Title: 12/13 PSI - Found Quantum Mechanics Lecture 9

Date: Jan 17, 2013 11:30 AM

URL: http://www.pirsa.org/13010076

Abstract:


Nonlocality in more depth

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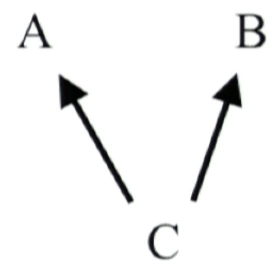
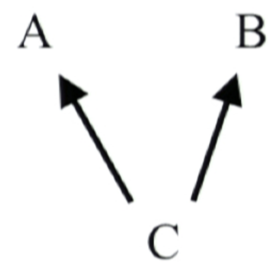
When correlations arise from common causes



When correlations arise from common causes



A and B are not independent

$$p(A, B) \neq p(A)p(B)$$

When correlations arise from common causes

$$
\begin{tikzcd}
A & & B \\
& C \arrow[ul] \arrow[ur] &
\end{tikzcd}
$$

A and B are not independent

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But A and B are conditionally independent given C

$$(i)\ \ p(A|B,C) = p(A|C)$$
$$(ii)\ \ p(B|A,C) = p(B|C)$$
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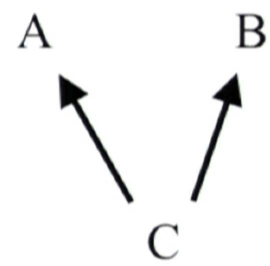

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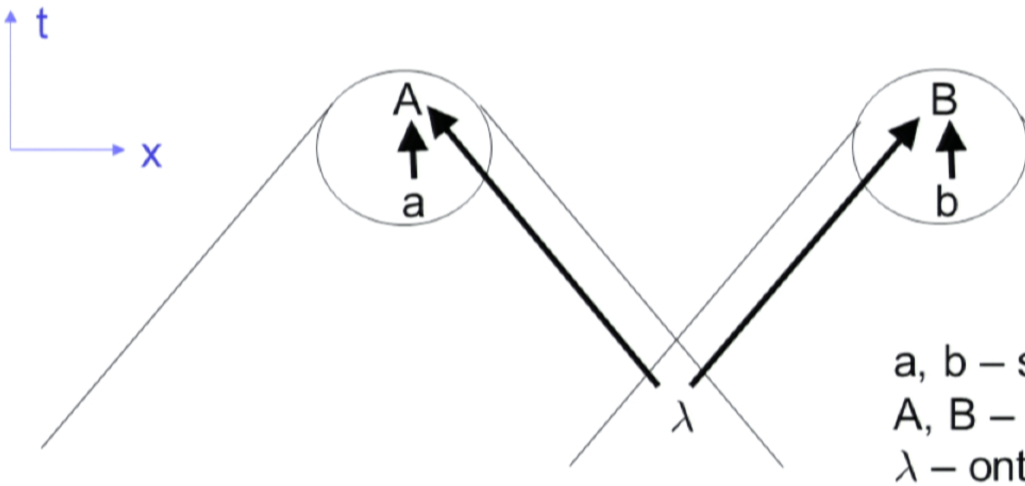
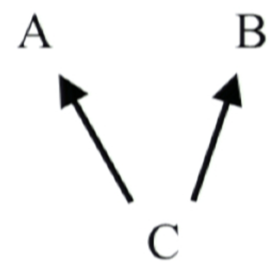
Formalizing the notion of locality



a, b – setting variables
A, B – outcome variables
λ – ontic state of system

When correlations arise from common causes



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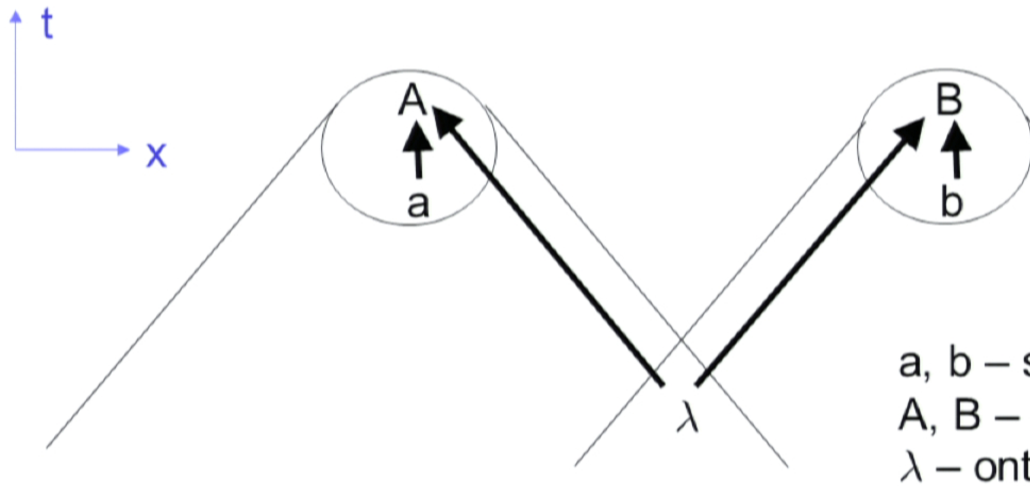

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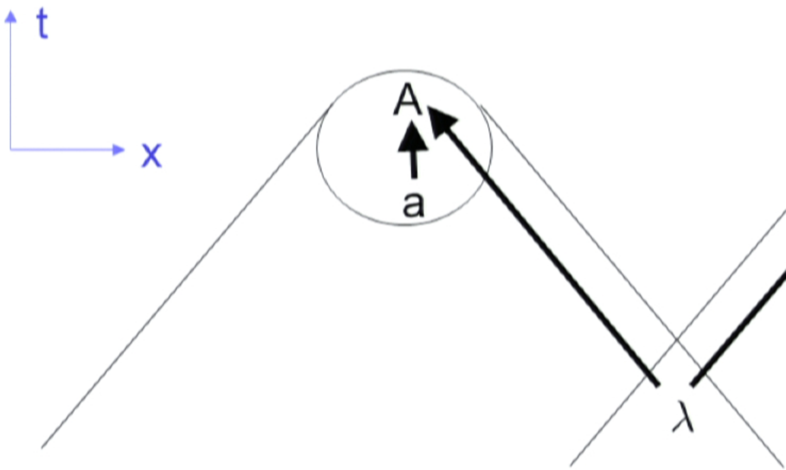

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This causal structure implies

$$p(A|a,b,B,\lambda) = p(A|a,\lambda)$$
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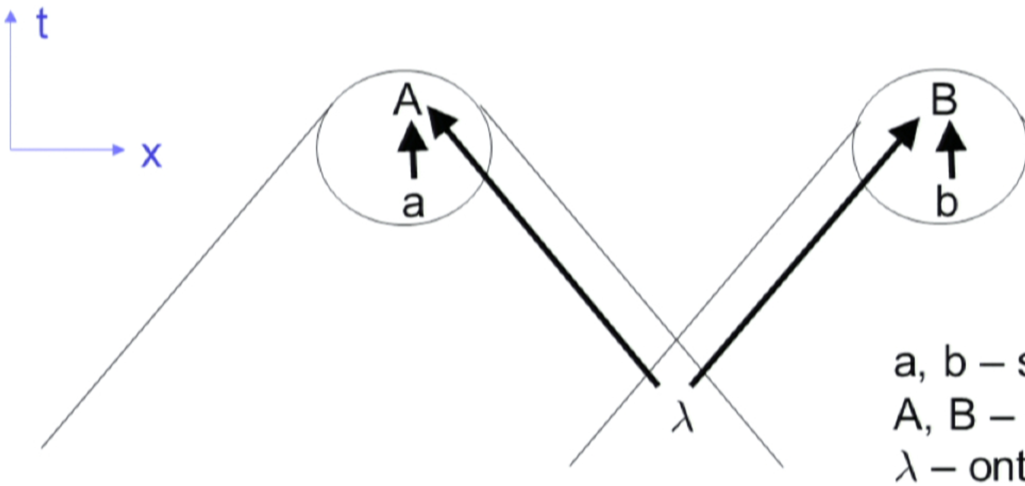
Formalizing the notion of locality



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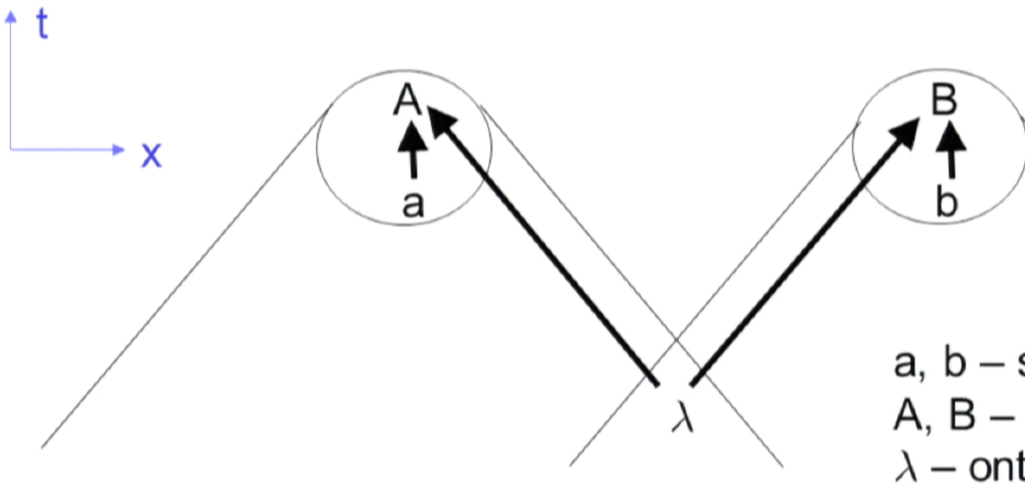
Bell called this assumption
Locality causality

Formalizing the notion of locality



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Bell called this assumption
Locality causality

This in turn implies *factorizability*

$$p(A,B|a,b,\lambda) = p(A|a,\lambda)p(B|b,\lambda)$$

Factorizability from local causality

Recall the relation between joint and conditional probability

$$p(A, B) = p(A|B)p(B)$$

Factorizability from local causality

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Factorizability from local causality

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therefore

$$p(A, B|a, b, \lambda) = p(A|B, a, b, \lambda)p(B|a, b, \lambda)$$

Factorizability from local causality

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By local causality

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Factorizability from local causality

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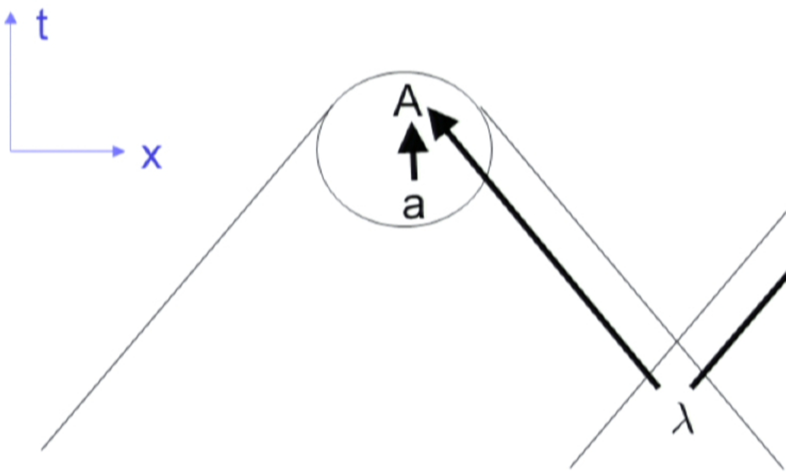

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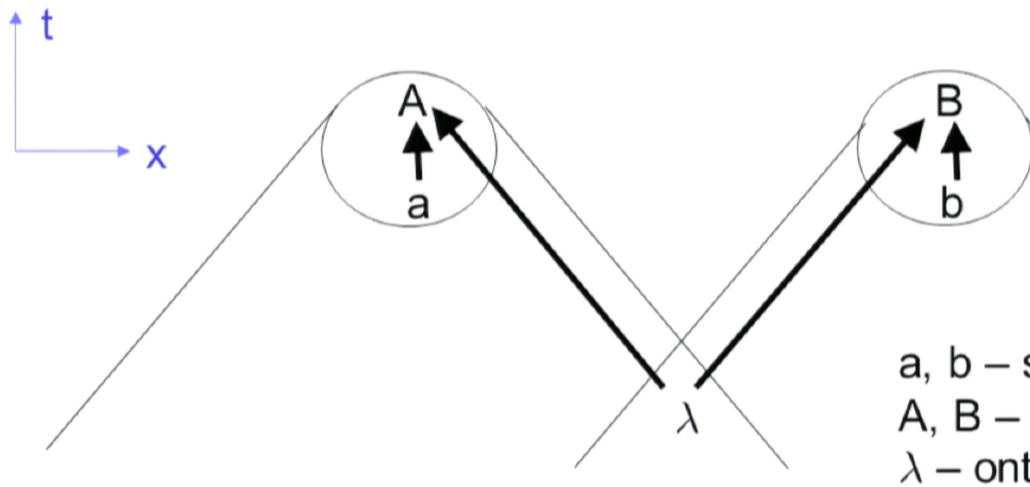
No superdeterminism

Independence of setting variables and ontic state of system

$$p(a, b, \lambda) = p(a)p(b)p(\lambda)$$



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Independence of setting variables and ontic state of system

$$p(a, b, \lambda) = p(a)p(b)p(\lambda)$$

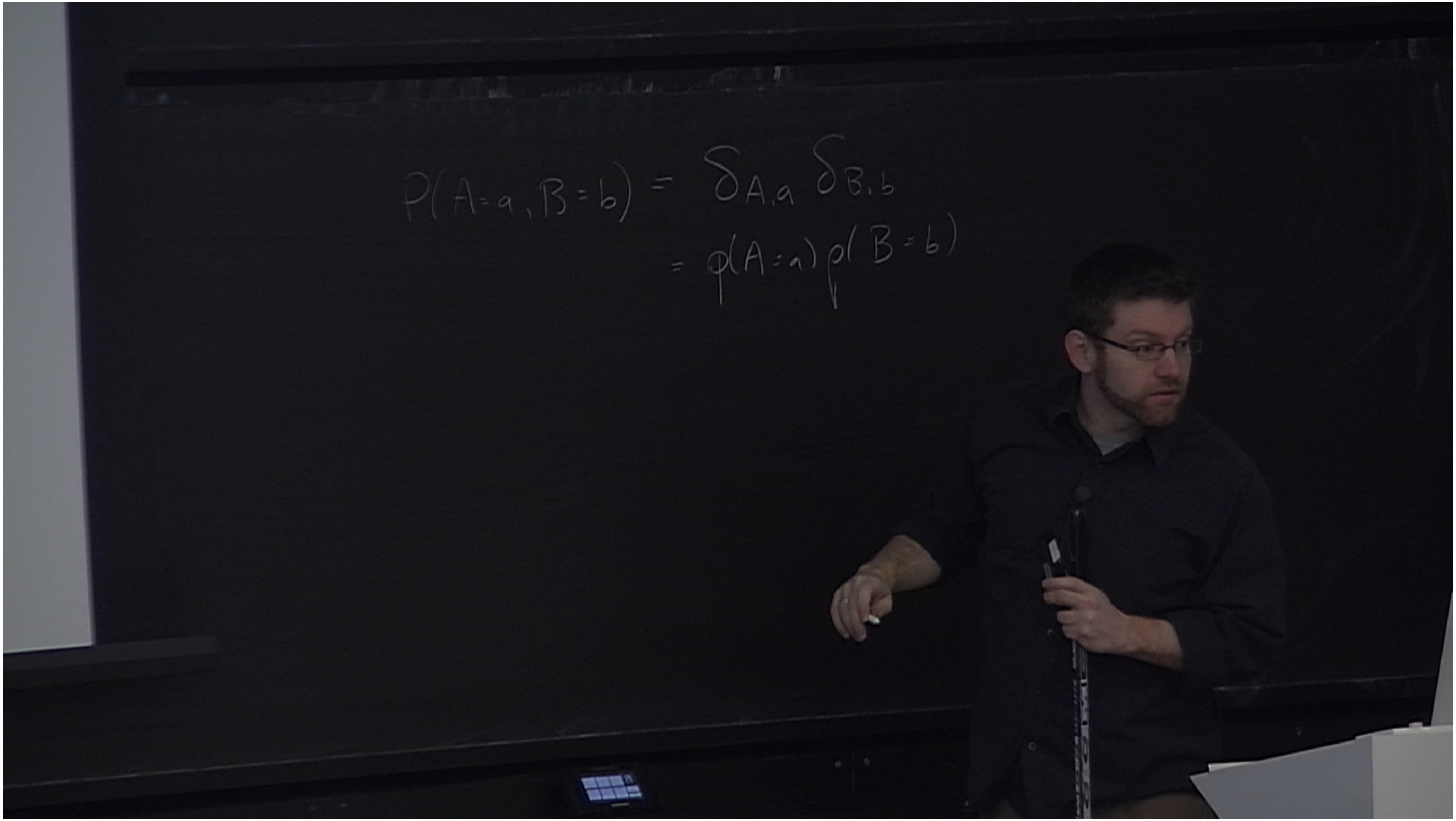
It follows that

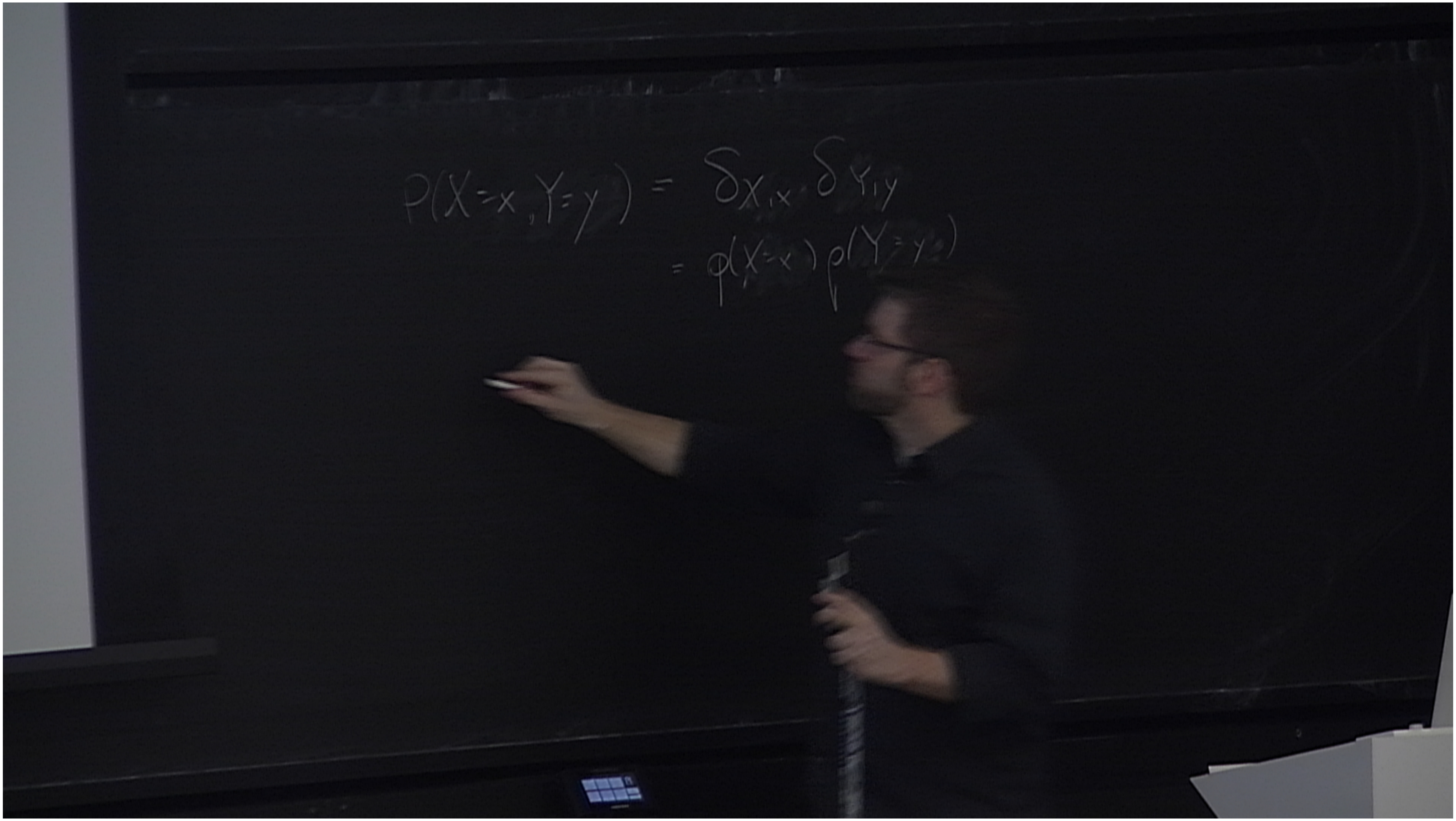
$$p(\lambda|a, b) = p(\lambda)$$

Consequence for experimentally observed correlations

$$p(A,B|a,b) = \int d\lambda p(A,B|a,b,\lambda)p(\lambda|a,b)$$

$$= \int d\lambda p(A|a,\lambda)p(B|b,\lambda)p(\lambda)$$


$P(A=a, B=b) = \delta_{A,a}\delta_{B,b}$
$= p(A=a)\, p(B=b)$


$p(X=x, Y=y) = \delta_{X,x}\,\delta_{Y,y}$
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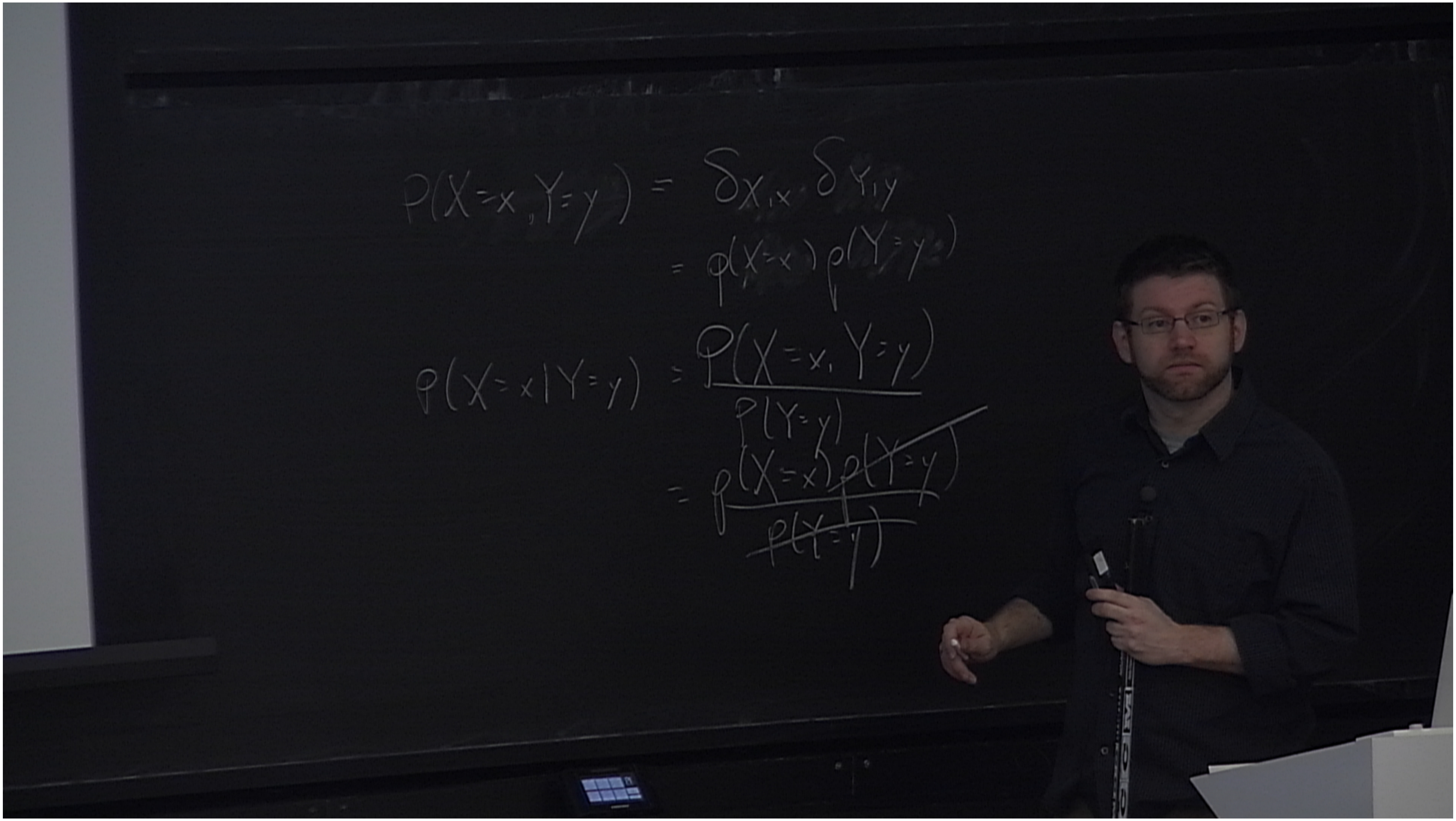
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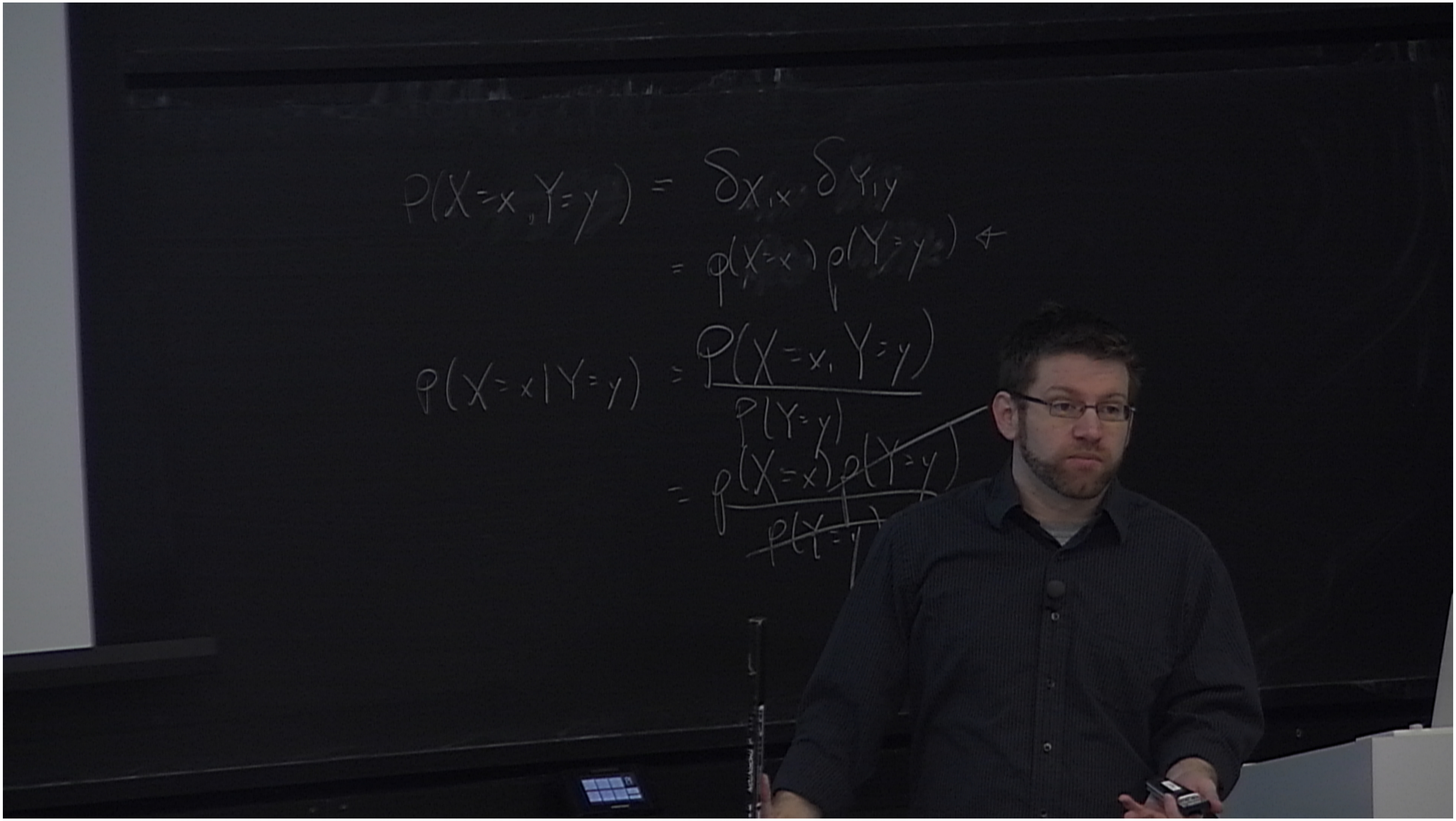
$p(X=x|Y=y) = \frac{p(X=x, Y=y)}{p(Y=y)}$
$= \frac{p(X=x)\,\cancel{p(Y=y)}}{\cancel{p(Y=y)}}$



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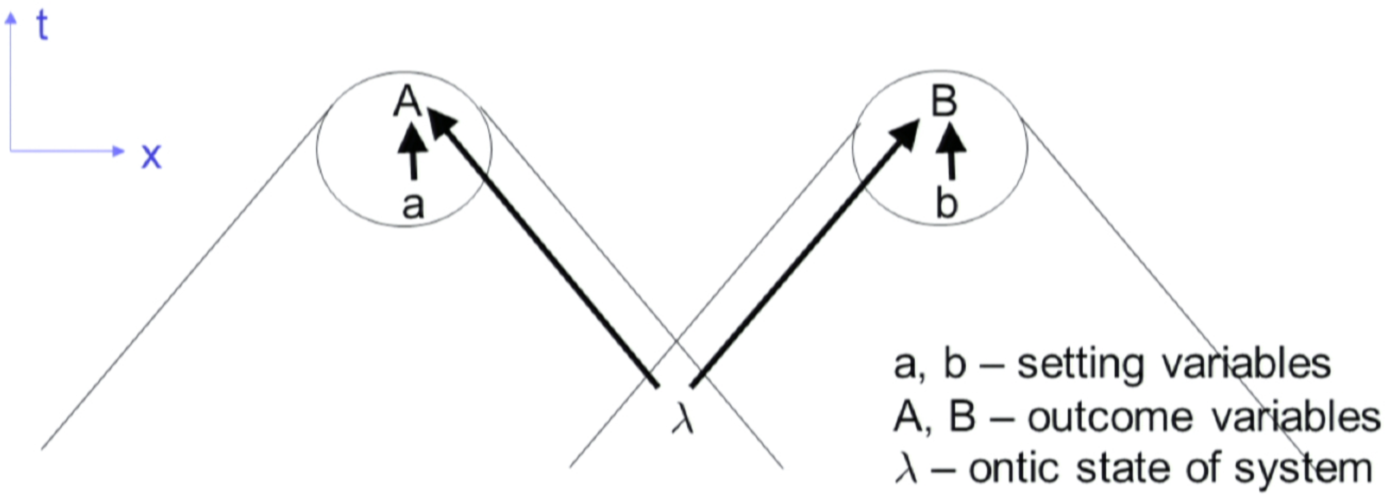

No superdeterminism

Independence of setting variables and ontic state of system

$$p(a,b,\lambda) = p(a)p(b)p(\lambda)$$

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Consequence for experimentally observed correlations

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Suppose: two settings $a \in \{S, T\}$ $b \in \{S, T\}$
And two outcomes $A \in \{r, g\}$ $B \in \{r, g\}$

$$p(\text{agree}|a, b) = p(r, r|a, b) + p(g, g|a, b)$$
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It is then not difficult to derive the Bell inequality

$$\frac{1}{4}[\, p(\text{agree}|SS) + p(\text{agree}|ST) + p(\text{agree}|TS) + p(\text{disagree}|TT)] \leq 3/4$$

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The difference between locality causality and no-signalling

Locality causality: $$p(A|a,b,B,\lambda) = p(A|a,\lambda)$$
$$p(B|a,b,A,\lambda) = p(B|b,\lambda)$$

No superluminal signalling: $$p(A|a,b) = p(A|a)$$
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Does the notion of "local causality" capture the content of relativity?

The difference between locality causality and no-signalling

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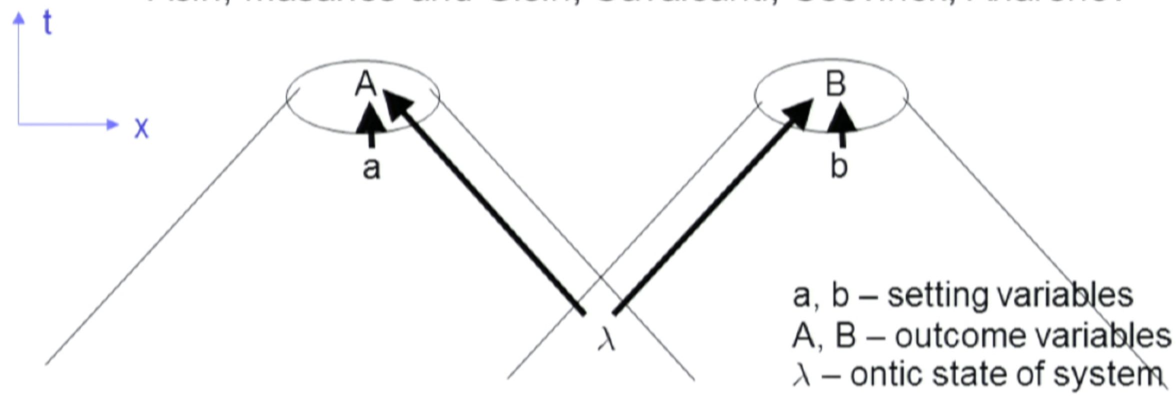
Does the notion of "local causality" capture the content of relativity?

Failure of predictability from Bell-inequality violations and no signalling

Acin, Masanes and Gisin; Cavalcanti; Seevinck; Aharonov



Predictability: $p(A,B|a,b) \in \{0,1\}$

No signalling: $p(A|a,b) = p(A|a)$ and $p(B|a,b) = p(B|b)$

Thm: No signalling + Bell-inequality violation → unpredictability

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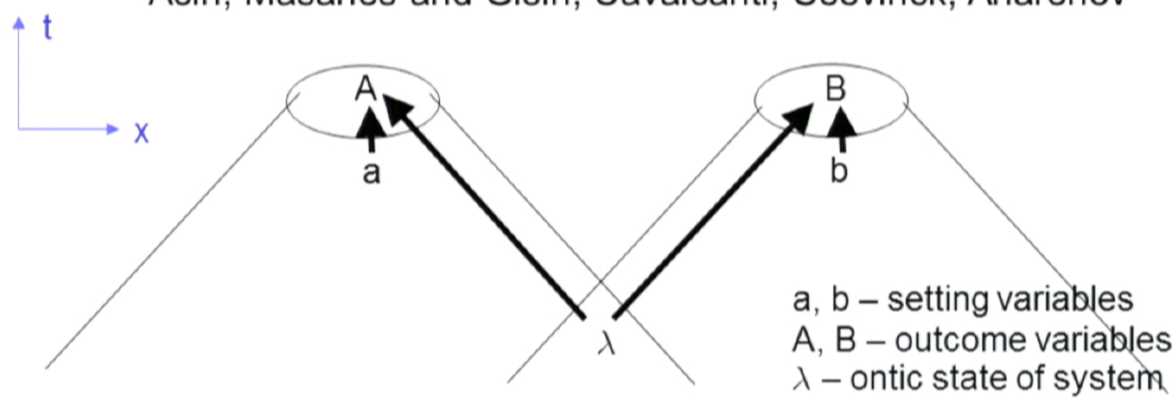

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Proof: $p(A,B|a,b,\lambda) = p(A,B|a,b) = p(A|a,b)\, p(B|a,b)$ (by predictability)

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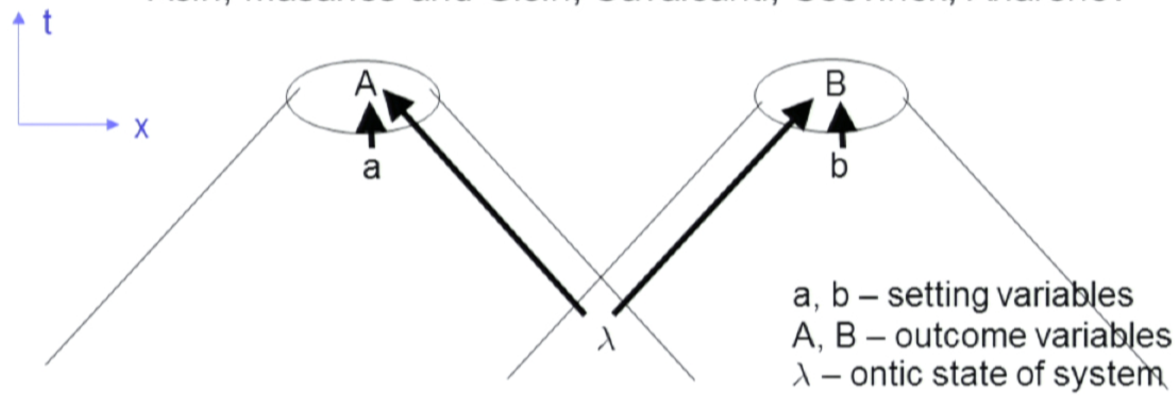

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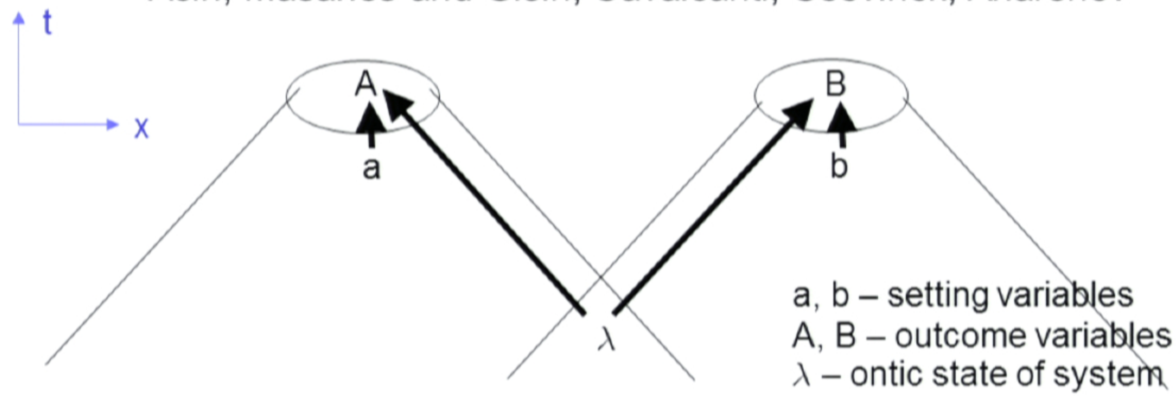

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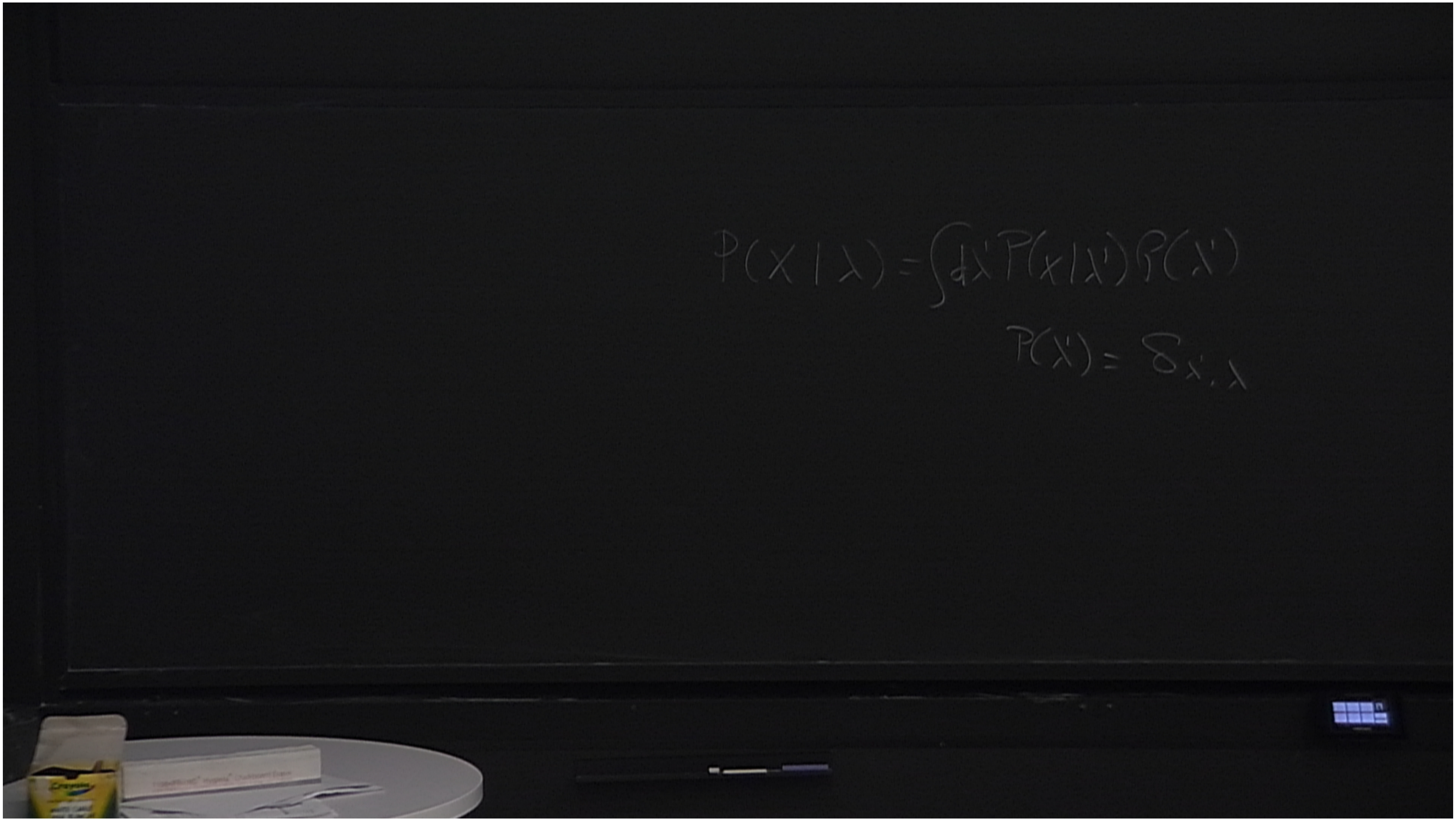
= p(A|a) p(B|b) (by no signalling)

But this is factorizability, from which the Bell inequalities follow.

Applications of nonlocality

Magic is a natural force that can be used to override the usual laws of nature.
-- Harry Potter entry in wikipedia


$P(x|\lambda) = \int d\lambda' \, P(x|\lambda') P(\lambda')$
$P(\lambda') = \delta_{\lambda',\lambda}$

Quantum Spellcraft

Based on Bell-inequality violation

Reduction in communication complexity

Buhrman, Cleve, van Dam, SIAM J.Comput. 30 1829 (2001)

Brassard, Found. Phys. 33, 1593 (2003)

Device-independent secure key distribution

Barrett, Hardy, Kent, PRL 95, 010503 (2005)

Acin, Gisin, Masanes, PRL. 97, 120405 (2006)

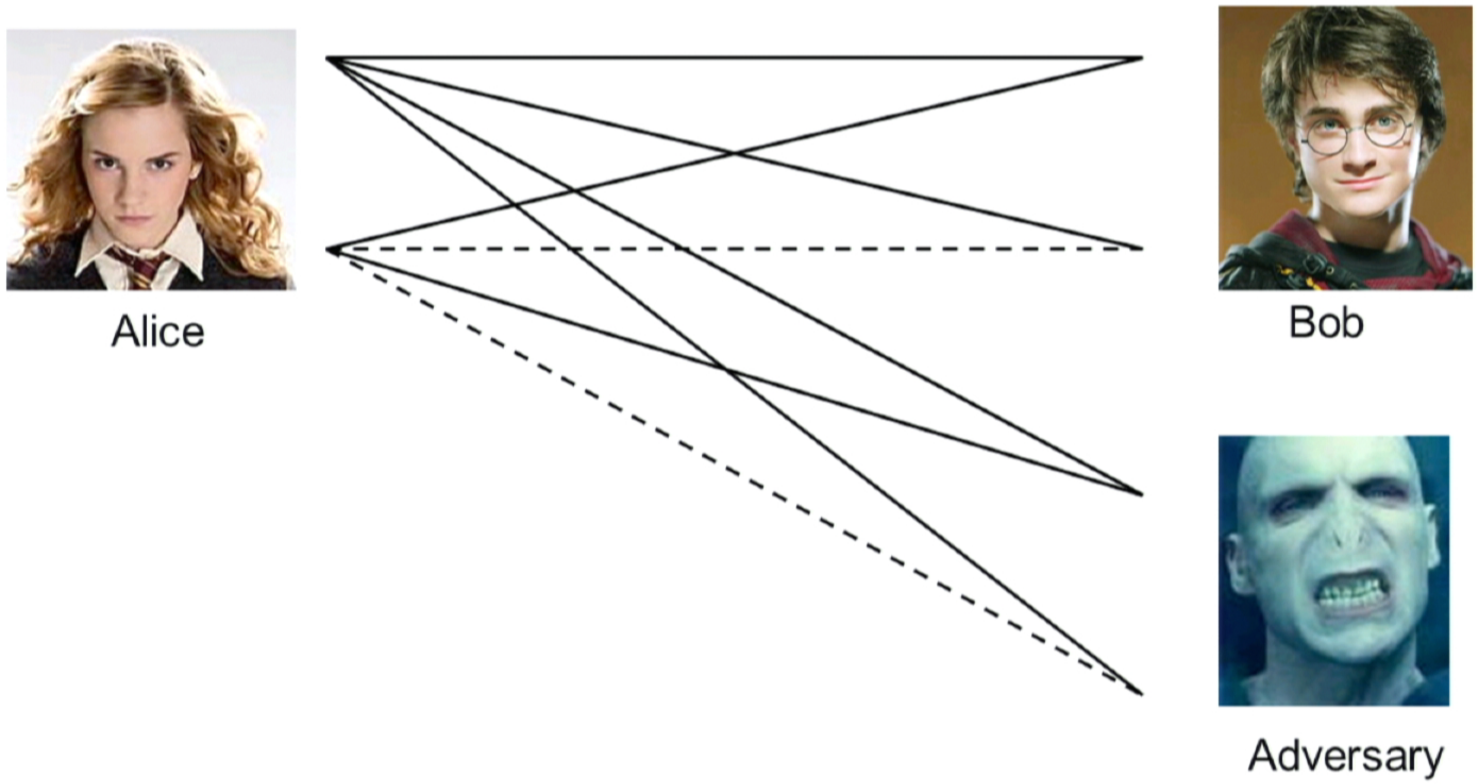
Randomness expansion

Colbeck, Kent, J. Phys. A, 44, 095305 (2011).

Enhancing zero-error channel capacity

Cubitt, Leung, Matthews, Winter, arXiv:0911.5300

Monogamy of Bell-inequality violating correlations



Recent trend in axiomatization:
Why isn't the world *more* nonlocal?

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Why isn't the world *more* nonlocal?