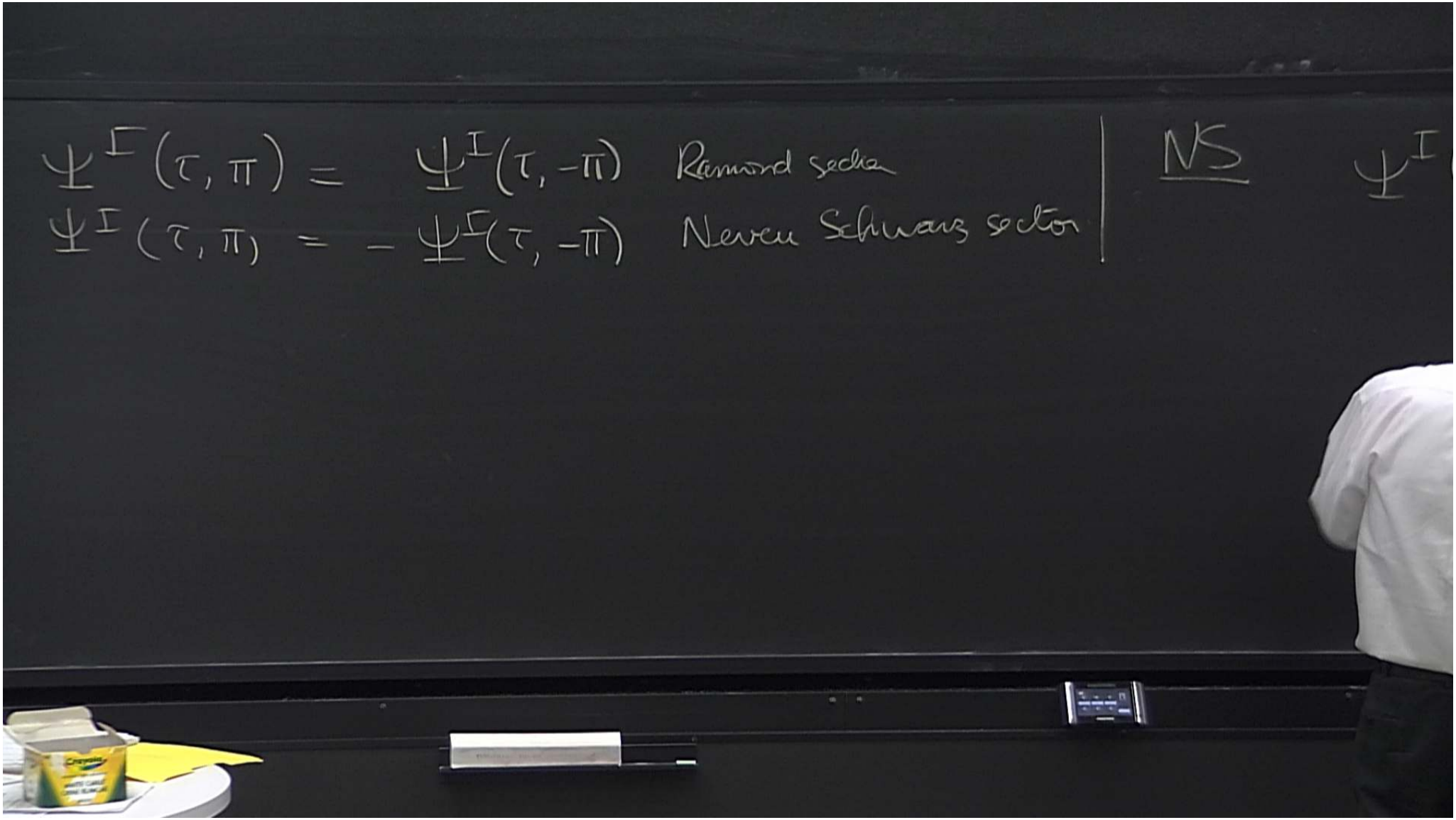
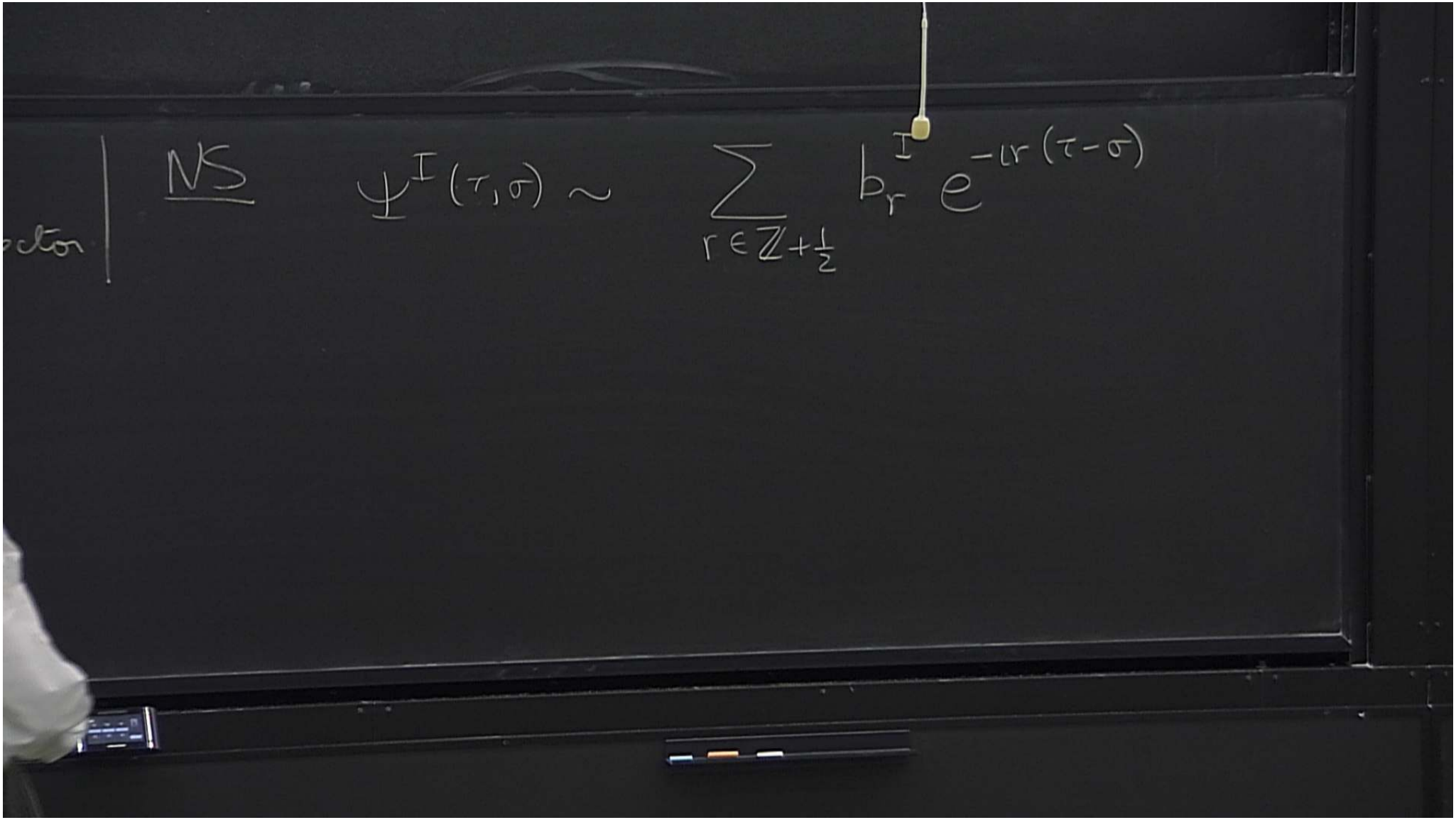
Title: 12/13 PSI - String Theory Review Lecture 14

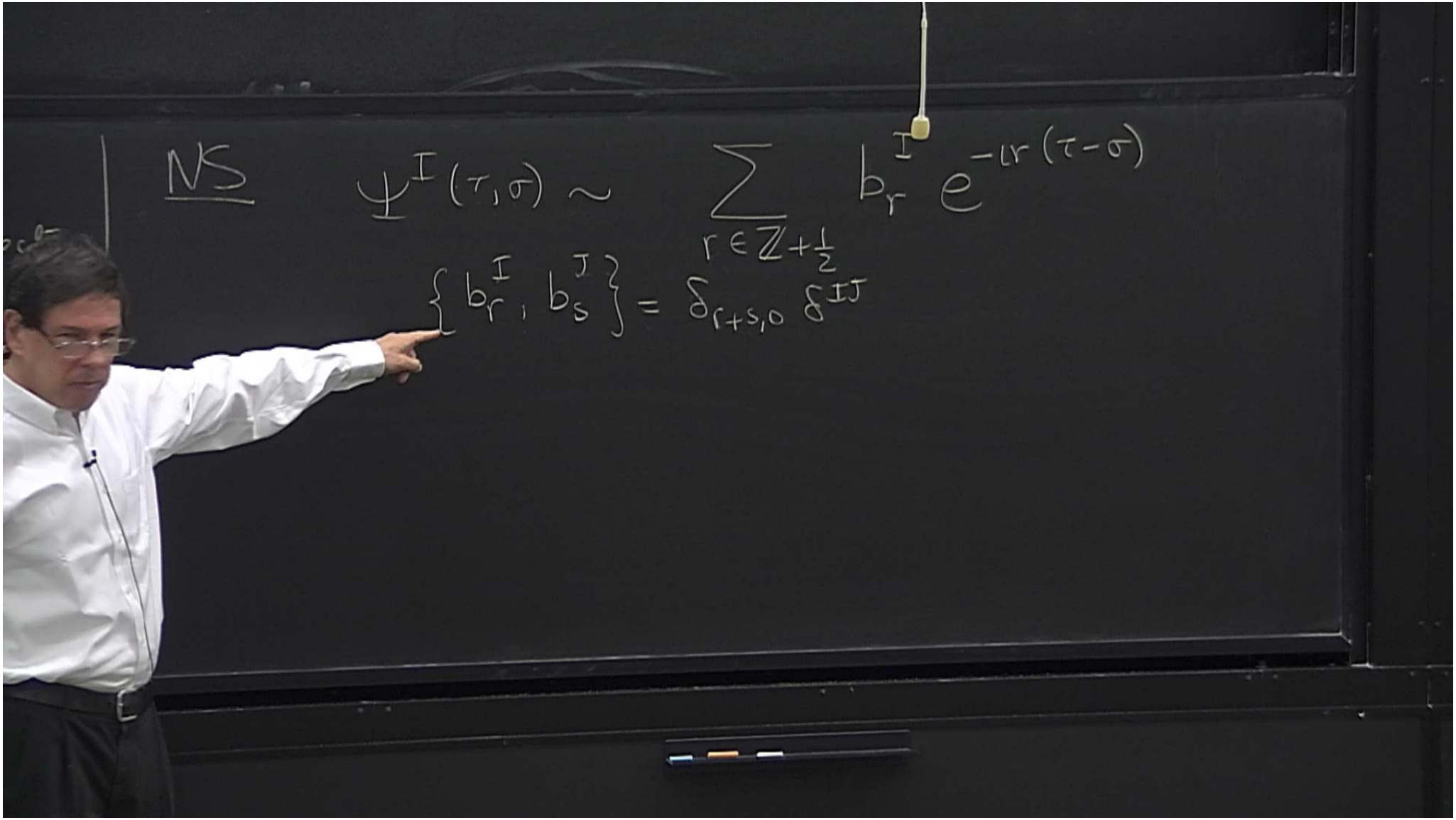
Date: Jan 24, 2013 10:15 AM

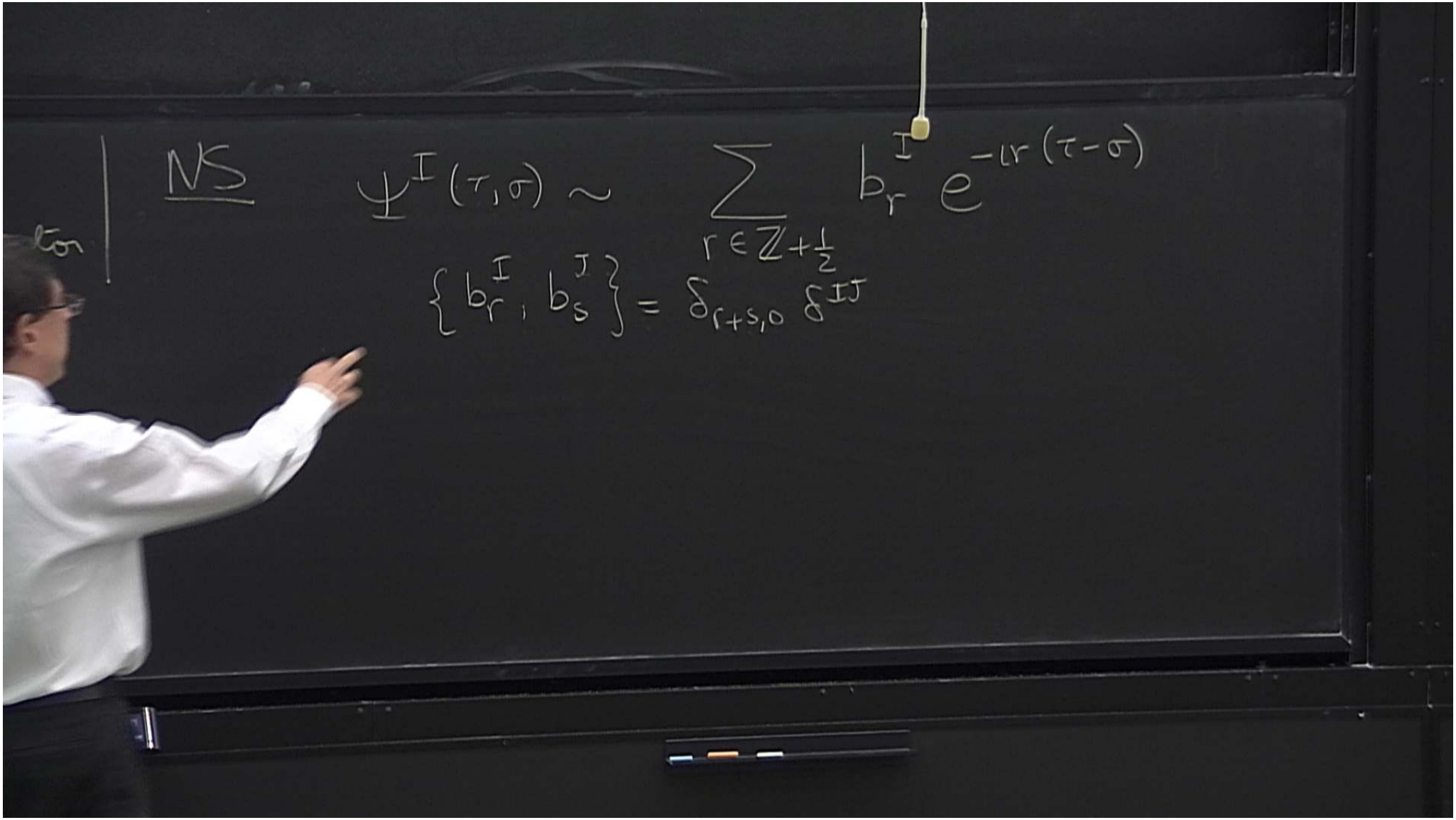
URL: http://pirsa.org/13010064

Abstract:


$\Psi^{I}(\tau,\pi) = \Psi^{I}(\tau,-\pi)$ Ramond sector
$\Psi^{I}(\tau,\pi) = -\Psi^{I}(\tau,-\pi)$ Neveu Schwarz sector
NS


NS
$\psi^I(\tau,\sigma) \sim \sum_{r \in \mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$


NS
$\psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$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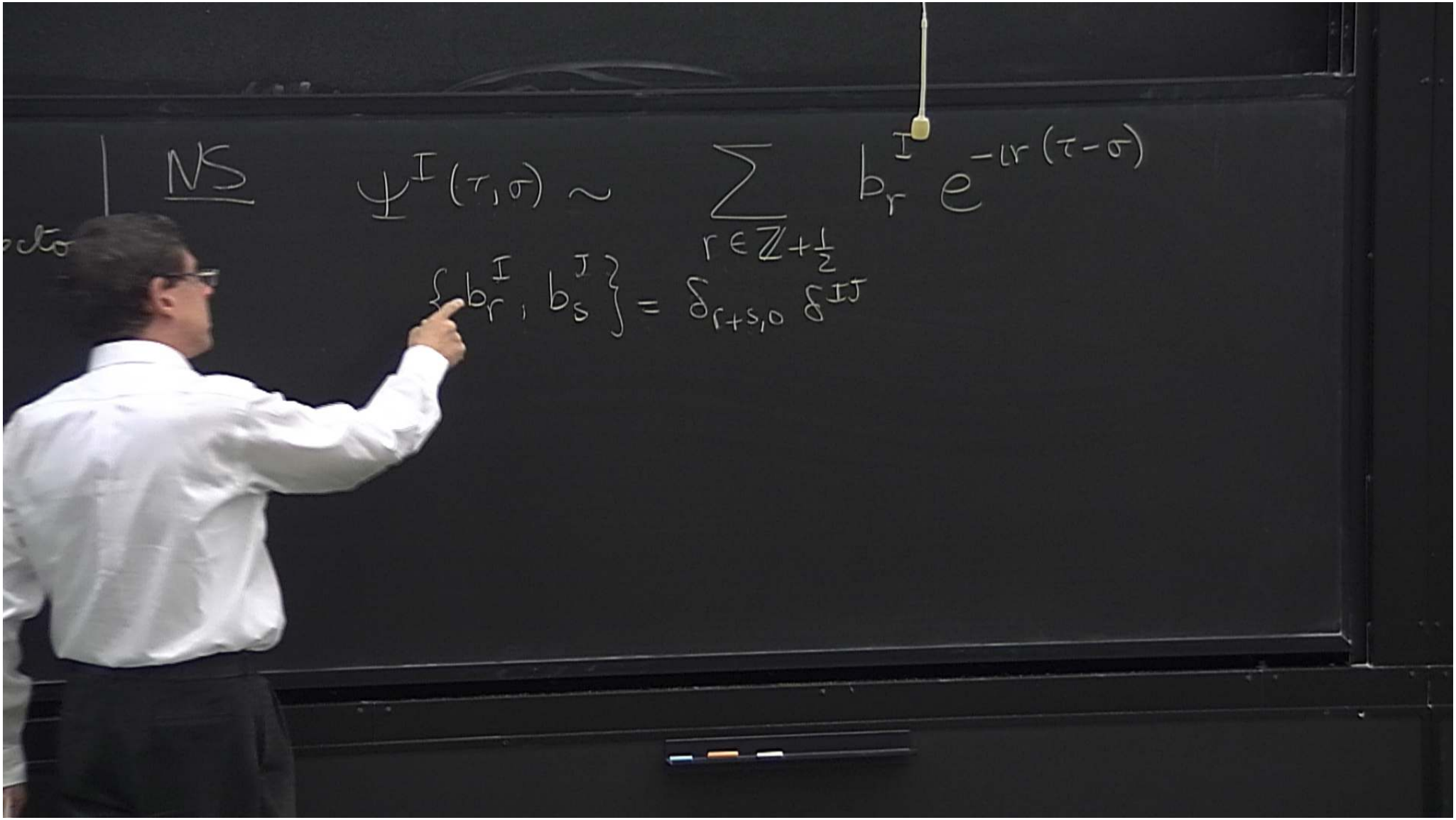
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$\psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$
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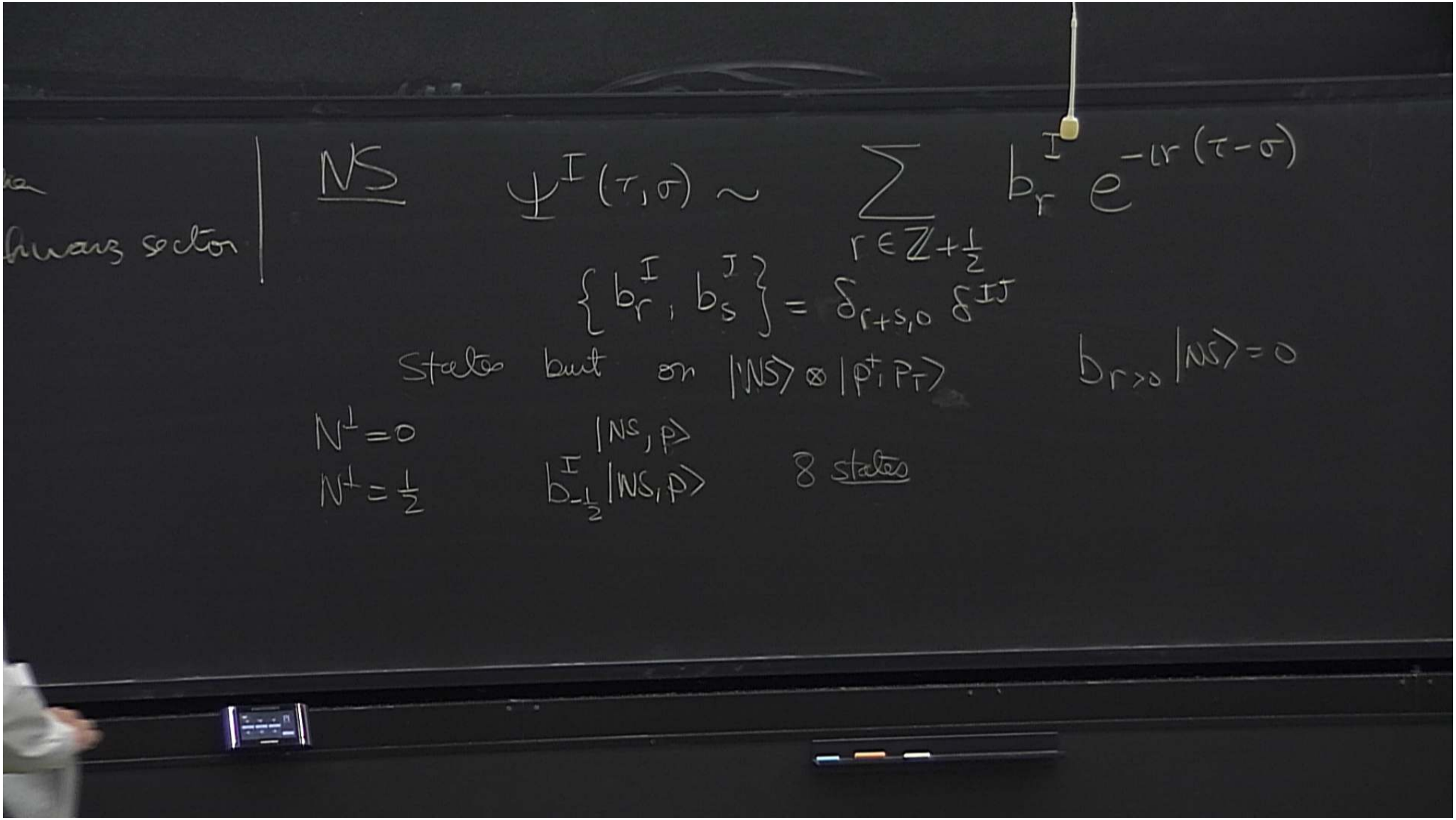


...ha

...hwarz sector

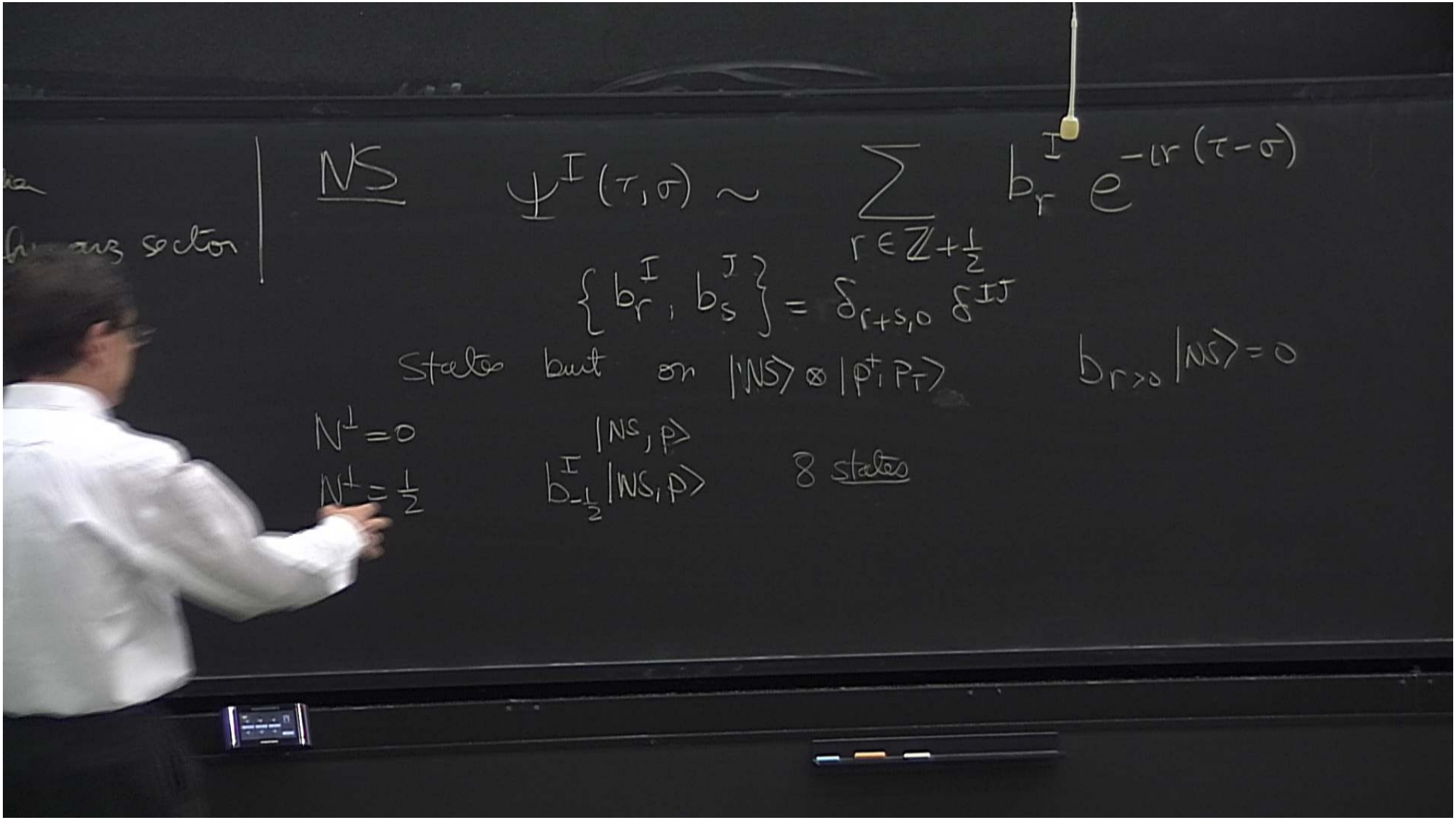
NS $\quad \Psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$

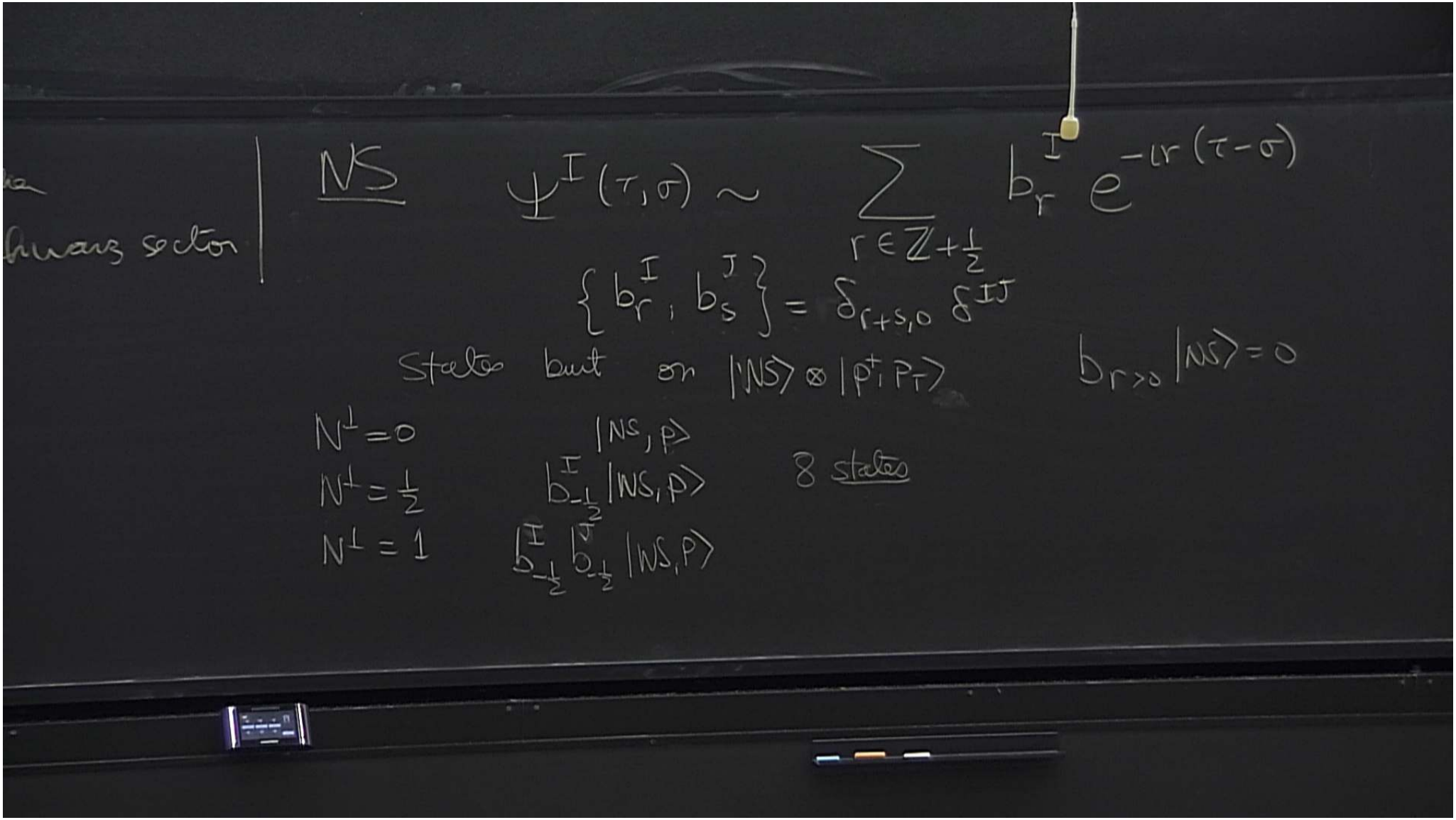
$$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$$

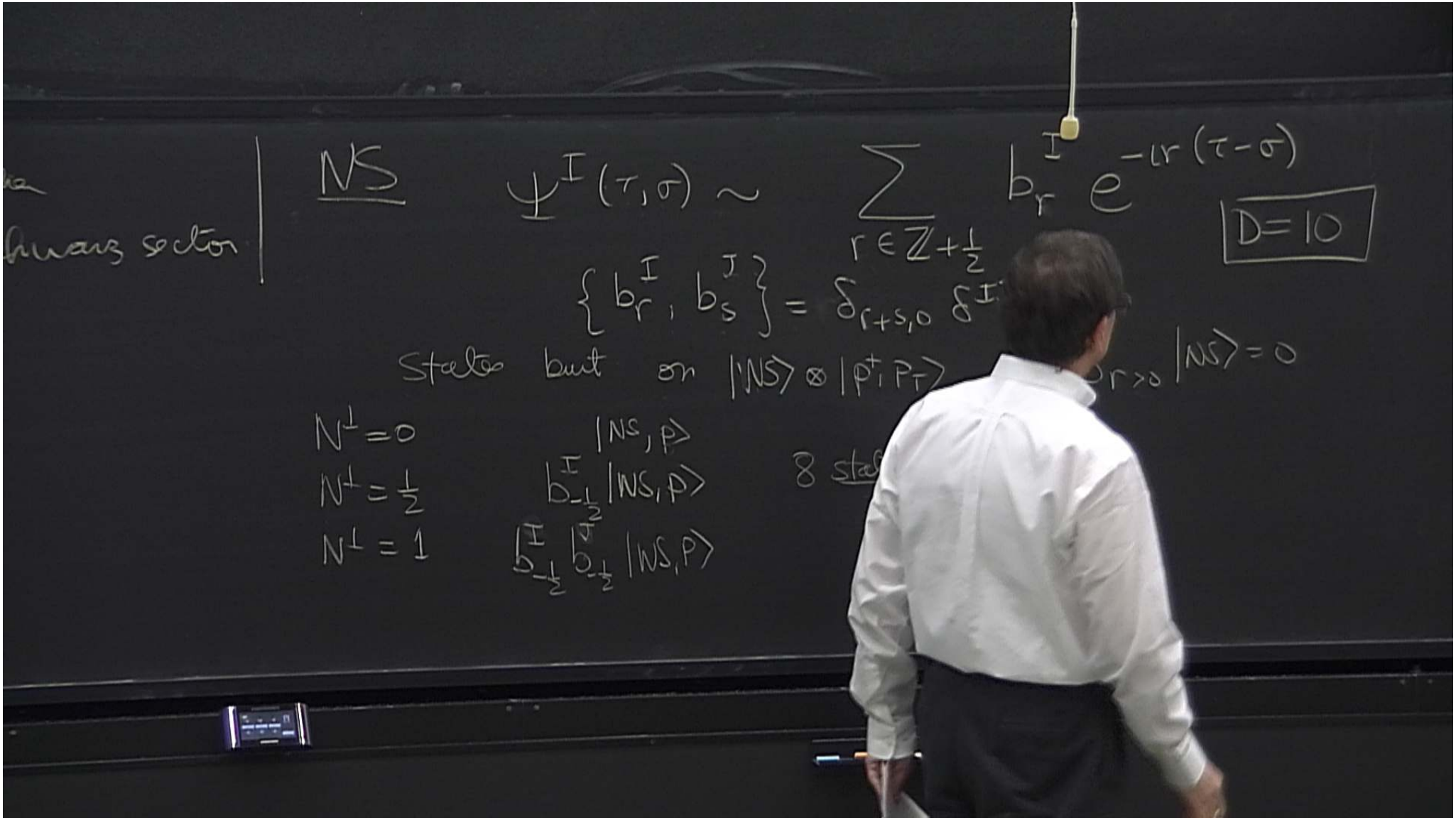
States built on $|NS\rangle \otimes |p^+, p_T\rangle$ $\qquad b_{r>0}|NS\rangle = 0$

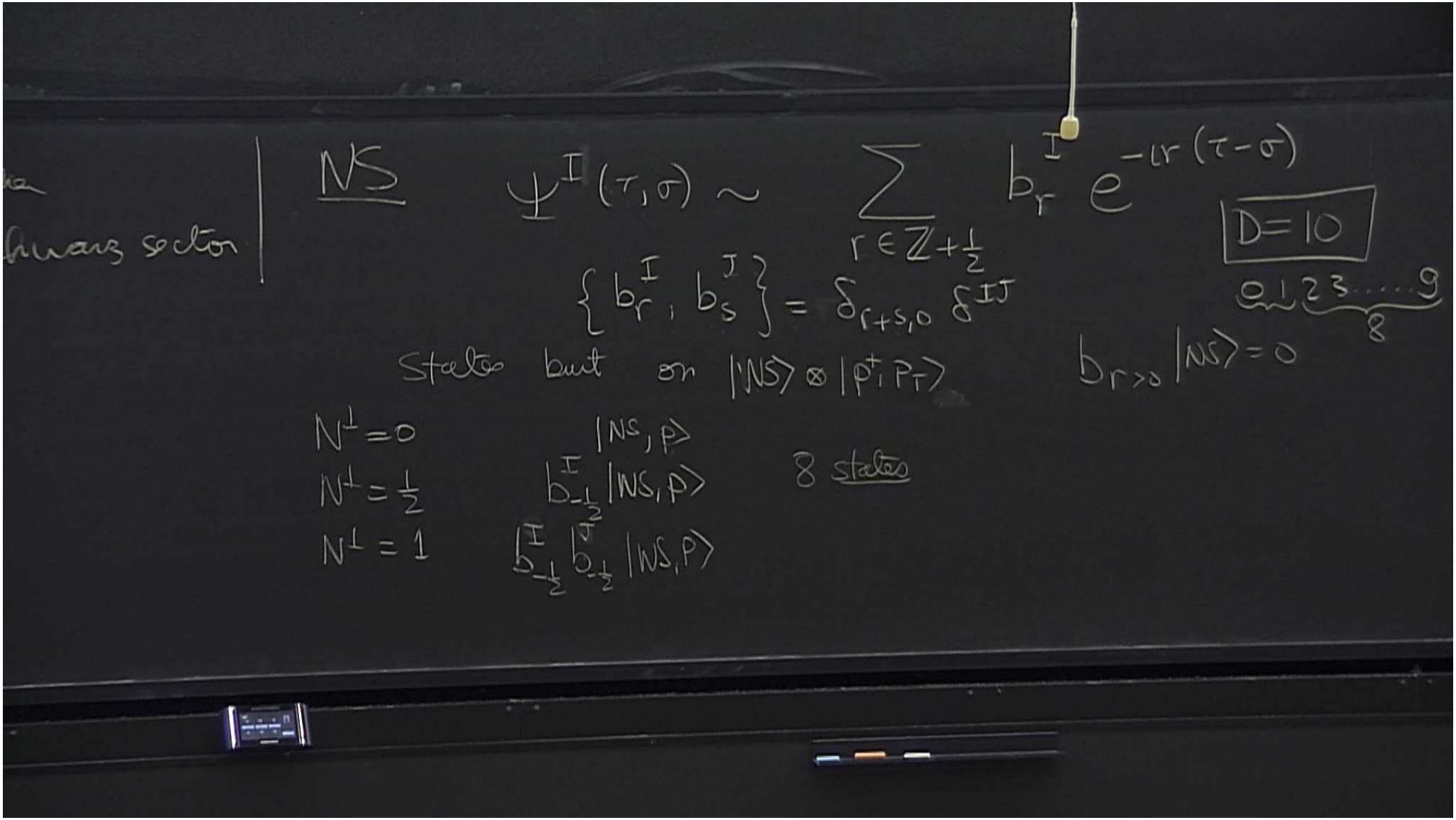
$N^\perp = 0 \qquad |NS, p\rangle$

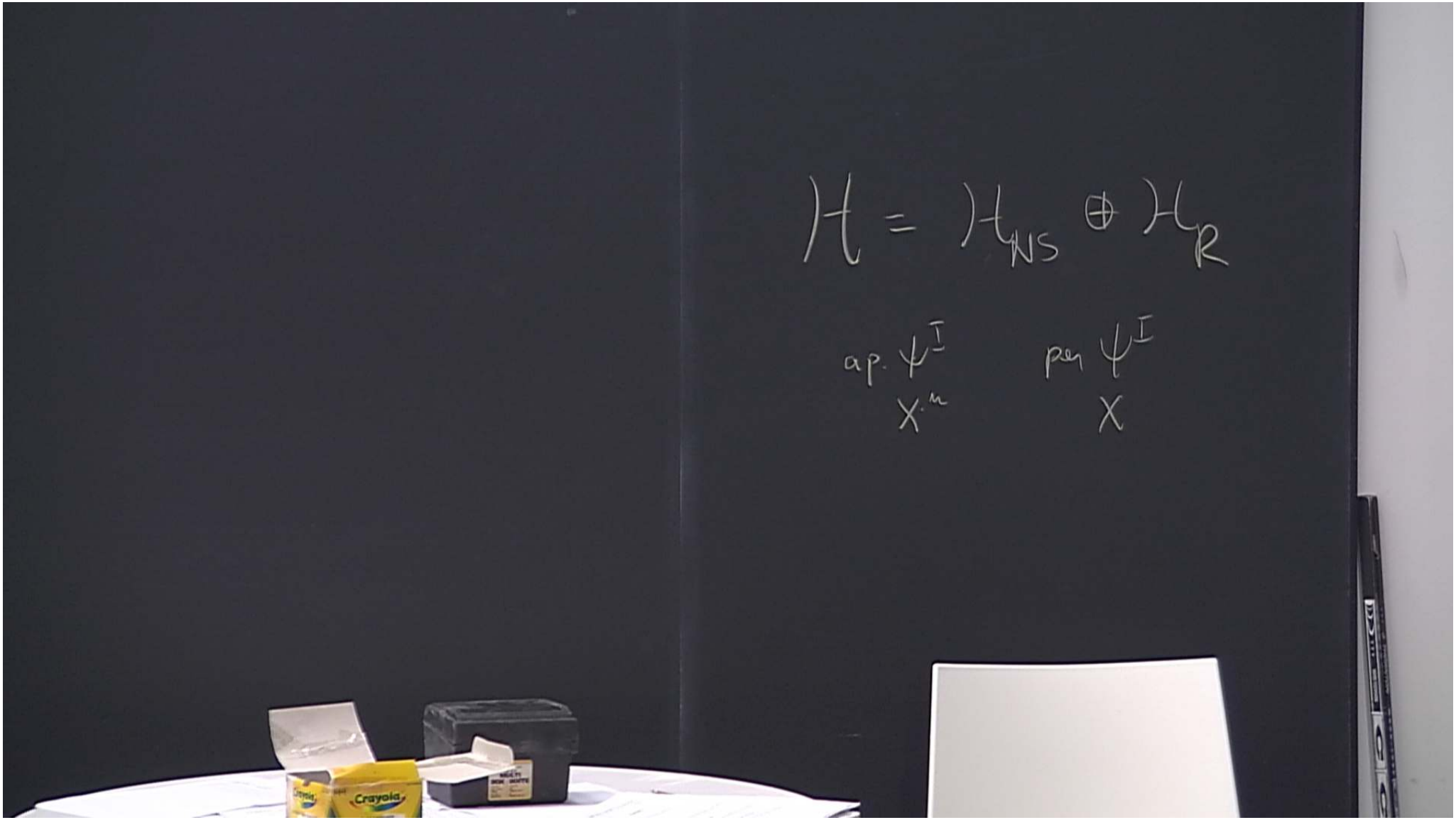
$N^\perp = \frac{1}{2} \qquad b^I_{-\frac{1}{2}}|NS, p\rangle \qquad$ 8 states


NS
$\Psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$
States built on $|NS\rangle \otimes |p^+, p_T\rangle$
$b_{r>0}|NS\rangle = 0$
$N^\perp = 0$
$|NS, p\rangle$
$N^\perp = \frac{1}{2}$
$b_{-\frac{1}{2}}^I |NS, p\rangle$
8 states


NS
$\psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$
States built on $|NS\rangle \otimes |p^+, p_T\rangle$
$b_{r>0}|NS\rangle = 0$
$N^\perp = 0$ $|NS,p\rangle$
$N^\perp = \frac{1}{2}$ $b_{-\frac{1}{2}}^I |NS,p\rangle$
8 states
$N^\perp = 1$ $b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J |NS,p\rangle$


NS
$\Psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$
D=10
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$
$N^\perp = 0$ $|NS,p\rangle$
$N^\perp = \frac{1}{2}$ $b_{-\frac{1}{2}}^I |NS,p\rangle$
$N^\perp = 1$ $b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J |NS,p\rangle$


NS
$\Psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$
D=10
0 1 2 3 9
8
States built on $|NS\rangle \otimes |p^+, p_T\rangle$
$b_{r>0}|NS\rangle = 0$
$N^\perp = 0$ $|NS,p\rangle$
$N^\perp = \frac{1}{2}$ $b_{-\frac{1}{2}}^I|NS,p\rangle$ 8 states
$N^\perp = 1$ $b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J|NS,p\rangle$


$\mathcal{H} = \mathcal{H}_{NS} \oplus \mathcal{H}_R$
ap. ψ^I
X^μ
per ψ^I
X
Crayola

NS $\quad \psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$

$D=10$

$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$ $\quad 0, \underbrace{1\,2\,3\,\ldots\,9}_{8}$

States built on $|NS\rangle \otimes |p^+, p_T\rangle$ $\quad b_{r>0}|NS\rangle = 0$

$N^\perp = 0$	$\|NS, p\rangle$	
$N^\perp = \frac{1}{2}$	$b_{-\frac{1}{2}}^I \|NS,p\rangle$	8 states
$N^\perp = 1$	$b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J \|NS,p\rangle$	$\alpha_{-1}^I \|NS,p\rangle$ $\quad \frac{8\cdot 7}{2} + 8 = 36$

NS section

NS $\quad \psi^I(\tau,\sigma) \sim \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^I e^{-ir(\tau-\sigma)}$

$D=10$

$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$ $\quad 0, 1, 2, 3, \ldots, 9$ (8)

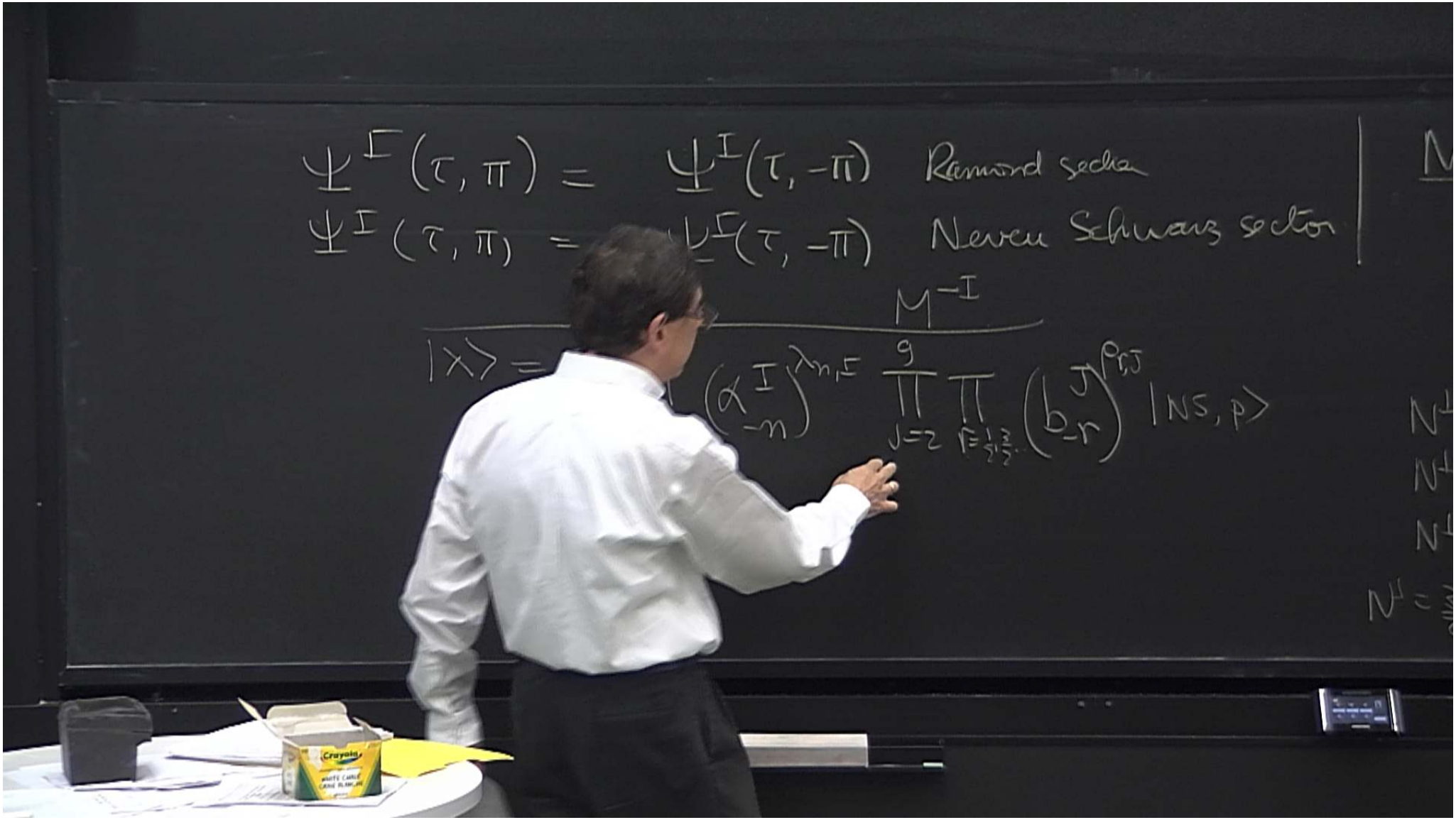
States built on $|NS\rangle \otimes |p^+, p_T\rangle$ $\quad b_{r>0}|NS\rangle = 0$

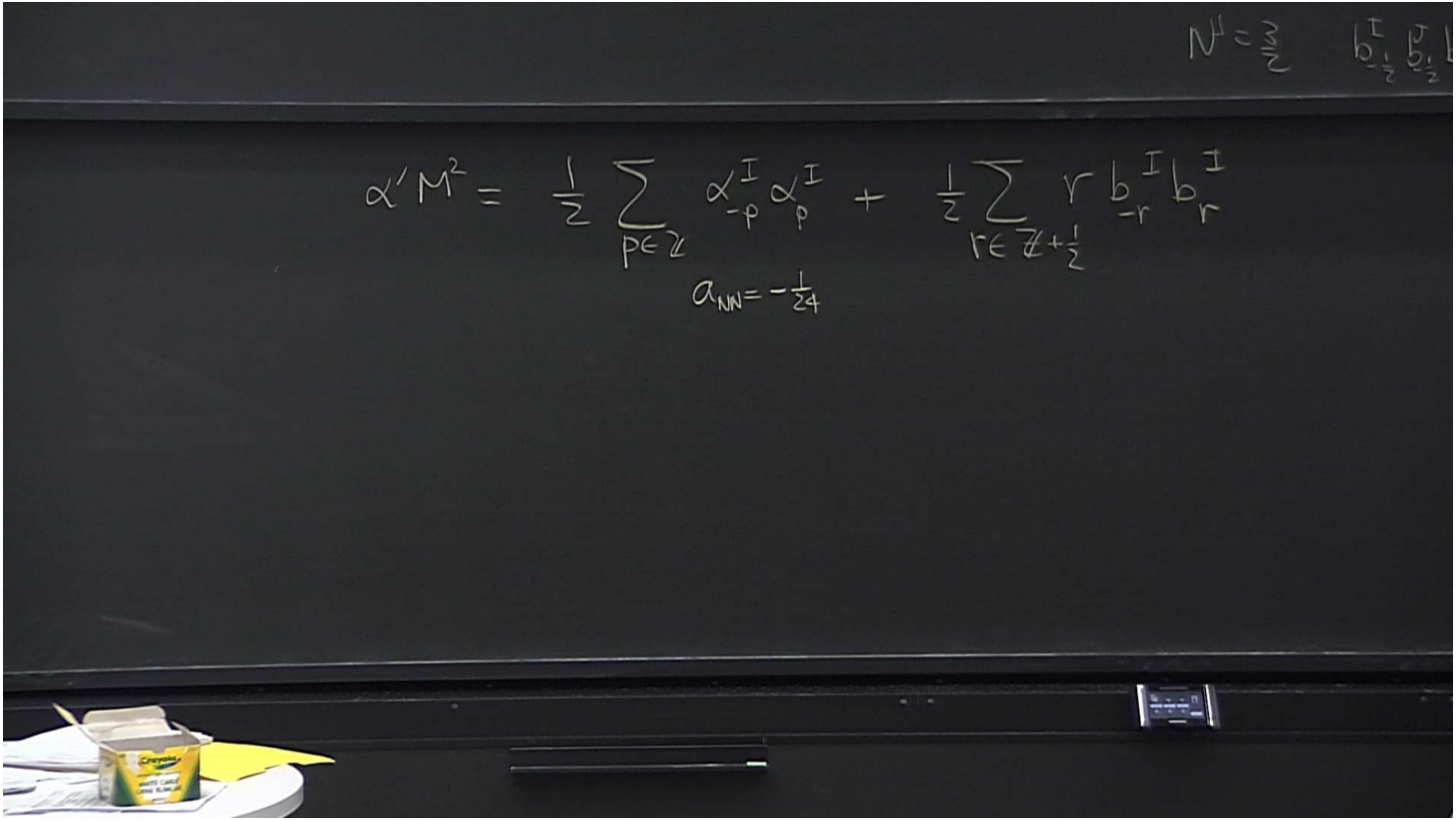
$N^\perp = 0$ $\quad |NS, p\rangle$

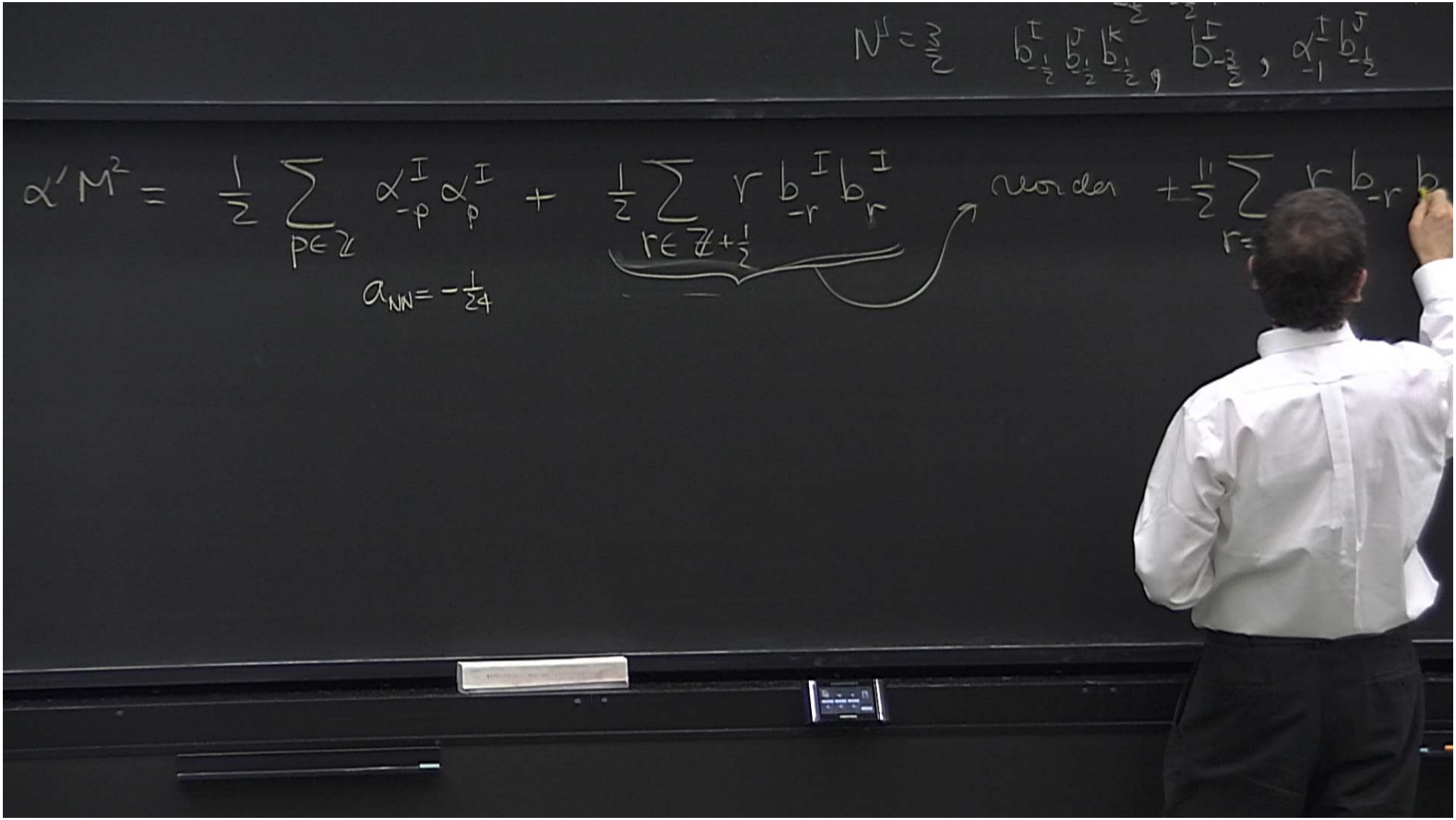
$N^\perp = \frac{1}{2}$ $\quad b_{-\frac{1}{2}}^I |NS,p\rangle$ $\quad 8$ states

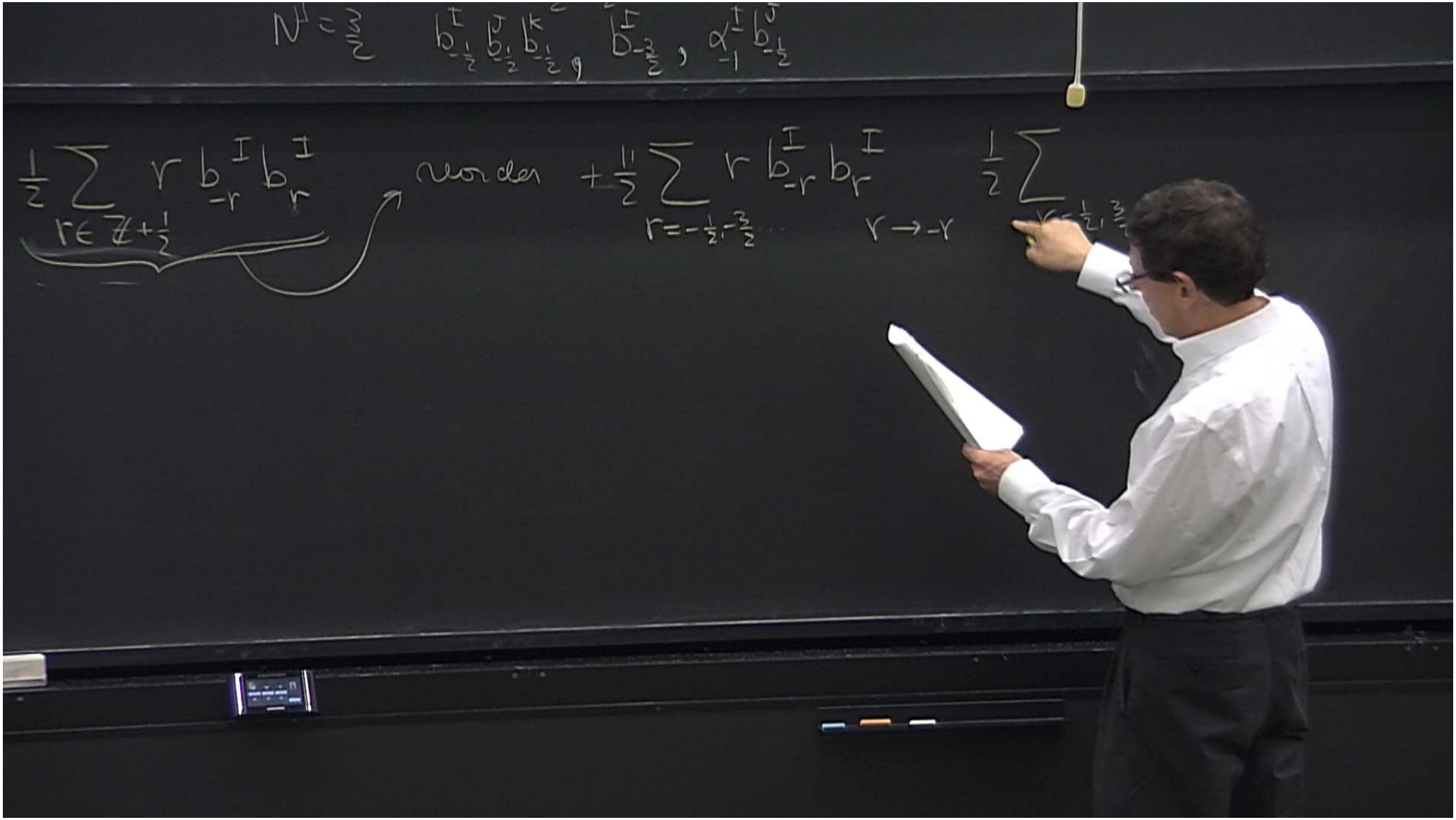
$N^\perp = 1$ $\quad b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J |NS,p\rangle$ $\quad \alpha_{-1}^I |NS,p\rangle$ $\quad \frac{8\cdot 7}{2} + 8 = 36$

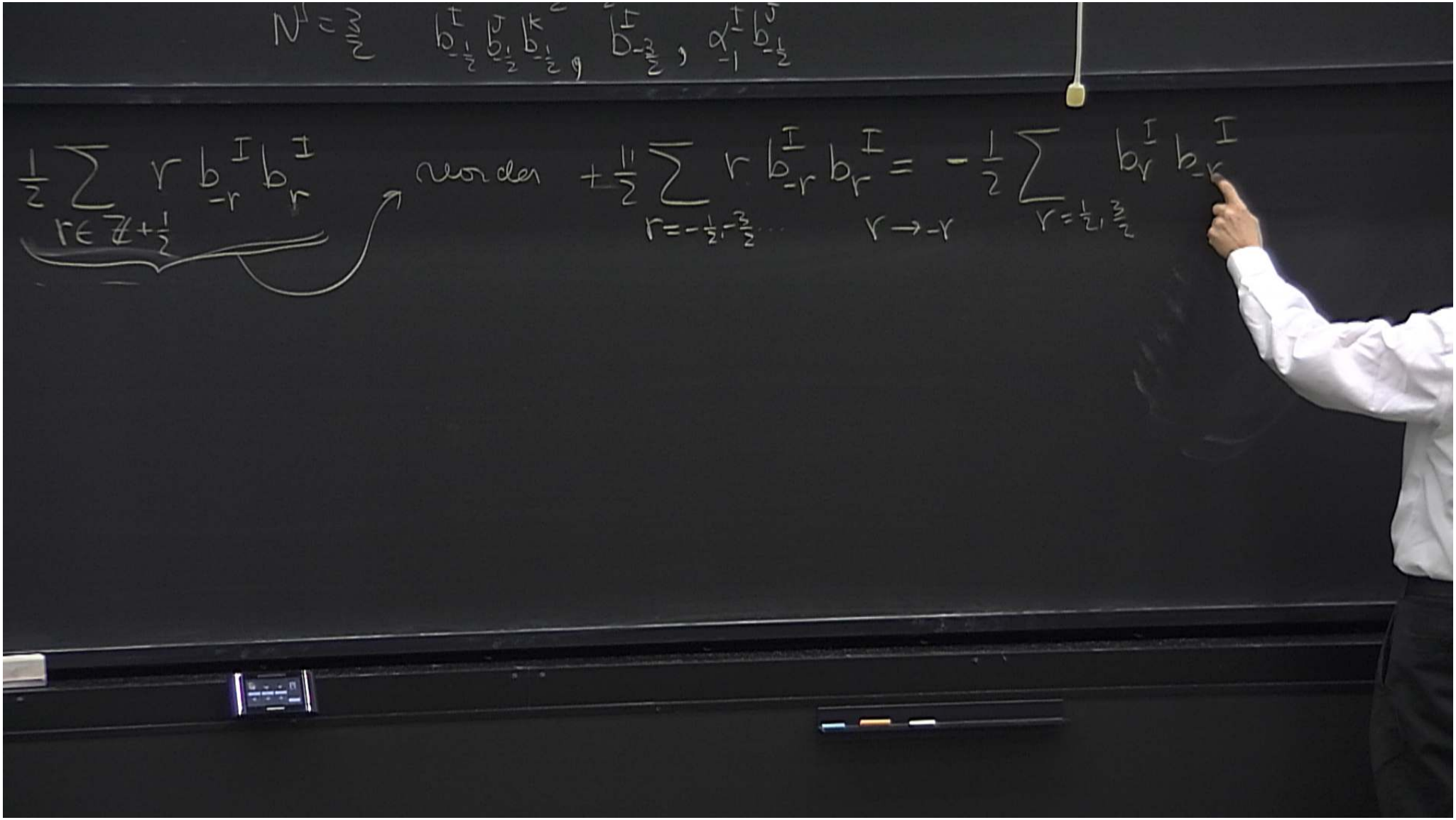
$N^\perp = \frac{3}{2}$ $\quad b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J b_{-\frac{1}{2}}^K |$, $b_{-\frac{3}{2}}^I |$, $\alpha_{-1}^I b_{-\frac{1}{2}}^J |$

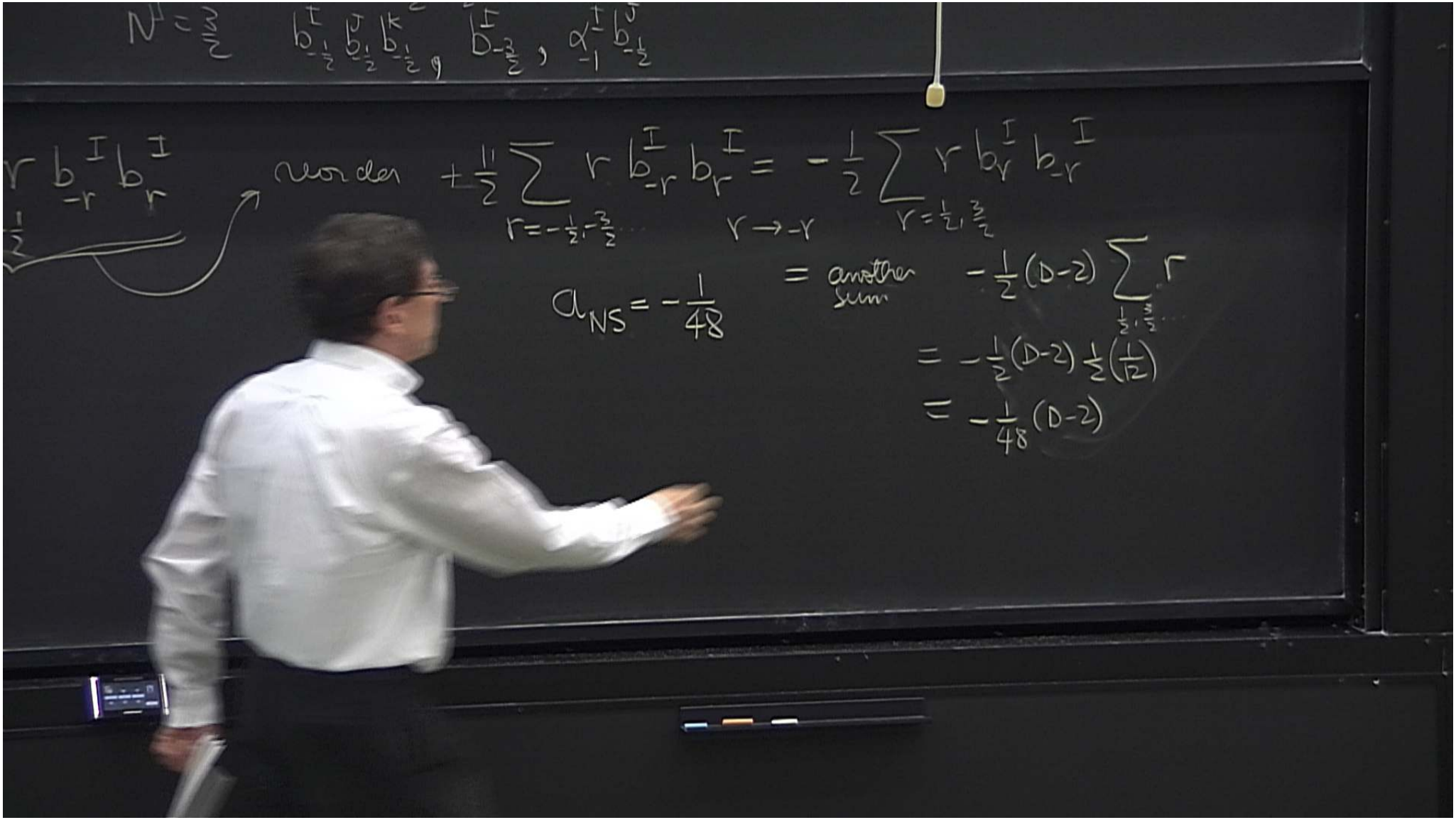

Ramond sector
Neveu Schwarz sector
Crayola

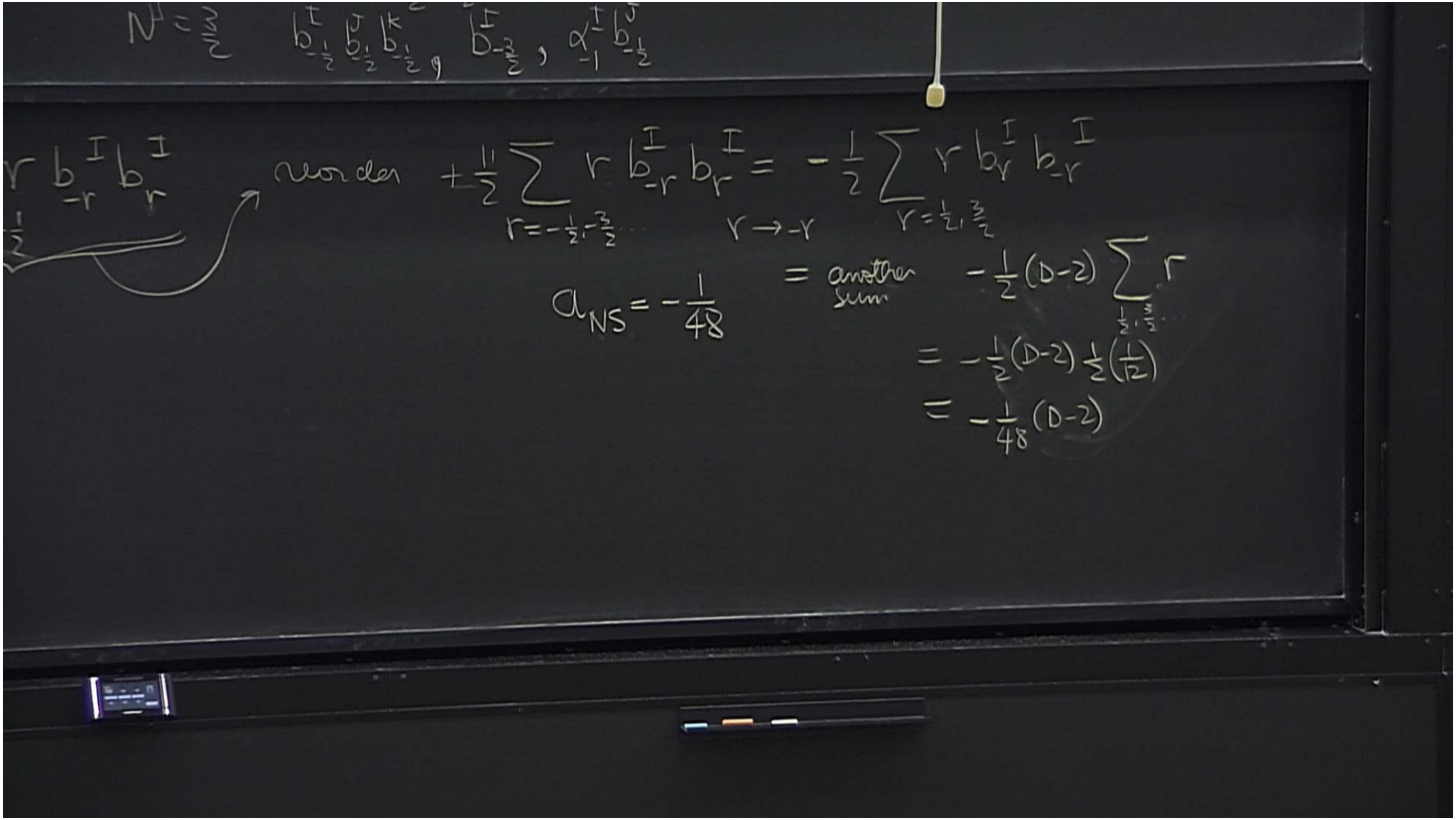

$N^I = \frac{3}{2}$
$\alpha' M^2 = \frac{1}{2} \sum_{p \in \mathbb{Z}} \alpha^I_{-p} \alpha^I_p + \frac{1}{2} \sum_{r \in \mathbb{Z}+\frac{1}{2}} r\, b^I_{-r} b^I_r$
$a_{NN} = -\frac{1}{24}$

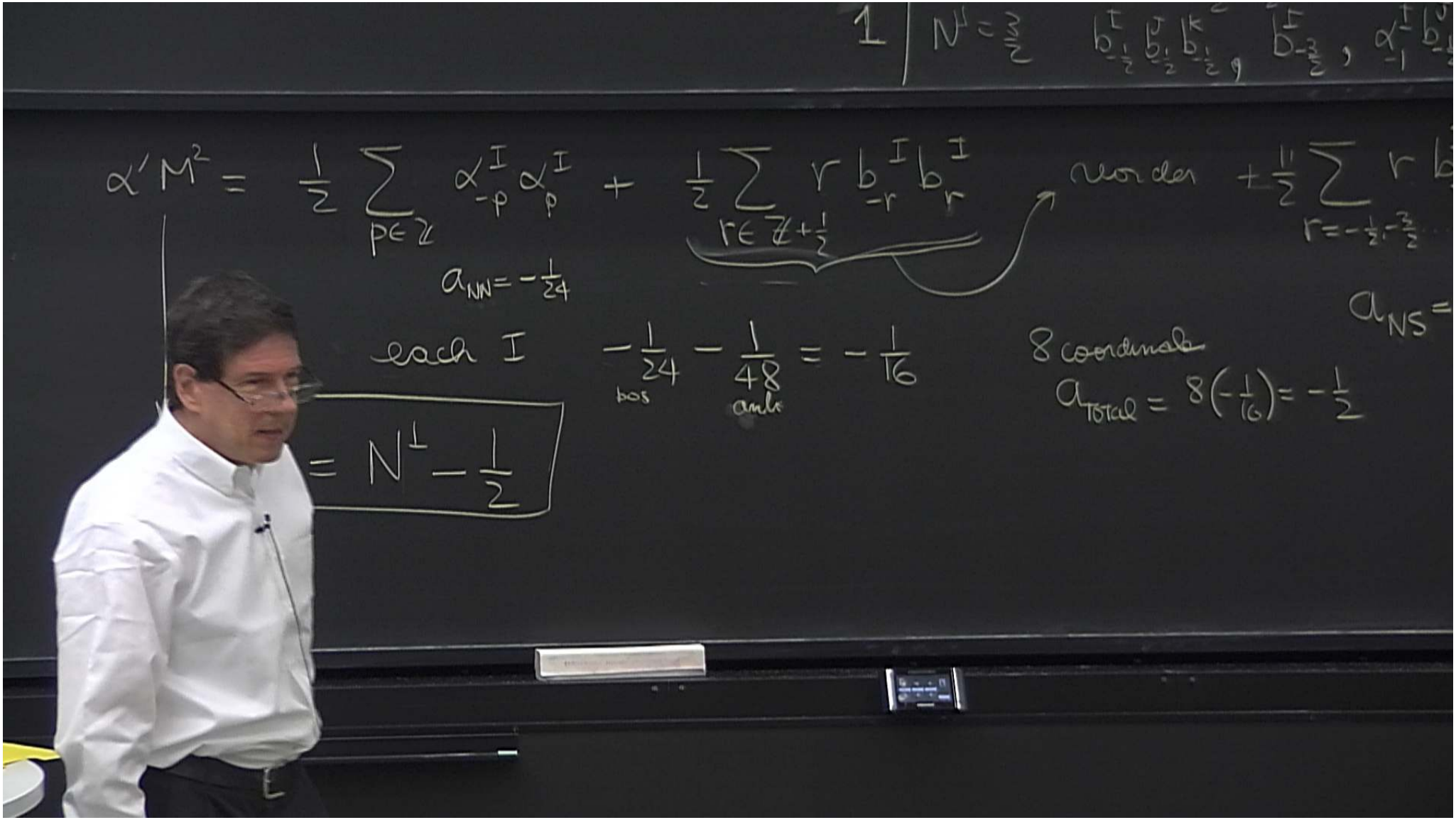

$N^I = \frac{3}{2}$ $b^I_{-\frac{1}{2}} b^J_{-\frac{1}{2}} b^K_{-\frac{1}{2}}$, $b^I_{-\frac{3}{2}}$, $\alpha^I_{-1} b^J_{-\frac{1}{2}}$
$\alpha' M^2 = \frac{1}{2} \sum_{p \in \mathbb{Z}} \alpha^I_{-p} \alpha^I_p + \frac{1}{2} \sum_{r \in \mathbb{Z}+\frac{1}{2}} r\, b^I_{-r} b^I_r$
reorder $+ \frac{1}{2} \sum_{r=} r\, b_{-r} b$
$a_{NN} = -\frac{1}{24}$

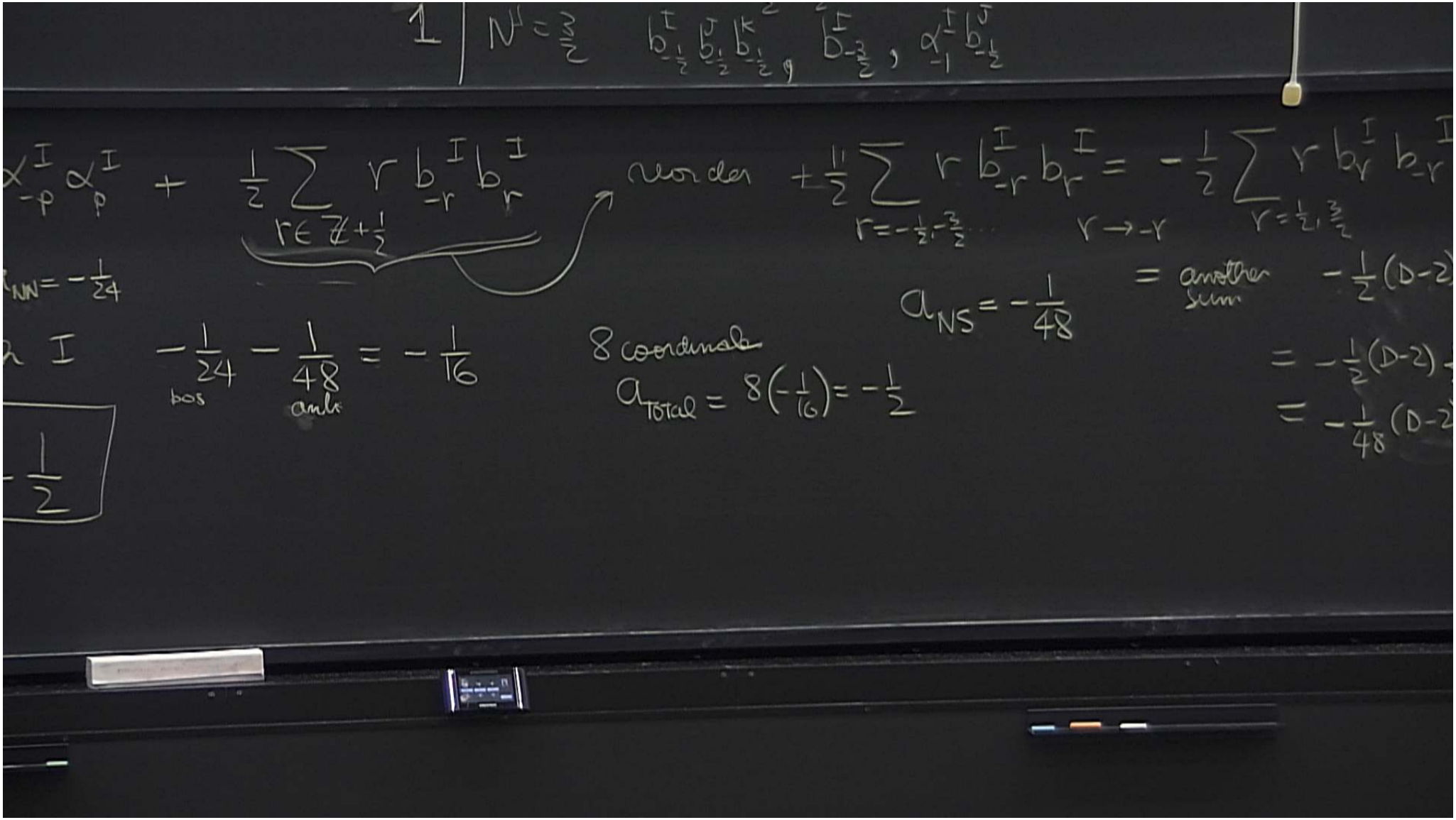

$\frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}} r\, b_{-r}^{I} b_{r}^{I}$
reorder
$+\frac{11}{2}\sum_{r=-\frac{1}{2},-\frac{3}{2}\ldots} r\, b_{-r}^{I} b_{r}^{I}$
$r\to -r$
$\frac{1}{2}\sum$


$N^I = \frac{3}{2}$ $b^I_{-\frac{1}{2}} b^J_{-\frac{1}{2}} b^K_{-\frac{1}{2}}$, $b^I_{-\frac{3}{2}}$, $\alpha^I_{-1} b^J_{-\frac{1}{2}}$
$\frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}} r\, b^I_{-r} b^I_r$
reorder
$\pm\frac{11}{2}\sum_{r=-\frac{1}{2},-\frac{3}{2}\ldots} r\, b^I_{-r} b^I_r = -\frac{1}{2}\sum_{r\to -r} b^I_r b^I_{-r}$
$r=\frac{1}{2},\frac{3}{2}$


$N = \frac{3}{2}$
$b^I_{-\frac{1}{2}}\, b^J_{-\frac{1}{2}}\, b^K_{-\frac{1}{2}}$, $b^I_{-\frac{3}{2}}$, $\alpha^I_{-1}\, b^J_{-\frac{1}{2}}$
$r\, b^I_{-r}\, b^I_r$
reorder
$\pm\frac{1}{2}\sum_{r=-\frac{1}{2},-\frac{3}{2}\cdots} r\, b^I_{-r}\, b^I_r = -\frac{1}{2}\sum_{r\to -r\ \ r=\frac{1}{2},\frac{3}{2}\cdots} r\, b^I_r\, b^I_{-r}$
$a_{NS} = -\frac{1}{48}$
$= $ another sum $-\frac{1}{2}(D-2)\sum_{r=\frac{1}{2},\frac{3}{2}\cdots} r$
$= -\frac{1}{2}(D-2)\,\frac{1}{2}\left(\frac{1}{12}\right)$
$= -\frac{1}{48}(D-2)$


$N = \frac{3}{2}$
$b^I_{-\frac{1}{2}}\, b^J_{-\frac{1}{2}}\, b^K_{-\frac{1}{2}}$, $D^{IJK}_{-\frac{3}{2}}$, $\alpha^I_{-1} b^J_{-\frac{1}{2}}$
$r\, b^I_{-r}\, b^I_r$
$r = \frac{1}{2}$
reorder
$\pm\frac{11}{2}\sum_{r=-\frac{1}{2},-\frac{3}{2}\ldots} r\, b^I_{-r} b^I_r = -\frac{1}{2}\sum_{r=\frac{1}{2},\frac{3}{2}} r\, b^I_r b^I_{-r}$
$r \to -r$
$a_{NS} = -\frac{1}{48}$
$= \text{another sum}\quad -\frac{1}{2}(D-2)\sum_{\frac{1}{2},\frac{3}{2}\ldots} r$
$= -\frac{1}{2}(D-2)\frac{1}{2}\left(\frac{1}{12}\right)$
$= -\frac{1}{48}(D-2)$


1
$N^1 = \frac{3}{2}$
$\alpha' M^2 = \frac{1}{2} \sum_{p \in \mathbb{Z}} \alpha^I_{-p} \alpha^I_p + \frac{1}{2} \sum_{r \in \mathbb{Z}+\frac{1}{2}} r\, b^I_{-r} b^I_r$
$a_{NN} = -\frac{1}{24}$
each I
$-\frac{1}{24} - \frac{1}{48} = -\frac{1}{16}$
bos
anti
$= N^1 - \frac{1}{2}$
normal order
$\pm \frac{11}{2} \sum_{r=-\frac{1}{2},-\frac{3}{2}} r$
a_{NS}
8 coordinate
$a_{Total} = 8\left(-\frac{1}{16}\right) = -\frac{1}{2}$



$1 \quad N^I = \frac{3}{2} \quad b^I_{-\frac{1}{2}} b^J_{-\frac{1}{2}} b^K_{-\frac{1}{2}}, \; b^I_{-\frac{3}{2}}, \; \alpha^I_{-1} b^J_{-\frac{1}{2}}$

$\alpha^I_{-p} \alpha^I_p + \underbrace{\frac{1}{2} \sum_{r \in \mathbb{Z}+\frac{1}{2}} r\, b^I_{-r} b^I_r}$ reorder $+\frac{1}{2} \sum_{r=-\frac{1}{2},-\frac{3}{2}\ldots} r\, b^I_{-r} b^I_r = -\frac{1}{2} \sum_{r=\frac{1}{2},\frac{3}{2}} r\, b^I_r b^I_{-r}$

$r \to -r$

$a_{NN} = -\frac{1}{24}$

$a_{NS} = -\frac{1}{48}$

$= \text{another sum} \quad -\frac{1}{2}(D-2)$

$= -\frac{1}{2}(D-2)$

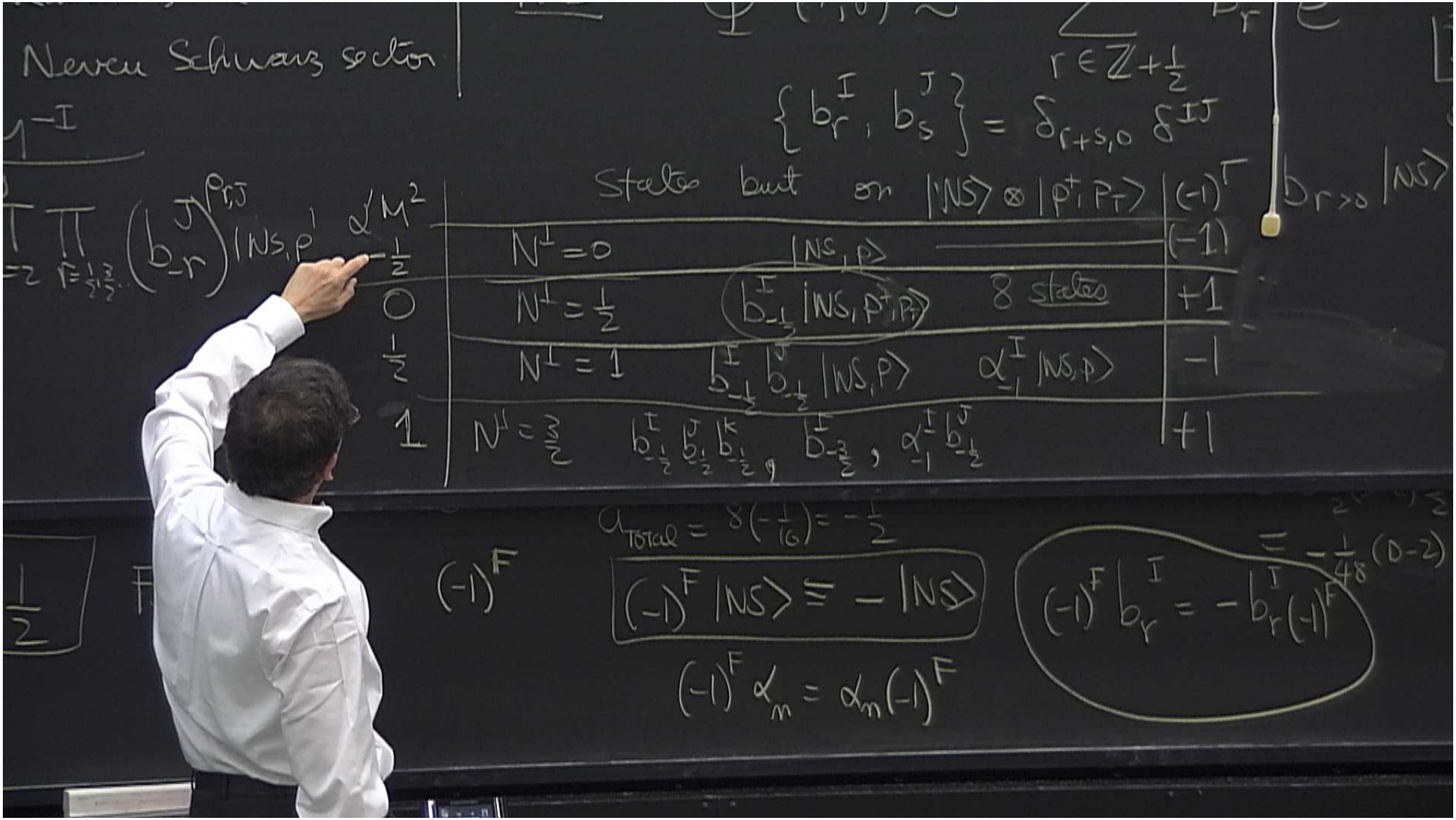
$= -\frac{1}{48}(D-2)$

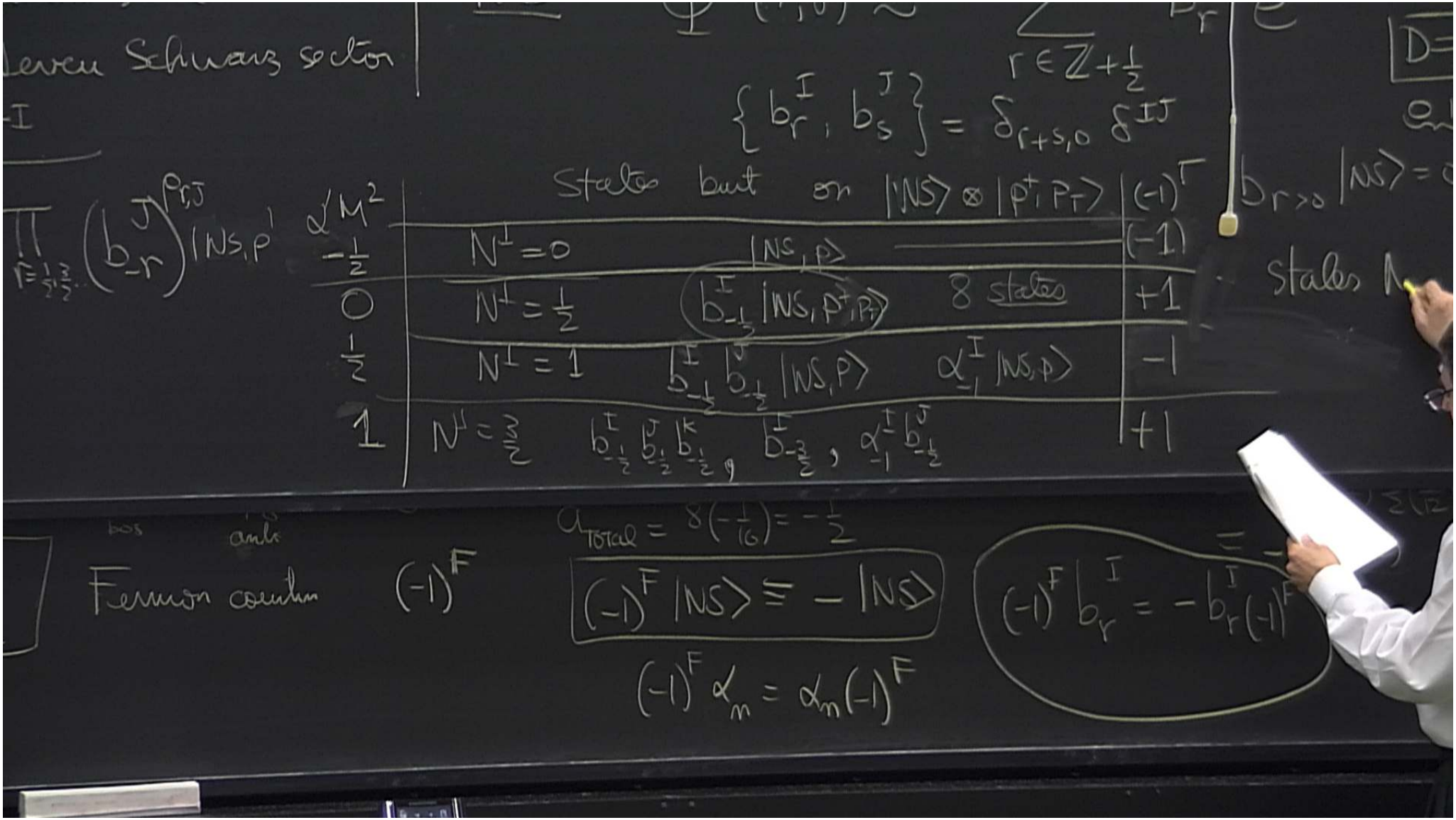
$\ldots I \quad \underset{\text{bos}}{-\frac{1}{24}} - \underset{\text{anti}}{\frac{1}{48}} = -\frac{1}{16}$

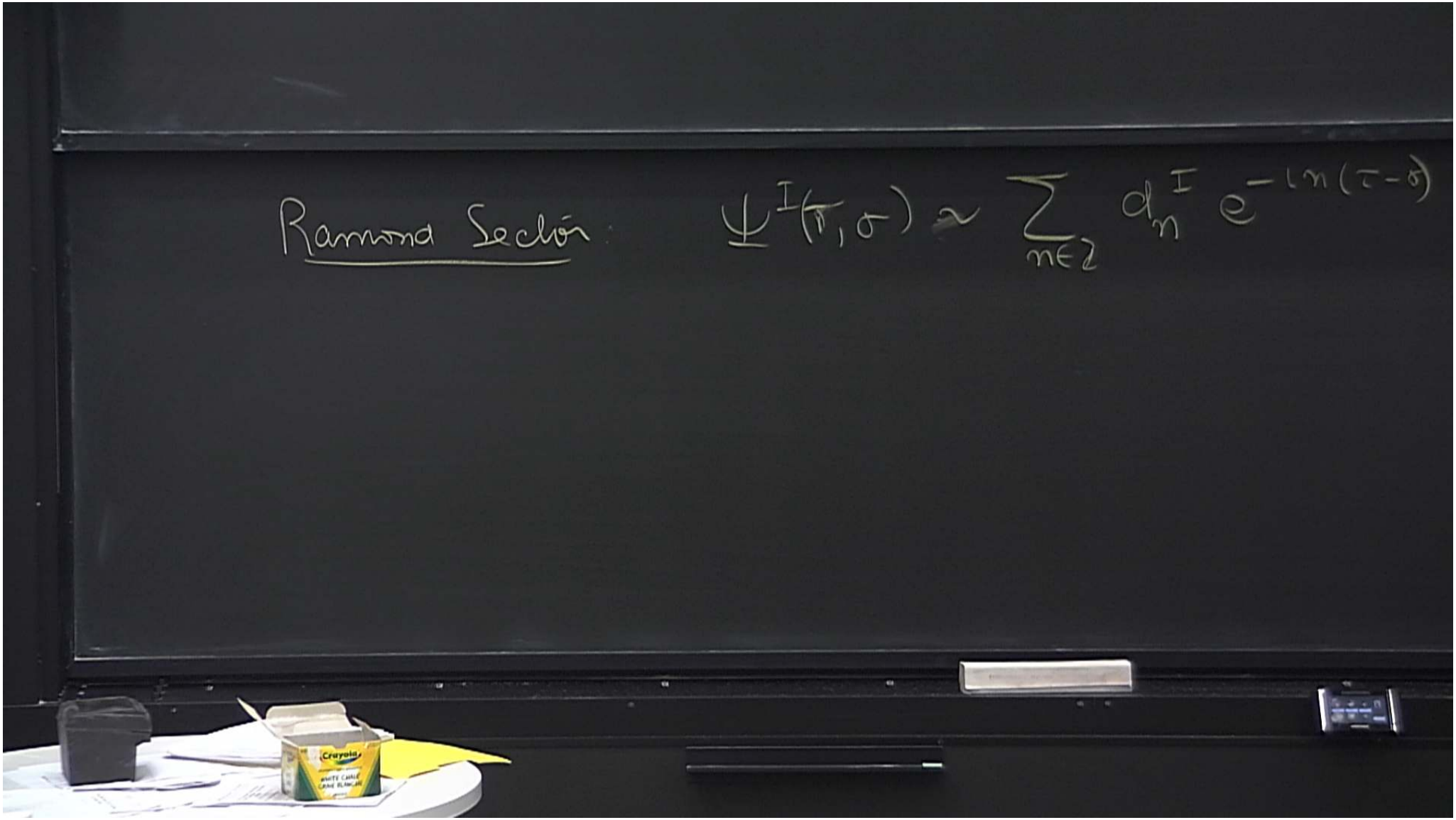
8 coordinates

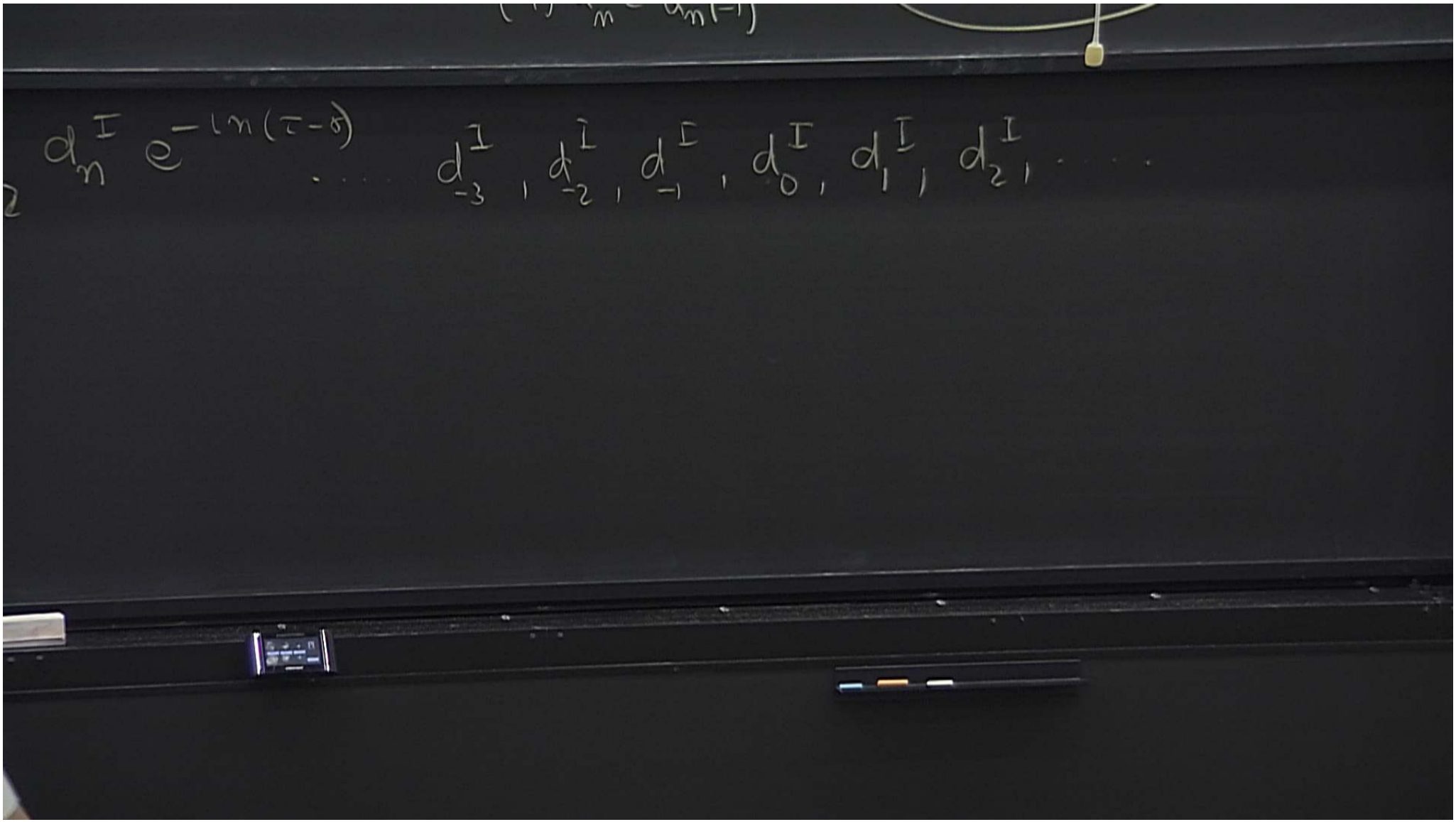
$a_{Total} = 8\left(-\frac{1}{16}\right) = -\frac{1}{2}$

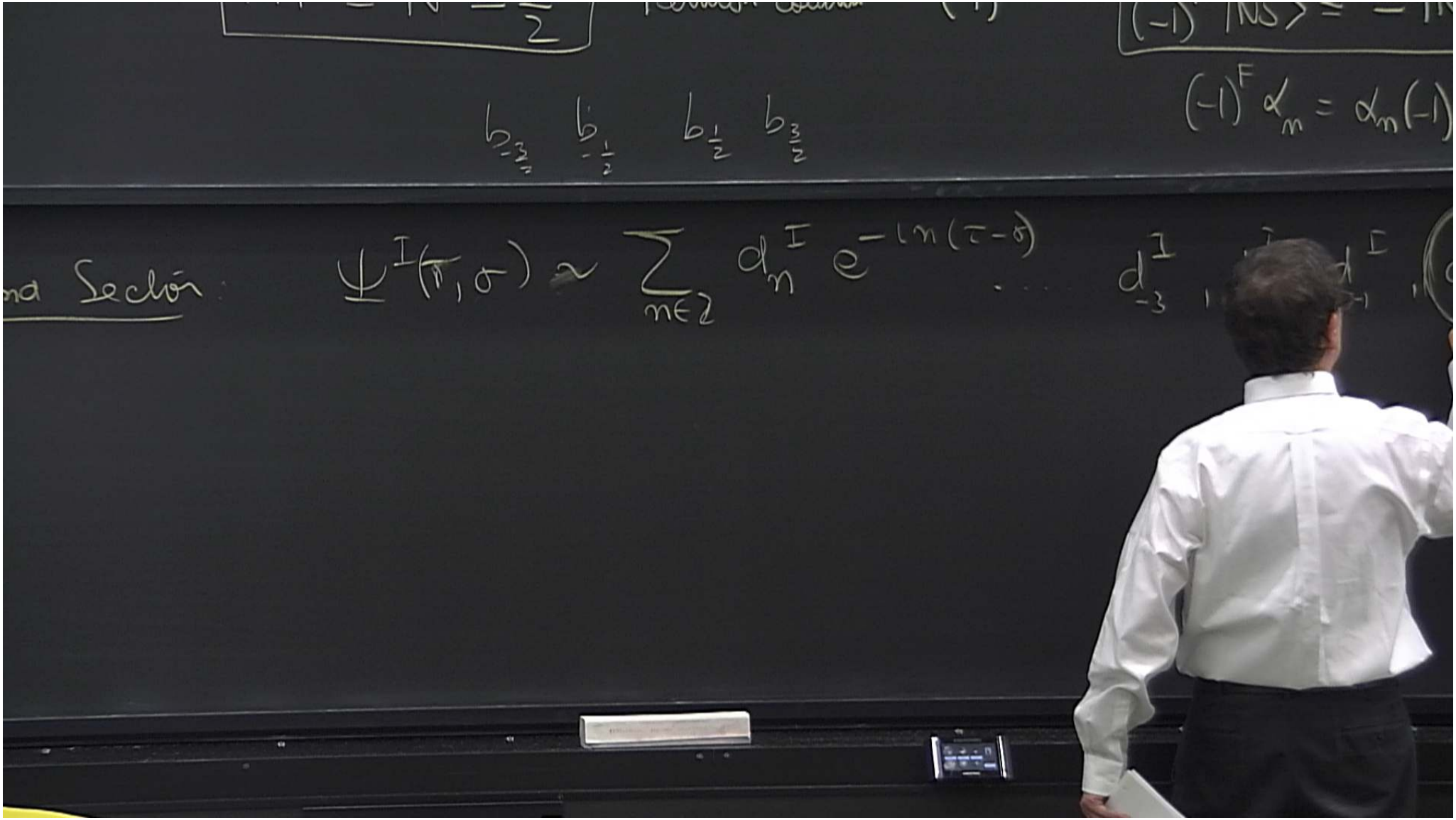
$\boxed{-\frac{1}{2}}$


Neveu Schwarz sector
$r \in \mathbb{Z}+\frac{1}{2}$
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$
States but on $|NS\rangle \otimes |p^+, p_T\rangle$
$N^{\perp}=0$ $|NS,p\rangle$
$N^{\perp}=\frac{1}{2}$ $b_{-\frac{1}{2}}^I |NS,p^+,p_T\rangle$ 8 states
$N^{\perp}=1$ $b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J |NS,p\rangle$ $\alpha_{-1}^I |NS,p\rangle$
$N^{\perp}=\frac{3}{2}$ $b_{-\frac{1}{2}}^I b_{-\frac{1}{2}}^J b_{-\frac{1}{2}}^K$, $b_{-\frac{3}{2}}^I$, $\alpha_{-1}^I b_{-\frac{1}{2}}^J$
$(-1)^F$
$(-1)^F |NS\rangle \equiv -|NS\rangle$
$(-1)^F \alpha_n = \alpha_n (-1)^F$
$(-1)^F b_r^I = -b_r^I (-1)^F$
$a_{Total} = 8\left(-\frac{1}{16}\right) = -\frac{1}{2}$
$\equiv -\frac{1}{48}(D-2)$


Fermion counting
$(-1)^F$
$(-1)^F |NS\rangle \equiv -|NS\rangle$
$(-1)^F \alpha_n = \alpha_n (-1)^F$
$(-1)^F b_r^I = -b_r^I (-1)^F$
$\{b_r^I, b_s^J\} = \delta_{r+s,0}\,\delta^{IJ}$
$r \in \mathbb{Z}+\frac{1}{2}$
$a_{Total} = 8(-\frac{1}{16}) = -\frac{1}{2}$
$N^L = 0$
$N^L = \frac{1}{2}$
$N^L = 1$
$N^L = \frac{3}{2}$
8 states
$b_r|NS\rangle = 0$


Ramond Sector
$\Psi^I(\tau,\sigma) \sim \sum_{n\in\mathbb{Z}} d_n^I e^{-in(\tau-\sigma)}$
Crayola


$d_n^I e^{-in(\tau-\delta)}$
$d_{-3}^I, d_{-2}^I, d_{-1}^I, d_0^I, d_1^I, d_2^I, \ldots$

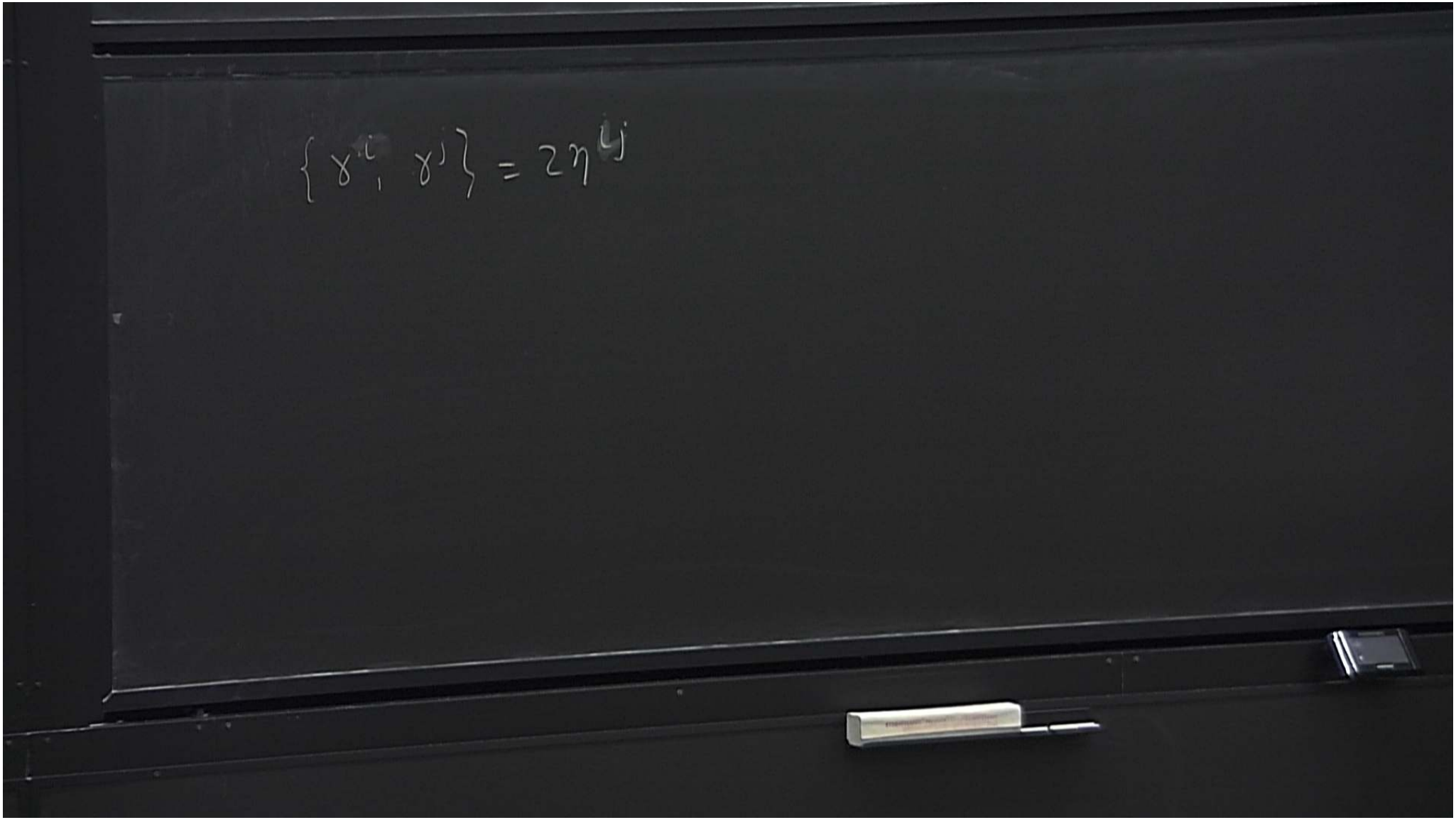

$b_{-\frac{3}{2}}$ $b_{-\frac{1}{2}}$ $b_{\frac{1}{2}}$ $b_{\frac{3}{2}}$
$(-1)^F \alpha_n = \alpha_n (-1)$
$\Psi^I(\tau,\sigma) \sim \sum_{n\in\mathbb{Z}} d_n^I e^{-in(\tau-\sigma)}$
d_{-3}^I

$(-1)^F \alpha_m = \alpha_m (-1)^F$

$e^{-\ln(\tau-\sigma)}$

$d^I_{-3}, d^I_{-2}, d^I_{-1}, d^I_0, d^I_1, d^I_2, \ldots$

$\{d^I_m, d^J_n\} = \delta_{m+n,0}\, \delta^{IJ} \longrightarrow \{d^I_0, d^J_0\} = \delta^{IJ}$


$\{\gamma^i, \gamma^j\} = 2\eta^{ij}$

$b_{\frac{1}{2}}$ $b_{\frac{3}{2}}$

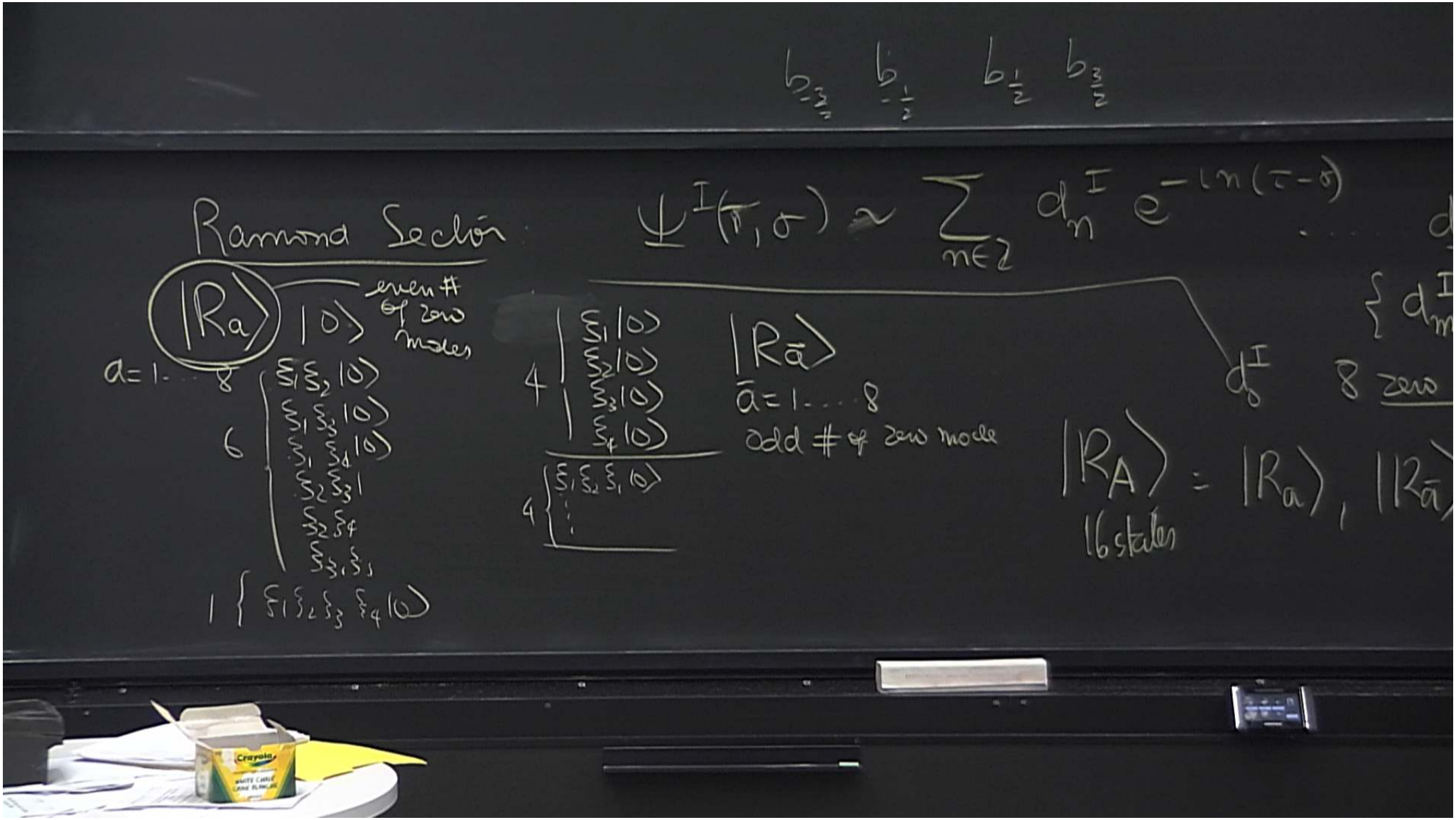
$(-1)^F \alpha_n = \alpha_n (-1)^F$

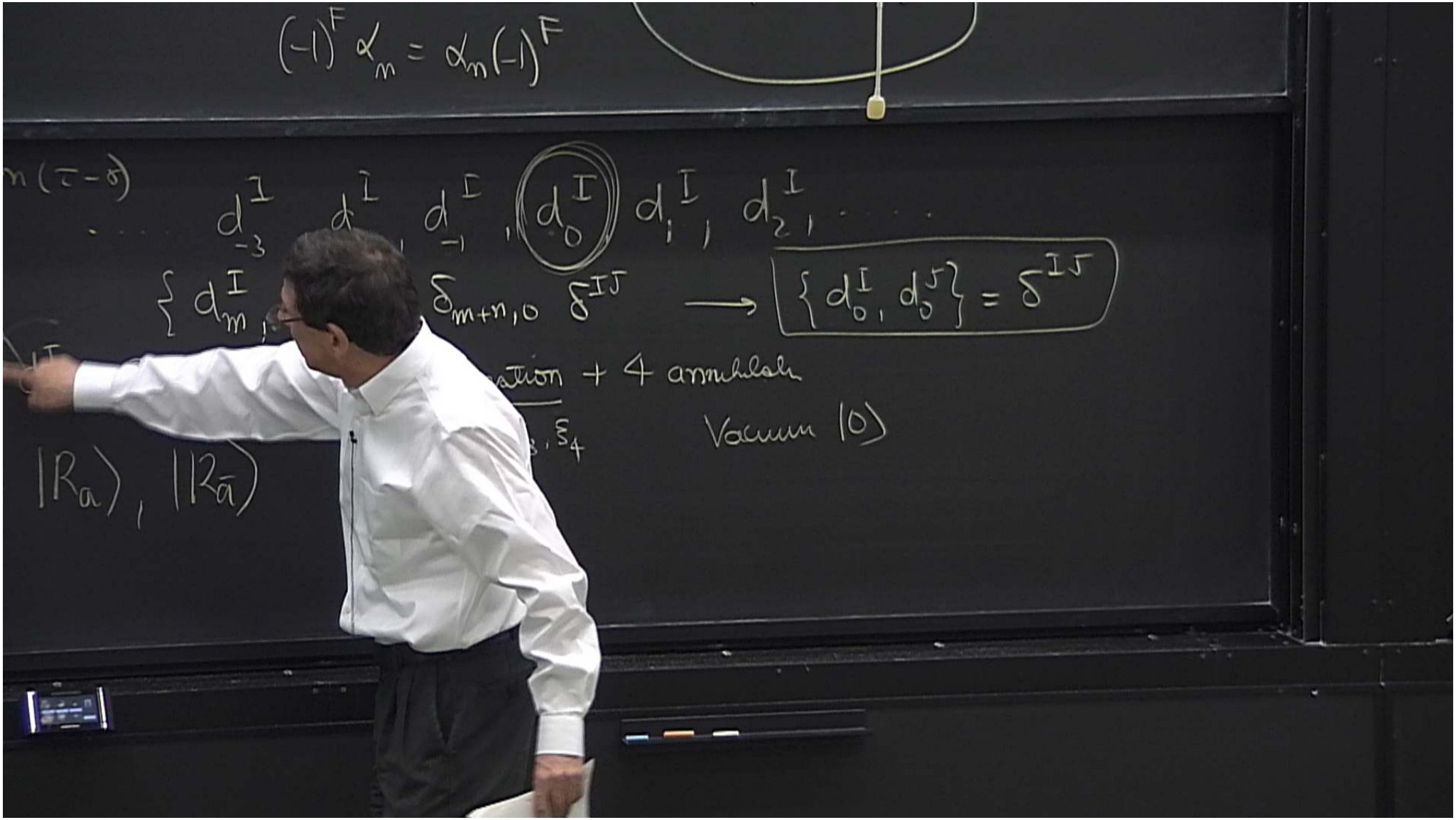
$\sum_{n \in \mathbb{Z}} d_n^I e^{-in(\tau-\sigma)}$ ... $d_{-3}^I, d_{-2}^I, d_{-1}^I, \boxed{d_0^I}, d_1^I, d_2^I, \ldots$

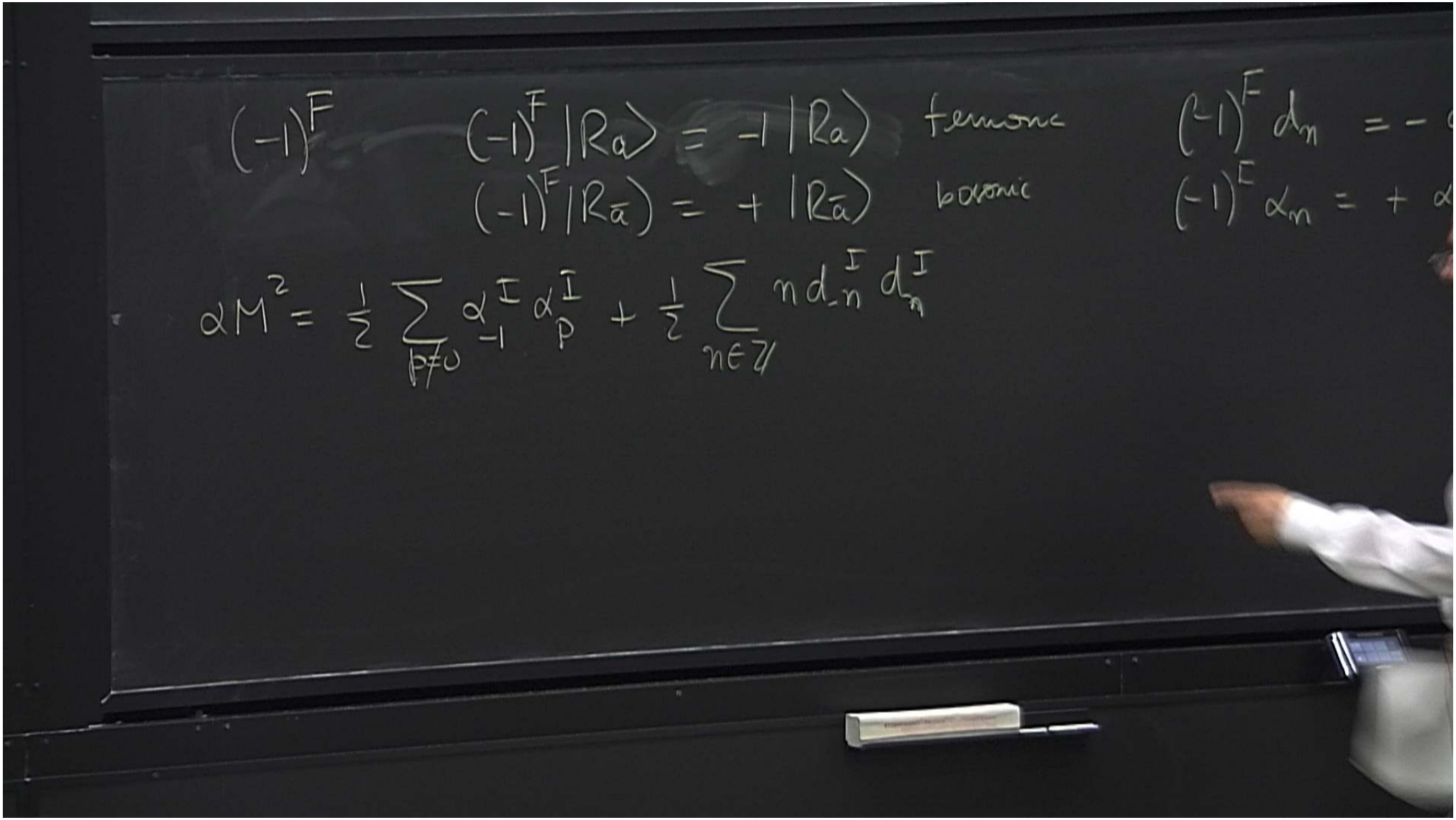
$\{d_m^I, d_n^J\} = \delta_{m+n,0}\, \delta^{IJ} \longrightarrow \boxed{\{d_0^I, d_0^J\} = \delta^{IJ}}$

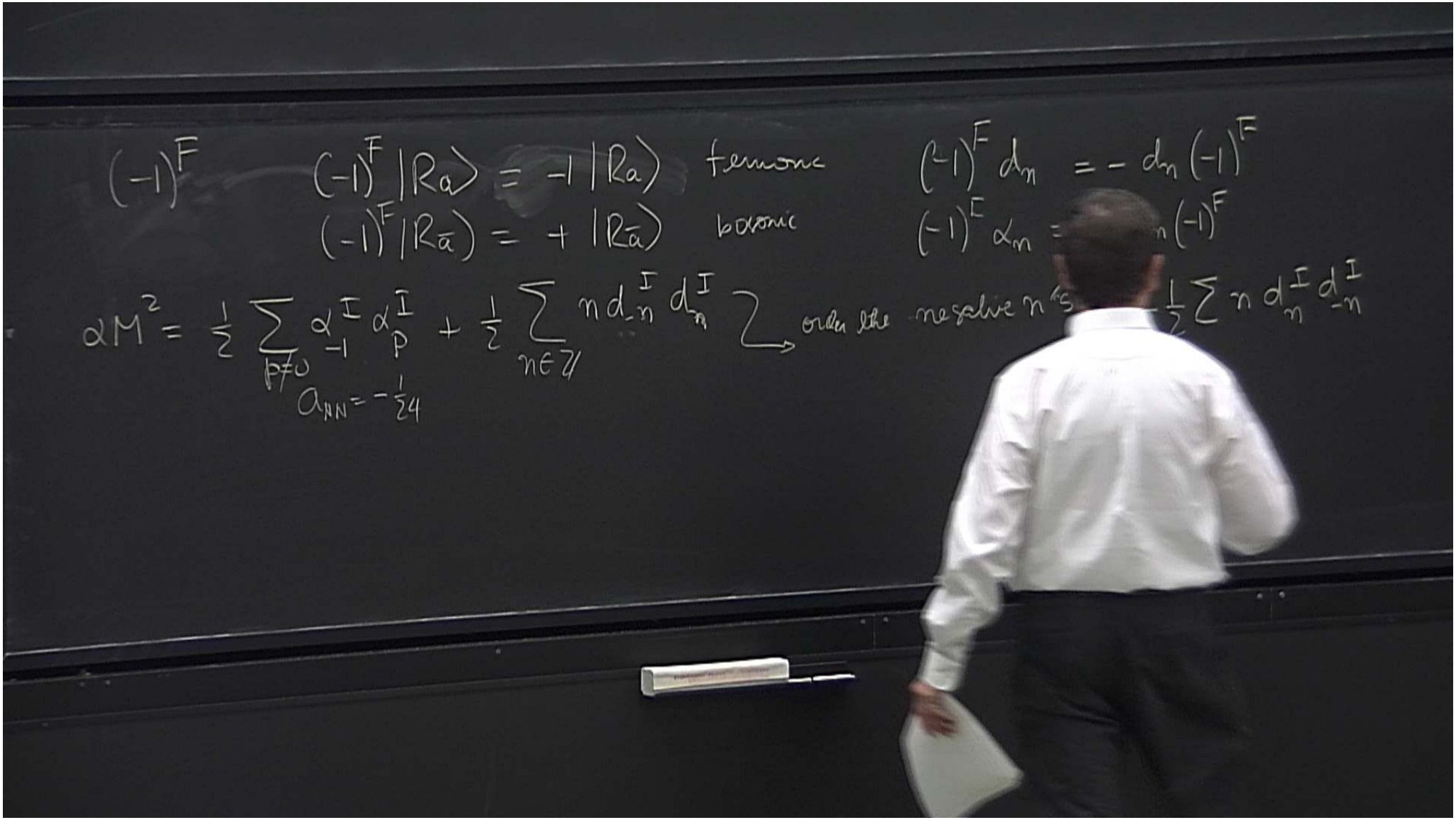
d_0^I 8 zero modes $\Rightarrow$ 4 creation + 4 annihilation

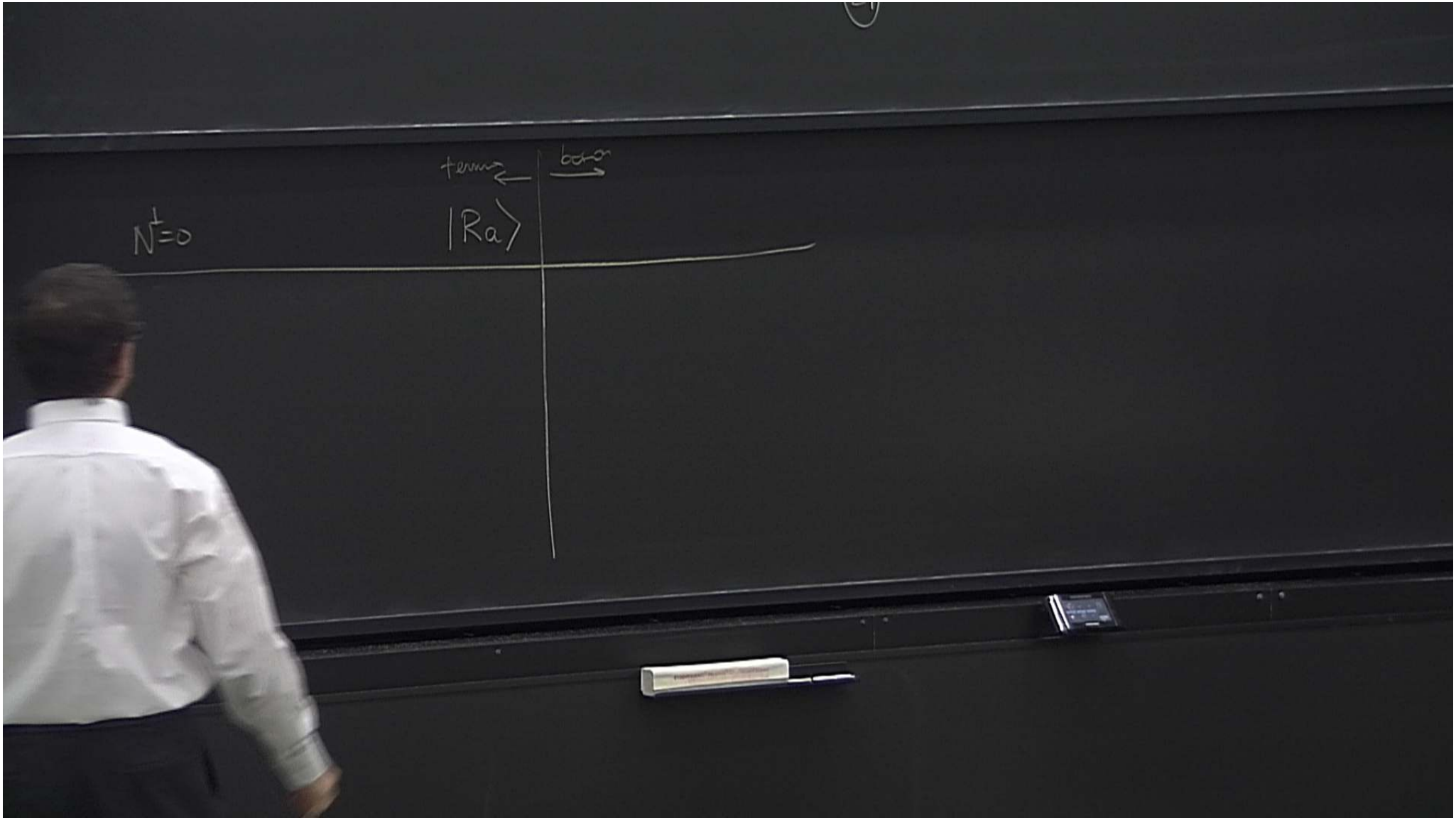
$\xi_1, \xi_2, \xi_3, \xi_4$ Vacuum $|0\rangle$

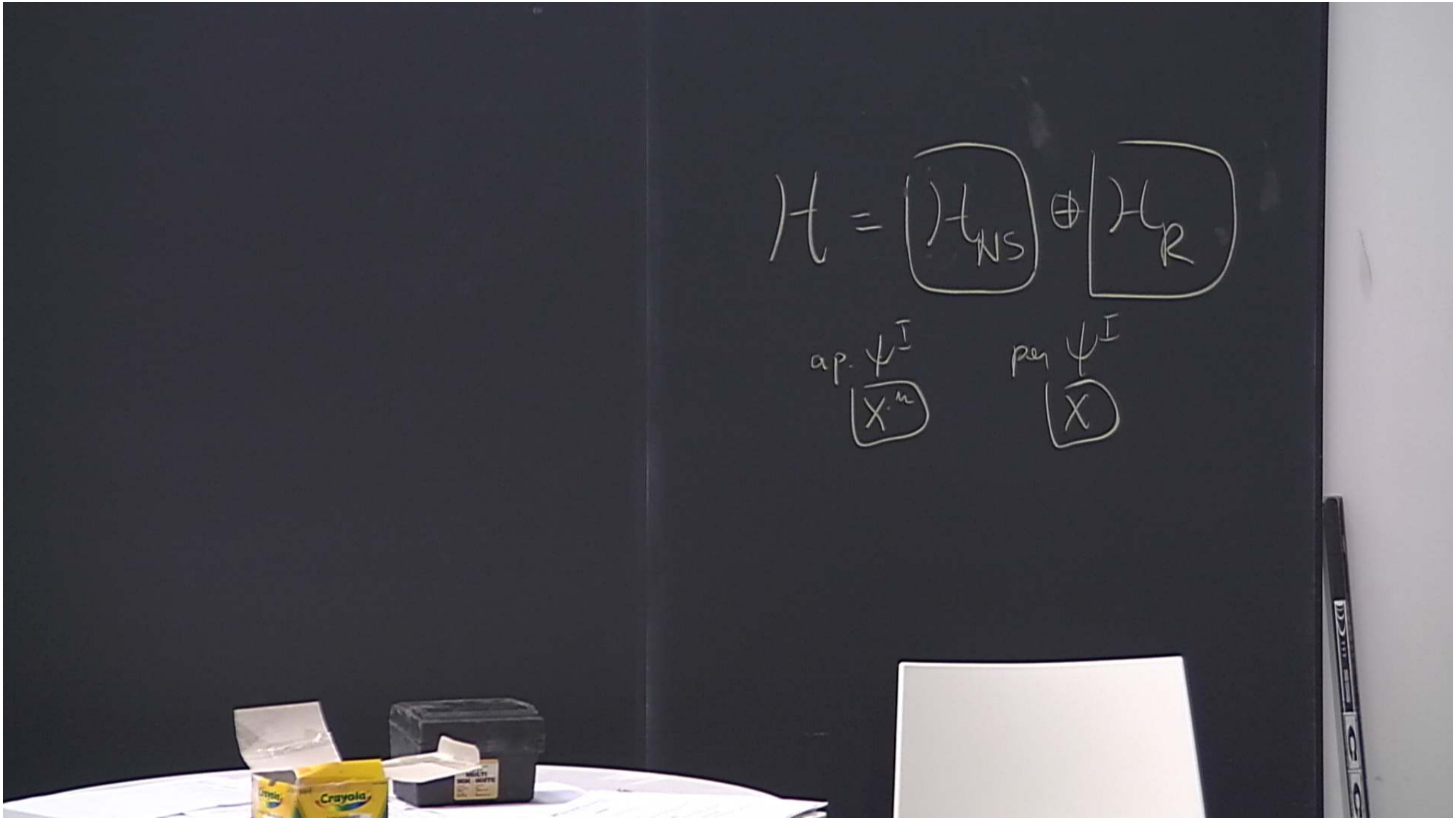

Ramond Sector
$|R_a\rangle$
$a = 1 \dots 8$
$|0\rangle$
even # of zero modes
6
$\xi_1\xi_2|0\rangle$
$\xi_1\xi_3|0\rangle$
$\xi_1\xi_4|0\rangle$
$\xi_2\xi_3|0\rangle$
$\xi_2\xi_4|0\rangle$
$\xi_3\xi_4|0\rangle$
1
$\xi_1\xi_2\xi_3\xi_4|0\rangle$
4
$\xi_1|0\rangle$
$\xi_2|0\rangle$
$\xi_3|0\rangle$
$\xi_4|0\rangle$
$|R_{\dot a}\rangle$
$\dot a = 1 \dots 8$
odd # of zero mode
4
$\xi_1\xi_2\xi_3|0\rangle$
$\Psi^I(\tau,\sigma) \sim \sum_{n\in\mathbb{Z}} d_n^I e^{-in(\tau-\sigma)}$
$b_{-\frac{3}{2}}$ $b_{-\frac{1}{2}}$ $b_{\frac{1}{2}}$ $b_{\frac{3}{2}}$
d_0^I
8 zero
$|R_A\rangle = |R_a\rangle, |R_{\dot a}\rangle$
16 states

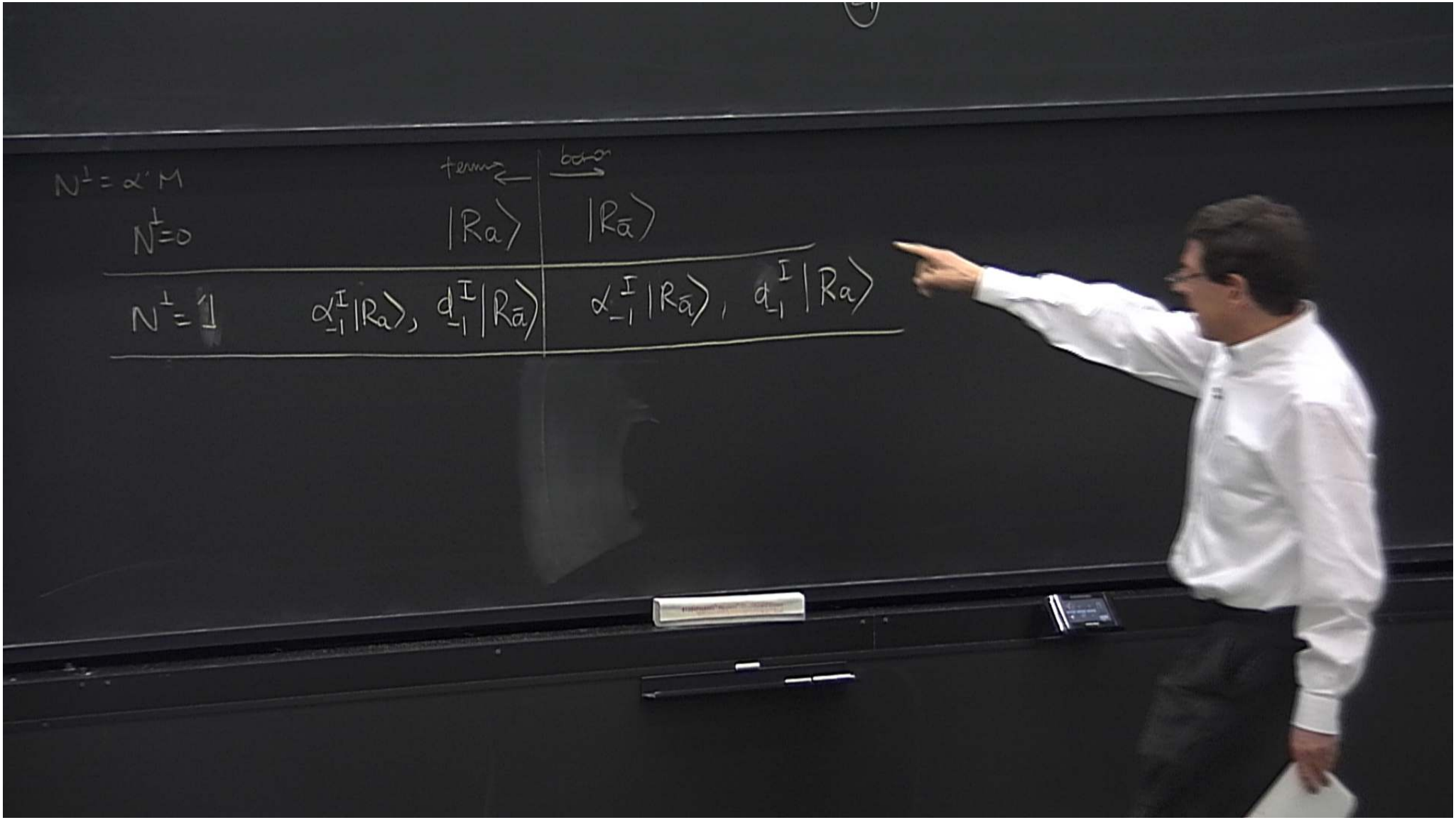

$(-1)^F \alpha_n = \alpha_n (-1)^F$
d_{-3}^I d_{-1}^I d_0^I d_1^I, d_2^I,
$\{ d_m^I$ $\delta_{m+n,0}$ δ^{IJ}
$\{ d_0^I, d_0^J \} = \delta^{IJ}$
+ 4 annihilation
Vacuum $|0\rangle$
$|R_a\rangle$, $|R_{\bar{a}}\rangle$

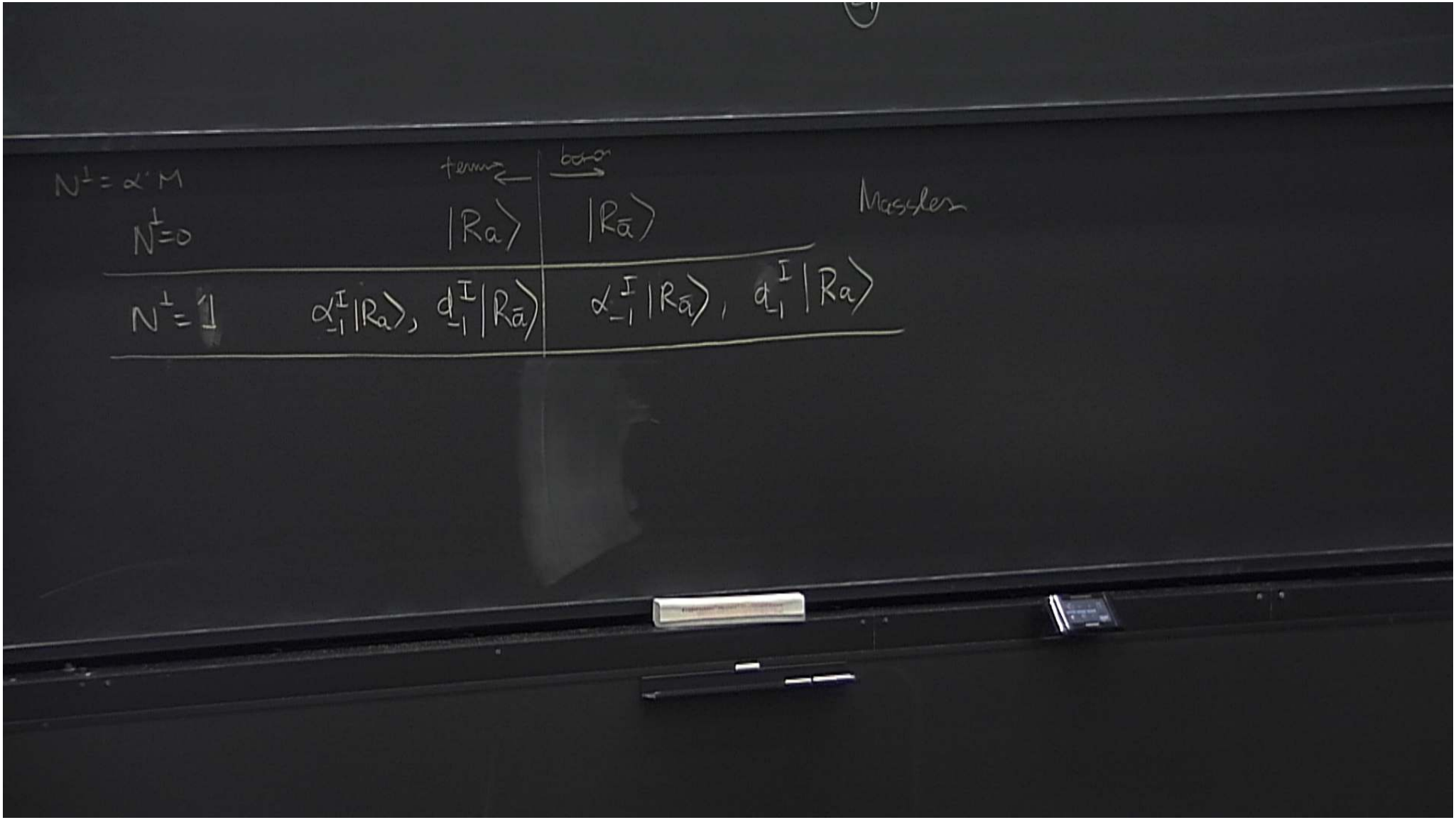

$(-1)^F$
$(-1)^F |Ra\rangle = -1 |Ra\rangle$ fermionic
$(-1)^F |R\bar{a}\rangle = +1 |R\bar{a}\rangle$ bosonic
$(-1)^F \alpha_n = -$
$(-1)^F \alpha_n = +$
$\alpha M^2 = \frac{1}{2} \sum_{p \neq 0} \alpha_{-1}^I \alpha_p^I + \frac{1}{2} \sum_{n \in \mathbb{Z}} n d_{-n}^I d_n^I$

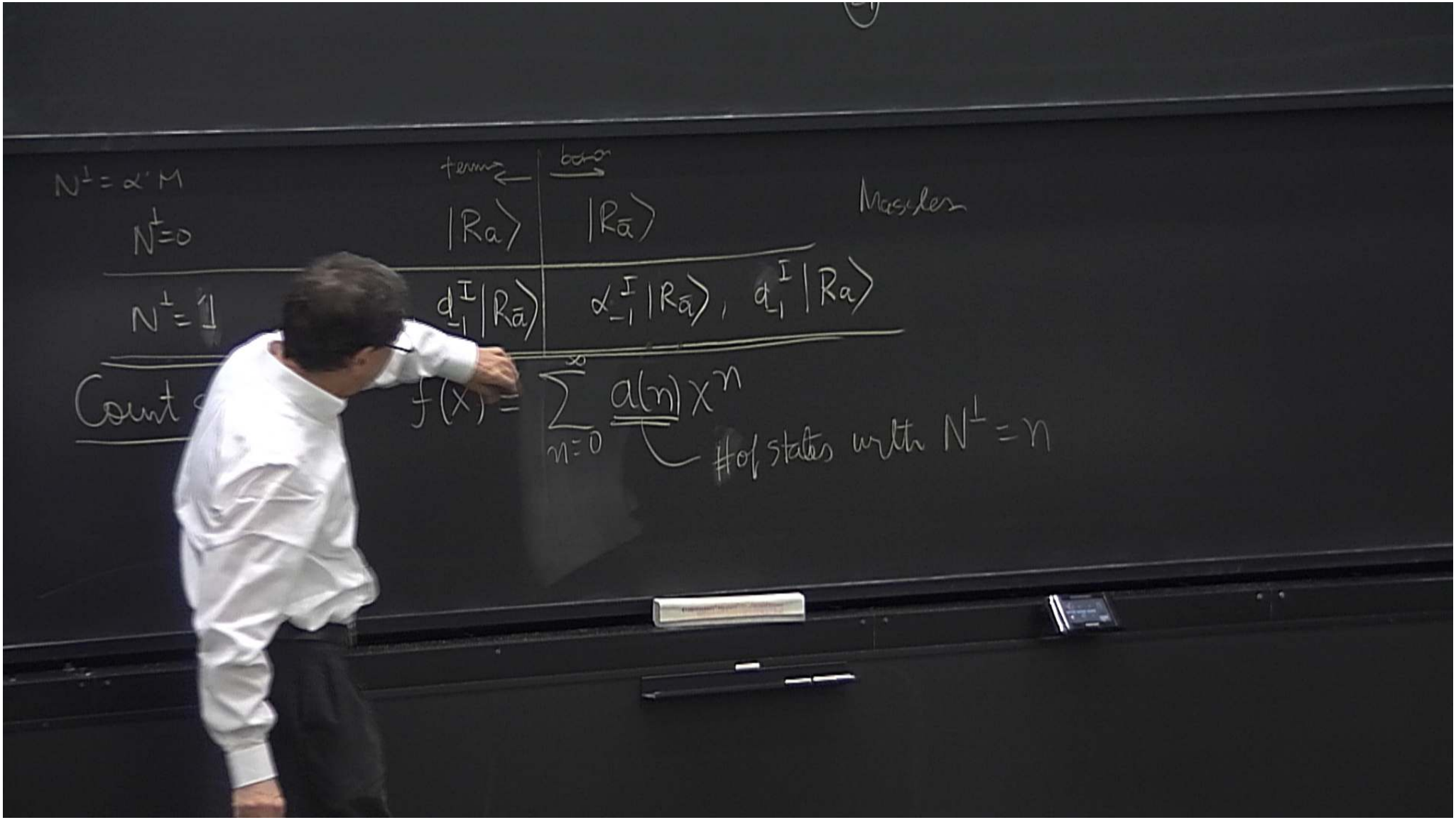

$(-1)^F$
$(-1)^F|Ra\rangle = -|Ra\rangle$ fermionic
$(-1)^F|R\bar{a}\rangle = +|R\bar{a}\rangle$ bosonic
$(-1)^F d_n = -d_n(-1)^F$
$(-1)^F \alpha_n = \alpha_n(-1)^F$
$\alpha M^2 = \frac{1}{2}\sum_{p\neq 0} \alpha_{-1}^I \alpha_p^I + \frac{1}{2}\sum_{n\in\mathbb{Z}} n\, d_{-n}^I d_n^I$
$a_{NN} = -\frac{1}{24}$
$-\frac{1}{2}\sum n\, d_n^I d_{-n}^I$

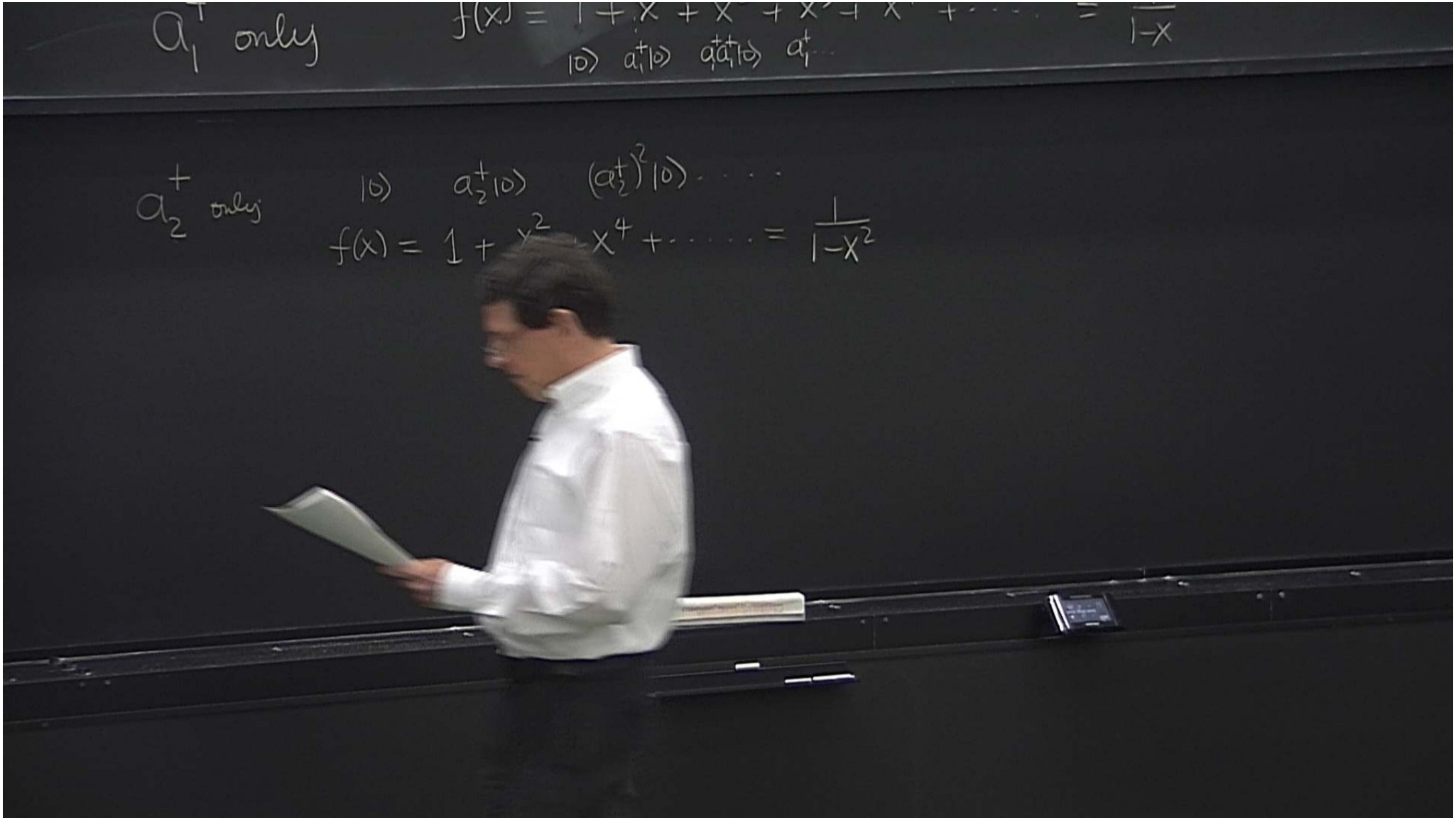

$N^{\dagger}=0$
$|Ra\rangle$

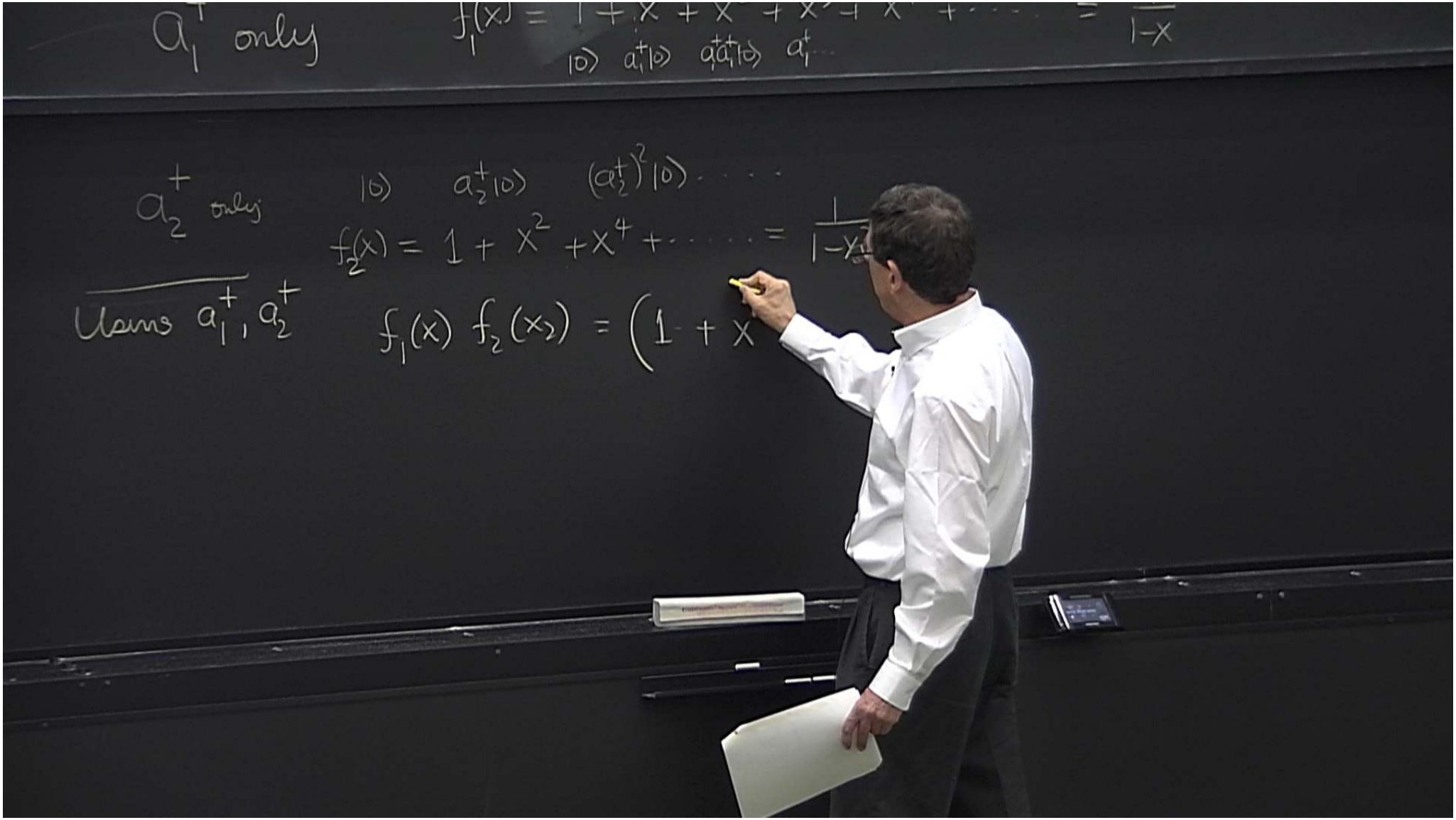

$\mathcal{H} = \mathcal{H}_{NS} \oplus \mathcal{H}_R$
ap. ψ^I
X^μ
per ψ^I
X


$N^\perp = \alpha' M$
fermi
bose
$N^\perp = 0$
$|Ra\rangle$
$|R\bar{a}\rangle$
$N^\perp = 1$
$\alpha_{-1}^I |Ra\rangle$, $\alpha_{-1}^I |R\bar{a}\rangle$
$\alpha_{-1}^I |R\bar{a}\rangle$, $\alpha_{-1}^I |Ra\rangle$


$N^\perp = \alpha' M$
$N^\perp = 0$
$|Ra\rangle$
$|R\bar{a}\rangle$
$N^\perp = 1$
$\alpha_{-1}^I |Ra\rangle$, $a_{-1}^I |R\bar{a}\rangle$
$\alpha_{-1}^I |R\bar{a}\rangle$, $a_{-1}^I |Ra\rangle$


$N^\perp = \alpha' M$
fermion
boson
$N^\perp = 0$
$|Ra\rangle$
$|R\bar{a}\rangle$
Massless
$N^\perp = 1$
$d_{-1}^I |R\bar{a}\rangle$
$\alpha_{-1}^I |R\bar{a}\rangle$, $d_{-1}^I |Ra\rangle$
Count
$f(x) = \sum_{n=0}^{\infty} a(n) x^n$
# of states with $N^\perp = n$


a_1^+ only
$f(x) = 1 + x + x^2 + x^3 + x^4 + \cdots = \frac{1}{1-x}$
$|0\rangle$ $a_1^+|0\rangle$ $a_1^+a_1^+|0\rangle$ $a_1^+ \cdots$
a_2^+ only
$|0\rangle$ $a_2^+|0\rangle$ $(a_2^+)^2|0\rangle \cdots$
$f(x) = 1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}$


$a_1^\dagger$ only
$f_1(x) = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$
$|0\rangle$ $a_1^\dagger|0\rangle$ $a_1^\dagger a_1^\dagger|0\rangle$ $a_1^\dagger \cdots$
$a_2^\dagger$ only
$|0\rangle$ $a_2^\dagger|0\rangle$ $(a_2^\dagger)^2|0\rangle \cdots$
$f_2(x) = 1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}$
Using $a_1^\dagger, a_2^\dagger$
$f_1(x)\, f_2(x_2) = \left(1 + x\right.$

$a_1^\dagger$ only $f_1(x) = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$

$|0\rangle \quad a_1^\dagger|0\rangle \quad a_1^\dagger a_1^\dagger|0\rangle \quad a_1^\dagger \cdots$

$a_2^\dagger$ only $|0\rangle \quad a_2^\dagger|0\rangle \quad (a_2^\dagger)^2|0\rangle \cdots$

$f_2(x) = 1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}$

Using $a_1^\dagger, a_2^\dagger$

$f_1(x)\, f_2(x_2) = \Big(1 + \underset{a_1^\dagger|0\rangle}{x} + \underset{a_1^\dagger a_1^\dagger|0\rangle}{x^2} + x^3 \cdots\Big)$

$\Big(1 + \underset{a_2^\dagger|0\rangle}{x^2} + \underset{(a_2^\dagger)^2|0\rangle}{x^4} + \cdots\Big)$

$a_1^\dagger$ only $f_1(x) = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$

$|0\rangle \quad a_1^\dagger|0\rangle \quad a_1^\dagger a_1^\dagger|0\rangle \quad a_1^\dagger \cdots$

$a_2^\dagger$ only $|0\rangle \quad a_2^\dagger|0\rangle \quad (a_2^\dagger)^2|0\rangle \cdots$

$f_2(x) = 1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}$

Using $a_1^\dagger, a_2^\dagger$ $f_{12}(x) = f_1(x)\, f_2(x_2) = \Big(1 + \underset{a_1^\dagger|0\rangle}{x} + \underset{a_1^\dagger a_1^\dagger|0\rangle}{x^2} + x^3 + \cdots\Big)$

$\Big(1 + \underset{a_2^\dagger|0\rangle}{x^2} + \underset{(a_2^\dagger)^2|0\rangle}{x^4} + \cdots\Big)$

$a_1^\dagger$ only $f_1(x) = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$

$|0\rangle \quad a_1^\dagger|0\rangle \quad a_1^\dagger a_1^\dagger|0\rangle \quad a_1^\dagger \cdots$

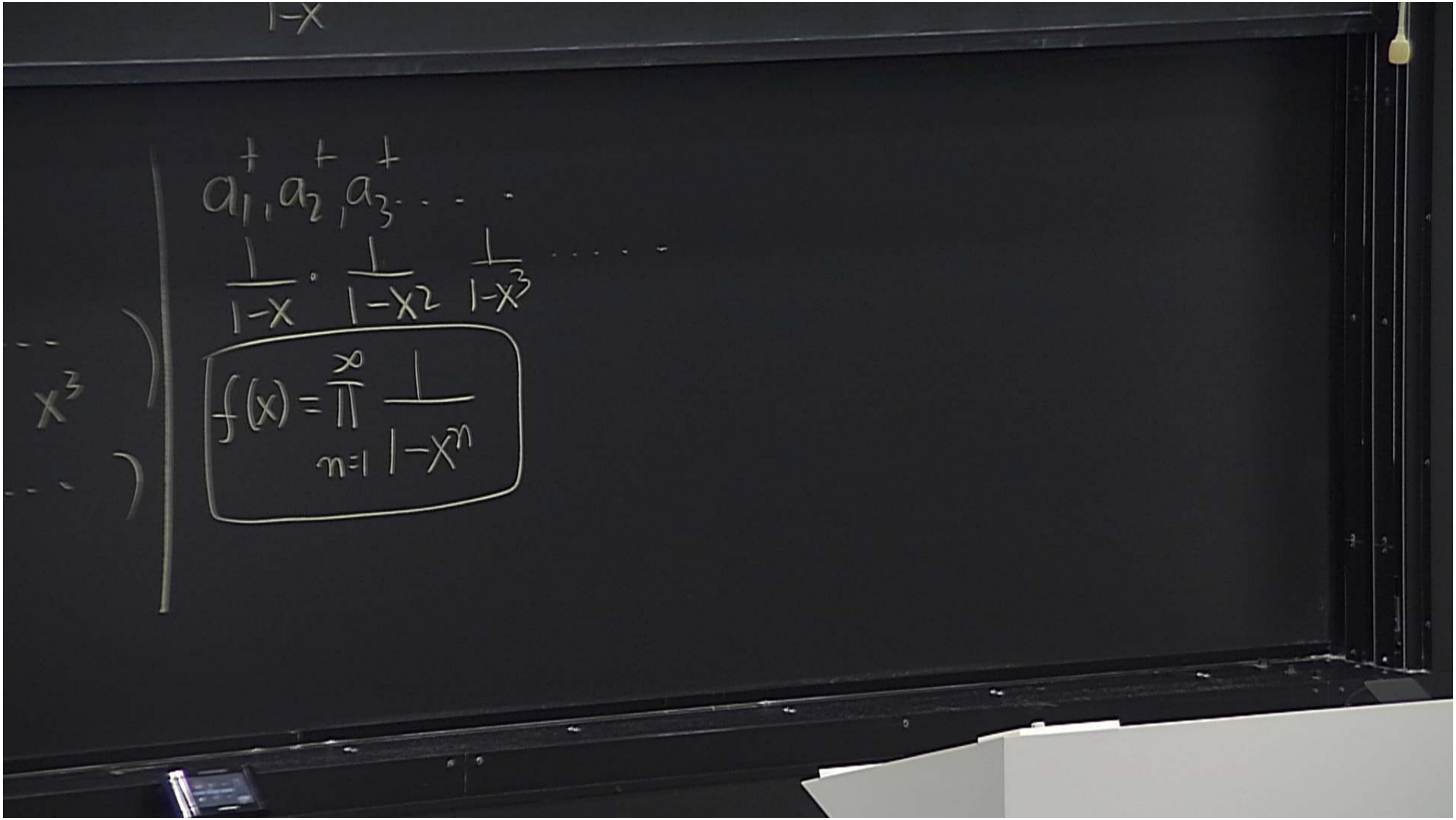
$a_2^\dagger$ only $|0\rangle \quad a_2^\dagger|0\rangle \quad (a_2^\dagger)^2|0\rangle \cdots$

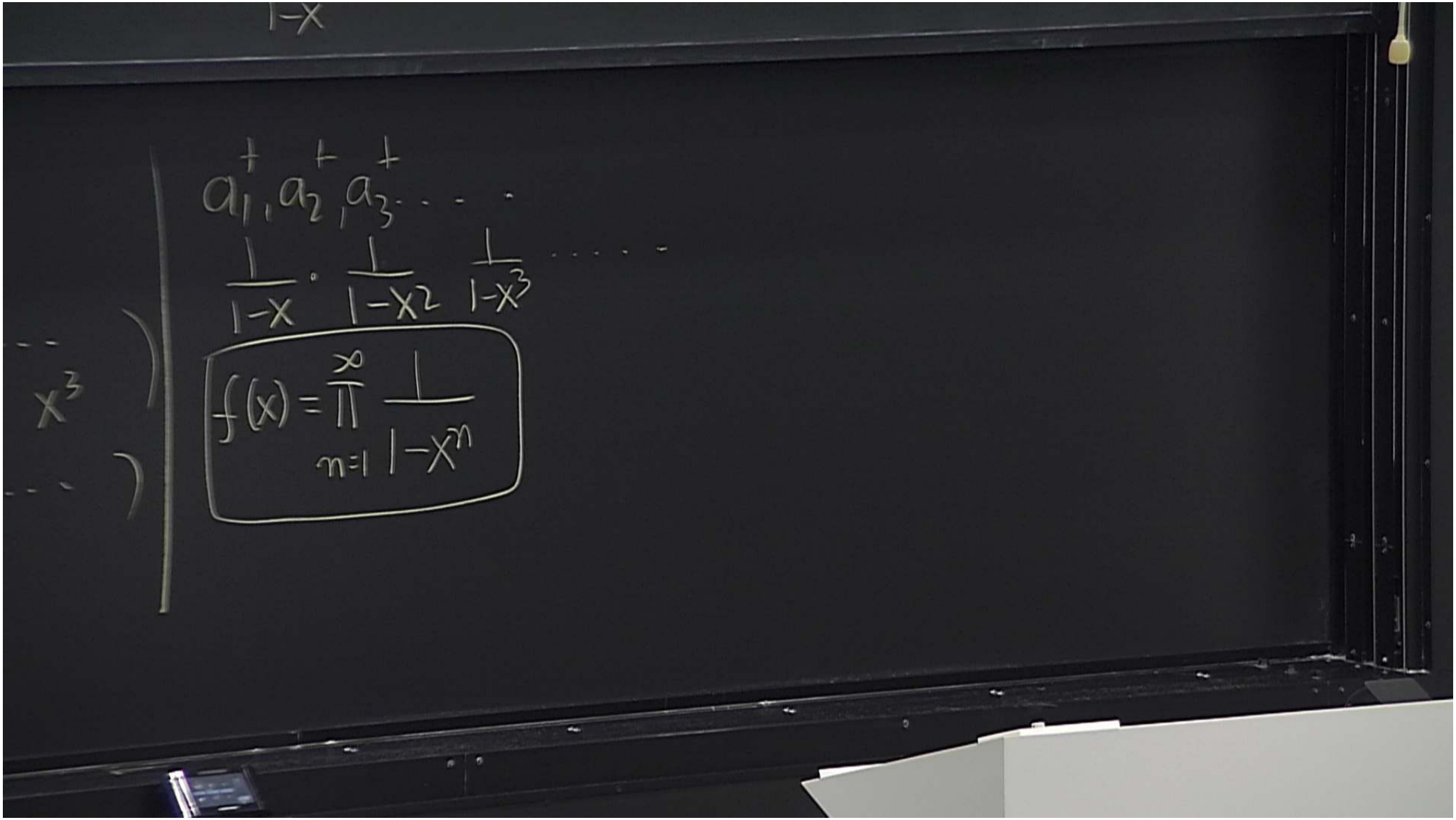
$f_2(x) = 1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}$

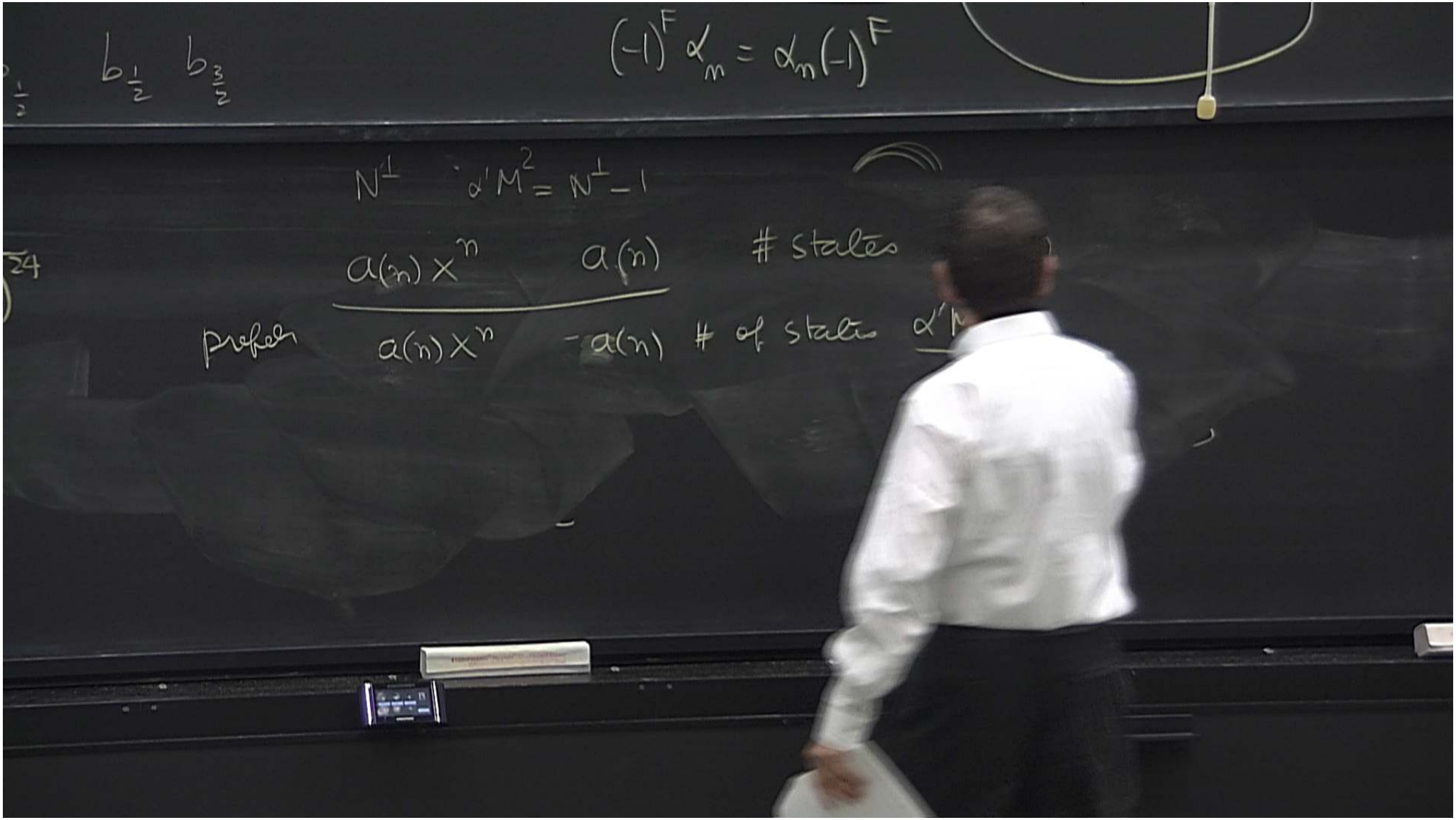
Using $a_1^\dagger, a_2^\dagger$ $f_{12}(x) = f_1(x)\, f_2(x_2) = \left(1 + \underset{a_1^\dagger|0\rangle}{x} + \underset{a_1^\dagger a_1^\dagger|0\rangle}{x^2} + x^3 + \cdots\right)$

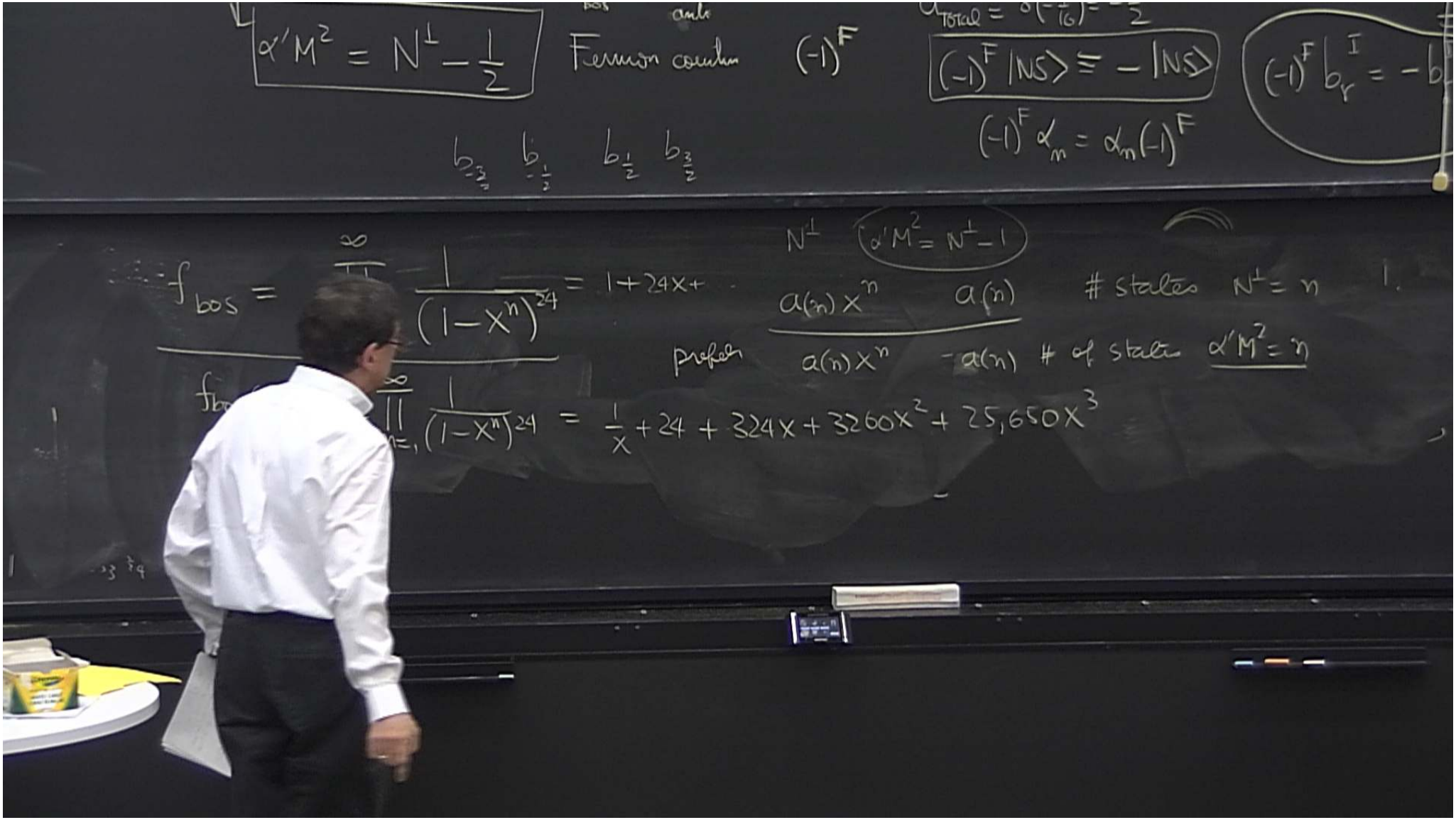
$\left(1 + \underset{a_2^\dagger|0\rangle}{x^2} + \underset{(a_2^\dagger)^2|0\rangle}{x^4} + \cdots\right)$

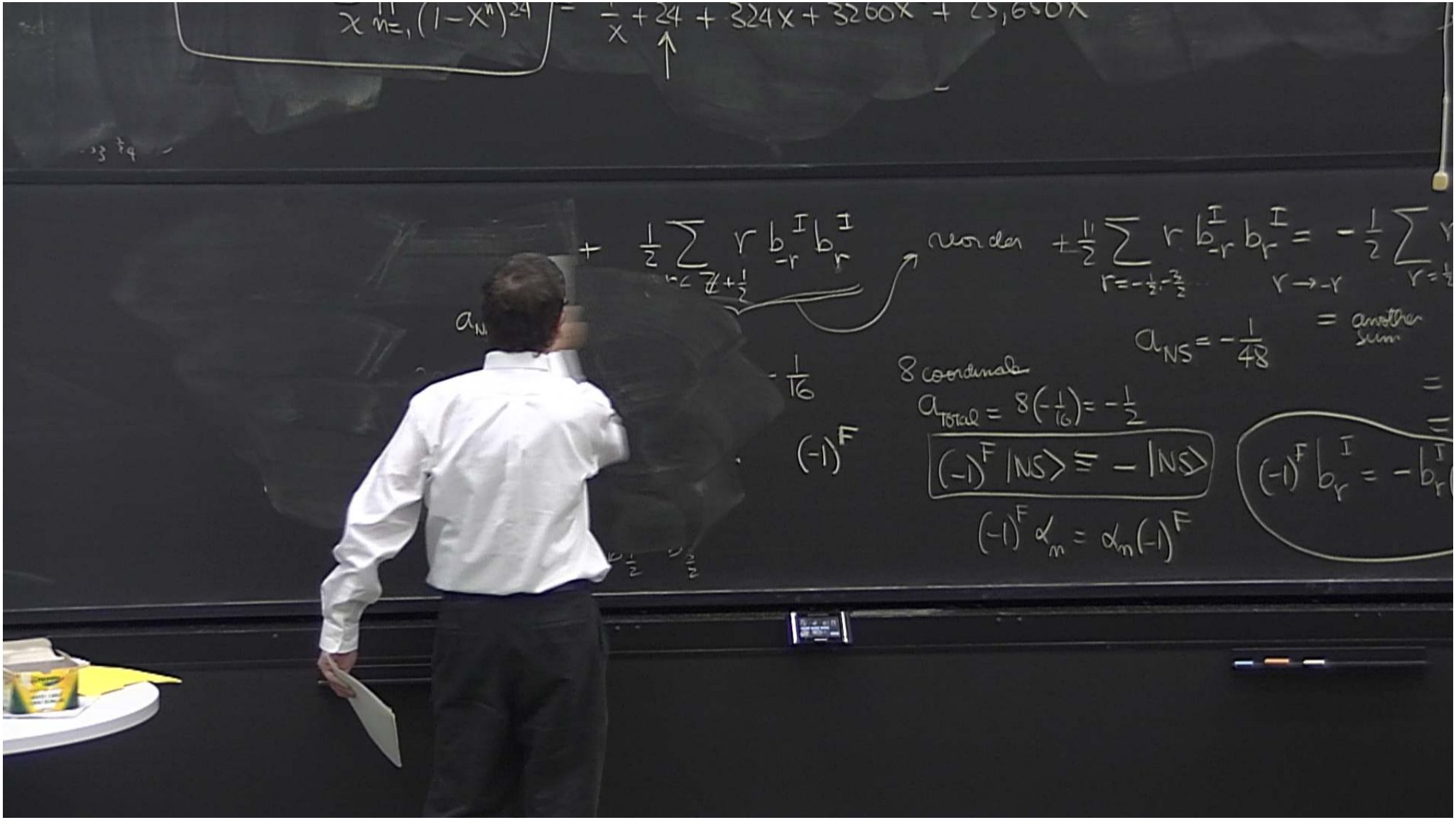
$a_1^\dagger, a_2^\dagger, a_3^\dagger \cdots$


1-x
$a_1, a_2, a_3 \ldots$
$\frac{1}{1-x} \cdot \frac{1}{1-x^2} \frac{1}{1-x^3} \ldots$
$f(x) = \prod_{n=1}^{\infty} \frac{1}{1-x^n}$
x^3

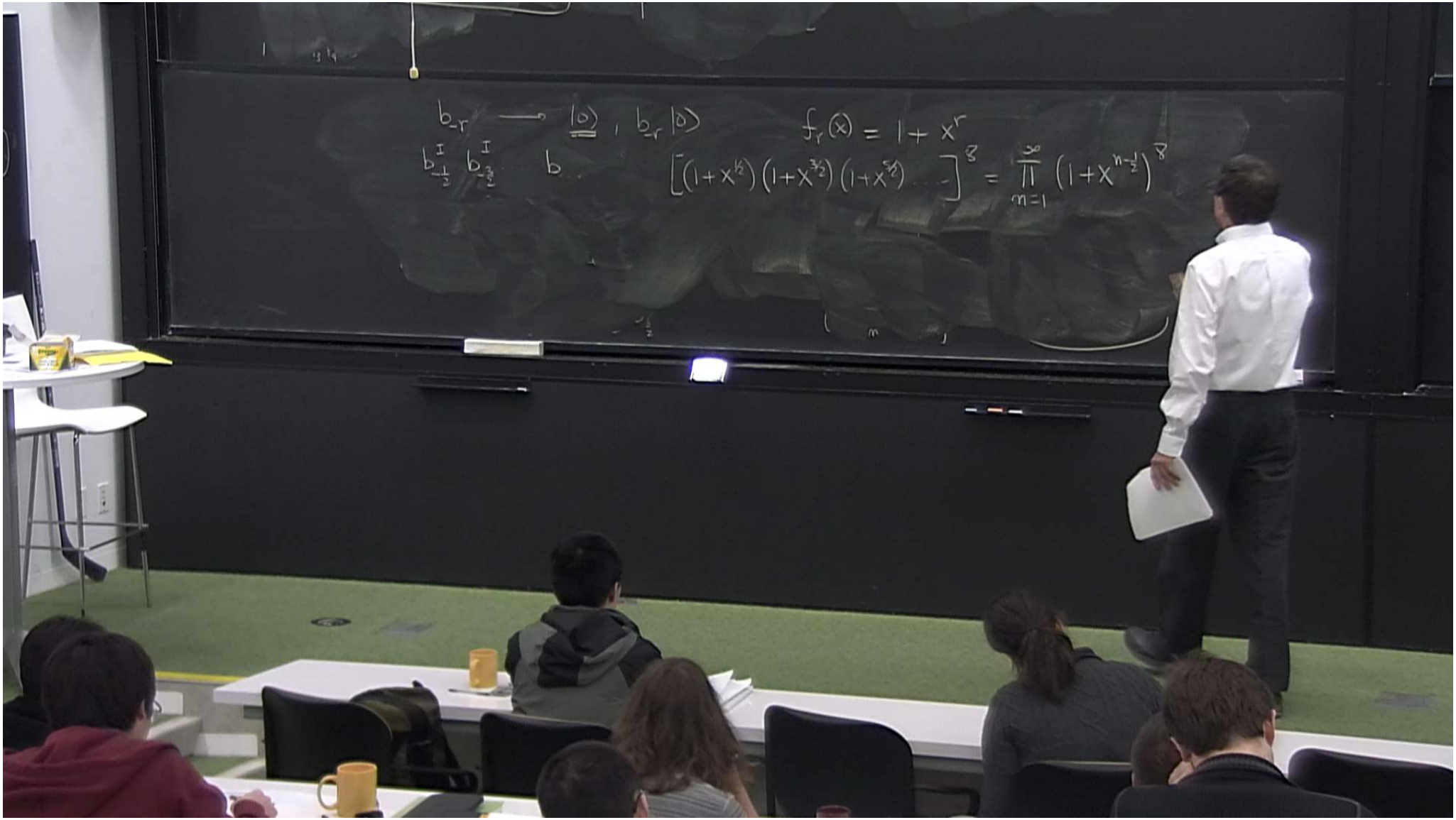

1-x
$a_1, a_2, a_3 \ldots$
$\frac{1}{1-x} \cdot \frac{1}{1-x^2} \frac{1}{1-x^3} \ldots$
$f(x) = \prod_{n=1}^{\infty} \frac{1}{1-x^n}$
x^3

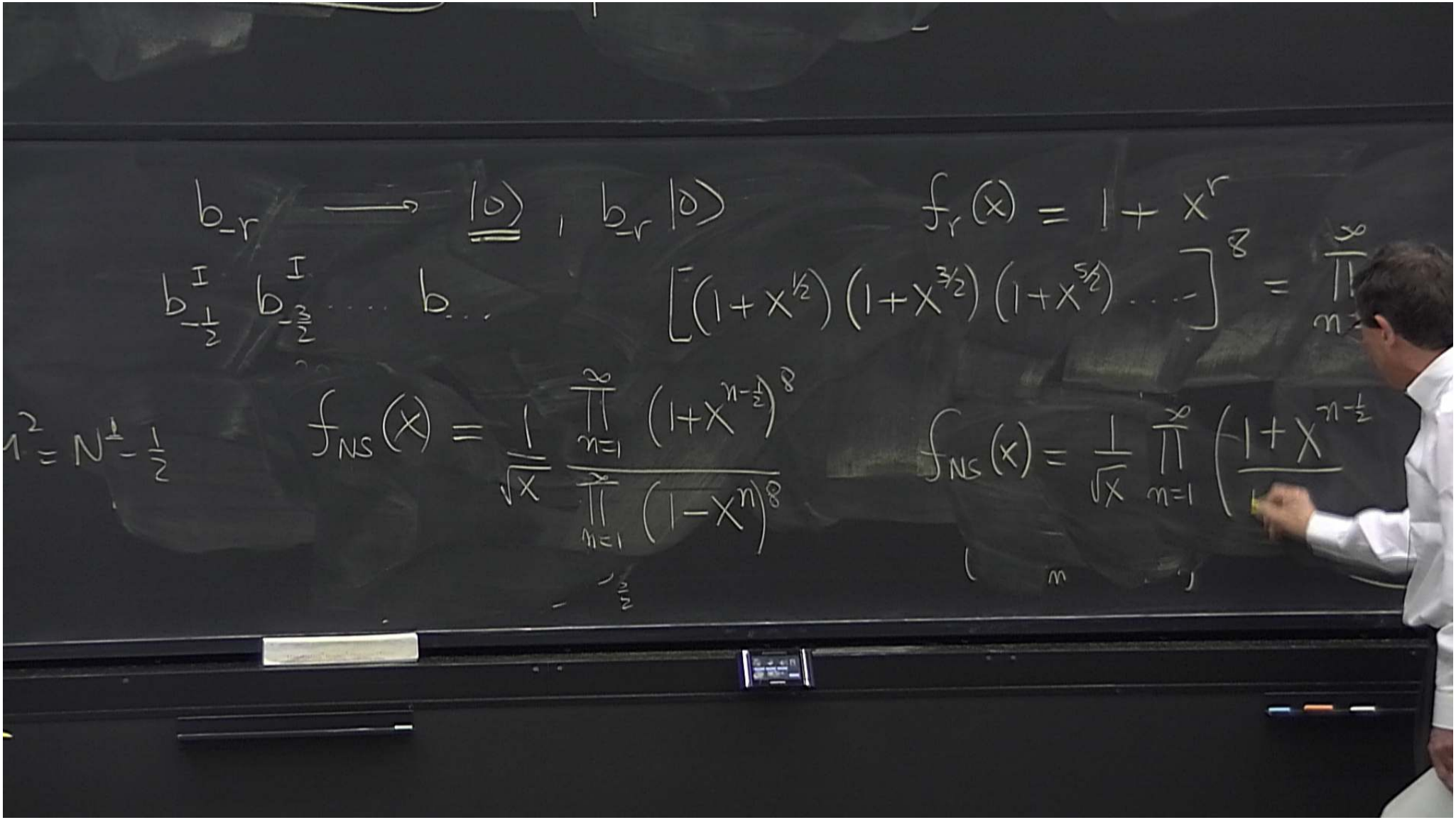

$b_{\frac{1}{2}}$ $b_{\frac{3}{2}}$
$(-1)^F \alpha_n = \alpha_n (-1)^F$
N^1 $\alpha' M^2 = N^1 - 1$
$a(n) x^n$ $a(n)$ # states
$a(n) x^n$ $-a(n)$ # of state

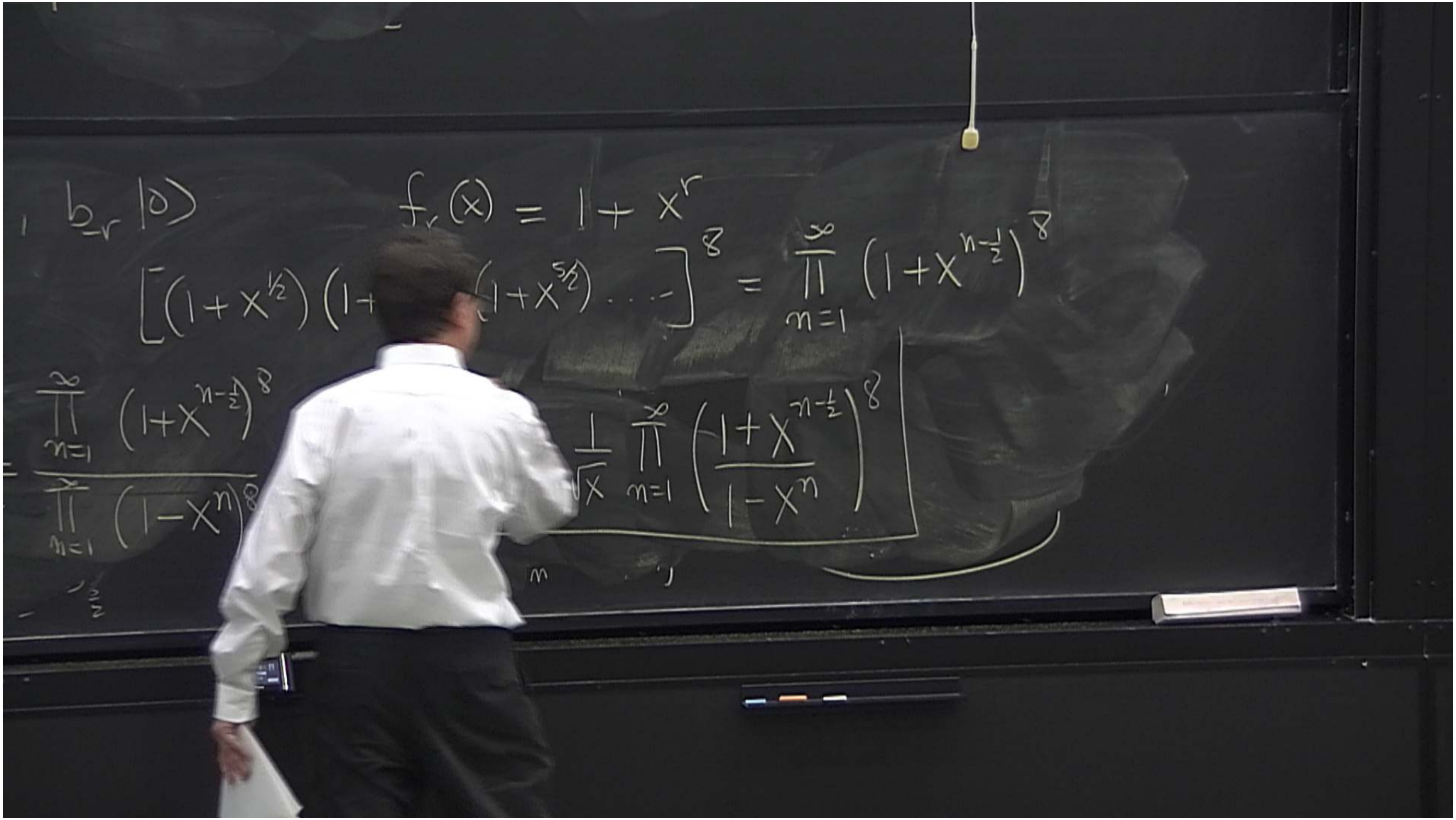

$\alpha' M^2 = N^\perp - \frac{1}{2}$
Fermion counting $(-1)^F$
$(-1)^F |NS\rangle \equiv -|NS\rangle$
$(-1)^F \alpha_n = \alpha_n (-1)^F$
$b_{-\frac{3}{2}}$ $b_{-\frac{1}{2}}$ $b_{\frac{1}{2}}$ $b_{\frac{3}{2}}$
$f_{bos} = \prod_{n=1}^{\infty} \frac{1}{(1-x^n)^{24}} = 1 + 24x +$
$N^\perp$ $\alpha' M^2 = N^\perp - 1$
$a(n) x^n$ $a(n)$ # states $N^\perp = n$
prefactor $a(n) x^n$ $a(n)$ # of states $\alpha' M^2 = n$
$\frac{1}{x} + 24 + 324x + 3260x^2 + 25{,}650x^3$

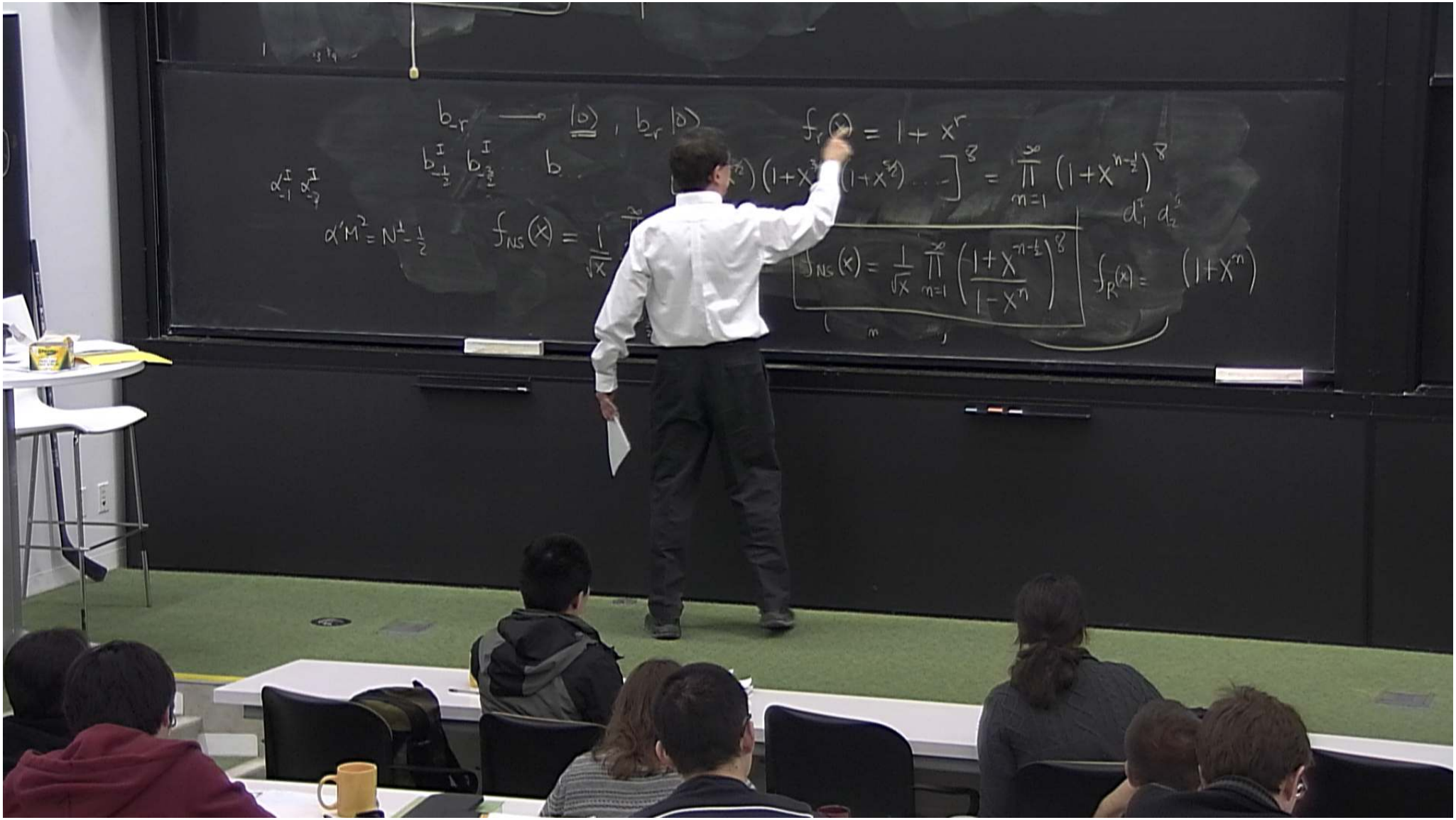


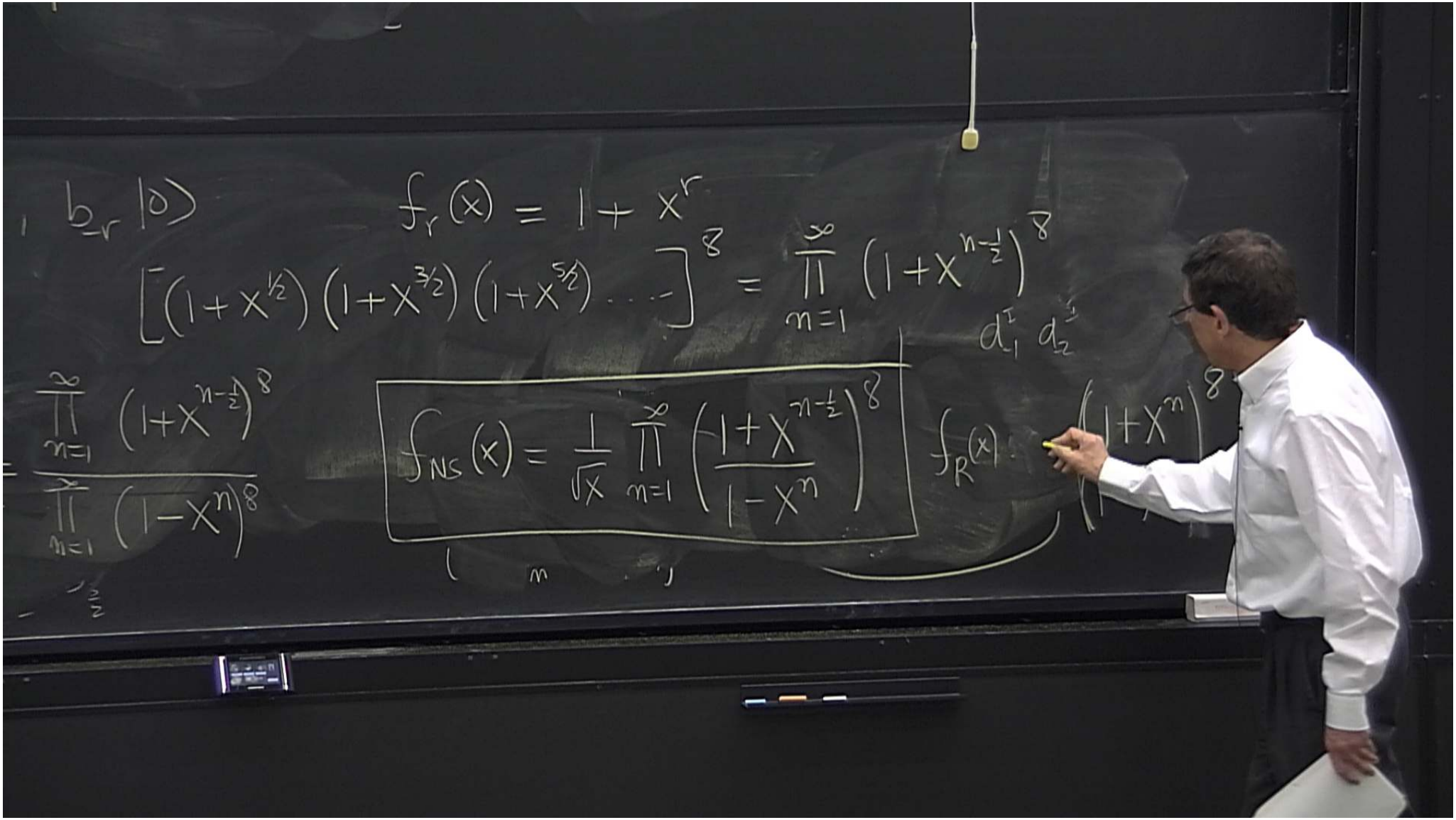
reorder
8 coordinates
$a_{NS} = -\frac{1}{48}$
$a_{Total} = 8(-\frac{1}{16}) = -\frac{1}{2}$
$(-1)^F |NS\rangle \equiv -|NS\rangle$
$(-1)^F \alpha_n = \alpha_n (-1)^F$
$(-1)^F b_r^I = -b_r^I$
= another sum

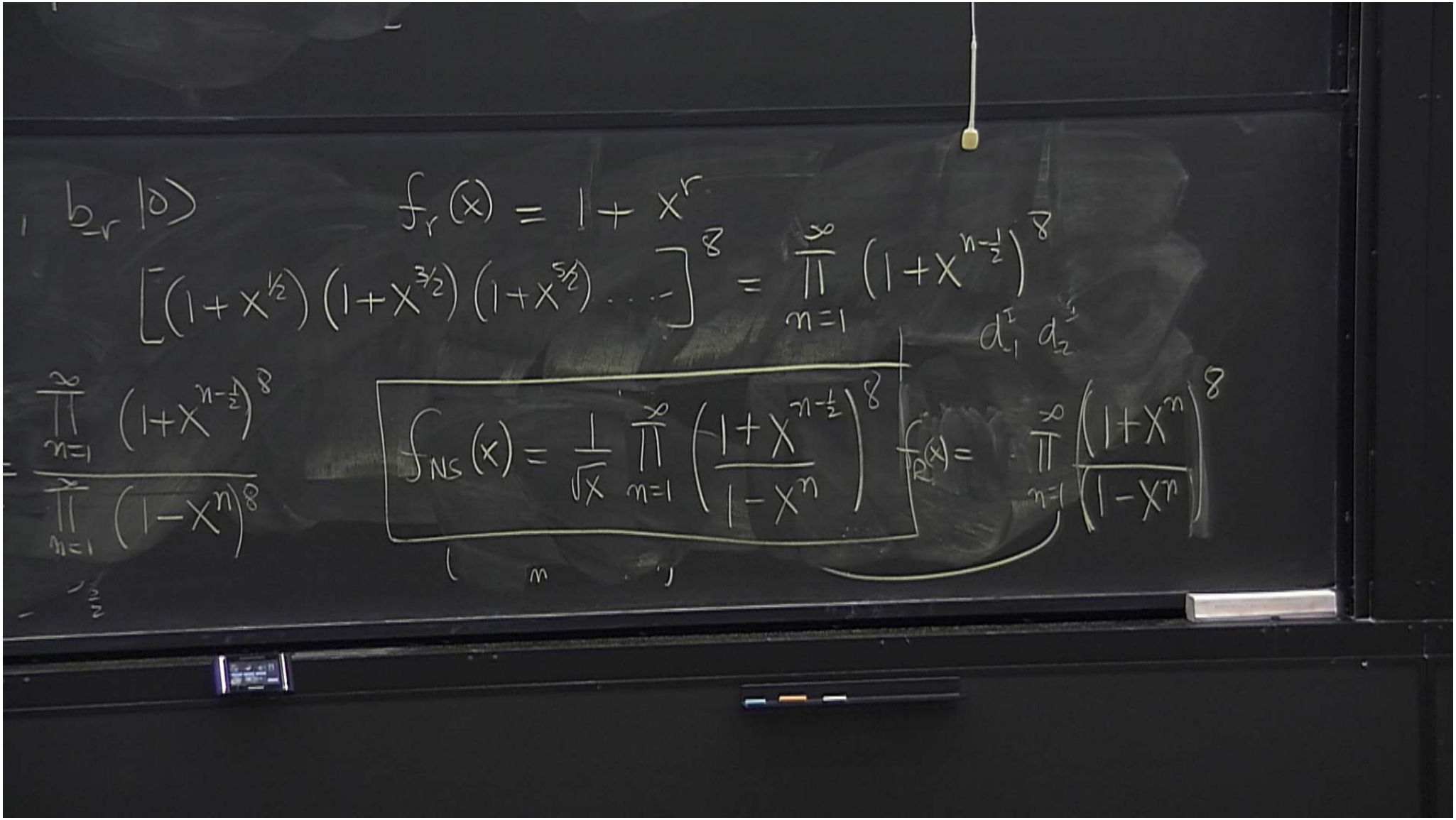

$b_{-r} \longrightarrow |0\rangle , b_{-r}|0\rangle$
$f_r(x) = 1 + x^r$
$b_{-\frac{1}{2}}^I \; b_{-\frac{3}{2}}^I \quad b$
$\left[(1+x^{\frac{1}{2}})(1+x^{\frac{3}{2}})(1+x^{\frac{5}{2}})\ldots\right]^8 = \prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8$

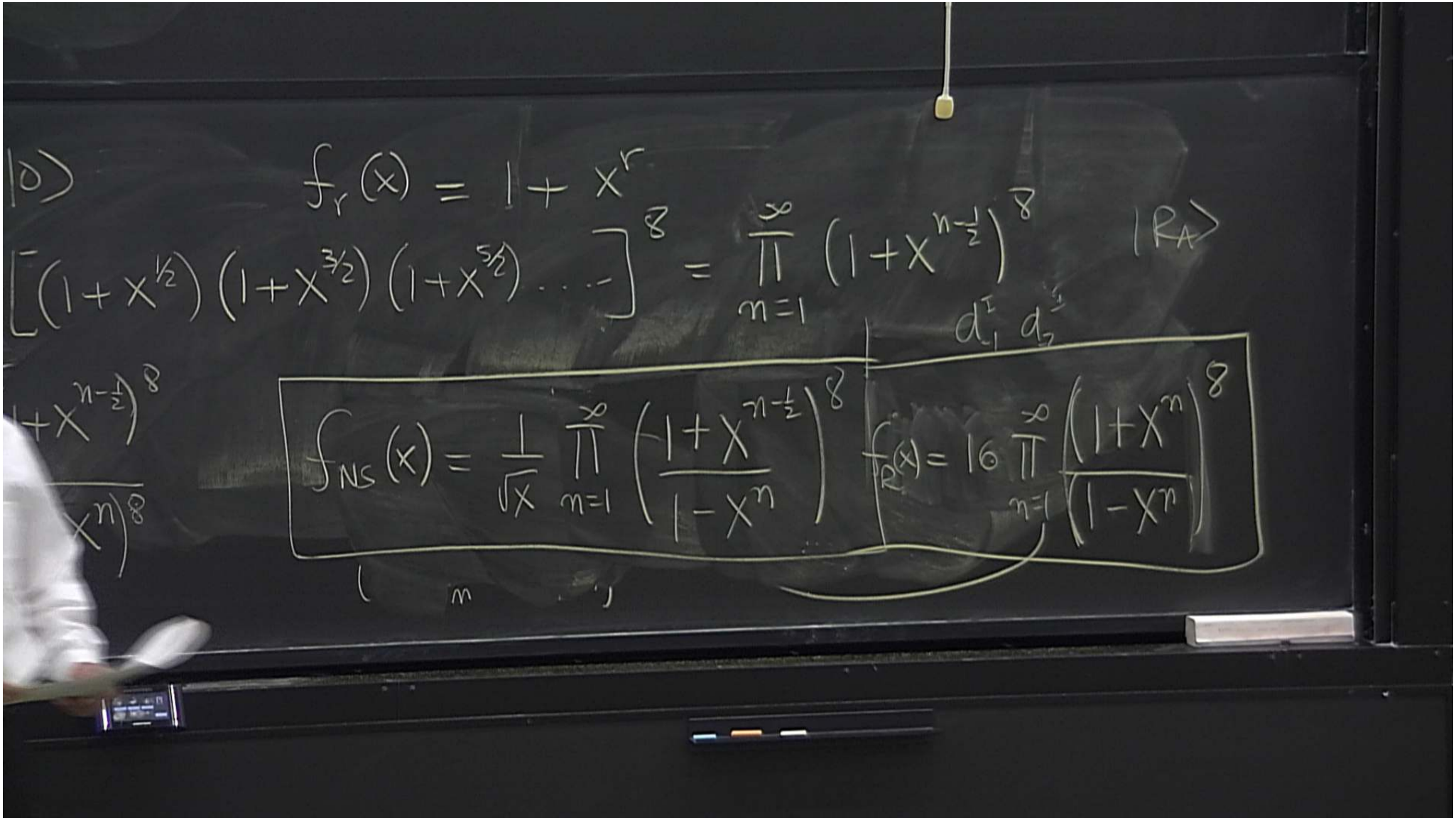

$b_{-r} \longrightarrow |0\rangle , \; b_{-r}|0\rangle$
$f_r(x) = 1 + x^r$
$b^I_{-\frac{1}{2}} \; b^I_{-\frac{3}{2}} \cdots b_{\cdots}$
$\left[(1+x^{1/2})(1+x^{3/2})(1+x^{5/2})\cdots\right]^8 = \prod_{n=}^{\infty}$
$1^2 = N^1_{-\frac{1}{2}}$
$f_{NS}(x) = \frac{1}{\sqrt{x}} \frac{\prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8}{\prod_{n=1}^{\infty}(1-x^n)^8}$
$f_{NS}(x) = \frac{1}{\sqrt{x}} \prod_{n=1}^{\infty}\left(\frac{1+x^{n-\frac{1}{2}}}{}\right.$


$b_r|0\rangle$
$f_r(x) = 1 + x^r$
$\left[(1+x^{1/2})(1+\quad)(1+x^{5/2})\cdots\right]^8 = \prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8$
$\frac{\prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8}{\prod_{n=1}^{\infty}(1-x^n)^8}$
$\frac{1}{\sqrt{x}}\prod_{n=1}^{\infty}\left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n}\right)^8$


$b_{-r}|0\rangle$, $b_{-r}|0\rangle$
$f_r(x) = 1 + x^r$
$\alpha' M^2 = N - \frac{1}{2}$
$f_{NS}(x) = \frac{1}{\sqrt{x}} \prod_{n=1}^{\infty} \left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n} \right)^8$
$f_R(x) = (1+x^n)^8$


$b_{-r}|0\rangle$
$f_r(x) = 1 + x^r$
$\left[(1+x^{1/2})(1+x^{3/2})(1+x^{5/2})\cdots\right]^8 = \prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8$
$f_{NS}(x) = \frac{1}{\sqrt{x}}\prod_{n=1}^{\infty}\left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n}\right)^8$
$f_R(x)$
$\frac{\prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8}{\prod_{n=1}^{\infty}(1-x^n)^8}$


$b_{-r}|0\rangle$
$f_r(x) = 1 + x^r$
$\left[(1+x^{1/2})(1+x^{3/2})(1+x^{5/2})\cdots\right]^8 = \prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8$
$f_{NS}(x) = \frac{1}{\sqrt{x}}\prod_{n=1}^{\infty}\left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n}\right)^8$
$f_R(x) = \prod_{n=1}^{\infty}\left(\frac{1+x^n}{1-x^n}\right)^8$
$\frac{\prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8}{\prod_{n=1}^{\infty}(1-x^n)^8}$


$|0\rangle$
$f_r(x) = 1 + x^r$
$\left[(1+x^{1/2})(1+x^{3/2})(1+x^{5/2})\cdots\right]^8 = \prod_{n=1}^{\infty}(1+x^{n-\frac{1}{2}})^8$
$|R_A\rangle$
$f_{NS}(x) = \frac{1}{\sqrt{x}}\prod_{n=1}^{\infty}\left(\frac{1+x^{n-\frac{1}{2}}}{1-x^n}\right)^8$
$f_2(x) = 16\prod_{n=1}^{\infty}\left(\frac{1+x^n}{1-x^n}\right)^8$