

Title: 12/13 PSI - Found Quantum Mechanics Lecture 7

Date: Jan 15, 2013 11:30 AM

URL: <http://pirsa.org/13010074>

Abstract:

Classical statistical theory
+
fundamental restriction on statistical distributions
↓
A large part of quantum theory



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Classical theory

Mechanics

Statistical theory for the classical theory

Liouville mechanics

Restricted Statistical theory for the classical theory

Restricted Liouville mechanics
= Gaussian quantum mechanics

Classical theory

Mechanics

Bits

Statistical theory for the classical theory

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Statistical theory of bits

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Restricted Liouville mechanics
= Gaussian quantum mechanics

Restricted statistical theory of bits
 \simeq Stabilizer theory for qubits



Classical theory

Mechanics

Bits

Trits

Statistical theory for the classical theory

Liouville mechanics

Statistical theory of bits

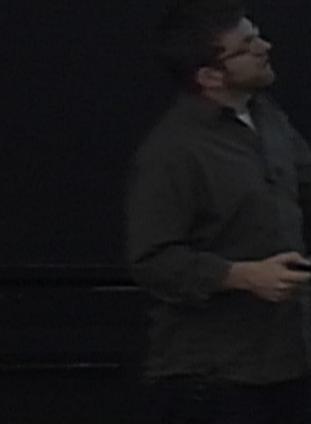
Statistical theory of trits

Restricted Statistical theory for the classical theory

Restricted Liouville mechanics
= Gaussian quantum mechanics

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Restricted statistical theory of trits
= Stabilizer theory for qutrits



Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics
Bits	Statistical theory of bits	Restricted statistical theory of bits \simeq Stabilizer theory for qubits
Trits	Statistical theory of trits	Restricted statistical theory of trits = Stabilizer theory for qutrits
Optics	Statistical optics	Restricted statistical optics = linear quantum optics

Classical theory

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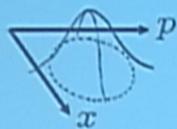
Restricted statistical theory of bits
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Restricted statistical optics
= linear quantum optics

Liouville mechanics

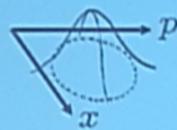
$$\mu(x, p)$$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Liouville mechanics

$$\mu(x, p)$$



What is a good epistemic restriction to apply?
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Quantum mechanics

Uncertainty principle:

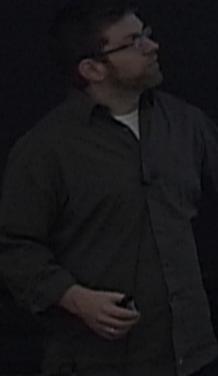
$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

where

$$\Delta^2 x \equiv \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$

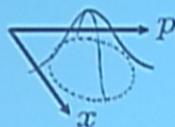
$$C_{x,p} \equiv \frac{1}{2} \langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle - \langle \hat{x} \rangle \langle \hat{p} \rangle$$

$$\langle \hat{A} \rangle \equiv \text{Tr}(\hat{A}\hat{\rho})$$



Liouville mechanics

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Quantum mechanics

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Liouville mechanics with an epistemic restriction

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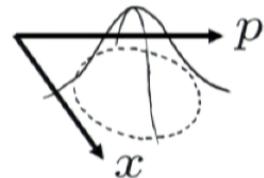
$$\Delta^2 x \equiv \langle x^2 \rangle - \langle x \rangle^2$$

$$C_{x,p} \equiv \langle xp \rangle - \langle x \rangle \langle p \rangle$$

$$\langle f(x, p) \rangle \equiv \int dx dp f(x, p) \mu(x, p)$$

Liouville mechanics

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Quantum mechanics

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Liouville mechanics with an epistemic restriction

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$$\langle f(x, p) \rangle \equiv \int dx dp f(x, p) \mu(x, p)$$

Liouville mechanics with an epistemic restriction

Assume:

The classical uncertainty principle (for a single particle in 1D):

The only Liouville distributions that can be prepared are those that satisfy

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

and that have maximal entropy for a given set of second-order moments.

The Wigner representation

Phase point operators

$$\hat{A}(x, p) = \frac{1}{2\pi\hbar} \int e^{ipy/\hbar} \left| x + \frac{1}{2}y \right\rangle \left\langle x - \frac{1}{2}y \right| dy.$$

Wigner representation of an operator

$$W_{\hat{O}}(x, p) = \text{Tr}[\hat{O}\hat{A}(x, p)]$$

The Wigner representation

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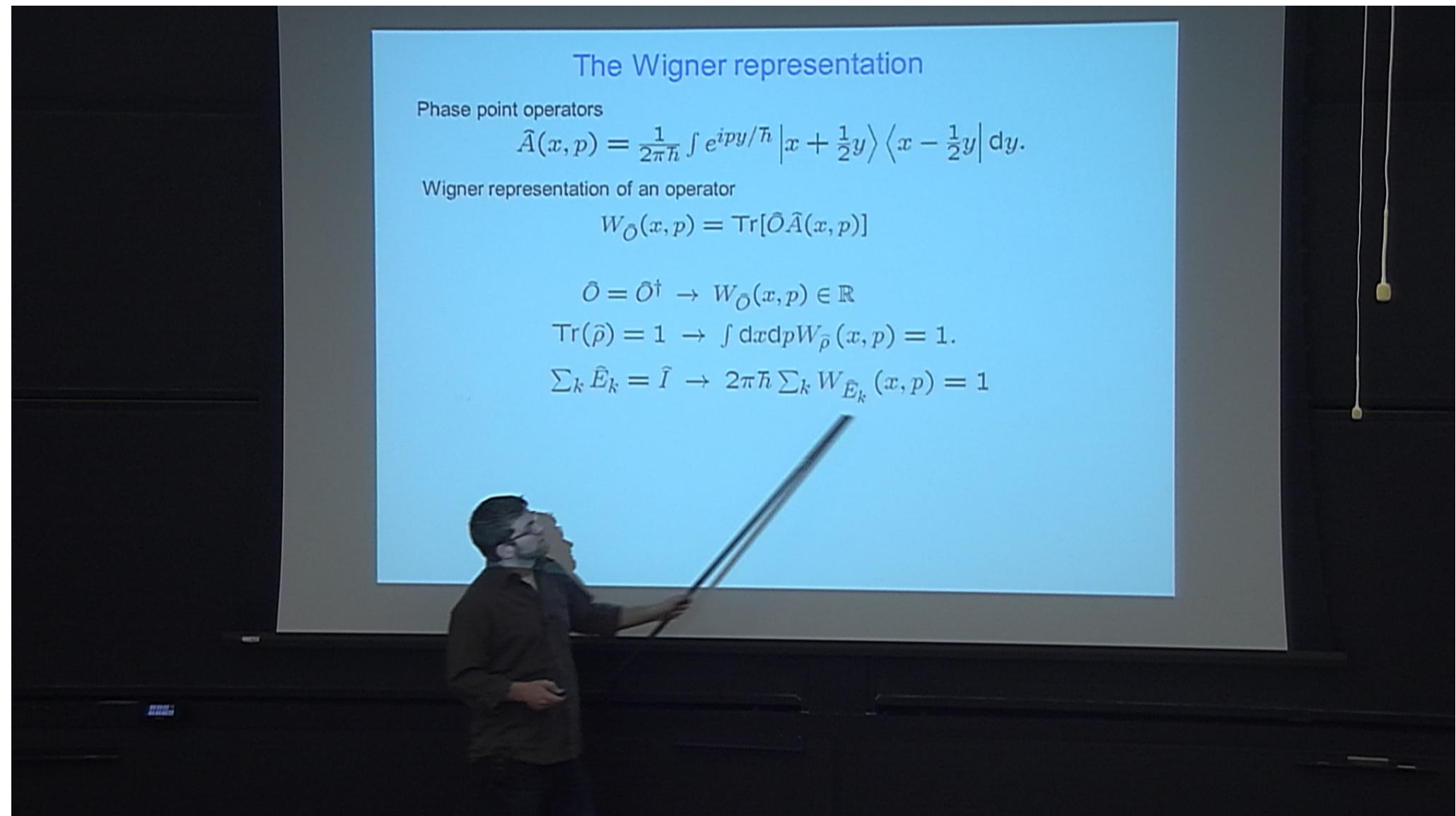
Wigner representation of an operator

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$$\hat{O} = \hat{O}^\dagger \rightarrow W_{\hat{O}}(x, p) \in \mathbb{R}$$

$$\text{Tr}(\hat{\rho}) = 1 \rightarrow \int dx dp W_{\hat{\rho}}(x, p) = 1.$$

$$\sum_k \hat{E}_k = \hat{I} \rightarrow 2\pi\hbar \sum_k W_{\hat{E}_k}(x, p) = 1$$



The Wigner representation

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$$\text{and } 2\pi\hbar \int dx dp W_{\hat{\rho}}(x, p) W_{\hat{E}_k}(x, p) = \text{Tr}[\hat{\rho}\hat{E}_k]$$

Wigner representation of a map \mathcal{E}

$$W_{\mathcal{E}}(x', p' | x, p) = \text{Tr}[\hat{A}(x', p') \mathcal{E}(\hat{A}(x, p))]$$

$$\text{and } W_{\mathcal{E}(\cdot, \cdot | \cdot, \cdot)}(x, p) = \int dx' dp' W_{\mathcal{E}}(x, p | x', p') W_{\hat{\rho}}(x', p')$$

The Wigner representation

Phase point operators

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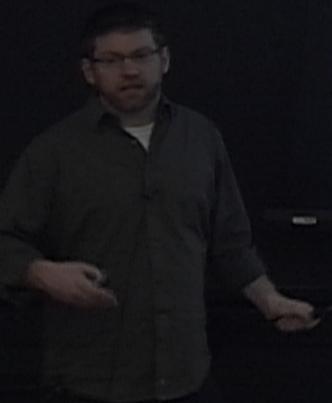
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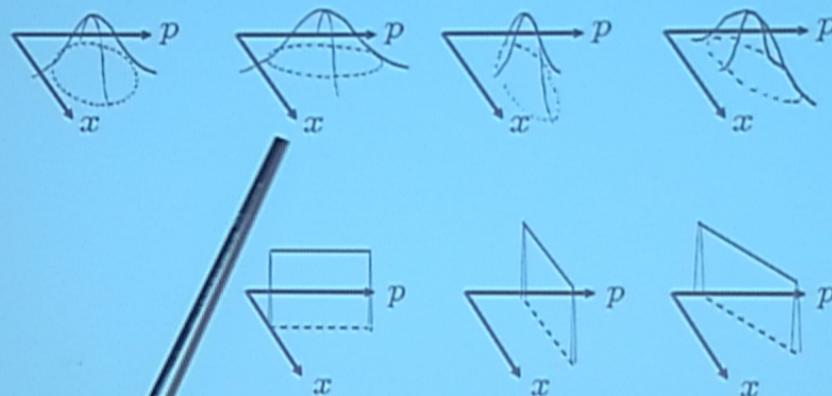
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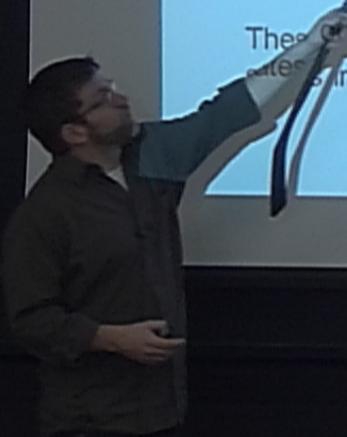
$$\text{and } W_{\mathcal{E}(\rho)}(x, p) = \int dx' dp' W_{\mathcal{E}}(x, p | x', p') W_{\hat{\rho}}(x', p')$$



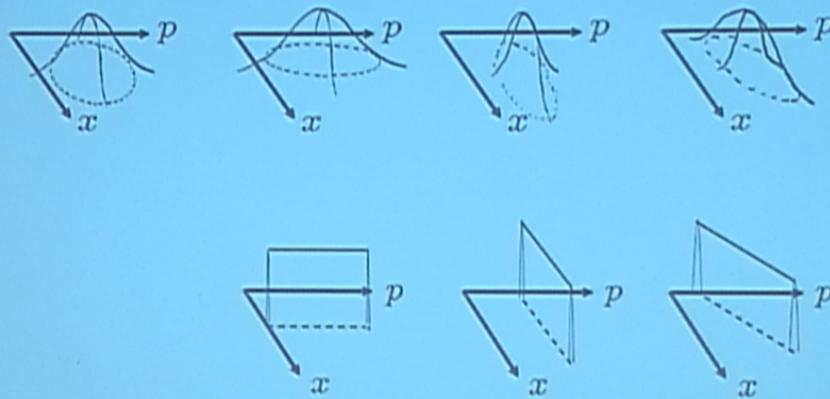
Valid pure epistemic states for one canonical system



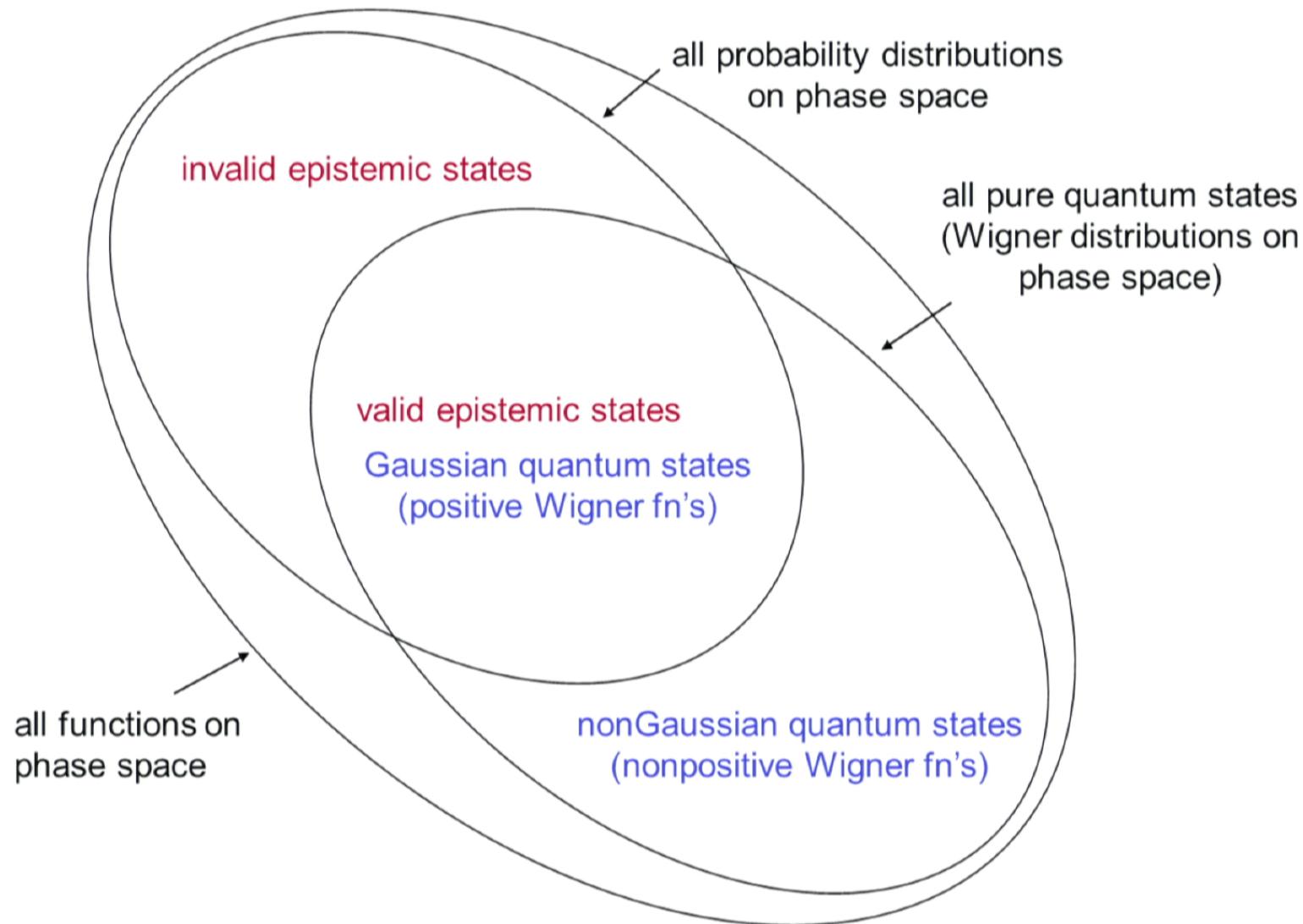
These correspond to the Wigner representations of the pure squeezed states in quantum mechanics



Valid pure epistemic states for one canonical system



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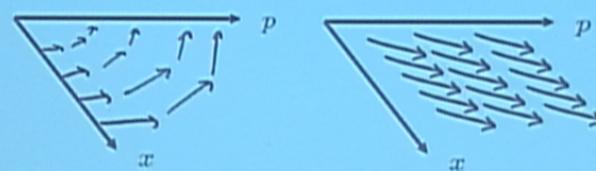


Valid deterministic transformations

The group of canonical transformations with quadratic Hamiltonian

Only canonical transformations preserve the uncertainty principle

Only quadratic Hamiltonians preserve the gaussianity

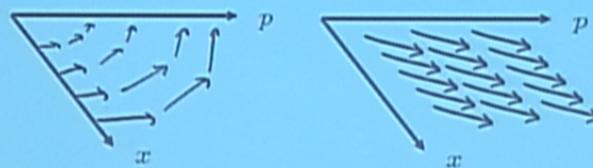


Valid deterministic transformations

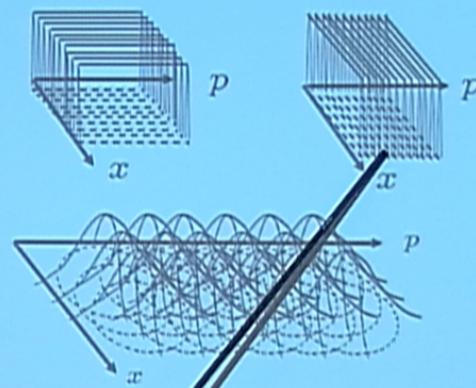
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Valid measurements



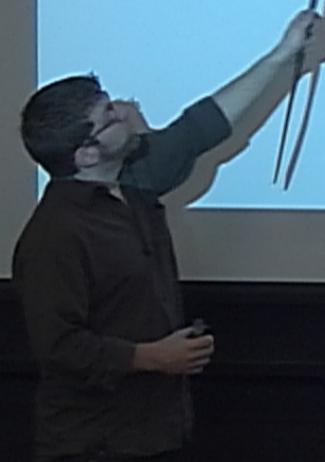
Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems

or:

Allow all products of valid epistemic states

Allow canonical transformations with quadratic Hamiltonians on these



Extension to multiple systems

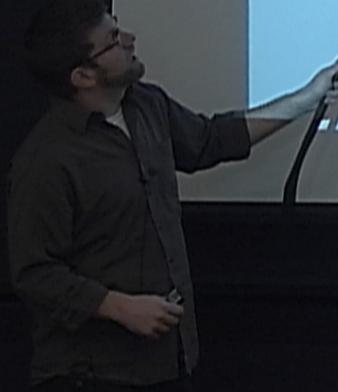
Use a generalization of the uncertainty principle for multiple systems
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$$\text{E.g. } x_A \rightarrow x_A - x_B \quad x_B \rightarrow x_A + x_B$$

$$p_A \rightarrow p_A - p_B \quad p_B \rightarrow p_A + p_B$$



Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems
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E.g.

$$\begin{array}{ll} x_A \rightarrow x_A - x_B & x_B \rightarrow x_A + x_B \\ p_A \rightarrow p_A - p_B & p_B \rightarrow p_A + p_B \end{array}$$

Extension to multiple systems

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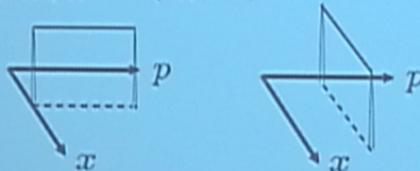
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$$p_A \rightarrow p_A - p_B \quad p_B \rightarrow p_A + p_B$$

know X_A and P_B

$$\mu(x_A, p_A) \propto \delta(x_A - a)$$



$$\mu(x_B, p_B) \propto \delta(x_B - b)$$



Extension to multiple systems

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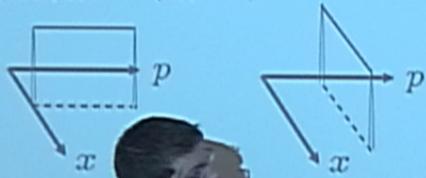
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know X_A and P_B

$$\mu(x_A, p_A) \propto \delta(x_A - a)$$



$$\mu(x_B, p_B) \propto \delta(x_B - b)$$

know $X_A - X_B$ and $P_A + P_B$

$$\mu(x_A, p_A, x_B, p_B)$$

$$\propto \delta(x_A - x_B - a)\delta(p_A + p_B - b)$$

corresponds to EPR state

How can one characterize the set of variables that can be jointly known?

They commute relative to the Poisson bracket!

$$[F, G](m) \equiv \left(\frac{\partial F}{\partial X} \frac{\partial G}{\partial P} - \frac{\partial F}{\partial P} \frac{\partial G}{\partial X} \right)(m)$$

Recall:

A canonically conjugate pair $[F, G] = 1$

e.g. $\{X_1, P_1\}, \{X_2, P_2\}$, and $\{X_1 + X_2, P_1 + P_2\}$



The Wigner representation for multiple systems

For a pair of systems

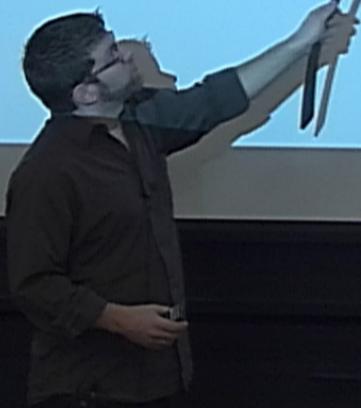
$$W_{\hat{\rho}}(x_1, p_1, x_2, p_2) = \text{Tr}[\hat{\rho}\hat{A}(x_1, p_1) \otimes \hat{A}(x_2, p_2)]$$



The Wigner representation for multiple systems

For a pair of systems

$$W_{\hat{\rho}}(x_1, p_1, x_2, p_2) = \text{Tr}[\hat{\rho}\hat{A}(x_1, p_1) \otimes \hat{A}(x_2, p_2)]$$



Gaussian Quantum Mechanics

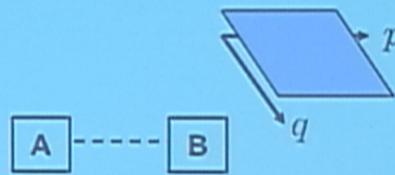
- = States with Gaussian Wigner rep'ns (a.k.a. squeezed states)
- + Measurements with Gaussian Wigner rep'ns
- + Transformations that preserve Gaussianity of states

Epistemically-restricted Liouville mechanics

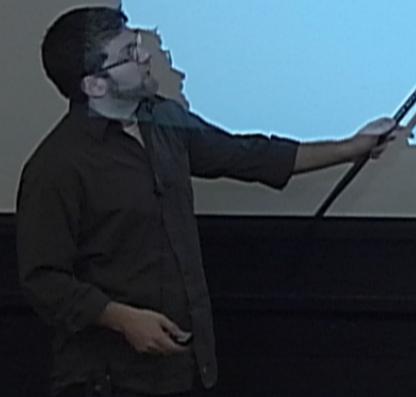
- = Gaussian quantum mechanics in the Wigner representation



EPR effect in Epistemically Restricted Liouville mechanics



$$\begin{aligned}\mu_{\text{EPR}}(q_A, p_A, q_B, p_B) &\propto \delta(q_A - q_B)\delta(p_A + p_B) \\ Q_B - Q_A &= 0 \\ P_B + P_A &= 0\end{aligned}$$



“bit mechanics” $\mathbb{Z}_2 = \{0, 1\}$

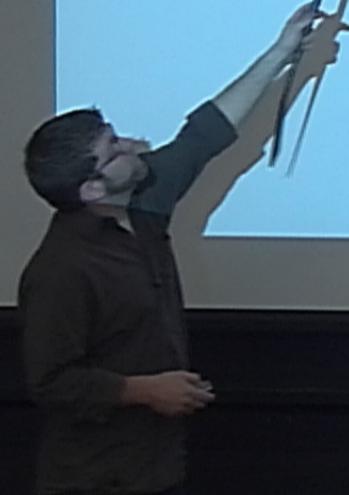
Configuration space: $(\mathbb{Z}_2)^n \ni (x_1, x_2, \dots, x_n)$

Phase space: $\Omega \equiv (\mathbb{Z}_2)^{2n} \ni (x_1, p_1, x_2, p_2, \dots, x_n, p_n) \equiv m$

Functionals on phase space: $F : \Omega \rightarrow \mathbb{Z}_2$

$$X_k(m) = \dots$$

$$P_k(m) = p_k$$



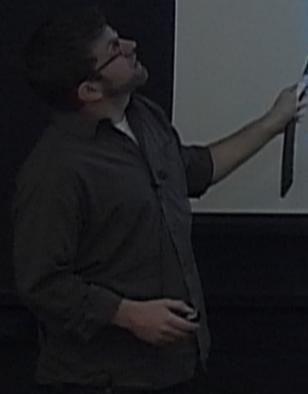
A single bit

$$\begin{array}{c} X \ 1 \\ 0 \\ \hline P \end{array}$$

Canonical variables

$$aX + bP \quad a, b \in \mathbb{Z}_2 \quad \text{Addition is mod2}$$

$$X, \ P, \ X + P$$



A single bit

X	1	
	0	
		0
		1

Canonical variables

$$aX + bP \quad a, b \in \mathbb{Z}_2 \quad \text{Addition is mod2}$$

$$X, P, X + P$$

Statistical distributions

X known

X	1	
	0	
		0
		1

P known

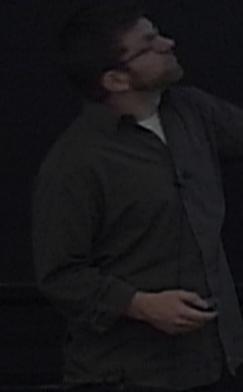
P	1	
	0	
		0
		1

$X + P$ known

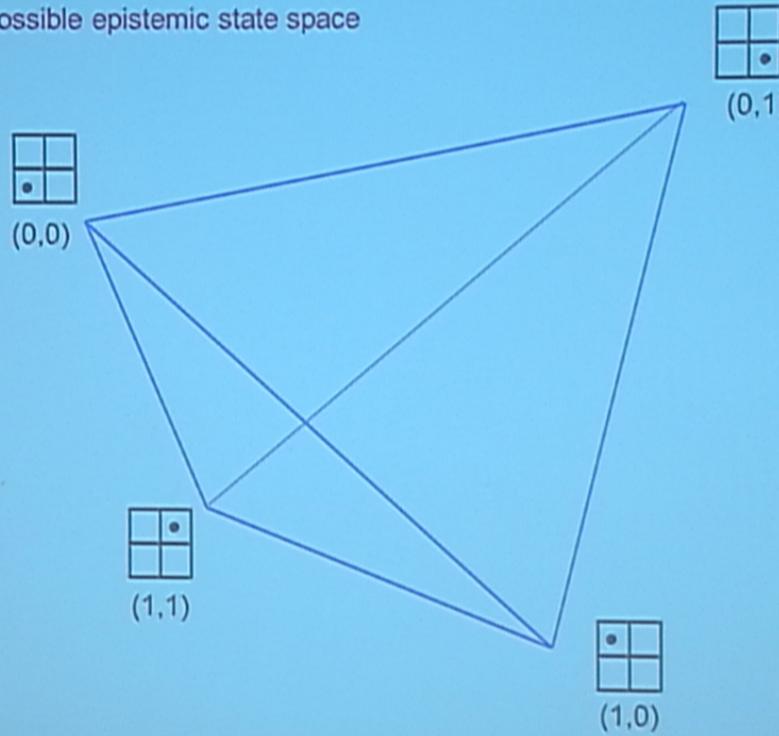
$X + P$	1	
	0	
		0
		1

Nothing known

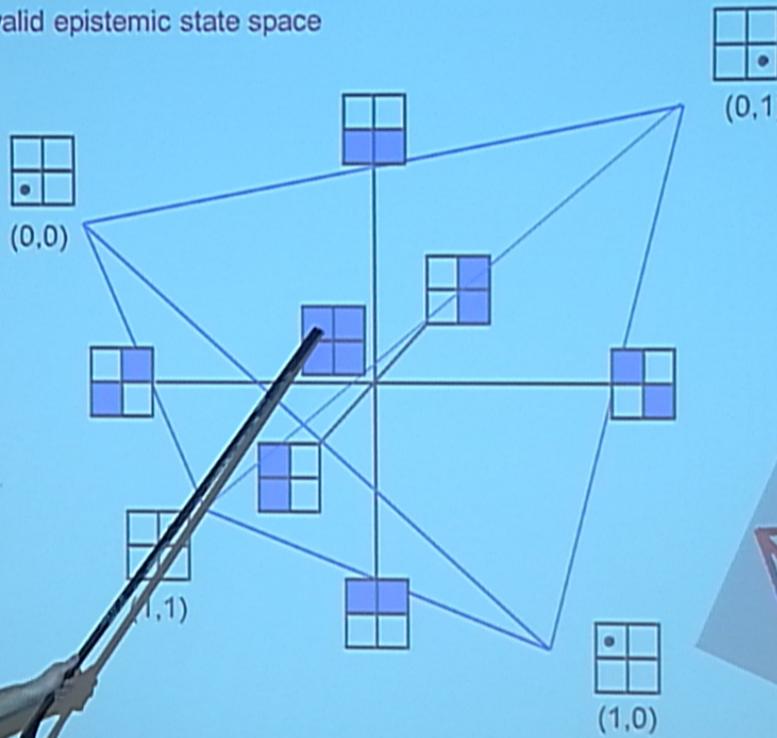
X	1	
	0	
		0
		1



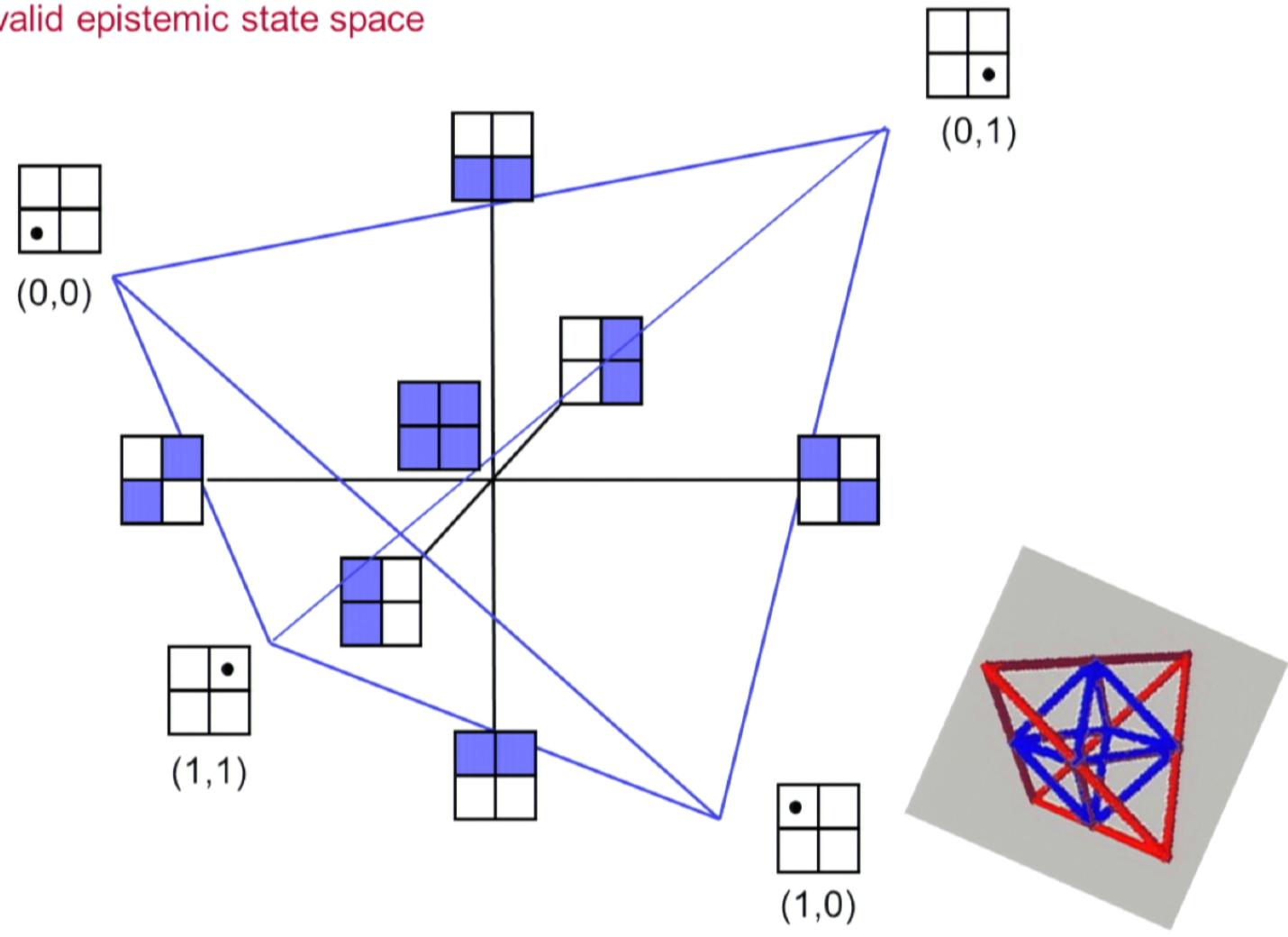
The possible epistemic state space



The valid epistemic state space

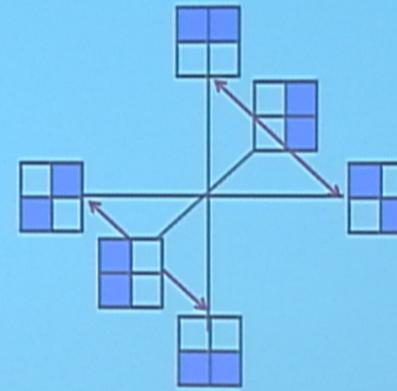


The valid epistemic state space



A 2-cycle

$$\begin{matrix} X & 1 \\ 0 & P \end{matrix}$$

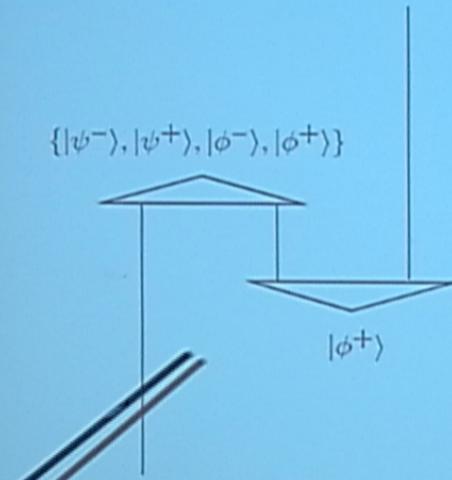


$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$$
$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle)$$

Teleportation

I, X, Y, Z

$$\{|\psi^-\rangle, |\psi^+\rangle, |\phi^-\rangle, |\phi^+\rangle\}$$

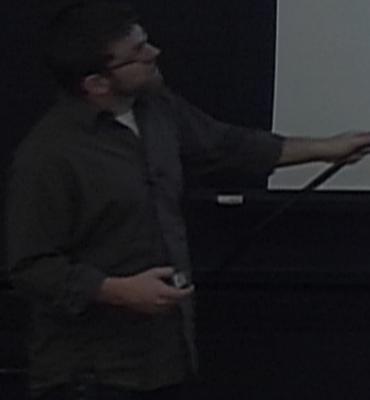
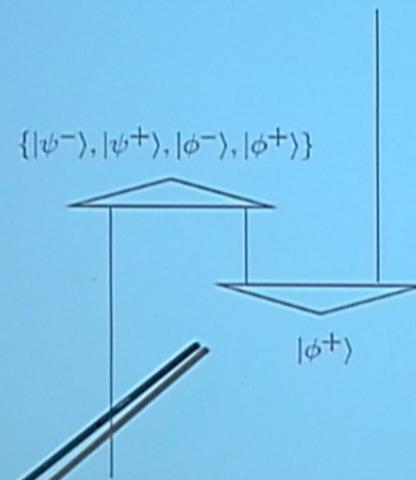


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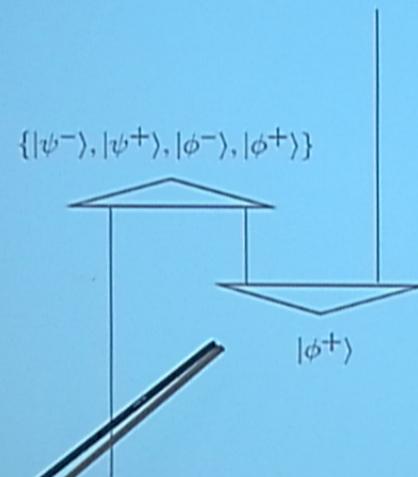
I, X, Y, Z



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Teleportation

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$\{|\psi^-\rangle, |\psi^+\rangle, |\phi^-\rangle, |\phi^+\rangle\}$

