

Title: 12/13 PSI - Found Quantum Mechanics Lecture 12

Date: Jan 22, 2013 11:30 AM

URL: <http://pirsa.org/13010081>

Abstract:

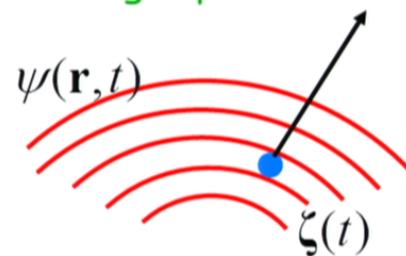
Responses to the measurement problem

1. Deny universality of quantum dynamics
 - Quantum-classical hybrid models
 - Collapse models
2. Deny representational completeness of ψ
 - ψ -ontic hidden variable models (e.g. deBroglie-Bohm)
 - ψ -epistemic hidden variable models
3. Deny that there is a unique outcome
 - Everett's relative state interpretation (many worlds)
4. Deny some aspect of classical logic or classical probability theory
 - Quantum logic and quantum Bayesianism
5. Deny some other feature of the realist framework?

The deBroglie-Bohm interpretation for a single particle

The ontic state: $(\psi(\mathbf{r}), \zeta)$

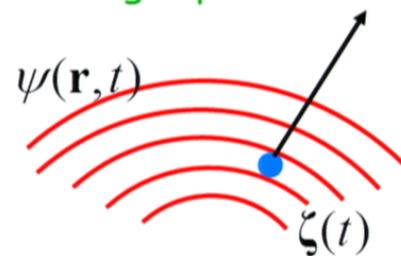
Wavefunction Particle position



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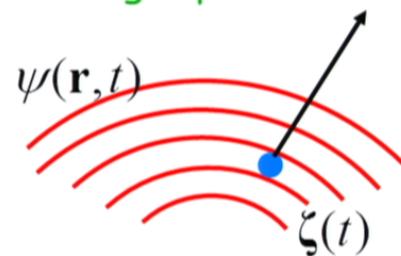
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The deBroglie-Bohm interpretation for a single particle

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The evolution equations:

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) + V(\mathbf{r})\psi(\mathbf{r}, t) \quad \text{Schrödinger's eq'n}$$

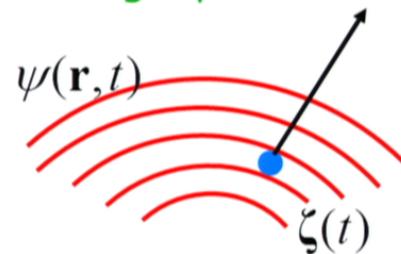
$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} \quad \text{The guidance eq'n}$$

where $\psi(\mathbf{r}, t) = R(\mathbf{r}, t)e^{iS(\mathbf{r}, t)/\hbar}$

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Given $\psi(\mathbf{r}, t) = R(\mathbf{r}, t)e^{iS(\mathbf{r}, t)/\hbar}$

The real part of the Schrodinger eq'n is:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$$

where $Q(\mathbf{r}, t) \equiv -\frac{\hbar^2}{2m} \frac{\nabla^2 R(\mathbf{r}, t)}{R(\mathbf{r}, t)}$ The "quantum potential"

The imaginary part of the Schrodinger eq'n is:

$$\frac{\partial}{\partial t}(R^2) + \nabla \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0$$

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$$\frac{\partial}{\partial t}(R^2) + \nabla \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0$$

Acting the ∇ operator on the real part of the Schrodinger eq'n gives:

$$\nabla \left[\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V \right] = 0$$

$$\left(\frac{\partial}{\partial t} + \frac{\nabla S \cdot \nabla}{m} \right) \nabla S = -\nabla(Q + V)$$

Taking the time derivative of the guidance equation gives:

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

$$\frac{d^2\zeta(t)}{dt^2} = \frac{1}{m} \left(\frac{\partial}{\partial t} + \frac{d\zeta}{dt} \cdot \nabla \right) \nabla S$$

Thus

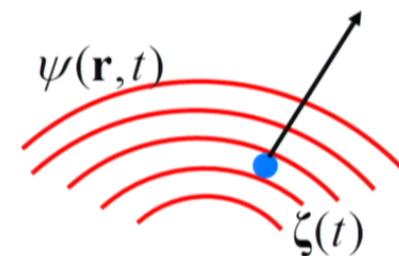
$$m \frac{d^2\zeta(t)}{dt^2} = -[\nabla V(\mathbf{r}) + \nabla Q(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

Newtonian form of the particle dynamics:

$$m \frac{d^2\zeta(t)}{dt^2} = -[\nabla V(\mathbf{r}) + \nabla Q(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

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(Note independence of quantum potential on amplitude)

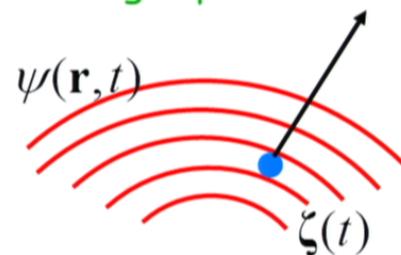


How else does deBroglie-Bohm differ from Newtonian mechanics?

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where $\psi(\mathbf{r}, t) = R(\mathbf{r}, t)e^{iS(\mathbf{r}, t)/\hbar}$

Note: There is no back-action on the wave

The amplitude of the wave is irrelevant \rightarrow a pilot wave

Proof of the preservation of the standard distribution:

The velocity field is

$$\mathbf{v}(\mathbf{r}, t) = \frac{1}{m} [\nabla S(\mathbf{r}, t)]$$

The probability current density is:

$$\mathbf{j}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t)$$

Conservation of probability implies

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = -\nabla \cdot \mathbf{j}(\mathbf{r}, t)$$

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$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = -\nabla \cdot \mathbf{j}(\mathbf{r}, t) = -\nabla \cdot \left(\frac{\rho(\mathbf{r}, t) \nabla S(\mathbf{r}, t)}{m} \right)$$

Recall the imaginary part of the Schrodinger eq'n:

$$\frac{\partial}{\partial t} (R^2) = -\nabla \cdot \left(\frac{R^2 \nabla S}{m} \right)$$

Therefore, if $\rho(\mathbf{r}, t) = R^2(\mathbf{r}, t)$ then $\frac{\partial}{\partial t} (\rho(\mathbf{r}, t) - R^2(\mathbf{r}, t)) = 0$

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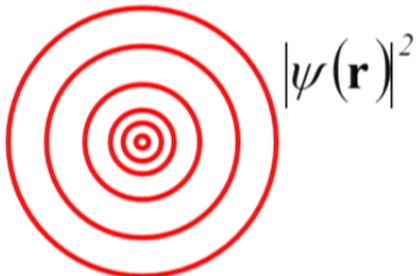
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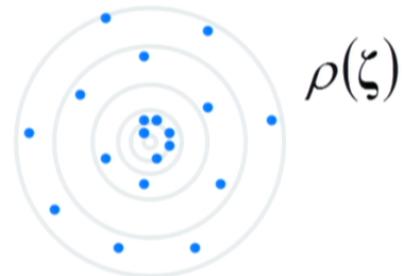
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1s orbital of Hydrogen atom

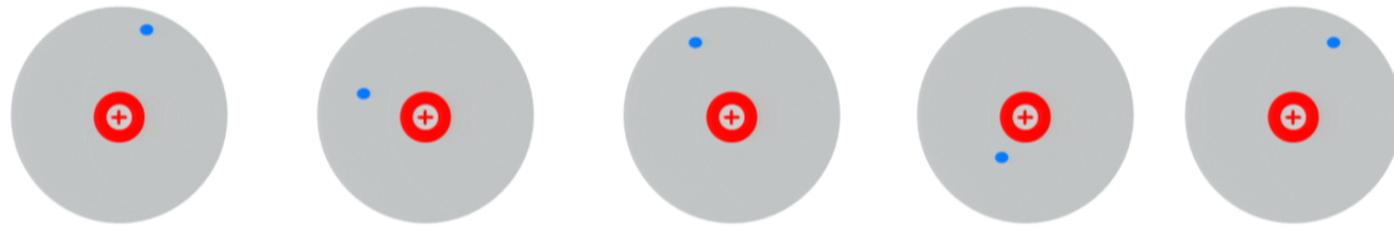


$$\psi(\mathbf{r}, t) = R(\mathbf{r})e^{-iEt/\hbar}$$

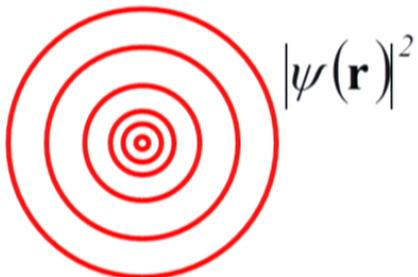


$$\rho(\zeta)$$

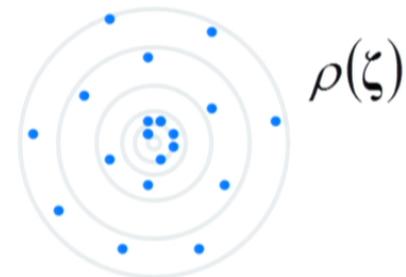
$$\text{so } \frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} = 0$$



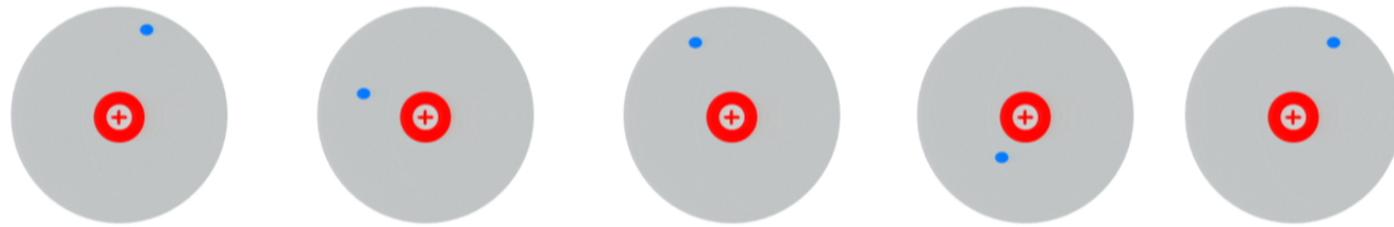
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"waves" of the decomposition

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$\zeta \in$ Spatial support of ψ_j *jth wave is occupied*

$\zeta \notin$ Spatial support of ψ_j *jth wave is empty*

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Proof of ineffectiveness of empty waves

$$\begin{aligned}\psi &= \psi_a + \psi_b \\ R e^{iS/\hbar} &= R_a e^{iS_a/\hbar} + R_b e^{iS_b/\hbar}\end{aligned}$$

$$R^2 = R_a^2 + R_b^2 + 2R_a R_b \cos[(S_a - S_b)/\hbar]$$

$$\nabla S = R^{-2} \left\{ \begin{array}{l} R_a^2 \nabla S_a + R_b^2 \nabla S_b + R_a R_b \cos[(S_a - S_b)/\hbar] \nabla (S_a + S_b) \\ - \hbar [R_a \nabla R_b - R_b \nabla R_a] \sin[(S_a - S_b)/\hbar] \end{array} \right\}$$

If $R_a R_b \approx 0$, $R_a \nabla R_b \approx 0$, $R_b \nabla R_a \approx 0$

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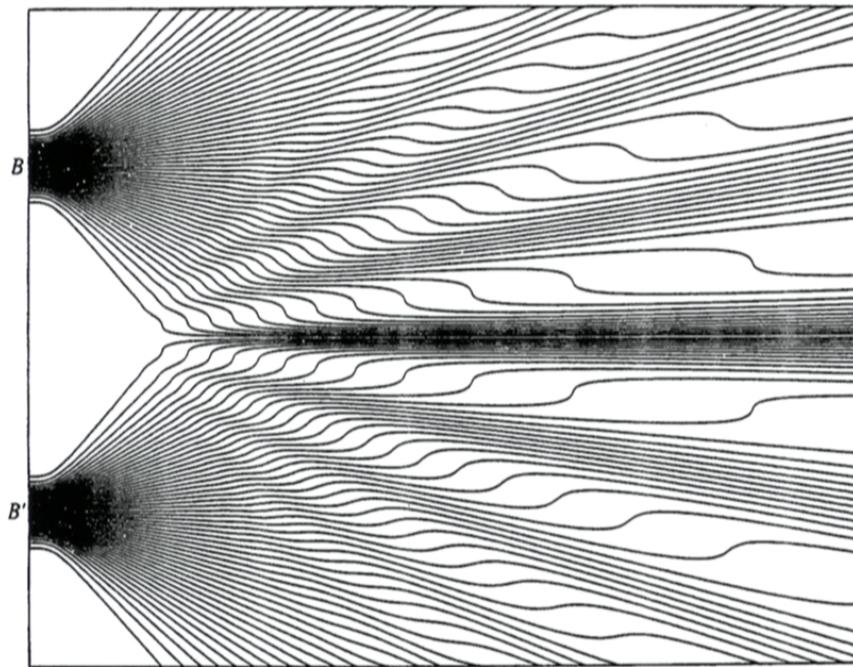
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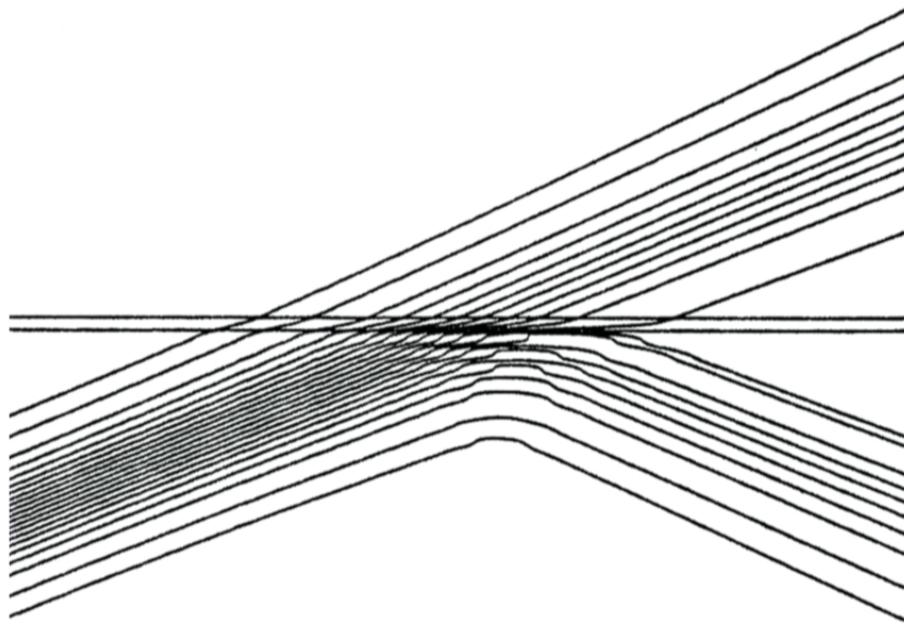
then $R^2 = R_a^2 + R_b^2$ and $\nabla S = \frac{R_a^2 \nabla S_a + R_b^2 \nabla S_b}{R_a^2 + R_b^2}$

$$\begin{aligned}\frac{d\zeta(t)}{dt} &= \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} = \frac{\nabla S_a}{m} && \text{If } \zeta \in \text{Support of } \psi_a \\ &= \frac{\nabla S_b}{m} && \text{If } \zeta \in \text{Support of } \psi_b\end{aligned}$$

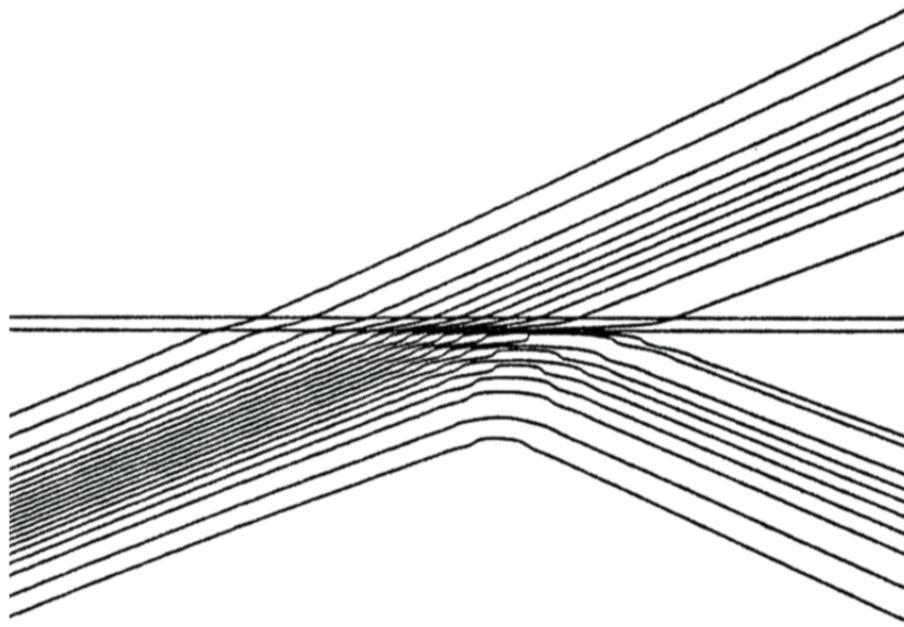


Double slit experiment

Note: Classically free motion is not rectilinear motion



Transmission through a barrier (probability $\frac{1}{2}$)



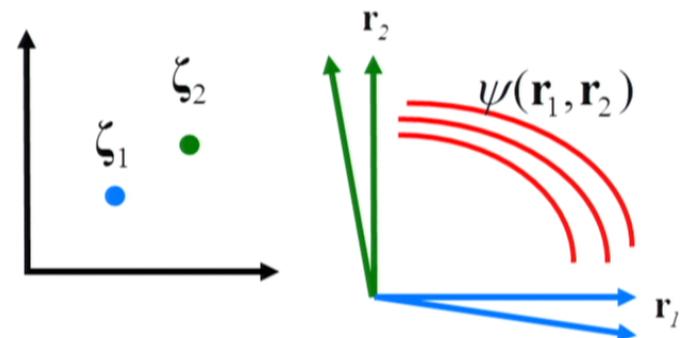
Transmission through a barrier (probability $\frac{1}{2}$)

The deBroglie-Bohm interpretation for many particles

The ontic state: $(\psi(\mathbf{r}_1, \mathbf{r}_2), \zeta_1, \zeta_2)$

↑
Wavefunction on
configuration space

↑
Particle
positions

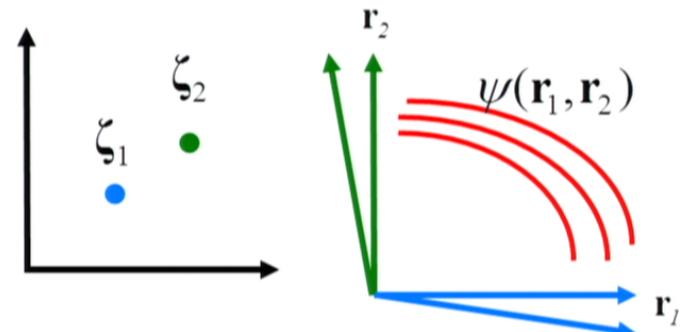


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The evolution equations:

Schrödinger's equation

$$i\hbar \frac{\partial \psi(\mathbf{r}_1, \mathbf{r}_2, t)}{\partial t} = -\frac{\hbar^2}{2m_1} \nabla_1^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) - \frac{\hbar^2}{2m_2} \nabla_2^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) + V(\mathbf{r}_1, \mathbf{r}_2) \psi(\mathbf{r}_1, \mathbf{r}_2, t)$$

$$\left. \begin{aligned} \frac{d\zeta_1(t)}{dt} &= \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \\ \frac{d\zeta_2(t)}{dt} &= \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \end{aligned} \right\}$$

The guidance equation

$$\text{where } \psi(\mathbf{r}_1, \mathbf{r}_2, t) = R(\mathbf{r}_1, \mathbf{r}_2, t) e^{iS(\mathbf{r}_1, \mathbf{r}_2, t)/\hbar}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

$$= R_1(\mathbf{r}_1, t) e^{iS_1(\mathbf{r}_1, t)/\hbar} R_2(\mathbf{r}_2, t) e^{iS_2(\mathbf{r}_2, t)/\hbar}$$

$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\frac{d\zeta_1(t)}{dt} = \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_1} [\nabla_1 S_1(\mathbf{r}_1, t)]_{\mathbf{r}_1=\zeta_1(t)}$$

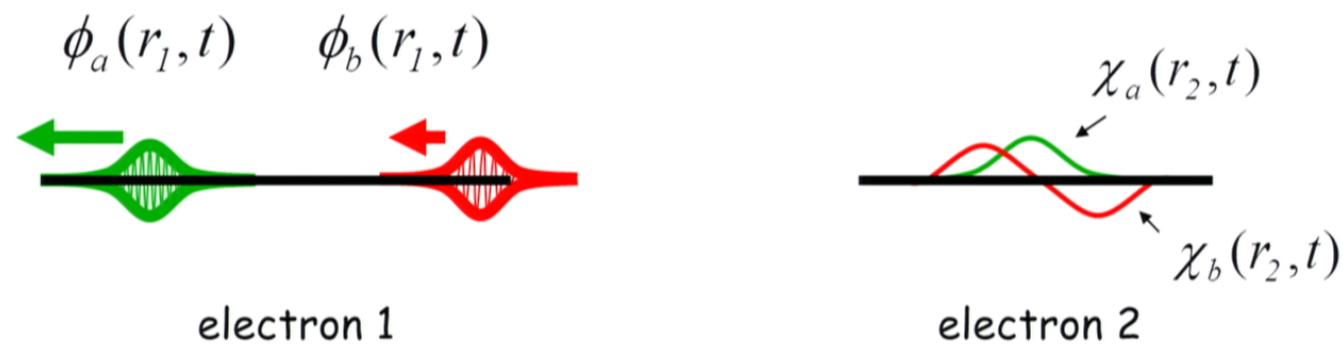
$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is occupied
 $(\zeta_1, \zeta_2) \notin$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is empty

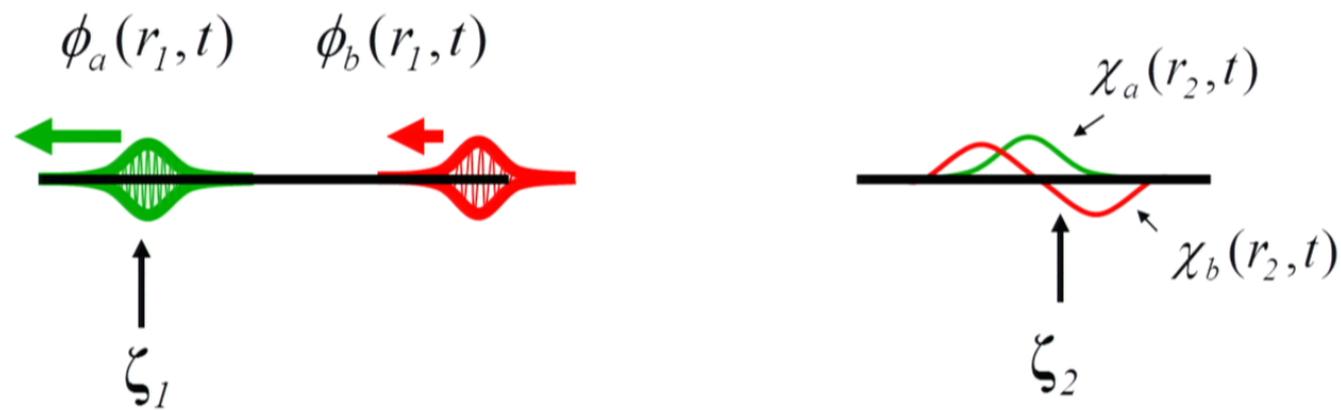
If only the k th wave is occupied

Then the particles evolve independently

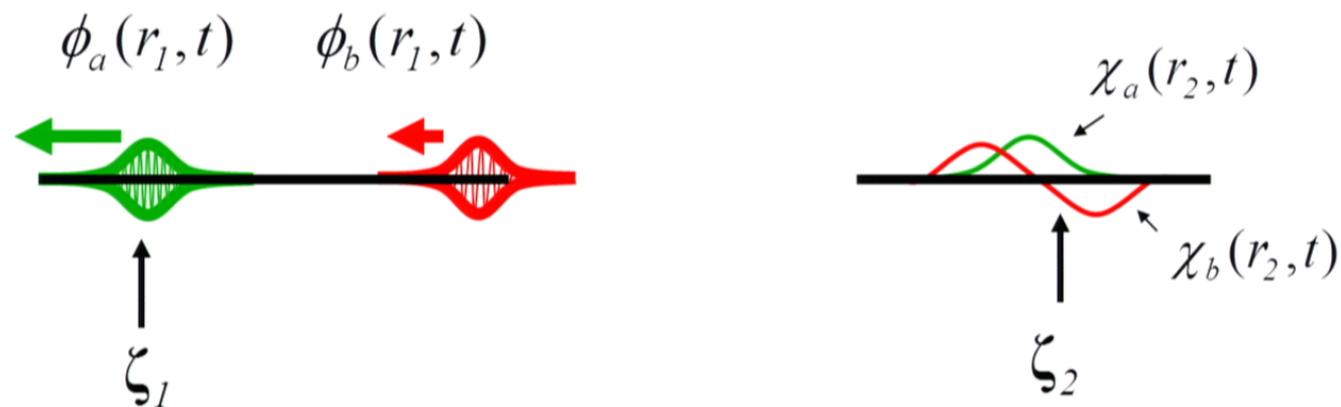
But in general, they do not



$$\psi(r_1, r_2; t) = c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$



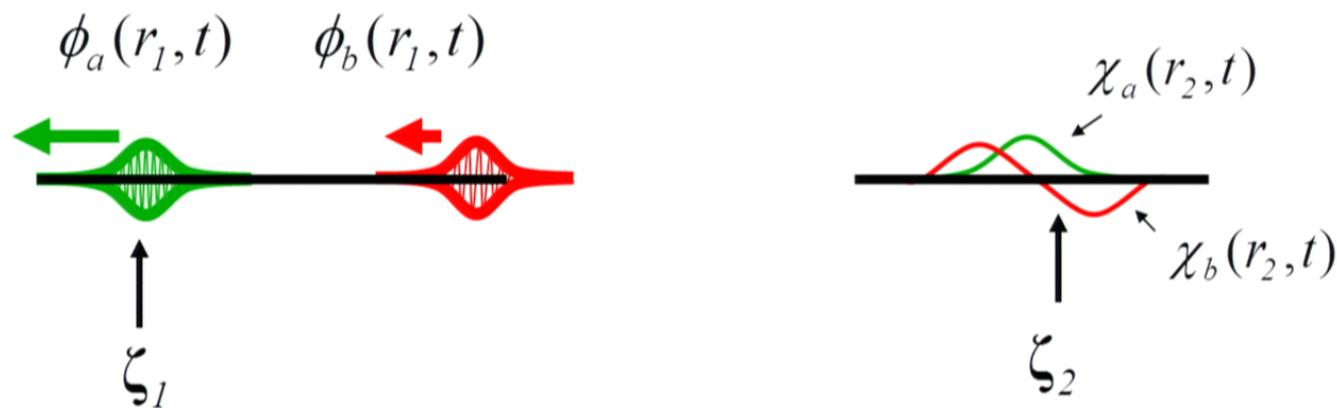
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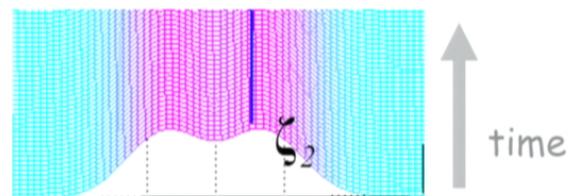
occupied wave

$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)}$$

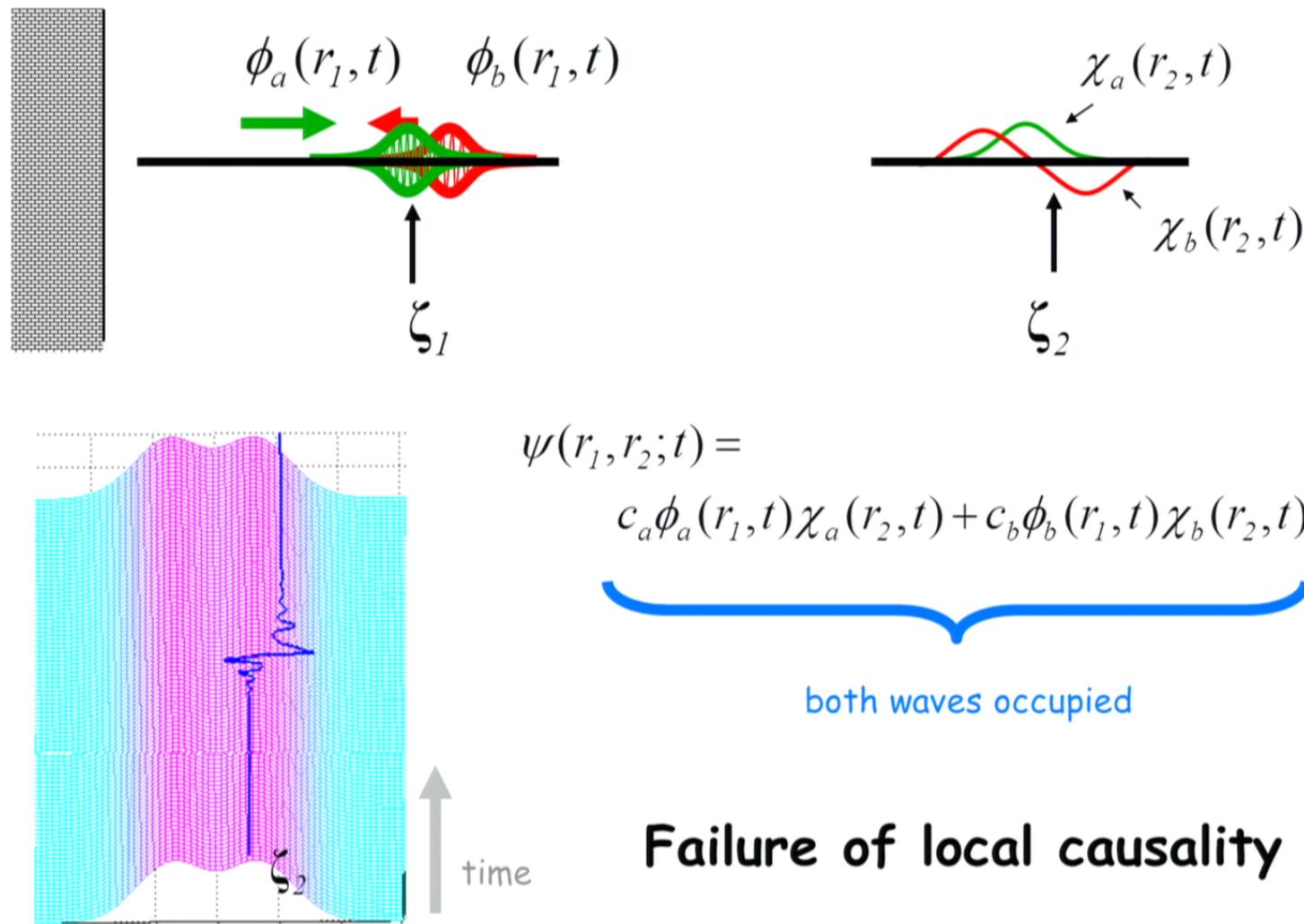


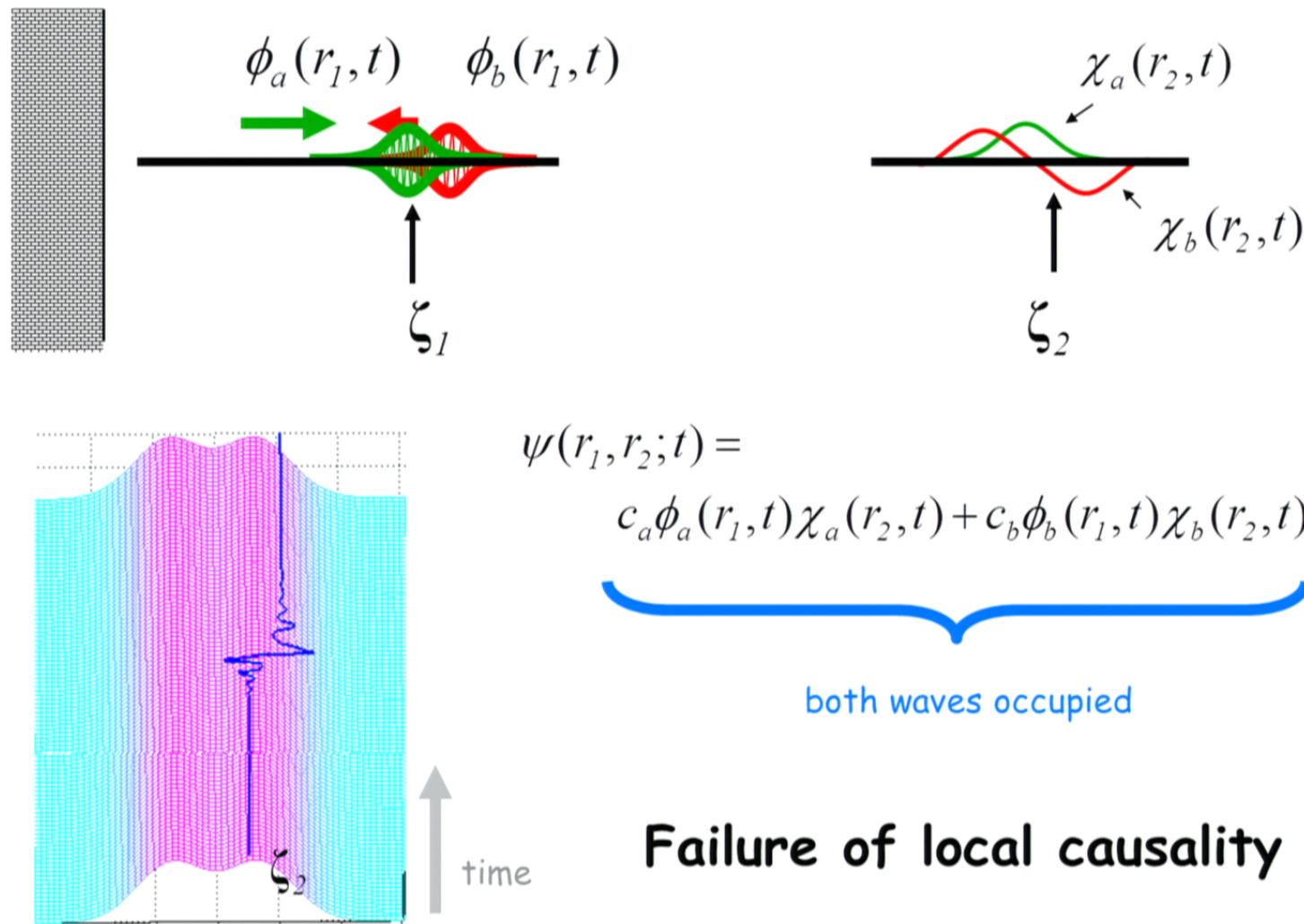
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occupied wave



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Reproducing the operational predictions

Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$

$$\phi_k(\mathbf{r})\chi(\mathbf{r}') \rightarrow \phi_k(\mathbf{r})\chi_k(\mathbf{r}')$$

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$$[\sum_k c_k \phi_k(\mathbf{r})]\chi(\mathbf{r}') \rightarrow \sum_k c_k \phi_k(\mathbf{r})\chi_k(\mathbf{r}')$$

Assumption: different outcomes of a measurement correspond to disjoint regions of the configuration space of the apparatus

$$\chi_j(\mathbf{r}')\chi_k(\mathbf{r}') \simeq 0 \text{ if } j \neq k$$

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Probability density of (ζ, ζ') being in the support of the j th wave

$$|c_j \phi_j(\zeta)\chi_j(\zeta')|^2$$

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$$|c_j \phi_j(\zeta)\chi_j(\zeta')|^2$$

Total probability of the j th wave being the occupied wave

$$|c_j|^2$$

If the j th wave comes to be occupied, then one can postulate an **effective collapse** of the guiding wave

$$\sum_k c_k \phi_k(\mathbf{r}) \rightarrow \phi_j(\mathbf{r})$$

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$$[\sum_k c_k \phi_k(\mathbf{r})]\chi(\mathbf{r}') \rightarrow \sum_k c_k \phi_k(\mathbf{r})\chi_k(\mathbf{r}')$$

Assumption: different outcomes of a measurement correspond to disjoint regions of the configuration space of the apparatus

$$\chi_j(\mathbf{r}')\chi_k(\mathbf{r}') \simeq 0 \text{ if } j \neq k$$

Probability density of (ζ, ζ') being in the support of the j th wave

$$|c_j \phi_j(\zeta)\chi_j(\zeta')|^2$$

Total probability of the j th wave being the occupied wave

$$|c_j|^2$$

If the j th wave comes to be occupied, then one can postulate an **effective collapse** of the guiding wave

$$\sum_k c_k \phi_k(\mathbf{r}) \rightarrow \phi_j(\mathbf{r})$$