

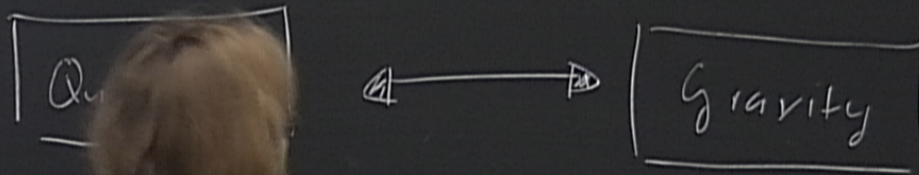
Title: 12/13 PSI - Quantum Gravity Review Lecture 1

Date: Jan 28, 2013 10:15 AM

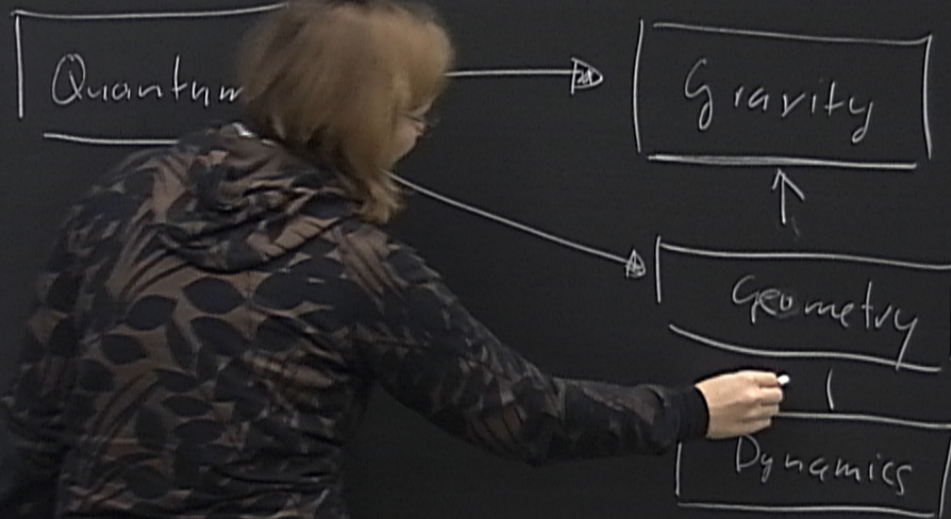
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Abstract:

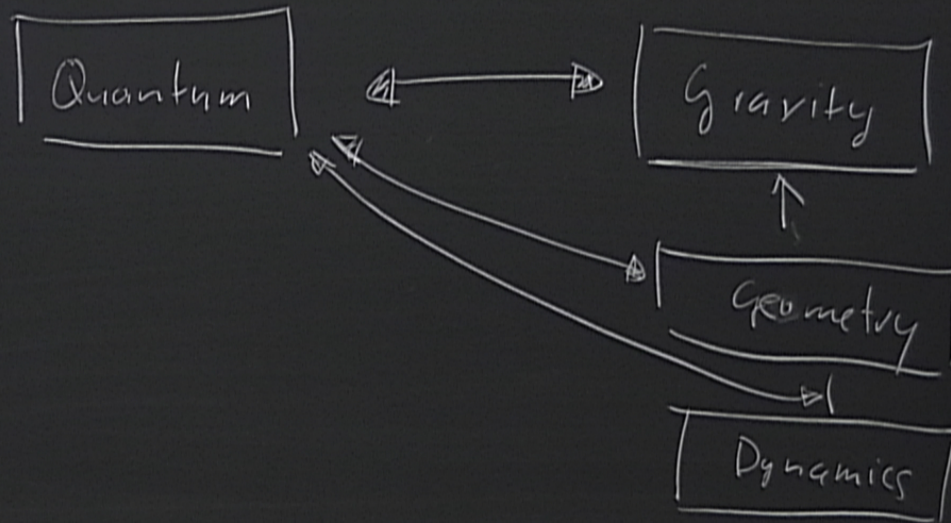
Quantum Gravity



Quantum Gravity

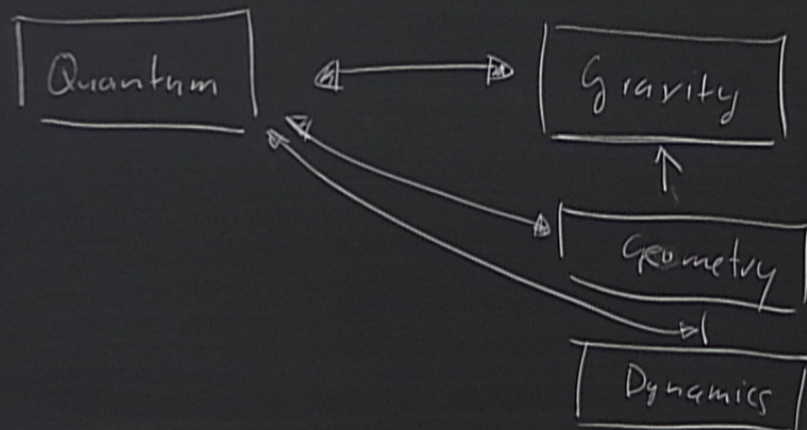


Quantum Gravity



classical background
(Minkowski space)

Quantum Gravity

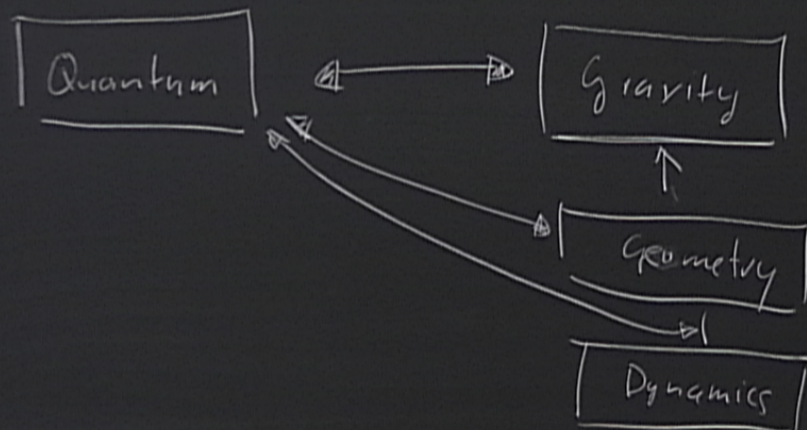


Classical background
(Minkowski space)

Quantum Field Theory

Geometro-
dynamics

Quantum Gravity



Classical background
(Minkowski)

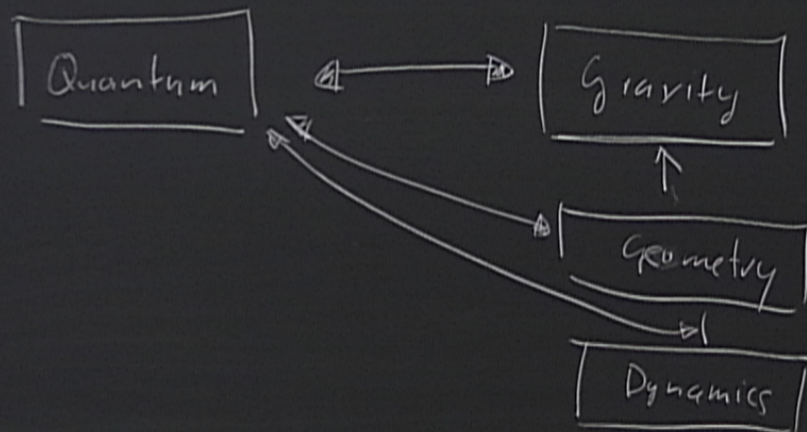
Quantum

larger
scale

Geometrodynamics

Quantum

Quantum Gravity



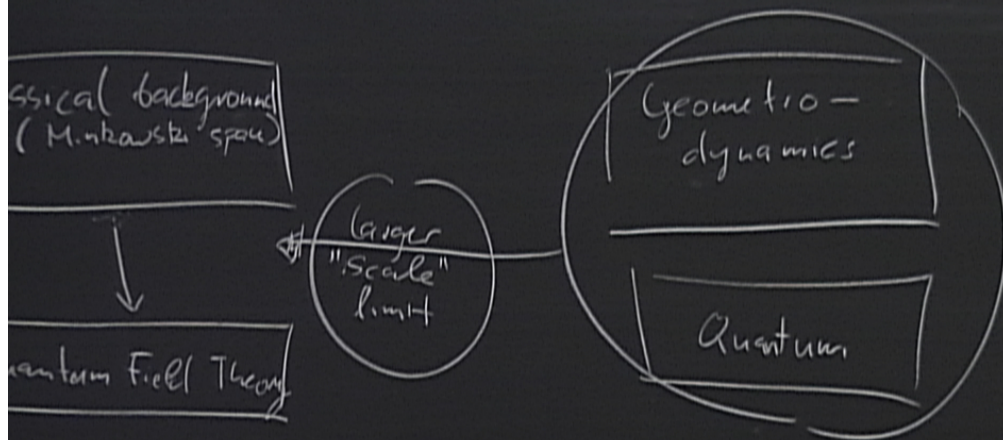
Classical background
(Minkowski space)

Quantum Field Theory

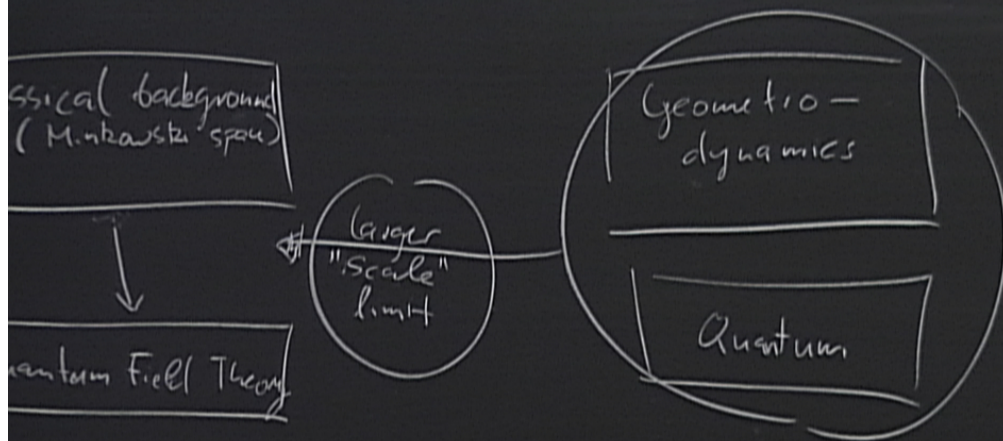
larger
"scale"
limit

Geometric-
dynamics

Quantum

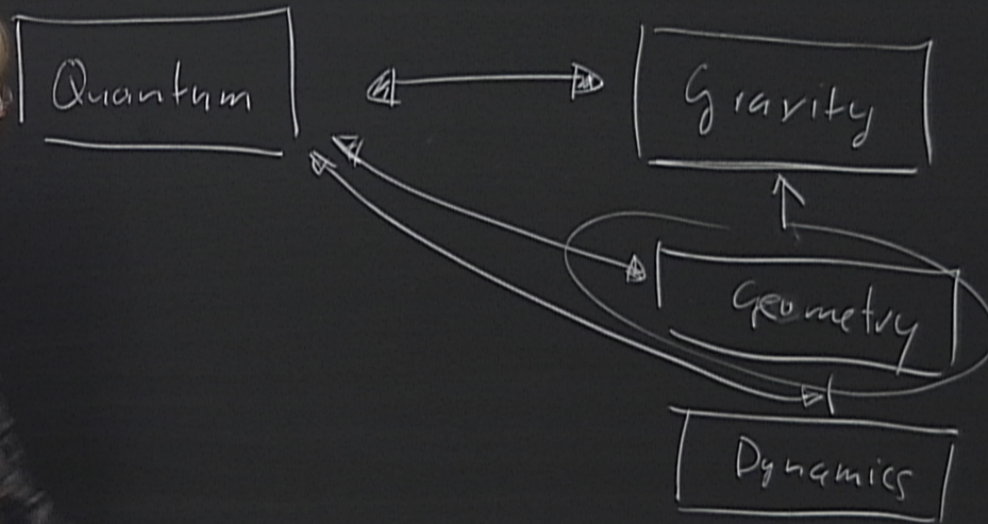


- Loop Quantum Gravity
- Spin Foams



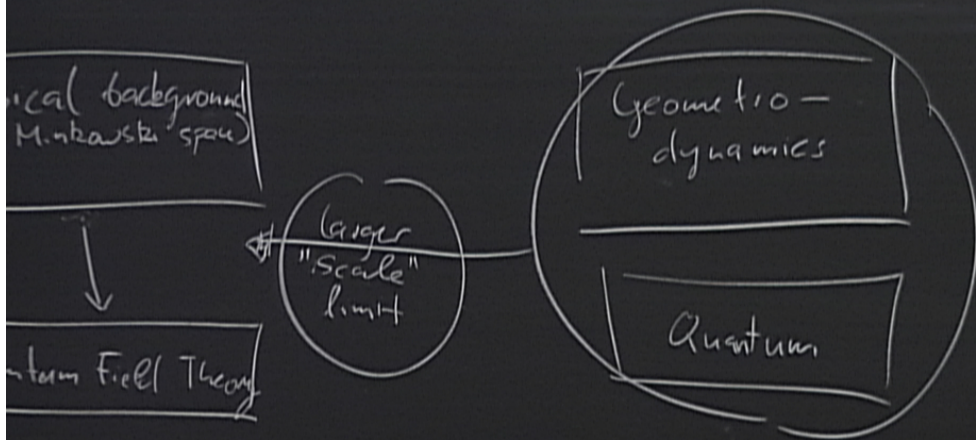
- Loop Quantum Gravity
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Quantum Gravity



Classical background
(Minkowski space)

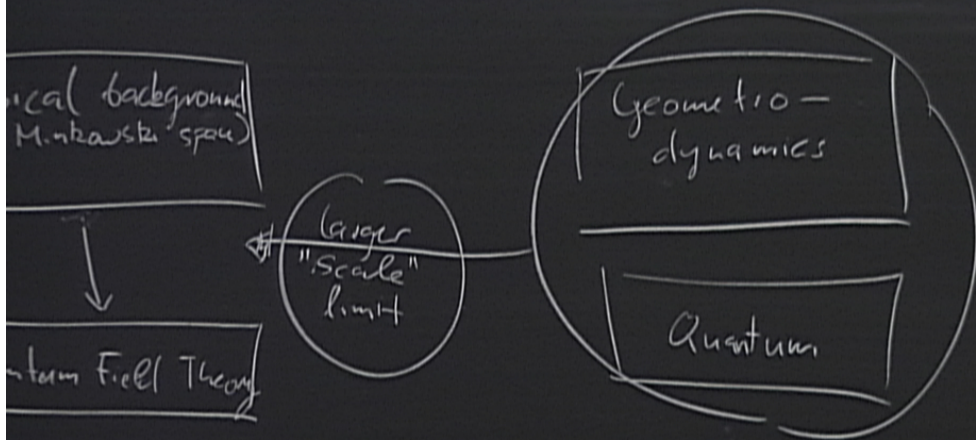
Quantum Field Theory



- Loop Quantum Gravity
- Spin Foams

3D QG

"Wave fd. of the universe"



- Loop Quantum Gravity
- Spin Foams

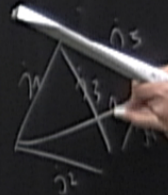
3D QG

"Wave fd. of the universe"

(spherical, Riemannian sign)

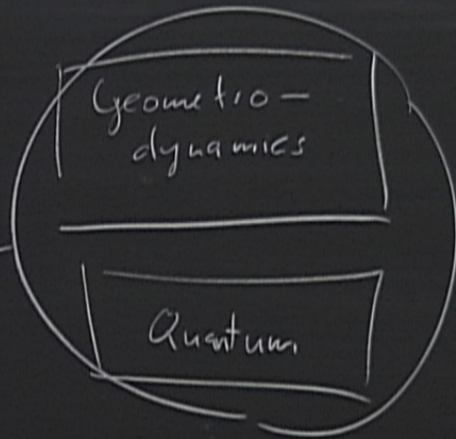
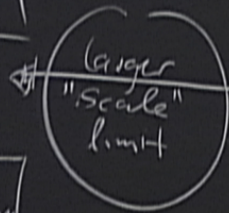
$$\psi(j) = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

Representation
theory of
Sl(2)



classical background
(Minkowski space)

Quantum Field Theory



- Loop Quantum Gravity
- Spin Foams

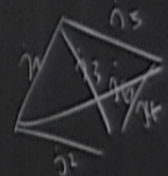
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(spherical, Riemannian sign)

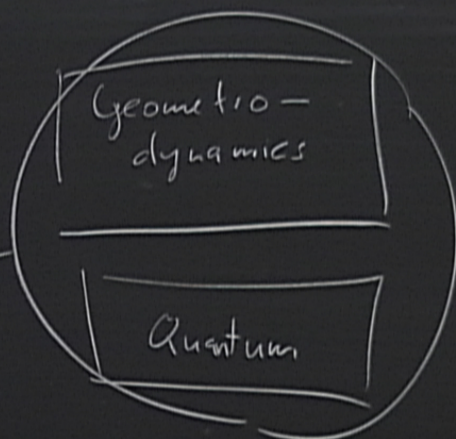
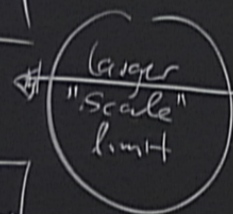
$$\psi(j) = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

Representation
theory of
 $Sl(2)$



Local background
Minkowski space

Quantum Field Theory



- Loop Quantum Gravity
- Spin Foams

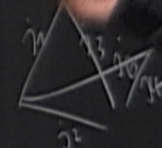
3D QG

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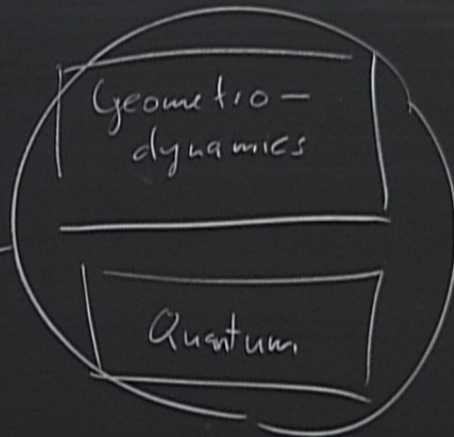
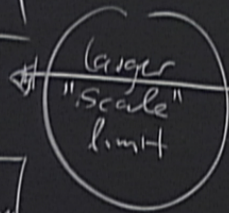
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Representation
theory of
Sl(2)



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Minkowski space

Quantum Field Theory



- Loop Quantum Gravity
- Spin Foams

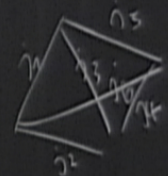
3D QG

"Wave fd. of the universe"

(spherical, Riemannian sign)

$$\psi(j) = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

Representation
theory of
 $Sl(2)$



Overview

1. Classical P

- Action
- First order
 - n-beins
 - spin conn
- Can

Overview

1. Classical Physics

- Action principle
- First order
- n-body
- spin
- Canonical
"dynamics"

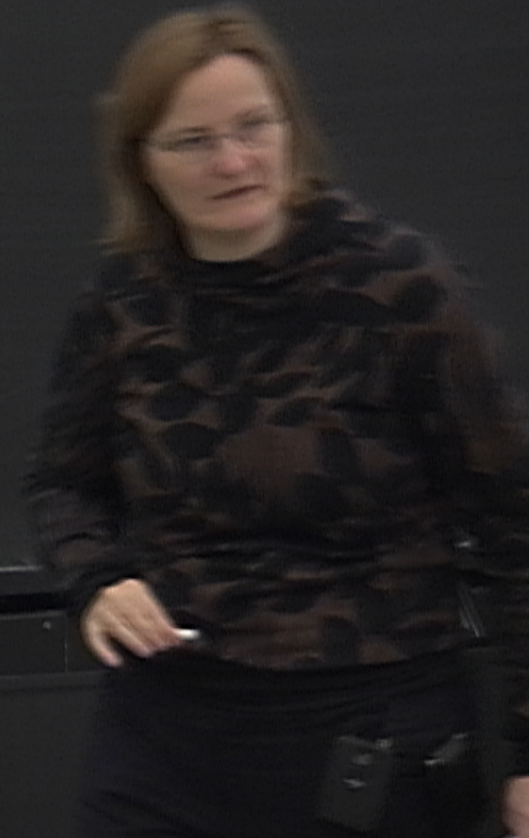
Overview

1 Classical Part

- Action principle
- First order action:
 - n -beins
 - spin connection
- Canonical formalism =
"dynamics of space in
time"

2 Quantum Part

-



Overview

1 Classical Part

- Action principle
- First order action:
 - n -beins
 - spin connection
- Canonical formalism =
"dynamics of space in time"

2 Quantum Part

- Canonical quantiz
 - spinnetwork: basis
of quantum geom.
states
- Quantum Geometric Operator

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- First order action:
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2 Quantum Part

- Canonical quantiz
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of quantum geom.
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- Quantum Geometric Operator
- Quantum Dynamics?

- Path integral approach
spin foam models

Quantum Part

non-cal quantiz

- Spinnetwork: basis
of quantum geom.
states

- Quantum Geometric Operator

- Quantum Dynamics?

- Path integral approach:
spin foam models

3. -4 D theory aspect
- applications

in integral approach:
spin foam models

+ D theory aspect
applications

- R Gambini, J Pullin: A first course in loop
quantum gravity
(2011)

- A. Perez: gr-qc/0409061
(more advanced)

2. 3D gravity

2.1. Gravity in n dimensions

2. 3D gravity

2.1. Gravity in n dimensions

(Euclidean signature $(+, +, +, \dots)$)

Einstein action

S

2. 3D gravity

2.1. Gravity in n dimensions

(Euclidean signature) $(+, +, +, \dots)$
Einstein-Hilbert action

$$S = \int R \sqrt{\det g} d^n x$$

2. 3D gravity

2.1. Gravity in n dimensions

(Euclidean signature $(+, +, + \dots)$)

Einstein-Hilbert action

$$S[\underset{\substack{\uparrow \\ \text{basic variable}}}{g_{\mu\nu}(x)}] = \int R_{\mu\nu} g^{\mu\nu} \underbrace{\sqrt{\det g}}_{\sqrt{|g|}} d^n x$$

2. 3D gravity

2.1. Gravity in n dimensions

(Euclidean signature $(+, +, + \dots)$):
Einstein-Hilbert action

$$g_{\mu\nu} U^\mu U^\nu > 0 \quad \forall U$$

$$S[\underset{\substack{\uparrow \\ \text{basic variable}}}{g_{\mu\nu}(x)}] = \int R_{\mu\nu} g^{\mu\nu} \underbrace{\sqrt{\det g}}_{\sqrt{|g|}} d^n x$$

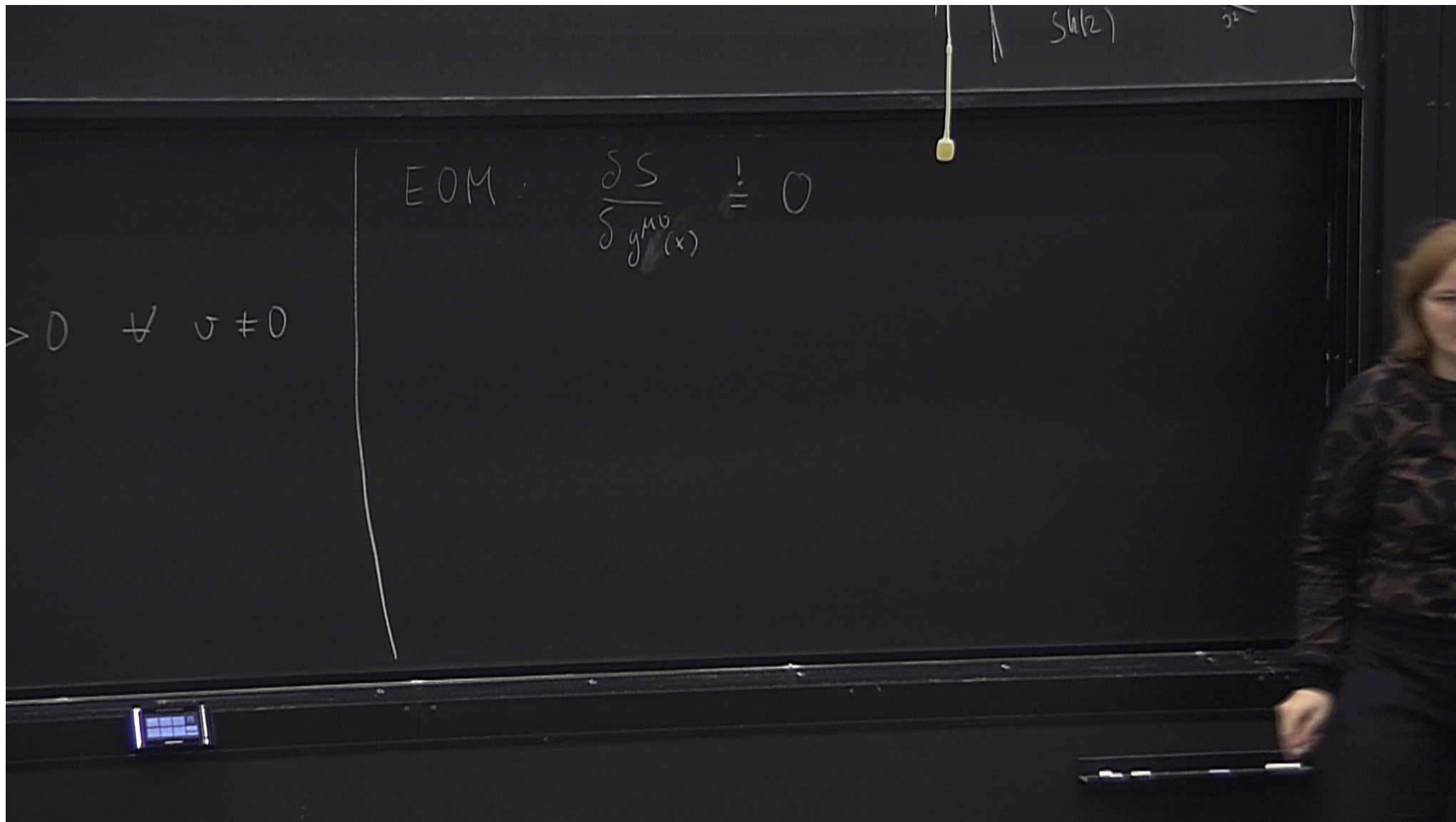
2. 3D gravity

2.1. Gravity in n dimensions

(Euclidean signature $(+, +, + \dots)$):
Einstein-Hilbert action

$$g_{\mu\nu} U^\mu U^\nu > 0 \quad \forall \quad U \neq 0$$

$$S[\underbrace{g_{\mu\nu}(x)}_{\text{base variable}}] = \int R_{\mu\nu} g^{\mu\nu} \underbrace{\sqrt{\det g}}_{\sqrt{|g|}} d^n x$$



$$> 0 \quad \forall \quad v \neq 0$$

$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\delta S = \int \mathcal{L}$$

$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\delta S = \int \left(\left[\delta R_{\mu\nu} \right] g^{\mu\nu} \sqrt{g} + R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + R \delta \sqrt{g} \right) d^4 x$$

$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\delta S = \int \left([\delta R_{\mu\nu}] g^{\mu\nu} \sqrt{g} + R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + R \delta \sqrt{g} \right) d^4 x$$

$$[\delta R_{\mu\nu}] g^{\mu\nu} =$$

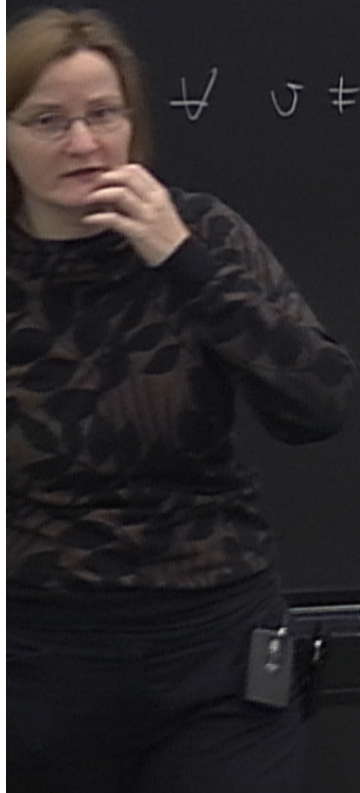
$$> 0 \quad \forall \quad \sigma \neq 0$$

$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\delta S = \int d^4x \left[\delta R_{\mu\nu} g^{\mu\nu} \sqrt{g} + R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + R \delta \sqrt{g} \right]$$

$$\nabla_\mu (T^{\mu\nu})$$

$$T^{\mu\nu} = (\nabla^\nu g^{\mu\sigma} \delta g_{\sigma\nu} - g^{\nu\sigma} g^{\mu\tau} \nabla_\tau \delta g_{\sigma\nu})$$



$\forall \quad \delta \neq 0$

EOM: $\frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$

$$\delta S = \int \left([\delta R_{\mu\nu}] g^{\mu\nu} \sqrt{g} + R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + R \delta \sqrt{g} \right) d^4 x$$

$$[\delta R_{\mu\nu}] g^{\mu\nu} = \nabla_\mu (\quad)^\mu$$

$$U^\mu = \left(\nabla^\nu g^{\mu\sigma} \delta g_{\sigma\nu} - g^{\nu\sigma} g^{\mu\tau} \nabla_\tau \delta g_{\nu\sigma} \right)$$

$$> 0 \quad \forall \quad \delta \neq 0$$

$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\delta S = \int \left([\delta R_{\mu\nu}] g^{\mu\nu} \sqrt{g} + R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + R \delta \sqrt{g} \right) d^4 x$$

$$[\delta R_{\mu\nu}] g^{\mu\nu} = \nabla_\mu \left(\delta g^{\mu\nu} \right) \quad \text{Total divergence}$$

$$U^\mu = \left(\nabla^\nu g^{\mu\sigma} \delta g_{\sigma\nu} - g^{\nu\sigma} g^{\mu\tau} \nabla_\tau \delta g_{\nu\sigma} \right)$$

$$> 0 \quad \forall \quad \delta \neq 0$$

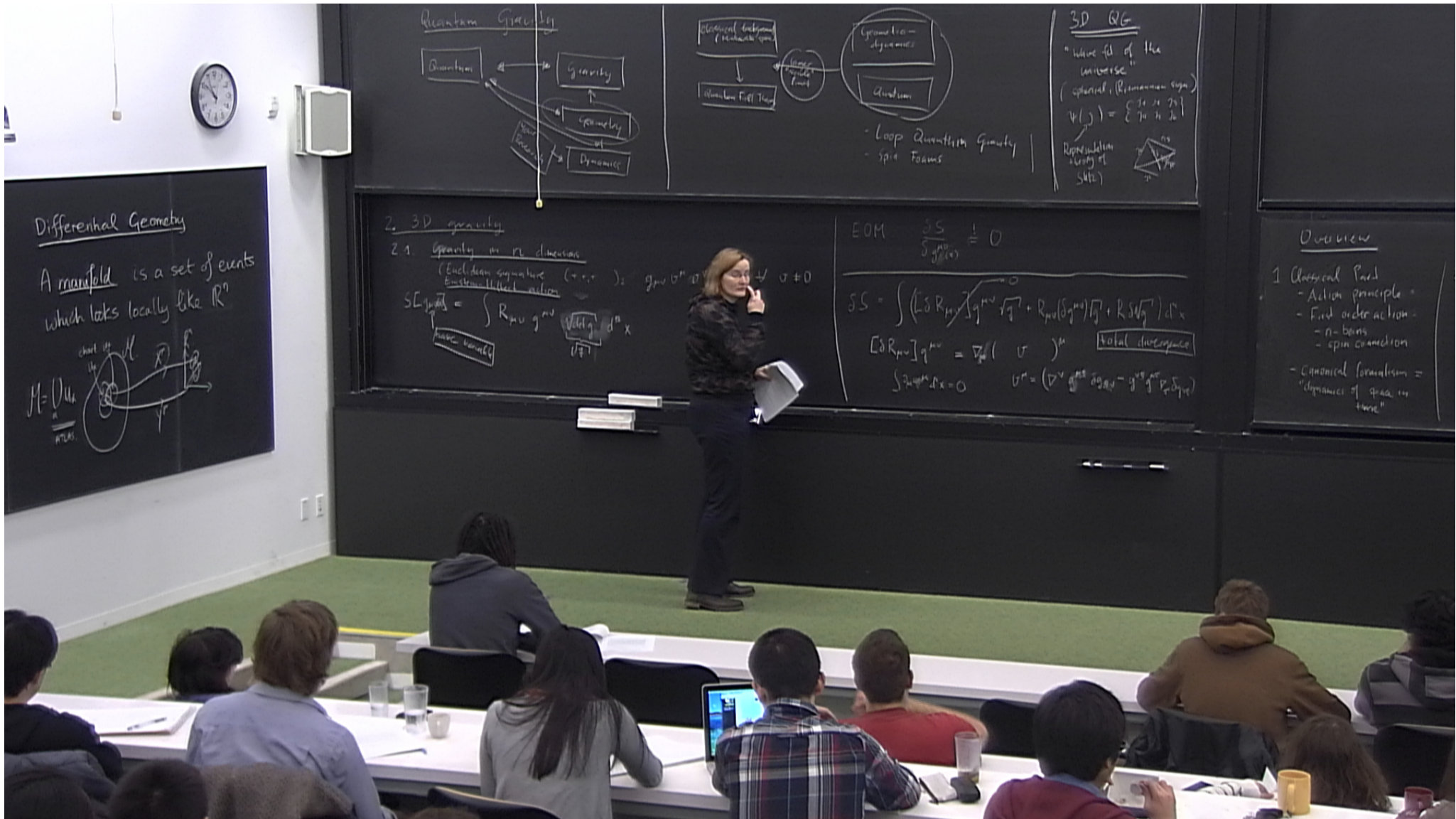
$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\delta S = \int \left(\cancel{[\delta R_{\mu\nu}] g^{\mu\nu} \sqrt{g}} + R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g} + R \delta \sqrt{g} \right) d^4 x$$

$$[\delta R_{\mu\nu}] g^{\mu\nu} = \nabla_\mu (\quad)^\mu$$

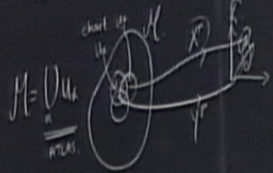
total divergence

$$U^\mu = \left(\nabla^\nu g^{\mu\sigma} \delta g_{\sigma\nu} - g^{\nu\sigma} g^{\mu\tau} \nabla_\tau \delta g_{\nu\sigma} \right)$$

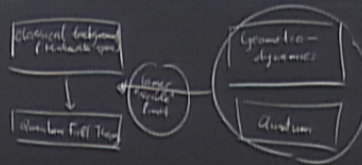
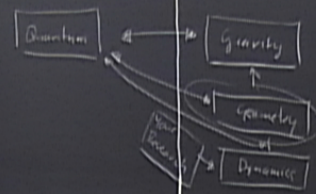


Differential Geometry

A manifold is a set of events which looks locally like $\mathbb{R}^n$



Quantum Gravity



- Loop Quantum Gravity
- Spin Foams

3D QG

"Volume of the universe"

(special: Riemannian sign)

$$\Psi(\gamma) = \begin{cases} 1 & \text{if } \gamma \text{ is trivial} \\ 0 & \text{otherwise} \end{cases}$$

Representation theory of SU(2)



2. 3D gravity

2.1. Gravity in 3 dimensions

(Euclidean signature (+,+,+)) $g_{\mu\nu} \partial^\mu \partial^\nu \psi \neq 0$

$$S_{\text{Einstein-Hilbert}} = \int R_{\mu\nu} g^{\mu\nu} \sqrt{|g|} d^3x$$

metric variable

Volume

$$\text{EOM} \quad \frac{\delta S}{\delta g^{\mu\nu}} = 0$$

$$S = \int (L(g, R_{\mu\nu}) \sqrt{|g|} + R_{\mu\nu} (g^{\mu\nu}) \sqrt{|g|} + R \sqrt{|g|}) d^3x$$

$$[D R_{\mu\nu}] g^{\mu\nu} = \nabla_\mu (\quad)^\mu \quad \text{total divergence}$$

$$\int \sqrt{|g|} d^3x = 0 \quad \nabla^\mu (\quad)_\mu = 0$$

Overview

1. Classical Part
 - Action principle
 - First order action
 - n-basis
 - spin connection
- Canonical formalism = "dynamics of phase space"

gravity

gravity in n dimensions

euclidean signature $(+, +, +, \dots)$:
stein-Hilbert action

$$S = \int R_{\mu\nu} g^{\mu\nu} \underbrace{\sqrt{\det g}}_{\sqrt{|g|}} d^n x$$

variable

$$g_{\mu\nu} U^\mu U^\nu > 0 \quad \forall U \neq 0$$

(Manifold without
boundaries)

EOM: $\frac{\delta S}{\delta g^{\mu\nu}}$

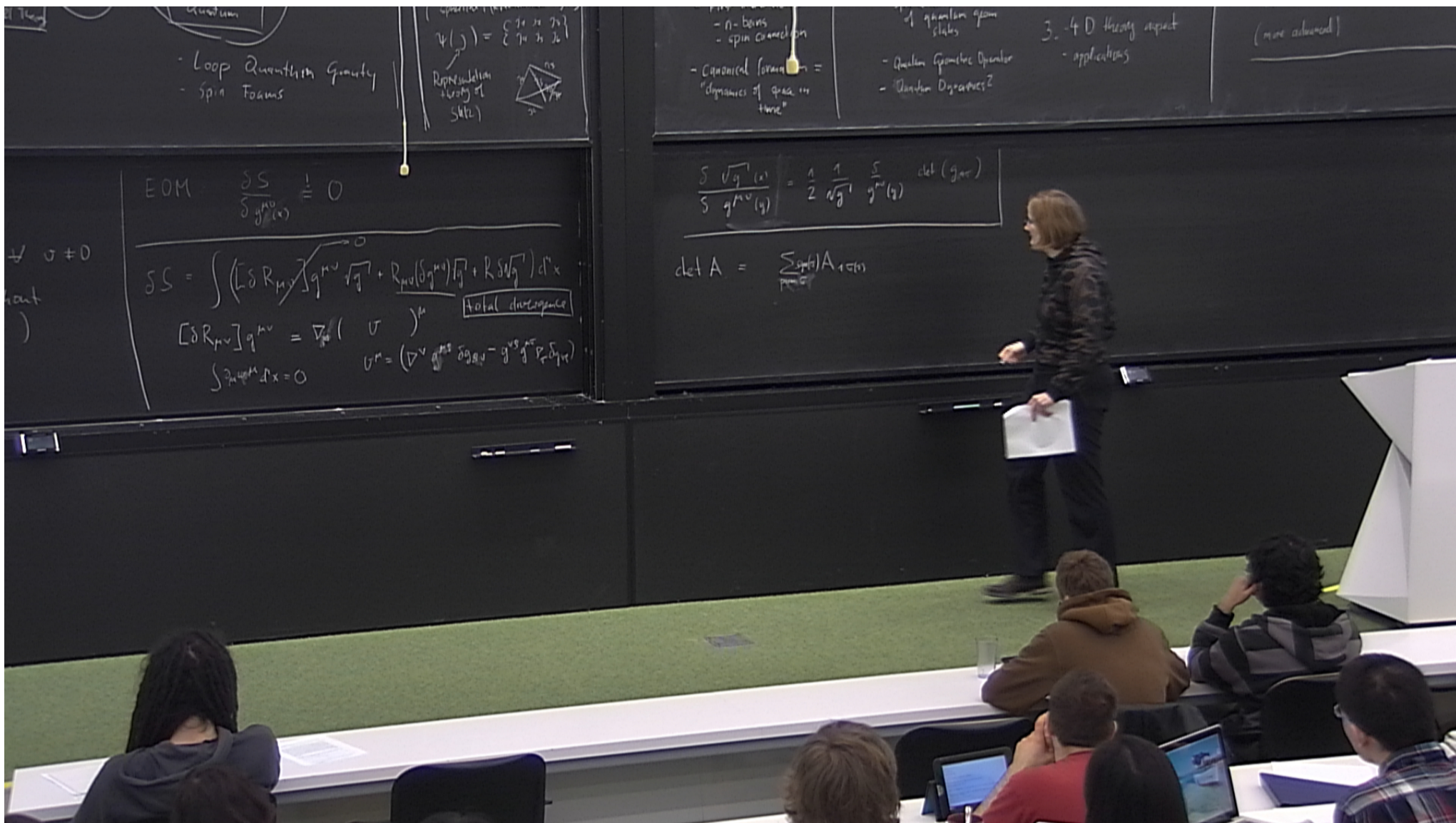
$$\delta S = \int (\delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu})$$

$$[\delta R_{\mu\nu}] g^{\mu\nu}$$

$$\int \delta g^{\mu\nu} d^n x$$

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)}$$

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta})$$



$$\frac{\delta \sqrt{g'}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g'}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta})$$

$$\det A = \sum_{\text{perm } \sigma} \text{sign}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \cdots A_{n\sigma(n)}$$

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta})$$

$$\det A = \sum_{\text{perm } \sigma} \text{sgn}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \cdots A_{n\sigma(n)}$$

$$=$$

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta})$$

$$\det A = \sum_{\text{perm } \sigma} \text{sgn}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \cdots A_{n\sigma(n)}$$

$$= \epsilon^{a_1 \dots a_n} a_1 \dots a_n$$

Levi-Civita symbol

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta})$$

$$\det A = \sum_{\text{perm } \sigma} \text{sign}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \cdots A_{n\sigma(n)}$$

$$= \sum_p a_1 \cdots a_n A_{1a_1} \cdots A_{na_n}$$

Levi-Civita symbol
 $\epsilon_{\sigma(1) \cdots \sigma(n)} = +1$
 $\epsilon_{\sigma(1) \cdots \sigma(n)} = \text{sign}(\sigma)$

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta})$$

$$\det A = \sum_{\text{perm } \sigma} \text{sign}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \dots A_{n\sigma(n)}$$

$$= \sum_{\sigma} \epsilon_{\sigma(1)\dots\sigma(n)} A_{1\sigma(1)} \dots A_{n\sigma(n)}$$

Levi-Civita symbol

$$\epsilon_{\sigma(1)\dots\sigma(n)} = \begin{cases} +1 & \text{if } \sigma \text{ is an even permutation} \\ -1 & \text{if } \sigma \text{ is an odd permutation} \\ 0 & \text{if } \sigma \text{ is not a permutation} \end{cases}$$

$$\det A = \frac{1}{n!} \epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

$$\frac{\partial \det A}{\partial A_{cd}} = (\det A) (A^{-1})^{cd}$$

$$\det A = \frac{1}{n!} \epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

$$\frac{\partial \det A}{\partial A_{cd}} = (\det A) (A^{-1})^{cd} \quad \leftarrow \text{check.}$$

$$g_{\mu\nu}(x) \delta(x, y) = \frac{1}{2} \frac{1}{\sqrt{g}} g_{\mu\nu}(x) \delta(x, y)$$

$$\det A = \frac{1}{n!} \epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

$$\frac{\partial \det A}{\partial A_{cd}} = (\det A) (A^{-1})^{cd} \leftarrow \text{check.}$$

- Canonical formalism =
"dynamics of space in
time"

- Quantum Dynamics?

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta}) = \frac{1}{2} \frac{1}{\sqrt{g}} g_{\mu\nu}(x) \delta(x,y)$$

$$\det A = \sum_{\text{perm } \sigma} \text{sgn}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \dots A_{n\sigma(n)}$$

$$= \epsilon^{a_1 \dots a_n} A_{a_1 a_1} \dots A_{a_n a_n}$$

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 $\epsilon_{\sigma(1) \dots \sigma(n)} = \text{sgn}(\sigma)$

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$$\frac{\partial \det A}{\partial A_{cd}} = (\det A) (A^{-1})^{cd}$$

- Canonical formalism =
"dynamics of space in
time"

- Quantum Dynamics?

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta}) = -\frac{1}{2} \frac{1}{\sqrt{g}} g_{\mu\nu}(x) \delta(x,y)$$

$$\det A = \sum_{\sigma \in S(n)} \text{sgn}(\sigma) A_{1\sigma(1)} \dots A_{n\sigma(n)}$$

$$= \epsilon^{a_1 \dots a_n} A_{1a_1} \dots A_{na_n}$$

Levi-Civita
1...n
 $\in S(n) \dots S(n)$

$$\det A = \frac{1}{n!} \epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

$$\frac{\partial \det A}{\partial A_{cd}} = (\det A) (A^{-1})^{cd} \leftarrow \text{check.}$$

- Canonical formalism =
"dynamics of space in
time"

Quantum Geometric Operators
- Quantum Dynamics?

$$\frac{\delta \sqrt{g}(x)}{\delta g^{\mu\nu}(y)} = \frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(y)} \det(g_{\alpha\beta}) = -\frac{1}{2} \frac{1}{\sqrt{g}} g_{\mu\nu}(x) \delta(x,y)$$

$\delta(g_{\mu\alpha} g^{\mu\nu}) | g_{\mu\nu} \delta g^{\mu\nu} = -g^{\mu\nu} \delta g_{\mu\nu}$

$$\det A = \sum_{\text{perm } \sigma} \text{sgn}(\sigma) A_{1\sigma(1)} A_{2\sigma(2)} \dots A_{n\sigma(n)}$$

$$= \epsilon^{a_1 \dots a_n} A_{a_1 a_1} \dots A_{a_n a_n}$$

Levi-Civita symbol
 $\epsilon^{1 \dots n} = +1$
 $\epsilon_{\sigma(1) \dots \sigma(n)} = \text{sgn}(\sigma)$

$$\det A = \frac{1}{n!} \epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

$$\frac{\partial \det A}{\partial A_{cd}} = (\det A) (A^{-1})^{cd} \leftarrow \text{check.}$$

$$\text{EOM: } \frac{\delta S}{\delta g^{\mu\nu}(x)} \stackrel{!}{=} 0$$

$$\forall \nu \neq 0$$

$$\delta S = \int \left(\cancel{[\delta R_{\mu\nu}] g^{\mu\nu}} \sqrt{g} + \underbrace{R_{\mu\nu} (\delta g^{\mu\nu}) \sqrt{g}}_{\text{total divergence}} + R \delta \sqrt{g} \right) d^4 x$$

$$[\delta R_{\mu\nu}] g^{\mu\nu} = \nabla_\mu (\quad)^\mu$$

$$\int \partial_\mu \quad d^4 x = 0$$

$$U^\mu = (\nabla^\nu g^{\mu\sigma} \delta g_{\sigma\nu} - g^{\nu\sigma} g^{\mu\tau} \nabla_\tau \delta g_{\nu\sigma})$$

$$g_{\mu\nu}(x) \delta(x, y)$$

$$\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}$$

$$\epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

$$A) (A^{-1})^{\mu}_{\nu}$$

$$\delta S = \int \left[R_{\mu\nu} \delta g^{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \delta g^{\mu\nu} \right] \sqrt{g} d^4x$$

$$\Rightarrow \begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \\ g_{\mu\nu} = 0 \end{cases}$$

$$g_{\mu\nu}(x) \delta(x, y)$$

$$\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}$$

$$\epsilon^{b_1 \dots b_n} \epsilon^{a_1 \dots a_n} A_{b_1 a_1} \dots A_{b_n a_n}$$

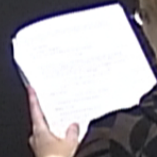
$$A) (A^{-1})^{cd} \leftarrow \text{check.}$$

$$\delta S = \int \left[R_{\mu\nu} \delta g^{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \delta g^{\mu\nu} \right] \sqrt{g} dx^4$$

$$\Rightarrow \begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \\ g_{\mu\nu} = 0 \end{cases}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

| g



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

| $g^{\mu\nu}$

$$R - \frac{1}{2} n R = 0 \Rightarrow R=0 \quad n > 2$$

$$\Rightarrow \boxed{R_{\mu\nu} = 0} \quad n=3,4$$

$$\boxed{3D} \quad R_{\mu\nu} = 0 \Rightarrow R_{\mu\nu\sigma\sigma}$$

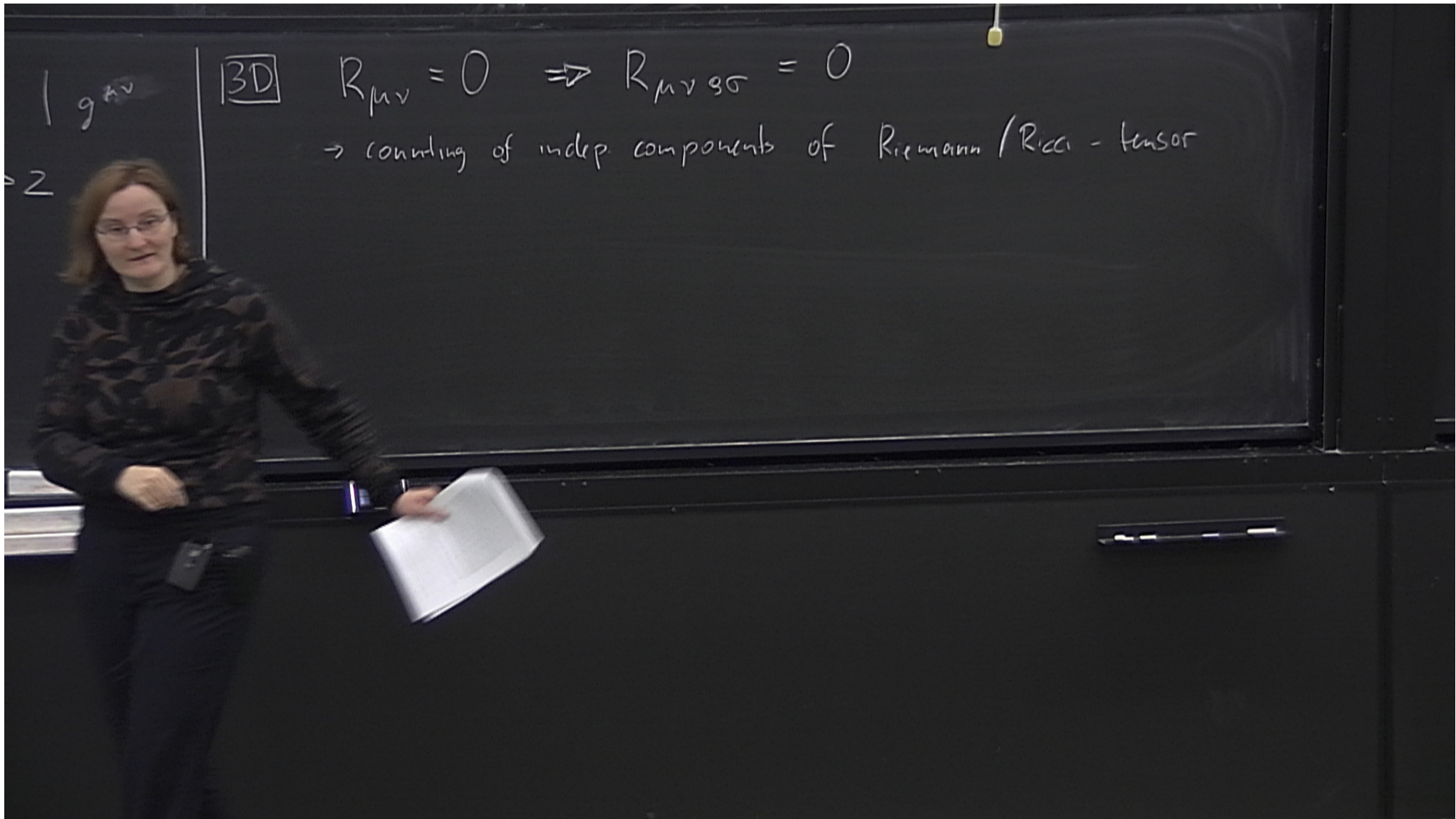
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

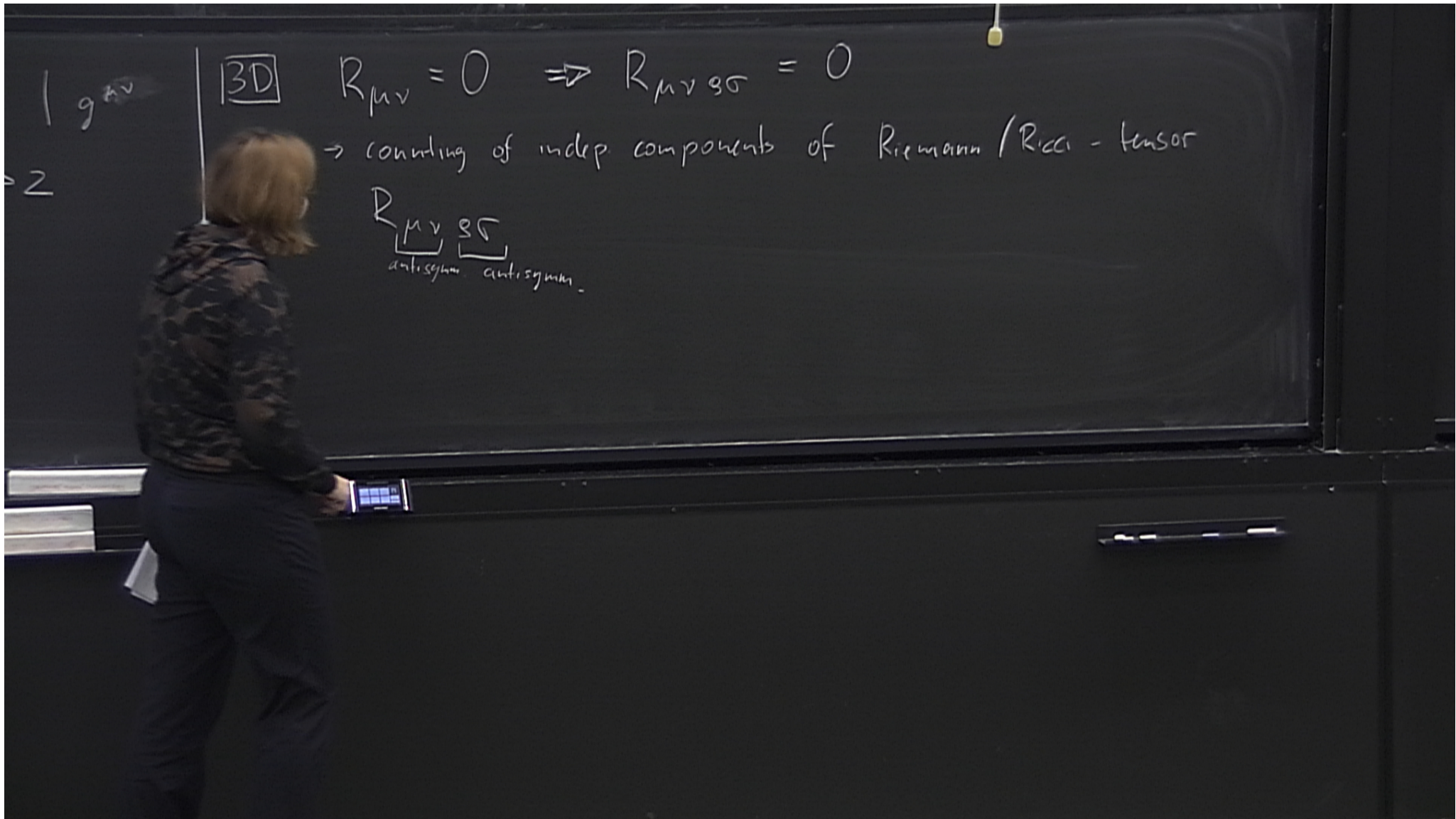
| $g^{\mu\nu}$

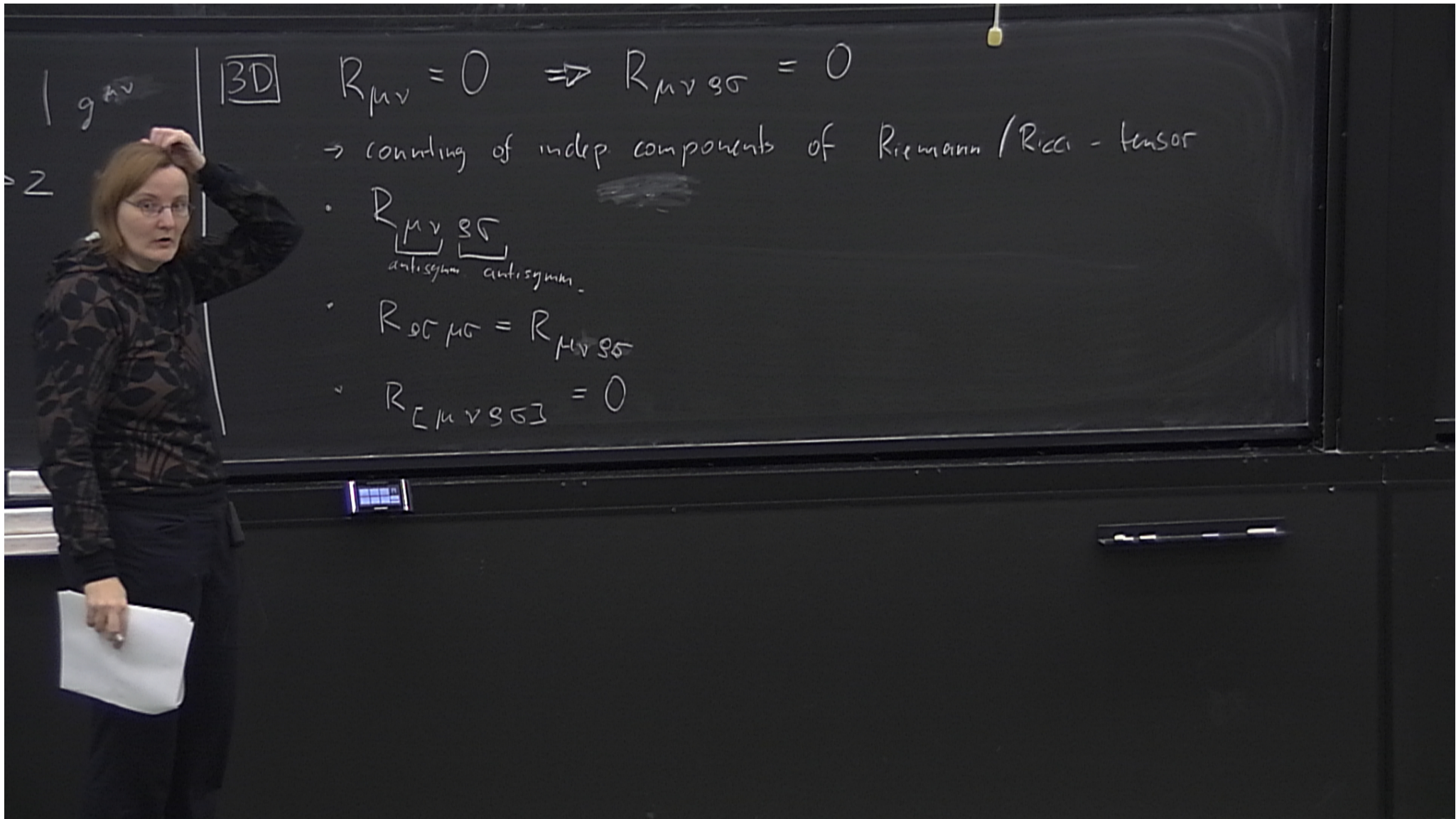
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$g_{\mu\nu}$

$$\boxed{3D} \quad R_{\mu\nu} = 0 \Rightarrow R_{\mu\nu\sigma\tau} = 0$$

→ counting of indep components of Riemann / Ricci - tensor

$12, 23, 13$

$$\cdot R_{\mu\nu\sigma\tau}$$

$\uparrow \quad \quad \uparrow$
 $1=1213 \quad 1'$
 $\underbrace{\mu\nu}_{\text{antisym}} \quad \underbrace{\sigma\tau}_{\text{antisym}}$

$$\cdot R_{\sigma\tau\mu\nu} = R_{\mu\nu\sigma\tau}$$

$$\cdot R_{[\mu\nu\sigma\tau]} = 0$$

$g_{\mu\nu}$
 $\rightarrow 2$

3D

 $R_{\mu\nu} = 0 \Rightarrow R_{\mu\nu\sigma\tau} = 0$
 $\rightarrow$ counting of indep components of Riemann / Ricci - tensor
 $R_{\mu\nu\sigma\tau}$ $\overset{1=1213}{\uparrow}$ $\overset{1}{\uparrow}$
antisym antisym
 $R_{\sigma\tau\mu\nu} = R_{\mu\nu\sigma\tau}$
 $R_{[\mu\nu\sigma\tau]} = 0$
 3×3 symmetric tensor $\rightarrow 6$
 $R_{\mu\nu} = R_{\nu\mu}$

3D: $R_{\mu\nu\sigma\tau} = 0$

$g_{\mu\nu}$
2

3D $R_{\mu\nu} = 0 \Rightarrow R_{\mu\nu\sigma\tau} = 0$

→ counting of indep components of Riemann / Ricci - tensor

$R_{\mu\nu\sigma\tau}$
 $\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ 1=12,13 & 2=23,12 & 3=13,21 & 4=21,32 \end{matrix}$
 antisym antisym

3x3 symmetric tensor → 6

$R_{\mu\nu} = R_{\nu\mu}$

3D $R_{\mu\nu\sigma\tau} = 0$

$R_{\sigma\tau\mu\nu} = R_{\mu\nu\sigma\tau}$

$R_{[\mu\nu\sigma\tau]} = 0$

topological
field
theory