n-\ell'} \end{pmatrix}$
$\mathrm{Tr}\, T_1^n \sigma^z \sigma^z$


- Scaling limit
ξ is very large
ℓ
ℓ+1
a
lattice constant

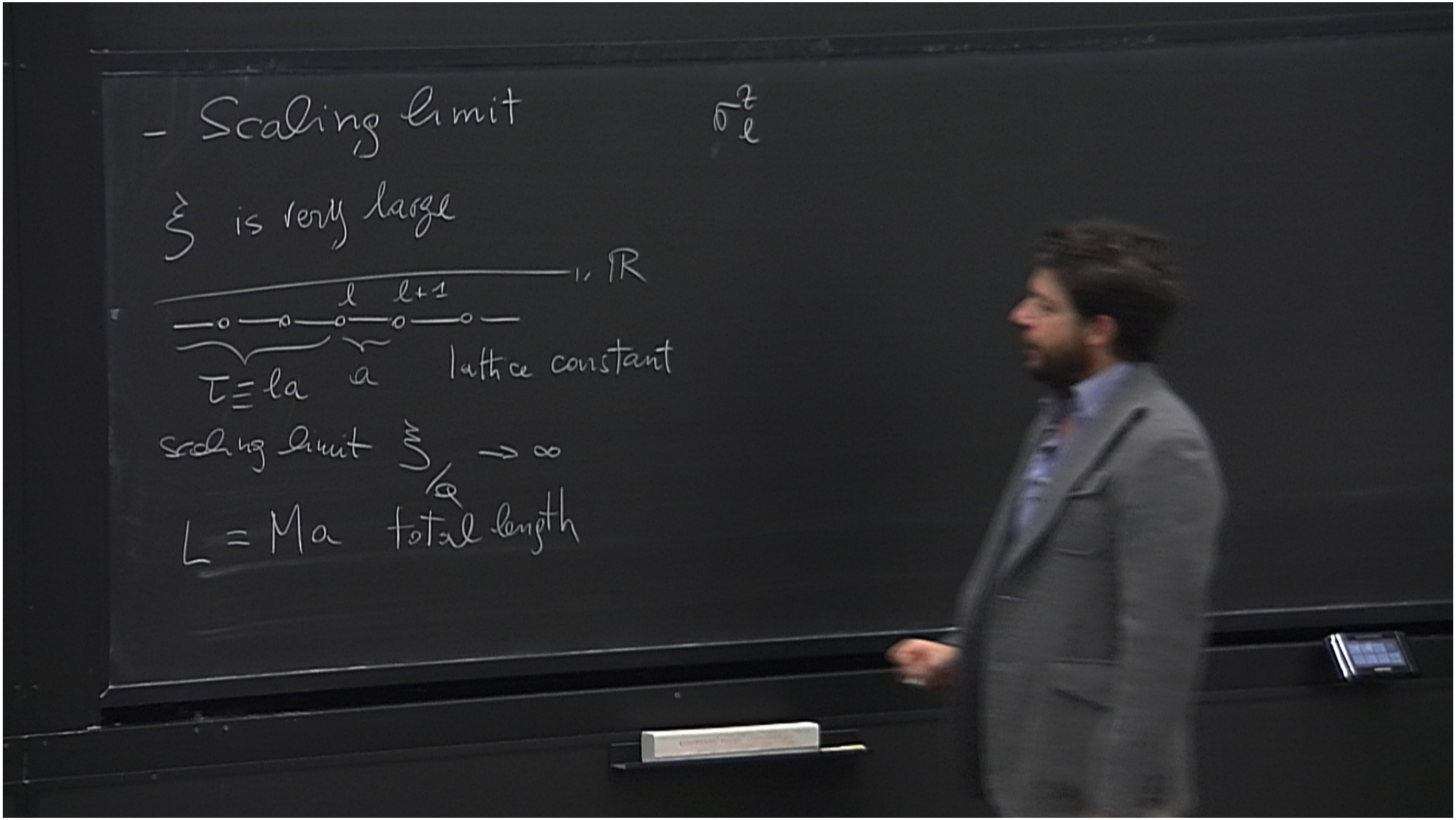
- Scaling limit

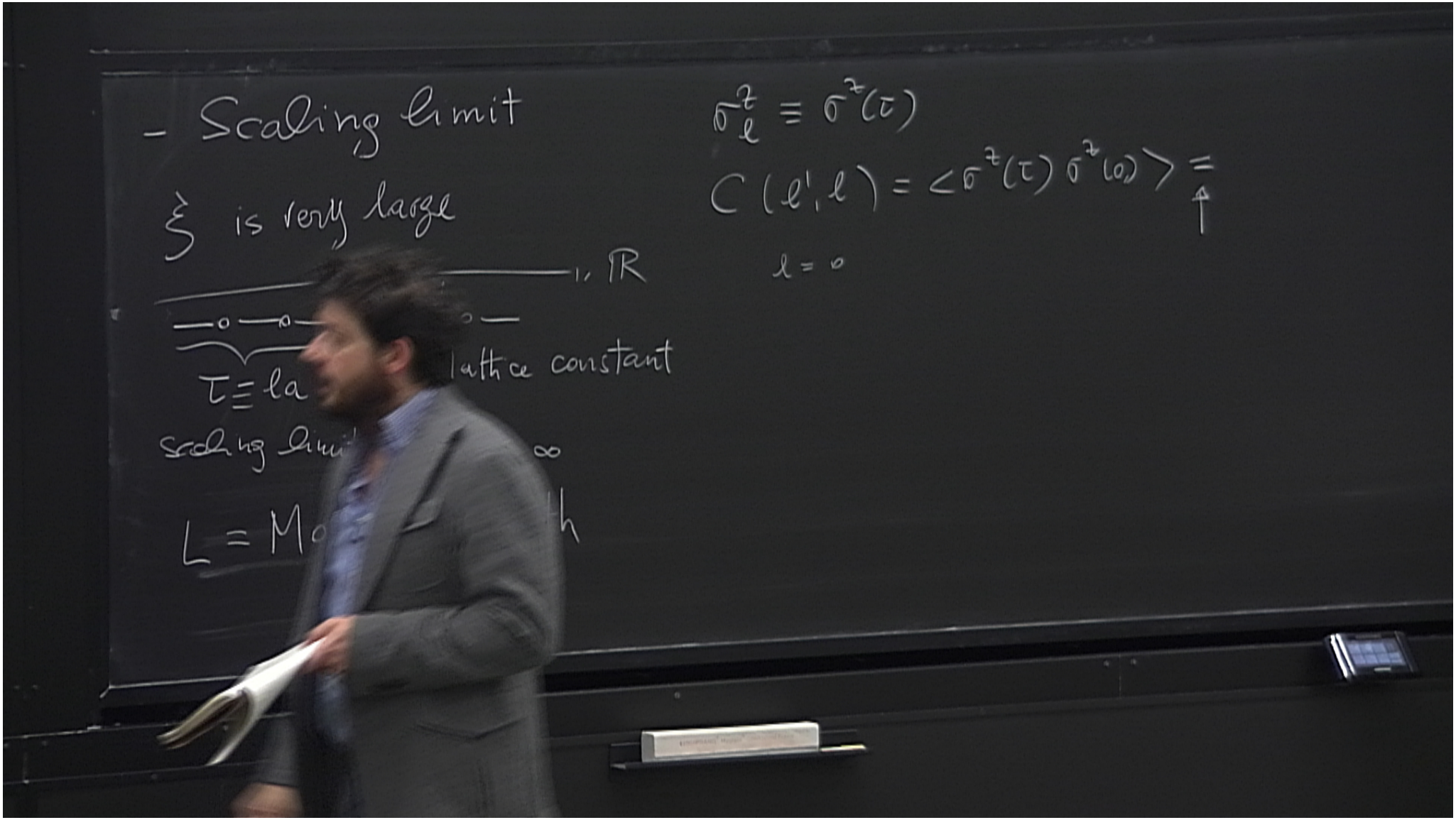
ξ is very large



lattice constant

scaling limit $\frac{\xi}{a} \to \infty$


- Scaling limit
σ^z_l
ξ is very large
$\mathbb{R}$
l $l+1$
$\tau \equiv la$
a
lattice constant
scaling limit $\xi/a \to \infty$
$L = Ma$ total length


- Scaling limit
ξ is very large
1, R
lattice constant
$\sigma_\ell^2 \equiv \sigma^2(\tau)$
$C(\ell', \ell) = \langle \sigma^2(\tau)\, \sigma^2(0) \rangle =$
$\ell = 0$

- Scaling limit

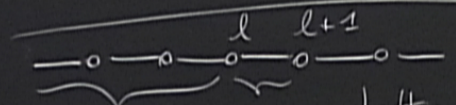
ξ is very large

$\sigma^z_\ell \equiv \sigma^z(\tau)$

$C(\ell', \ell) = \langle \sigma^z(\tau)\, \sigma^z(0) \rangle = e^{-\tau/\xi}$

$\ell = 0$

$\xi^{-1} = \frac{1}{a} \log \coth K$



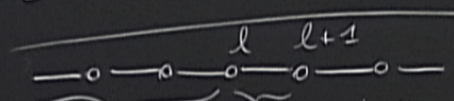
$\tau \equiv \ell a$ a lattice constant

scaling limit $\xi/a \to \infty$

$L = Ma$ total length

- Scaling limit

ξ is very large



$\tau \equiv la$ a lattice constant

scaling limit $\xi/a \to \infty$

$L = Ma$ total length

$\sigma^z_l \equiv \sigma^z(\tau)$

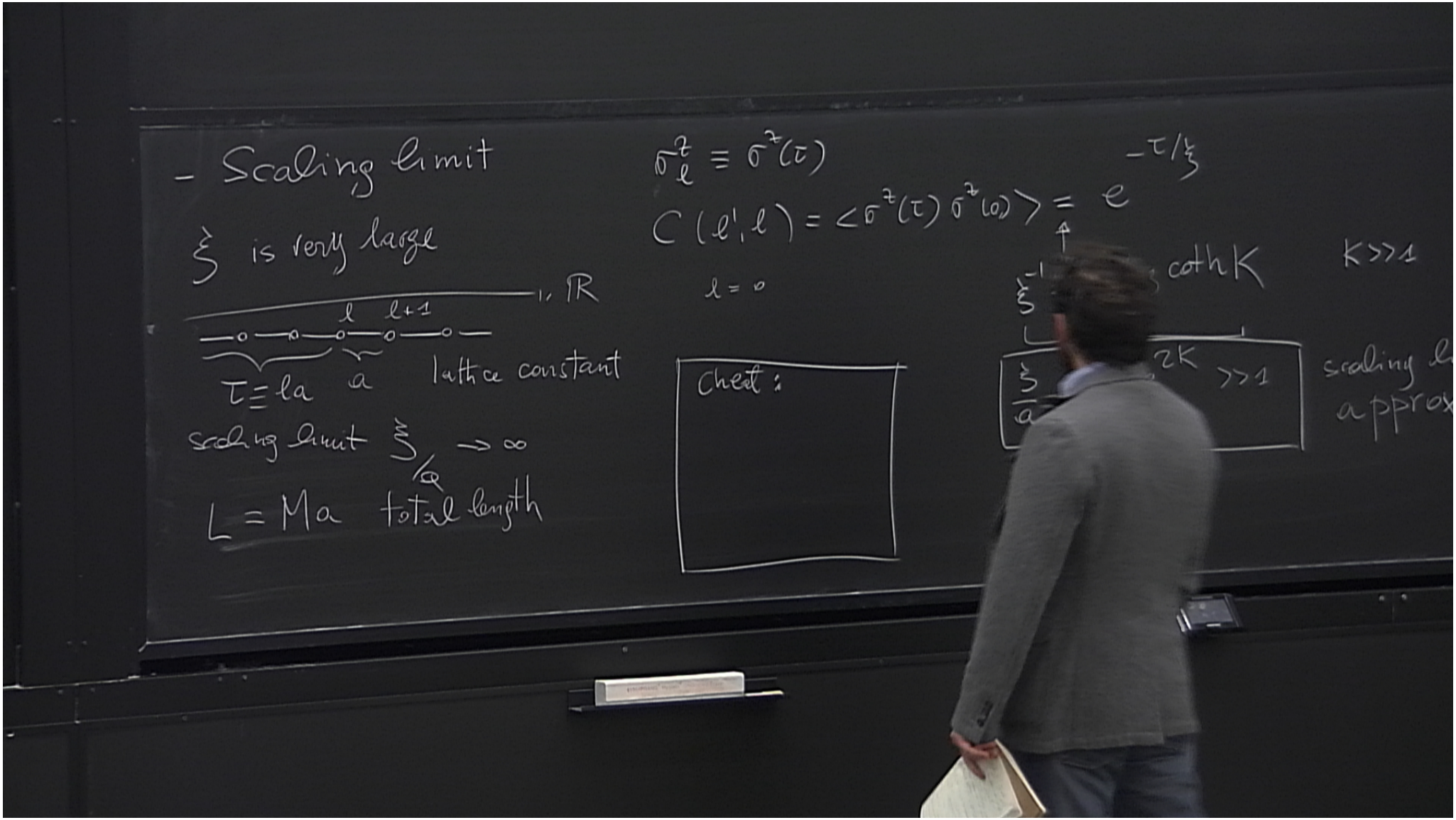
$C(l', l) = \langle \sigma^z(\tau)\, \sigma^z(0) \rangle = e^{-\tau/\xi}$

$l = 0$

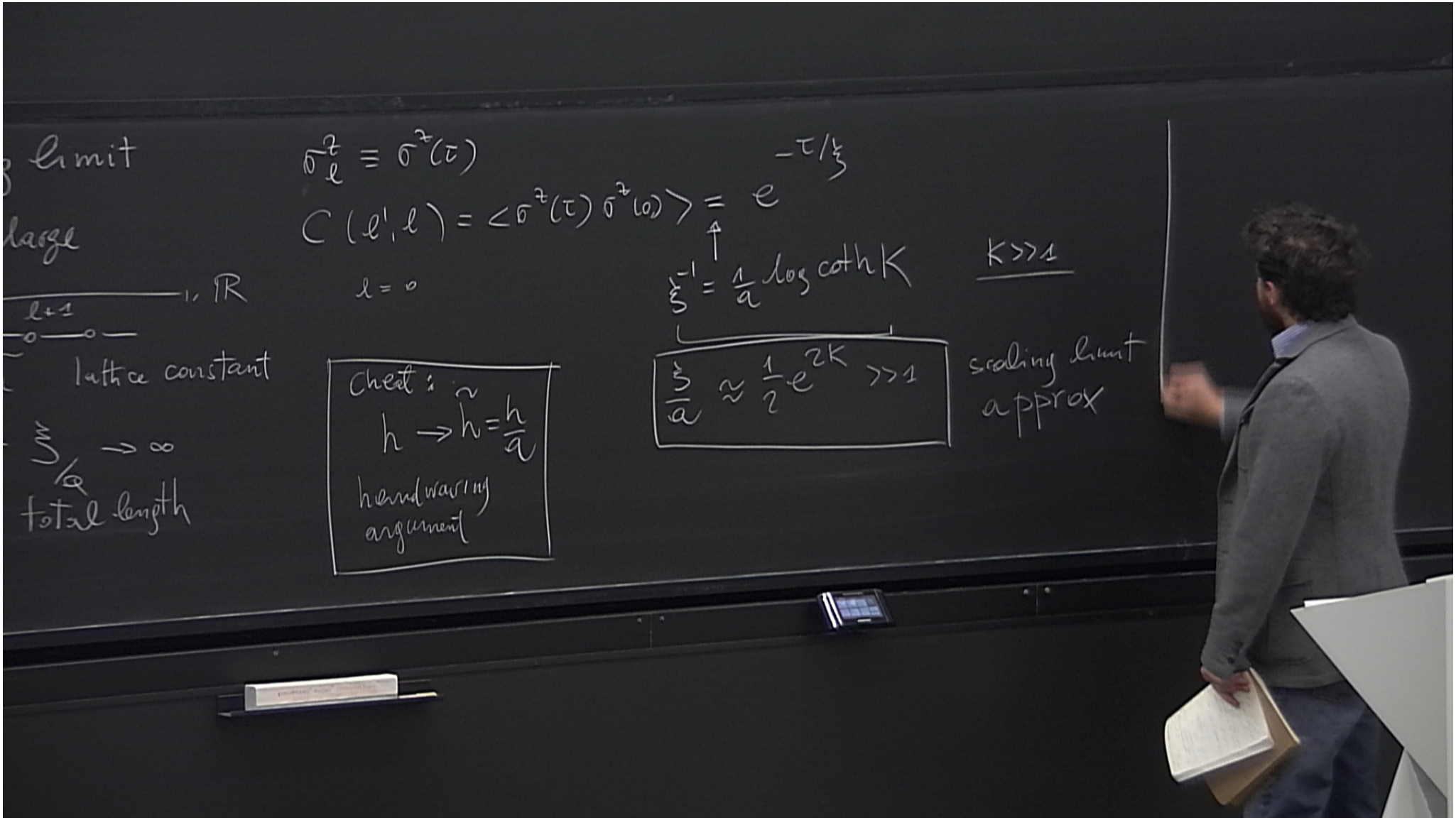
$\xi^{-1} = \frac{1}{a} \log \coth K$ $K \gg 1$

$\frac{\xi}{a} \approx \frac{1}{2} e^{2K} \gg 1$

scal

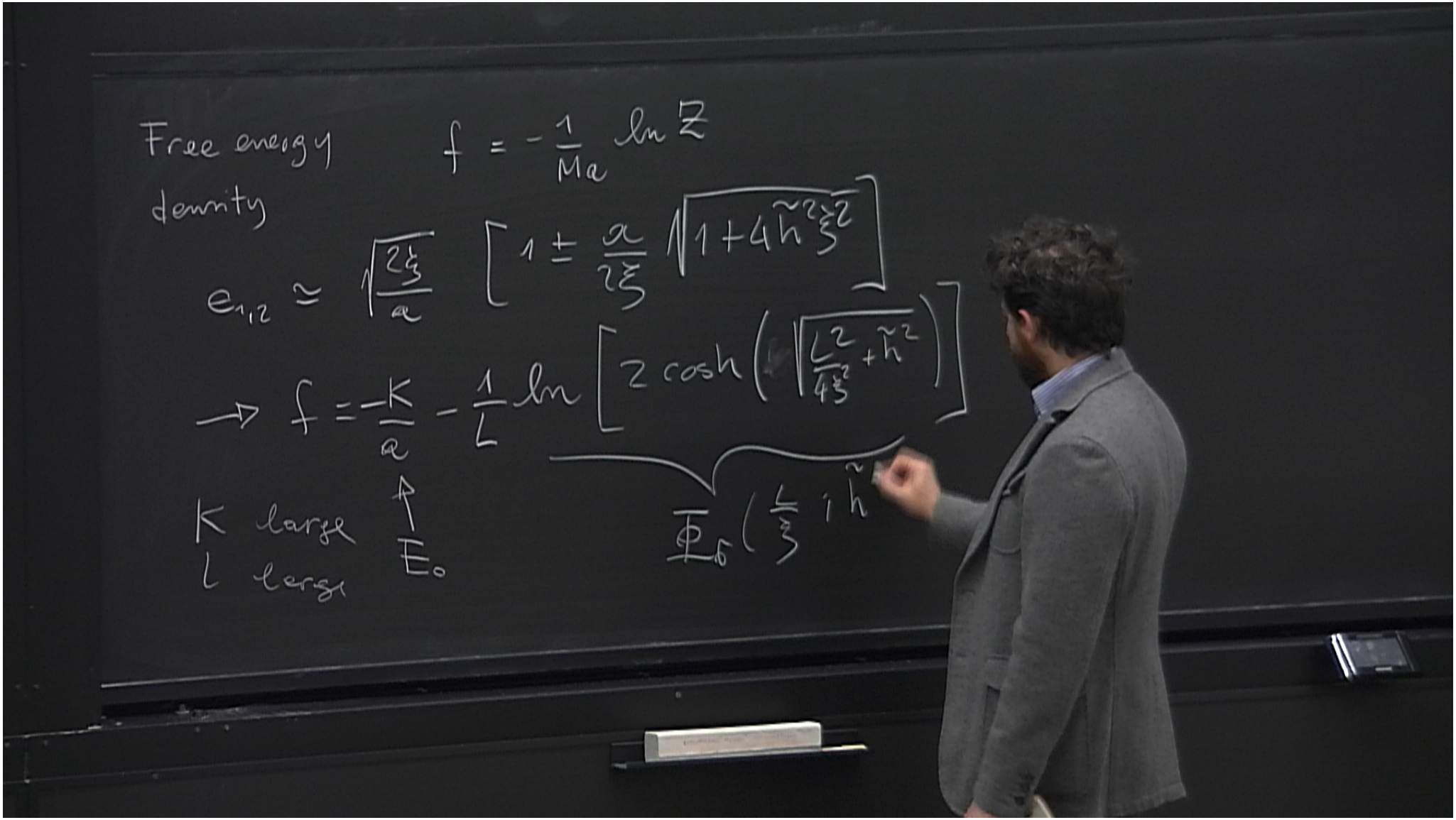

- Scaling limit
ξ is very large
$\sigma^z_\ell \equiv \sigma^z(\tau)$
$C(\ell', \ell) = \langle \sigma^z(\tau)\, \sigma^z(0) \rangle = e^{-\tau/\xi}$
$\ell = 0$
$\coth K$
$K \gg 1$
ℓ $\ell+1$
$\tau \equiv \ell a$
a
lattice constant
scaling limit $\xi/a \to \infty$
$L = Ma$ total length
Cheat:
scaling
approx

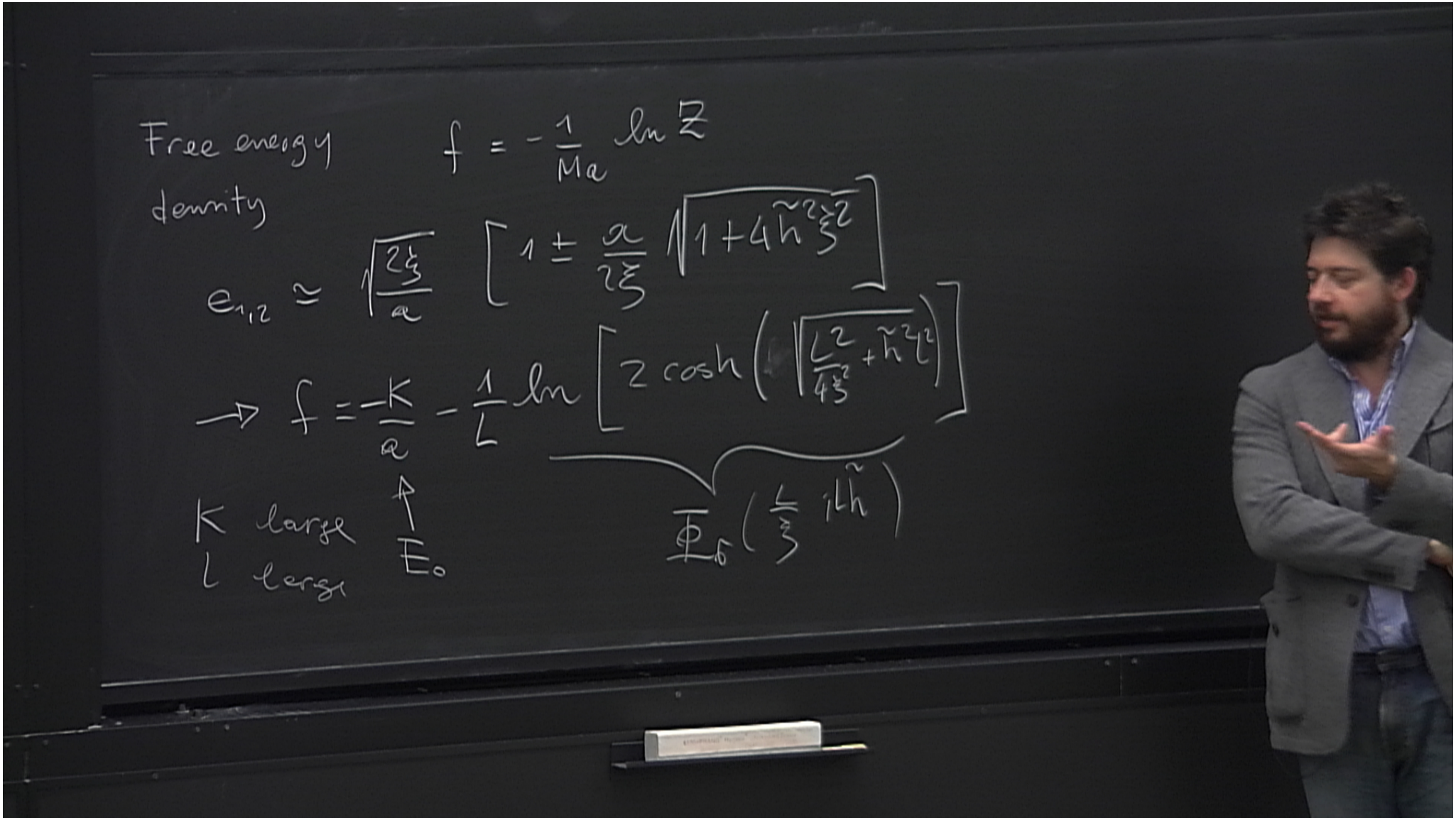

limit
large
lattice constant
total length
Cheat :
scaling limit
approx
K >> 1

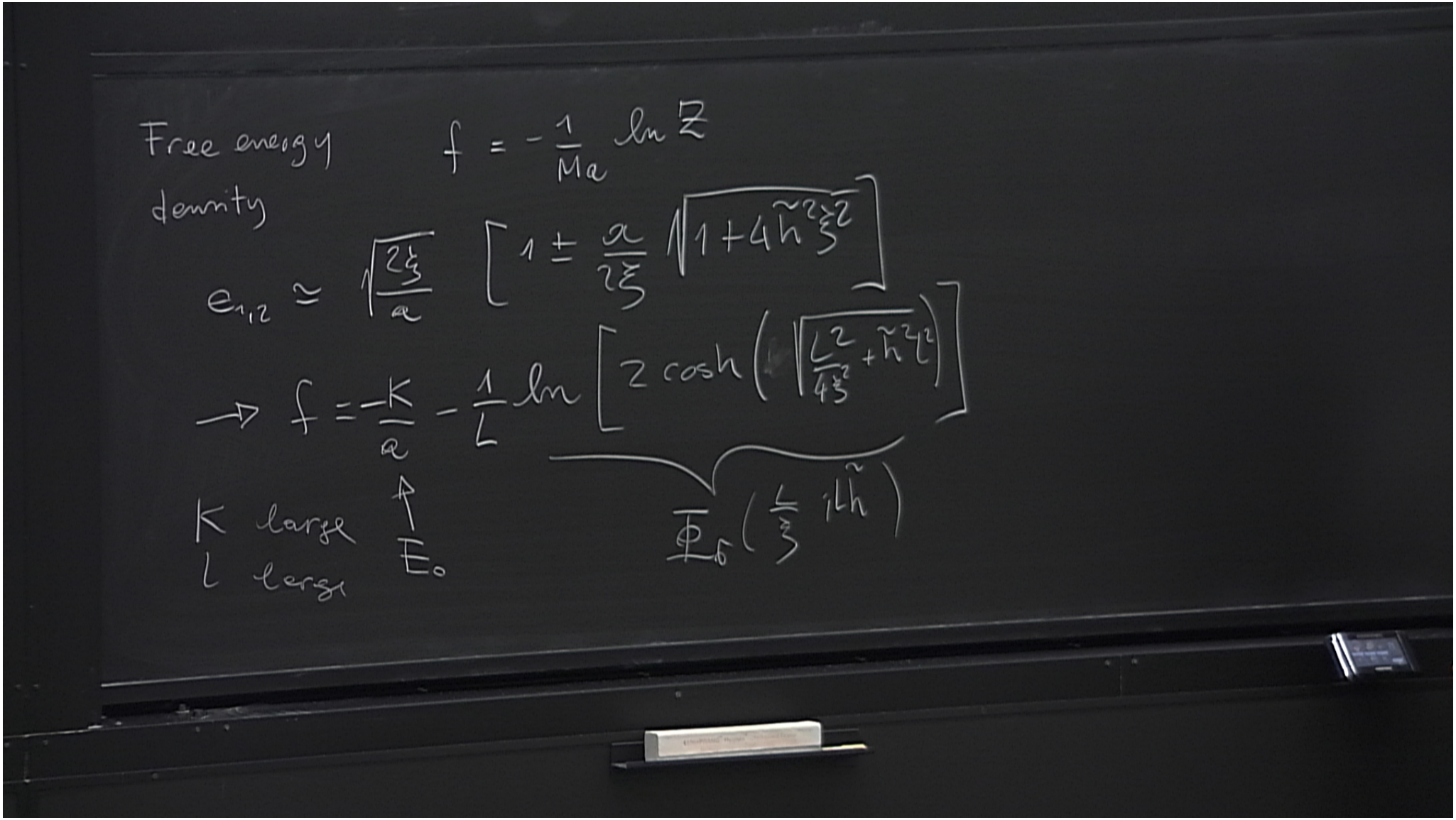

limit
large
$\sigma^2_\ell \equiv \sigma^2(\tau)$
$C(\ell', \ell) = \langle \sigma^2(\tau)\, \sigma^2(0) \rangle = e^{-\tau/\xi}$
$\ell = 0$
$\xi^{-1} = \frac{1}{a} \log \coth K$
$K \gg 1$
$\ell+1$
ℓ
lattice constant
$\to \infty$
total length
cheat: $h \to \tilde{h} = \frac{h}{a}$
handwaving argument
$\frac{\xi}{a} \approx \frac{1}{2} e^{2K} \gg 1$
scaling limit
approx

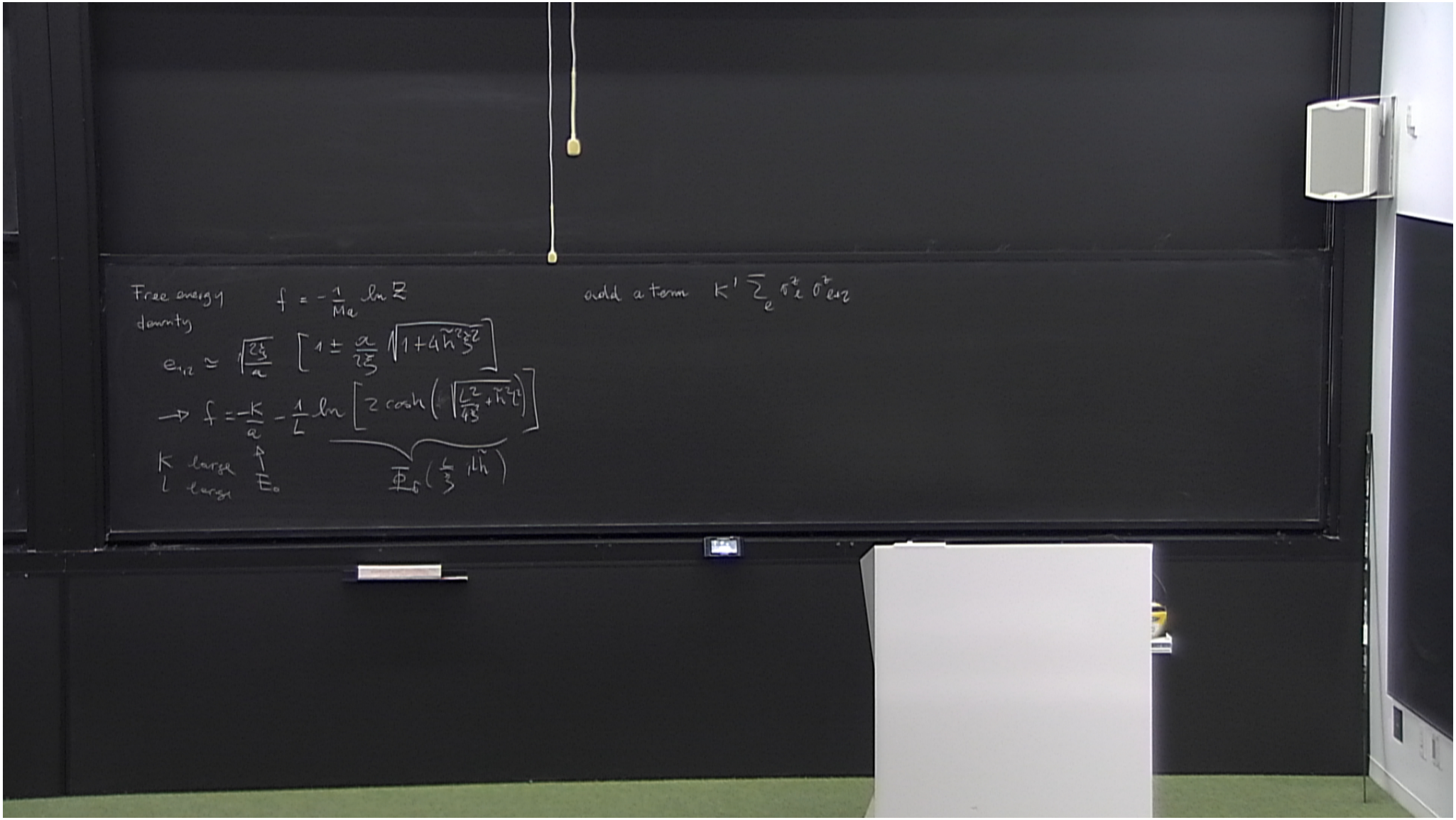
Free energy density $f = -\frac{1}{Ma} \ln Z$

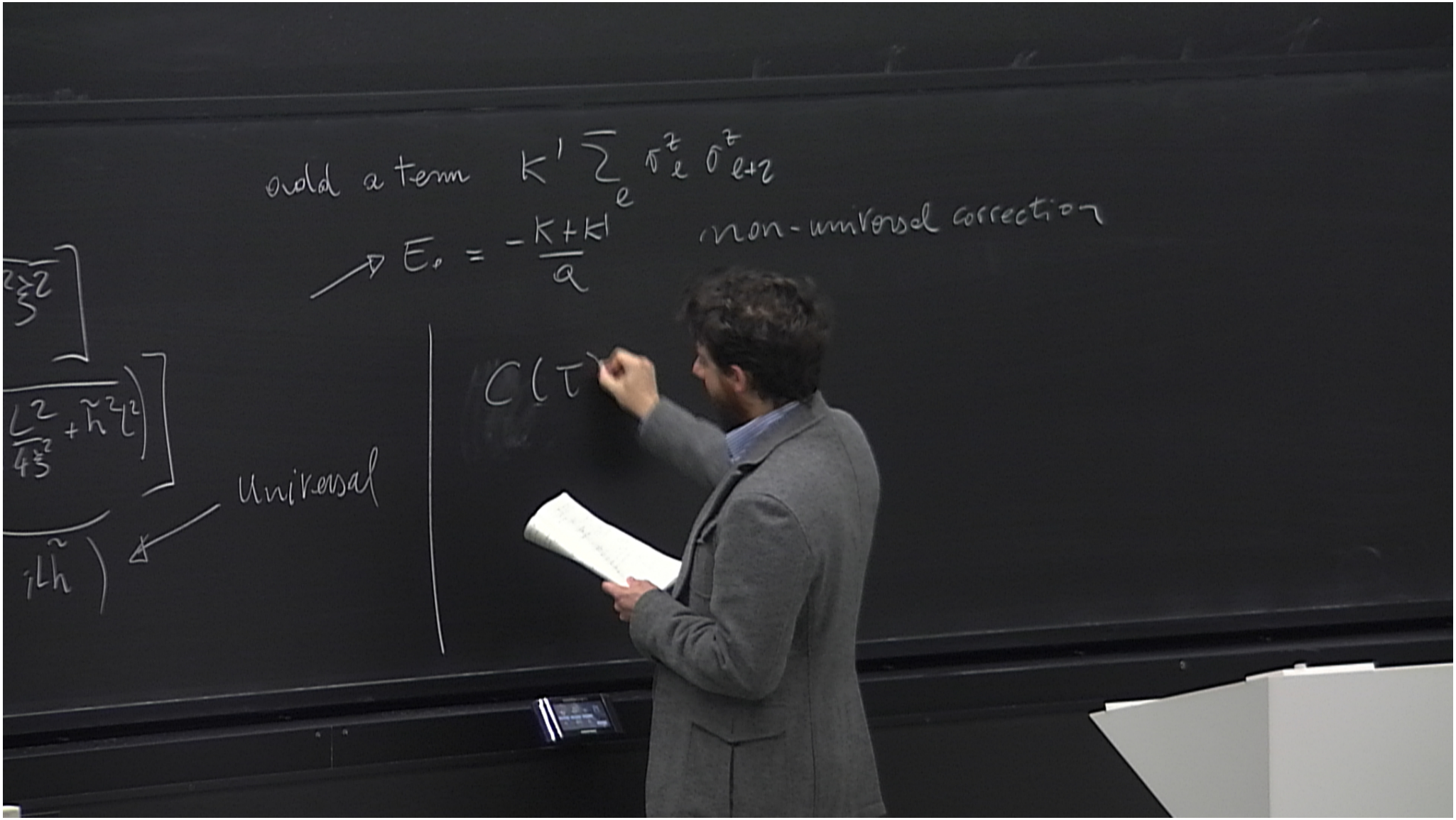
$e_{1,2} \approx$


Free energy density
$f = -\frac{1}{Ma} \ln Z$
$e_{1,2} \simeq \sqrt{\frac{2\xi}{a}} \left[1 \pm \frac{a}{2\xi} \sqrt{1+4\tilde{h}^2\xi^2}\right]$
$\rightarrow f = -\frac{K}{a} - \frac{1}{L} \ln \left[2\cosh\left(\sqrt{\frac{L^2}{4\xi^2}+\tilde{h}^2}\right)\right]$
K large
L large
E_0
$\Phi_0\left(\frac{L}{\xi}, \tilde{h}\right.$


Free energy density
$f = -\frac{1}{Ma} \ln Z$
$e_{1,2} \simeq \sqrt{\frac{2\xi}{a}} \left[1 \pm \frac{a}{2\xi} \sqrt{1 + 4\tilde{h}^2 \xi^2} \right]$
$\rightarrow f = -\frac{K}{a} - \frac{1}{L} \ln \left[2 \cosh \left(\sqrt{\frac{L^2}{4\xi^2} + \tilde{h}^2 L^2} \right) \right]$
$\Phi_0 \left(\frac{L}{\xi}, iL\tilde{h} \right)$
K large
L large
E_0


Free energy density
$f = -\frac{1}{Ma} \ln Z$
$e_{1,2} \simeq \sqrt{\frac{2\xi}{a}} \left[1 \pm \frac{a}{2\xi} \sqrt{1+4\tilde{h}^2\xi^2}\right]$
$\rightarrow f = -\frac{K}{a} - \frac{1}{L} \ln \left[2\cosh\left(\sqrt{\frac{L^2}{4\xi^2} + \tilde{h}^2 L^2}\right)\right]$
$\Phi_0\left(\frac{L}{\xi}, iL\tilde{h}\right)$
K large
L large
E_0


Free energy density
$f = -\frac{1}{Ma} \ln Z$
add a term $K' \sum_e \sigma_e^z \sigma_{e+2}^z$
$e_{\pm 2} \simeq \sqrt{\frac{2J}{a}} \left[1 \pm \frac{a}{2J} \sqrt{1+4\tilde{h}^2 J^2}\right]$
$\rightarrow f = -\frac{K}{a} - \frac{1}{L} \ln \left[2 \cosh\left(\sqrt{\frac{L^2}{4J^2} + \tilde{h}^2 L^2}\right)\right]$
K large
L large
E_0
$\Phi_0(\frac{1}{3}, i\tilde{h})$


could a term $K' \sum_e \sigma^z_e \sigma^z_{e+2}$
$\rightarrow E_0 = -\frac{K+K'}{a}$
non-universal correction
Universal