

Title: 12/13 PSI - Condensed Matter Review Lecture 1

Date: Jan 28, 2013 11:30 AM

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Abstract:

Quantum Phase Transitions

- phase transitions some non analyticity in the free energy $F = E - TS$

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fluctuations? $\uparrow$ $T=0$ only energy)

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 - fluctuations
 - quantum fluctuations
 - large superpositions

Phase Transitions happen

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- fluctuations? $\leftarrow$ fluctuations
- $\leftarrow$ superpositions

Phase Transitions happen in thermodyn. limit

ψ is a many-body wave function

$\mathcal{H}_N$

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Hilbert space of n bodies

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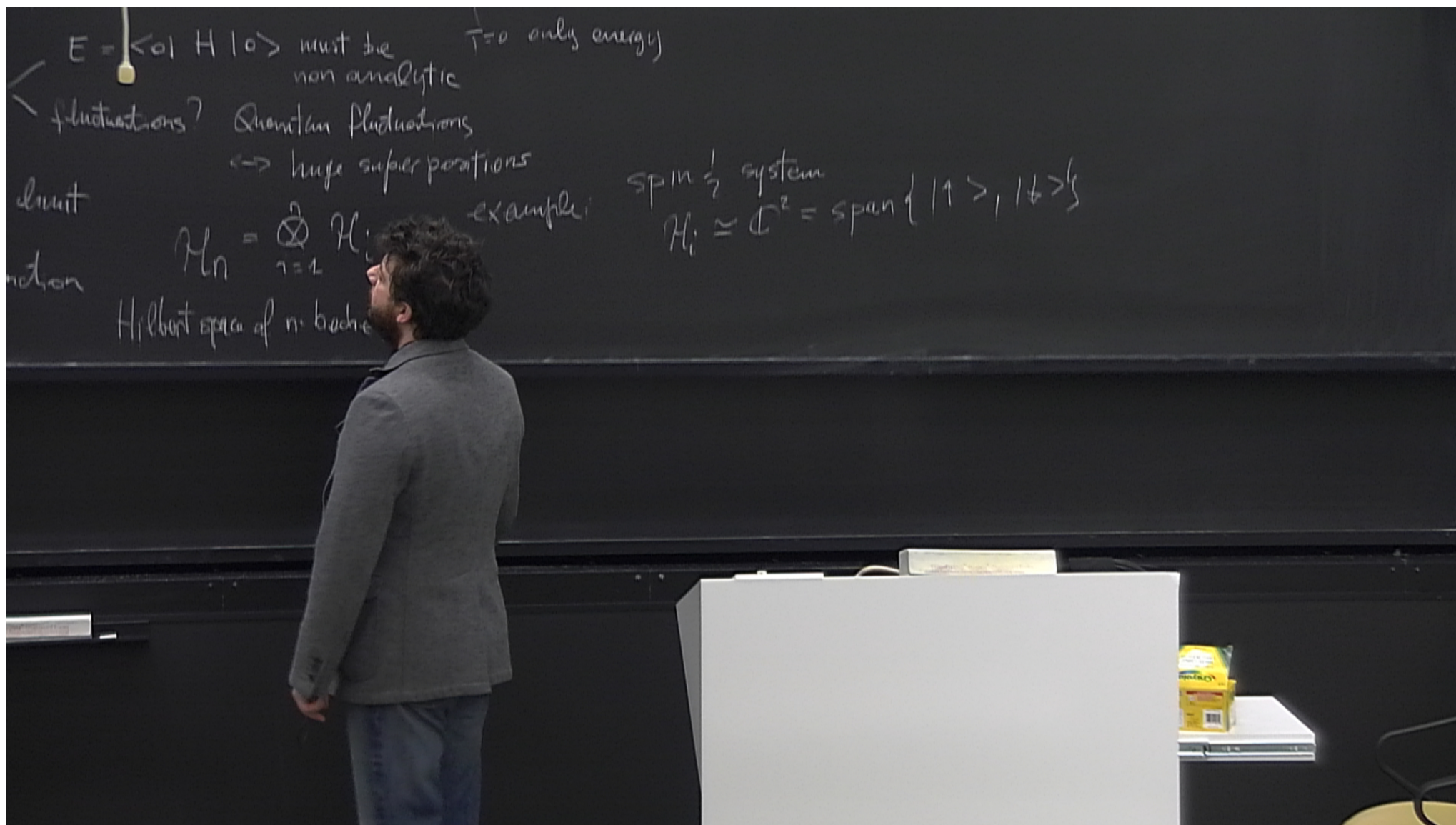
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example: spin $\frac{1}{2}$ system
 $\mathcal{H}_i \simeq \mathbb{C}^2 = \text{span} \{$



$$\mathcal{H}_n = \mathbb{C}^{2n}$$

$$\mathcal{H}_\infty = \lim_{n \rightarrow \infty} \mathcal{H}_n$$

$$\psi, \phi \in \mathcal{H}_n$$

$$\psi =$$

$$\mathcal{H}_n = \mathbb{C}^{2^n}$$

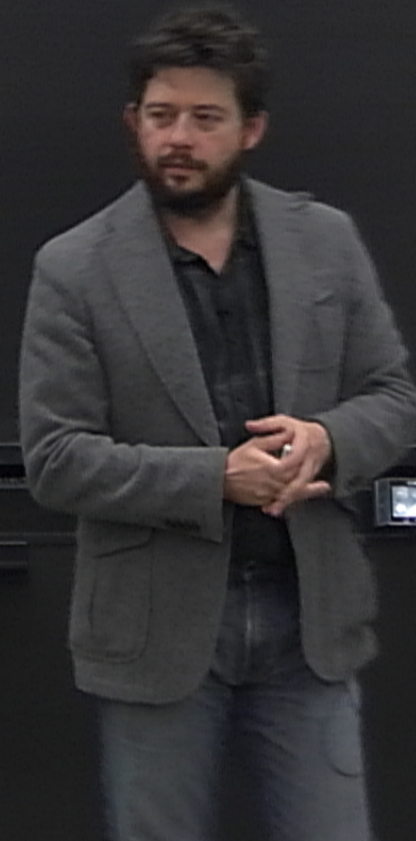
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$$\psi = \psi_1 \otimes \psi_2 \otimes \dots \otimes \psi_n$$

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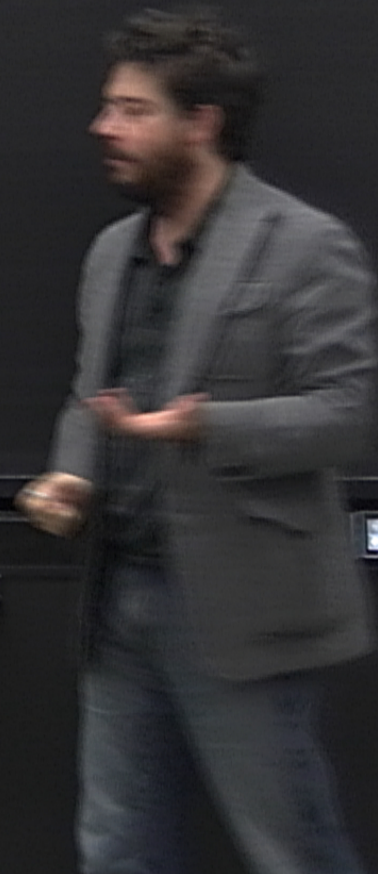
$$\psi, \phi \in \mathcal{H}_n$$

$$|\langle \psi_i, \phi_i \rangle| \sim 1 - \epsilon$$

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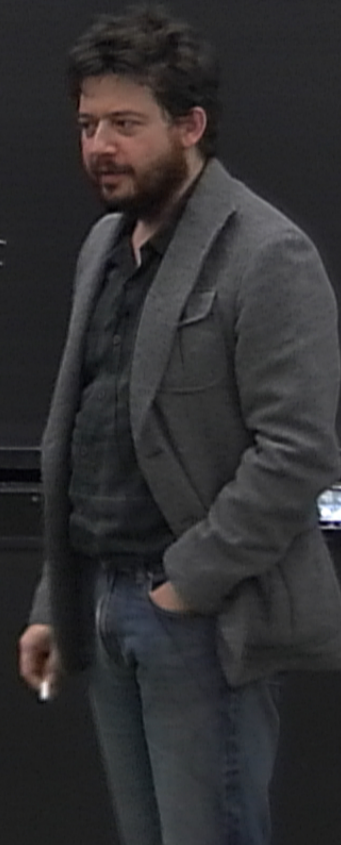
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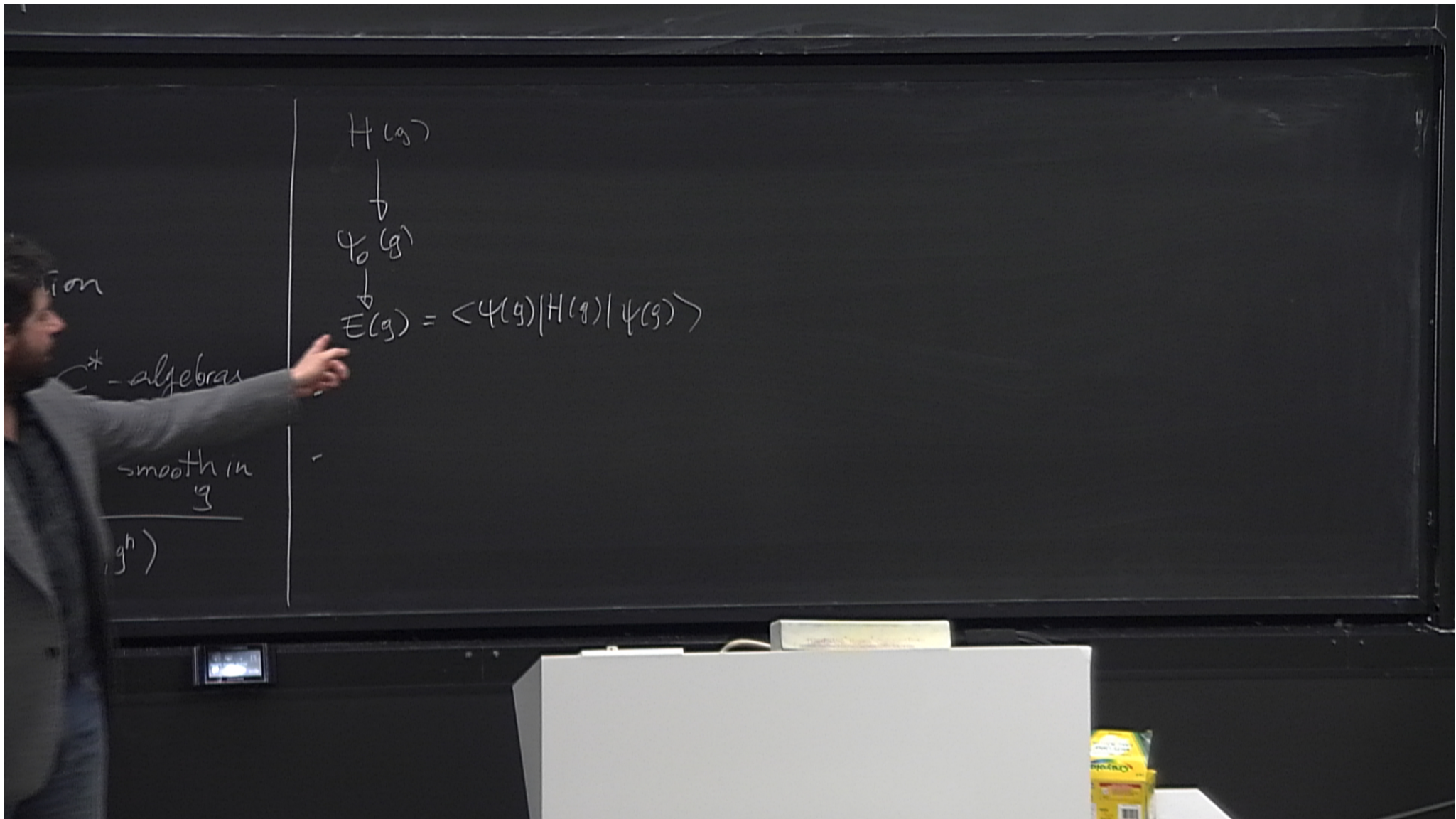
different
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$$|\langle \psi_i, \phi_i \rangle| \sim 1-\epsilon$$

Orthogonality Catastrophe

C^* -algebras

$$\frac{H(g)}{g = (g^1, \dots, g^n)} \text{ smooth in } g$$



+
lection

C^* -algebras

$H(g)$ smooth in g
 $(g^1, \dots, g^n)$

$$H(g)$$



$$\psi_0(g)$$



$$E_0(g) = \langle \psi(g) | H(g) | \psi(g) \rangle \quad \text{this is cont. always}$$

$$- \frac{dE_0(g)}{dg} = \langle d\psi | H | \psi \rangle + \langle \psi | H | d\psi \rangle + \langle \psi | \dot{H} | \psi \rangle$$

$$E_0(\langle d\psi | + \langle \psi | d\psi \rangle) + \langle \psi | \dot{H} | \psi \rangle$$

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$$H(g)$$

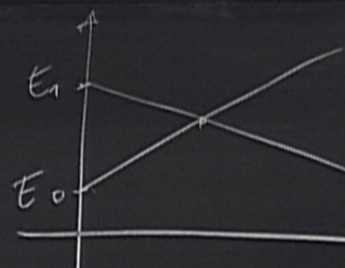
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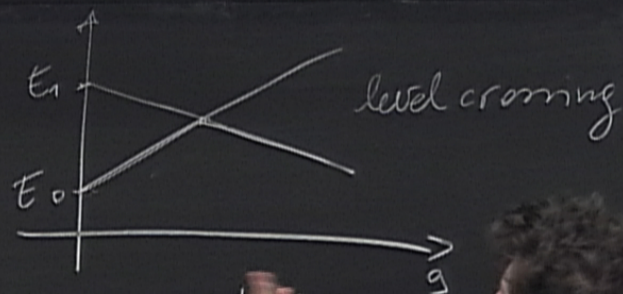
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C^* -algebras

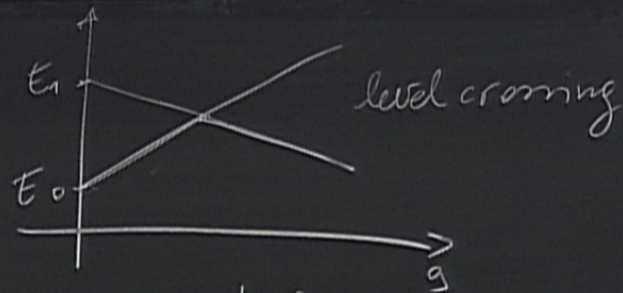
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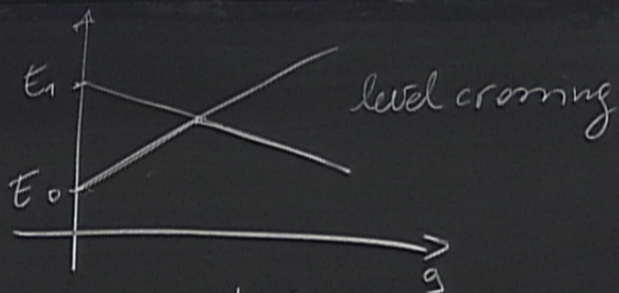
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3

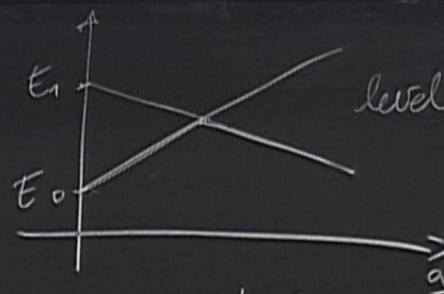
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NOT INTERESTING

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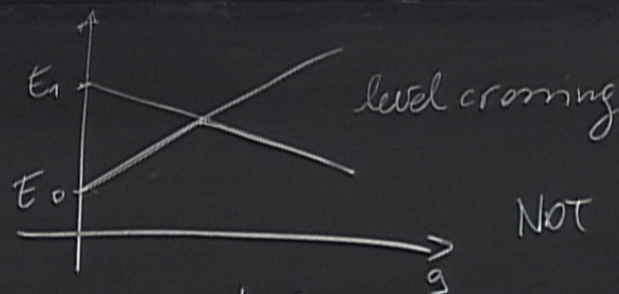
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$$- \frac{d^2 E}{dg^2}$$



NOT INTERESTING

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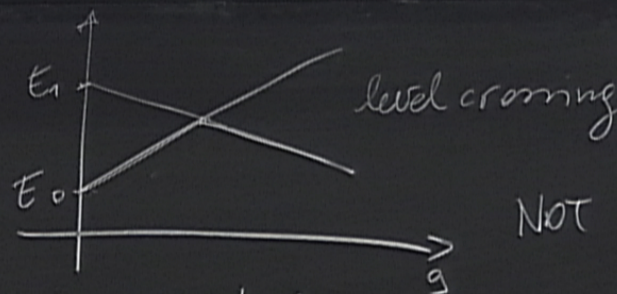
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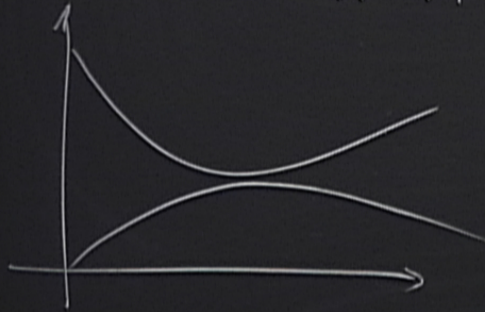
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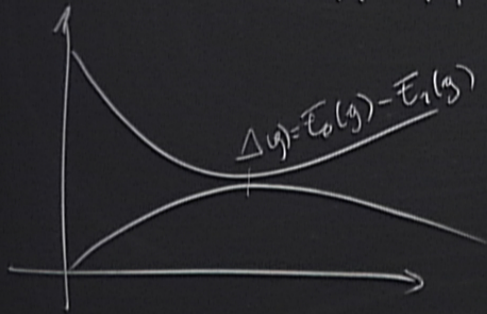
$$\frac{d^2 E}{dg^2}$$

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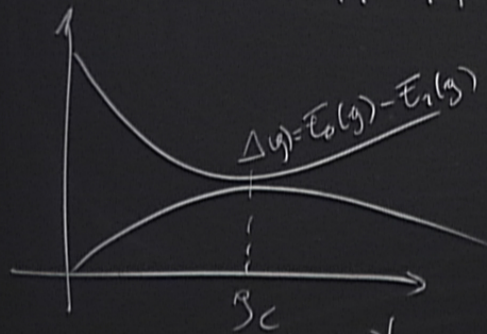
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$$\lim_{n \rightarrow \infty} \Delta_n(g) = 0$$

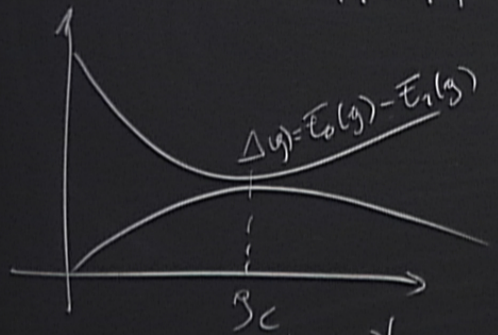
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g_c quantum critical point



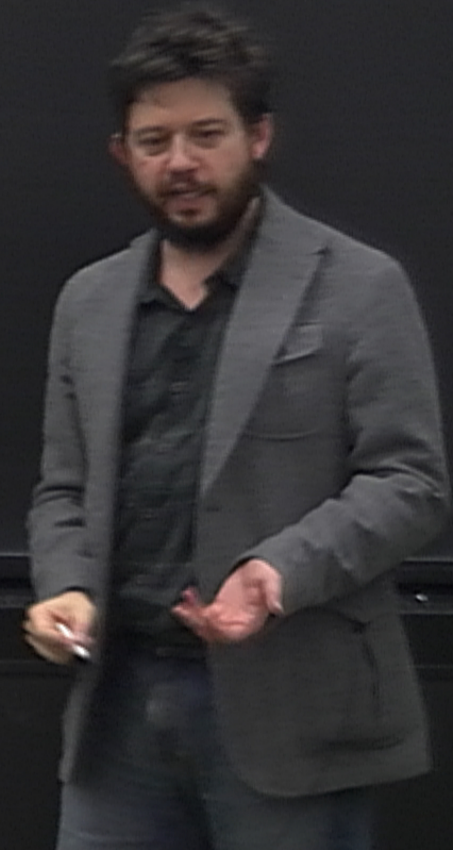
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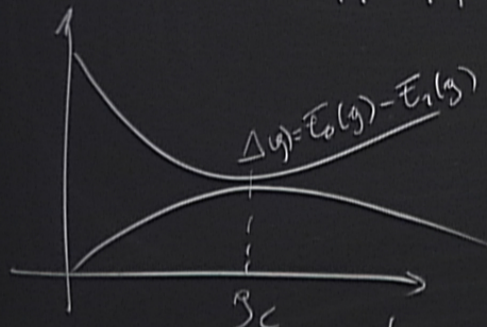


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g_c quantum critical point
 $g \sim g_c \quad \Delta(g) \sim |g - g_c|^{\frac{2\nu}{\nu+1}}$



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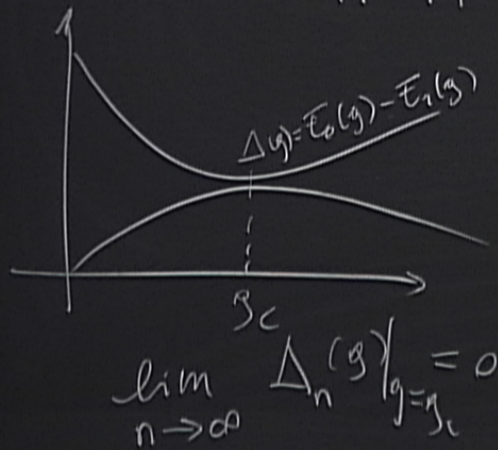
$$g \sim g_c \quad \Delta(g) \sim |g - g_c|^{\nu}$$

ν critical exponent

diverging length scale

$$C_{ij} = \langle \psi(g) | \mathcal{O}_i \mathcal{O}_j$$

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$$g \sim g_c \quad \Delta(g) \sim J |g - g_c|^{\frac{z\nu}{2}}$$

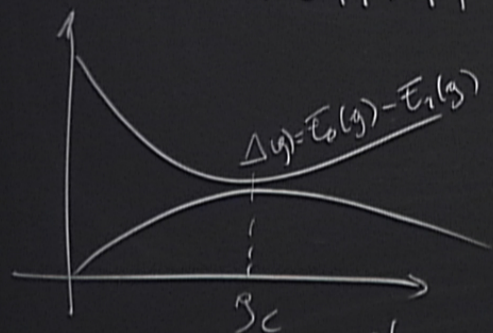
critical exponent

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$$\sim \Lambda |g - g_c|^\nu$$

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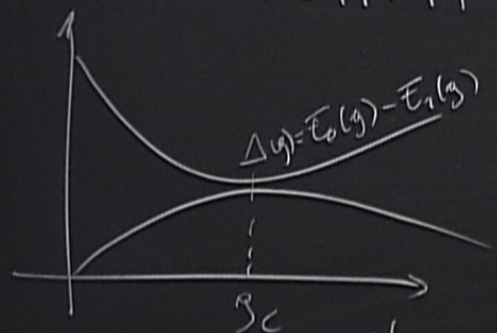
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$$\sim \Lambda |g - g_c|^{-\nu_R} \quad \text{dynamic critical exp.}$$

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critical exponent

$$\Delta \sim \xi^{-z}$$

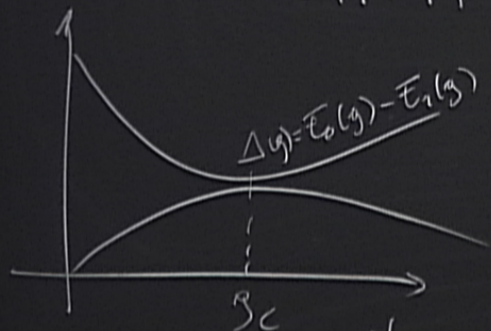
diverging length scale

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$$g \sim g_c$$

$$\sim \Lambda |g - g_c|^{\nu_R} \quad \text{dynamical critical exp.}$$

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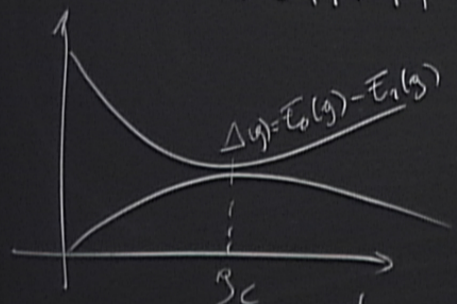
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diverging length scale

$$C_{ij} = \langle \psi(g) | \mathcal{O}_i \mathcal{O}_j | \psi(g) \rangle \sim \tau^{-|i-j|/\Delta}$$

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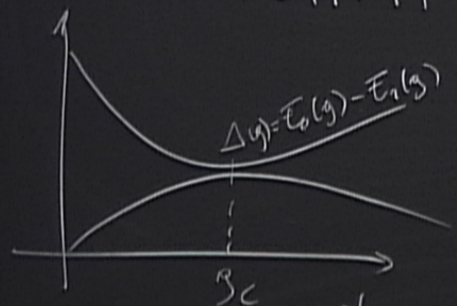
$$C_{ij} = \langle \psi(g) | \mathcal{O}_i \mathcal{O}_j | \psi(g) \rangle \sim \frac{1}{|i-j|^\alpha}$$

$$-|i-j|^{-\alpha}$$

away from criticality

$\frac{1}{3}$ finite

$$\frac{d^2 E}{dg^2} = \langle d\psi | \hat{H} | \psi \rangle + \text{h.c.} + \langle \psi | \hat{H} | \psi \rangle$$



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$$C_{ij} = \langle \psi(g) | \mathcal{O}_i \mathcal{O}_j | \psi(g) \rangle \sim \frac{1}{|i-j|^\alpha}$$

$$-1 < -\alpha < 1$$

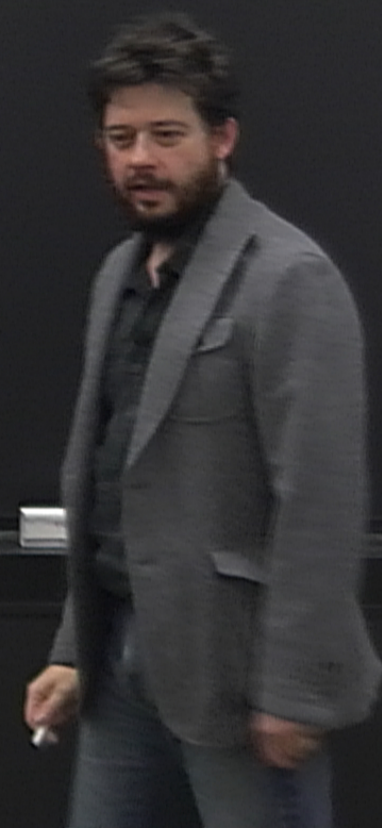
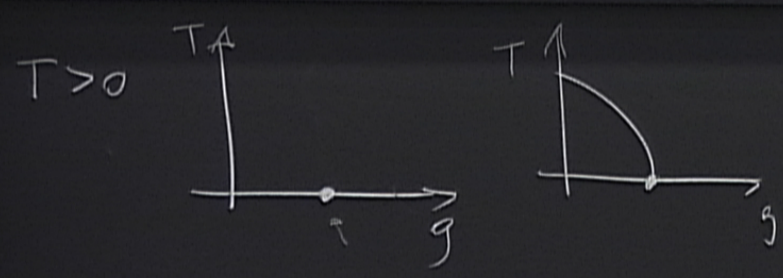
away from criticality

$\frac{1}{2}$ finite

power law

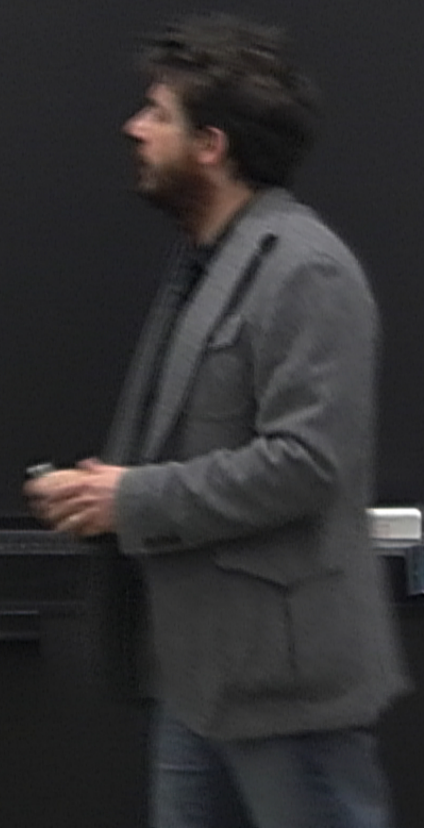
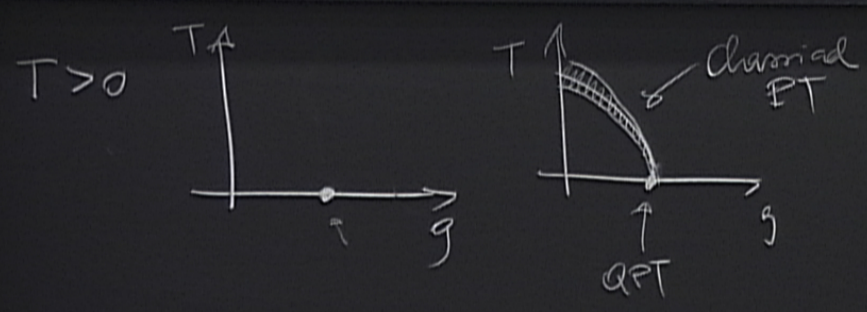
$n \rightarrow \infty$ n $\eta = \eta_c$

$|i-j|$ power law



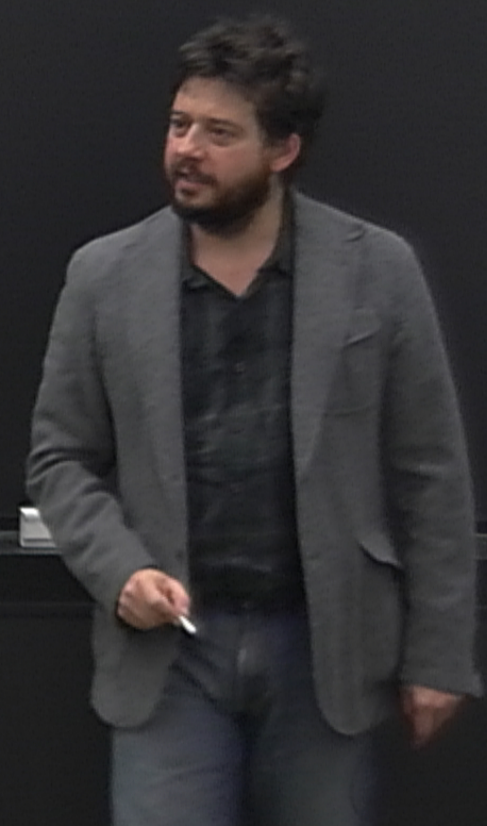
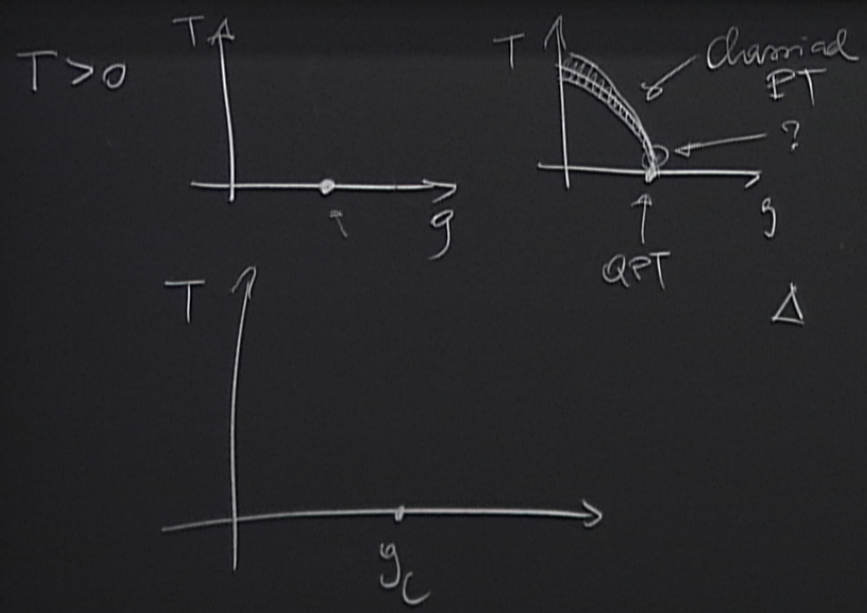
$n \rightarrow \infty$ n $\mu = \mu_c$

$|i-j|$ power law



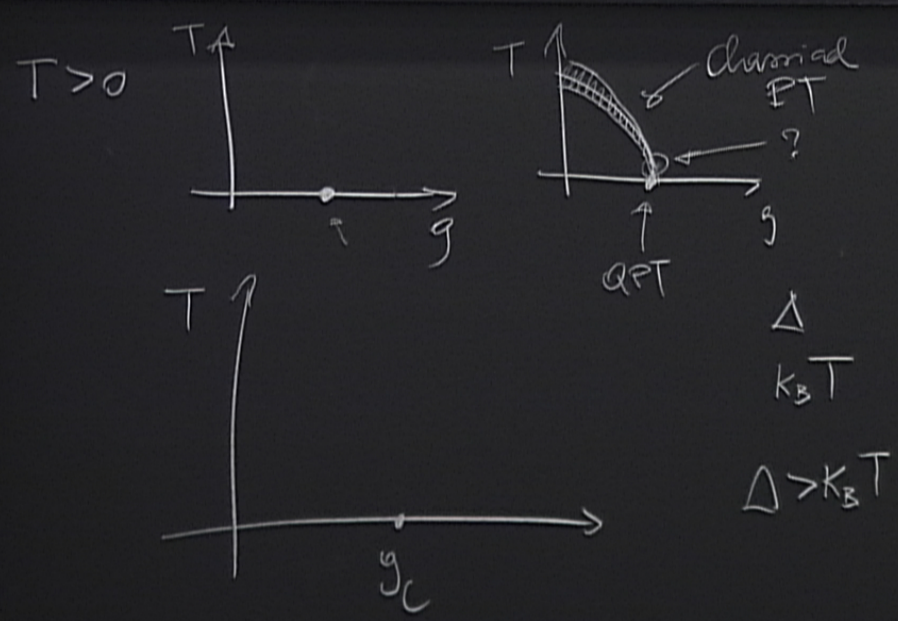
$n \rightarrow \infty$ n $y = y_c$

$|i-j|$ power law



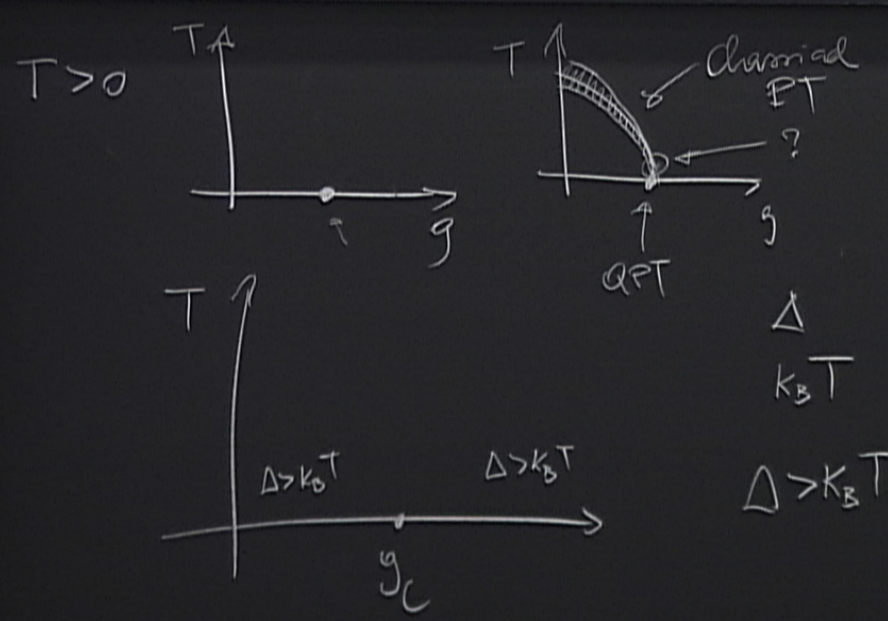
$n \rightarrow \infty$ n $\mu = \mu_c$

$|i-j|$ power law



$n \rightarrow \infty$ n $\mu = \mu_c$

$|i-j|$ power law

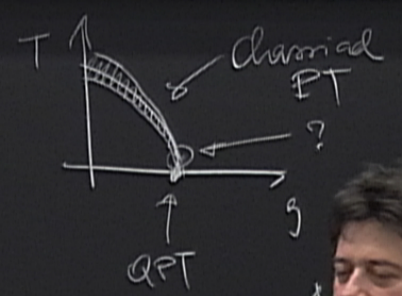
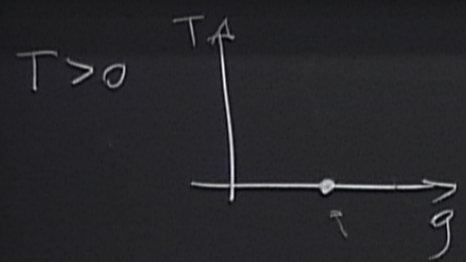


$$\Delta > k_B T$$

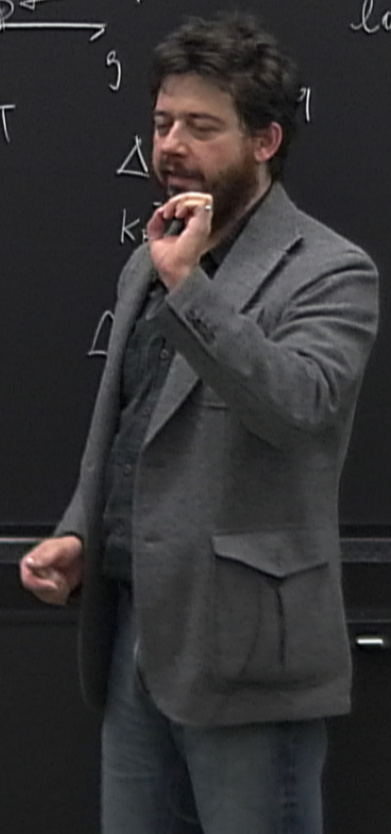
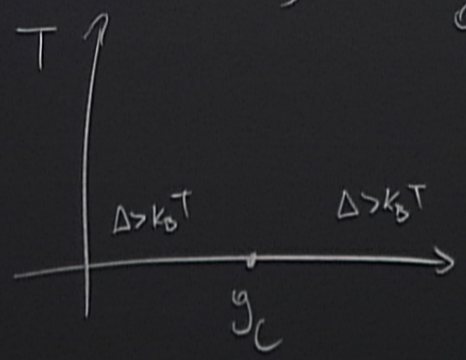
$$\Delta > k_B T$$

$n \rightarrow \infty$ n $y = y_c$

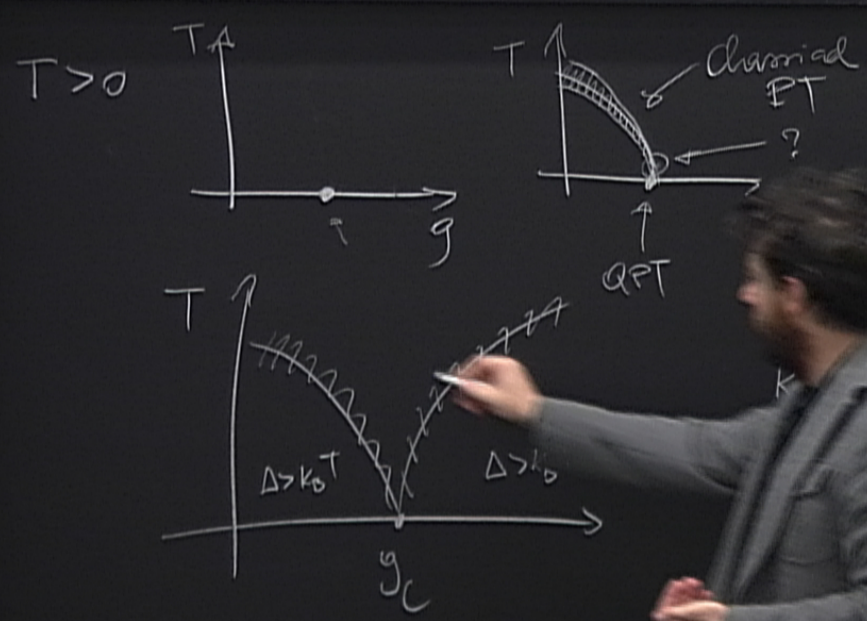
$|i-j|$ power law



local eq. time for local observables

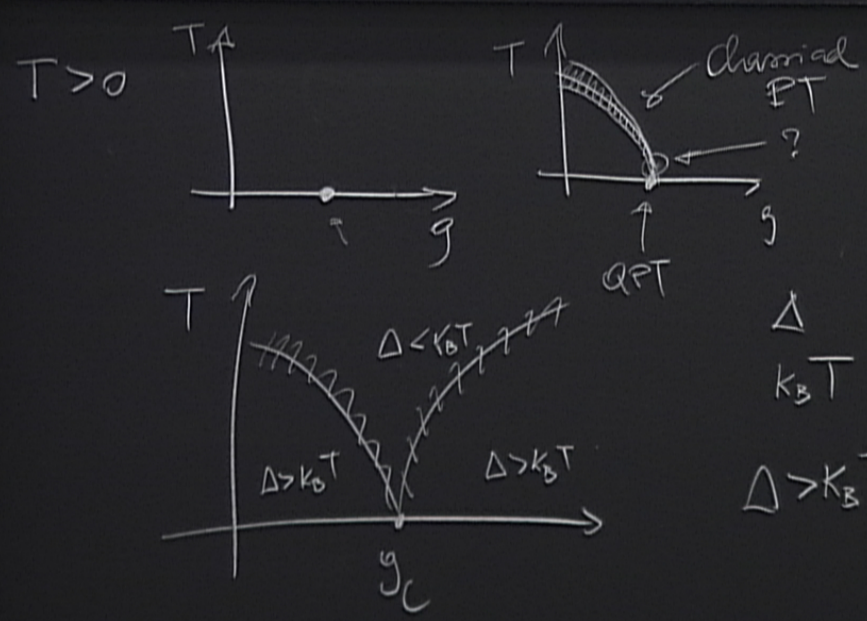


$n \rightarrow \infty$ n $\mu = \mu_c$ $|i-j|$ power law



local eq. time for local observables
 $\tau_{eq} \gg \frac{\hbar}{k_B T} \leadsto$ classical dynamics

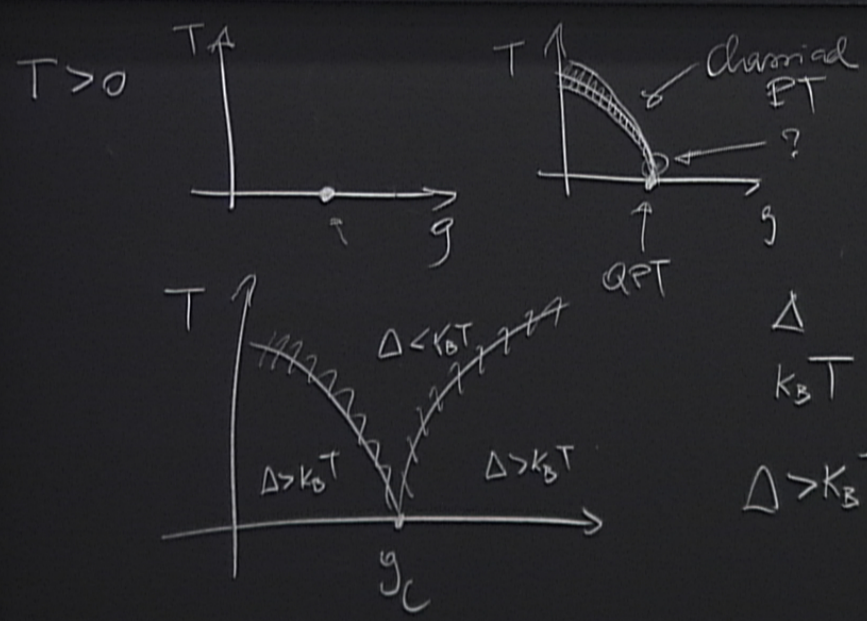
$n \rightarrow \infty$ n $y = g_c$ $|i-j|$ power law



local eq. time for local observables
 $T_{eq} \gg \frac{\hbar}{k_B T} \leadsto$ classical dynamics
 $\Delta < k_B T$ T_{eq} Quantum critical

$n \rightarrow \infty$ $n = g_c$

$|i-j|$ power law



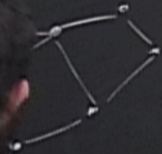
local eq. time for local observables
 $T_{eq} \gg \frac{\hbar}{k_B T} \leadsto$ classical dynamics
 $\Delta < k_B T \quad T_{eq} \sim \frac{\hbar}{k_B T}$ Quantum critical

Quantum Ising Model

$$\mathcal{H}_n = \mathbb{C}^{2^n}$$

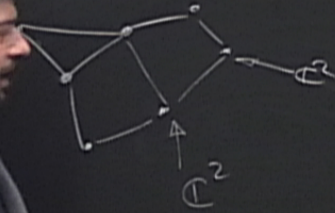
quantum spin system

$$\Gamma = (V, E)$$



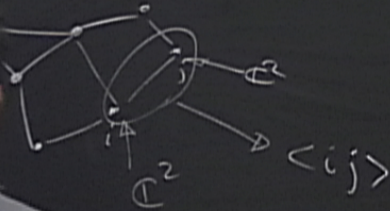
Quantum Ising Model
 $\mathcal{H}_n = \mathbb{C}^{2^n}$ quantum spin system

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Quantum Ising Model
 $\mathcal{H}_n = \mathbb{C}^{2^n}$ quantum spin system

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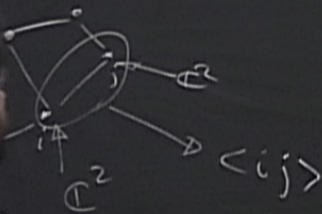


Quantum Ising Model

$\mathcal{H}_n = \mathbb{C}^{2^n}$ quantum spin system

$$\Gamma = (V, E)$$

$$H_I = -J_x \sum_i \hat{\sigma}_i^x - J_z \sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$$



Quantum Ising Model

$\mathcal{H}_n = \mathbb{C}^{2^n}$ quantum spin system

$\Gamma = (V, E)$



$$H_I = -J_x \sum_i \hat{\sigma}_i^x - J_z \sum_{\langle i,j \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

Example 1D

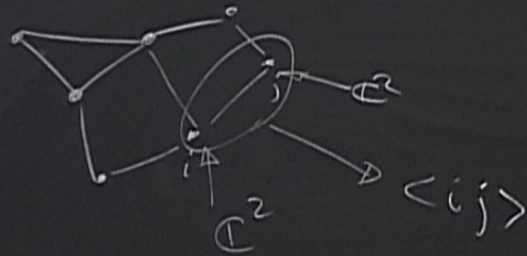
$$-J_x \sum_i \hat{\sigma}_i^x - J_z \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Quantum Ising Model

$$\mathcal{H}_n = \mathbb{C}^{2^n}$$

quantum spin system (Ising Model)

$$\Gamma = (V, E)$$



$$H_I = -J_g \sum_i \hat{\sigma}_i^x - J \sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

Example 1D

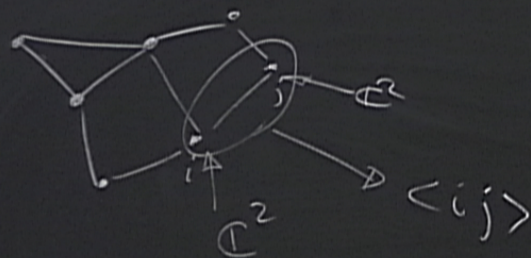
$$-J_g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Quantum Ising Model

$$\mathcal{H}_n = \mathbb{C}^{2^n}$$

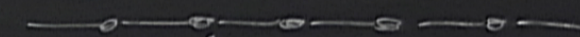
quantum spin system (Ising Model)

$$\Gamma = (V, E)$$



$$H_I = -J_g \sum_i \hat{\sigma}_i^x - J \sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

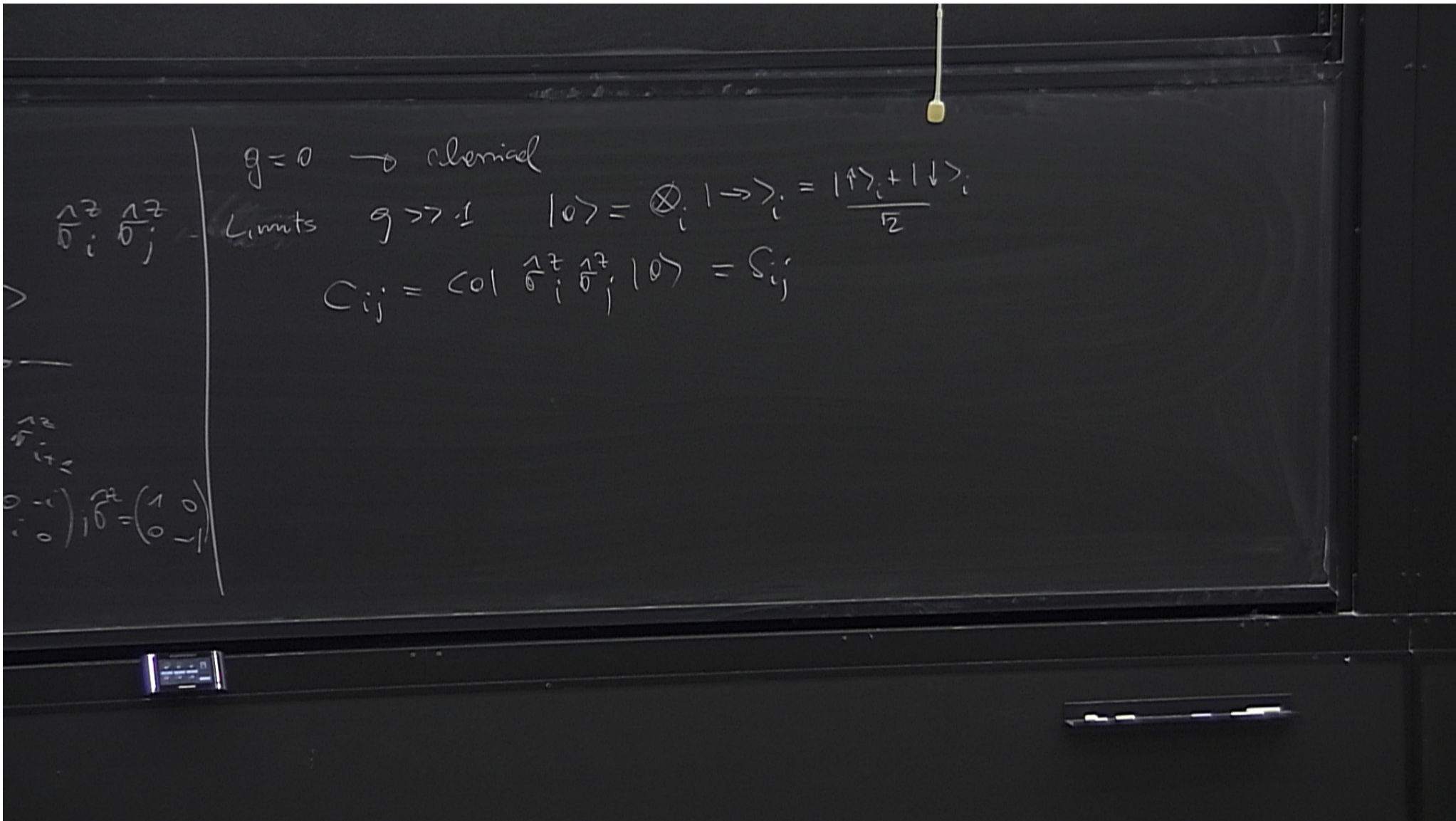
Example 1D

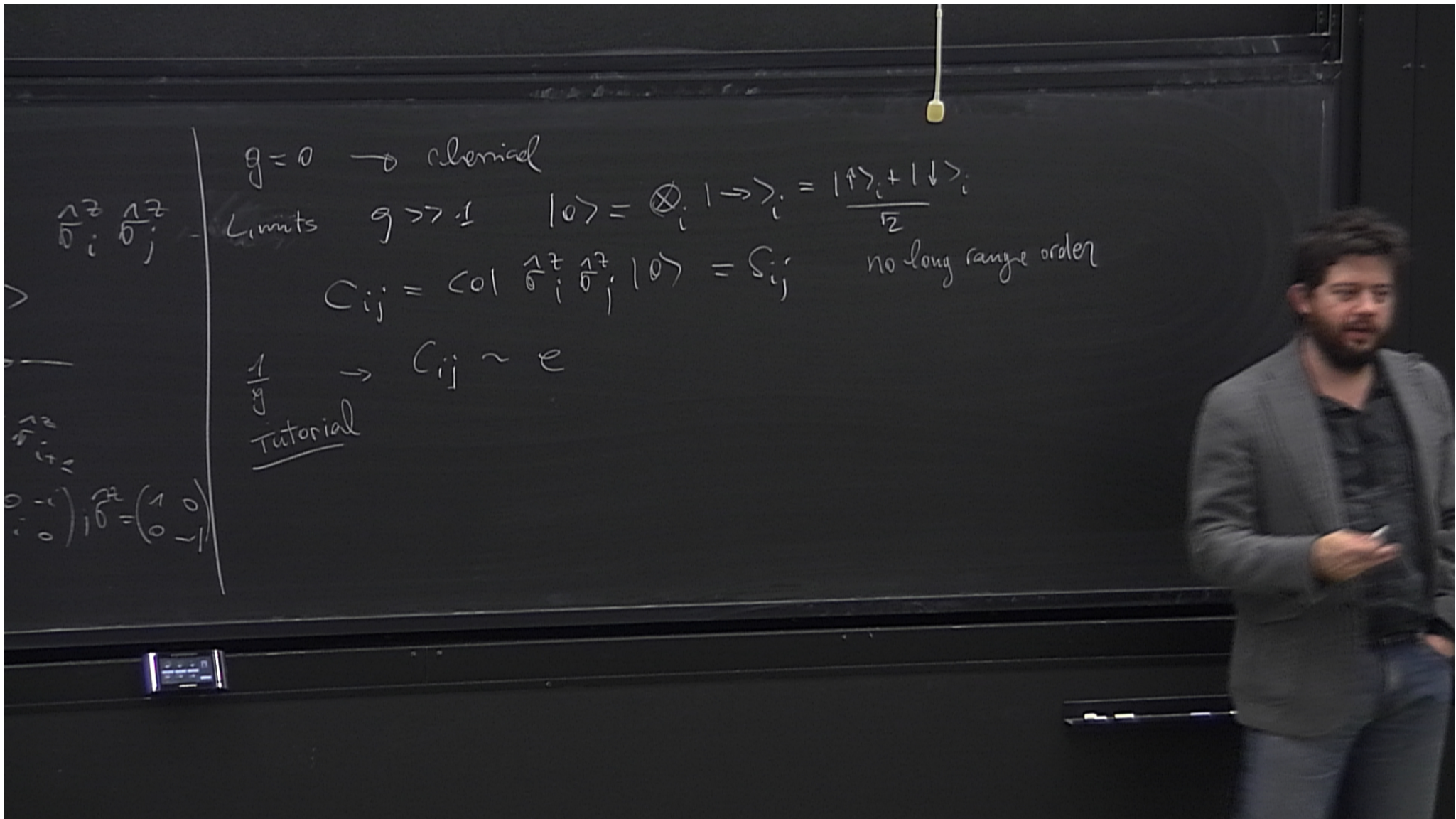


$$-J_g \sum_i \hat{\sigma}_i^x - J \sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

$$\hat{\sigma}^z |\uparrow\rangle = |\uparrow\rangle, \quad \hat{\sigma}^z |\downarrow\rangle = -|\downarrow\rangle, \quad \hat{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \hat{\sigma}^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$g=0 \rightarrow$
Limits g





$$\frac{1}{b_i} \frac{1}{b_j}$$

$g=0 \rightarrow$ chemical

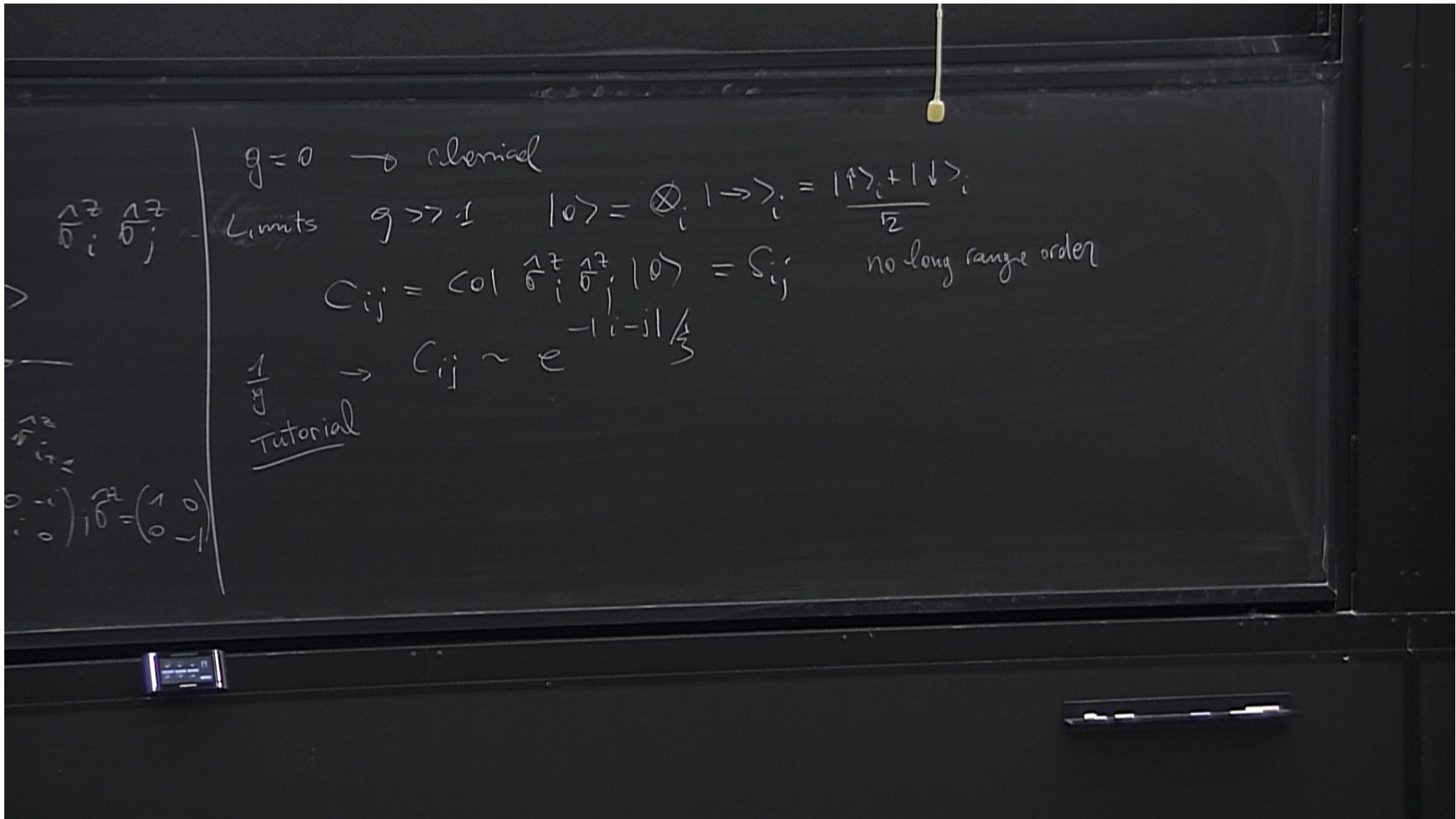
Limits $g \gg 1$ $|0\rangle = \bigotimes_i | \rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$

$$C_{ij} = \langle 0 | \frac{1}{b_i} \frac{1}{b_j} | 0 \rangle = \delta_{ij} \quad \text{no long range order}$$

$\frac{1}{g} \rightarrow C_{ij} \sim e$

Tutorial

$$\begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix} \frac{1}{b} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$g=0 \rightarrow$ chemical

Limits

$g \gg 1$

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$$

no long range order

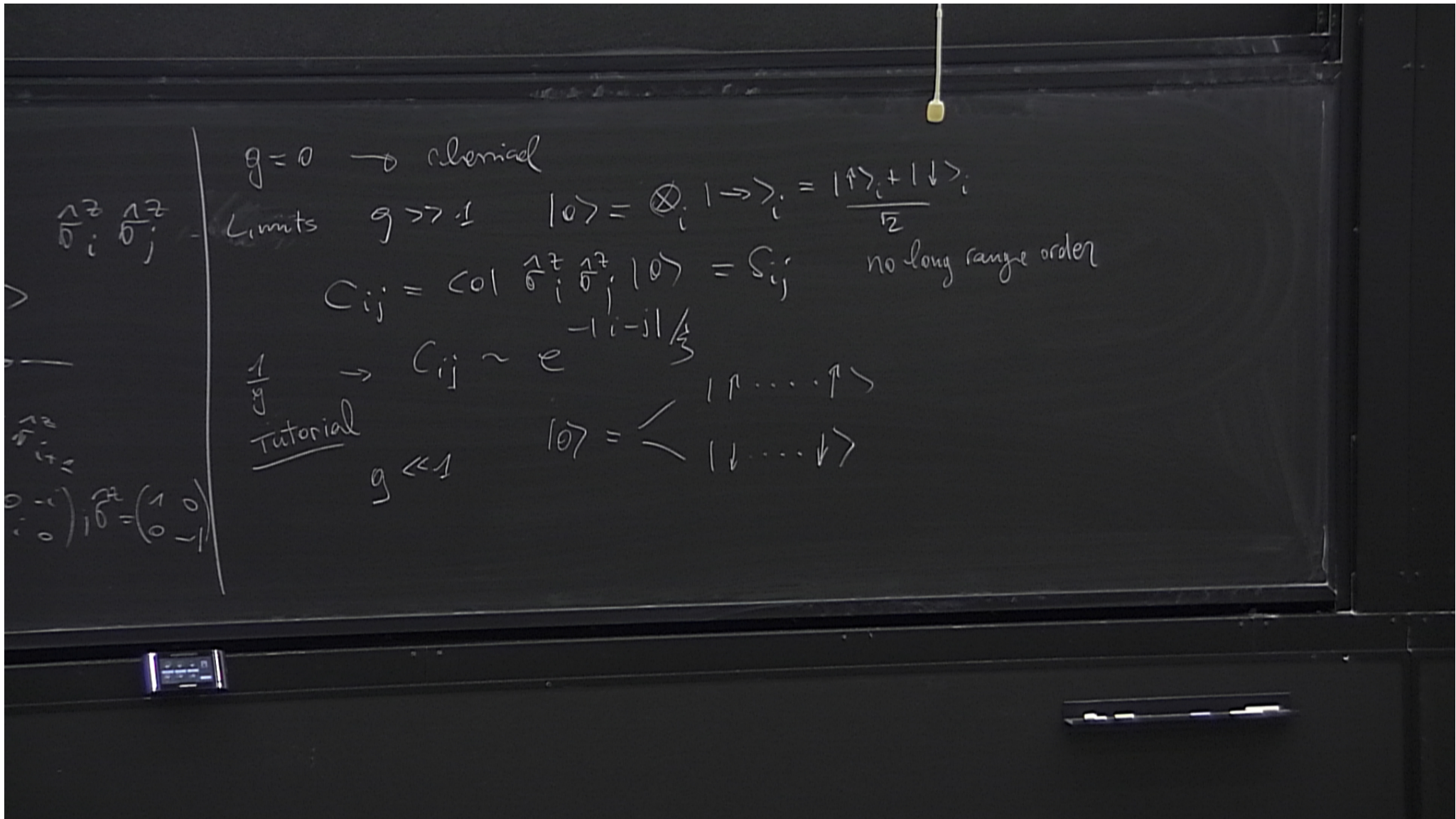
$$C_{ij} = \langle 0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | 0 \rangle = \delta_{ij}$$

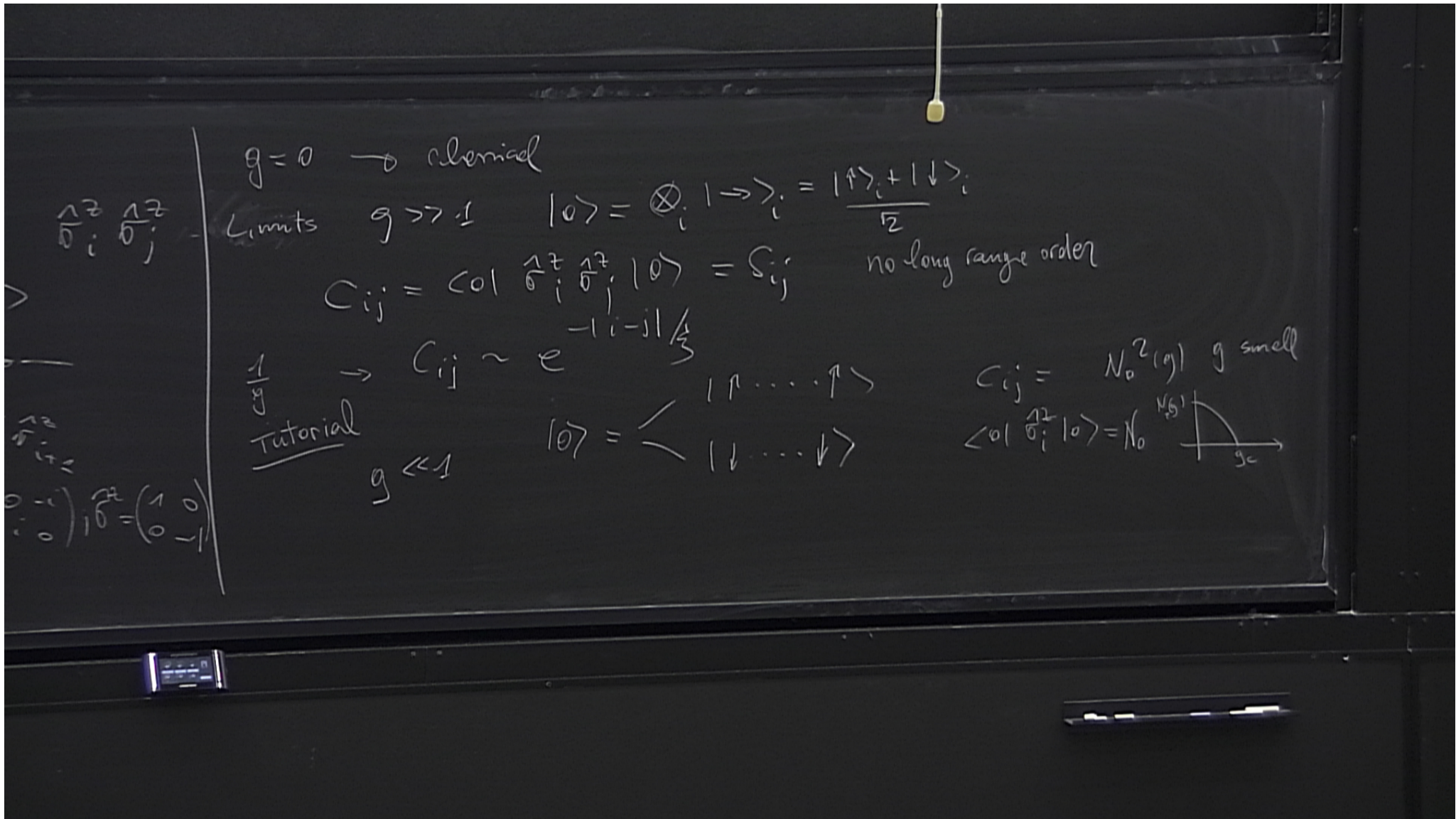
$$\frac{1}{g} \rightarrow C_{ij} \sim e^{-|i-j|/3}$$

Tutorial

$$\frac{1}{2} \hat{\sigma}_i^z \frac{1}{2} \hat{\sigma}_j^z$$

$$\frac{1}{2} \hat{\sigma}_i^z = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$





$g=0 \rightarrow$ classical

$$\hat{\sigma}_i^z \hat{\sigma}_j^z$$

Limits

$$g \gg 1$$

$$|0\rangle = \bigotimes_i |\rightarrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$$

$$C_{ij} = \langle 0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | 0 \rangle = \delta_{ij}$$

no long range order

$$\frac{1}{g}$$

$$C_{ij} \sim e^{-|i-j|/3}$$

Tutorial

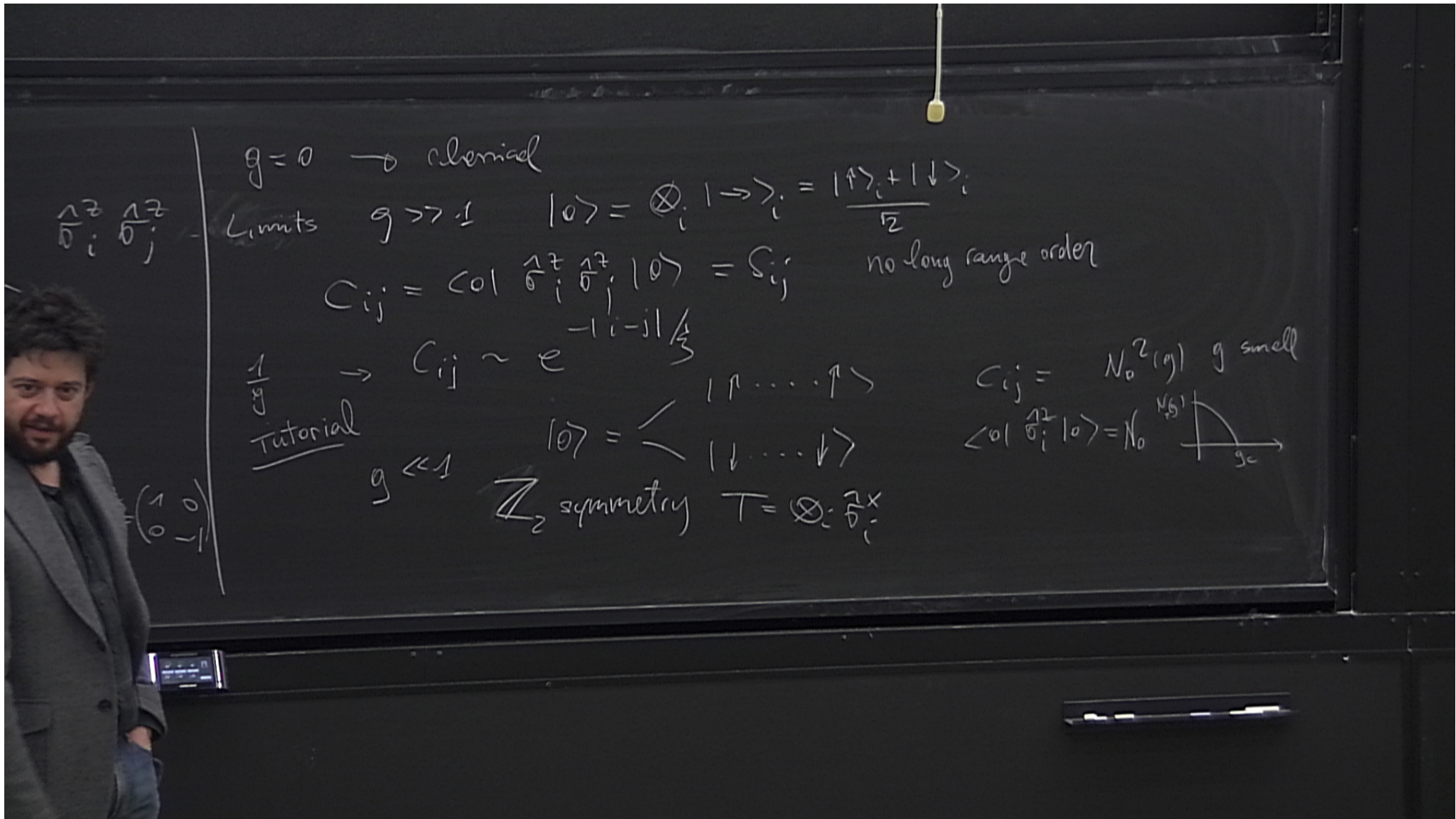
$$g \ll 1$$

$$|0\rangle = \begin{pmatrix} |\uparrow \dots \uparrow\rangle \\ |\downarrow \dots \downarrow\rangle \end{pmatrix}$$

$$C_{ij} = N_0^2(g) \quad g \text{ small}$$

$$\langle 0 | \hat{\sigma}_i^z | 0 \rangle = N_0 \quad \begin{array}{c} N_0 \\ \downarrow \\ g_c \end{array}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hat{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$$\frac{1}{\sigma_i} \frac{1}{\sigma_j}$$

$g=0 \rightarrow$ classical

Limits

$$g \gg 1$$

$$|0\rangle = \bigotimes_i |\rightarrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$$

$$C_{ij} = \langle 0 | \frac{1}{\sigma_i} \frac{1}{\sigma_j} | 0 \rangle = \delta_{ij}$$

no long range order

$$\frac{1}{g}$$

Tutorial

$$C_{ij} \sim e^{-|i-j|/3}$$

$$|\uparrow \dots \uparrow\rangle$$

$$|0\rangle = \frac{|\uparrow \dots \uparrow\rangle + |\downarrow \dots \downarrow\rangle}{\sqrt{2}}$$

$$g \ll 1$$

$$\mathbb{Z}_2 \text{ symmetry } T = \bigotimes_i \sigma_i^x$$

$$C_{ij} = N_0^2(g) \quad g \text{ small}$$

$$\langle 0 | \frac{1}{\sigma_i} | 0 \rangle = N_0 \quad \text{triangle diagram with horizontal axis } g_c$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[H, T] = 0$$

$T | \uparrow \uparrow \uparrow \dots \uparrow \rangle = | \downarrow \downarrow \downarrow \dots \downarrow \rangle$ $\mathbb{Z}_2$ is broken
 breaking of ergodicity.

$$\Delta(g) \sim J|1-g|$$

$$[H, T] = 0$$

$T | \uparrow \uparrow \uparrow \dots \uparrow \rangle = | \downarrow \downarrow \downarrow \dots \downarrow \rangle$ $\mathbb{Z}_2$ is broken
 breaking of ergodicity.

$$\Delta(g) \sim J|1-g|$$

Duality

$$[H, T] = 0$$

$$T | \uparrow \uparrow \uparrow \dots \uparrow \rangle = | \downarrow \downarrow \downarrow \dots \downarrow \rangle \quad \mathbb{Z}_2 \text{ is broken}$$

breaking of ergod

$$\Delta(g) \sim J|1-g|$$

Duality

$$\tau_{\mu}^z = \sigma_i^z \sigma_{i+1}^z$$

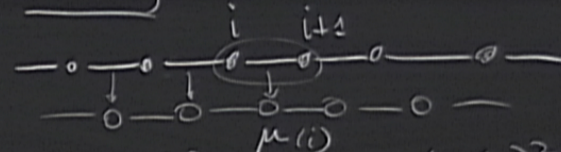
$$[H, T] = 0$$

$$T | \uparrow \uparrow \uparrow \dots \uparrow \rangle = | \downarrow \downarrow \downarrow \dots \downarrow \rangle \quad \mathbb{Z}_2 \text{ is broken}$$

breaking of ergodicity.

$$\Delta(g) \sim J|1-g|$$

Duality



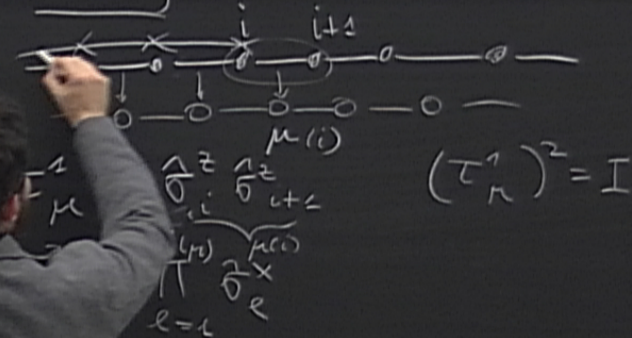
$$\tau_{\mu}^1 = \underbrace{\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z}_{\mu(i)} \quad (\tau_{\mu}^1)^2 = I$$

$$\tau_{\mu}^3$$

$$[H, T] = 0$$

$$\Delta(q) \sim 5/1-q!$$

Duality



$$|\downarrow\rangle = -|\downarrow\rangle$$

$$(0, 1, 0) = (1, 0, 1) = (0, -1, 1)$$

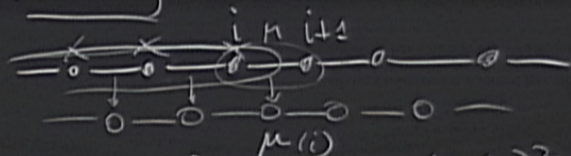
$\mathbb{Z}_2$ symmetry

$$[H, T] = 0$$

$T|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle = |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$ $\mathbb{Z}_2$ is broken
breaking of ergodicity.

$$\Delta(g) \sim J|1-g|$$

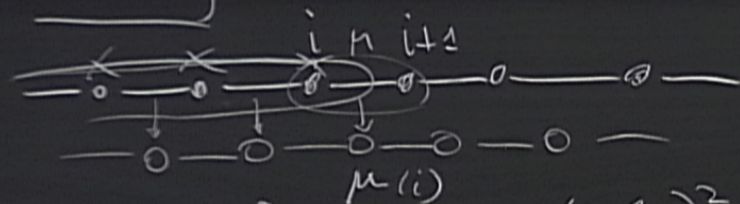
Duality



$$\tau_\mu^1 = \prod_{i \in \mu} \sigma_i^z \quad (\tau_\mu^1)^2 = I$$

$$\tau_\mu^3 = \prod_{i \in \mu} \sigma_i^x \quad (\tau_\mu^3)^2 = I$$

Duality



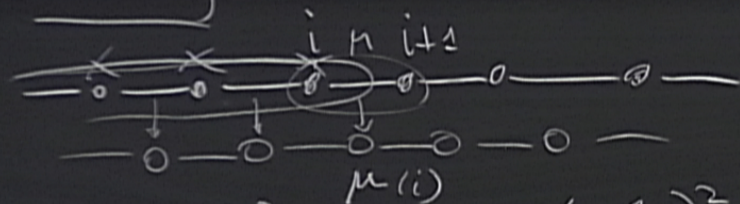
$$\tau_{\mu}^1 = \frac{\hat{\sigma}_{i,i}^z}{\hat{\sigma}_{i+1,i}^z} \quad (\tau_{\mu}^1)^2 = I$$

$$\tau_{\mu}^3 = \prod_{\ell=1}^{i(\mu)} \hat{\sigma}_{\ell}^x \quad (\tau_{\mu}^3)^2 = I$$

$$\{\tau_{\mu}^3, \tau_{\mu'}^1\} = \delta_{\mu\mu'}$$

$$H_I \xrightarrow{\text{duality}} H_E = -Jg$$

Duality



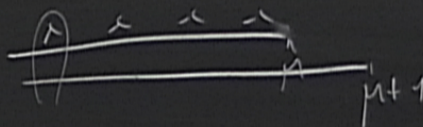
$$\tau_{\mu}^1 = \frac{\hat{\sigma}_i^z}{\hat{\sigma}_{i+1}^z} \quad (\tau_{\mu}^1)^2 = I$$

$$\tau_{\mu}^3 = \prod_{\ell=1}^{i(\mu)} \hat{\sigma}_{\ell}^x \quad (\tau_{\mu}^3)^2 = I$$

$$\{\tau_{\mu}^3, \tau_{\mu'}^1\} = \delta_{\mu\mu'}$$

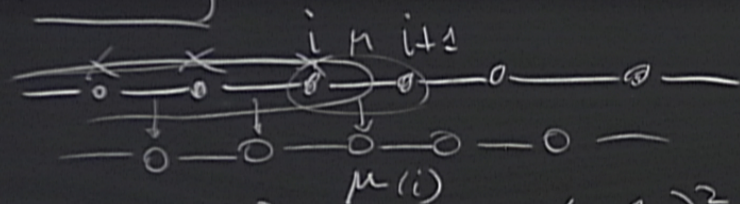
$$H_I \xrightarrow{\text{duality}} H_C = -Jg$$

$$\tau_{\mu}^3 \tau_{\mu+1}^3 = \hat{\sigma}_{\mu+1}^x$$



$$\sum_i \hat{\sigma}_i^x$$

Duality



$$\tau_{\mu}^1 = \frac{\hat{\sigma}_i^z}{\hat{\sigma}_{i+1}^z} \quad (\tau_{\mu}^1)^2 = I$$

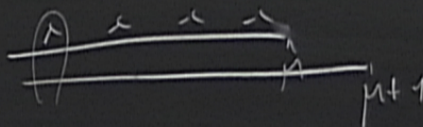
$$\tau_{\mu}^3 = \prod_{l=1}^{i(\mu)} \hat{\sigma}_l^x \quad (\tau_{\mu}^3)^2 = I$$

$$\{\tau_{\mu}^3, \tau_{\mu+1}^1\} = \delta_{\mu, \mu+1}$$

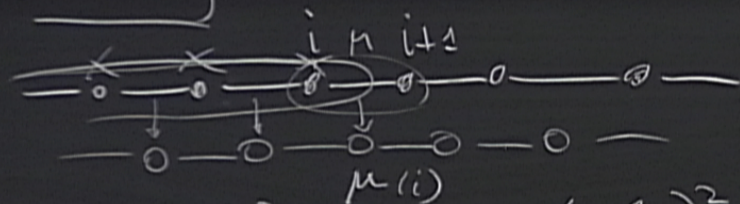
$$H_I \xrightarrow{\text{duality}} H_C = -Jg \bar{\tau}_{\mu}^3 \tau_{\mu+1}^3 - J \sum_{\mu} \tau_{\mu}^1$$

$$\tau_{\mu}^3 \tau_{\mu+1}^3 = \hat{\sigma}_{\mu+1}^x$$

$$\bar{\tau}_i^3 \hat{\sigma}_i^x$$



Duality



$$\tau_{\mu}^1 = \frac{\hat{\sigma}_i^z}{\hat{\sigma}_{i+1}^z} \quad (\tau_{\mu}^1)^2 = I$$

$$\tau_{\mu}^3 = \prod_{l=1}^{\mu(i)} \hat{\sigma}_l^x \quad (\tau_{\mu}^3)^2 = I$$

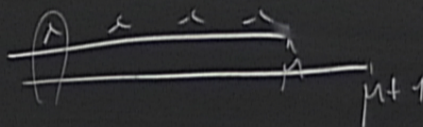
$$\{\tau_{\mu}^3, \tau_{\mu+1}^1\} = \delta_{\mu, \mu+1}$$

$$H_I \xrightarrow{\text{duality}} H_C = -Jg \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - J \sum_{\mu} \tau_{\mu}^1$$

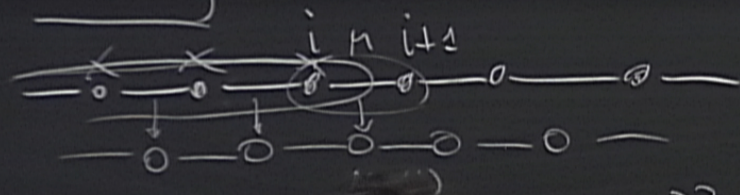
$$= g \left(-J \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - \frac{J_1}{g} \sum_{\mu} \tau_{\mu}^1 \right)$$

$$\tau_{\mu}^3 \tau_{\mu+1}^3 = \hat{\sigma}_{\mu+1}^x$$

$$\sum_i \hat{\sigma}_i^x$$



Duality



$$\tau_{\mu}^1 = \frac{\hat{\sigma}^z}{i}$$

$$(\tau_{\mu}^1)^2 = I$$

$$\tau_{\mu}^3 = \frac{i(n)}{11}$$

$$(\tau_{\mu}^3)^2 =$$

$$\{\tau_{\mu}^3, \tau_{\mu+1}^3\}$$

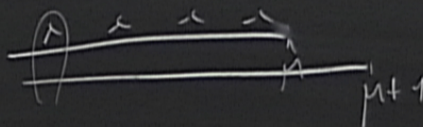
$$H_I \xrightarrow{\text{duality}} H_C = -Jg \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - J \sum_{\mu} \tau_{\mu}^1$$

$$= g \left(-J \sum_{\mu} \tau_{\mu}^3 \tau_{\mu}^3 - \frac{J_1}{g} \sum_{\mu} \tau_{\mu}^1 \right)$$

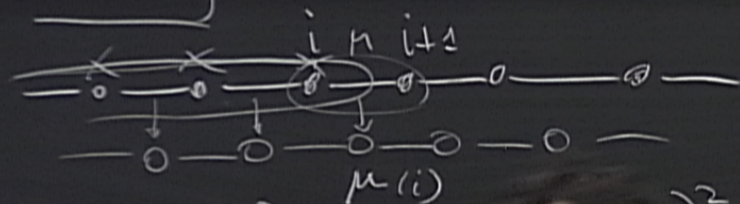
$$H_I(\sigma, g) = g H_{\pm}(\tau, \frac{1}{g})$$

$$\tau_{\mu}^3 \tau_{\mu+1}^3 = \hat{\sigma}_{\mu+1}^x$$

$$\sum_i \hat{\sigma}_i^x$$



Duality



$$\tau_{\mu}^1 = \frac{\partial}{\partial \sigma_i} \frac{\partial}{\partial \sigma_{i+1}} \quad \tau_{\mu}^3 = \frac{\partial}{\partial \sigma_i} \frac{\partial}{\partial \sigma_{i+1}} \quad \tau_{\mu}^1 \tau_{\mu}^3 = I$$

$$\tau_{\mu}^3 = \frac{\partial}{\partial \sigma_i} \frac{\partial}{\partial \sigma_{i+1}} \quad \tau_{\mu}^1 = \frac{\partial}{\partial \sigma_i} \frac{\partial}{\partial \sigma_{i+1}} \quad \tau_{\mu}^1 \tau_{\mu}^3 = I$$

$$\{\tau_{\mu}^3, \tau_{\mu}^1\} = \delta_{\mu, \mu'}$$

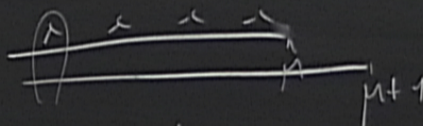
$$H_I \xrightarrow{\text{duality}} H_C = -Jg \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - J \sum_{\mu} \tau_{\mu}^1$$

$$= g \left(-J \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - \frac{J_1}{g} \sum_{\mu} \tau_{\mu}^1 \right)$$

$$H_I(\sigma, g) = g H_{\pm}(\tau, \frac{1}{g})$$

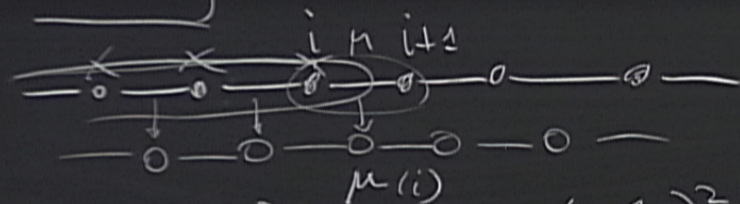
$$\tau_{\mu}^3 \tau_{\mu+1}^3 = \sigma_{\mu+1}^x$$

$$\sum_i \sigma_i^x$$



$$\Delta E(\sigma, g) = g \Delta E(\tau, \frac{1}{g})$$

Duality



$$\tau_{\mu}^1 = \frac{\hat{\sigma}_{i,i}^z}{\hat{\sigma}_{i+1,i+1}^z} \quad (\tau_{\mu}^1)^2 = I$$

$$\tau_{\mu}^3 = \prod_{l=1}^{i(\mu)} \hat{\sigma}_{\mu(l)}^x \quad (\tau_{\mu}^3)^2 = I$$

$$\{\tau_{\mu}^3, \tau_{\mu+1}^1\} = \delta_{\mu, \mu+1}$$

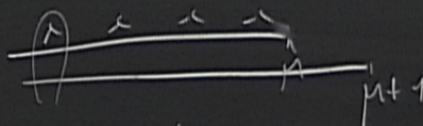
$$H_I \xrightarrow{\text{duality}} H_C = -Jg \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - J \sum_{\mu} \tau_{\mu}^1$$

$$= g \left(-J \sum_{\mu} \tau_{\mu}^3 \tau_{\mu+1}^3 - \frac{J_1}{g} \sum_{\mu} \tau_{\mu}^1 \right)$$

$$H_I(\sigma, g) = g H_C(\tau, \frac{1}{g})$$

$$\tau_{\mu}^3 \tau_{\mu+1}^3 = \hat{\sigma}_{\mu+1}^x$$

$$\sum_i \hat{\sigma}_i^x$$



$$\Delta E(\sigma, g) = g \Delta E(\tau, \frac{1}{g})$$