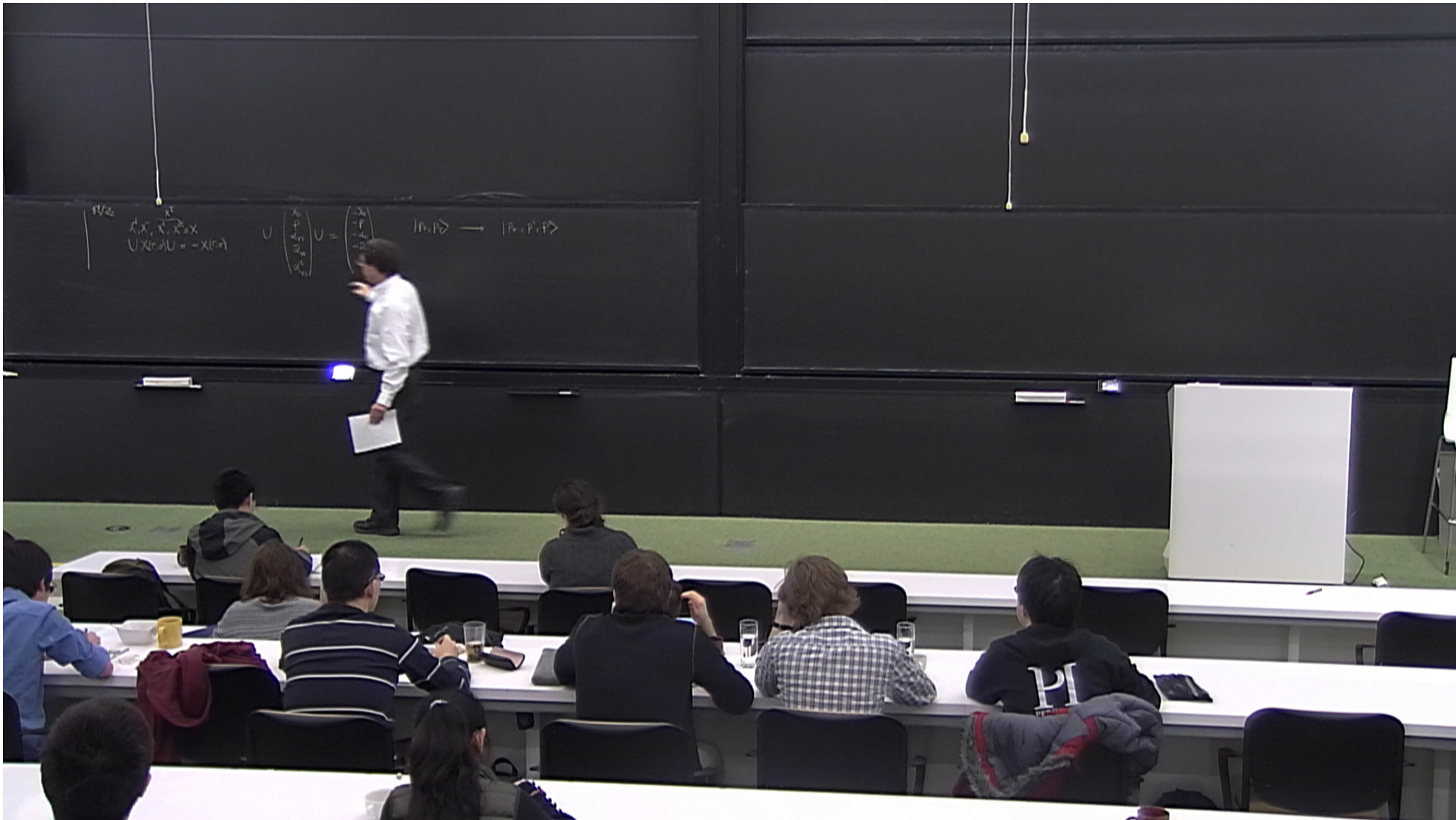


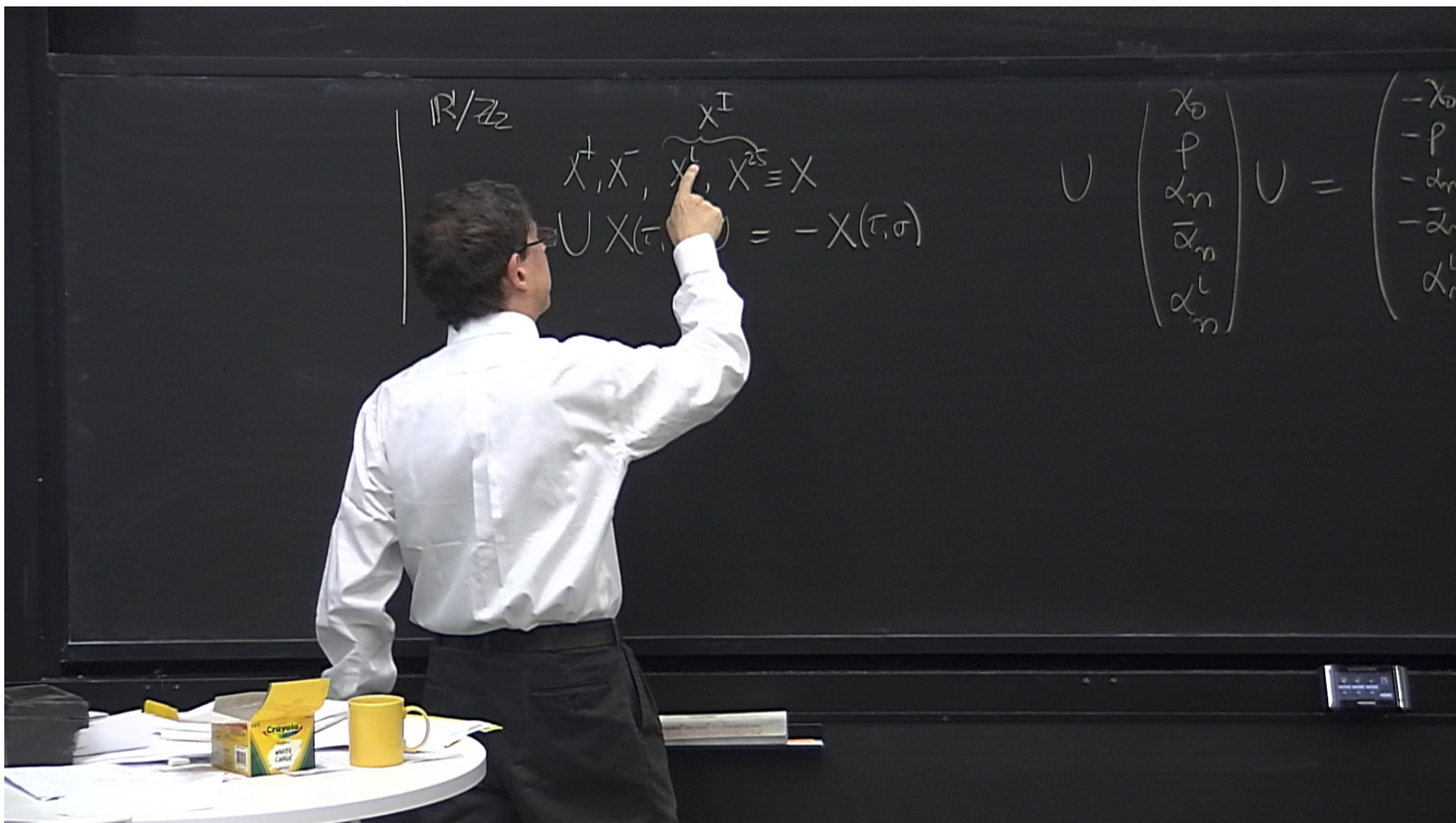
Title: 12/13 PSI - String Theory Review Lecture 13

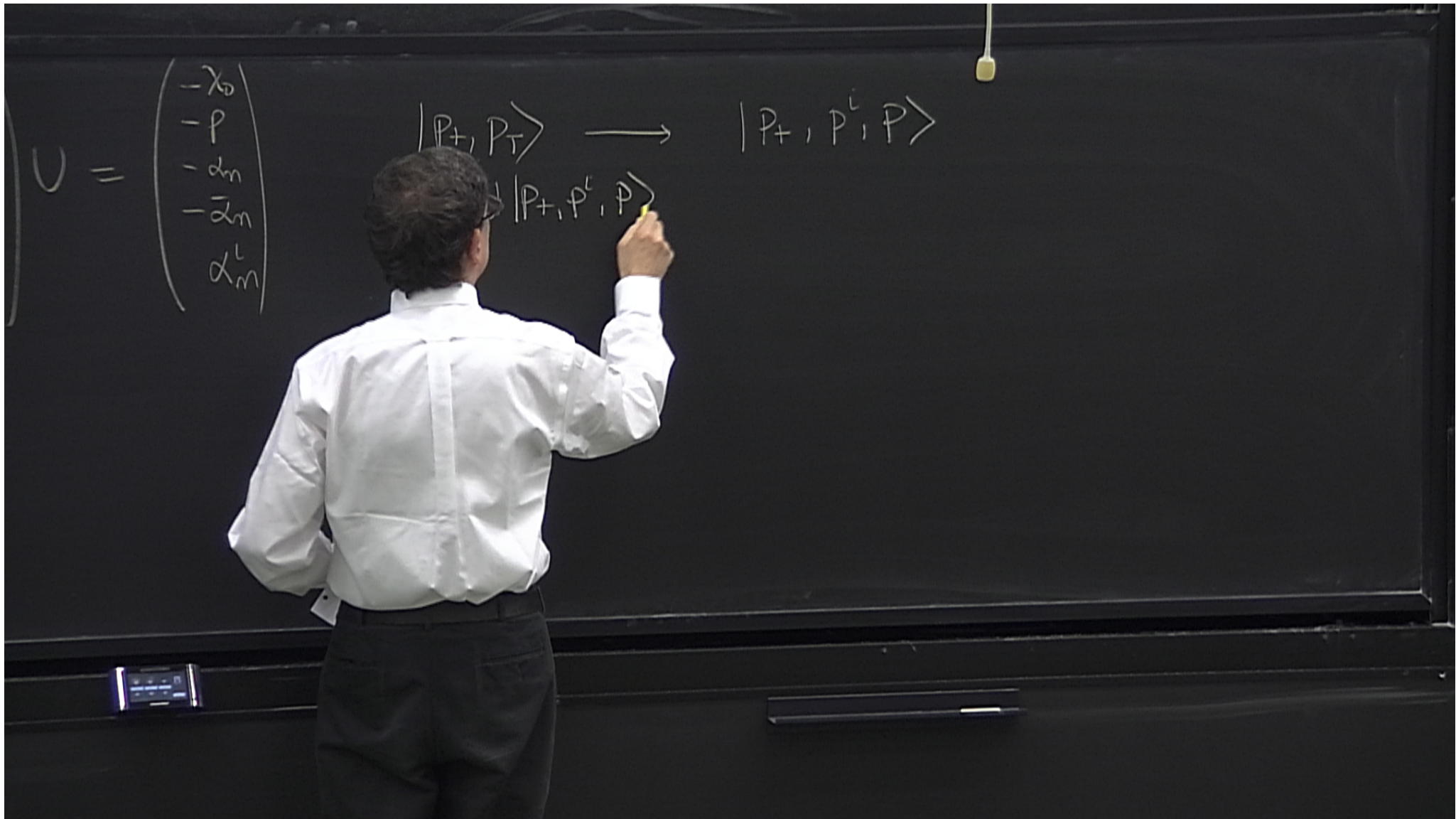
Date: Jan 23, 2013 10:15 AM

URL: <http://pirsa.org/13010063>

Abstract:







$$U = \begin{pmatrix} -\chi_0 \\ -P \\ -\alpha_m \\ -\bar{\alpha}_m \\ \alpha_m^L \end{pmatrix}$$

$$|P_+, P_T\rangle \longrightarrow |P_+, P^L, P\rangle$$
$$|P_+, P^L, P\rangle$$

$$U = \begin{pmatrix} -\chi_0 \\ -p \\ -\alpha_m \\ -\bar{\alpha}_m \\ \alpha_m^L \end{pmatrix}$$

$$|P_+, P_T\rangle \longrightarrow |P_+, P^L, P\rangle$$

$$U |P_+, P^L, P\rangle = |P_+, P^L, -P\rangle$$

$$U = \begin{pmatrix} -\chi_0 \\ -p \\ -\alpha_m \\ -\bar{\alpha}_m \\ \alpha_m^L \end{pmatrix}$$

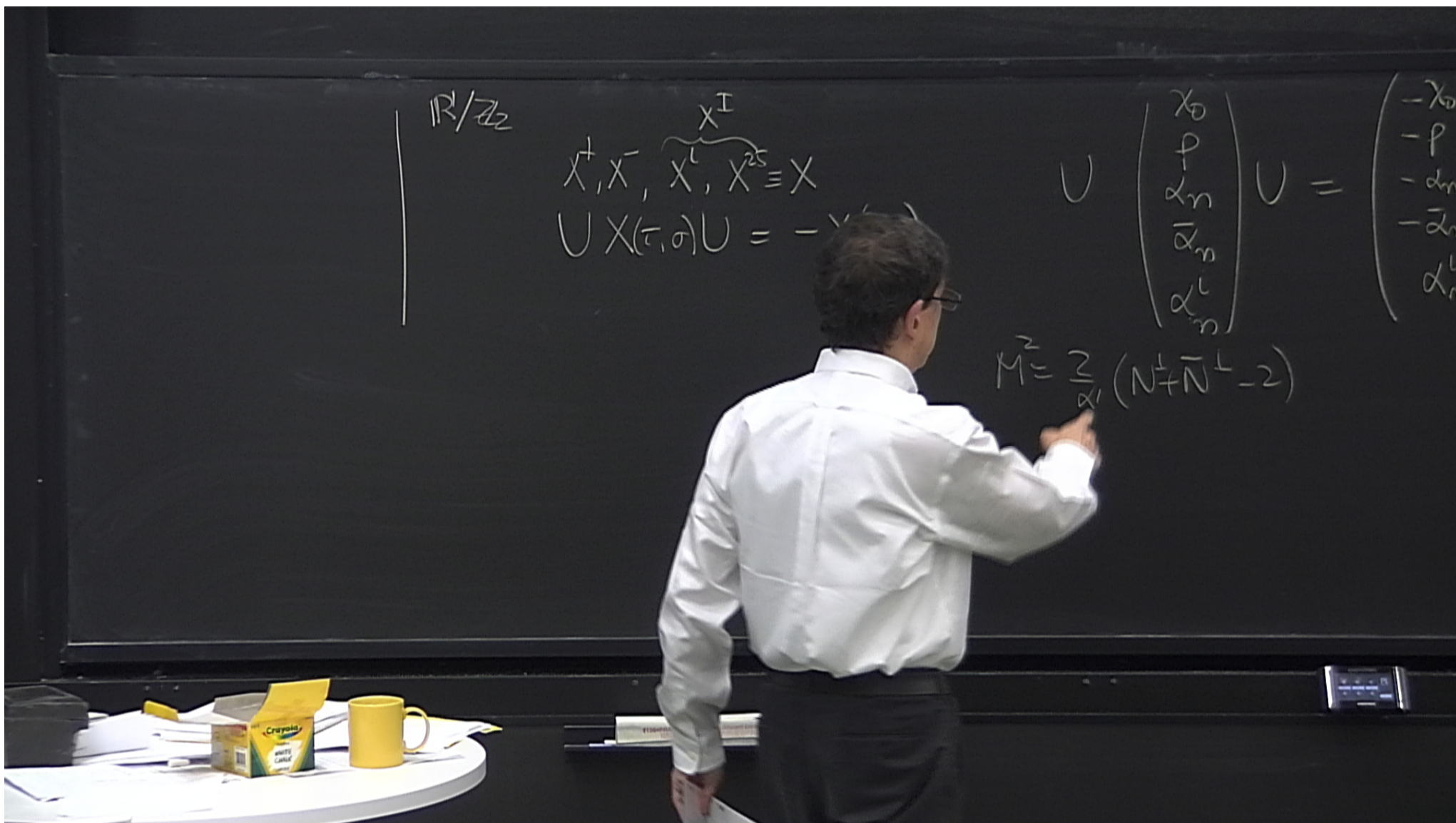
$$|P_+, P_T\rangle \longrightarrow |P_+, P^L, P\rangle$$

$$U |P_+, P^L, P\rangle = |P_+, P^L, -P\rangle$$

true ground states

$$|\oplus_+\rangle = |P^+, P^L, P\rangle + |P^+, P^L, -P\rangle$$

Massless states



$$U = \begin{pmatrix} -\lambda_0 \\ -p \\ -\alpha_m \\ -\bar{\alpha}_m \\ \alpha_m^L \\ \alpha_m^L \\ \alpha_m^L \\ \alpha_m^L \end{pmatrix}$$

$$|P_+, P_T\rangle \longrightarrow |P_+, P^L, P\rangle$$

$$U |P_+, P^L, P\rangle = |P_+, P^L, -P\rangle$$

two ground states
tachyon.

$$|0_+\rangle = |P^+, P^L, P\rangle + |P^+, P^L, -P\rangle$$

Massless states

$$\alpha_{-1}^i \bar{\alpha}_{-1}^j |0_+\rangle$$

$$U = \begin{pmatrix} -\lambda_0 \\ -p \\ -\alpha_m \\ -\bar{\alpha}_m \\ \alpha_m^L \\ \alpha_m^- \end{pmatrix}$$

-2)

$$|P_+, P_-\rangle \longrightarrow |P_+, P^L, P\rangle$$

$$U |P_+, P^L, P\rangle = |P_+, P^L, -P\rangle$$

true ground state
tachyon

$$|0_+\rangle = |P^+, P^L, P\rangle + |P^+, P^L, -P\rangle$$

Massless state

$$\alpha_{-1}^j |0_+\rangle$$

$$\bar{\alpha}_{-1}^j |0_+\rangle$$

$$\alpha_{-1}^i$$

$$U = \begin{pmatrix} -\lambda_0 \\ -p \\ -\alpha_m \\ -\bar{\alpha}_m \\ \alpha_m^L \\ \bar{\alpha}_m^L \end{pmatrix}$$

$$|P_+, P_T\rangle \longrightarrow |P_+, P^L, P\rangle$$

$$U |P_+, P^L, P\rangle = |P_+, P^L, -P\rangle$$

two ground states
tachyon.

Massless states

$$|Q_+\rangle = |P^+, P^L, P\rangle + |P^+, P^L, -P\rangle$$

$$|Q_-\rangle = |P^+, P^L, P\rangle - |P^+, P^L, -P\rangle$$

$$\alpha_{-1}^i \bar{\alpha}_{-1}^j |Q_+\rangle$$

$$\alpha_{-1} \bar{\alpha}_{-1} |Q_+\rangle$$

$$\alpha_{-1}^i \bar{\alpha}_{-1}^i |Q_-\rangle$$

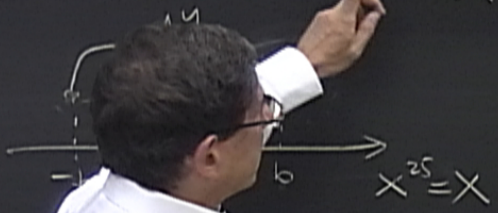
$$\alpha_{-1}^L \bar{\alpha}_{-1}^L |Q_-\rangle$$

Twisted states

$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma)$$

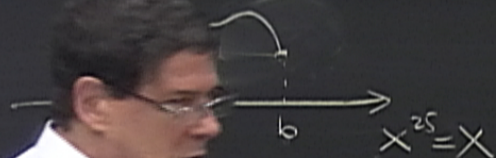
X periodic period 4π

$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$



Twisted states

14



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma + 2\pi)$$

X periodic period 4π

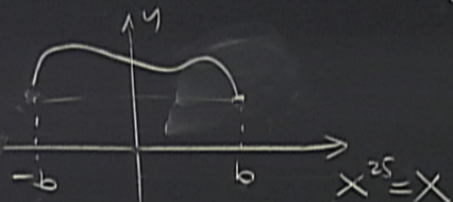
$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$

$$X = X_L\left(\frac{\tau + \sigma}{u}\right) + X_R\left(\frac{\tau - \sigma}{v}\right)$$

$$X_L(u + 2\pi) + X_R(v - 2\pi) = -X_L(u) - X_R(v)$$

$$X_L(u + 2\pi) + X_L(u) = -(X_R(v) + X_R(v - 2\pi))$$

Twisted states



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma + 2\pi)$$

X periodic period 4π

$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$

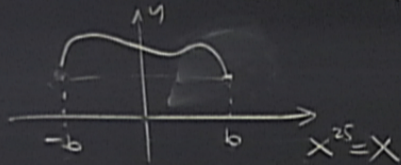
$$X = X_L\left(\frac{\tau + \sigma}{u}\right) + X_R\left(\frac{\tau - \sigma}{v}\right)$$

$$X_L(u + 2\pi) + X_R(v - 2\pi) = -X_L(u) - X_R(v)$$

$$X_L(u + 2\pi) + X_L(u) = -(X_R(v) + X_R(v - 2\pi))$$

$$X'_L(u + 2\pi) + X'_L(u) = 0$$

Twisted states



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma)$$

X periodic period 4π .

$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$

$$X'_L(u) = \sqrt{\frac{\alpha'}{2}} \sum_n \alpha_n e^{-inu}$$

$$X = X_L\left(\frac{\tau+\sigma}{u}\right) + X_R\left(\frac{\tau-\sigma}{v}\right)$$

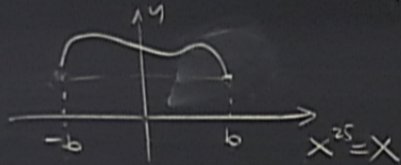
$$X_L(u+2\pi) + X_R(v-2\pi) = -X_L(u) - X_R(v)$$

$$X_L(u+2\pi) + X_L(u) = -(X_R(v) + X_R(v-2\pi))$$

$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

$$X'_L(u+2\pi) = -X'_L(u)$$

Twisted states



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma)$$

X periodic period 4π

$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$

$$X = X_L\left(\frac{\tau+\sigma}{2}\right) + X_R\left(\frac{\tau-\sigma}{2}\right)$$

$$X_L(u+2\pi) + X_R(v-2\pi) = -X_L(u) - X_R(v)$$

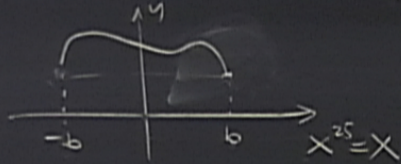
$$X_L(u+2\pi) + X_L(u) = -(X_R(v) + X_R(v-2\pi))$$

$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

$$X'_L(u+2\pi) = -X'_L(u)$$

$$X'_L(u) = \sum_n \alpha_n e^{-\frac{in}{2}u}$$

Twisted States



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma)$$

X periodic period 4π

$$X(\tau, \sigma_*) = 0$$

$$X'_L(u) = \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}_{\text{odd}}} \alpha_{\frac{n}{2}} e^{-\frac{in}{2}u}$$

$$X = X_L(\tau - \sigma) + X_R(\tau + \sigma)$$

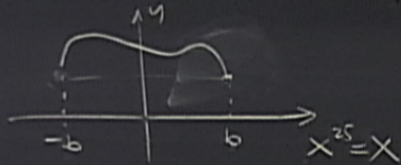
$$X_L(u + 2\pi) = -X_L(u) - X_R(v)$$

$$X_L(v) = -(X_R(v) + X_R(v - 2\pi))$$

$$X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

$$X'_L(u)$$

Twisted states



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma) \quad X \text{ periodic period } 4\pi$$

$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$

$$X = X_L\left(\frac{\tau+\sigma}{u}\right) + X_R\left(\frac{\tau-\sigma}{v}\right)$$

$$X_L(u+2\pi) + X_R(v-2\pi) = -X_L(u) - X_R(v)$$

$$X_L(u+2\pi) + X_L(u) = -(X_R(v) + X_R(v-2\pi))$$

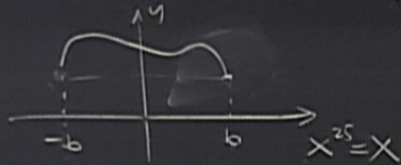
$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

$$X'_L(u+2\pi) = -X'_L(u)$$

$$X'_L(u) = \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \bar{\alpha}_n e^{-\frac{in}{2}u}$$

$$X'_R(v) = \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \alpha_n e^{-\frac{in}{2}v}$$

Twisted states



$$X(\tau, \sigma + 2\pi) = -X(\tau, \sigma)$$

X periodic period 4π

$$\exists \sigma_* \quad X(\tau, \sigma_*) = 0$$

$$X = X_L\left(\frac{\tau+\sigma}{u}\right) + X_R\left(\frac{\tau-\sigma}{v}\right)$$

$$X_L(u+2\pi) + X_R(v-2\pi) = -X_L(u) - X_R(v)$$

$$\boxed{X_L(u+2\pi) + X_L(u) = -(X_R(v) + X_R(v-2\pi))}$$

$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

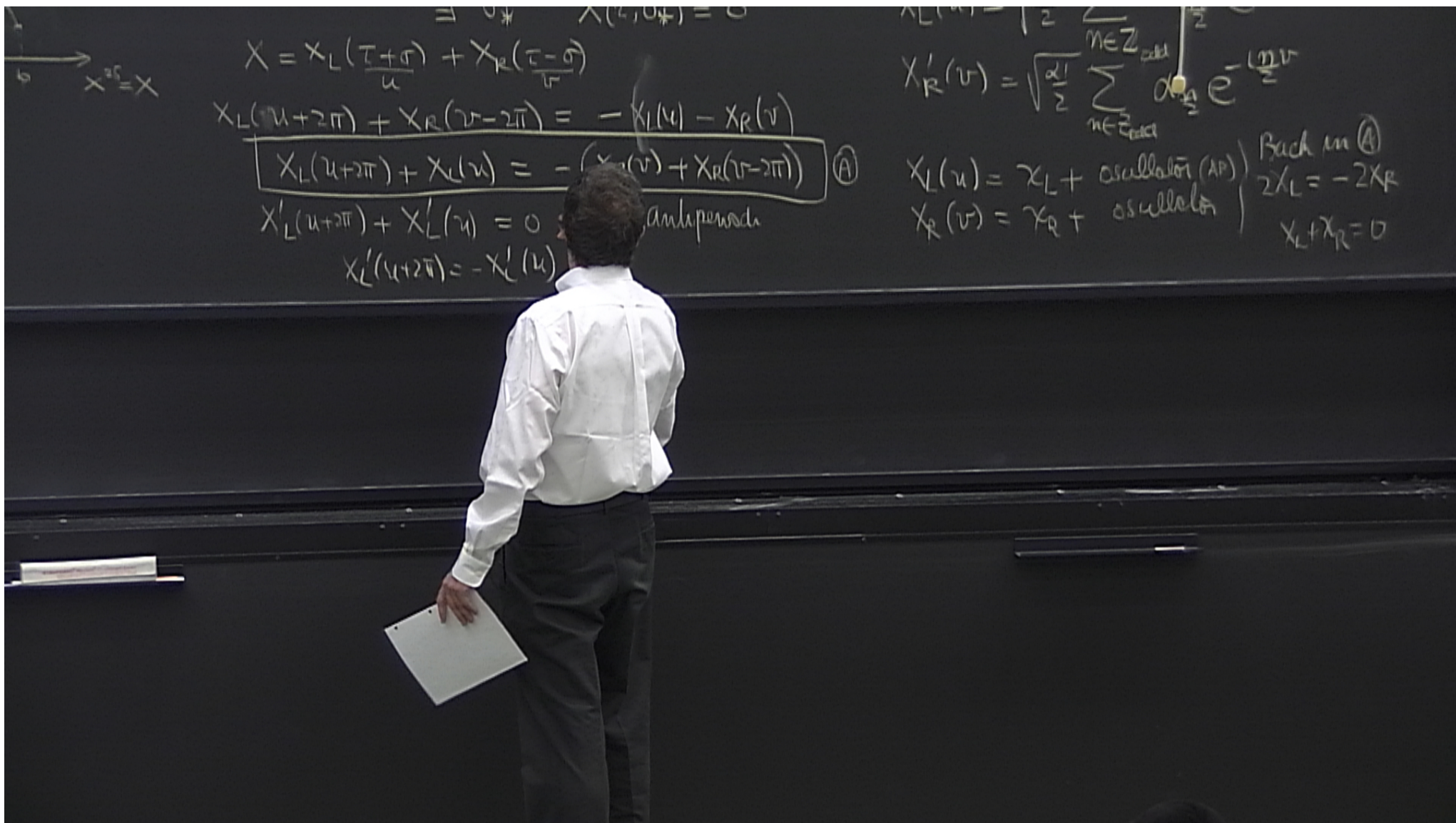
$$X'_L(u+2\pi) = -X'_L(u)$$

$$X'_L(u) = \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \bar{\alpha}_n e^{-\frac{in}{2}u}$$

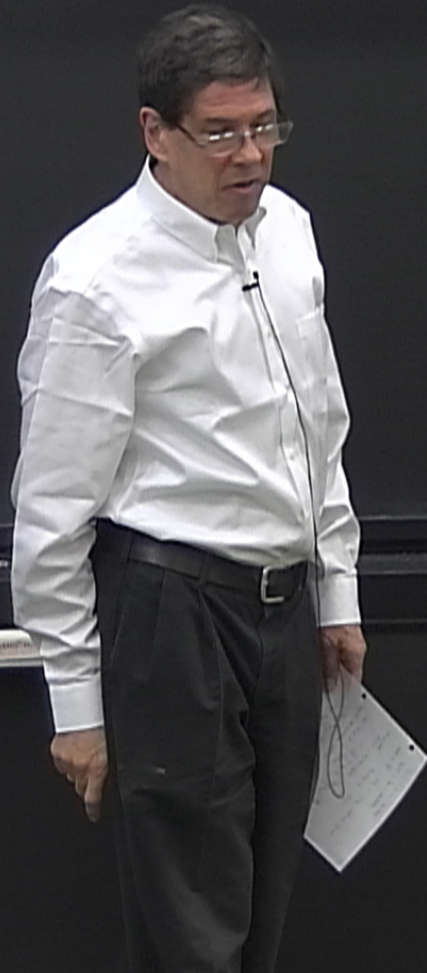
$$X'_R(v) = \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}_{\text{odd}}} \alpha_n e^{-\frac{in}{2}v}$$

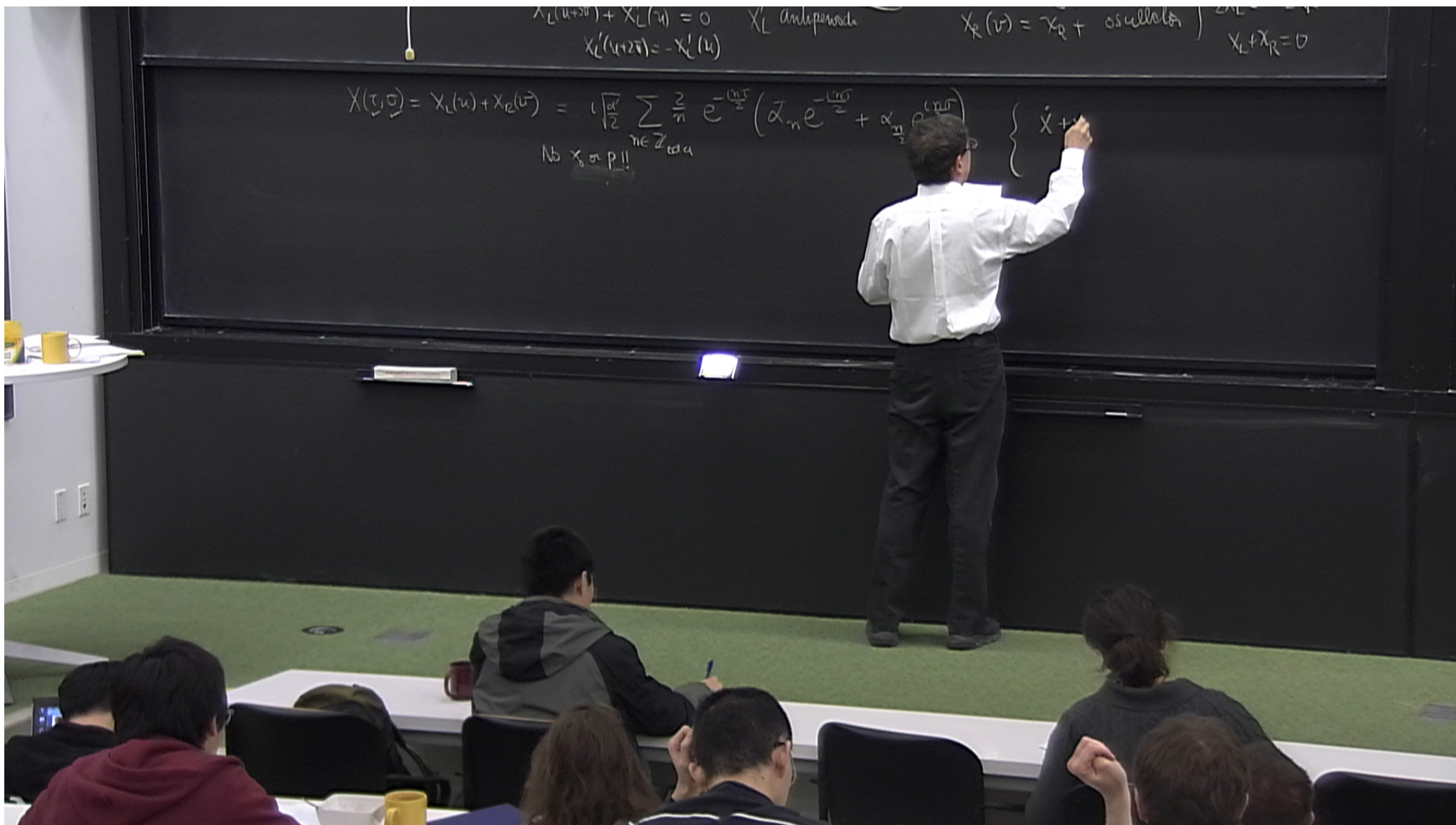
$$X_L(u) = x_L + \text{oscillator (AP)}$$

$$X_R(v) = x_R + \text{oscillator}$$



$$X(\underline{\tau}, \underline{\sigma}) = X_L(u) + X_R(u) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \frac{2}{n} e^{-\frac{(n\tau)}{2}} \left(\alpha_n e^{-\frac{(n\sigma)}{2}} + \alpha_{\frac{n}{2}} e^{\frac{(n\sigma)}{2}} \right)$$





$$X(\underline{\tau}, \underline{\sigma}) = X_L(u) + X_R(u) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \frac{2}{n} e^{-\frac{(n\tau)}{2}} \left(\bar{\alpha}_n e^{-\frac{(n\sigma)}{2}} + \alpha_{\frac{n}{2}} e^{\frac{(n\sigma)}{2}} \right) \quad \left\{ \begin{array}{l} \vdots \end{array} \right.$$

Nb x_0 or p_0 !!

$$X_L(u+2\pi) + X_L(-u) = 0 \quad X_L' \text{ antiperiodic}$$

$$X_L'(u+2\pi) = -X_L'(u)$$

$$X_R(v) = X_R + \dots \quad X_L + X_R = 0$$

$$X(\tau, \sigma) = X_L(u) + X_R(v) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z} \setminus \{0\}} \frac{1}{n} e^{-\frac{n\tau}{2}} \left(\bar{\alpha}_n e^{-\frac{in\sigma}{2}} + \alpha_{\frac{n}{2}} e^{\frac{in\sigma}{2}} \right)$$

No X_0 or P_0 !!

$$\begin{cases} \dot{X} + X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \bar{\alpha}_{\frac{n}{2}} e^{-\frac{n\tau}{2}} \\ \dot{X} - X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \alpha_{\frac{n}{2}} e^{\frac{n\tau}{2}} \end{cases}$$

$$X_L(u+2\pi) + X_L(-u) = 0 \quad X_L' \text{ antiperiodic}$$

$$X_L'(u+2\pi) = -X_L'(u)$$

$$X_R(v) = X_R + \dots \quad X_L + X_R = 0$$

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No X_0 or P_0 !!

$$[\bar{\alpha}_{\frac{m}{2}}, \bar{\alpha}_{\frac{n}{2}}] = \frac{m}{2} \delta_{m+n,0}$$

$$[\alpha_{\frac{m}{2}}, \alpha_{\frac{n}{2}}] = \dots$$

$$\begin{cases} \dot{X} + X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \bar{\alpha}_{\frac{n}{2}} e^{\dots} \\ \dot{X} - X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \alpha_{\frac{n}{2}} \dots \end{cases}$$

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$$\begin{cases} \dot{X} + X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \bar{\alpha}_{\frac{n}{2}} e^{\dots} \\ \dot{X} - X' = \sqrt{2\alpha'} \sum \alpha_{\frac{n}{2}} \dots \end{cases}$$

$$U X U = -X$$

$$U \begin{pmatrix} \alpha_{\frac{n}{2}} \\ \bar{\alpha}_{\frac{n}{2}} \end{pmatrix} U = - \begin{pmatrix} \alpha_{\frac{n}{2}} \\ \bar{\alpha}_{\frac{n}{2}} \end{pmatrix}$$

$$X_L(u+2\pi) + X_L(-u) = 0 \quad X_L' \text{ antiperiodic}$$

$$X_L'(u+2\pi) = -X_L'(u)$$

$$X_R(v) = X_R + \dots \quad X_L + X_R = 0$$

$$X(\tau, \sigma) = X_L(u) + X_R(v) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \frac{1}{n} e^{-\frac{(n-i\tau)}{2}} \left(\bar{\alpha}_n e^{-\frac{(n-i\tau)}{2}} + \alpha_{\frac{n}{2}} e^{\frac{(n-i\tau)}{2}} \right)$$

No X_0 or P_0 !!

$$[\bar{\alpha}_{\frac{m}{2}}, \bar{\alpha}_{\frac{n}{2}}] = \frac{m}{2} \delta_{m+n,0}$$

$$[\alpha_{\frac{m}{2}}, \alpha_{\frac{n}{2}}] = \quad \quad \quad "$$

$$\begin{cases} \dot{X} + X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \bar{\alpha}_{\frac{n}{2}} e^{\dots} \\ \dot{X} - X' = \sqrt{2\alpha'} \sum_{n \text{ even}} \alpha_{\frac{n}{2}} e^{\dots} \end{cases}$$

$$\frac{1}{2} \alpha' M^2 = N^\perp + \bar{N}^\perp + 2a \quad a = -1 = \left(-\frac{1}{24}\right) 24$$

$$U X U = -X$$

$$U \begin{pmatrix} \alpha_{\frac{n}{2}} \\ \bar{\alpha}_{\frac{n}{2}} \end{pmatrix} U = - \begin{pmatrix} \alpha_{\frac{n}{2}} \\ \bar{\alpha}_{\frac{n}{2}} \end{pmatrix}$$

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No x_0 or p_0 !!

$$\begin{cases} \dot{X} + X' = \sqrt{2\alpha'} \sum_{n \text{ odd}} \bar{\alpha}_{\frac{n}{2}} e^{\frac{n\sigma}{2}} \\ \dot{X} - X' = \sqrt{2\alpha'} \sum_{n \text{ even}} \alpha_{\frac{n}{2}} e^{\frac{n\sigma}{2}} \end{cases}$$

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$$[\alpha_{\frac{m}{2}}, \alpha_{\frac{n}{2}}] = \frac{n}{2} \delta_{m+n,0}$$

$$\frac{1}{2} \alpha' M^2 = N + \bar{N} + 2a \quad a = -1 = \left(-\frac{1}{24}\right) 24$$

$$U X U = -X$$

$$U \begin{pmatrix} \alpha_{\frac{n}{2}} \\ \bar{\alpha}_{\frac{n}{2}} \end{pmatrix} U = - \begin{pmatrix} \alpha_{\frac{n}{2}} \\ \bar{\alpha}_{\frac{n}{2}} \end{pmatrix}$$

$$L_0^\perp = \frac{1}{2} \sum_{p \in \mathbb{Z}} \alpha_p^i \alpha_{-p}^i + \frac{1}{2} \sum_{k \in \text{odd}} \alpha_{\frac{k}{2}}^\perp \alpha_{-\frac{k}{2}}^\perp$$

$$\alpha = 23 \left(-\frac{1}{24} \right) + \text{extne from } \text{twisted}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{k \in \mathbb{Z}^+_{\text{odd}}} \alpha_{-\frac{k}{2}} \alpha_{\frac{k}{2}} + \frac{1}{2} \sum_{k \text{ (odd+)}} [\alpha_{\frac{k}{2}}, \alpha_{-\frac{k}{2}}]$$

$$\alpha = 23 \left(-\frac{1}{24} \right) + \text{extra from twist}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{k \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{k}{2}} \alpha_{\frac{k}{2}} + \left(\frac{1}{2} \sum_{k \text{ (odd)}} \left[\alpha_{\frac{k}{2}}, \alpha_{-\frac{k}{2}} \right] \right)$$

$$a = 23 \left(-\frac{1}{24} \right) + \text{extra from twisted}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{k \in \mathbb{Z}^+_{\text{odd}}} \alpha_{-\frac{k}{2}} \alpha_{\frac{k}{2}} + \left(\frac{1}{2} \sum_{k \text{ (odd+)}} [\alpha_{\frac{k}{2}}, \alpha_{-\frac{k}{2}}] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots)$$

$$1+2+3+4+5+6+7+8+\dots = -\frac{1}{12}$$

$$a = 23 \left(-\frac{1}{24} \right) + \text{extra from twisted}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{k \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{k}{2}} \alpha_{\frac{k}{2}} + \left(\frac{1}{2} \sum_{k \text{ (odd)}} [\alpha_{\frac{k}{2}}, \alpha_{-\frac{k}{2}}] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots)$$

$$1+2+3+4+5+6+7+8+\dots = -\frac{1}{12}$$

$$1+3+5+7+\dots + 2+4+6+8+\dots$$

$$\sum_{k \in \mathbb{Z}_{\text{odd}}^+} + 2$$

$$a = 23 \left(-\frac{1}{24} \right) + \text{extra from twisted}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{k \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{k}{2}} \alpha_{\frac{k}{2}} + \left(\frac{1}{2} \sum_{k \text{ (odd)}} \left[\alpha_{\frac{k}{2}}, \alpha_{-\frac{k}{2}} \right] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots) = \frac{1}{48}$$

$$1+2+3+4+5+6+7+8+\dots = -\frac{1}{12}$$

$$1+3+5+7+\dots + 2+4+6+8+\dots$$

$$\sum_{k \in \mathbb{Z}_{\text{odd}}^+} + 2 \left(-\frac{1}{12} \right) = -\frac{1}{12} \quad \boxed{1+3+5+\dots = +\frac{1}{12}}$$

$$a = 23 \left(-\frac{1}{24} \right) + \text{extra from } \text{tunnel} = \frac{1}{48}$$

$$a = -\frac{15}{16}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{k \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{k}{2}} \alpha_{\frac{k}{2}} + \left(\frac{1}{2} \sum_{k \text{ (odd)}} \left[\alpha_{\frac{k}{2}}, \alpha_{-\frac{k}{2}} \right] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots) = \frac{1}{48}$$

$$1+2+3+4+5+6+7+8+\dots = -\frac{1}{12}$$

$$\frac{1}{2} \alpha' M^2 = N + \bar{N} - \frac{15}{8}$$

$$1+3+5+7+\dots + 2+4+6+8+\dots$$

$$\sum_{k \in \mathbb{Z}_{\text{odd}}^+} + 2 \left(-\frac{1}{12} \right) = -\frac{1}{12}$$

$$1+3+5+\dots = +\frac{1}{12}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{p \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{p}{2}} \alpha_{\frac{p}{2}} + \left(\frac{1}{2} \sum_{k \text{ (odd)}} [\alpha_{\frac{k}{2}}, \alpha_{\frac{k}{2}}] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots) = \frac{1}{48}$$

$$1+2+3+4+5+6+7+8+\dots = -\frac{1}{12}$$

$$1+3+5+7+\dots + 2+4+6+8+\dots$$

$$\sum_{k \in \mathbb{Z}_{\text{odd}}^+} + 2\left(-\frac{1}{12}\right) = -\frac{1}{12}$$

$$1+3+5+\dots = +\frac{1}{12}$$

$$\frac{1}{2} \alpha' M^2 = N + \bar{N} - \frac{15}{8}$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{p \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{p}{2}} \alpha_{\frac{p}{2}} + \left(\frac{1}{2} \sum_{p \text{ (odd)}} [\alpha_{\frac{p}{2}}, \alpha_{-\frac{p}{2}}] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots) = \frac{1}{48}$$

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$$\frac{1}{2} \alpha' M^2 = N + \bar{N} - \frac{15}{8}$$

ground states

$$|P^+, P^-, \cancel{P} \rangle$$

$$\frac{1}{2} \sum_{k \text{ odd}} \alpha_{\frac{k}{2}} \alpha_{-\frac{k}{2}} = \sum_{p \in \mathbb{Z}_{\text{odd}}^+} \alpha_{-\frac{p}{2}} \alpha_{\frac{p}{2}} + \left(\frac{1}{2} \sum_{p \text{ (odd)}} [\alpha_{\frac{p}{2}}, \alpha_{\frac{p}{2}}] \right) = \frac{1}{2} \sum_{k \text{ odd}} \frac{k}{2} = \frac{1}{4} (1+3+5+7+\dots) = \frac{1}{48}$$

$$1+2+3+4+5+6+7+8+\dots = -\frac{1}{12}$$

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$$\sum_{p \in \mathbb{Z}_{\text{odd}}^+} + 2\left(-\frac{1}{12}\right) = -\frac{1}{12}$$

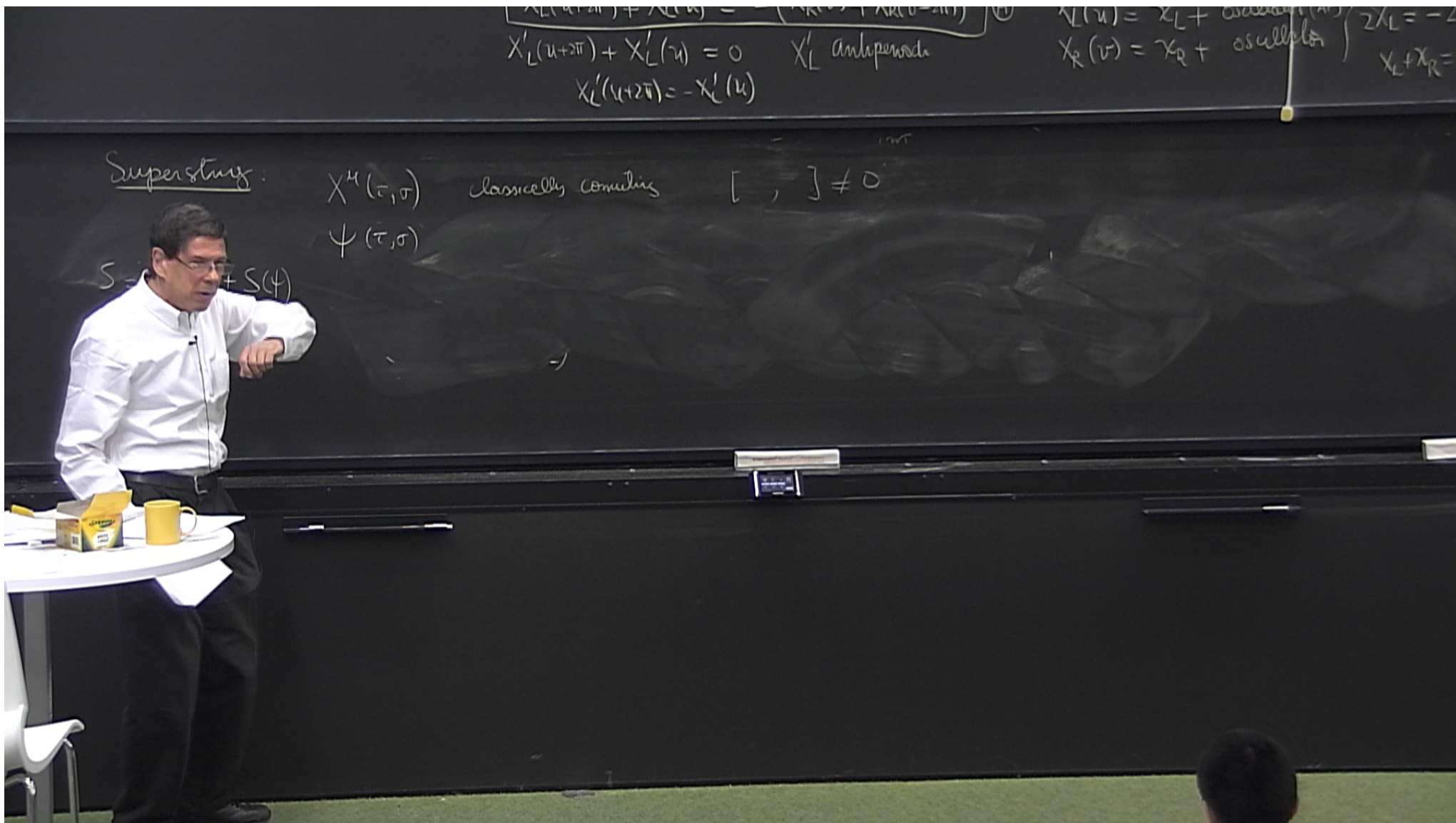
$$1+3+5+\dots = +\frac{1}{12}$$

$$\frac{1}{2} \alpha' M^2 = N + \bar{N} - \frac{15}{8}$$

ground states

$$|p^+, p^-, p\rangle = |p^+, p^-\rangle$$

$$\frac{1}{2} \alpha' M^2 = \frac{1}{2} + \frac{1}{2} - \frac{15}{8} = -\frac{7}{8} \alpha_{-\frac{1}{2}} \bar{\alpha}_{-\frac{1}{2}} |p^+, p^-\rangle$$



$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

$$X'_L(u+2\pi) = -X'_L(u)$$

$$X_R(v) = X_R + \text{oscillations} \quad X_L + X_R = 0$$

Superstring:

$$X^\mu(\tau, \sigma)$$

classically commuting

$$\psi^\mu_1(\tau, \sigma)$$

anti-commuting

$$\psi^\mu_2(\tau, \sigma)$$

Light-cone variables

Quantum

$$[,] \neq 0$$

$$\{ , \} \neq 0$$

$$S = S_{NS}(X) + S(\psi)$$

$$S(\psi) = \frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \left[\psi_1^I \left(\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma} \right) \psi_1^I + \psi_2^I \left(\frac{\partial}{\partial \tau} - \frac{\partial}{\partial \sigma} \right) \psi_2^I \right]$$

$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ antiperiodic}$$

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Superstring

$$X^\mu(\tau, \sigma)$$

classically commuting

|| anti-commuting

Light-cone variables:

$$\psi_1^I \frac{\partial}{\partial \tau} \psi_1^I = \frac{1}{2} \frac{\partial}{\partial \tau} \psi_1^I \psi_1^I$$

EQM

Quantum

$$[,] \neq 0$$

$$\{ \} \neq 0$$

$$S = S_{NS}(X) + S(\psi)$$

$$S(\psi) = \frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \left[\underbrace{\psi_1^I \left(\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma} \right) \psi_1^I}_{\text{vary this}} + \psi_2^I \left(\frac{\partial}{\partial \tau} - \frac{\partial}{\partial \sigma} \right) \psi_2^I \right]$$

$$X'_L(u+2\pi) + X'_L(u) = 0 \quad X'_L \text{ Antiperiodic}$$

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Superstring:

$$X^\mu(\tau, \sigma)$$

classically commuting

$$\psi^\mu_1(\tau, \sigma)$$

anti-commuting

$$\psi^\mu_2(\tau, \sigma)$$

Light-cone variables:

Quantum

$$\left[\begin{array}{c} X^\mu \\ \psi^\mu \end{array} \right] \neq 0$$

$$S = S_{NS}(X) + S(\psi)$$

$$S(\psi) = \frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \left[\psi_1^I \left(\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma} \right) \psi_1^I + \psi_2^I \left(\frac{\partial}{\partial \tau} - \frac{\partial}{\partial \sigma} \right) \psi_2^I \right]$$

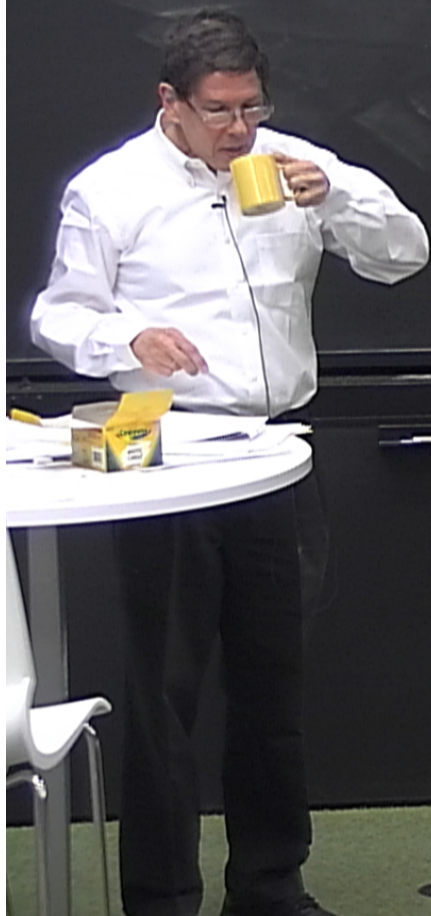
vary this

$$\psi_1^I \frac{\partial}{\partial \tau} \psi_1^I = \frac{1}{2} \frac{\partial}{\partial \tau} \psi_1^I \psi_1^I$$

EOM

$$\delta S(\psi) = \frac{1}{\pi} \int d\tau \int d\sigma \delta \psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I$$

$$\delta S = \frac{1}{\pi} \int d\tau \int_0^\pi d\sigma \left[\delta\psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta\psi_2^I (\partial_\tau - \partial_\sigma) \psi_2^I \right] + \frac{1}{2\pi} \int d\tau \left[\psi_1^I \delta\psi_1^I - \psi_2^I \delta\psi_2^I \right]_0^\pi$$



$$\delta S = \frac{1}{\pi} \int d\tau \int_0^\pi d\sigma \left[\delta \psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta \psi_2^I (\partial_\tau - \partial_\sigma) \psi_2^I \right] + \frac{1}{2\pi} \int d\tau \left[\psi_1^I \delta \psi_1^I - \psi_2^I \delta \psi_2^I \right]_0^\pi$$

$$\begin{array}{l} \text{EOM} \\ \sigma \in [0, \pi] \end{array} \quad \begin{array}{l} (\partial_\tau + \partial_\sigma) \psi_1^I = 0 \\ (\partial_\tau - \partial_\sigma) \psi_2^I = 0 \end{array} \quad \begin{array}{l} \longrightarrow \psi_1^I(\tau, \sigma) = \chi_1^I(\tau - \sigma) \\ \psi_2^I(\tau, \sigma) = \chi_2^I(\tau + \sigma) \end{array}$$

$$\text{BC:} \quad \psi_1^I(\tau, \sigma_*) \delta \psi_1^I(\tau, \sigma_*) - \psi_2^I(\tau, \sigma_*) \delta \psi_2^I(\tau, \sigma_*) = 0$$

$$\delta S = \frac{1}{\pi} \int d\tau \int_0^\pi d\sigma \left[\delta \psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta \psi_2^I (\partial_\tau - \partial_\sigma) \psi_2^I \right] + \frac{1}{2\pi} \int d\tau \left[\psi_1^I \delta \psi_1^I - \psi_2^I \delta \psi_2^I \right]_0^\pi$$

$$\begin{array}{l} \text{EOM} \\ \sigma \in [0, \pi] \end{array} \quad \begin{array}{l} (\partial_\tau + \partial_\sigma) \psi_1^I = 0 \\ (\partial_\tau - \partial_\sigma) \psi_2^I = 0 \end{array} \quad \begin{array}{l} \longrightarrow \psi_1^I(\tau, \sigma) = \chi_1^I(\tau - \sigma) \\ \psi_2^I(\tau, \sigma) = \chi_2^I(\tau + \sigma) \end{array}$$

$$\begin{array}{l} \text{BC} \\ \psi_1^I(\tau, \sigma_*) \delta \psi_1^I(\tau, \sigma_*) - \psi_2^I(\tau, \sigma_*) \delta \psi_2^I(\tau, \sigma_*) = 0 \\ \psi_1^I(\tau, 0) = 0 \quad \psi_1^I(\tau, \pi) = \chi_1^I(\tau) = 0 \end{array}$$

$$\delta S = \frac{1}{\pi} \int d\tau \int_0^\pi d\sigma \left[\delta \psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta \psi_2^I (\partial_\tau - \partial_\sigma) \psi_2^I \right] + \frac{1}{2\pi} \int d\tau \left[\psi_1^I \delta \psi_1^I - \psi_2^I \delta \psi_2^I \right]_0^\pi$$

EOM $(\partial_\tau + \partial_\sigma) \psi_1^I = 0 \rightarrow \psi_1^I(\tau, \sigma) = \chi_1^I(\tau - \sigma)$
 $\sigma \in [0, \pi] \quad (\partial_\tau - \partial_\sigma) \psi_2^I = 0 \quad \psi_2^I(\tau, \sigma) = \chi_2^I(\tau + \sigma)$

BC: $\psi_1^I(\tau, \sigma_*) \delta \psi_1^I(\tau, \sigma_*) - \psi_2^I(\tau, \sigma_*) \delta \psi_2^I(\tau, \sigma_*) = 0$
 $\psi_1^I(\tau, 0) = 0 \quad \psi_1^I(\tau, \pi) = \chi_1^I(\tau) = 0$

$\psi_1^I(\tau, \sigma_*) = \pm \psi_2^I(\tau, \sigma_*)$
 $\delta \psi_1^I(\tau, \sigma_*) = \pm \delta \psi_2^I(\tau, \sigma_*)$

$$\delta S = \frac{1}{\pi} \int d\tau \int_0^\pi d\sigma \left[\delta \psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta \psi_2^I (\partial_\tau - \partial_\sigma) \psi_2^I \right] + \frac{1}{2\pi} \int d\tau \left[\psi_1^I \delta \psi_1^I - \psi_2^I \delta \psi_2^I \right]_0^\pi$$

$\text{EOM} \quad (\partial_\tau + \partial_\sigma) \psi_1^I = 0 \rightarrow \psi_1^I(\tau, \sigma) = \chi_1^I(\tau - \sigma)$
 $\sigma \in [0, \pi] \quad (\partial_\tau - \partial_\sigma) \psi_2^I = 0 \quad \psi_2^I(\tau, \sigma) = \chi_2^I(\tau + \sigma)$

$\text{BC:} \quad \psi_1^I(\tau, \sigma_*) \delta \psi_1^I(\tau, \sigma_*) - \psi_2^I(\tau, \sigma_*) \delta \psi_2^I(\tau, \sigma_*) = 0$

$\psi_1^I(\tau, 0) = 0 \quad \psi_1^I(\tau, \pi) = \chi_1^I(\pi) = 0$

$\psi_1^I(\tau, \sigma_*) = \pm \psi_2^I(\tau, \sigma_*)$
 $\delta \psi_1^I(\tau, \sigma_*) = \pm \delta \psi_2^I(\tau, \sigma_*)$
 $\psi_1^I \delta \psi_1^I(\tau, \sigma_*) = + \psi_2^I \delta \psi_2^I(\tau, \sigma_*)$

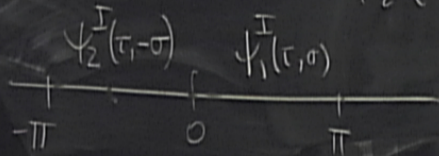
$$\left. \begin{aligned} \psi_1^I(\tau, 0) &= \psi_2^I(\tau, 0) \\ \psi_1^I(\tau, \pi) &= \pm \psi_2^I(\tau, \pi) \\ &\quad - \psi_2^I(\tau, \pi) \end{aligned} \right\} R$$

$$\psi_1^I(\tau, 0) = \psi_2^I(\tau, 0)$$

$$\psi_1^I(\tau, \pi) = \pm \psi_2^I(\tau, \pi) \left\{ \begin{array}{l} \text{Ramond sector} \\ \text{Neveu-Schwartz sector} \end{array} \right.$$

$$- \psi_2^I(\tau, \pi)$$

$$\psi^I(\tau, \sigma) = \begin{cases} \psi_1^I(\tau, \sigma) & \sigma \in [0, \pi) \\ \psi_2^I(\tau, -\sigma) & \sigma \in [-\pi, 0) \end{cases}$$



$$\delta S = \frac{1}{\pi} \int d\tau \int_0^\pi d\sigma \left[\delta \psi_1^I (\partial_\tau + \partial_\sigma) \psi_1^I + \delta \psi_2^I (\partial_\tau - \partial_\sigma) \psi_2^I \right] + \frac{1}{2\pi} \int d\tau \left[\psi_1^I \delta \psi_1^I - \psi_2^I \delta \psi_2^I \right]_0^\pi$$

EOM $(\partial_\tau + \partial_\sigma) \psi_1^I = 0$ $(\partial_\tau - \partial_\sigma) \psi_2^I = 0$
 $\sigma \in [0, \pi]$ $\psi_1^I(\tau, \sigma) = \chi_1^I(\tau - \sigma)$ $\psi_2^I(\tau, \sigma) = \chi_2^I(\tau + \sigma)$

BC: $\psi_1^I(\tau, \sigma_*) \delta \psi_1^I(\tau, \sigma_*) = 0$ $\psi_2^I(\tau, \sigma_*) \delta \psi_2^I(\tau, \sigma_*) = 0$
 $\psi_1^I(\tau, 0) = 0$ $\psi_2^I(\tau, \pi) = 0$

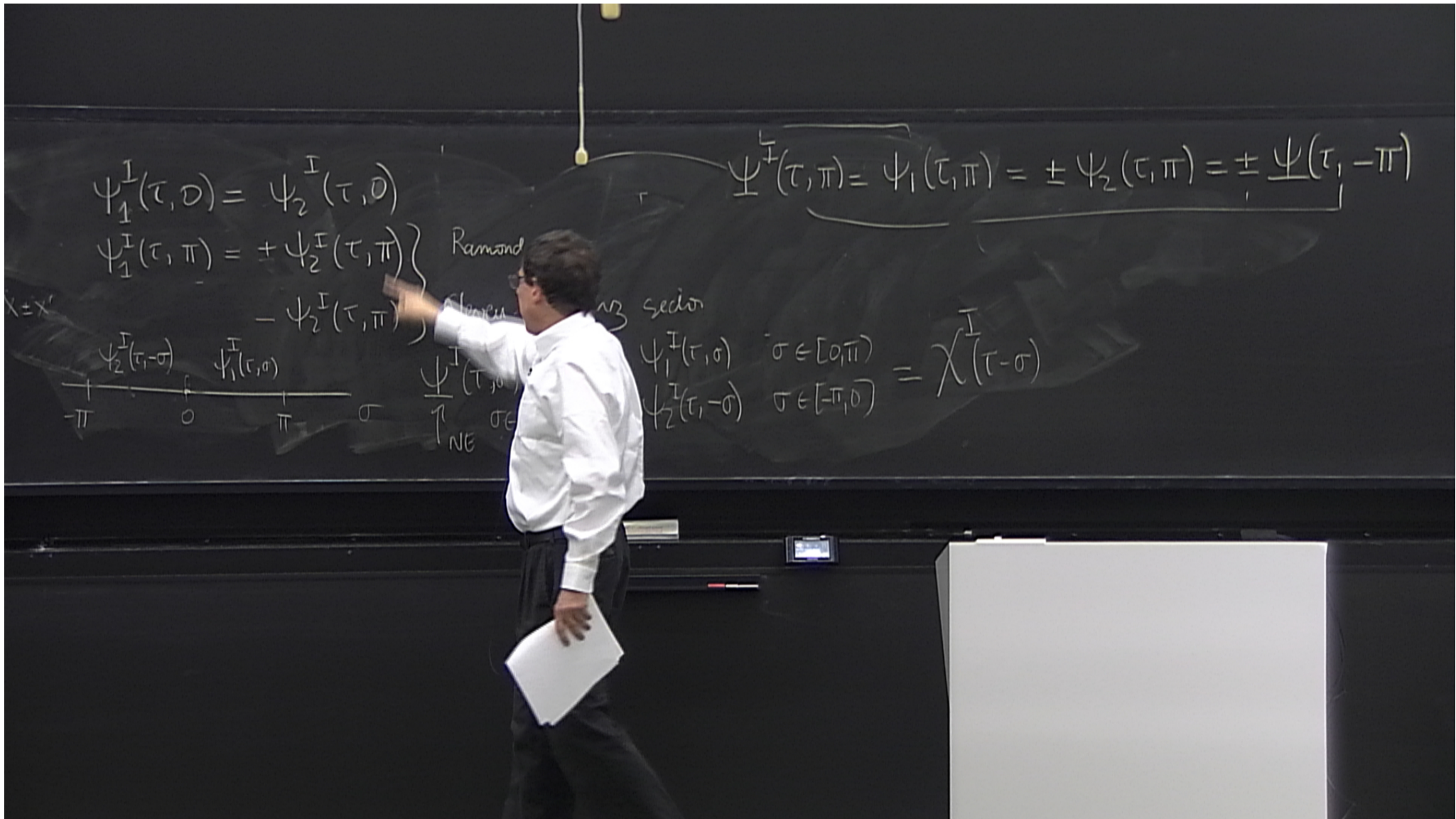
$\psi_1^I(\tau, \sigma_*) = \pm \psi_2^I(\tau, \sigma_*)$
 $\delta \psi_1^I(\tau, \sigma_*) = \pm \delta \psi_2^I(\tau, \sigma_*)$
 $\psi_1^I \delta \psi_1^I(\tau, \sigma_*) = + \psi_2^I \delta \psi_2^I(\tau, \sigma_*)$ ✓

$$\psi_1^I(\tau, 0) = \psi_2^I(\tau, 0)$$

$$\psi_1^I(\tau, \pi) = \pm \psi_2^I(\tau, \pi) \quad \left. \begin{array}{l} \text{Ramond sector} \\ -\psi_2^I(\tau, \pi) \end{array} \right\} \text{Neveu-Schwartz sector}$$

$$\psi_2^I(\tau, -\sigma) \quad \psi_1^I(\tau, \sigma)$$

$$\psi_{NE}^I(\tau, \sigma) = \begin{cases} \psi_1^I(\tau, \sigma) & \sigma \in [0, \pi) \\ \psi_2^I(\tau, -\sigma) & \sigma \in [-\pi, 0) \end{cases} = \chi^I(\tau, -\sigma)$$



$\pm \chi^I$

$\left\{ \begin{array}{l} -\psi_2^I(\tau, \pi) \\ \psi_1^I(\tau, 0) \end{array} \right\}$ Neveu-Schwartz sector

$\psi_2^I(\tau, -\sigma)$ $\psi_1^I(\tau, \sigma)$ $\psi^I(\tau, \sigma)$

$\sigma \in [0, \pi)$

$\uparrow_{NE}$

$\left\{ \begin{array}{l} \psi_1^I(\tau, \sigma) \quad \sigma \in [0, \pi) \\ \psi_2^I(\tau, -\sigma) \quad \sigma \in [-\pi, 0) \end{array} \right\} = \chi^I(\tau - \sigma)$

$\psi^I(\tau, \pi) = \psi^I(\tau, -\pi)$ Ramond

$\psi^I(\tau, \pi) = -\psi^I(\tau, -\pi)$ NS sector