

Title: 12/13 PSI - Found Quantum Mechanics Lecture 13

Date: Jan 23, 2013 11:30 AM

URL: <http://pirsa.org/13010082>

Abstract:

The deBroglie-Bohm interpretation, part 2



Louis deBroglie
(1892-1987)



David Bohm
(1917-1992)

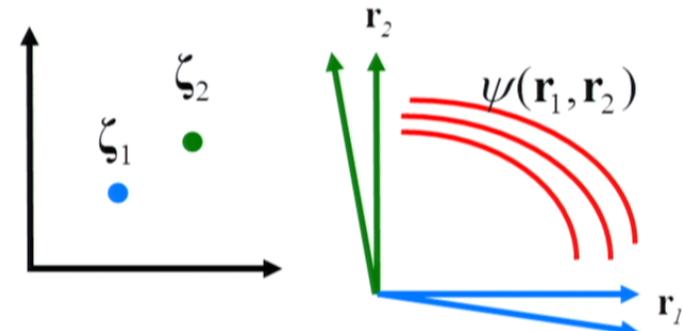
"I saw the impossible done..."
John Bell

The deBroglie-Bohm interpretation for many particles

The ontic state: $(\psi(\mathbf{r}_1, \mathbf{r}_2), \zeta_1, \zeta_2)$

Wavefunction on
configuration space

Particle
positions



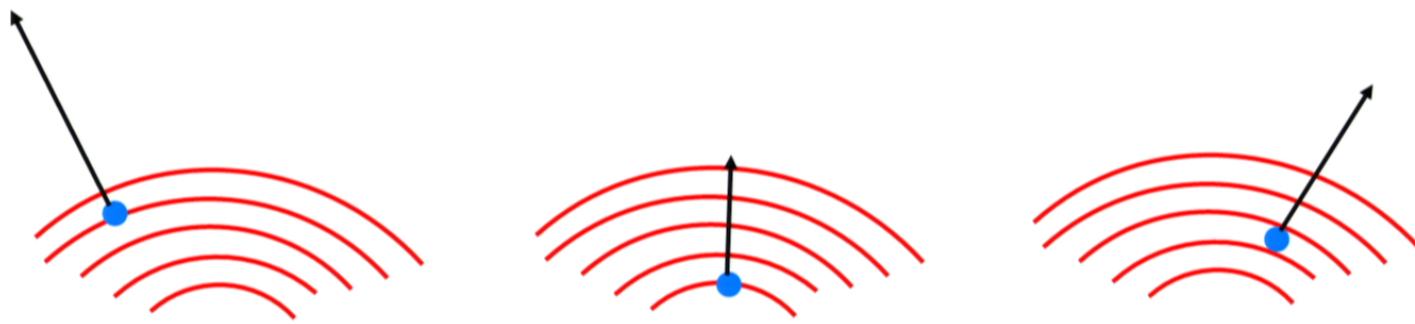
The evolution equations:

Schrödinger's equation

$$i\hbar \frac{\partial \psi(\mathbf{r}_1, \mathbf{r}_2, t)}{\partial t} = -\frac{\hbar^2}{2m_1} \nabla_1^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) - \frac{\hbar^2}{2m_2} \nabla_2^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) + V(\mathbf{r}_1, \mathbf{r}_2) \psi(\mathbf{r}_1, \mathbf{r}_2, t)$$

$$\left. \begin{aligned} \frac{d\zeta_1(t)}{dt} &= \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \\ \frac{d\zeta_2(t)}{dt} &= \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \end{aligned} \right\} \text{The guidance equation}$$

where $\psi(\mathbf{r}_1, \mathbf{r}_2, t) = R(\mathbf{r}_1, \mathbf{r}_2, t) e^{iS(\mathbf{r}_1, \mathbf{r}_2, t)/\hbar}$



Epistemic state (assuming perfect knowledge of $\psi(\mathbf{r}, t)$)

$\rho(\zeta) d\zeta$ = the probability the particle is within $d\zeta$ of ζ .

The "standard distribution"

$$\rho(\zeta, t) = |\psi(\zeta, t)|^2$$

Note: it is preserved by the dynamics:

$$\text{if } \rho(\zeta, 0) = |\psi(\zeta, 0)|^2 \text{ then } \rho(\zeta, t) = |\psi(\zeta, t)|^2$$

$$\psi = \sum_j c_j \psi_j$$

"waves" of the decomposition

$\zeta \in$ Spatial support of ψ_j jth wave is occupied

$\zeta \notin$ Spatial support of ψ_j jth wave is empty

If only the k th wave is occupied

Then the guidance equation depends only on the k th wave

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If only the k th wave is occupied
Then the guidance equation depends only on the k th wave

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

$$= R_1(\mathbf{r}_1, t) e^{iS_1(\mathbf{r}_1, t)/\hbar} R_2(\mathbf{r}_2, t) e^{iS_2(\mathbf{r}_2, t)/\hbar}$$

$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\frac{d\zeta_1(t)}{dt} = \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_1} [\nabla_1 S_1(\mathbf{r}_1, t)]_{\mathbf{r}_1=\zeta_1(t)}$$

$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_2} [\nabla_2 S_2(\mathbf{r}_2, t)]_{\mathbf{r}_2=\zeta_2(t)}$$

The two particles evolve independently



Reproducing the operational predictions

Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$

$$\phi_k(\mathbf{r})\chi(\mathbf{r}')\eta(\mathbf{r}'', \mathbf{r}''', \dots) \rightarrow \phi_k(\mathbf{r})\chi_k(\mathbf{r}')\eta_k(\mathbf{r}'', \mathbf{r}''', \dots)$$

$$[\sum_k c_k \phi_k(\mathbf{r})]\chi(\mathbf{r}')\eta(\mathbf{r}'', \mathbf{r}''', \dots) \rightarrow \sum_k c_k \phi_k(\mathbf{r})\chi_k(\mathbf{r}')\eta_k(\mathbf{r}'', \mathbf{r}''', \dots)$$

Distinct states of environment correspond to disjoint regions of the configuration space

$$\eta_j(\mathbf{r}'', \mathbf{r}''', \dots)\eta_k(\mathbf{r}'', \mathbf{r}''', \dots) \simeq 0 \text{ if } j \neq k$$

If the j th wave comes to be occupied, then one can postulate an **effective collapse** of the guiding wave

$$\sum_k c_k \phi_k(\mathbf{r}) \rightarrow \phi_j(\mathbf{r})$$

To have re-interference with the empty waves, it would be necessary to map all the η_k back to η

The "standard distribution" as quantum equilibrium

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A. Valentini and H. Westman

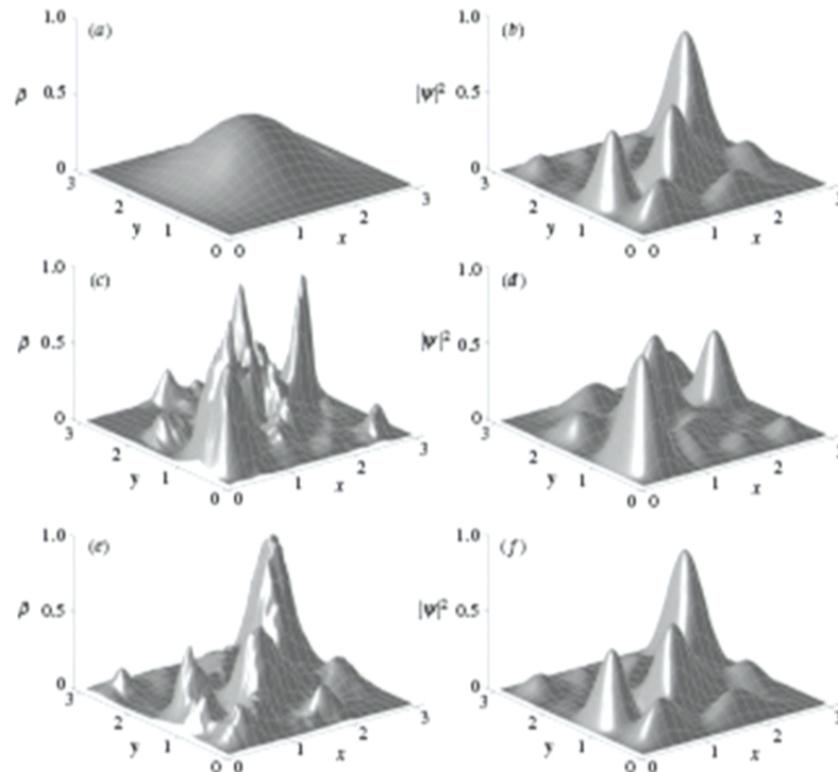


Figure 7. Smoothed $\tilde{\rho}$ ((a), (c) and (e)), compared with $|\psi|^2$ ((b), (d) and (f)), at times $t = 0$ ((a), (b)), 2π ((c), (d)) and 4π ((e), (f)). While $|\psi|^2$ recovers to its initial value, the smoothed $\tilde{\rho}$ shows a remarkable evolution towards equilibrium.



Do measurements reveal attributes of the particles?

Position measurements generate the statistics one would expect:

$$\langle \psi | F(\mathbf{R}) | \psi \rangle = \int d\zeta F(\zeta) \rho(\zeta)$$

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But momentum measurements do not:

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$$\langle \psi | \mathbf{P} | \psi \rangle = \int d\zeta \left(m \frac{d\zeta}{dt} \right) \rho(\zeta)$$

we have, for example:

$$\langle \psi | \frac{\mathbf{P}^2}{2m} | \psi \rangle \neq \int d\zeta \left(\frac{1}{2} m \left(\frac{d\zeta}{dt} \right)^2 \right) \rho(\zeta)$$

If we define energy as a function of the configuration and the wavefunction

$$\mathcal{E}_\psi(\zeta) = - \frac{\partial S(\mathbf{r}, t)}{\partial t} \Big|_{\mathbf{r}=\zeta(t)} = \left[\frac{(\nabla S)^2}{2m} + Q(\mathbf{r}) + V(\mathbf{r}, t) \right]_{\mathbf{r}=\zeta(t)}$$

Recall imaginary part of S.E. is $\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$



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$$\int d\zeta \rho(\zeta, t) \mathcal{E}_\psi(\zeta) = \int d\mathbf{r} |\psi(\mathbf{r}, t)|^2 \left[\frac{(\nabla S)^2}{2m} + Q + V \right](\mathbf{r}, t)$$

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$$\begin{aligned} \int d\zeta \rho(\zeta, t) \mathcal{E}_\psi(\zeta) &= \int d\mathbf{r} |\psi(\mathbf{r}, t)|^2 \left[\frac{(\nabla S)^2}{2m} + Q + V \right](\mathbf{r}, t) \\ &= \int d\mathbf{r} \psi^*(\mathbf{r}, t) \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t) \end{aligned}$$

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$$\varepsilon_{\psi}(\zeta) = - \frac{\partial S(\mathbf{r}, t)}{\partial t} \Big|_{\mathbf{r}=\zeta(t)} = \left[\frac{(\nabla S)^2}{2m} + Q(\mathbf{r}) + V(\mathbf{r}, t) \right]_{\mathbf{r}=\zeta(t)}$$

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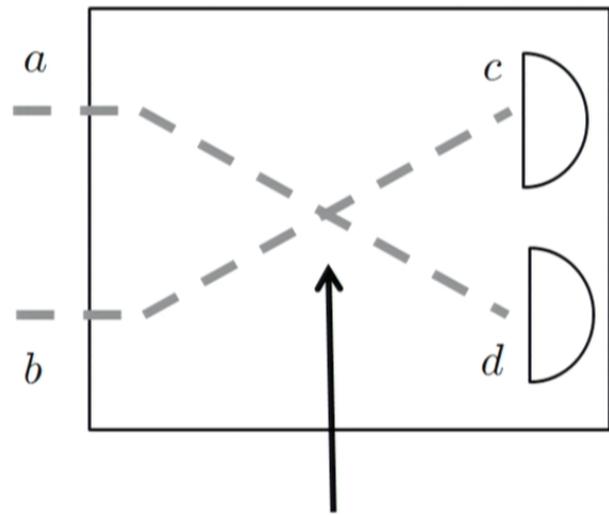
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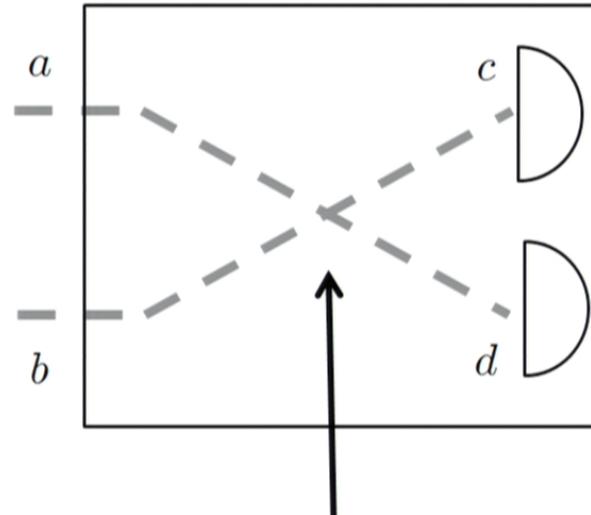
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Expression for expectation values in terms of ζ, ψ depends on mmt context

Contextuality

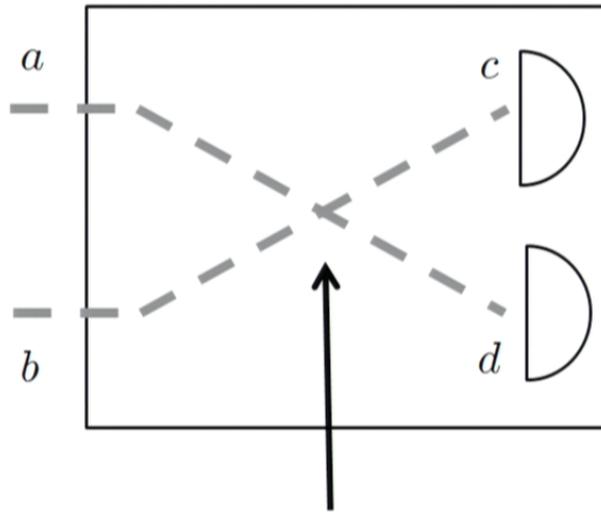


No overlap (not confined to 2d plane)



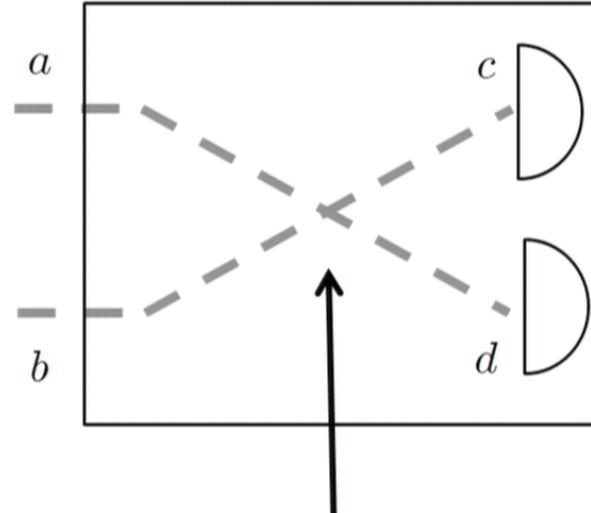
Overlap (confined to 2d plane)

Contextuality



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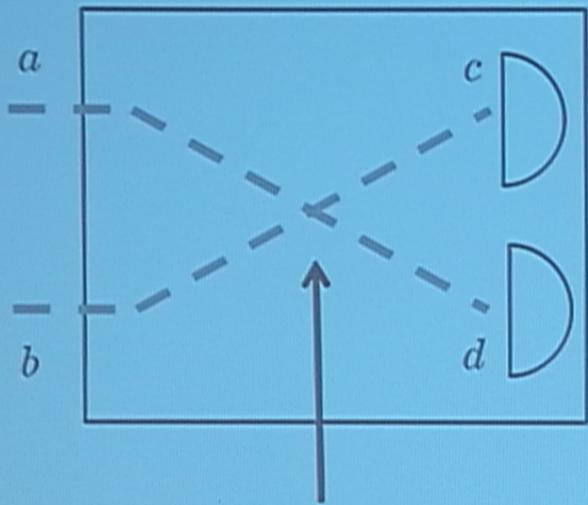
$$U(\cdot)U^\dagger \quad \{|\phi_c\rangle\langle\phi_c|, |\phi_d\rangle\langle\phi_d|\}$$
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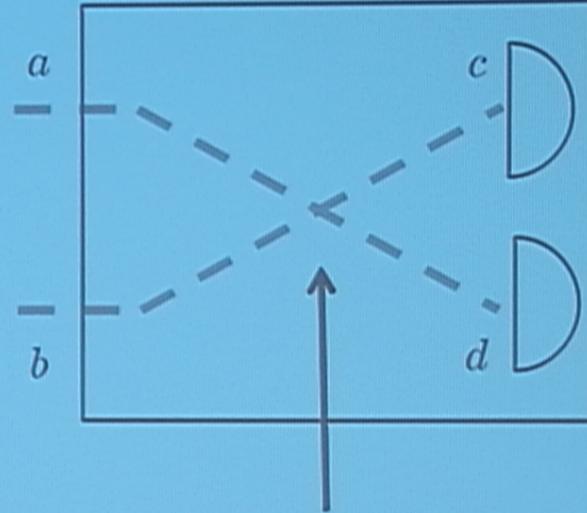
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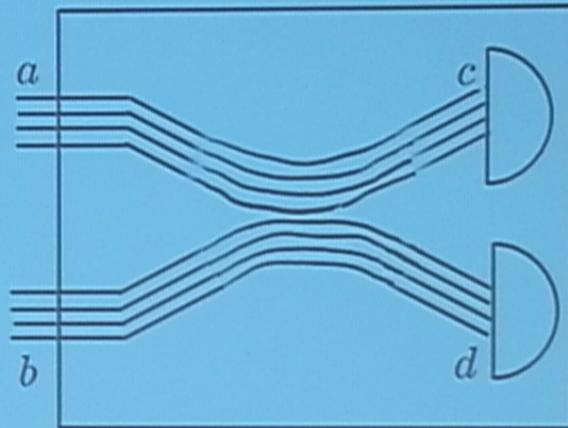
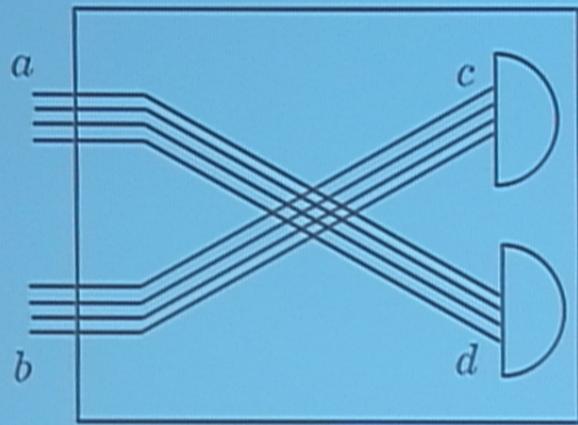
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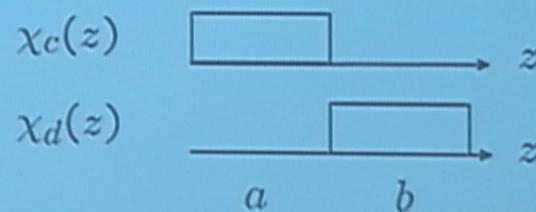
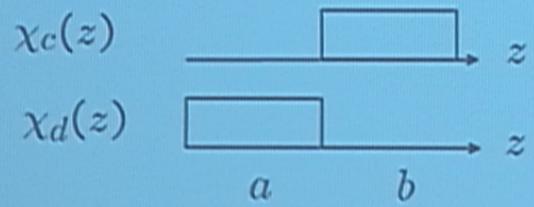
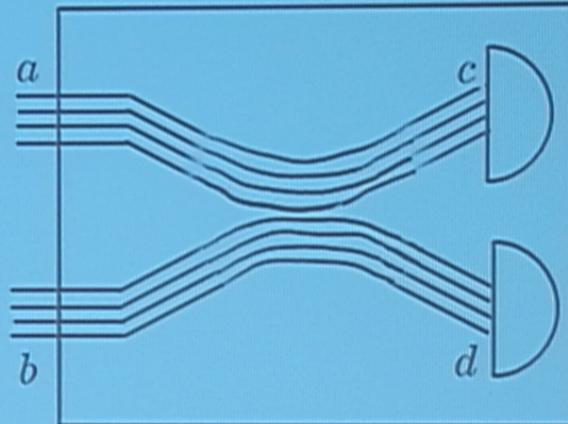
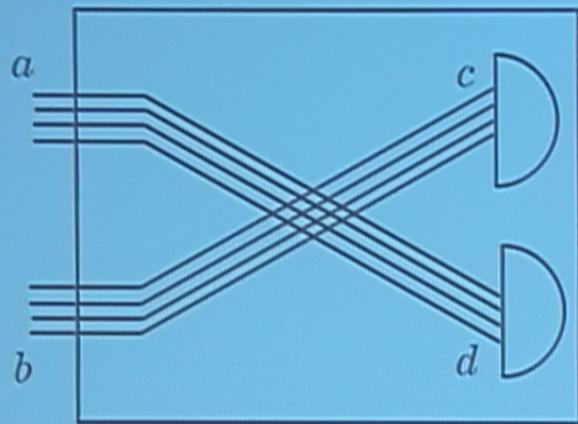
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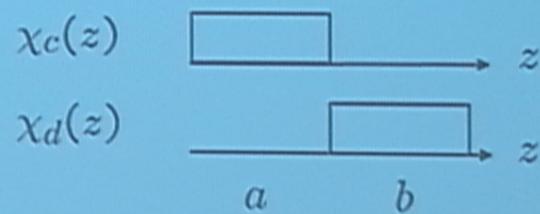
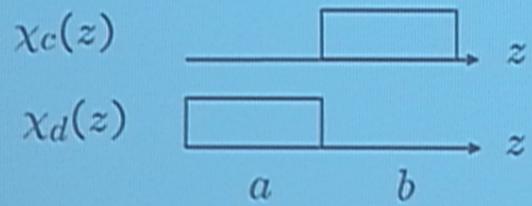
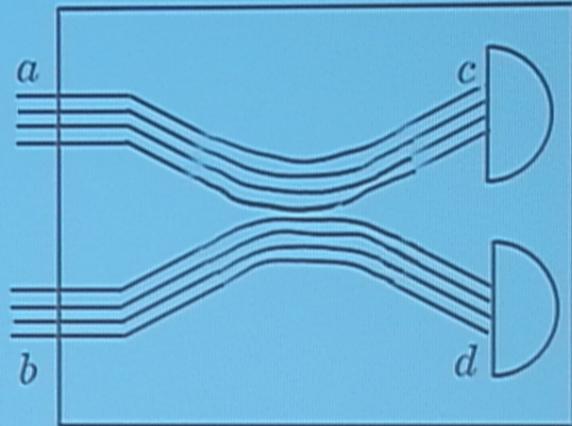
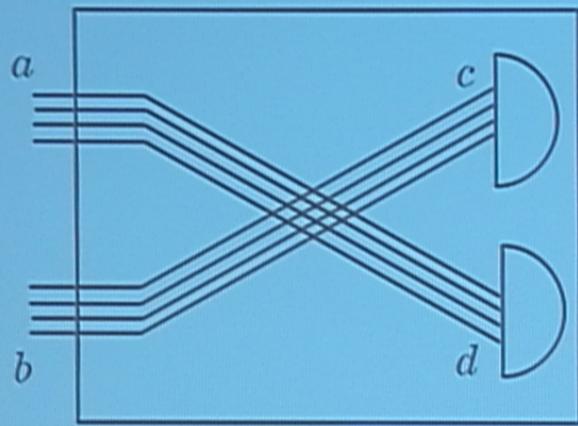
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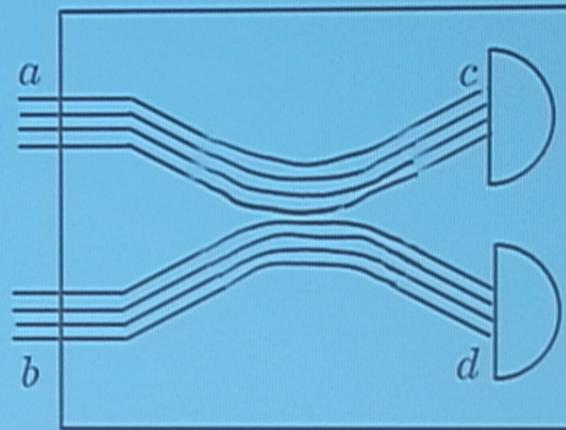
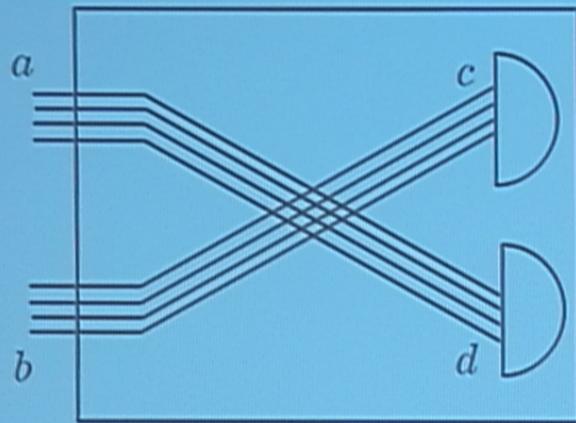
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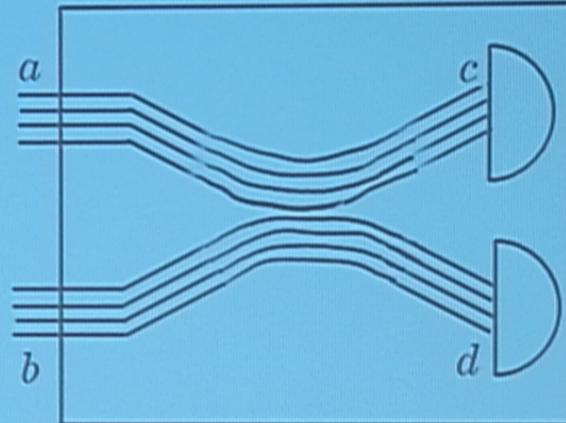
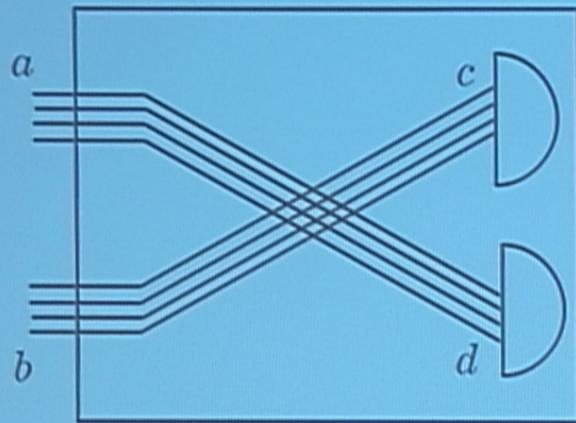
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A dependence on this sort of context is not required of any ontological model

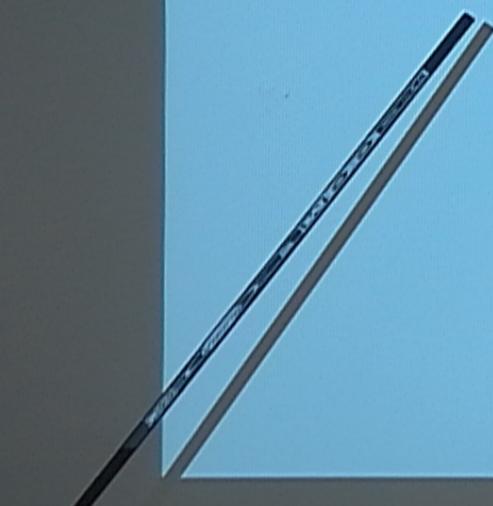
Criticism

The Bohmian treatment of contextuality and nonlocality does not resolve the tension between operational and ontological levels (e.g. the existence of superluminal influences and the impossibility of superluminal signalling)

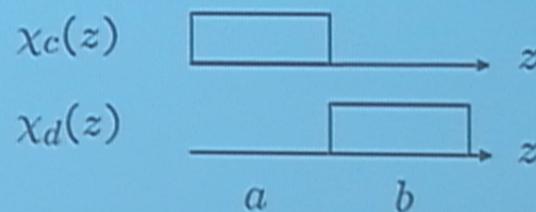
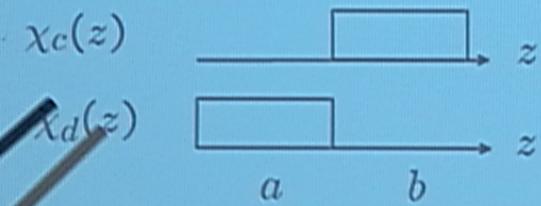
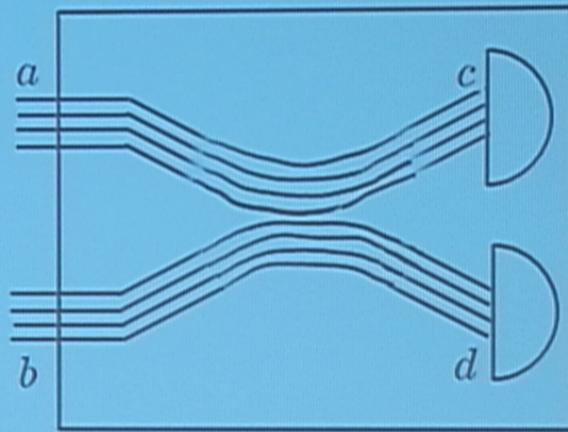
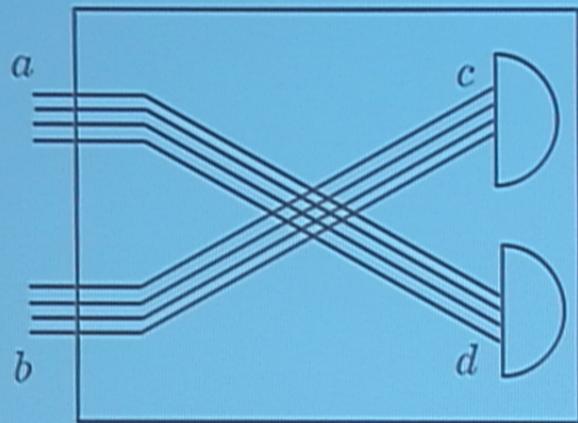
Response:

Eliminate tension via nonstandard distributions.

The impossibility of superluminal signalling is an artifact of quantum equilibrium

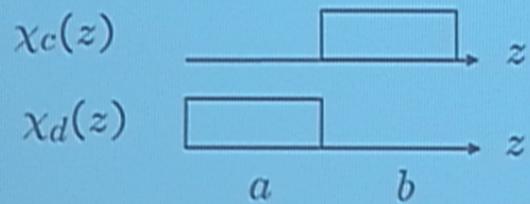
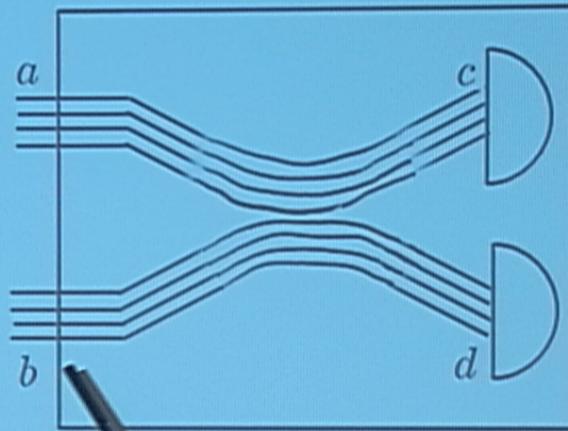
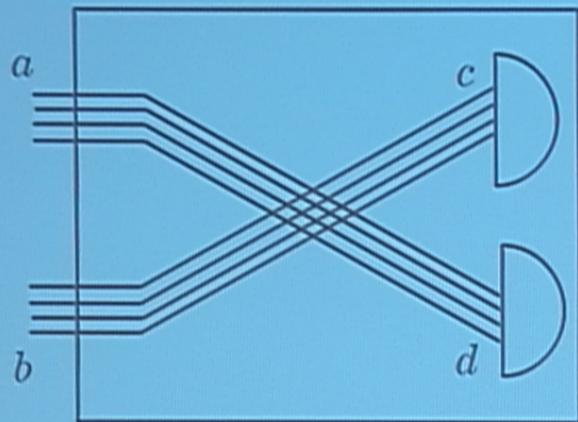


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Classical limit

Operational correspondence vs. Ontological correspondence

In the Ehrenfest regime, $\lambda \ll L$, Bohmian trajectories are Newtonian (Allori, Durr, Goldstein, Zanghi)

But even the dynamics of macroscopic objects need not stay in the Ehrenfest regime (ex: chaotic tumble of Hyperion)





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Possible solution: Decoherence in configuration space
Eliminates interference, thereby allowing crossing

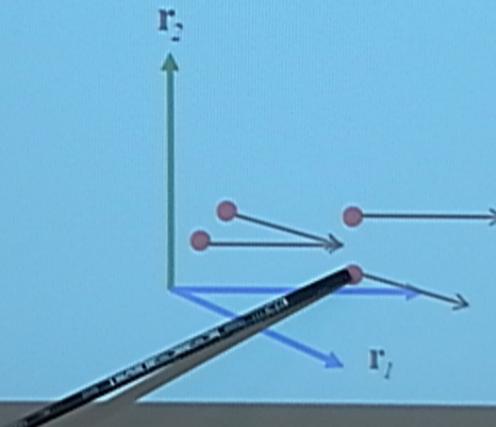
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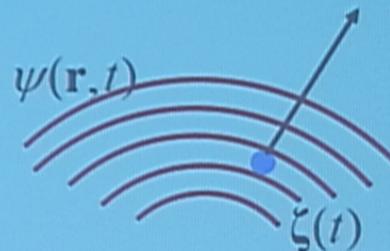
The underdetermination criticism

Underdetermination: when there are many possible choices of ontological structure that are consistent with observations

Underdetermination of the supplementary variables

Standard approach - Position preferred

The ontic state: $(\psi(\mathbf{r}), \zeta)$
↗ ↙
Wavefunction
in position rep'n Particle
position



The evolution equations:

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) + V(\mathbf{r})\psi(\mathbf{r}, t) \quad \text{Schrödinger's eq'n}$$

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Underdetermination of the supplementary variables

Struyve (2010) has argued that this reproduces the predictions of operational quantum theory

Other choices are possible



Underdetermination of the supplementary variables

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Other choices are possible

The problem returns for the case of a deBroglie-Bohm theory of the electromagnetic field:

Supplementary variable: Electric field or Magnetic field?

Underdetermination of the supplementary variables

Multiple treatments of spin:

Bohm, Schiller and Tiomno approach

Supplementary variables: particle position and orientation

The particle is taken to be an extended rigid object which makes a 'spin' contribution to the total angular momentum

Underdetermination of the supplementary variables

Multiple treatments of spin:

Bohm, Schiller and Tiomno approach

Supplementary variables: particle position and orientation

The particle is taken to be an extended rigid object which makes a 'spin' contribution to the total angular momentum

or

Bell's minimalist approach

Supplementary variables: particle position

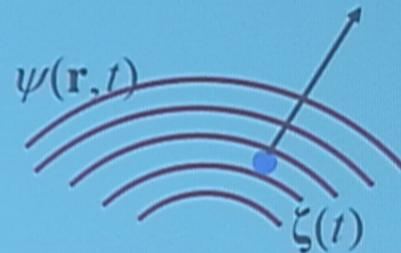
The effect of spin is seen only in the dynamics of the particle positions

The operational predictions are reproduced by virtue of localization of pointers

Bell's minimalist approach to spin

The ontic state: $(\psi(\mathbf{r}), \zeta)$

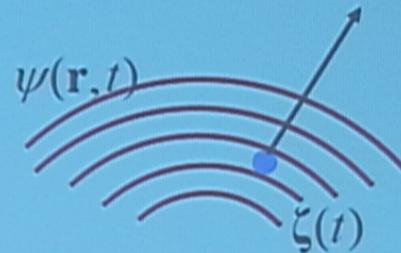
Two-component wavefunction Particle position



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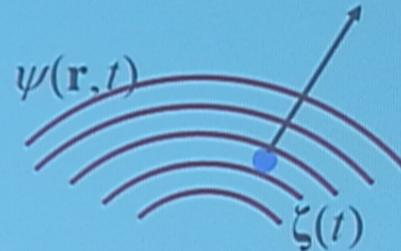


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Bell's minimalist approach to spin

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Two-component wavefunction Particle position



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Underdetermination of the supplementary variables

Multiple treatments of quantum electrodynamics:

Bohm's model of the free electromagnetic field

Supplementary variables: electric field (or magnetic field)

combined with

Bell's model of fermions (indeterministic, discrete) or Colin's continuum version of it

Supplementary variables: fermion number at each lattice point
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Underdetermination of the dynamics: guidance equation

(Deotto and Ghirardi, 1998)

Consider a modified guidance equation

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} + \frac{\mathbf{j}_0(\mathbf{r}, t)}{R^2(\mathbf{r}, t)} \Big|_{\mathbf{r}=\zeta(t)}$$

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Proof of equivariance for standard guidance equation:

The velocity field is

$$\mathbf{v}(\mathbf{r}, t) = \frac{1}{m} [\nabla S(\mathbf{r}, t)]$$

The current density is:

$$\mathbf{j}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t)$$

Conservation of probability implies

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = -\nabla \cdot \mathbf{j}(\mathbf{r}, t) = -\nabla \cdot \left(\frac{\rho(\mathbf{r}, t) \nabla S(\mathbf{r}, t)}{m} \right)$$

Recall the imaginary part of the Schrodinger eq'n:

$$\frac{\partial}{\partial t} (R^2) = -\nabla \cdot \left(\frac{R^2 \nabla S}{m} \right)$$

Therefore, if $\rho(\mathbf{r}, t) = R^2(\mathbf{r}, t)$ then $\frac{\partial}{\partial t} (\rho(\mathbf{r}, t) - R^2(\mathbf{r}, t)) = 0$

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Responses to criticism of underdetermination

Appeal to simplicity

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Appeal to symmetry

It is argued that symmetry principles can narrow down the choice of guidance equation (Holland and Philippidis)

Eliminate underdetermination via nonstandard distributions

Experiments on "quantum nonequilibrium matter" could see the difference between the choices of dynamics

Failure of Lorentz invariance

There are deBroglie-Bohm interpretations of relativistic quantum field theories.

They are Lorentz-invariant at the operational level, but not at the ontological level

This implies an unobservable preferred rest frame, which is a kind of underdetermination

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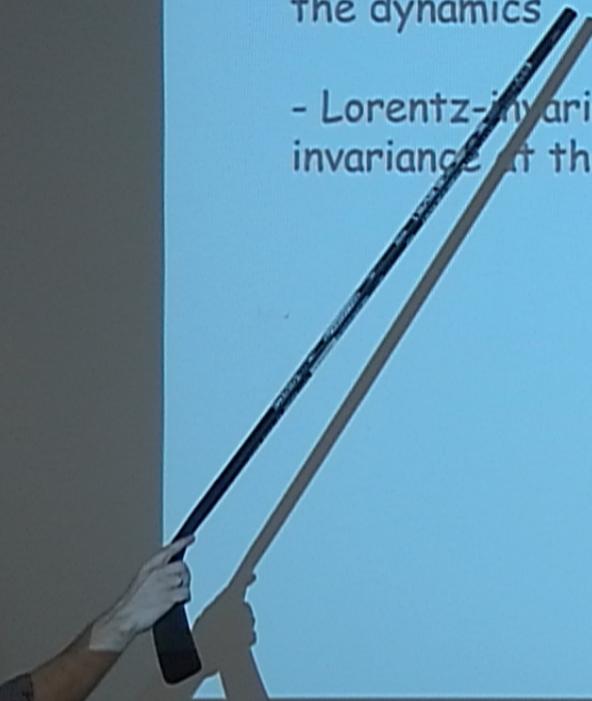
Response:

Lorentz invariance is an emergent symmetry - a statistical consequence of quantum equilibrium

Nonequilibrium matter would reveal the failure of Lorentz invariance

Summary of criticisms

- Contextuality and nonlocality are hardwired into the theory
More contextuality and nonlocality than strictly necessary
The tension between ontological and operational levels persists
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- Lorentz-invariance at the operational level, failure of Lorentz invariance at the ontological level



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All these criticisms point to failures of the methodological principle: situations that are operationally indistinguishable should not be distinguished in the ontological model