

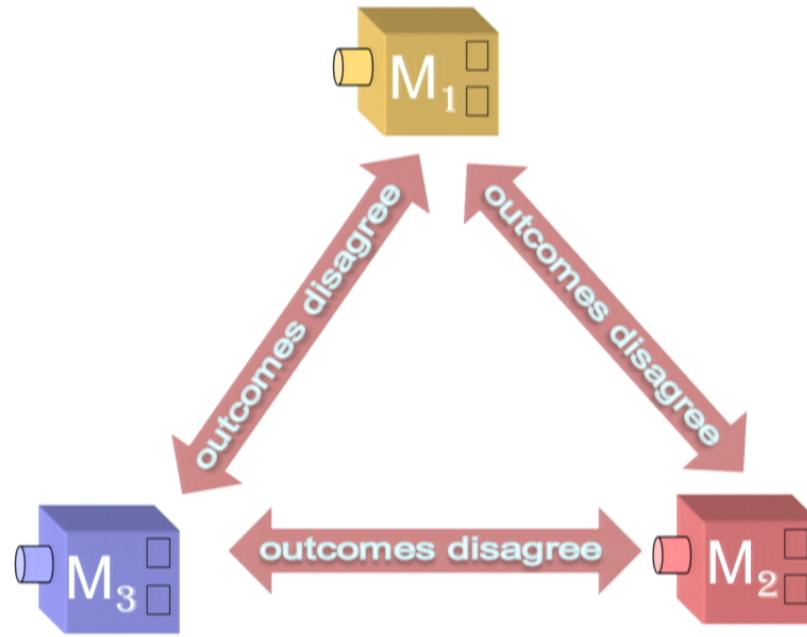
Title: 12/13 PSI - Found Quantum Mechanics Lecture 10

Date: Jan 18, 2013 11:30 AM

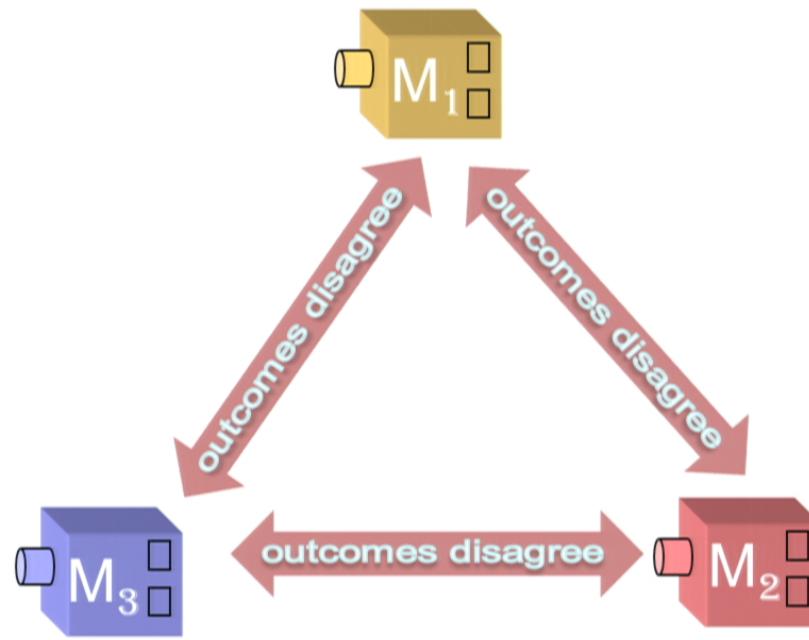
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Abstract:

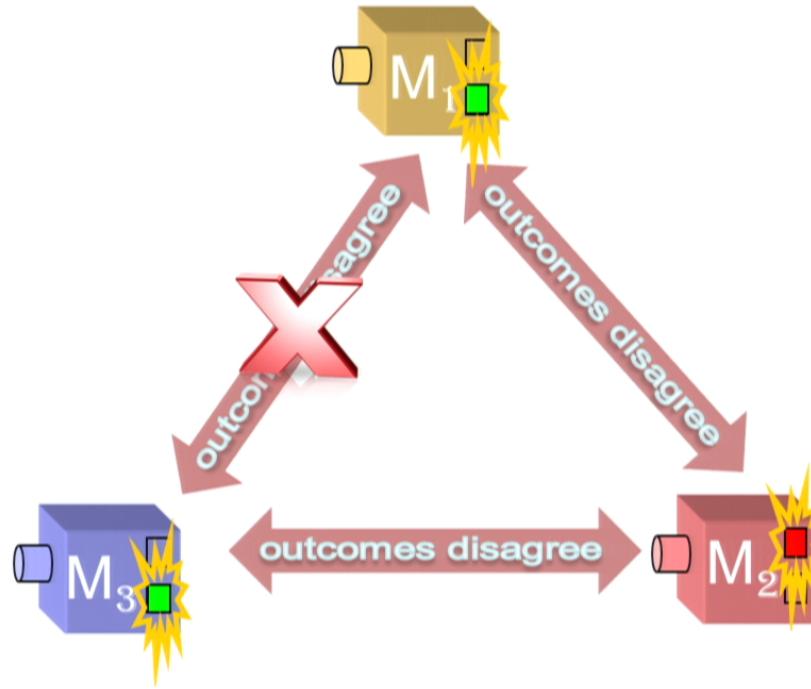
Specker's example



Specker's example



Specker's example



If the outcomes are fixed **deterministically** by the ontic state and are **independent of the context** in which the measurement is performed, then

$$p(\text{success}) \leq \frac{2}{3}$$

Frustrated Networks

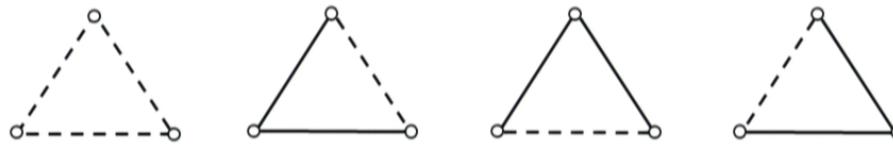
Nodes are binary variables

Edges imply joint measurability

—○— Outcomes agree

—○---○ Outcomes disagree

Frustration = no valuation satisfying all correlations



Frustrated Networks

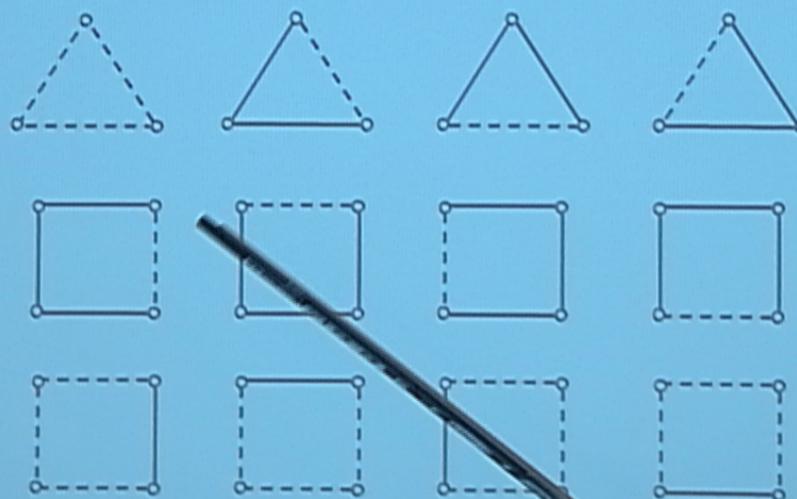
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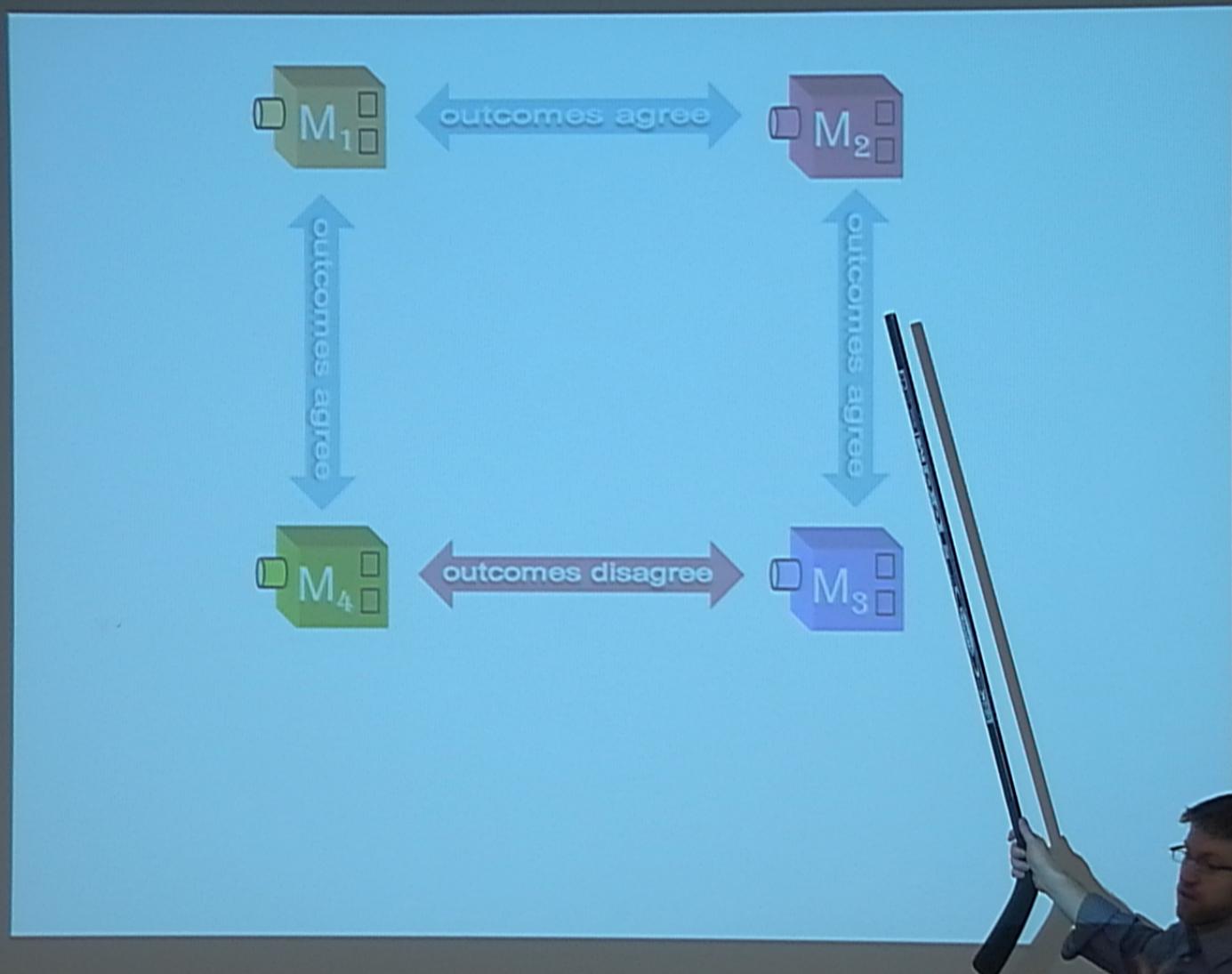
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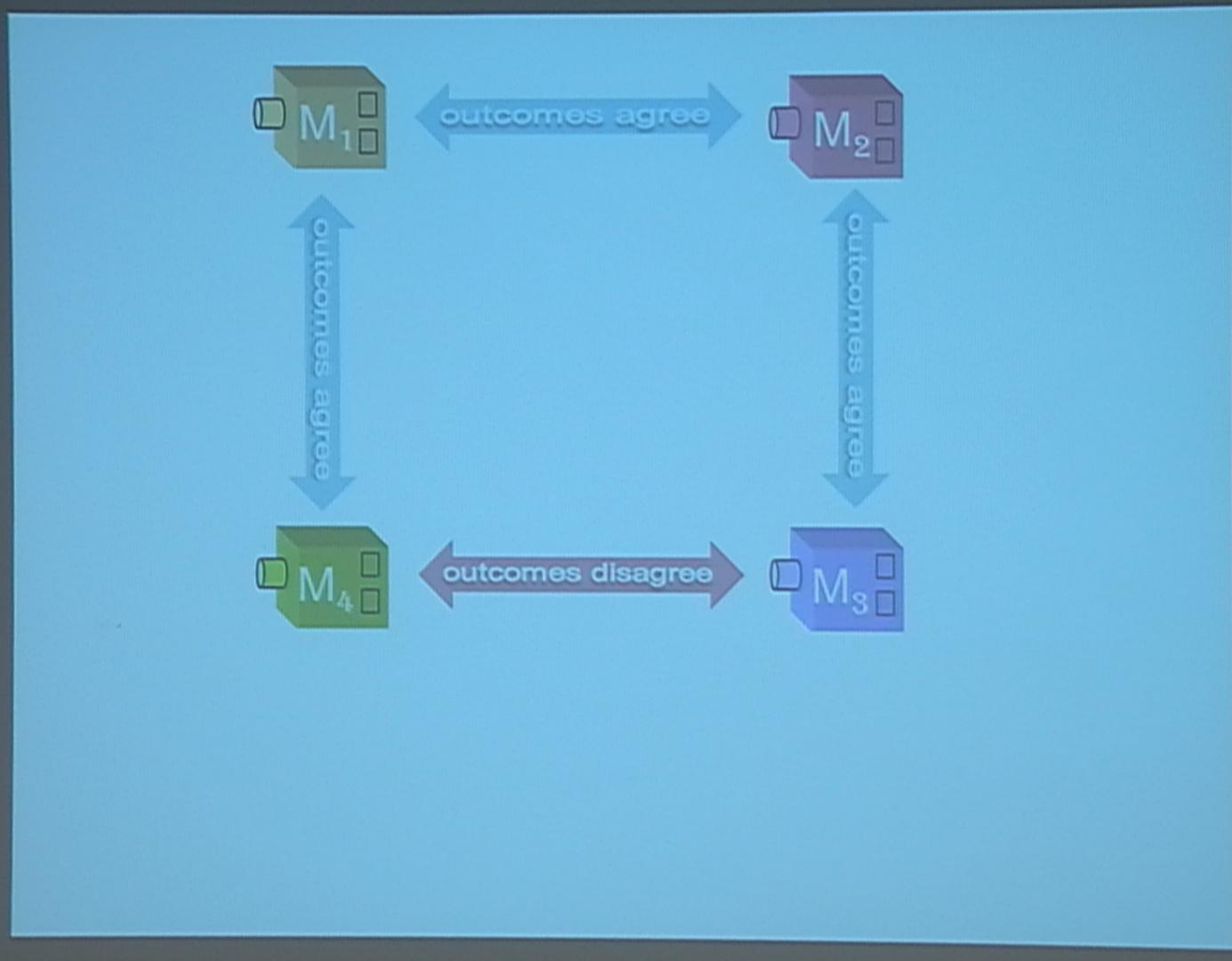
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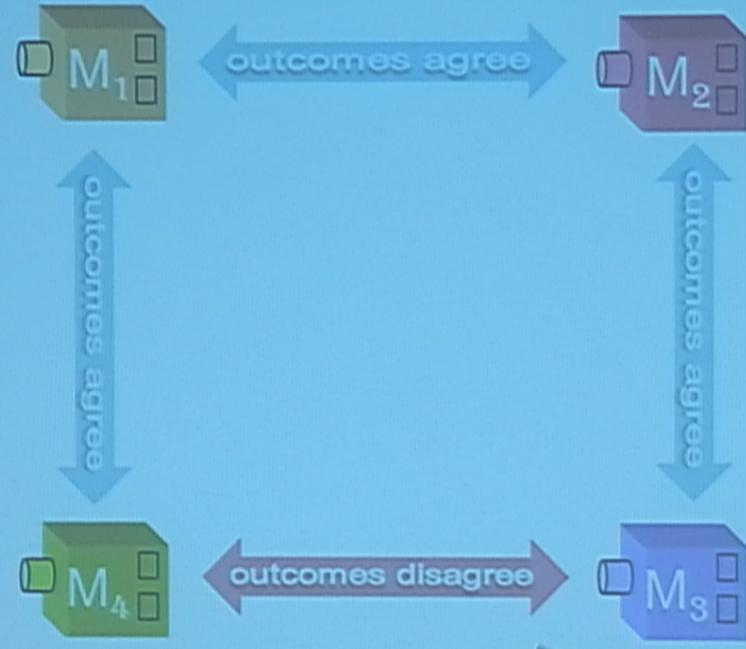
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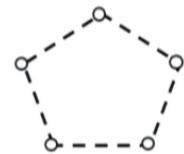




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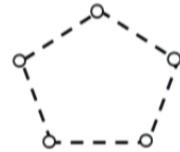
$$p(\text{success}) \leq \frac{3}{4}$$

Klyachko's example



$$p(\text{success}) \leq \frac{4}{5}$$

Klyachko's example



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5 projective mmts:

$$\{|l_1\rangle\langle l_1|, I - |l_1\rangle\langle l_1|\}$$

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$$\{|l_5\rangle\langle l_5|, I - |l_5\rangle\langle l_5|\}$$

where $\langle l_i | l_{i \oplus 1} \rangle = 0 \quad i \in \{1, \dots, 5\}$

Klyachko's example
 $p(\text{success}) \leq \frac{4}{5}$

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Klyachko's example

$$\text{Diagram showing two configurations of dashed lines connecting five points around a circle, separated by an equals sign.} \quad p(\text{success}) \leq \frac{4}{5}$$

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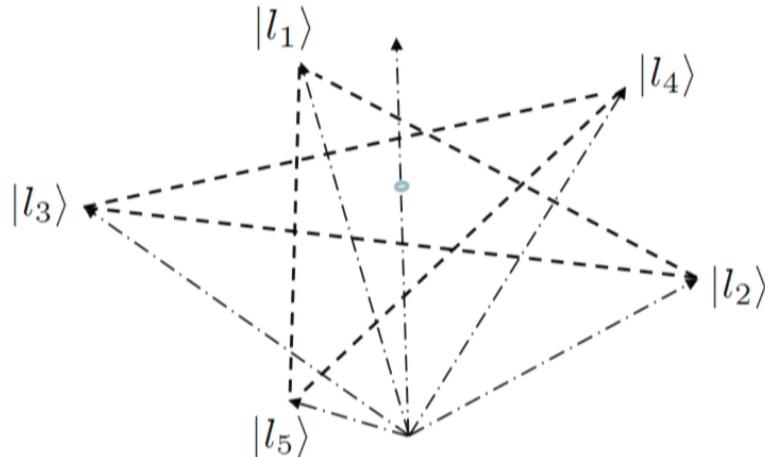
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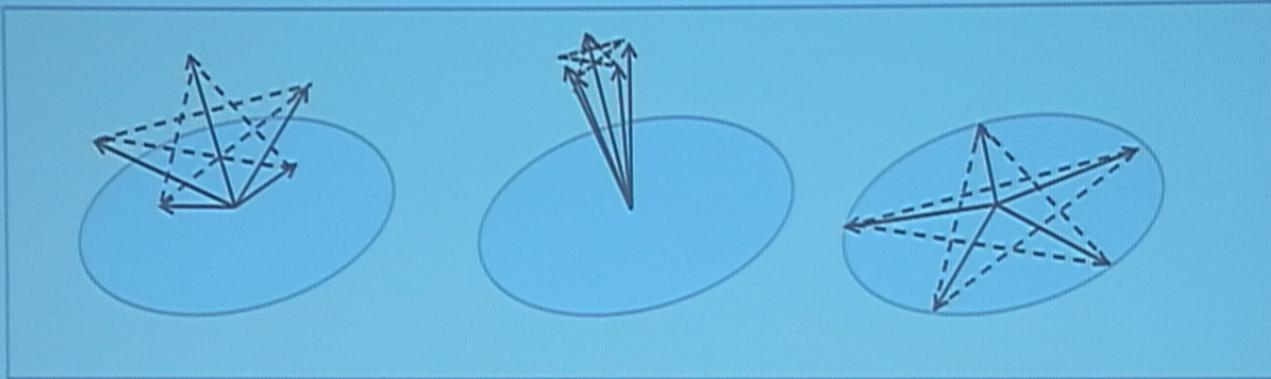
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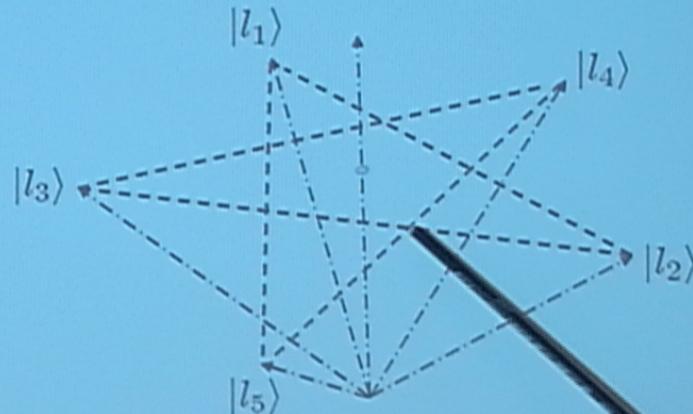
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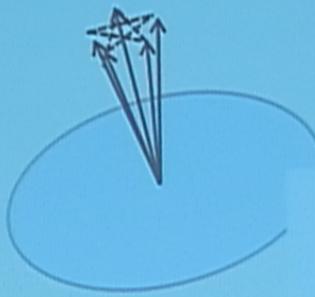
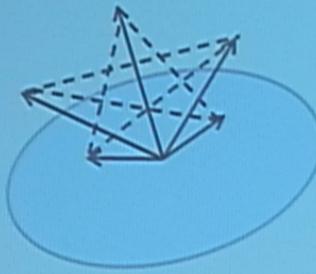
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$$\cos^2 \theta = \frac{1}{\sqrt{5}}$$

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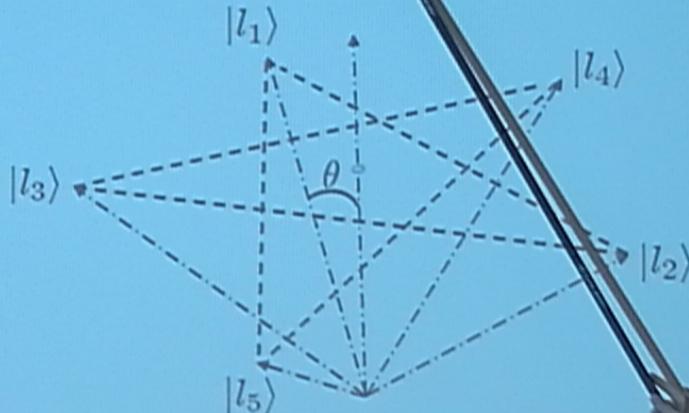
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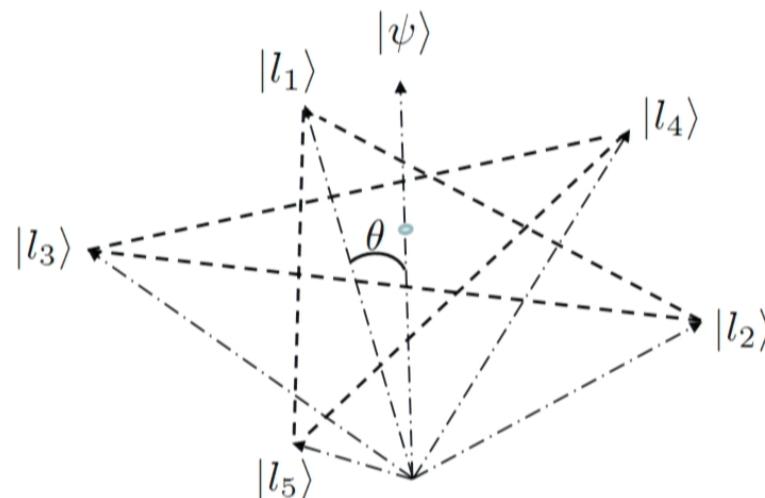
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Preparation: the ψ that lies on the symmetry axis

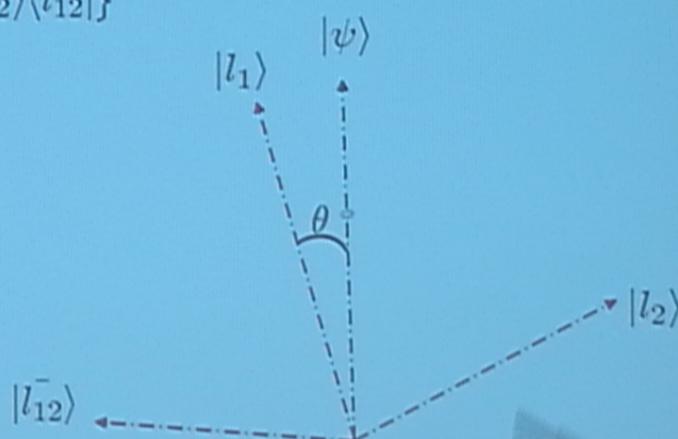
Klyachko's example

Consider measuring:

$$\begin{aligned}\{|l_1\rangle\langle l_1|, I - |l_1\rangle\langle l_1|\} \\ \{|l_2\rangle\langle l_2|, I - |l_2\rangle\langle l_2|\}\end{aligned}$$

$$\cos^2 \theta = \frac{1}{\sqrt{5}}$$

Equivalently: $\{|l_1\rangle\langle l_1|, |l_2\rangle\langle l_2|, |\bar{l}_{12}\rangle\langle \bar{l}_{12}|\}$



Klyachko's example

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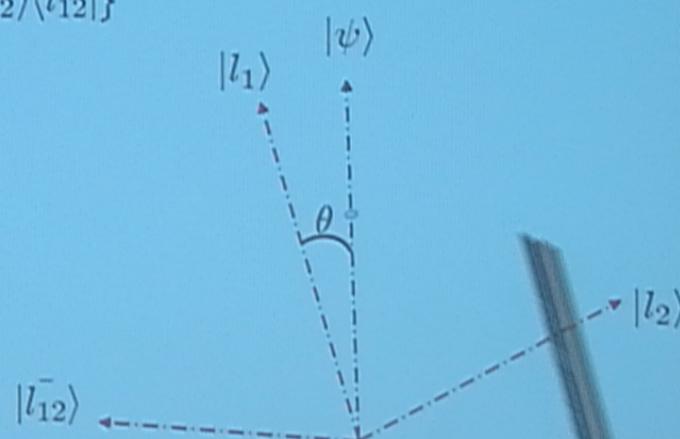
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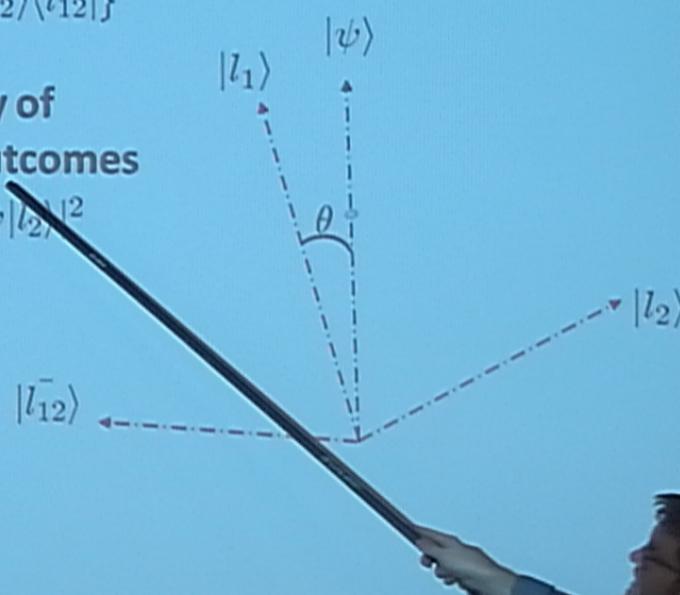
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**Probability of
anticorrelated outcomes**

$$|\langle\psi|l_1\rangle|^2 + |\langle\psi|l_2\rangle|^2 = \frac{2}{\sqrt{5}}$$

$$\text{prob.} |\langle\psi|l_{12}^-\rangle|^2 = 1 - \frac{2}{\sqrt{5}}$$



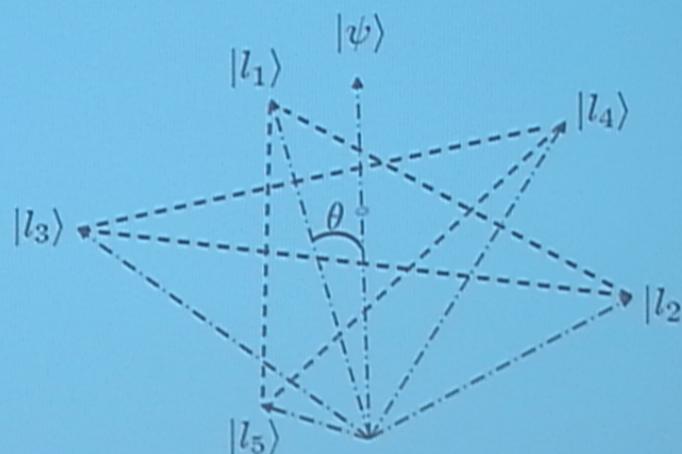
Klyachko's example

$$\cos^2 \theta = \frac{1}{\sqrt{5}}$$

Similarly for any pair of measurements...

Probability of anticorrelated outcomes

$$= \frac{2}{\sqrt{5}}$$



Quantum probability of success

$$p(\text{success}) = \frac{2}{\sqrt{5}} \simeq 0.89 > \frac{4}{5}$$

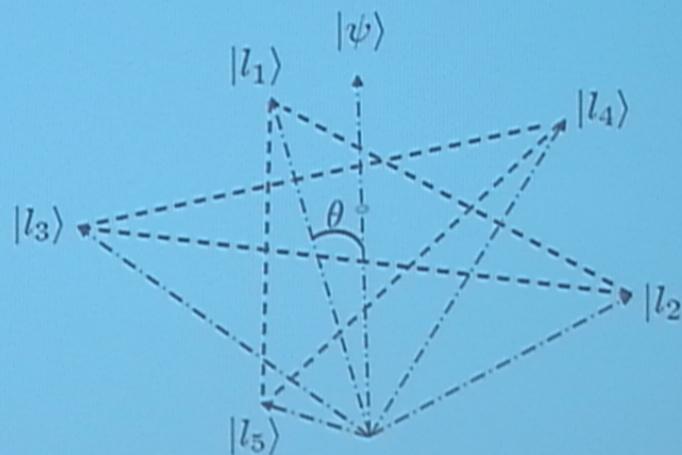
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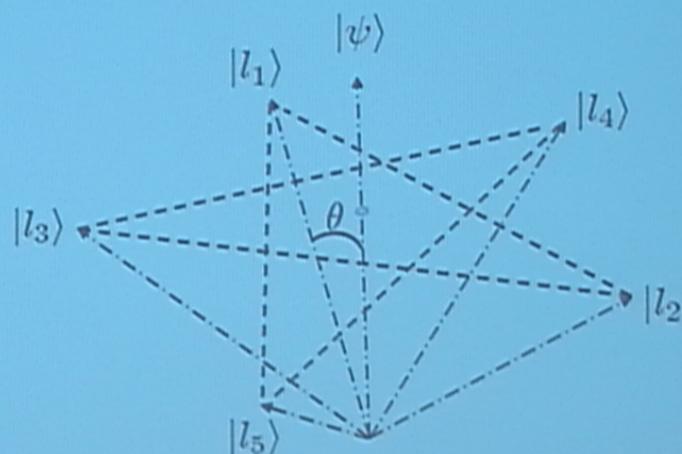
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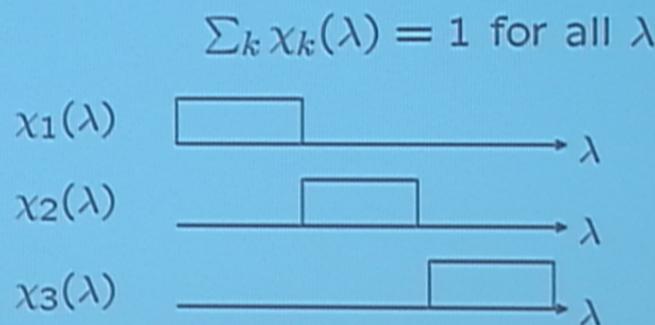
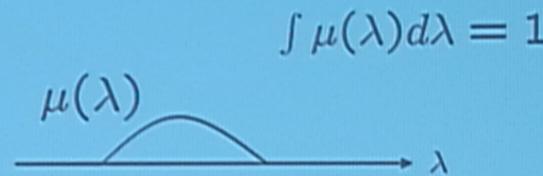
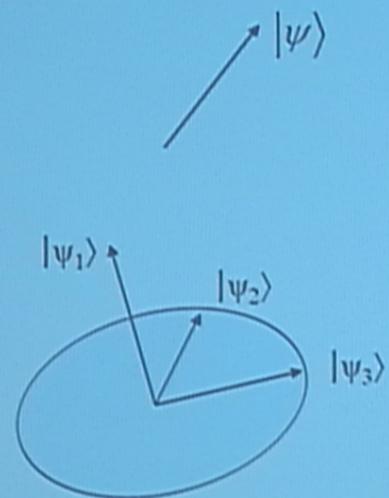
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Deterministic hidden variable model for pure states and projective measurements

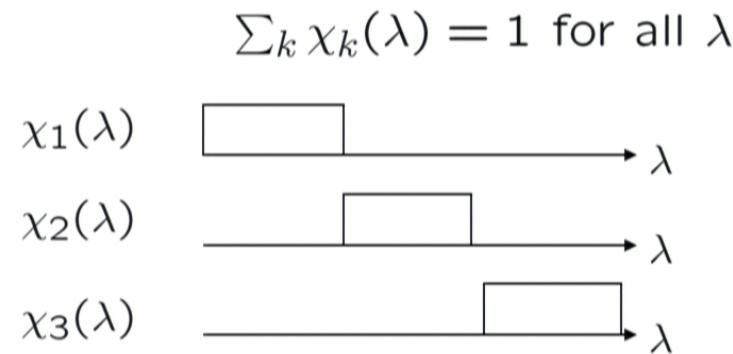
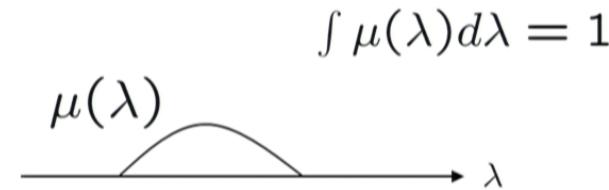
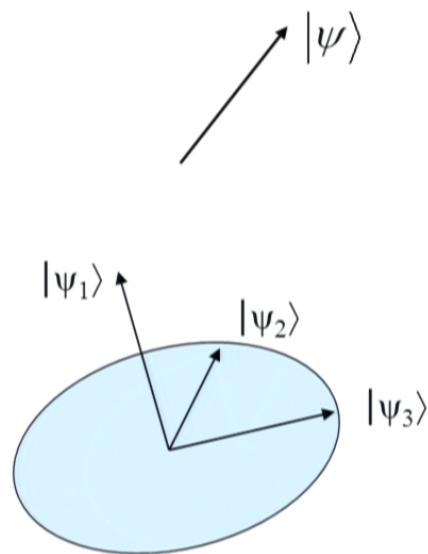


It is assumed that the outcomes are deterministic given λ

$$|\langle \psi | \psi_k \rangle|^2 = \int d\lambda \mu(\lambda) \chi_k(\lambda)$$

The traditional definition of noncontextuality in quantum theory

Deterministic hidden variable model for pure states and projective measurements

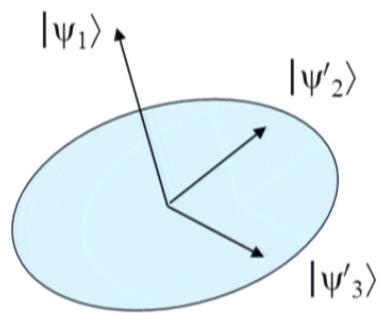
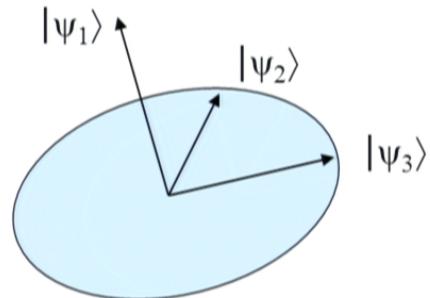


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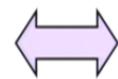
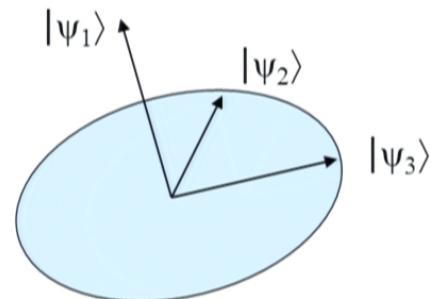
Traditional notion of noncontextuality

A given vector may appear in many different measurements



Traditional notion of noncontextuality

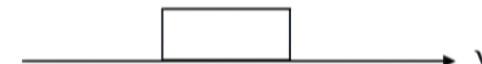
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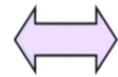
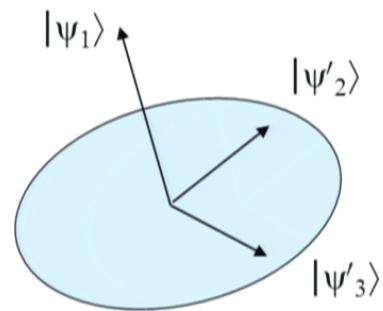
$$\chi_1(\lambda)$$



$$\chi_2(\lambda)$$



$$\chi_3(\lambda)$$



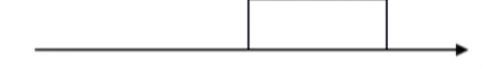
$$\chi_1(\lambda)$$



$$\chi'_2(\lambda)$$

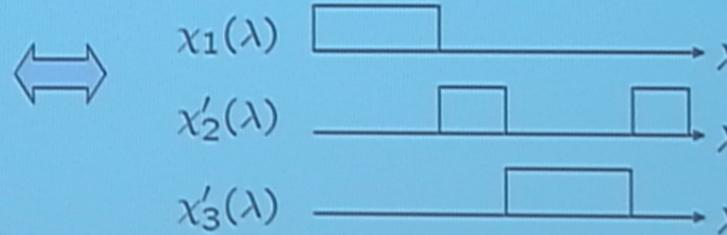
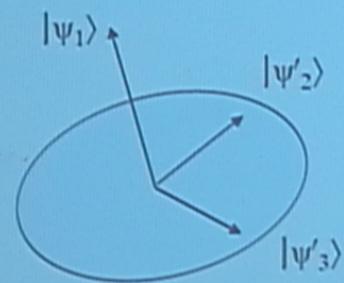
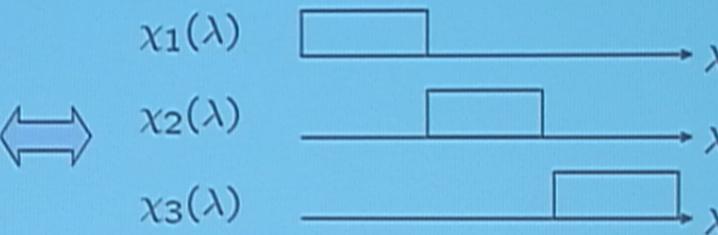
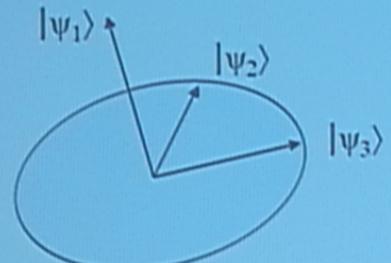


$$\chi'_3(\lambda)$$



Traditional notion of noncontextuality

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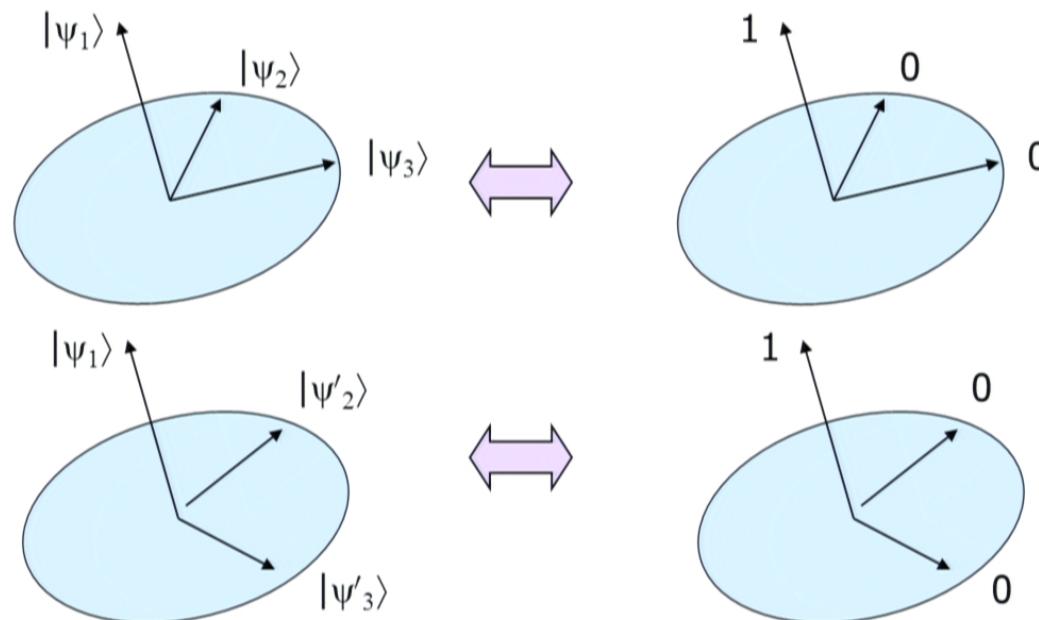


The traditional notion of noncontextuality:

Every vector is associated with the same $\chi(\lambda)$
regardless of how it is measured (i.e. the context)

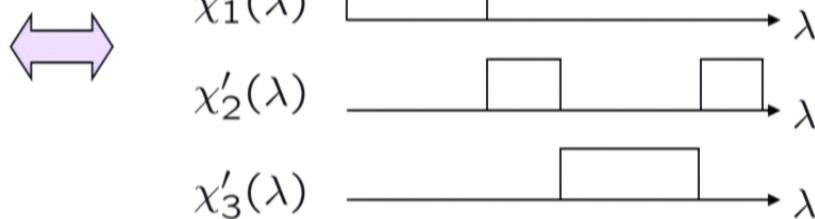
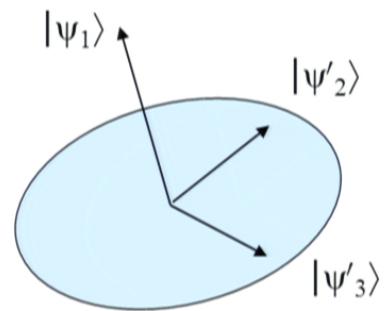
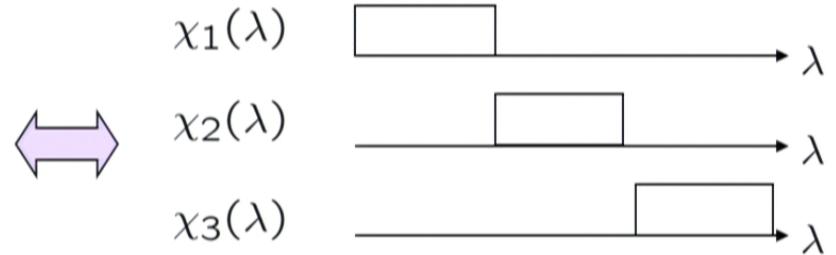
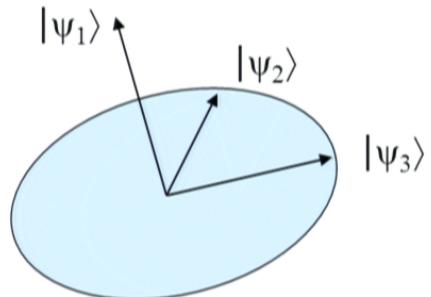
The traditional notion of noncontextuality (take 2):

For every λ , every basis of vectors receives a 0-1 valuation, wherein exactly one element is assigned the value 1 (corresponding to the outcome that would occur for λ), and every vector is assigned the same value regardless of the basis it is considered a part (i.e. **the context**).

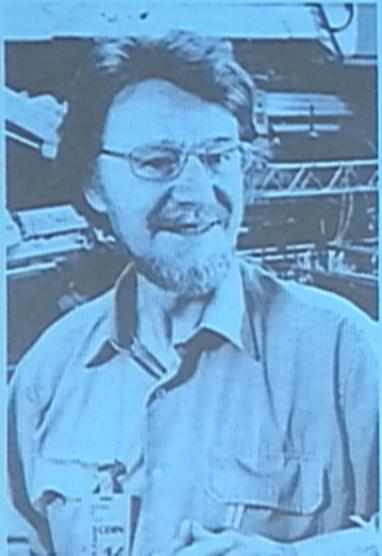


Traditional notion of noncontextuality

A given vector may appear in many different measurements



The traditional notion of noncontextuality:
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John S. Bell

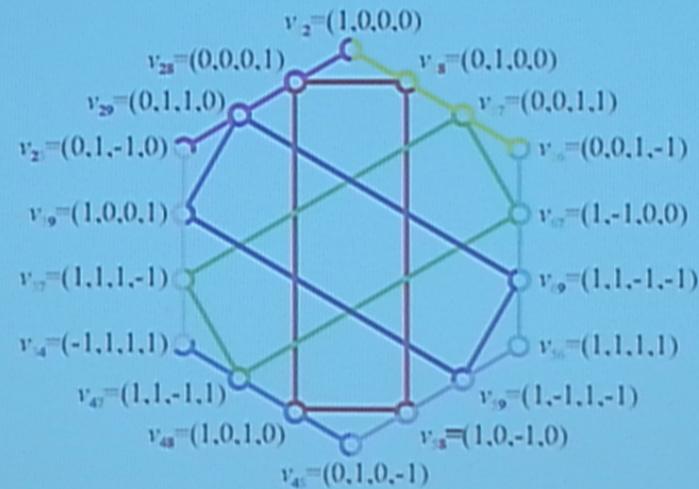


Ernst Specker (with son) and
Simon Kochen

Bell-Kochen-Specker theorem: A noncontextual hidden variable model of quantum theory for Hilbert spaces of dimension 3 or greater is impossible.

Example: The CEGA 18 ray proof in 4d:

Cabello, Estebaranz, Garcia-Alcaine, Phys. Lett. A 212, 183 (1996)



Example: The CEGA 18 ray proof in 4d:

Cabello, Estebaranz, Garcia-Alcaine, Phys. Lett. A 212, 183 (1996)

If we list all 9 orthogonal quadruples, each ray appears twice in the list

0,0,0,1	0,0,0,1	1,-1,1,-1	1,-1,1,-1	0,0,1,0	1,-1,-1,1	1,1,-1,1	1,1,-1,1	1,1,1,-1
0,0,1,0	0,1,0,0	1,-1,-1,1	1,1,1,1	0,1,0,0	1,1,1,1	1,1,1,-1	-1,1,1,1	-1,1,1,1
1,1,0,0	1,0,1,0	1,1,0,0	1,0,-1,0	1,0,0,1	1,0,0,-1	1,-1,0,0	1,0,1,0	1,0,0,1
1,-1,0,0	1,0,-1,0	0,0,1,1	0,1,0,-1	1,0,0,-1	0,1,-1,0	0,0,1,1	0,1,0,-1	0,1,-1,0

Example: The CEGA 18 ray proof in 4d:

Cabello, Estebaranz, Garcia-Alcaine, Phys. Lett. A 212, 183 (1996)

If we list all 9 orthogonal quadruples, each ray appears twice in the list

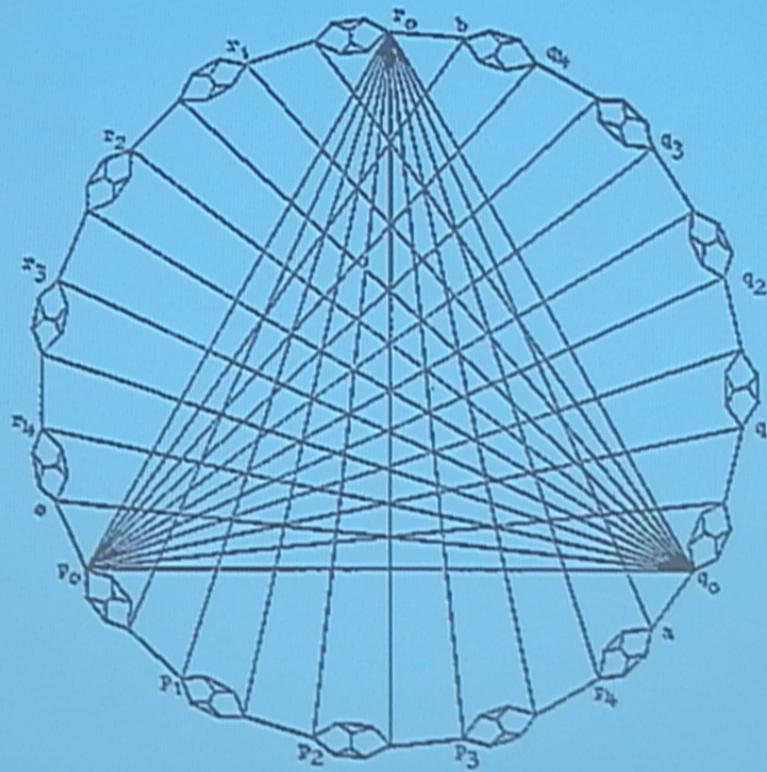
0,0,0,1	0,0,0,1	1,-1,1,-1	1,-1,1,-1	0,0,1,0	1,-1,-1,1	1,1,-1,1	1,1,-1,1	1,1,1,-1
0,0,1,0	0,1,0,0	1,-1,-1,1	1,1,1,1	0,1,0,0	1,1,1,1	1,1,1,-1	-1,1,1,1	-1,1,1,1
1,1,0,0	1,0,1,0	1,1,0,0	1,0,-1,0	1,0,0,1	1,0,0,-1	1,-1,0,0	1,0,1,0	1,0,0,1
1,-1,0,0	1,0,-1,0	0,0,1,1	0,1,0,-1	1,0,0,-1	0,1,-1,0	0,0,1,1	0,1,0,-1	0,1,-1,0

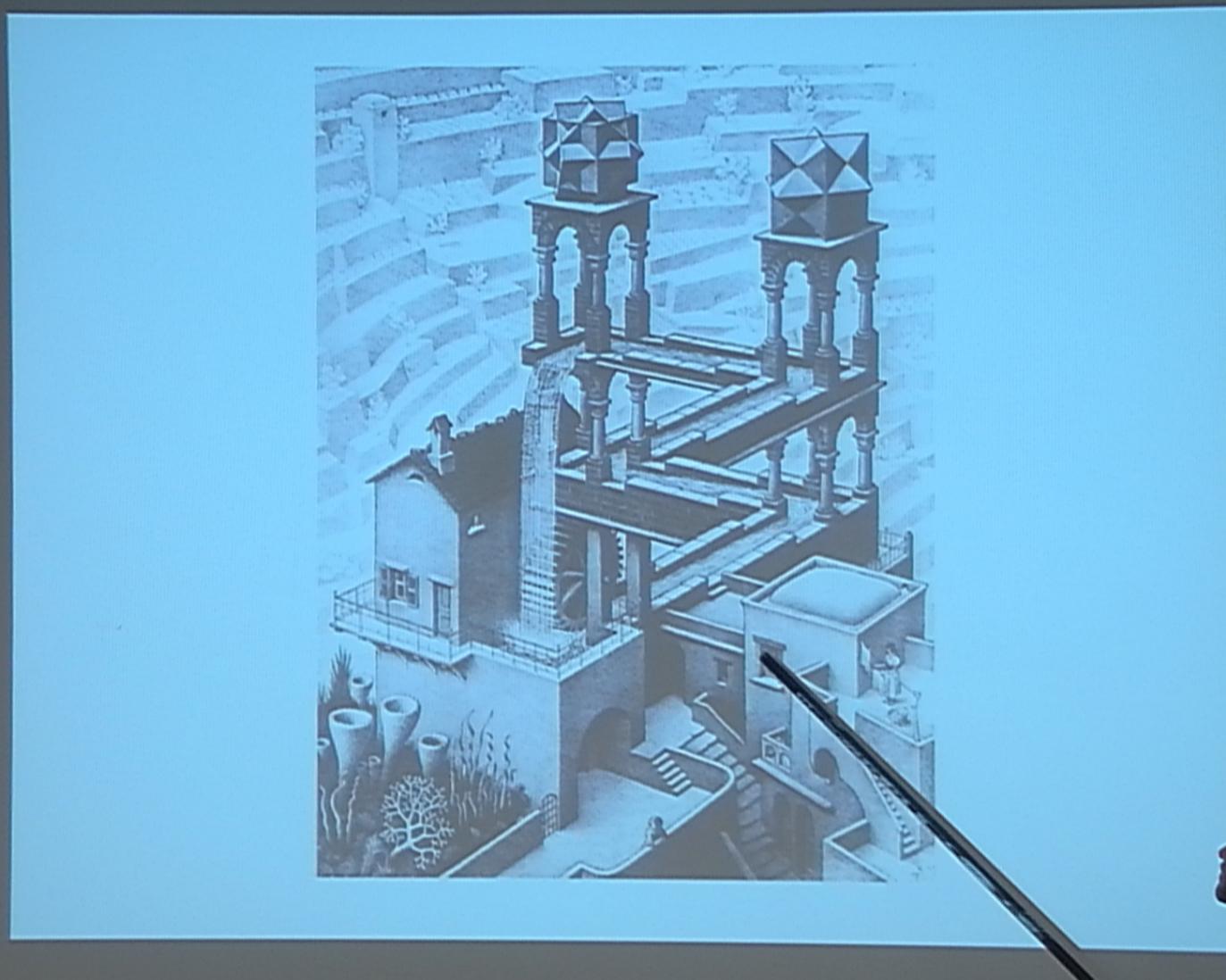
In each of the 9 quadruples, one ray is assigned 1, the other three 0
Therefore, 9 rays must be assigned 1

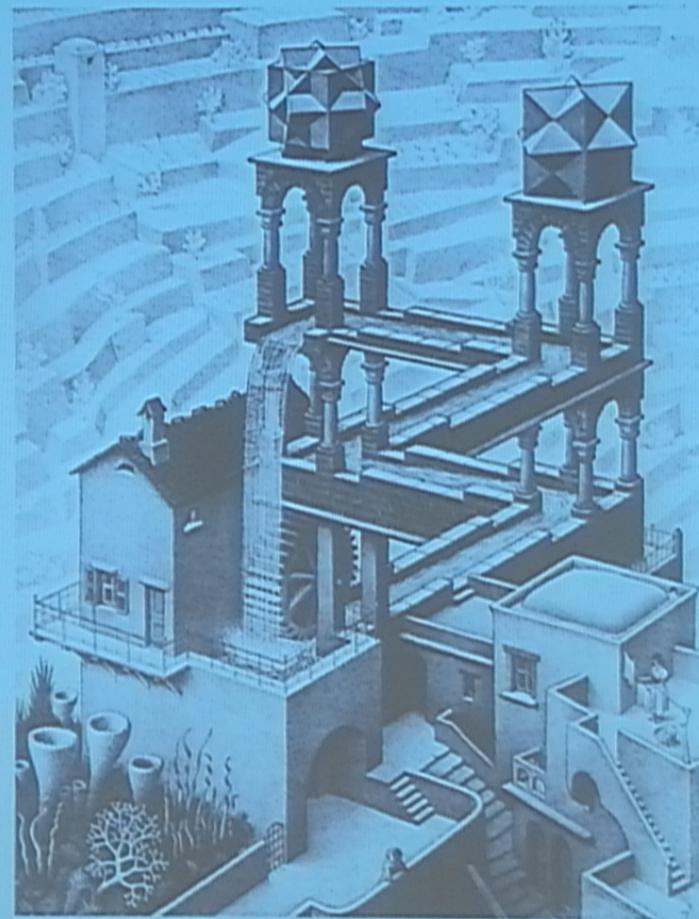
But each ray appears twice and so there must be an even number
of rays assigned 1

CONTRADICTION!

Example: Kochen and Specker's original 117 ray proof in 3d







Problems with the traditional definition of noncontextuality:

- applies only to *projective* measurements
- applies only to *deterministic* hidden variable models
- applies only to models of *quantum theory*

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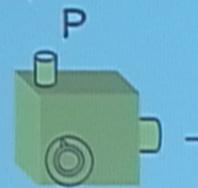
An operational notion of noncontextuality would determine

- whether any given *operational theory* admits of a noncontextual model
- whether any given *experimental data* can be explained by a noncontextual model

An ontological model of an operational theory

Specifies an ontic state space Λ

Preparation



$$\int \mu_P(\lambda) d\lambda = 1$$



An ontological model of an operational theory

Specifies an ontic state space Λ

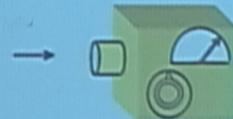
Preparation

P



Measurement

M



$$\int \mu_P(\lambda) d\lambda = 1$$

$\mu_P(\lambda)$

λ

$$0 \leq \xi_{M,k} \leq 1$$

$$\sum_k \xi_{M,k}(\lambda) = 1 \text{ for all } \lambda$$



An ontological model of an operational theory

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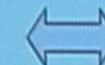


$$\int \mu_P(\lambda) d\lambda = 1$$



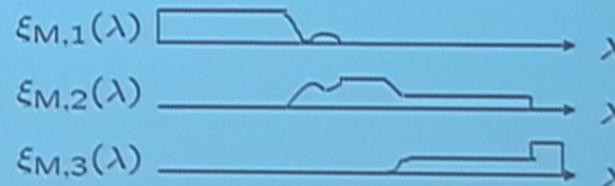
Measurement

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$$\sum_k \xi_{M,k}(\lambda) = 1 \text{ for all } \lambda$$



$$p(k|P, M) = \int d\lambda \xi_{M,k}(\lambda) \mu_P(\lambda)$$

Generalized definition of noncontextuality:

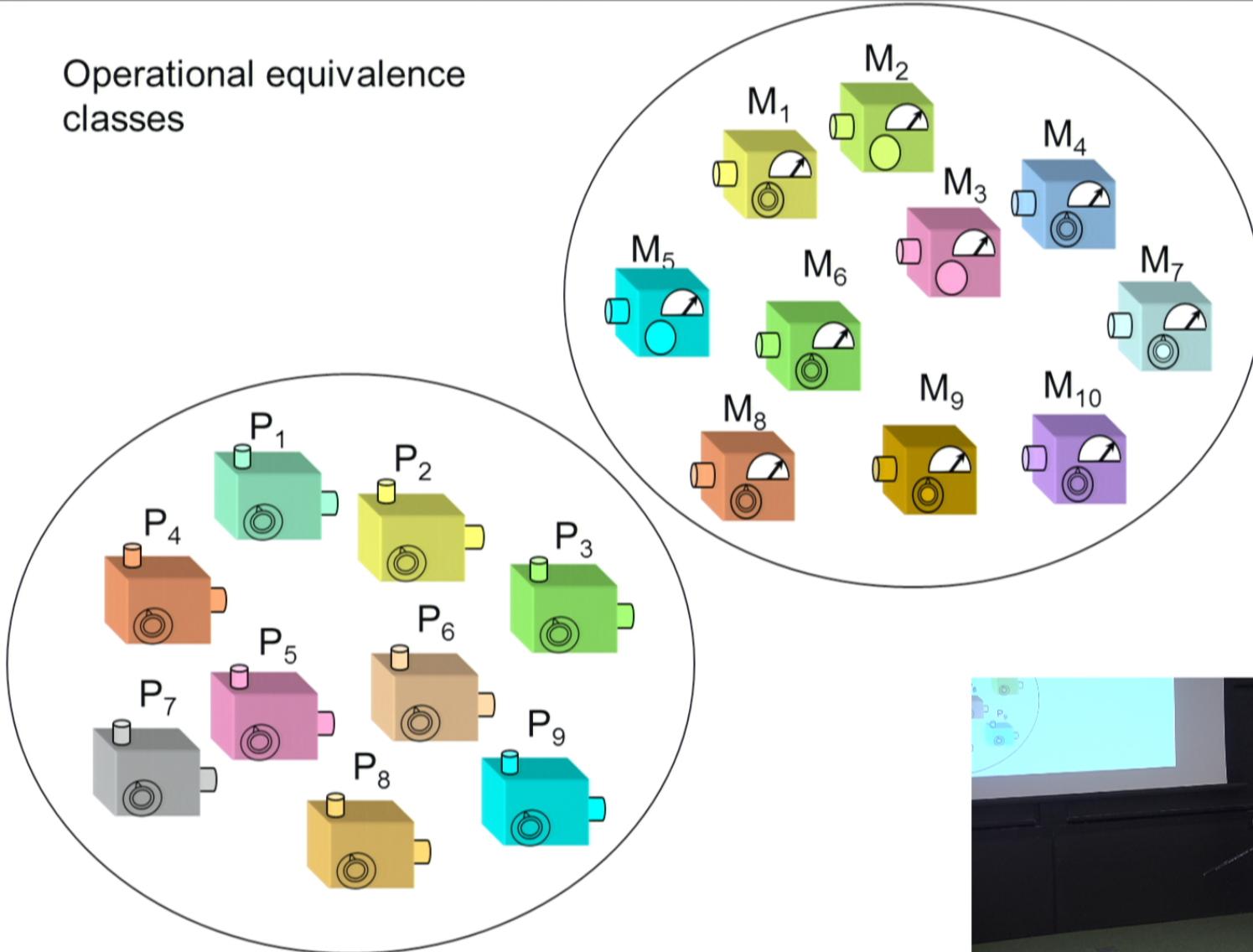
An ontological model of an operational theory is **noncontextual** if

Operational equivalence
of two experimental
procedures

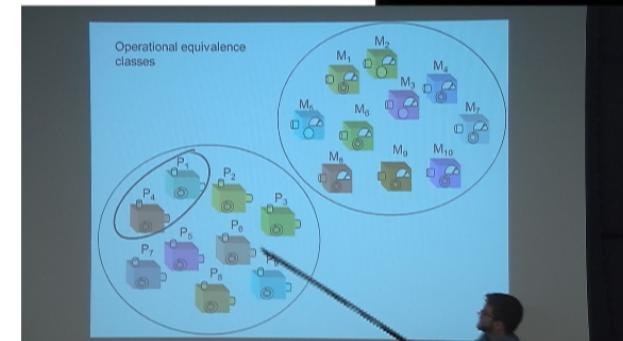
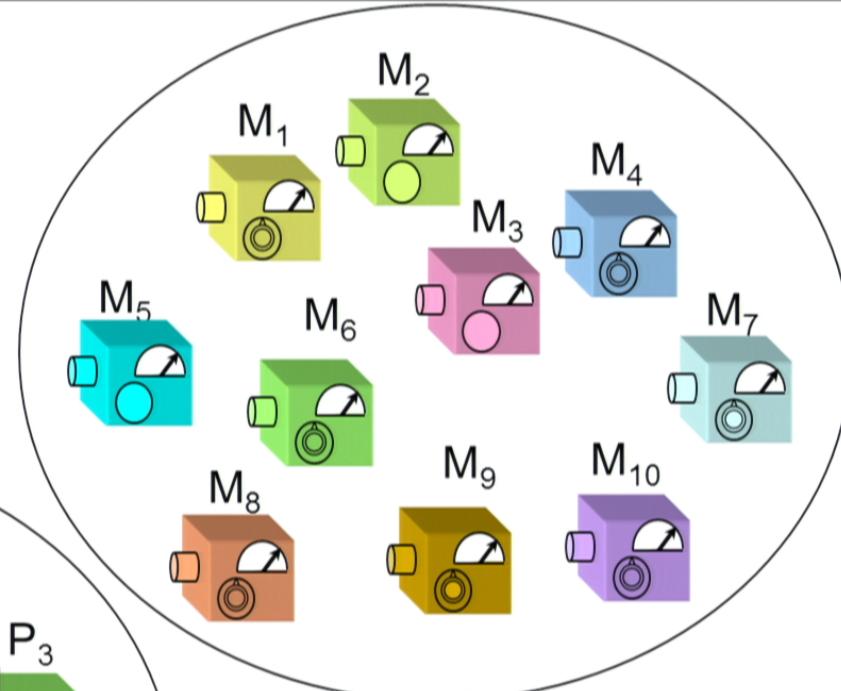
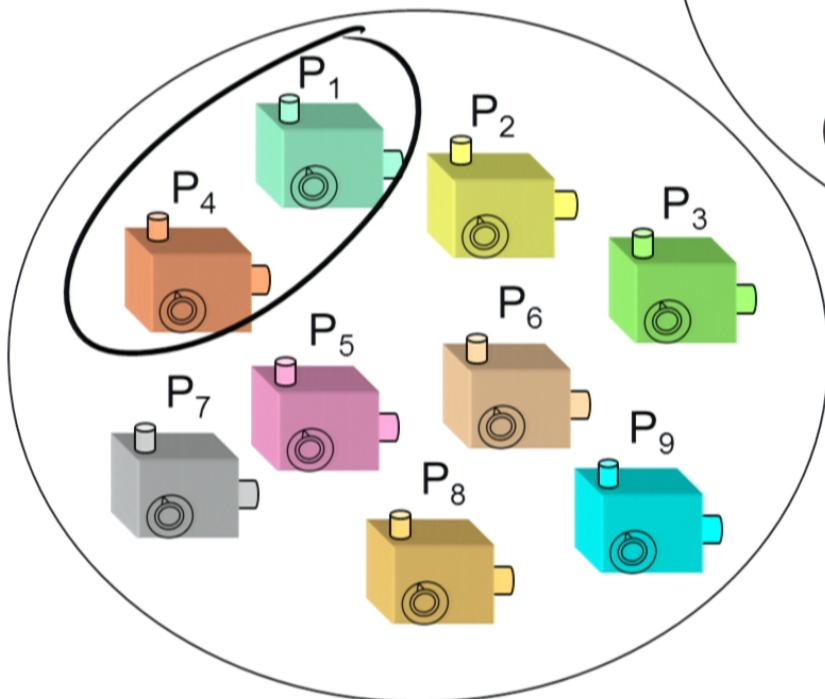


Equivalent
representations
in the ontological model

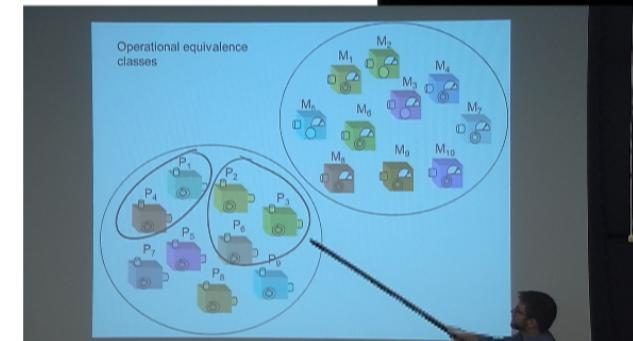
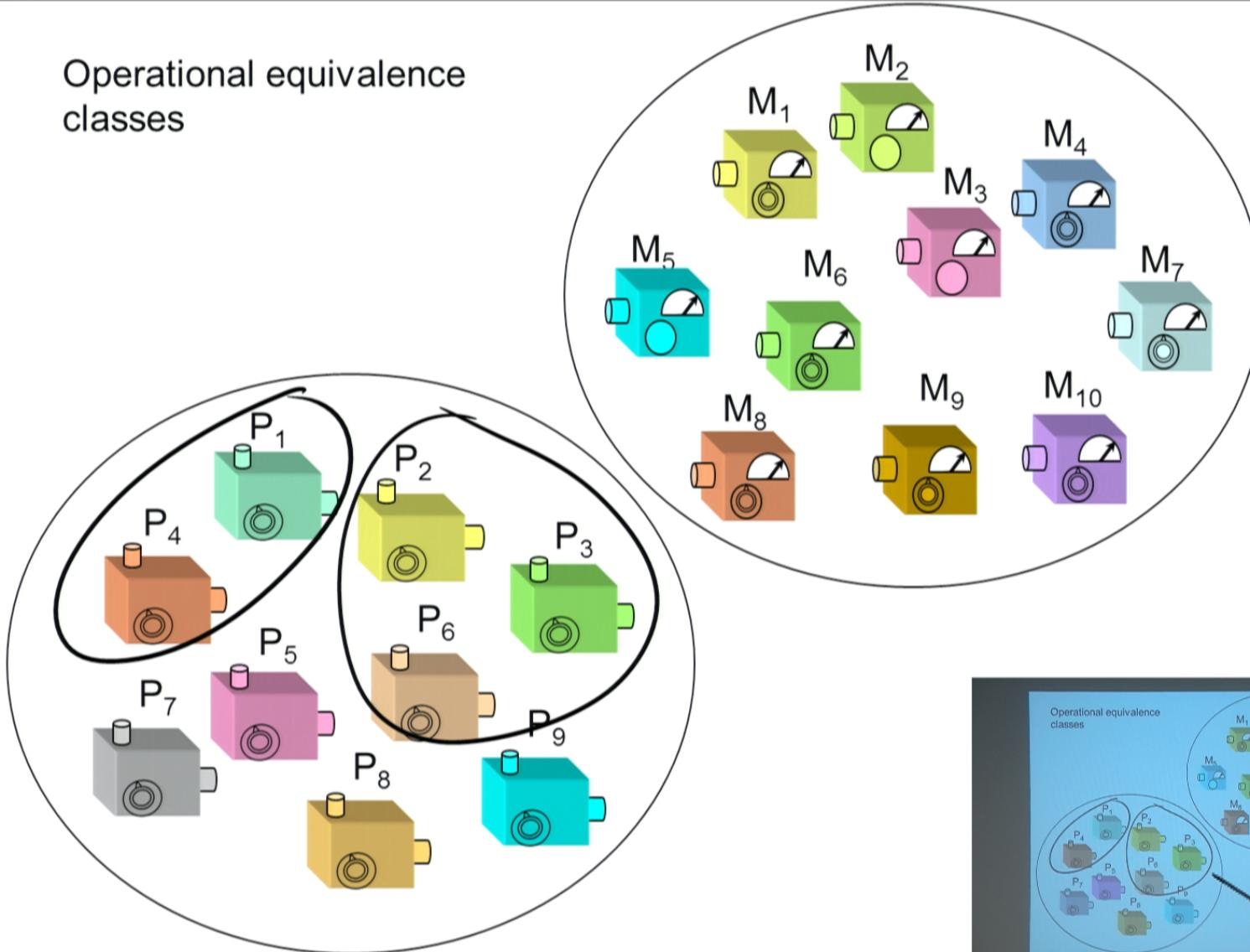
Operational equivalence classes



Operational equivalence classes



Operational equivalence classes

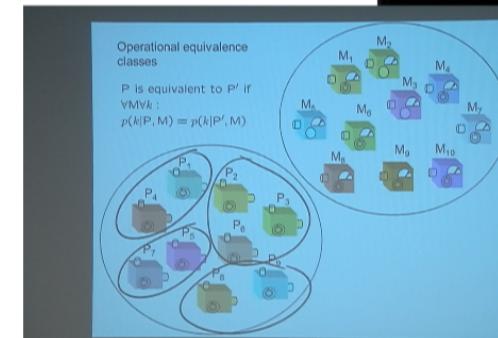
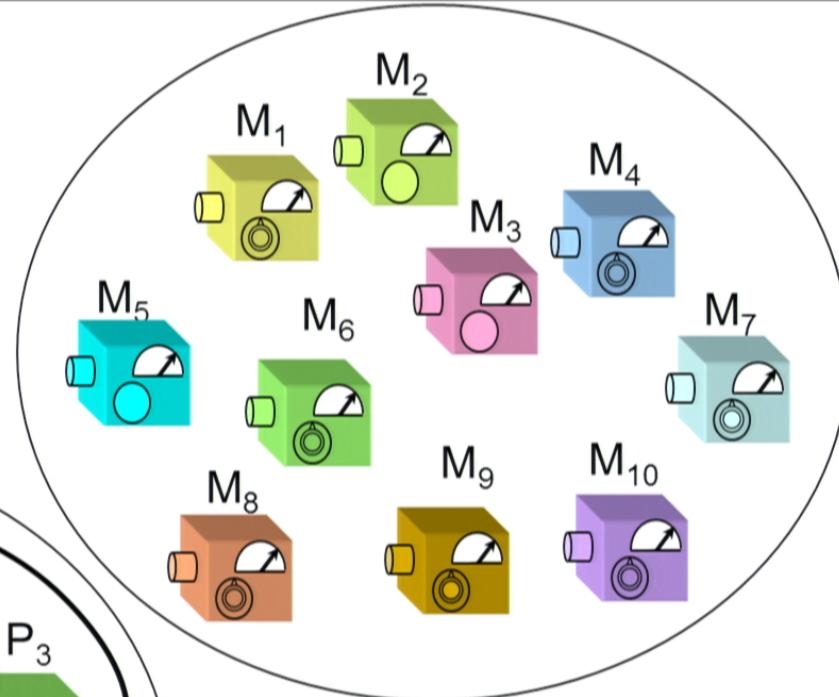
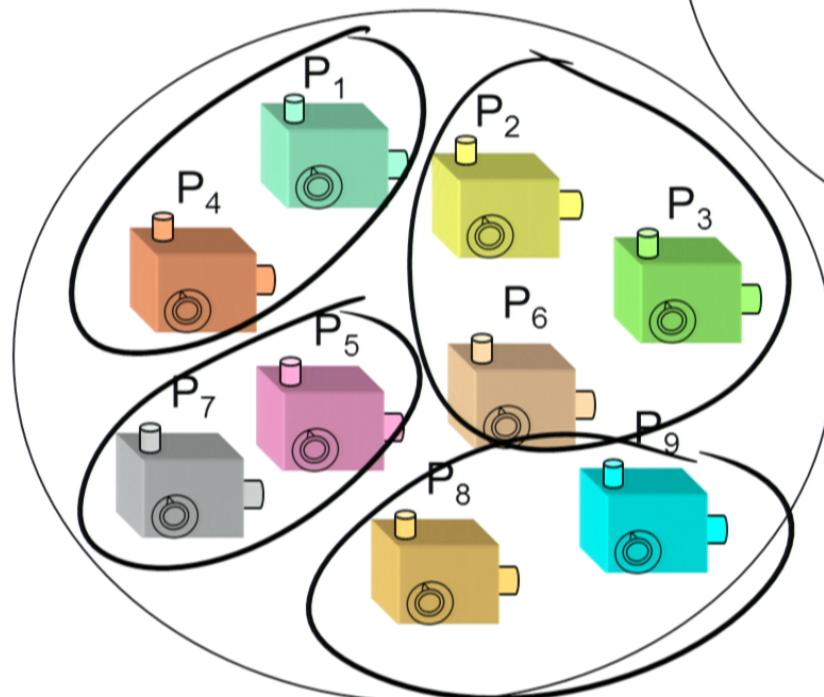


Operational equivalence classes

P is equivalent to P' if

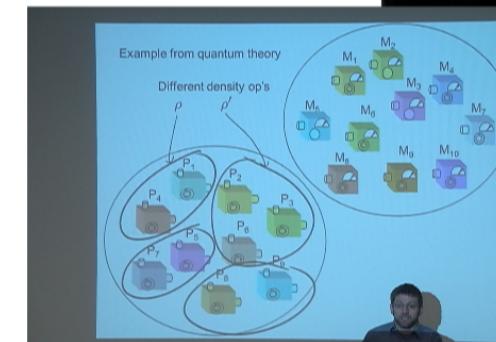
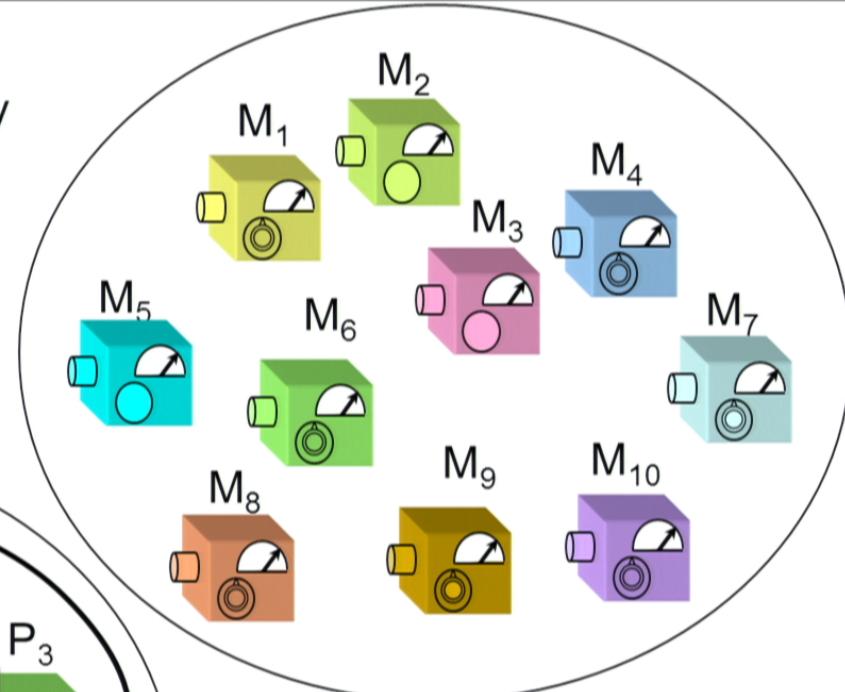
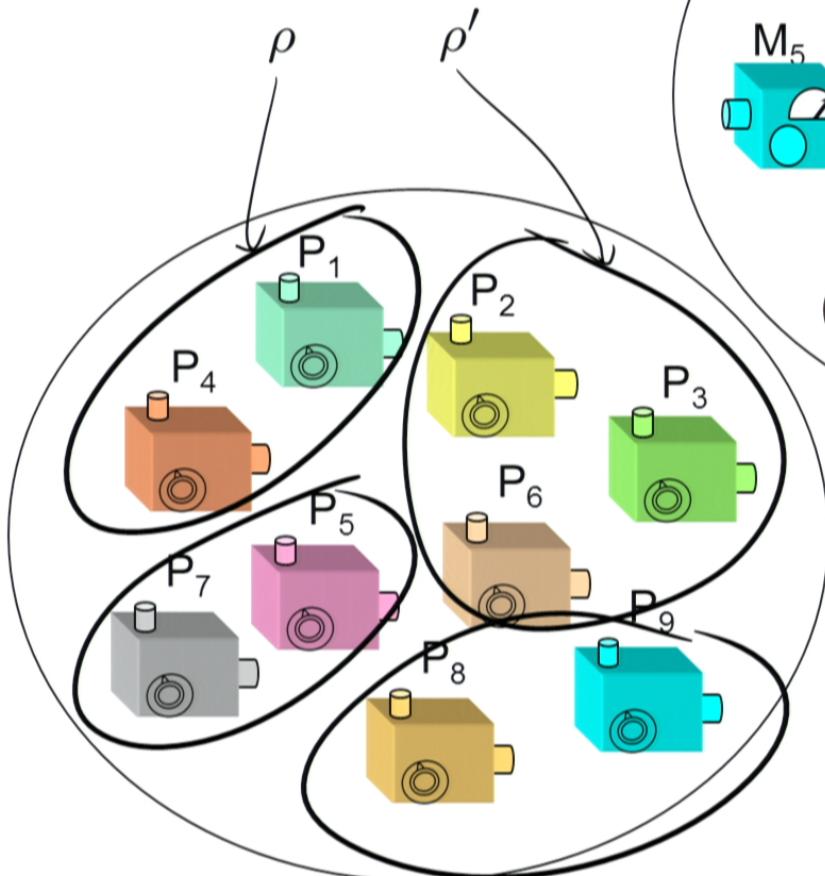
$\forall M \forall k :$

$$p(k|P, M) = p(k|P', M)$$



Example from quantum theory

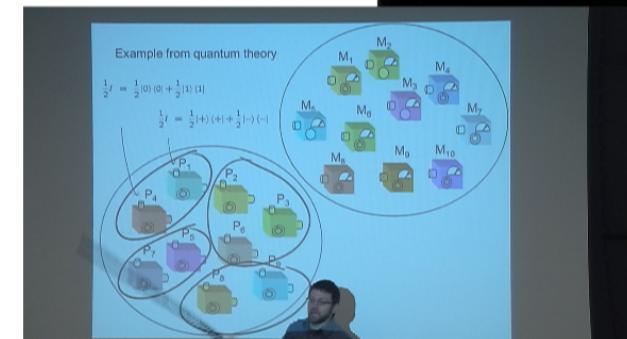
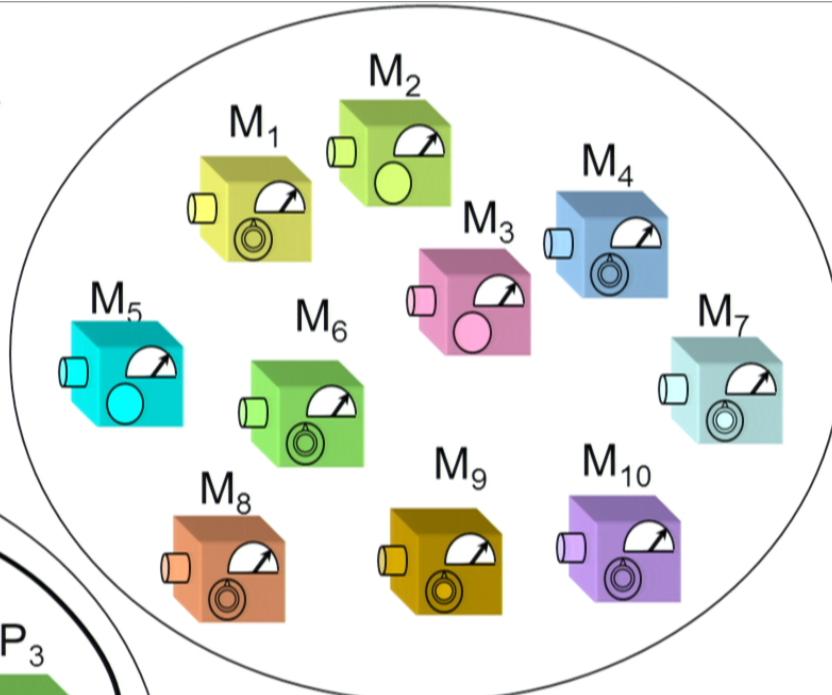
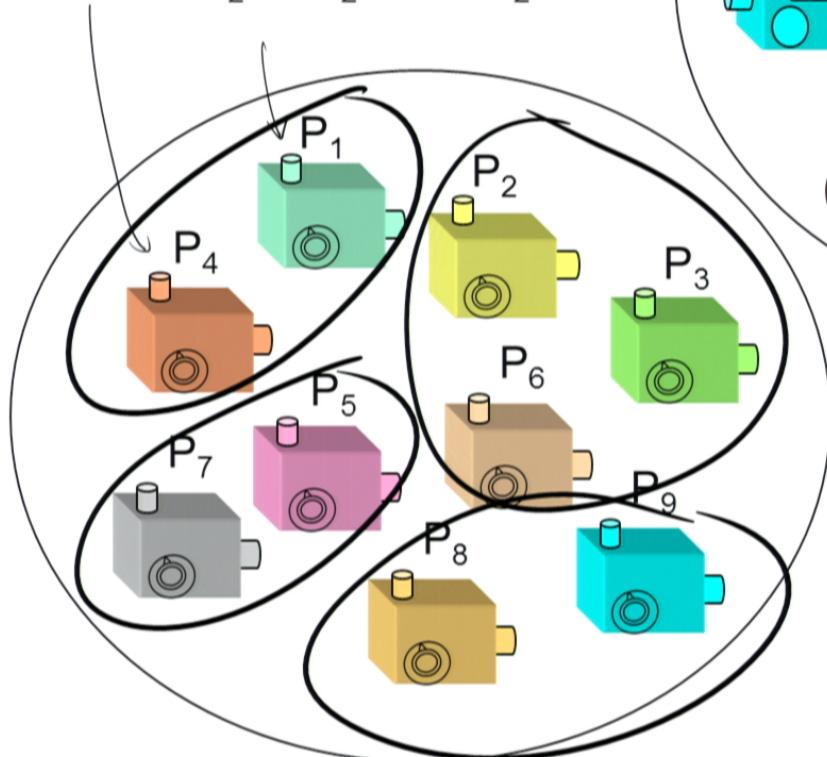
Different density op's



Example from quantum theory

$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

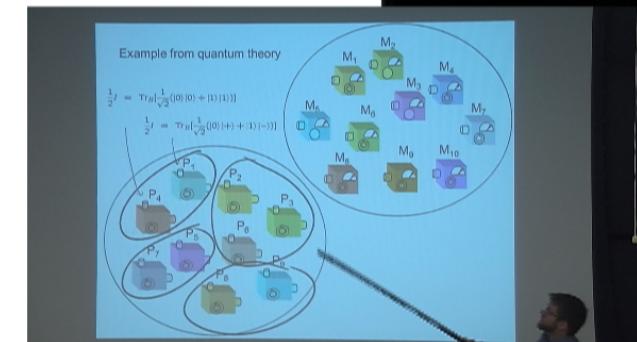
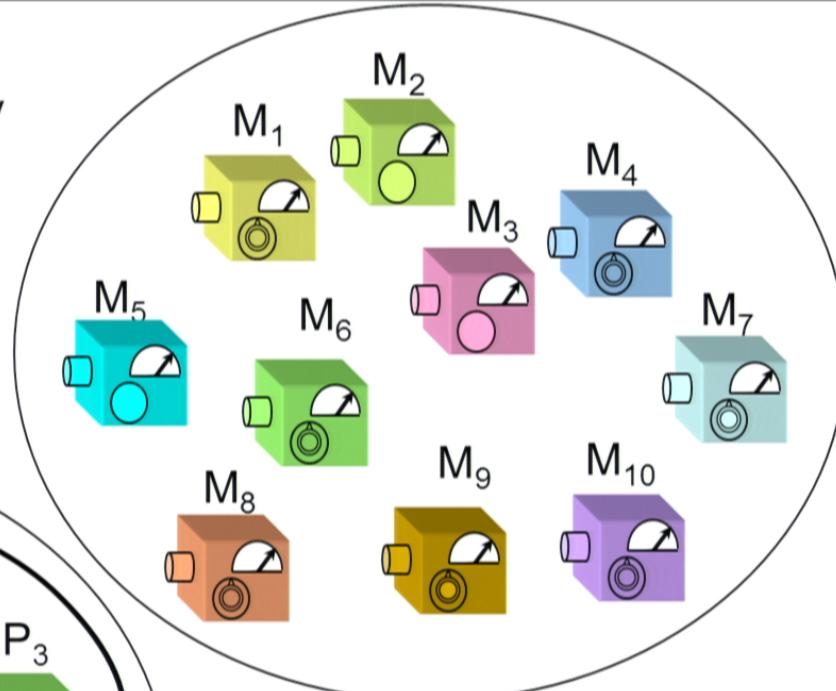
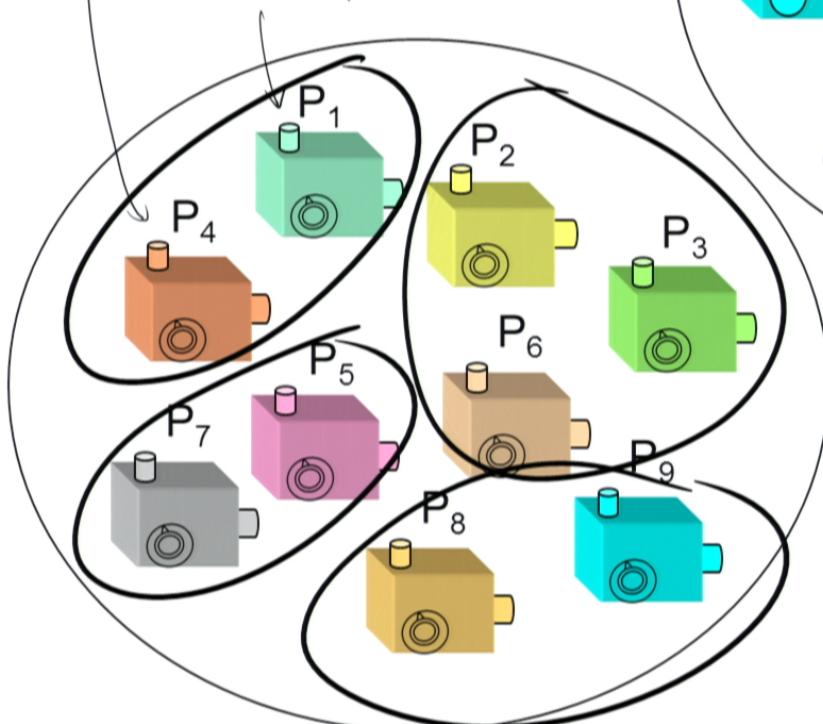
$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$



Example from quantum theory

$$\frac{1}{2}I = \text{Tr}_B\left[\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)\right]$$

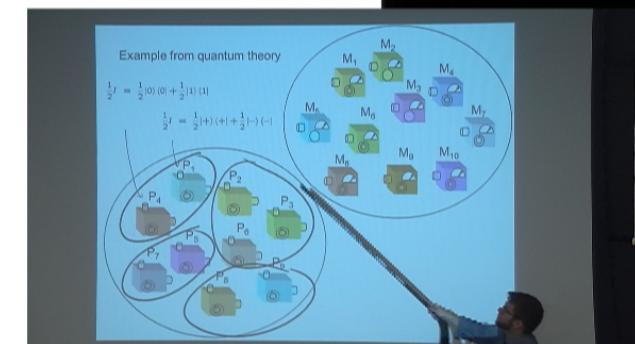
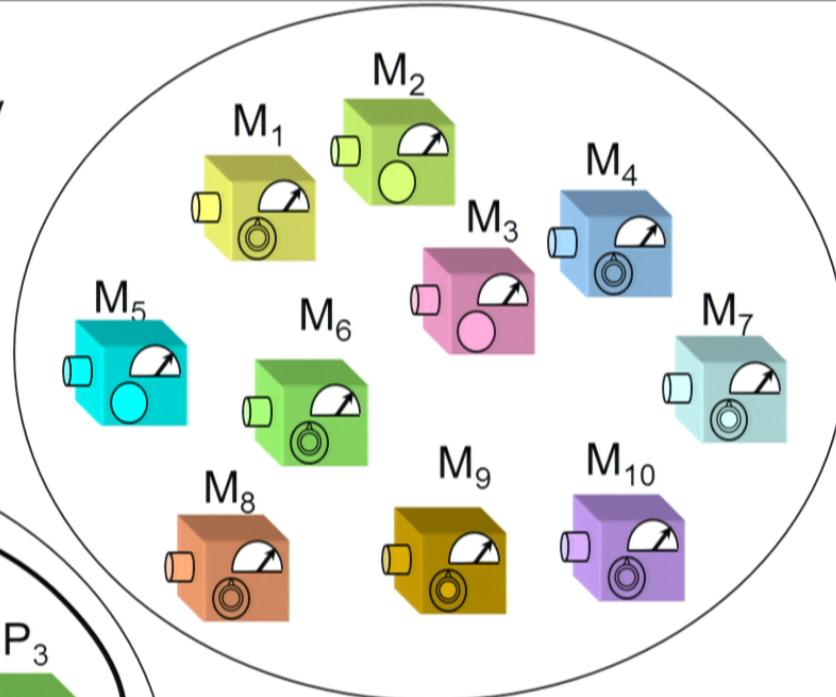
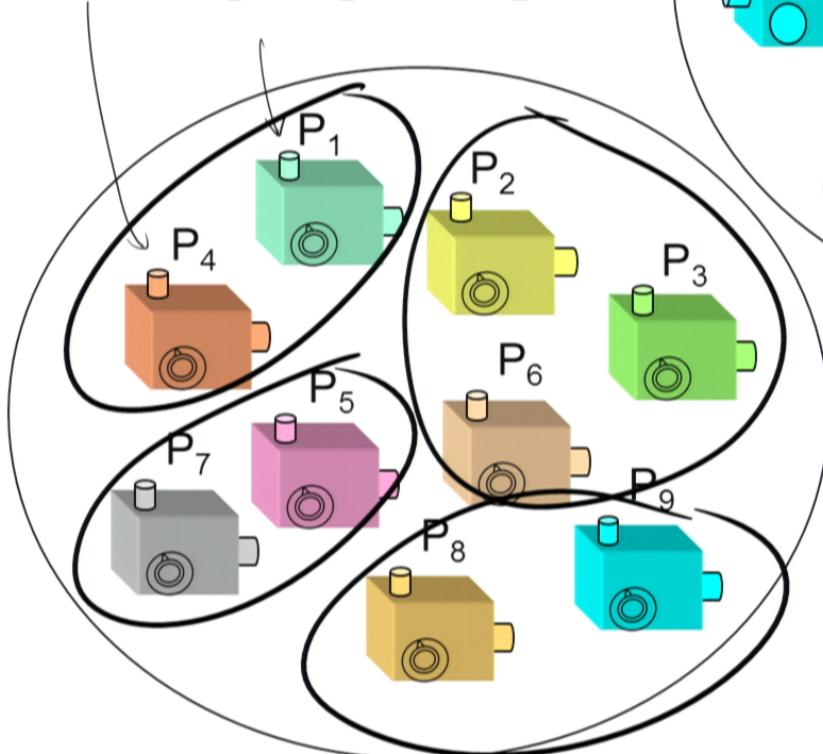
$$\frac{1}{2}I = \text{Tr}_B\left[\frac{1}{\sqrt{2}}(|0\rangle|+\rangle + |1\rangle|-\rangle)\right]$$



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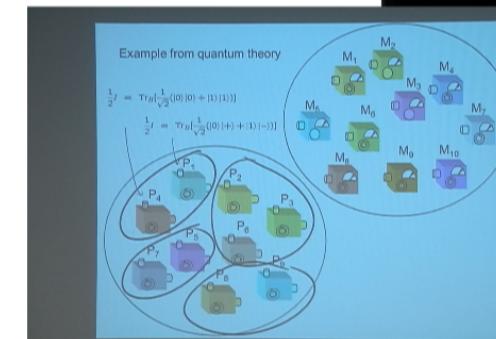
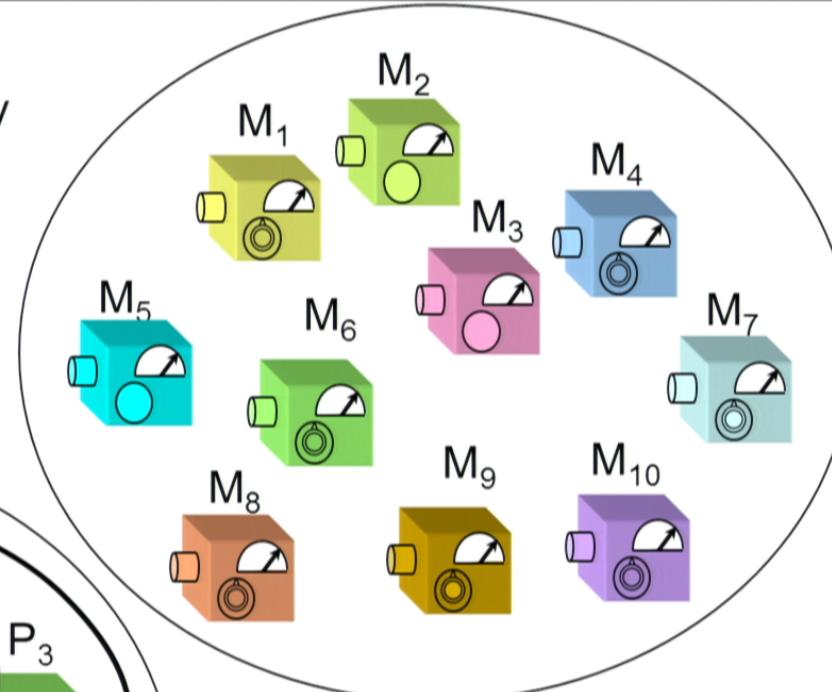
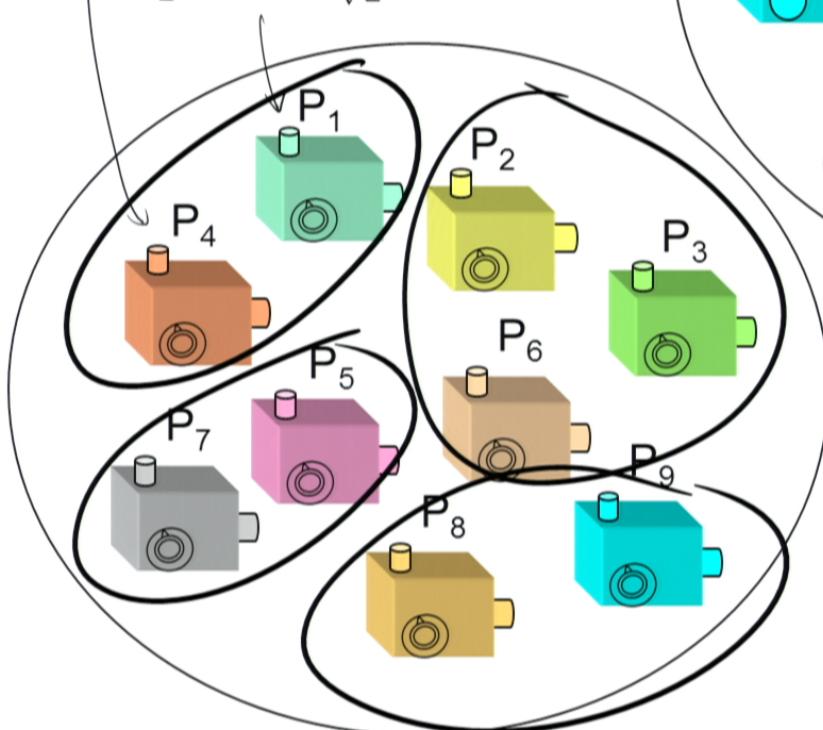
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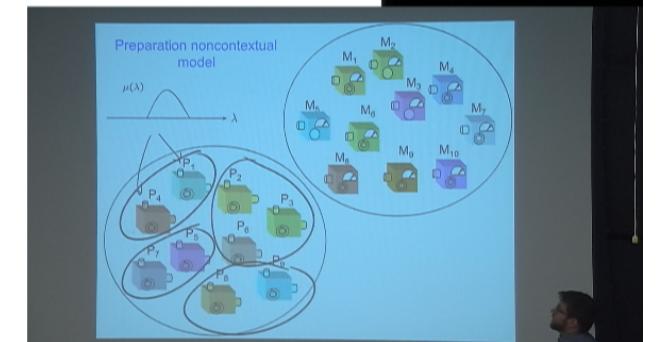
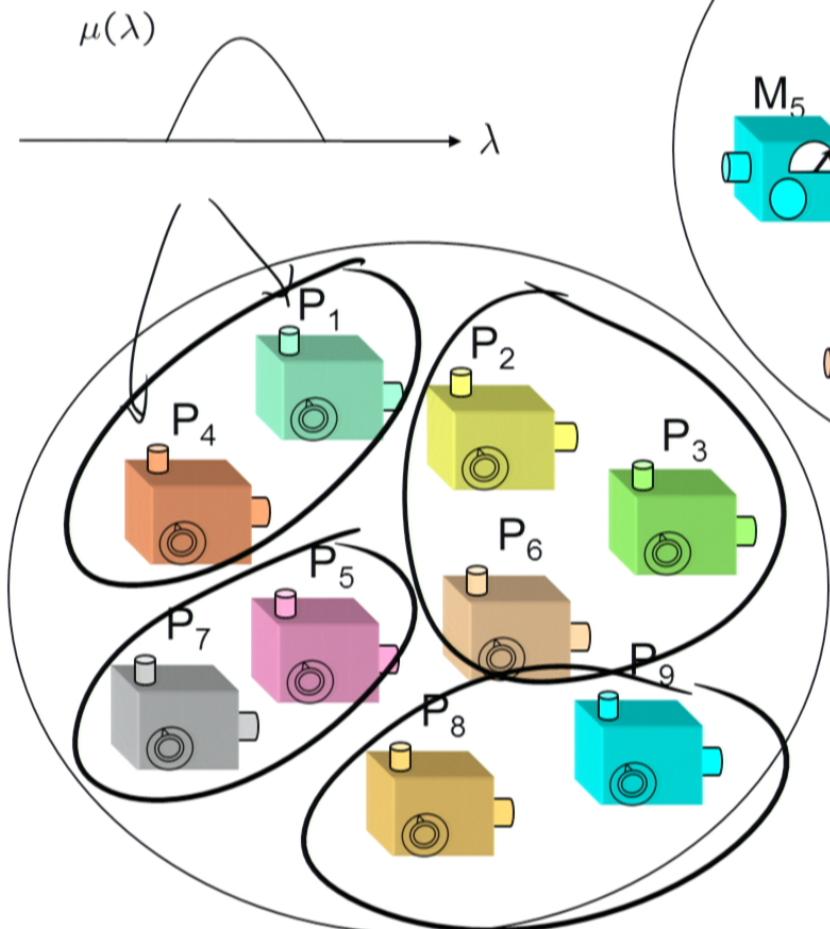
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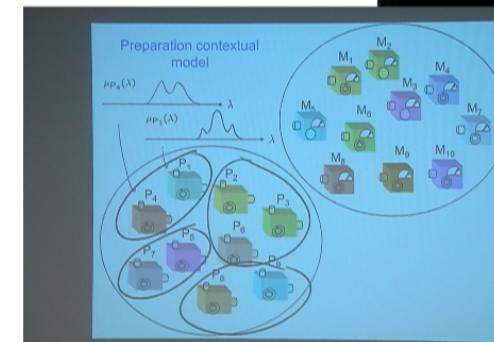
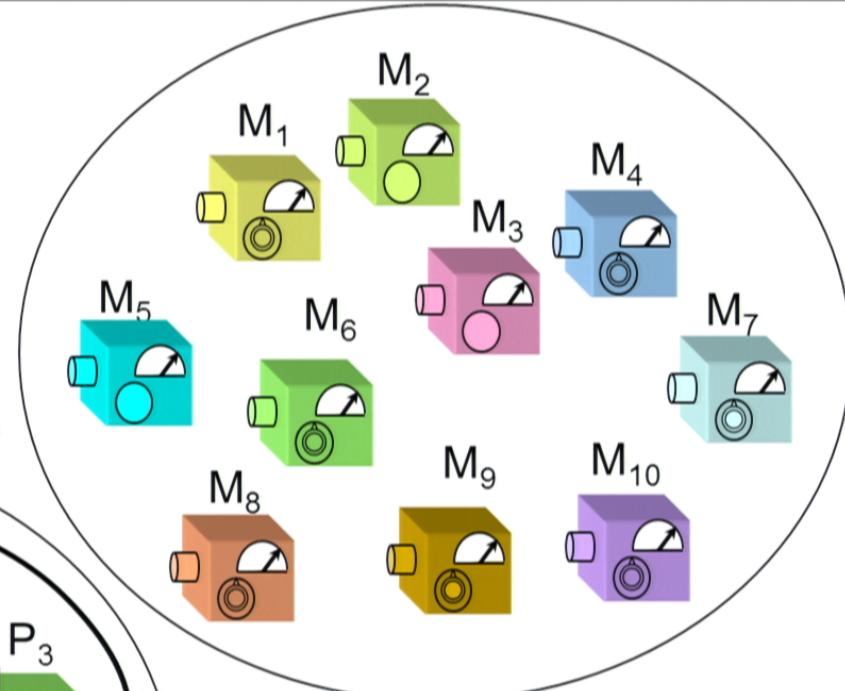
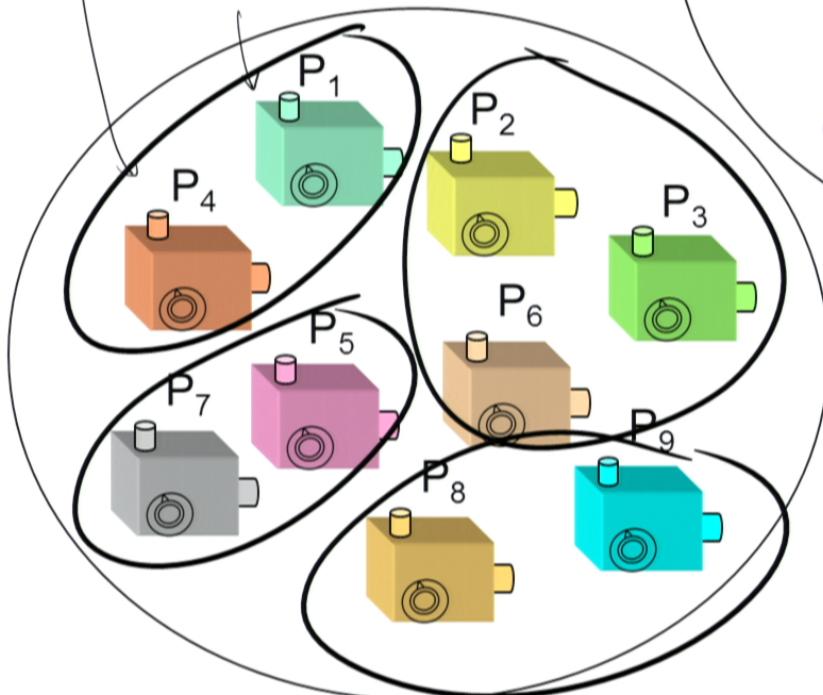
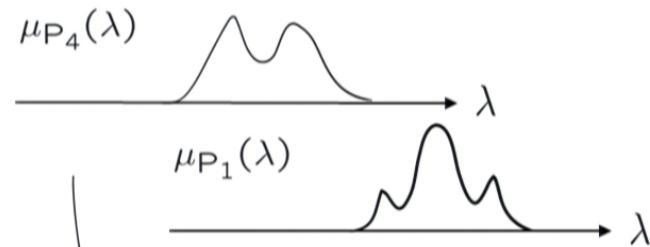
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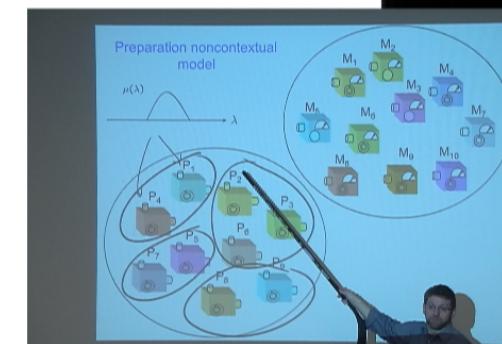
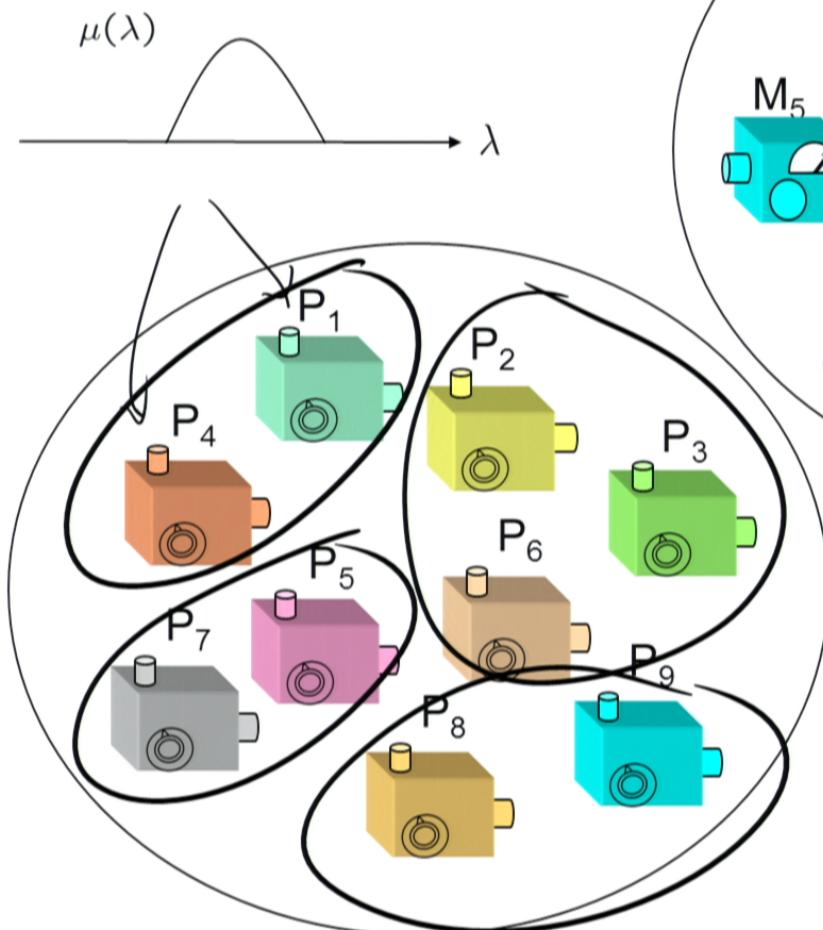
Preparation noncontextual model



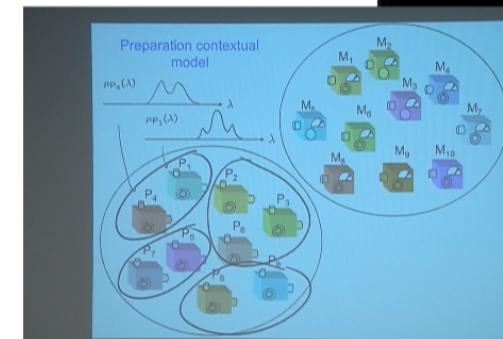
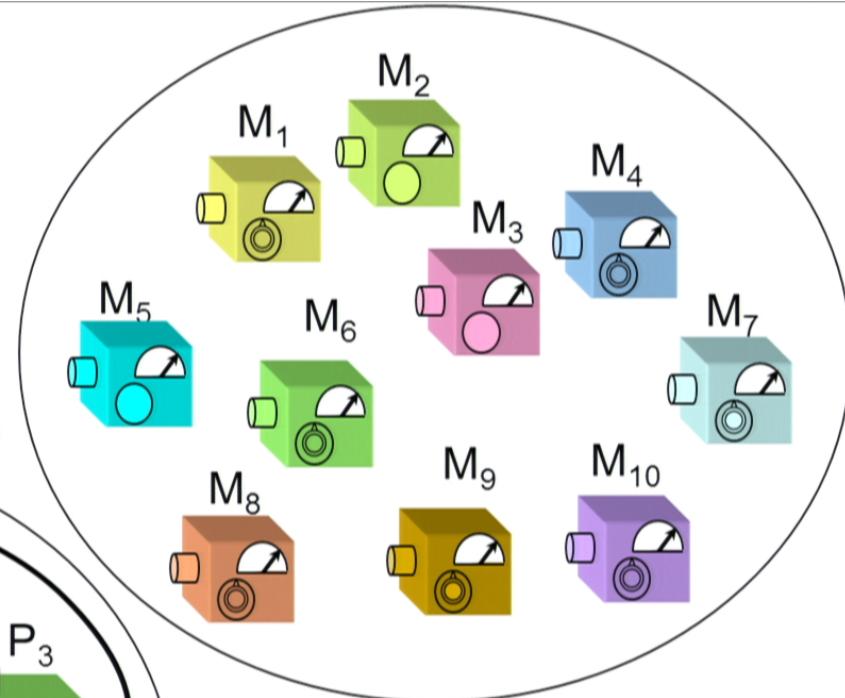
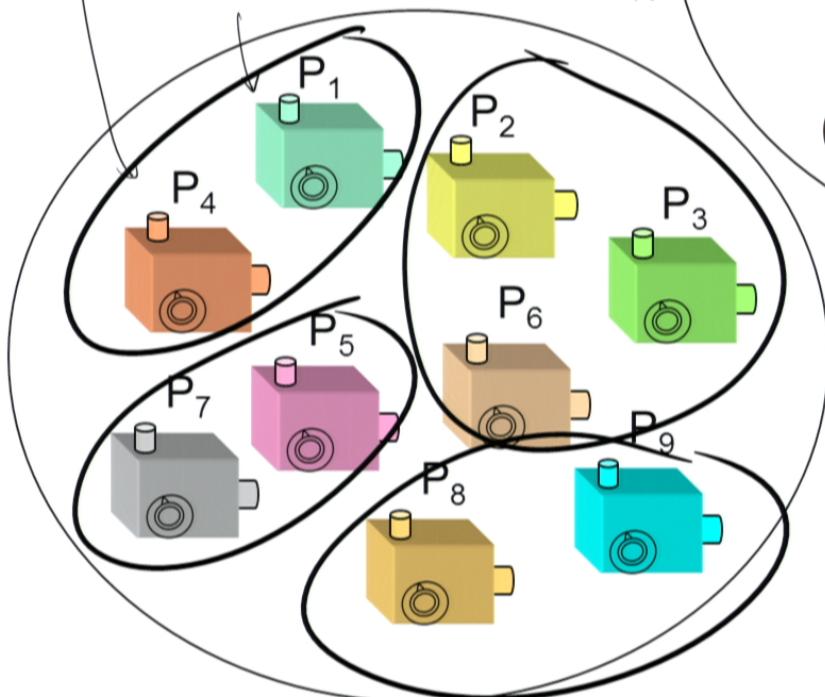
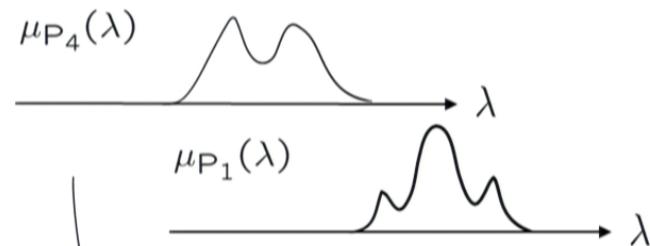
Preparation contextual model



Preparation noncontextual model

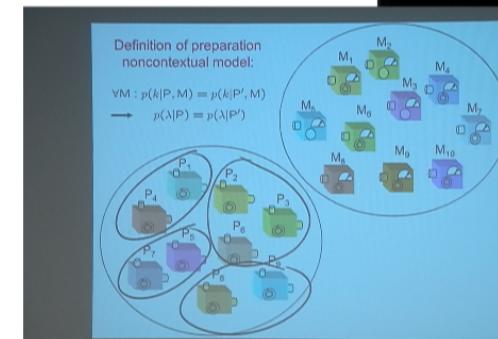
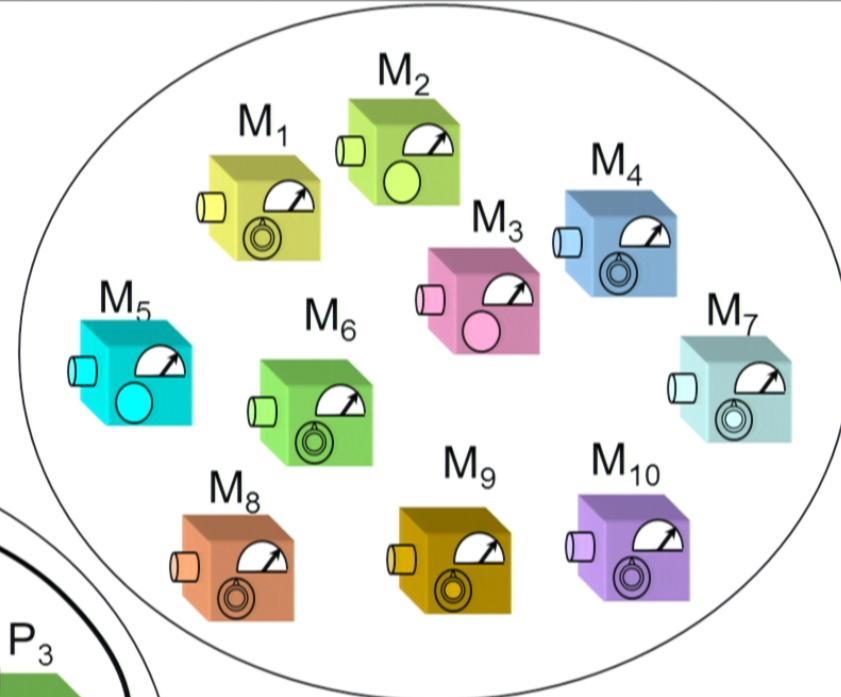
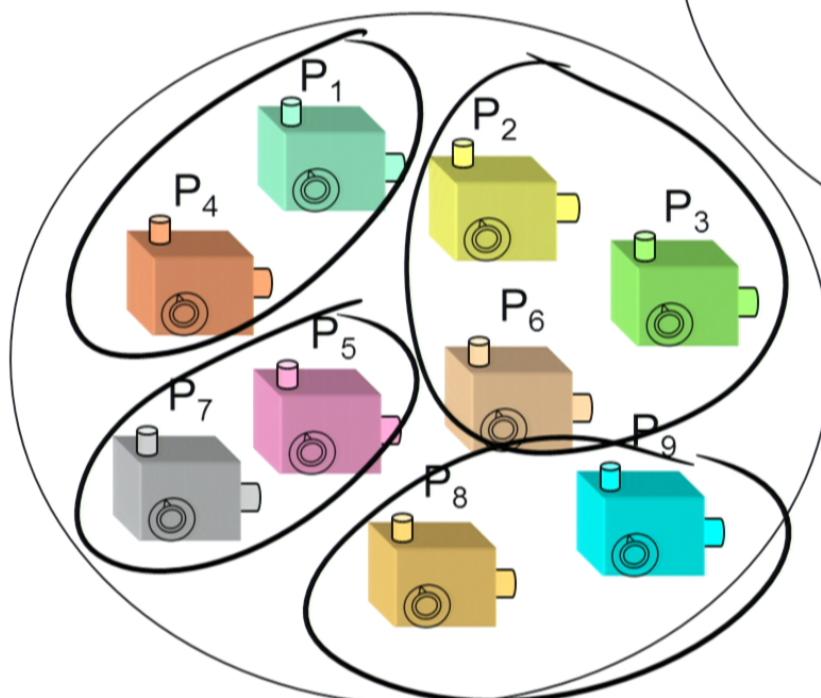


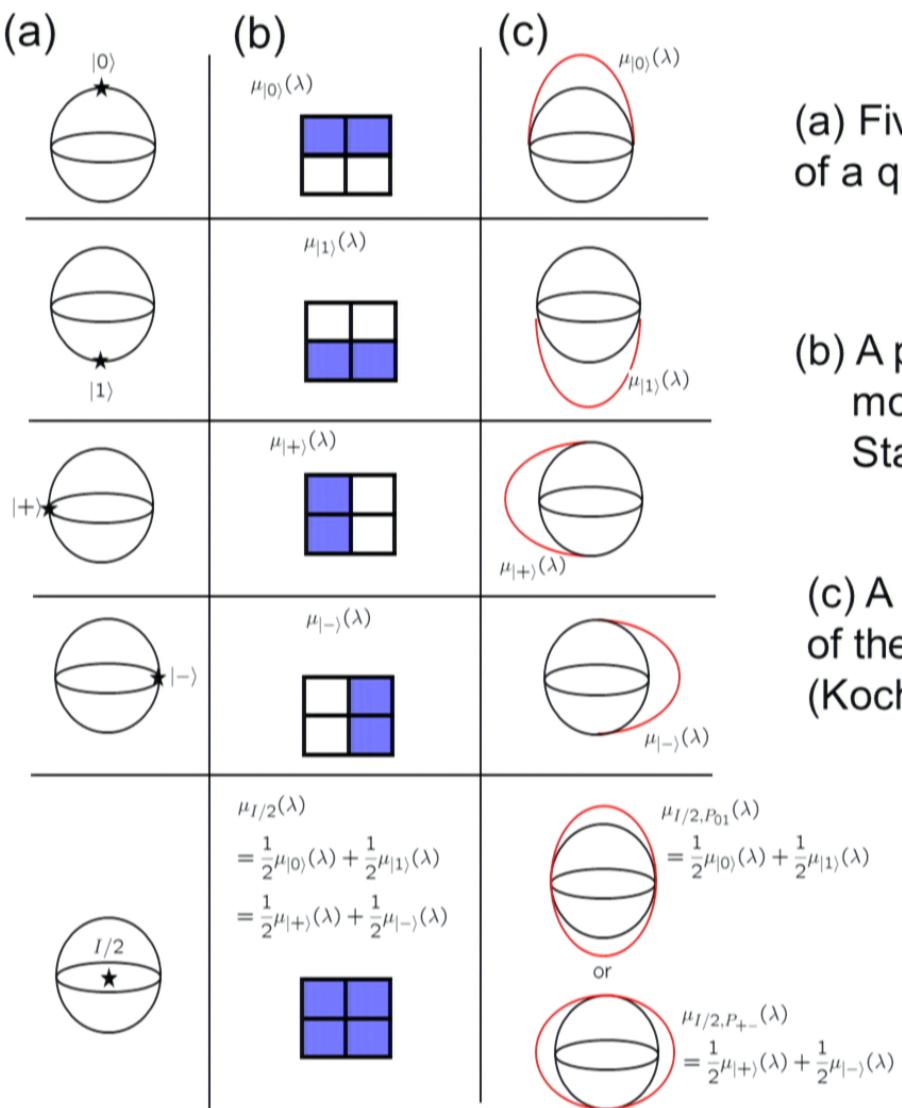
Preparation contextual model



Definition of preparation noncontextual model:

$$\forall M : p(k|P, M) = p(k|P', M)$$
$$\rightarrow p(\lambda|P) = p(\lambda|P')$$



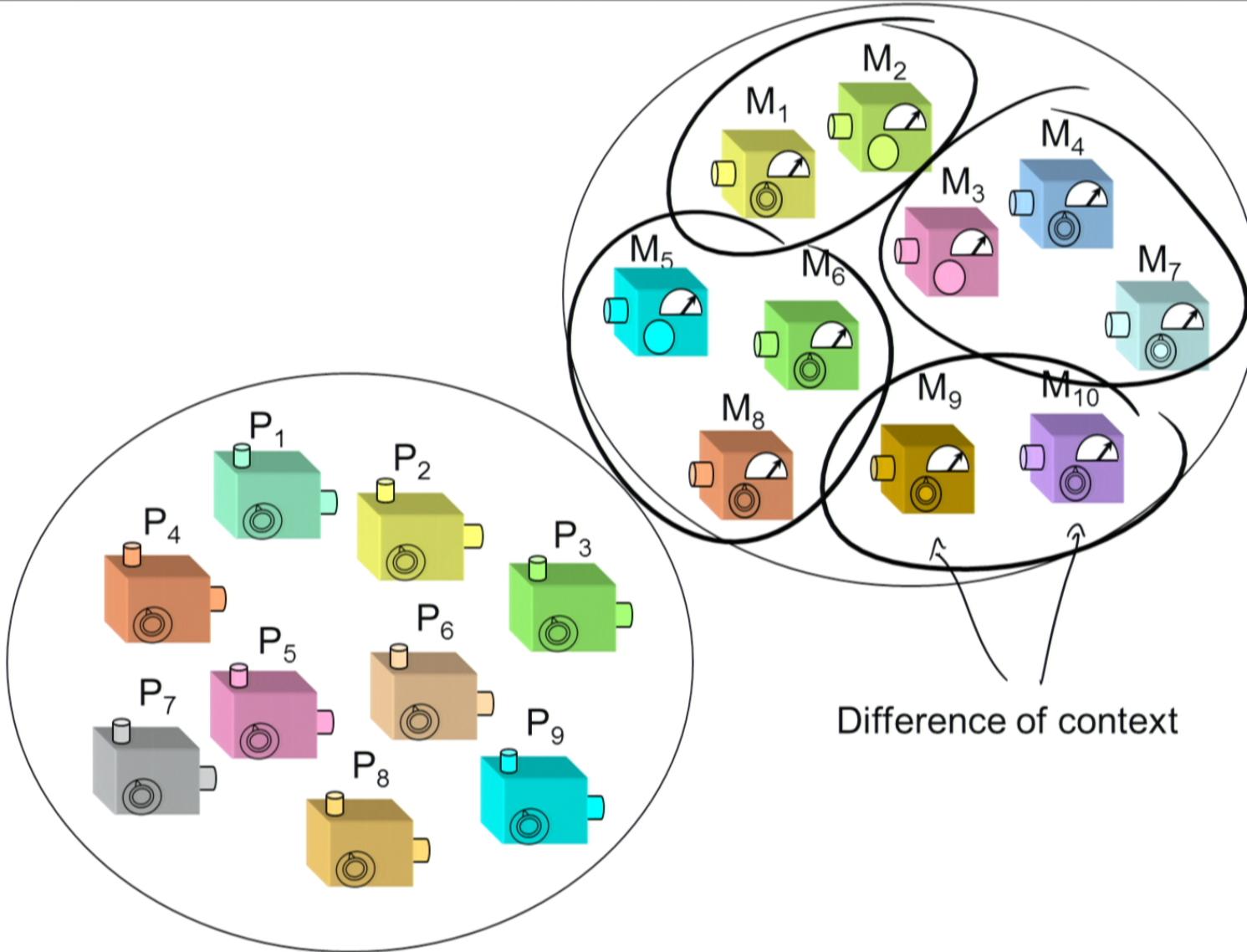


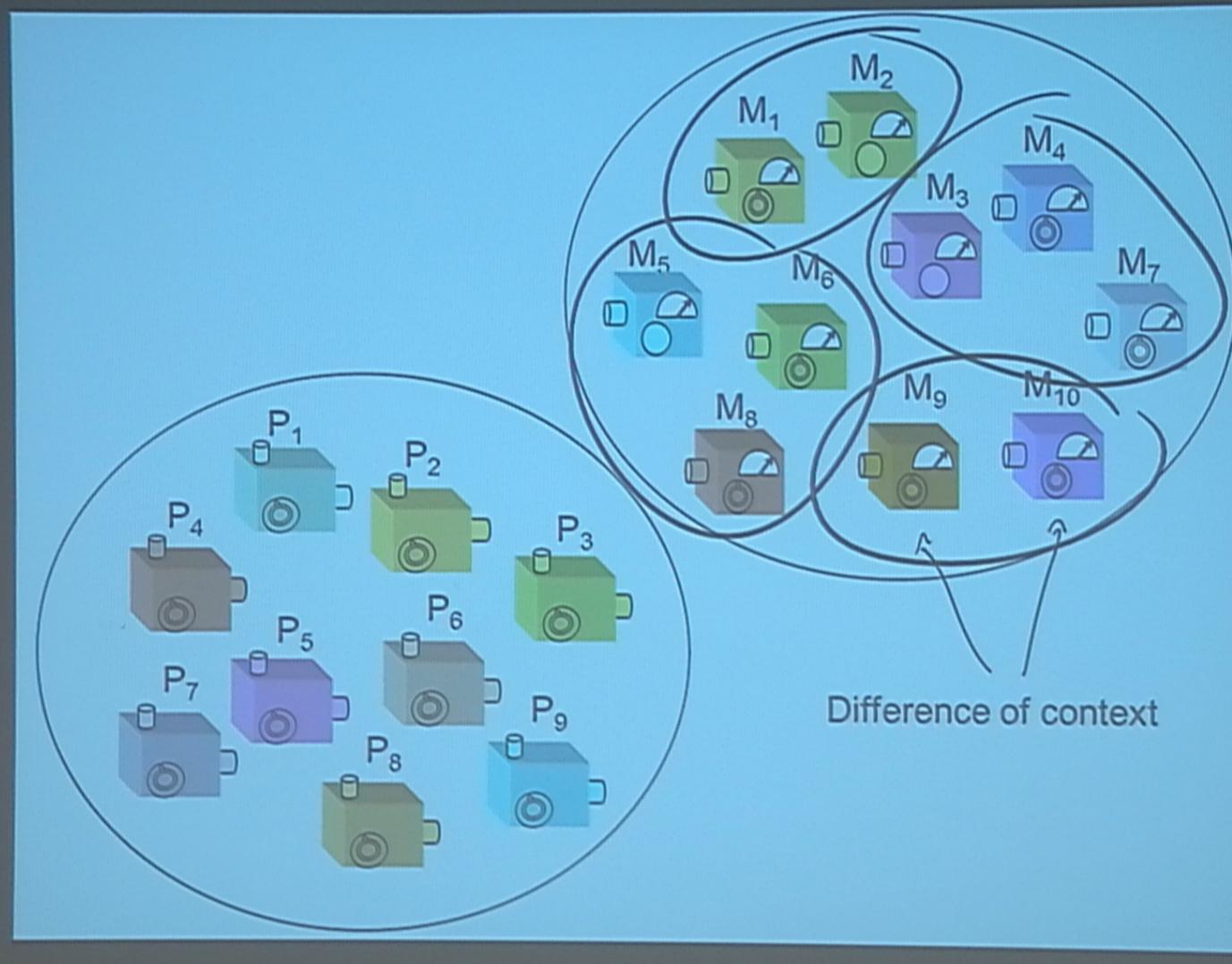
(a) Five operational states of a qubit

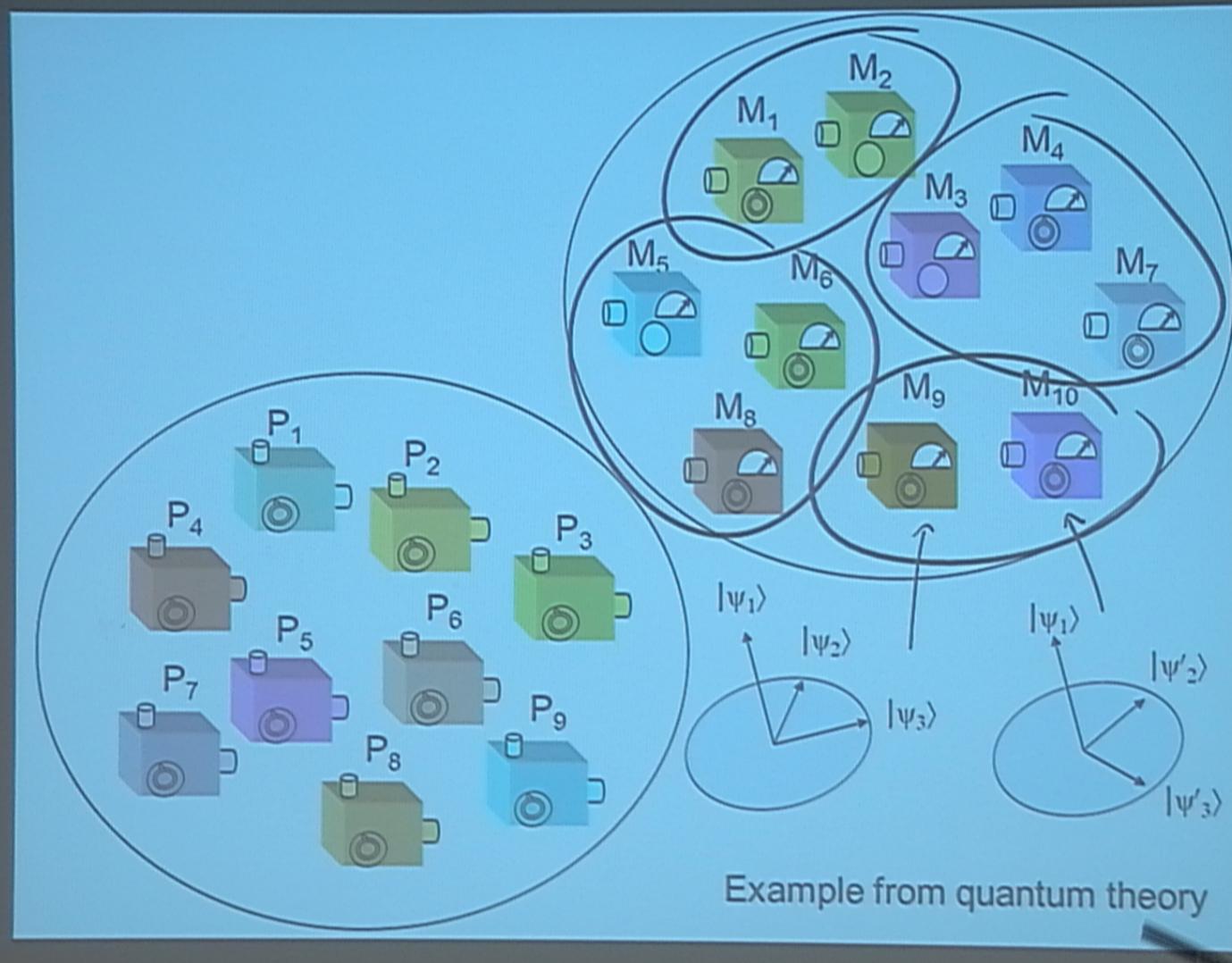
(b) A preparation **noncontextual** model of these (Restricted Statistical theory of bits)

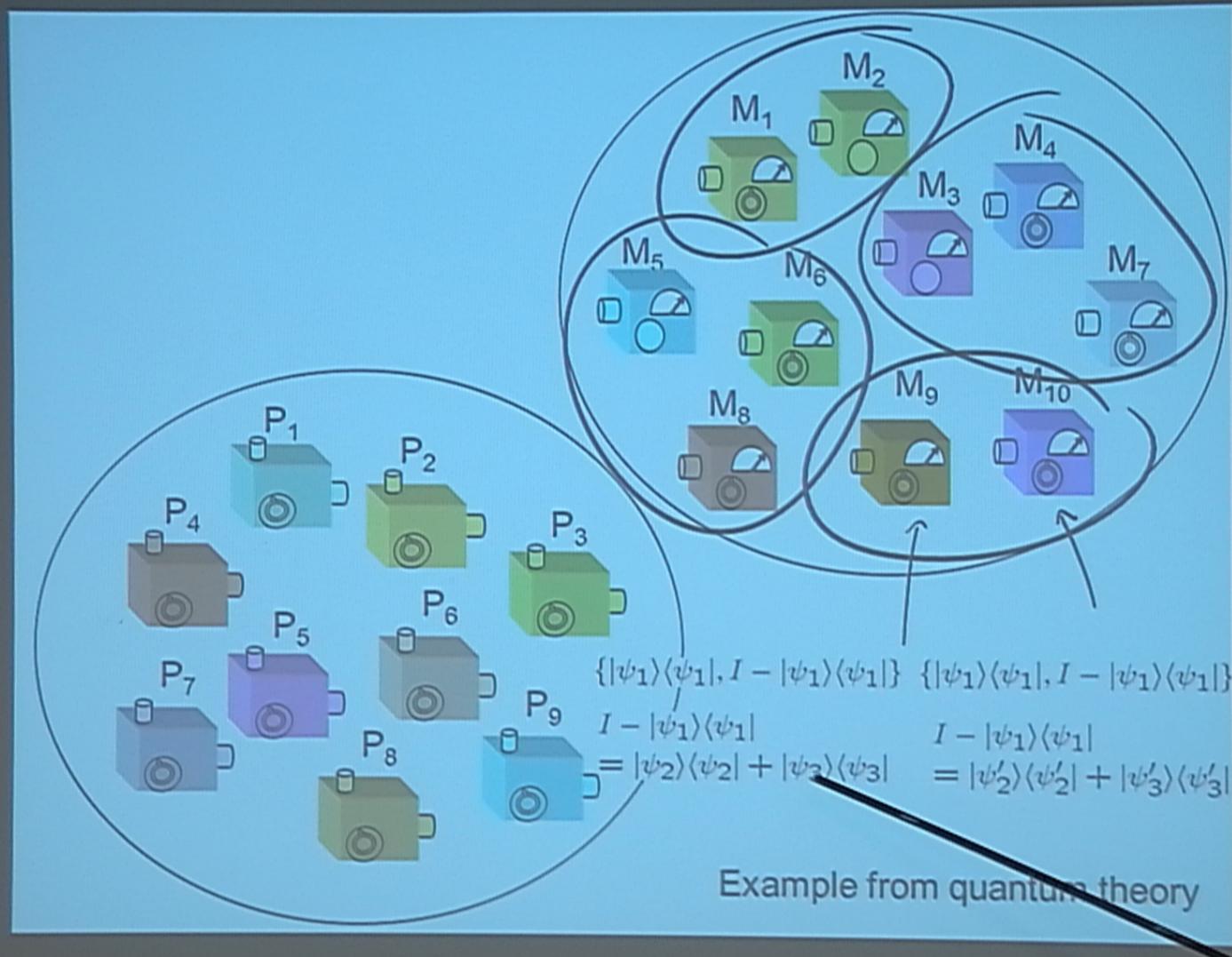
(c) A preparation **contextual** model of these
(Kochen-Specker, 1967)

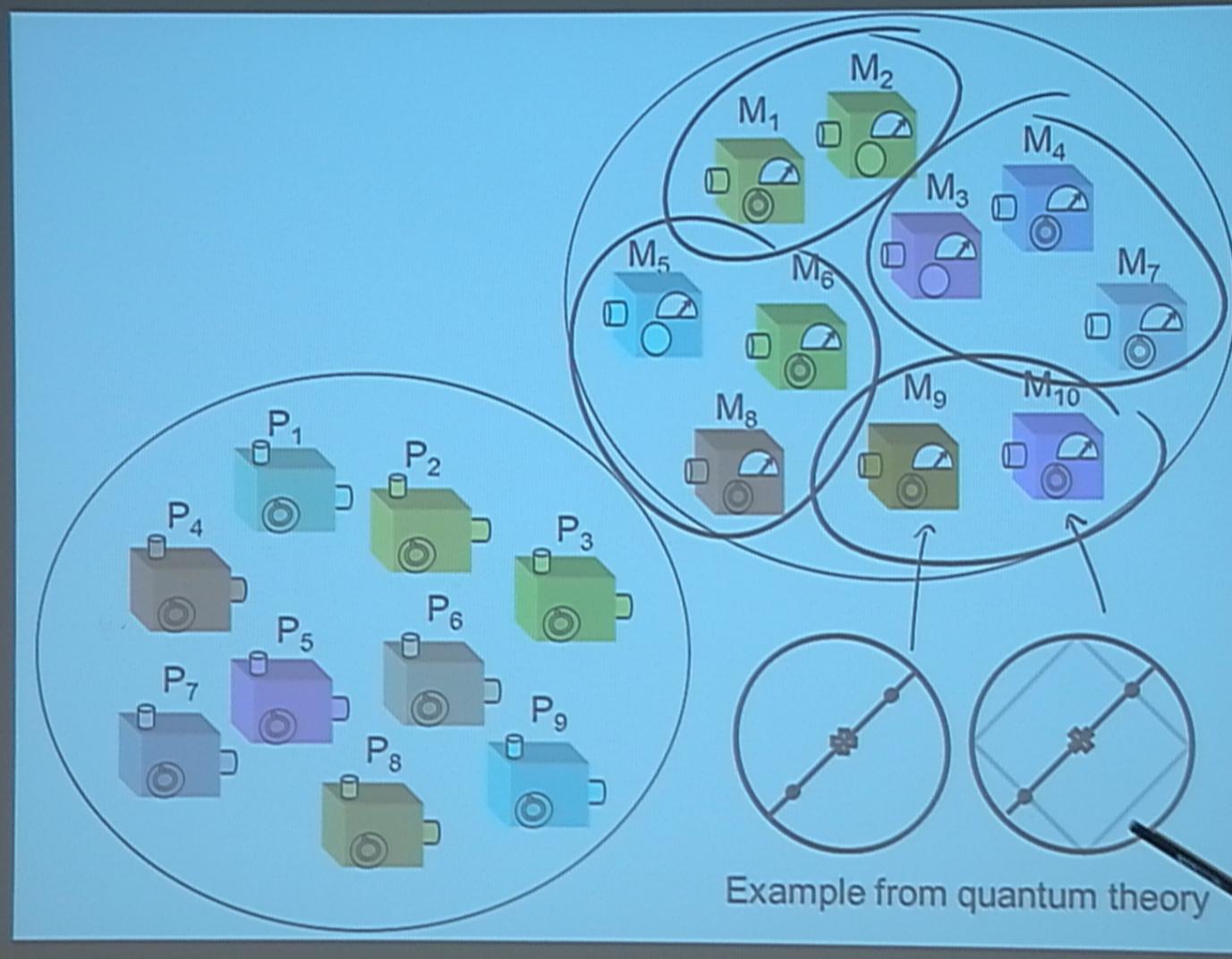


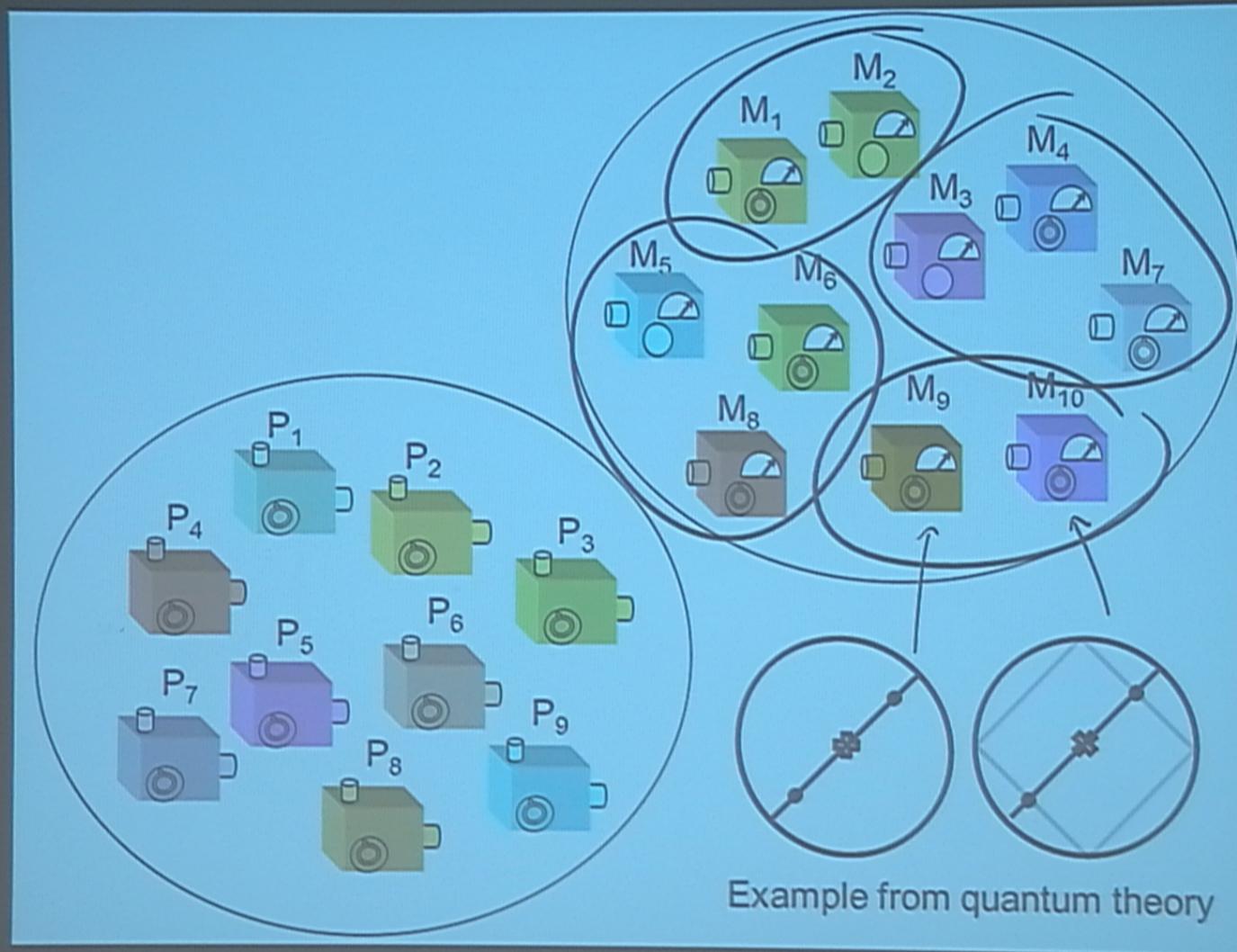


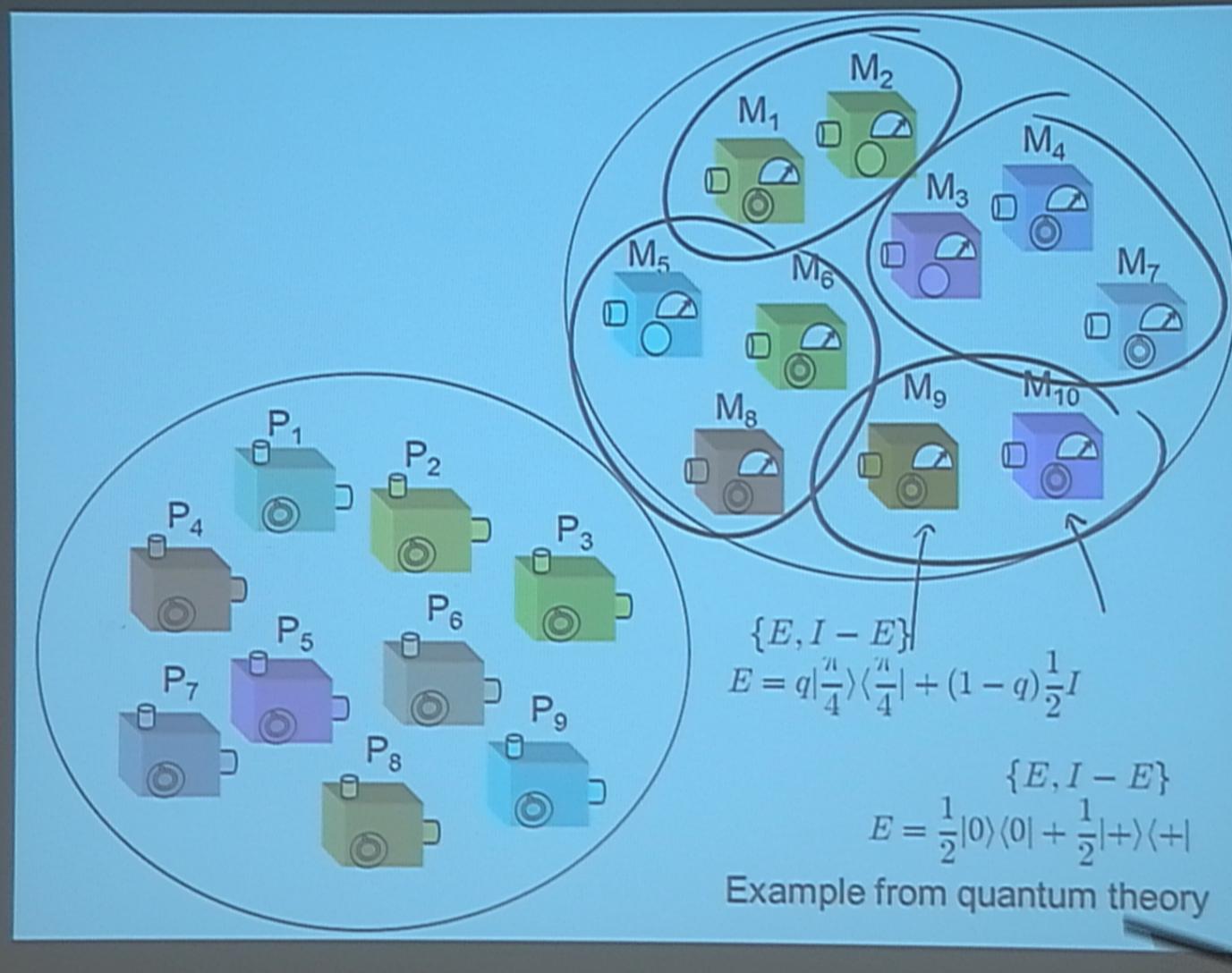


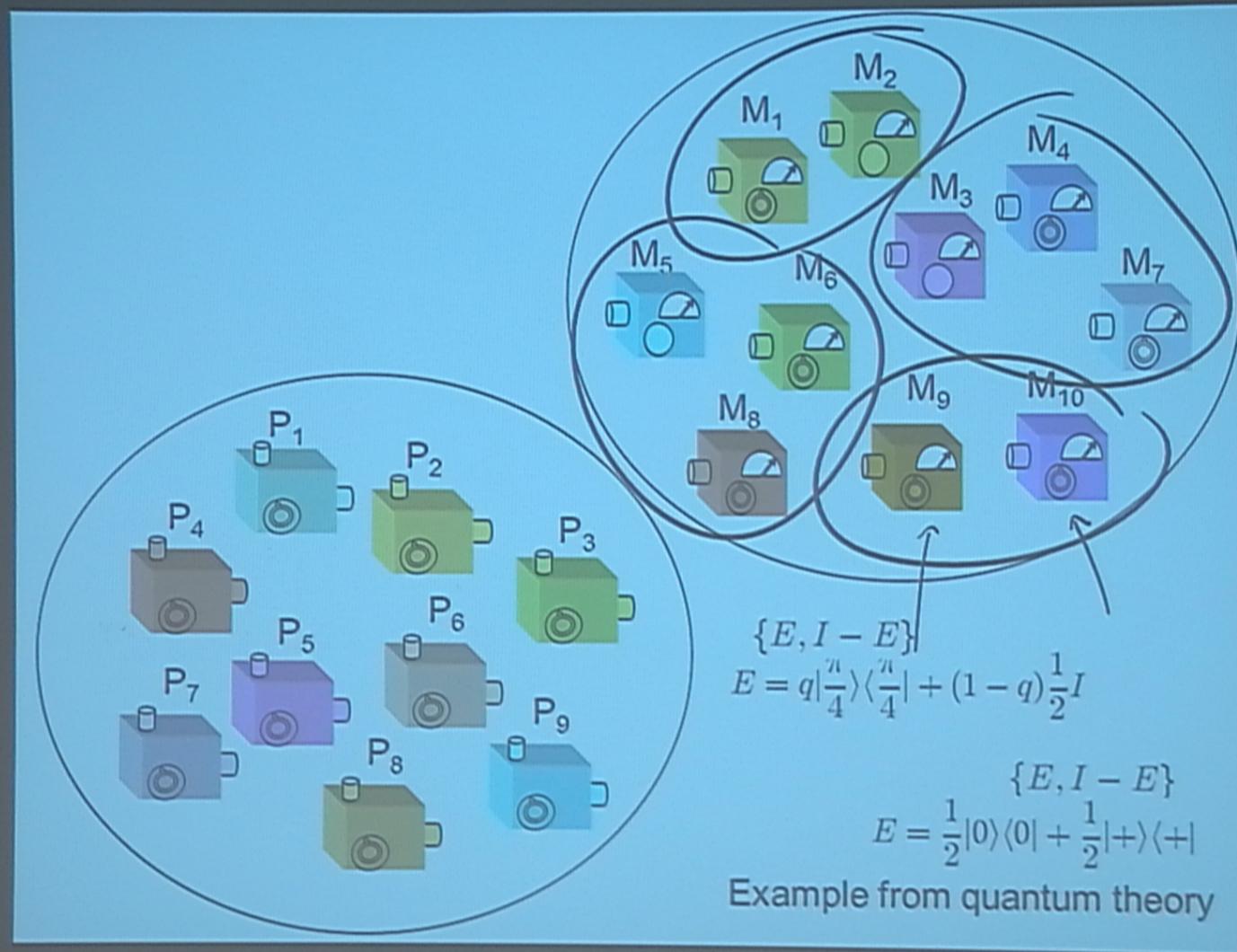


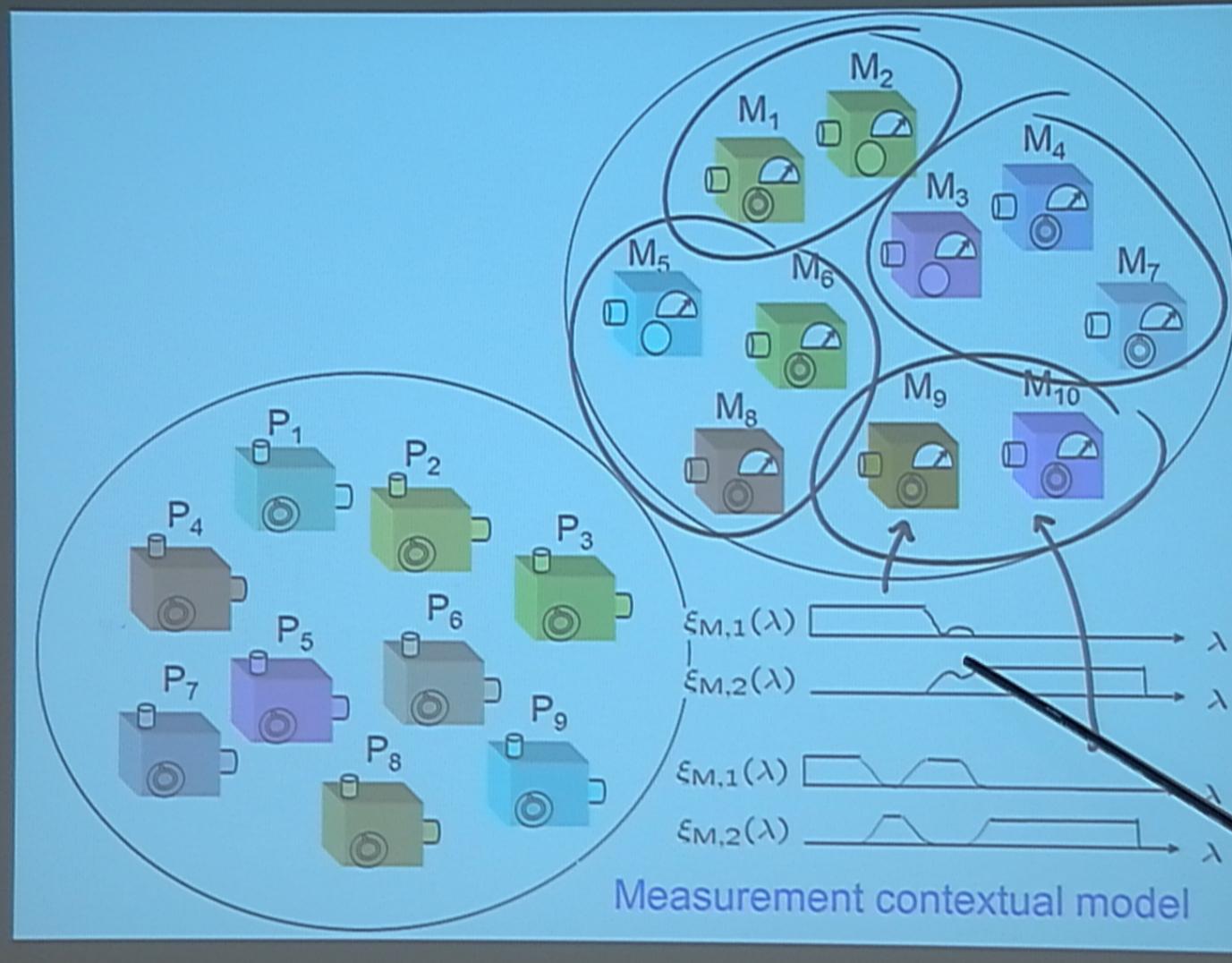


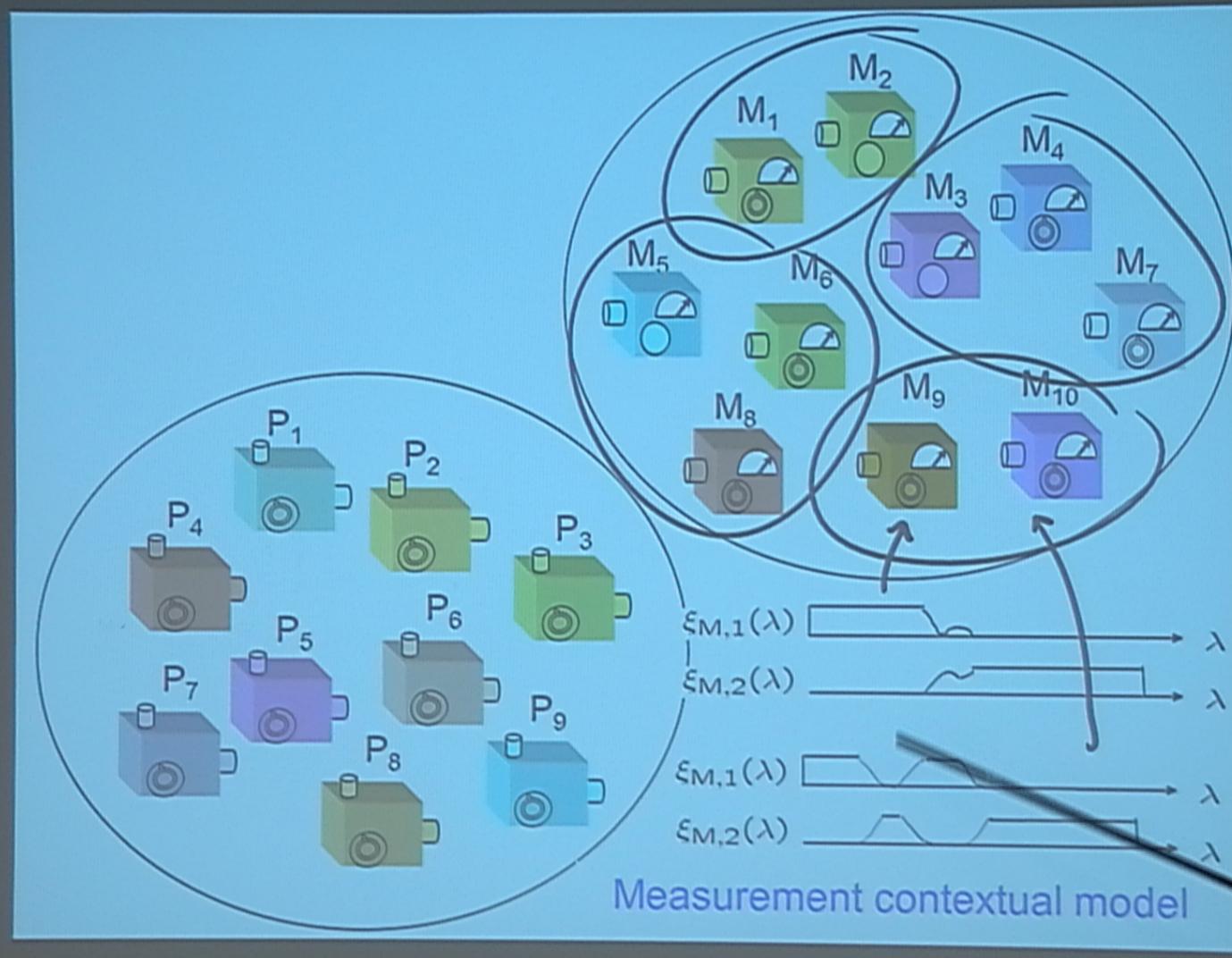












Generalized
noncontextuality

=

measurement
noncontextuality
and
preparation
noncontextuality

Claim: Preparation noncontextuality is as natural (or unnatural) as measurement noncontextuality



Generalized noncontextuality = measurement noncontextuality
and preparation noncontextuality

Claim: Preparation noncontextuality is as natural (or unnatural) as measurement noncontextuality

Q: Why is noncontextuality plausible at all?

A: The methodological equivalence principle: if a difference in set-up is not distinguished in the observable phenomena then it should not be distinguished in the ontological picture either

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