

Title: 12/13 PSI - String Theory Review Lecture 15

Date: Jan 25, 2013 10:15 AM

URL: <http://pirsa.org/13010065>

Abstract:

NS

$$\alpha_{-n}^I, b_{-r}^I$$

$$\alpha' M^2 = N^\perp - \frac{1}{2}$$

$$(-1)^F |NS\rangle = -1$$

$$f_{NS}(x) = \frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8$$

R

$$\alpha_{-n}^I, d_{-n}^I$$

$$\alpha' M^2 = N^\perp$$

$$|R_A\rangle \rightarrow |R_a\rangle, |R_{\bar{a}}\rangle$$

$$1-16$$

$$1-8$$

$$1-8$$

$$(-1)^F = -1$$

$$(-1)^F = +1$$

$$f_R(x) = 16 \prod_{n=1}^{\infty} \left(\frac{1+x^n}{1-x^n} \right)^8$$

$$a_{NN} = -\frac{1}{24}$$

$$a_{NS} = -\frac{1}{48}$$

$$a_R = +\frac{1}{24}$$

NS

$$\alpha_m^I, b_r^I$$
$$\alpha'^2 M^2 = N^2 - \frac{1}{2}$$

$$(-1)^F |NS\rangle = -1$$

$$f_{NS}(x) = \frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8$$

R

$$\alpha_m^I$$
$$\alpha'^2 M^2 = N^2$$

$$|R_A\rangle \rightarrow |R_a\rangle, |R_{\bar{a}}\rangle$$
$$1 \rightarrow 16 \quad 1 \rightarrow 8 \quad 1 \rightarrow 8$$

$$8 = (4+4)$$
$$(-1)^F = -1 \quad (-1)^F = +1$$

$$f_R(x) = 16 \prod_{n=1}^{\infty} \left(\frac{1+x^n}{1-x^n} \right)^8$$

NS

$$\alpha_{-n}^I, b_{-r}^I$$

$$\alpha' M^2 = N^\perp - \frac{1}{2}$$

$$(-1)^F |NS\rangle = -1$$

$$f_{NS}(x) = \frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8 = \frac{1}{\sqrt{x}} + 8 + 36\sqrt{x} + 128x + \dots$$

R

$$\alpha_{-n}^I, d_{-n}^I$$

$$\alpha' M^2 = N^\perp$$

$$|R_A\rangle \rightarrow |R_a\rangle, |R_{\bar{a}}\rangle$$

$$1 \dots 16$$

$$1 \dots 8$$

$$1 \dots 8$$

$$8 = (4) + (4)$$

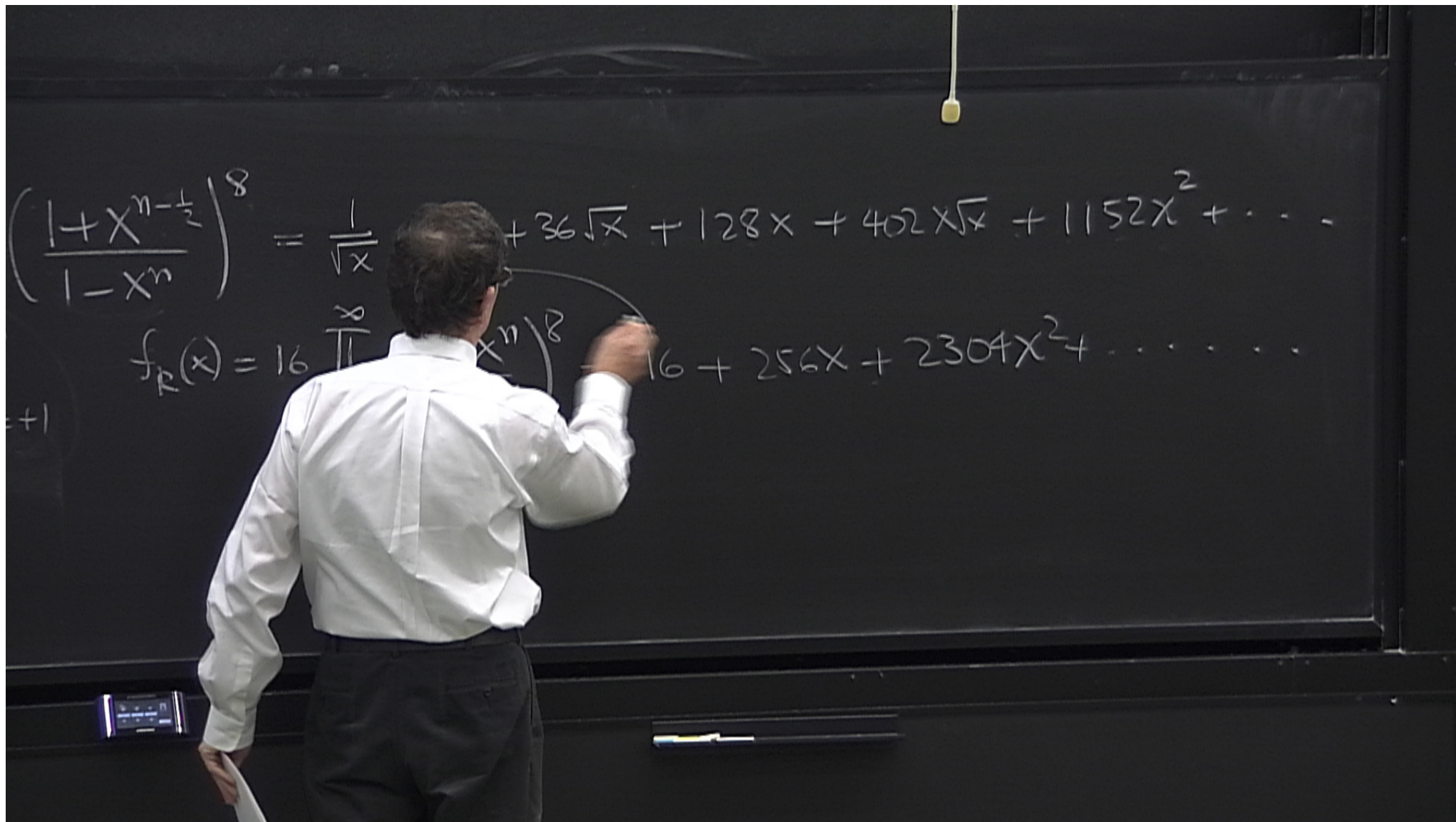
$$(-1)^F = -1$$

$$(-1)^F = +1$$

$$f_R(x) = 16 \prod_{n=1}^{\infty} \left(\frac{1+x^n}{1-x^n} \right)^8$$

$$\left(\frac{1+X^{n-\frac{1}{2}}}{1-X^n} \right)^8 = \frac{1}{\sqrt{x}}^8 + 36\sqrt{x} + 128x + 402x\sqrt{x} + 1152x^2 + \dots$$

$$f_R(x) = \left(\frac{1+X^n}{1-X^n} \right)^8 = 16 + 256x + 2304x^2 + \dots$$



NS

$$\alpha_{-n}^I, b_{-r}^I$$
$$\alpha' M^2 = N^I - \frac{1}{2}$$

$$(-1)^F |NS\rangle = -1$$

$$f_{NS}(x) = \frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)$$

R

$$\alpha_{-n}^I, d_{-n}^I$$
$$\alpha' M^2 = N^I$$

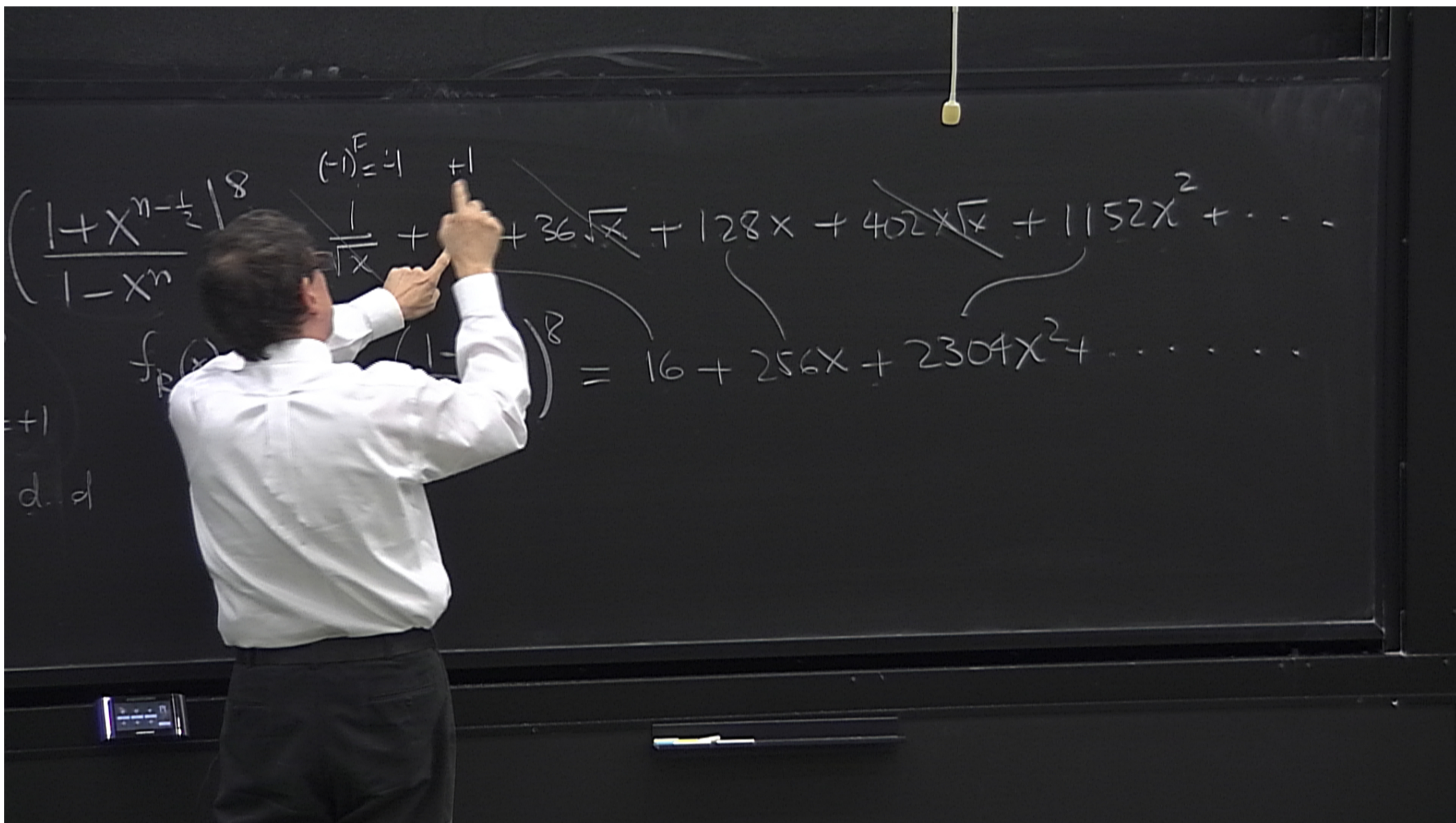
$$|R_A\rangle \rightarrow \begin{matrix} |R_a\rangle, & |R_{\bar{a}}\rangle \\ 1 \dots 16 & 1 \dots 8 & 1 \dots 8 \end{matrix}$$

$$(-1)^F = -1 \quad (-1)^F = +1$$

$$f_R(x)$$

$(-1)^F$ commutes with H

$$\alpha_{-n}^I d_{-n}^I \quad b_{-r} b_r \quad d_{-r} d_r$$



$$\left(\frac{1+X^{n-\frac{1}{2}}}{1-X^n} \right)^8 = \frac{(-1)^F = -1}{\sqrt{x}} + 8 + 36\sqrt{x} + 128x + 402x\sqrt{x} + 1152x^2 + \dots$$

$$f_R(x) = 16 \prod_{n=1}^{\infty} \left(\frac{1+X^n}{1-X^n} \right)^8 = 16 + 256x + 2304x^2 + \dots$$

$$NS+ \quad \left\{ NS \text{ states } (-1)^F = +1 \right\}$$

$$\frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8 = \frac{(-1)^F}{\sqrt{x}} + 8 + 36\sqrt{x} + 128x + 402x\sqrt{x} + 1152x^2 + \dots$$

$(-1)^F = -1 \quad +1 \quad -1 \quad +1 \quad -1 \quad +1$

$$f_R(x) = 16 \prod_{n=1}^{\infty} \left(\frac{1+x^n}{1-x^n} \right)^8 = 16 + 256x + 2304x^2 + \dots$$

$|R_a\rangle$
 $1 \dots 8$
 $(-1)^F = +1$

$b_r b_r$ d, d

NS+ { NS states $(-1)^F = +1$ }

R- { R states $(-1)^F = -1$ }

$|R_a\rangle, \alpha_i^I |R_a\rangle, d_{\frac{1}{2}L}^I$

$$\left(\frac{1+x^{n-\frac{1}{2}}}{1-x^m} \right)^8 = \frac{1}{\sqrt{x}} + 8 + 36\sqrt{x} + 128x + 402x\sqrt{x} + 1152x^2 + \dots$$

$(-1)^F = -1 \quad +1 \quad -1 \quad +1 \quad -1 \quad +1$

$$Z(x) = 16 \prod_{n=1}^{\infty} \left(\frac{1+x^n}{1-x^n} \right)^8 = 16 + 256x + 2304x^2 + \dots$$

NS + { NS states $(-1)^F = +1$ }

{ R states $(-1)^F = -1$ }

$|R_a\rangle, \alpha_{-1}^I |R_a\rangle, d_{-1}^I |R_a\rangle$

keep states with an odd # of b's

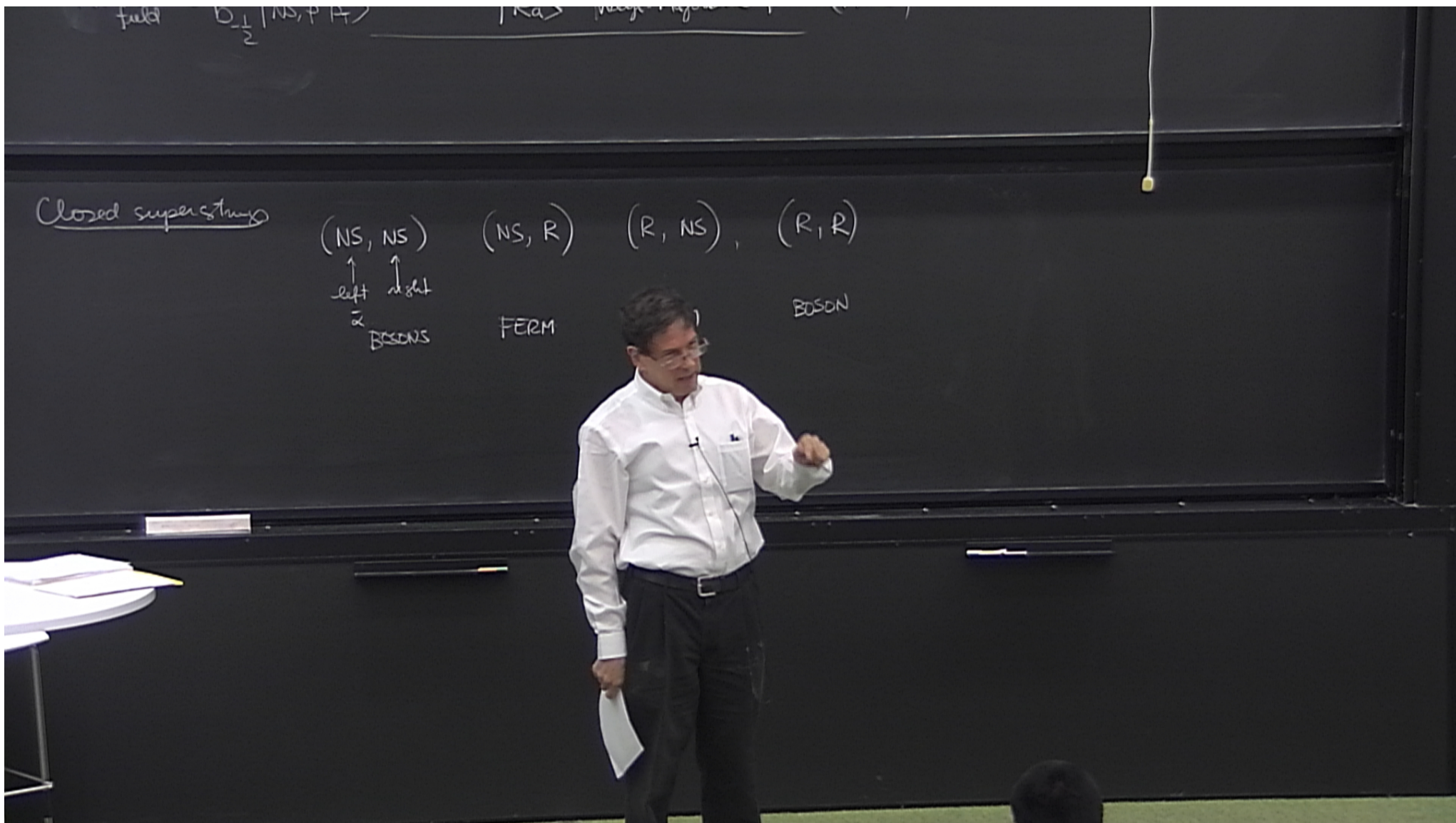
$$\frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8$$

$(-1)^F$ commutes with H $\Rightarrow$ $\alpha_m^\dagger \alpha_m$ $b_r b_r$ $d^\dagger d$ $(-1)^F = -1$ $(-1)^F = +1$ $(1-x)$
 $NS+$ $\{NS \text{ states } (-1)^F = +1\}$
 $R-$ $\{R \text{ states } (-1)^F = -1\}$ $|R_a\rangle, \alpha_m^\dagger |R_a\rangle, d^\dagger |R_a\rangle$

keep states with an odd # of b 's

$$f_{NS+}(x) \equiv \frac{1}{2\sqrt{x}} \left[\prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8 - \prod_{n=1}^{\infty} \left(\frac{1-x^{n-\frac{1}{2}}}{1-x^n} \right)^8 \right]$$

$$8 \prod_{n=1}^{\infty} \left(\frac{1-x^{n-\frac{1}{2}}}{1-x^n} \right)^8 = f_{R-}(x)$$



Closed superstrings

(NS, NS)
↑ left ↑ right
α
BOSONS

(NS, R)
FERM

(R, NS) ,
FERM

(R, R)
BOSON

Closed superstrings

left
 $(NS+)$
 $(R-)$

right
 $(NS+)$
 $(R+)$

$(NS+, NS+)$
 $(NS+, R+)$
 $(R-, NS+)$
 $(R-, R+)$

(NS, NS)
 $\uparrow$ left $\uparrow$ right
 $\bar{\alpha}$
 BOSONS

(NS, R)

FERM

$\bar{b}_{-\frac{1}{2}}^I |NS\rangle_L$ $b_{-\frac{1}{2}}^J |NS\rangle_R$

(R, NS)

FERM

(R, R)

BOSON

(R, NS) , (R, R)

$$\alpha_{-1}^I \bar{\alpha}_{-1}^J | p_L^+ p_T^- \rangle$$

FERM

BOSON

64 states

Closed superstrings

left	right
$\begin{pmatrix} NS+ \\ R- \end{pmatrix}$	$\begin{pmatrix} NS+ \\ R+ \end{pmatrix}$

type IIA
superstrings!

$$\begin{pmatrix} NS+, NS+ \\ NS+, R+ \\ R-, NS+ \\ R-, R+ \end{pmatrix}$$

$$\begin{pmatrix} NS, NS \end{pmatrix}$$

$\uparrow$ left $\uparrow$ right
 α BOSONS

$$\begin{pmatrix} NS, R \end{pmatrix}$$

$$\begin{pmatrix} R, NS \end{pmatrix}$$

FERM

FERM

$\bar{b}_{-\frac{1}{2}}^I NS\rangle_L$	$b_{-\frac{1}{2}}^J NS\rangle_R$	64
$\bar{b}_{-\frac{1}{2}}^I NS\rangle_L$	$ R\rangle_R$	64
$ R\rangle_L$	$b_{-\frac{1}{2}}^I NS\rangle_R$	64
$ R\rangle_L$	$ R\rangle_R$	

Closed superstrings

left right
 $(NS+)$ $(NS+)$
 $(R-)$ $(R+)$

type IIA
 superstrings!
 $(NS+, NS+)$
 $(NS+, R+)$
 $(R-, NS+)$
 $(R-, R+)$

(NS, NS)
 $\uparrow$ left $\uparrow$ right
 $\bar{\alpha}$ BOSONS

FERM

$\bar{b}_{-1/2}^I |NS\rangle_L$ $b_{-1/2}^J |NS\rangle_R$
 $\bar{b}_{-1/2}^I |NS\rangle_L$ $|R\rangle_R$
 $|R\rangle_L$ $b_{-1/2}^I |NS\rangle_R$
 $|R\rangle_L \otimes |R\rangle_R$

FERM

64 st
 64
 64
 64

$$\alpha_{-1}^I \bar{\alpha}_{-1}^J |P^+ P^- \rangle$$

gauge $\downarrow$ 35
 $\downarrow$ $28 + 1$
 del

Closed superstrings

left right
 $(NS+)$ $(NS+)$
 $(R-)$ $(R+)$

type IIA
 superstrings!
 $(NS+, NS+)$
 $(NS+, R+)$
 $(R-, NS+)$
 $(R-, R+)$

(NS, NS)
 $\uparrow$ left $\uparrow$ right
 $\bar{\alpha}$ BOSONS

FERM

$b_{-\frac{1}{2}}^{-I} |NS\rangle_L$ $b_{-\frac{1}{2}}^J |NS\rangle_R$
 $\bar{b}_{-\frac{1}{2}}^{-I} |NS\rangle_L$ $|R\rangle_R$
 $|R\rangle_L$ $b_{-\frac{1}{2}}^{-I} |NS\rangle_R$
 $|R\rangle_L \otimes |R\rangle_R$

(NS, R) (R, NS) (R, R)

FERM

64 states
 64 states fermi
 64 states fermi
 64 bosons

BOSON

ST $\frac{8 \times 9}{2} - 1 = 35$

gravitino

fermion

del

28 + 1

Majorana

gravitino

dilatino

$$\alpha_{-1}^{-I} \bar{\alpha}_{-1}^J |P^+ P^-\rangle$$

Closed superstrings

left right
 $(NS+)$ $(NS+)$
 $(R-)$ $(R+)$

type IIA
 superstrings!
 $(NS+, NS+)$
 $(NS+, R+)$
 $(R-, NS+)$
 $(R-, R+)$

(NS, NS) (NS, R) (R, NS) (R, R)
 $\uparrow$ $\uparrow$
 left right
 $\bar{\alpha}$ α
 BOSONS FERM FERM BOSON

$b_{-1/2}^{-I} |NS\rangle_L$ $b_{-1/2}^J |NS\rangle_R$
 $\bar{b}_{-1/2}^{-I} |NS\rangle_L$ $|R\rangle_R$
 $|R\rangle_L$ $b_{-1/2}^I |NS\rangle_R$
 $|R\rangle_L \otimes |R\rangle_R$

ST $\frac{8 \times 9}{2} - 1 = 35$, $28 + 1$
 Majorana
 " gravitino
 dilaton

$$\alpha_{-1}^I \bar{\alpha}_{-1}^J |P^+ P^- \rangle$$

$(R, NS), (R, R)$

$$\alpha_{-1}^I, \bar{\alpha}_{-1}^J | P^+ P^- \rangle$$

FERM

BOSON

gravitino, Kalb-R, dil

64 states

$$ST \frac{8 \times 9}{2} - 1 = 35, 28 + 1$$

64 states fermi

64 states fermi

Majorana, gravitino, dilations

64 bosons

$$8 \times 8' = 8 + 56$$

$A_\mu, A_{\mu\nu\rho}$

$A_{\mu\nu\rho}$

A_{IJK}

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3}$$

$(R, NS), (R, R)$

$$\alpha_{-1}^I, \bar{\alpha}_{-1}^J | P^+ P^- \rangle$$

FERM

BOSON

gravitino

Kalb-R

dil

$A_{\mu\nu\rho}$

A_{IJK}

64 states

$$ST \frac{8 \times 9}{2} - 1 = 35$$

$$28 + 1$$

$g_{\mu\nu}, b_{\mu\nu}, \phi$

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3}$$

64 states fermi

64 states fermi

Majorana

gravitino

ψ_α

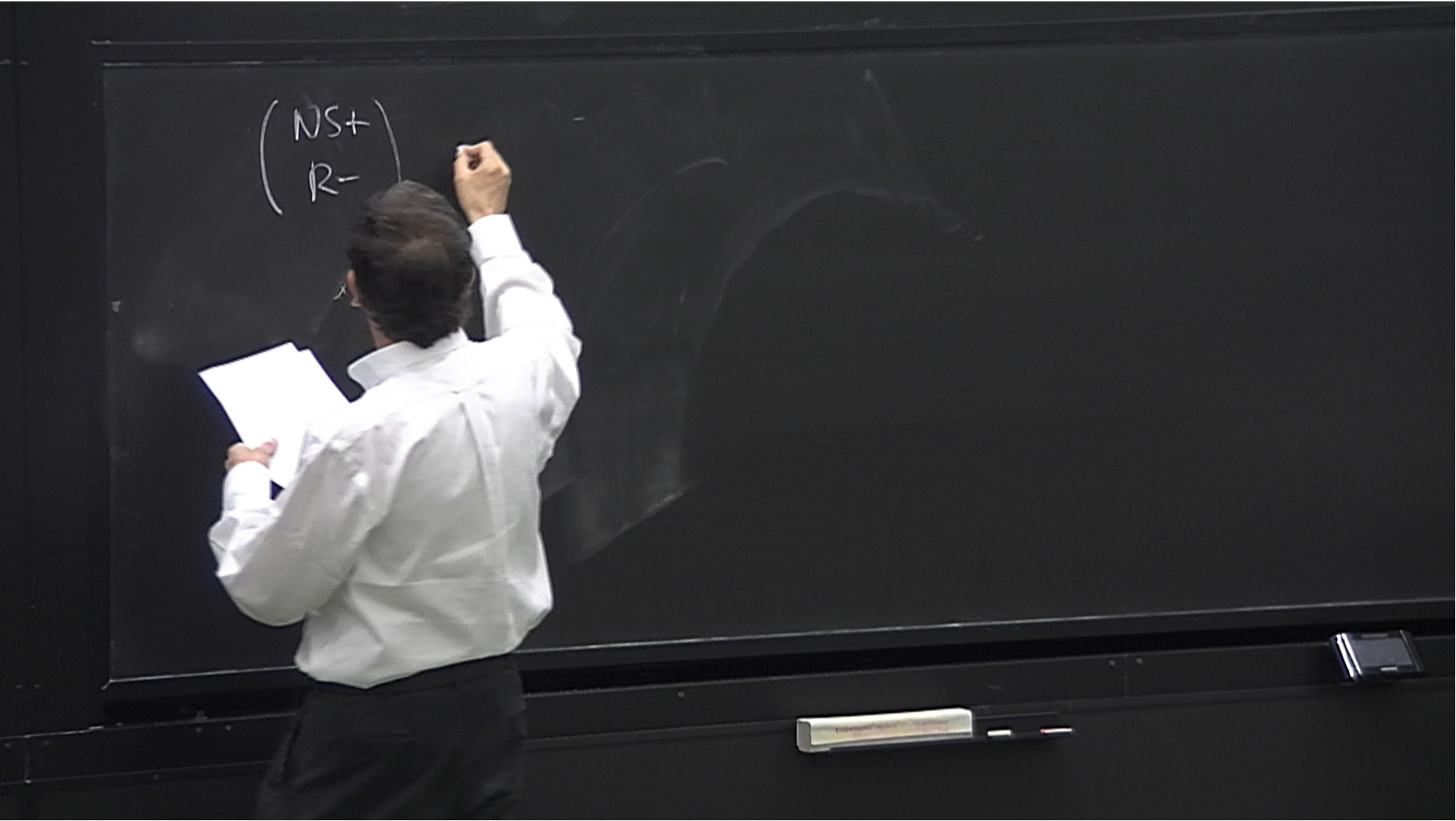
dilatino

χ_α

64 bosons

$$8 \times 8' = 8 + 56$$

$A_\mu, A_{\mu\nu\rho}$



$\begin{pmatrix} NS+ \\ R- \end{pmatrix}$	$\begin{pmatrix} NS+ \\ R- \end{pmatrix}$	$(NS+, NS+)$	$\longrightarrow$ same		
left		$(NS+, R-)$	NS	$ Ra\rangle_R$	64 fm
		$(R-, NS+)$	$ Ra\rangle_L$	NS	64 fm

$$\begin{array}{ccc}
 \begin{pmatrix} NS+ \\ R- \end{pmatrix} & \begin{pmatrix} NS+ \\ R- \end{pmatrix} & (NS+, NS+) \longrightarrow \text{same} \\
 \text{left} & & \\
 & & \begin{array}{ccc}
 (NS+, R-) & NS & |Ra\rangle_R & 64 \text{ ferm} \\
 (R-, NS+) & |Ra\rangle_L & NS & 64 \text{ ferm} \\
 (R-, R-) & |Ra\rangle_L & |Rb\rangle_R & 8 \times 8 = 1 + 28 + 35 + \\
 & & & A
 \end{array}
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Weyl fermions} \\ \text{" dilatino"} \end{array}$$

$$\begin{array}{ccc}
 \begin{pmatrix} NS+ \\ R- \\ \text{left} \end{pmatrix} & \begin{pmatrix} NS+ \\ R- \end{pmatrix} & (NS+, NS+) \longrightarrow \text{same} \\
 & & \begin{array}{ccc}
 (NS+, R-) & NS & |Ra\rangle_R \\
 (R-, NS+) & |Ra\rangle_L & NS \\
 (R-, R-) & |Ra\rangle_L & |Rb\rangle_R
 \end{array}
 \end{array}
 \left. \begin{array}{l} 64 \text{ ferm} \\ 64 \text{ ferm} \end{array} \right\} \begin{array}{l} \text{Weyl fermions} \\ \text{" dilatino"} \end{array}$$

$$8 \times 8 = 1 + 28 + 35 +$$

$$A, A_{\mu\nu}, A$$

DI-branes

$$\frac{2}{8 \cdot 7 \cdot 6 \cdot 5} = 70$$

$$\frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4}$$

64 ferm } Weyl gravitino
64 ferm } " dilatino

$$8 \times 8 = 1 + 28 + 35 +$$

Self dual
4 form

$A, A_{\mu\nu}, A_{\mu\nu\rho\sigma}$
↓
DI-branes

$\begin{pmatrix} NS+ \\ R- \end{pmatrix}$

left

type II B

$\begin{pmatrix} NS+ \\ R- \end{pmatrix}$

$(NS+, NS+)$

→ same

$(NS+, R-)$

NS

$|Ra\rangle_R$

64 fem

$(R-, NS+)$

$|Ra\rangle_L$

NS

64 fem

$(R-, R-)$

$|Ra\rangle_L$

$|Rb\rangle_R$

8x8

$(R, NS), (R, R)$

$$\alpha_{-1}^I, \bar{\alpha}_{-1}^J | P^+ P^- \rangle$$

$$F^{(5)} = dA^{(4)}$$

$$F^{(5)} = F^{*(5)}$$

FERM

BOSON

gravitino, Kalb-R, dil

64 states

$$ST \frac{8 \times 9}{2} - 1 = 35$$

$$28 + 1$$

$g_{\mu\nu}, b_{\mu\nu}, \phi$

64 states fermi

64 states fermi

Majorana

gravitino
dilatinos

ψ_α
 χ_α

table branes

$$64 \text{ bosons} \quad 8 \times 8' = 8 + 56$$

$$A_\mu, A_{\mu\nu}$$

Closed superstrings

left right
 $\begin{pmatrix} NS+ \\ R- \end{pmatrix}$ $\begin{pmatrix} NS+ \\ R+ \end{pmatrix}$

type IIA
 superstrings!

$\begin{pmatrix} NS+, NS+ \\ NS+, R+ \\ R-, NS+ \\ R-, R+ \end{pmatrix}$

$\begin{pmatrix} NS, NS \end{pmatrix}$
 $\uparrow \quad \uparrow$
 left right
 α BOSONS

$\begin{pmatrix} NS, R \end{pmatrix}$

$\begin{pmatrix} R, NS \end{pmatrix}$

FERM

ERM

$\bar{b}_{-\frac{1}{2}}^I |NS\rangle_L$
 $\bar{b}_{-\frac{1}{2}}^I |NS\rangle_L$
 $|R_\alpha\rangle$
 $|R_\alpha\rangle$

64 sta

64 sta

4 sta

4

Hellenistic Stanzas Left & Right



Heterotic String

Left $\otimes$ Right
(bosonic string) (superstring)

Heterotic String

$24(X^{I'}\text{'s})$

Exotic $16 X^{I'}$
Lattice $\rightleftharpoons$

Left $\otimes$ Right
(bosonic string) (superstring)

$8 X^{I'}\text{'s}$

32 fermions $\lambda^{A=1..32}$

NS+

R-

Heterotic String

$24(X^{I'}\text{'s})$

Left & Right

(bosonic
string)

$8 X^{I'}\text{'s}$

Exotic $16 X^{I'}$

Lattice

$N^I=0$ 1
 $N^I=1$ 504
 $N^I=2$ 40,996

$\Rightarrow$ 32 fermions

$A=1.32$

NS'

R'

Heterotic String

24(X^I's)

Left ⊗ Right
(bosonic string)

(superstring)

Exotic 16 X^I's
Lattice

N⁺=0 1
N⁺=1 504
N⁺=2 40,996

8 X^I's

32 fermions $\lambda^{A=1..32}$

NS⁺

R⁺

NS⁺

R⁻

$$(-1)^F |NS'\rangle = + |NS'\rangle$$

Heterotic String

$24(X^{I'}s)$

Left $\otimes$ Right
(bosonic string) (superstring)

$8 X^{I'}s$

Exotic $16 X^{I'}$

Lattice

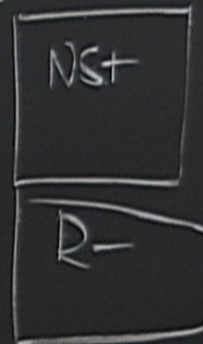
$N^I=0$ 1
 $N^I=1$ 504
 $N^I=2$ 40,996

32 fermions

$\lambda^{A=1..32}$

NS^+

R^+



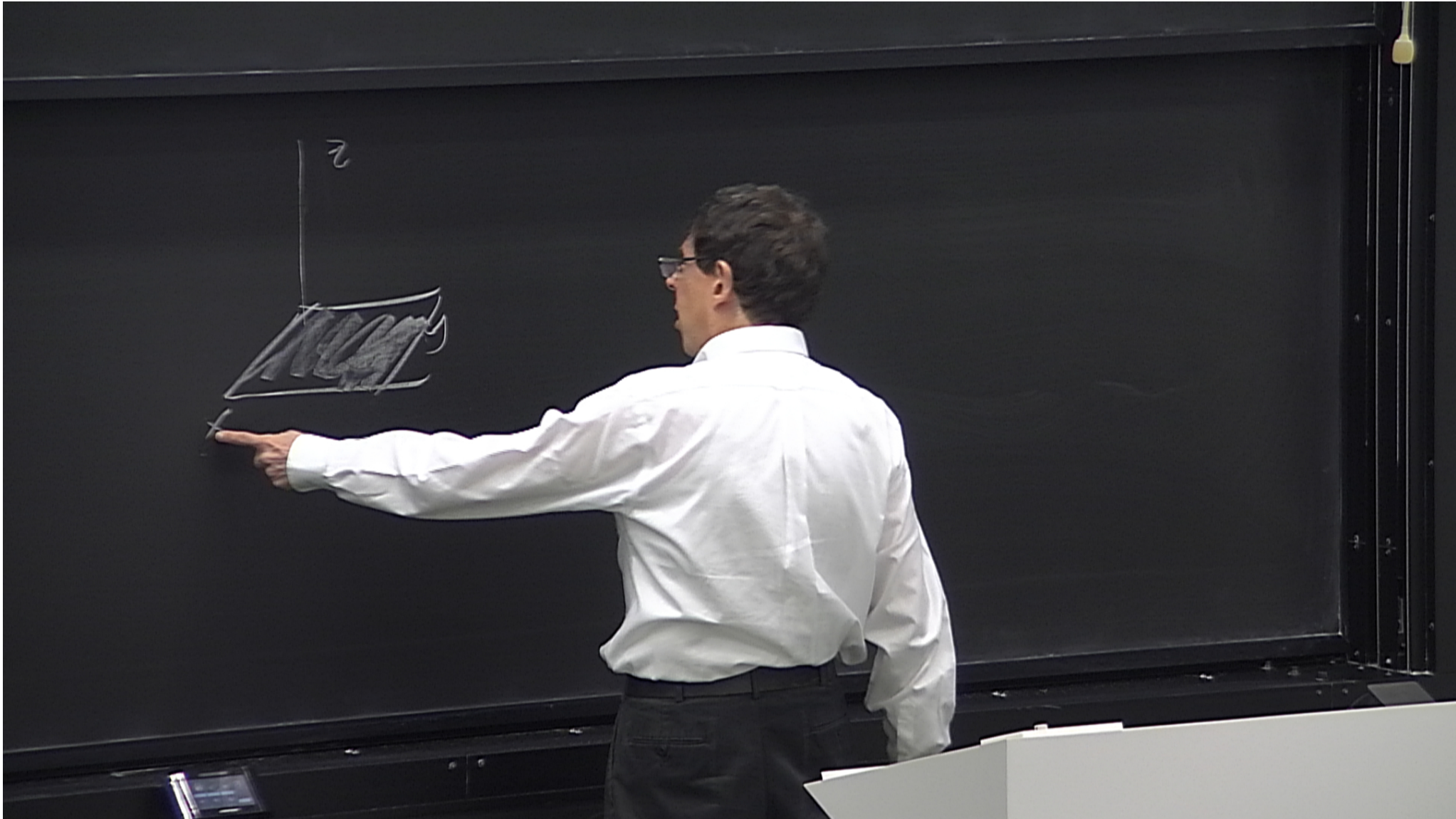
$$(-1)^F |NS^+\rangle = + |NS^+\rangle$$

Heterotic

Dp-brane

$x^0 x^1 \dots x^p$ $x^{p+1} \dots x^d$
Dp tangential
coordinates





Dp-brane

$$X^M(\tau, \sigma)$$

$$\underbrace{x^0 x^1 \dots x^p}_{\text{Dp tangential coordinates}} \quad \underbrace{x^{p+1} \dots x^d}_{\text{normal coordinates}}$$

$$x^0, x^1, x^2, \dots, x^p, \underbrace{x^{p+1}, \dots, x^d}_{x^a}$$

Dp-brane

$$X^M(\tau, \sigma)$$

$$\underbrace{x^0 x^1 \dots x^p}_{\text{Dp tangential coordinates}} \quad \underbrace{x^{p+1} \dots x^d}_{\text{normal coordinates}}$$

$$\underbrace{x^0, x^1, x^2, \dots, x^p}_{\substack{x^+, x^- \\ i \text{ runs over } (p-1) \text{ values}}}, \underbrace{x^1, \dots, x^p}_{x^L}, \underbrace{x^{p+1}, \dots, x^d}_{x^a \text{ a over } (d-p) \text{ values}}$$

Dp-brane

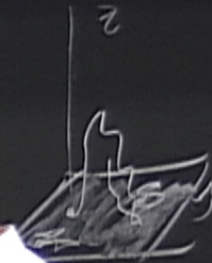
$$X^M(\tau, \sigma)$$

$$\underbrace{x^0 x^1 \dots x^p}_{\text{Dp tangential coordinates}} \underbrace{x^{p+1} \dots x^d}_{\text{normal coordinates}}$$

$$\underbrace{x^0, x^1}_{x^+, x^-}, \underbrace{x^2 \dots x^p}_{x^L}, \underbrace{x^{p+1} \dots x^d}_{x^a} \text{ a over } (d-p) \text{ values}$$

(runs over (p-1) values)

$$X^d(\tau, \sigma_*)$$



Dp-brane

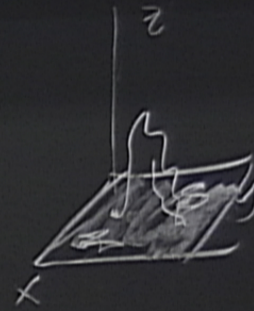
$$X^M(\tau, \sigma)$$

$$\underbrace{x^0 x^1 \dots x^p}_{\text{Dp tangential coordinates}} \quad \underbrace{x^{p+1} \dots x^d}_{\text{normal coordinates}}$$

$$\underbrace{x^0 x^1}_{X^+, X^-}, \underbrace{x^2 \dots x^p}_{X^L}, \underbrace{x^{p+1} \dots x^d}_{X^a}$$

i runs over $(p-1)$ values
 Neumann, free

Dirichlet
 a over $(d-p)$ values
 $X^a(\tau, \sigma_*) = \bar{X}^a$
 $\frac{0}{\pi}$



$$X^a(\tau, \sigma) = \bar{X}^a + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^a e^{-in\tau} \sin n\sigma \quad \text{no } p^a$$

$$|p^+ p^- \rangle \rightarrow |p^+, p^-, p^+ \rangle$$

$$\begin{aligned} & |p^+, p^- \rangle \xrightarrow{L=2 \dots p} \text{tach} \\ & \psi(\tau, p^+, p^-) \\ & \psi(x^+, x^-, x^+) \end{aligned}$$

$$\alpha' M^2 = -1 + N^L$$

$$\begin{aligned} & \alpha_{-1}^L |p^+, \vec{p}\rangle \\ & \alpha_{-1}^a |p^+, \vec{p}\rangle \end{aligned}$$

Massless $p-1$ values = $(p+1)-2$
 one massless scalar for each normal direction

Maxwell for brane

