

Title: 12/13 PSI - String Theory Review Lecture 11

Date: Jan 21, 2013 10:15 AM

URL: <http://pirsa.org/13010061>

Abstract:

$$L_0 = \frac{1}{2} \sum \alpha_{-n}^P \alpha_n^P$$

$$L_0^\perp = \frac{1}{2} \sum_{n \in \mathbb{Z}} \alpha_{-n}^I \alpha_n^I = \sum_{n \geq 1} \alpha_{-n}^I \alpha_n^I + a$$

V ops?

$$(L_n^\perp)^\dagger = L_{-n}^\perp$$

(α_n^I)

$$L_0^\perp = \frac{1}{2} \sum_{n \in \mathbb{Z}} \alpha_{-n}^I \alpha_n^I = \sum_{n \geq 1} \alpha_{-n}^I \alpha_n^I + a$$

Verify pps:

$$(L_n^\perp)^\dagger = L_{-n}^\perp$$

(α_n^I)

$$L_0^\perp = \frac{1}{2} \sum_{n \in \mathbb{Z}} \alpha_{-n}^I \alpha_n^I = \sum_{n \geq 1} \alpha_{-n}^I \alpha_n^I + a$$

Virasoro ops:

$$(L_n^\perp)^\dagger = L_{-n}^\perp$$

$$[L_m^\perp, L_n^\perp] = (m-n) L_{m+n}^\perp$$

(α_n^I)

$$L_0^\perp = \frac{1}{2} \sum_{n \in \mathbb{Z}} \alpha_{-n}^\perp \alpha_n^\perp = \sum_{n \geq 1} \alpha_{-n}^\perp \alpha_n^\perp + a$$

Virasoro ops

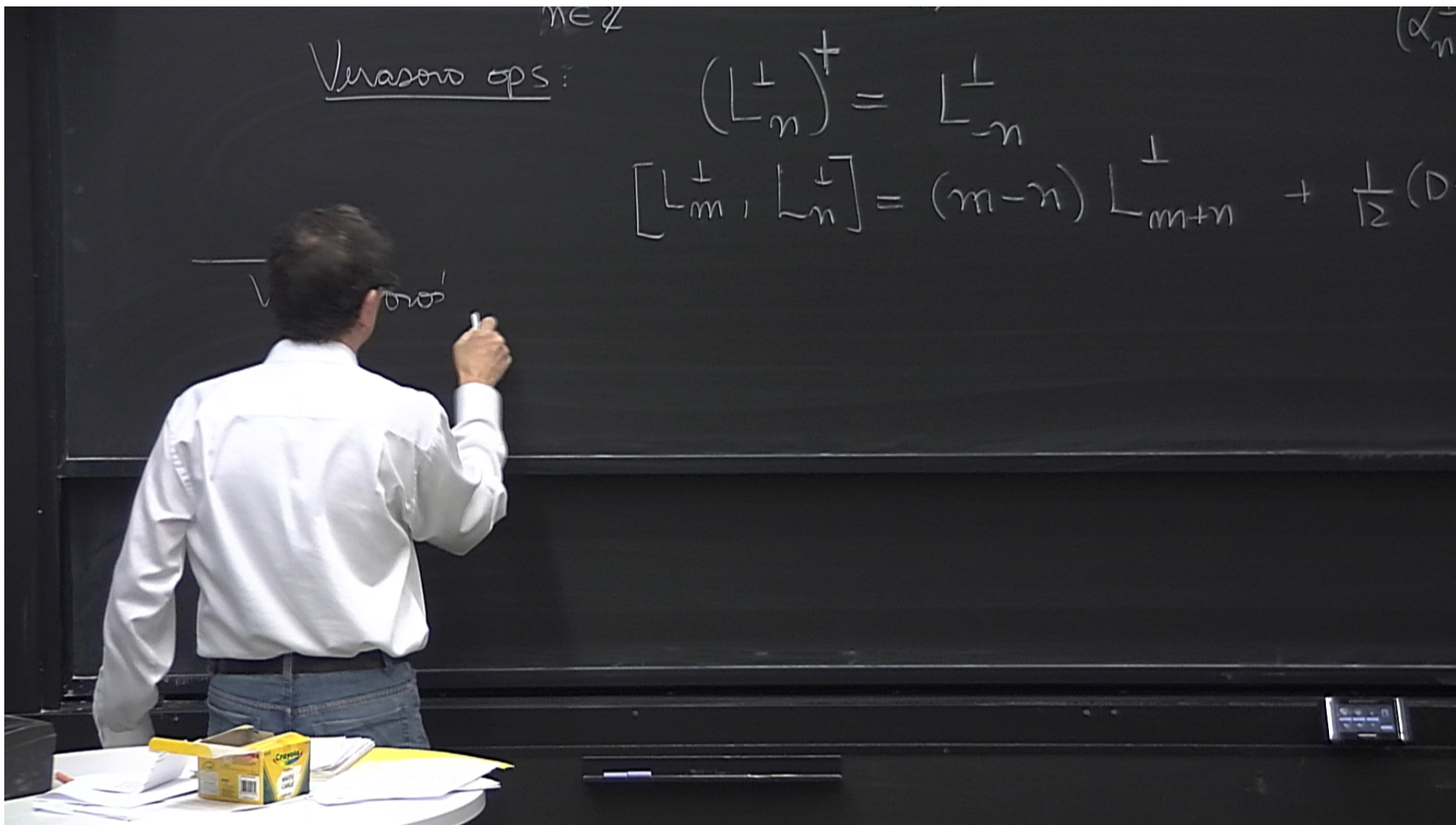
$$(L_m^\perp)^\dagger = L_{-n}^\perp$$

$$[L_m^\perp, L_n^\perp] = (m-n) L_{m+n}^\perp + \frac{1}{12} (D-2) (m^3 - m) \delta_{m+n,0} \mathbb{1}$$

$$a = \frac{(D-2)}{24} \left(\frac{1}{2} \right) \left(-\frac{1}{12} \right) = -\frac{(D-2)}{24}$$

$$(\alpha_n^\perp)^\dagger = \alpha_{-n}^\perp$$

$$a_{NN} = -\frac{1}{24}$$



$$L_0^\perp = \frac{1}{2} \sum_{n \in \mathbb{Z}} \alpha_{-n}^\perp \alpha_n^\perp = \sum_{n \geq 1} \alpha_{-n}^\perp \alpha_n^\perp + a$$

$$a = \frac{(D-2)}{2} \left(\frac{1}{2} \right) \left(-\frac{1}{12} \right) = -\frac{(D-2)}{24}$$

$$(\alpha_n^\perp)^\dagger = \alpha_{-n}^\perp$$

$$a_{NN} = -\frac{1}{24}$$

Virasoro ops:

$$(L_m^\perp)^\dagger = L_{-m}^\perp$$

$$[L_m^\perp, L_n^\perp] = (m-n) L_{m+n}^\perp + \frac{1}{12} (D-2) (m^3 - m) \delta_{m+n,0} \mathbb{1}$$

Virasoro generate reparameterizations of the world sheet:

$$[L_m^\perp, X^I(\tau, \sigma)] = \sum_n \xi_m^\tau \dot{X}^I + \sum_n \xi_m^\sigma X^{I'}$$

$$\int dq \psi(q) \delta(q)$$

$$\int dq \psi(q) e^{-i b_k \hat{q}} e^{i a_k \hat{p}} \int dp e^{-i q p k} |p\rangle$$

$$\int dp \psi(p) e^{-i b_k \hat{q}} e^{i a_k \hat{p}} |p\rangle$$

$$\psi(q) e^{-i b_k \hat{q}} |q-a\rangle$$

$$e^{-i(\hat{p} \cdot \hat{x})}$$

$$e^{A+B} = e^A e^B e^{A^H} = U$$

$$[A, B] \quad [A, [A, B]]$$

$$[L_m^\perp, L_n^\perp] = (m-n) L_{m+n}^\perp + \frac{1}{2}(D-2)(m^3-m)\delta_{m+n,0} \mathbb{1}$$

Virasoro generate reparameterizations of the world sheet

$$[L_m^\perp, X^I(\tau, \sigma)] = \xi_m^\tau \dot{X}^I + \xi_m^\sigma X^{I'}$$

$$\xi_m^\tau(\tau, \sigma) = -L e^{im\tau} \cos m\sigma$$

$$\xi_m^\sigma(\tau, \sigma) = e^{im\tau} \sin m\sigma$$

$$X^I(\tau + \epsilon \xi_m^\tau, \sigma + \epsilon \xi_m^\sigma) = X^I(\tau, \sigma) + \epsilon (\xi_m^\tau \dot{X}^I + \xi_m^\sigma X^{I'})$$

$$= X^I(\tau, \sigma) + \epsilon [L_m^\perp, X^I(\tau, \sigma)]$$

Anti-hem ops generate transp

These rep pres

$$L_m^\perp - L_{-m}^\perp \quad i(L_m^\perp + L_{-m}^\perp)$$

$$\dot{X}^2 + 1$$

$$[L_m^\perp, L_n^\perp] = (m-n) L_{m+n}^\perp + \frac{1}{2}(D-2)(m^3-n^3)\delta_{m+n,0} \mathbb{1}$$

Virasoro generate reparameterizations of the world sheet

$$[L_m^\perp, X^I(\tau, \sigma)] = \xi_m^\tau \dot{X}^I + \xi_m^\sigma X^{I'}$$

$$\xi_m^\tau(\tau, \sigma) = -L e^{im\tau} \cos m\sigma$$

$$\xi_m^\sigma(\tau, \sigma) = e^{im\tau} \sin m\sigma$$

$$X^I(\tau + \epsilon \xi_m^\tau, \sigma + \epsilon \xi_m^\sigma) = X^I(\tau, \sigma) + \epsilon (\xi_m^\tau \dot{X}^I + \xi_m^\sigma X^{I'})$$

$$= X^I(\tau, \sigma) + \epsilon [L_m^\perp, X^I(\tau, \sigma)]$$

Anti-hem ops generate reparam.

These rep preserve:

$$L_m^\perp - L_{-m}^\perp \quad i(L_m^\perp + L_{-m}^\perp)$$

$$L_0^\perp : \quad \xi_0^\tau = -1 \quad \xi_0^\sigma = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$[L_m^\pm, [L_k^\pm, L_p^\pm]]$$

$$[L_m^\pm, L_p^\pm] = 0$$

$$L_i^2$$

$$[L_i, L_j] = i\epsilon_{ijk} L_k + \dots$$

$$[L_m^\pm, [L_k^\perp, L_p^\perp]]$$

$$L_v^2$$

$$[L_m^\perp, \dots, L_p^\perp] = 0$$

$$[L_i, L_j] = i\epsilon_{ijk} L_k + \dots$$

$$[L_m^\perp, L_n^\perp] = (m-n) L_{m+n}^\perp + \frac{1}{12} (D-2) (m^3 - m) \delta_{m+n,0} \mathbb{1}$$

Virasoro' generate reparameterizations of the world sheet:

$$[L_m^\perp, X^I(\tau, \sigma)] = \xi_m^\tau \dot{X}^I + \xi_m^\sigma X^{I'}$$

$$\xi_m^\tau(\tau, \sigma) = -l e^{im\tau} \cos m\sigma$$

$$\xi_m^\sigma(\tau, \sigma) = e^{im\tau} \sin m\sigma$$

$$X^I(\tau + \epsilon \xi_m^\tau, \sigma + \epsilon \xi_m^\sigma) = X^I(\tau, \sigma) + \epsilon (\xi_m^\tau \dot{X}^I + \xi_m^\sigma X^{I'}) \\ = X^I(\tau, \sigma) + \epsilon [L_m^\perp, X^I(\tau, \sigma)]$$

Anti-hem. ops generate real transp.

The rep. preserve:

$$\eta_{\mu\nu} X^\mu X^\nu$$

$$\partial_\tau X \cdot \partial_\sigma X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_m^\perp - L_{-m}^\perp \quad i(L_m^\perp + L_{-m}^\perp)$$

$$L_0^\perp: \quad \xi_0^\tau = -l \quad \xi_0^\sigma = 0$$

These rep. preserve:

$$\partial_\mu X \cdot \partial_\mu X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_m^\perp - L_{-m}^\perp$$

$$i(L_m^\perp + L_{-m}^\perp)$$

$$L_0^\perp: \xi_0^\pi = -1 \quad \xi_0^\pi = 0$$

Lorentz Generator

$$M^{\mu\nu} = \int_0^\pi$$

These rep. preserve:

$$\partial_\mu X \cdot \partial^\mu X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_m^\perp - L_{-m}^\perp$$

$$i(L_m^\perp + L_{-m}^\perp)$$

$$L_0^\perp: \xi_0^\tau = -1 \quad \xi_0^\sigma = 0$$

Lorentz Generator

$$M^{\mu\nu} = \int_{-\pi}^{\pi} (x^\mu p^{\nu} - x^\nu p^{\mu}) d\sigma$$

$$p^\nu - x_0^\nu p^\mu - i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^\mu \alpha_n^\nu - \alpha_{-n}^\nu \alpha_n^\mu)$$

$$\partial_\tau X \cdot \partial_\sigma X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_0^{\perp} : \xi_0^T = -i \quad \xi_0^T = 0$$

Lorentz Generator

$$M^{\mu\nu} = \int_0^\pi (x^\mu p^{\tau\nu} - x^\nu p^{\tau\mu}) d\sigma$$

$$= x_0^\mu p^\nu - x_0^\nu p^\mu - i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^\mu \alpha_n^\nu - \alpha_{-n}^\nu \alpha_n^\mu)$$

$$M^{-I} = x_0^- p^I - x_0^I p^- - i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^- \alpha_n^I - \alpha_{-n}^I \alpha_n^-)$$

$$\alpha_n^- = \frac{1}{\sqrt{2\alpha'}} \frac{1}{p^+} L_n^{\perp}$$

$$p^- = \frac{1}{2\alpha' p^+} (L_0^{\perp} + a)$$

$$- X^\nu p^\mu) d\sigma$$

$$- i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^\mu \alpha_n^\mu - \alpha_{-n}^\nu \alpha_n^\nu)$$

$$\frac{1}{n} (\alpha_{-n}^- \alpha_n^+ - \alpha_{-n}^+ \alpha_n^-)$$

$$\alpha_n^- = \frac{1}{\sqrt{2\alpha'}} \frac{1}{p^+} L_n^+$$

$$p^- = \frac{1}{2\alpha' p^+} (L_0^+ + a)$$

$$p^+ p^+$$

$$\partial_\tau X \cdot \partial_\sigma X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_0^{\perp}: \xi_0^T = -i \quad \xi_0^T = 0$$

Lorentz Generator

$$M^{\mu\nu} = \int_0^\pi (x^\mu p^{\nu} - x^\nu p^{\mu}) d\sigma$$

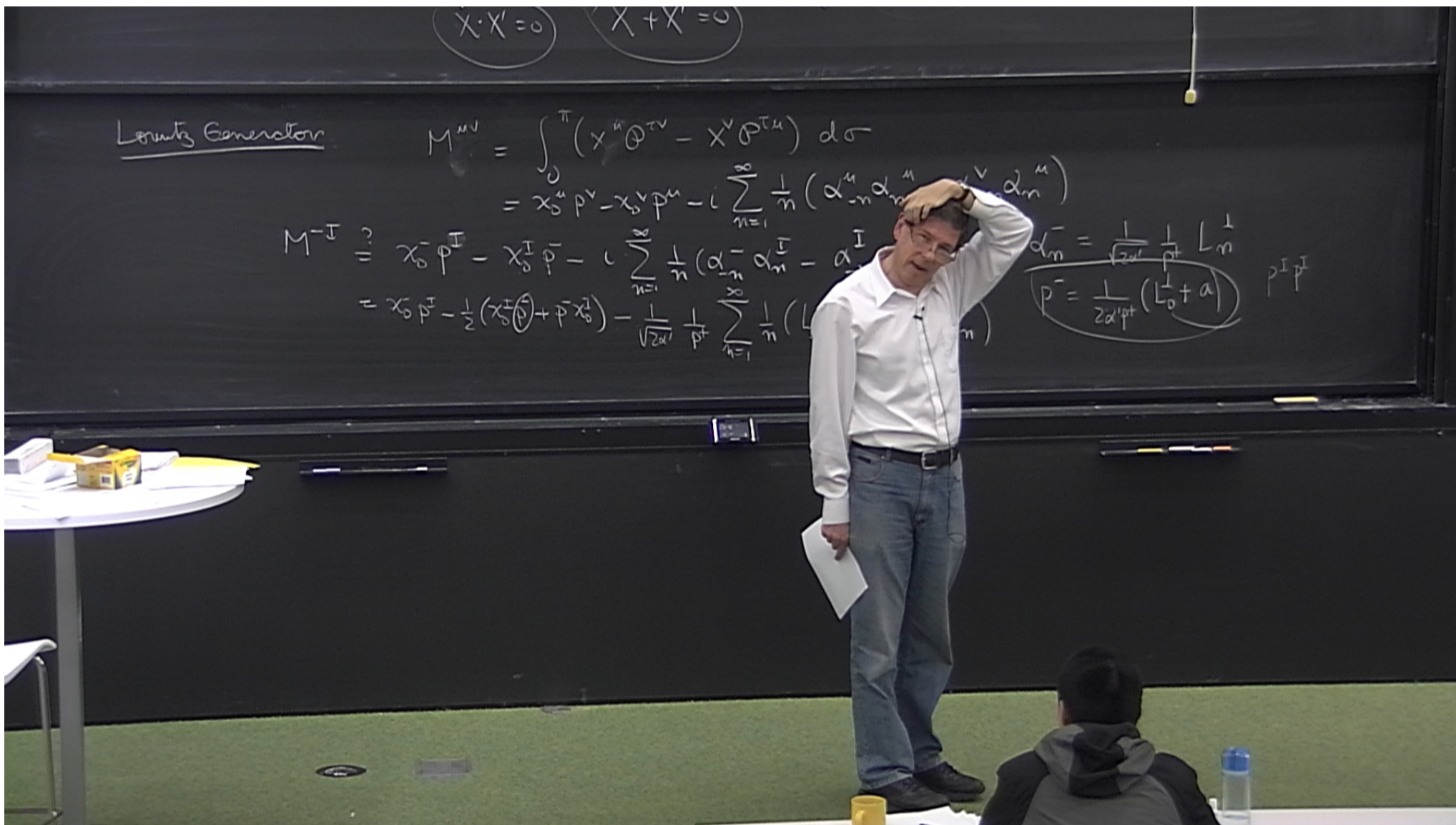
$$= x_0^\mu p^\nu - x_0^\nu p^\mu - i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^\mu \alpha_n^\nu - \alpha_{-n}^\nu \alpha_n^\mu)$$

$$M^{-I} = x_0^- p^I - x_0^I p^- - i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^- \alpha_n^I - \alpha_{-n}^I \alpha_n^-)$$

$$= x_0^- p^I - \frac{1}{2} (x_0^I \dot{p} + \dot{p} x_0^I) - \frac{1}{\sqrt{2\alpha'}} \frac{1}{p^+} \sum_{n=1}^{\infty} \frac{1}{n} (L_n^+ \alpha_n^I - \alpha_n^I L_n^+)$$

$$\alpha_n^- = \frac{1}{\sqrt{2\alpha'}} \frac{1}{p^+} L_n^+$$

$$p^- = \frac{1}{2\alpha' p^+} (L_0^+ + a)$$



$$\sum_{n=-1}^{\infty} \alpha_n \alpha_{-n} = \frac{1}{\alpha'} (L_0 - 1)$$

$$H = L_0 - 1$$

$\forall m$

$$1 = \frac{1}{24}(D-2)$$

$$D=26$$

$$1+a=0$$

$$a=-1$$

$$M^2 = \frac{1}{\alpha'} (-1 + N^\perp)$$

$$N^\perp = \sum_{k=1}^{\infty} k a_k^{\perp\dagger} a_k^{\perp}$$

$$[N^\perp, a_k^{\perp\dagger}] = k a_k^{\perp\dagger}$$

$$N = a^\dagger a$$

$$M^2 = \frac{1}{\alpha'} (-1 + N^\perp)$$

$$p^\perp, x_0^-, p^+, a_n^\perp, a_n^{\perp\dagger}$$

$$|p^+, \vec{p}_T\rangle$$

$$N^\perp = \sum_{k=1}^{\infty} k a_n^{\perp\dagger} a_n^\perp$$

$$[N^\perp, a_n^{\perp\dagger}] = k a_n^{\perp\dagger}$$

$$\sum_{n=1}^{\infty} \alpha_n \alpha_{-n} \quad \forall n$$

$$\boxed{2\alpha' p^- = \frac{1}{p^+} (L_0 - 1)}$$

$$\boxed{H = L_0 - 1}$$

$$1 = \frac{1}{24}(D-2)$$

$$\boxed{D=26}$$

$$1+a=0$$

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$$M^2 = \frac{1}{\alpha'} (-1 + N^\perp)$$

$$N^\perp = \sum_{k=1}^{\infty} k a_k^{I\dagger} a_k^I$$

$$[N^\perp, a_k^{I\dagger}] = k a_k^{I\dagger}$$

$$N = a^\dagger a$$

$\alpha_0^I, p^I, \alpha_0^{I\dagger}$
 $\alpha_n^I, a_n^{I\dagger}$
 killed by all a_n^I

$$| \lambda \rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} (a_n^{I\dagger})^{\lambda_{n,I}} | p^+ \vec{p}_T \rangle$$

$$\sum_{m=-1}^{\infty} \alpha_m \alpha_{-m} \quad \forall m$$

$$\boxed{2\alpha' p^- = \frac{1}{p^+} (L_0 - 1)}$$

$$\boxed{H = L_0 - 1}$$

$$\boxed{1 = \frac{1}{24}(D-2)}$$

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$$[N^\perp, a_k^{I\dagger}] = k a_k^{I\dagger}$$

$$N = a^\dagger a$$

$$\alpha_0^I, p^I, \alpha_0^I, p^I, a_n^I, a_n^{I\dagger}$$

$$|p^+, \vec{p}_T\rangle \text{ killed by all } a_n^I$$

$$a_1^{(2)\dagger}, a_1^{(3)\dagger}, \dots, a_1^{(25)\dagger}$$

$$a_2^{(1)\dagger}, a_2^{(1)\dagger}$$

$$|\lambda\rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} (a_n^{I\dagger})^{\lambda_{n,I}} |p^+, \vec{p}_T\rangle$$

$$\sum_{m=1}^{\infty} \alpha_m \alpha_{-m} \quad \forall m$$

$$\boxed{2\alpha' p^- = \frac{1}{p^+} (L_0 - 1)}$$

$$\boxed{H = L_0 - 1}$$

$$\boxed{1 = \frac{1}{24}(D-2)}$$

$$\boxed{D=26}$$

$$\boxed{1+a=0}$$

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$$M^2 = \frac{1}{\alpha'} (-1 + N^\perp)$$

$$N^\perp = \sum_{k=1}^{\infty} k a_k^{I\dagger} a_k^I$$

$$[N^\perp, a_k^{I\dagger}] = k a_k^{I\dagger}$$

$$N = a^\dagger a$$

$$\alpha_0^I, p^I, \alpha_0^-, p^+, a_n^I, a_n^{I\dagger}$$

$$|p^+, \vec{p}_T\rangle \text{ killed}$$

$$a_1^{(2)\dagger}, a_1^{(3)\dagger}, \dots, a_2^{(1)\dagger}, a_2^{(1)}$$

$$|\lambda\rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} (a_n^{I\dagger})^{\lambda_{n,I}} |p^+, \vec{p}_T\rangle$$

$$\sum_{m=1}^{\infty} \alpha_m \alpha_{-m} \quad \forall m$$

$$\boxed{2\alpha' p^- = \frac{1}{p^+} (L_0 - 1)}$$

$$\boxed{H = L_0 - 1}$$

$$1 = \frac{1}{24}(D-2)$$

$$\boxed{D=26}$$

$$1+a=0$$

$$\boxed{a=-1}$$

$$M^2 = \frac{1}{\alpha'} (-1 + N^\perp)$$

$$N^\perp = \sum_{k=1}^{\infty} k a_k^{I\dagger} a_k^I$$

$$N = a^\dagger a$$

$$\alpha_0^I, p^I, \alpha_0^{I\dagger}, p^{I\dagger}, a_n^I, a_n^{I\dagger}$$

$$[N^\perp, a_k^{I\dagger}] = k a_k^{I\dagger}$$

$|p^+, \vec{p}_T\rangle$ killed by all a_n^I

$$a_1^{(2)\dagger}, a_1^{(3)\dagger}, \dots, a_1^{(25)\dagger}$$

$$a_2^{(1)\dagger}, a_2^{(1)\dagger}$$

$$|\lambda\rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} (a_n^{I\dagger})^{\lambda_{n,I}} |p^+, \vec{p}_T\rangle$$

$$\begin{aligned}
 & \underbrace{x_0^I, p_0^I, x_0^-, p_0^+, a_n^I, a_n^{I\dagger}} \quad [N^I, a_k^{I\dagger}] = \sqrt{2} a_k^{I\dagger} \quad n, I \\
 & |p^+, \vec{p}_T\rangle \text{ killed by all } a_n^I \\
 & \underbrace{a_1^{(2)\dagger}, a_1^{(3)\dagger}, \dots, a_1^{(25)\dagger}}_{a_2^{(1)\dagger}, a_2^{(1)\dagger}} \quad |\lambda\rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} (a_n^{I\dagger}) |p^+, \vec{p}_T\rangle
 \end{aligned}$$

$$\begin{aligned}
 & \boxed{N^I = 0} \quad M^2 |p^+, p_T\rangle = \frac{1}{\alpha'} (-1 + N^I) |p^+, p_T\rangle = -\frac{1}{\alpha'} |p^+, p_T\rangle \\
 & M^2 = -\frac{1}{\alpha'}
 \end{aligned}$$

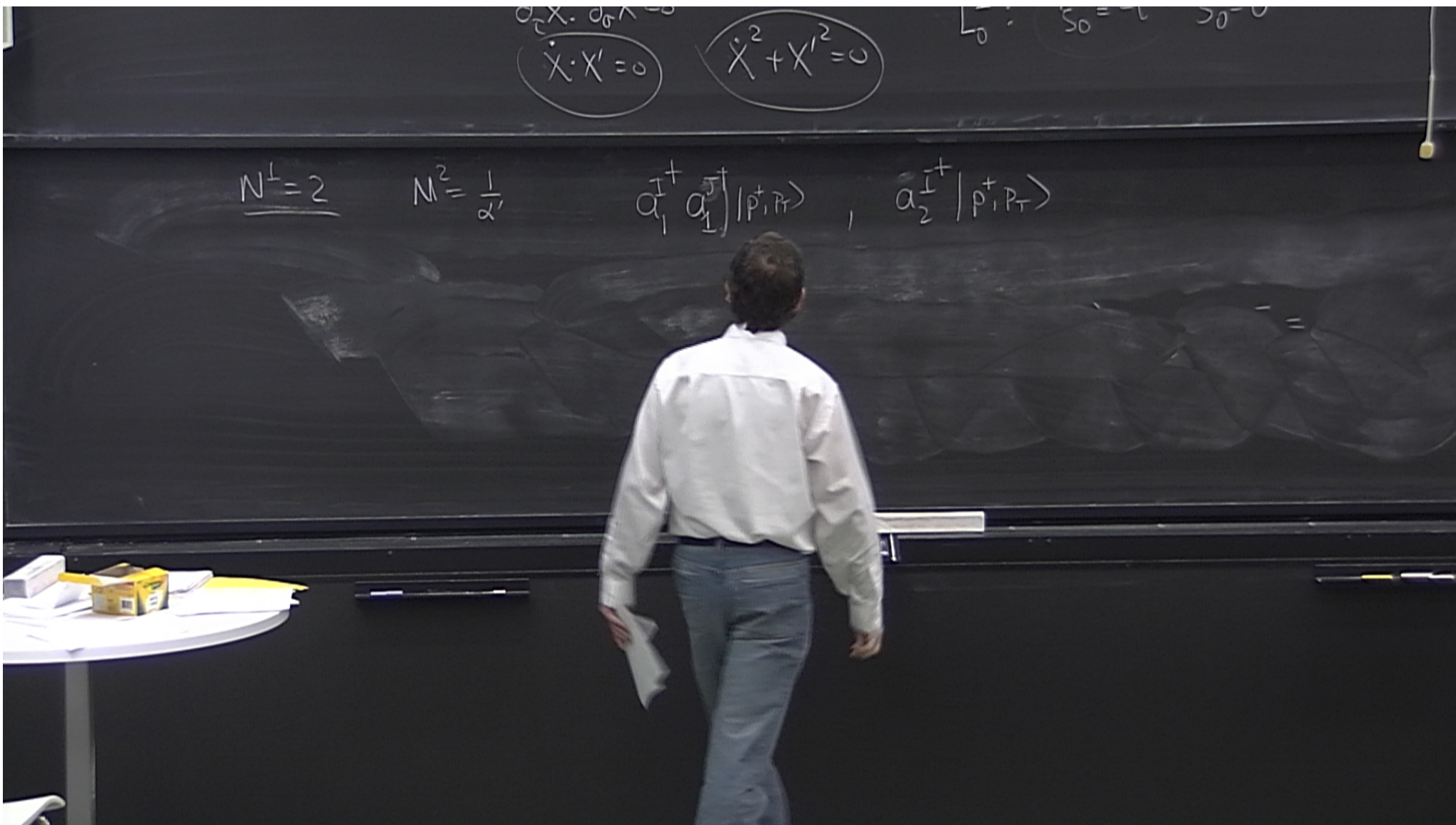
$$\begin{aligned}
 & \underline{x_0^I, p^I, x_0^-, p^+, a_n^I, a_n^{I\dagger}} \quad [N^I, a_k^{I\dagger}] = \sqrt{2\alpha'} a_k^{I\dagger} \\
 & |p^+, \vec{p}_T\rangle \text{ killed by all } a_n^I \\
 & a_1^{(2)\dagger} a_1^{(3)\dagger} \dots a_1^{(25)\dagger} \quad |\lambda\rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} (a_n^{I\dagger}) |p^+, \vec{p}_T\rangle \\
 & a_2^{(1)\dagger} a_2^{(1)\dagger} \quad \lambda_{n,I} \in \mathbb{Z}
 \end{aligned}$$

$$\boxed{N^I = 0} \quad M^2 |p^+, p_T\rangle = \frac{1}{\alpha'} (-1 + N^I) |p^+, p_T\rangle = -\frac{1}{\alpha'} |p^+, p_T\rangle$$

$$M^2 = -\frac{1}{\alpha'} \quad \text{tachyon} \quad \leftarrow \text{scalar field}$$

$$\underline{N^I = 1} \quad M^2 = 0 \quad (\alpha_1^{I\dagger}) |p^+, p_T\rangle$$

$$D=2 \text{ states } \underline{a_{p^+, p_T}^{I\dagger} |\Omega\rangle} \quad \text{Maxwell field}$$



$$\partial_\mu X \cdot \partial_\mu X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_0 : S_0 = -1 \quad S_0 = 0$$

$$N^L = 2$$

$$M^2 = \frac{1}{\alpha'}$$

$$a_1^{\dagger} a_1^{\dagger} |p^+, p_T\rangle$$

$$a_2^{\dagger} |p^+, p_T\rangle$$

$$\frac{24 \times 25}{2} = 300$$

$$24 \text{ states} = 324 \text{ states}$$

$$|P^+, P_T\rangle \text{ raised by } a_m^{\dagger} \lambda_{m,I}$$

$$a_1^{(2)\dagger} a_1^{(3)\dagger} \dots a_1^{(25)\dagger} \quad | \lambda \rangle = \prod_{m=1}^{\infty} \prod_{I=2}^{25} (a_m^{I\dagger}) |P^+, P_T\rangle$$

$$\boxed{N^L=0}$$

$$M^2 |P^+, P_T\rangle = \frac{1}{\alpha'} (-1 + N^L) |P^+, P_T\rangle = -\frac{1}{\alpha'} |P^+, P_T\rangle$$

$$M^2 = -\frac{1}{\alpha'}$$

tachyon

scalar field

$$\underline{N^L=1}$$

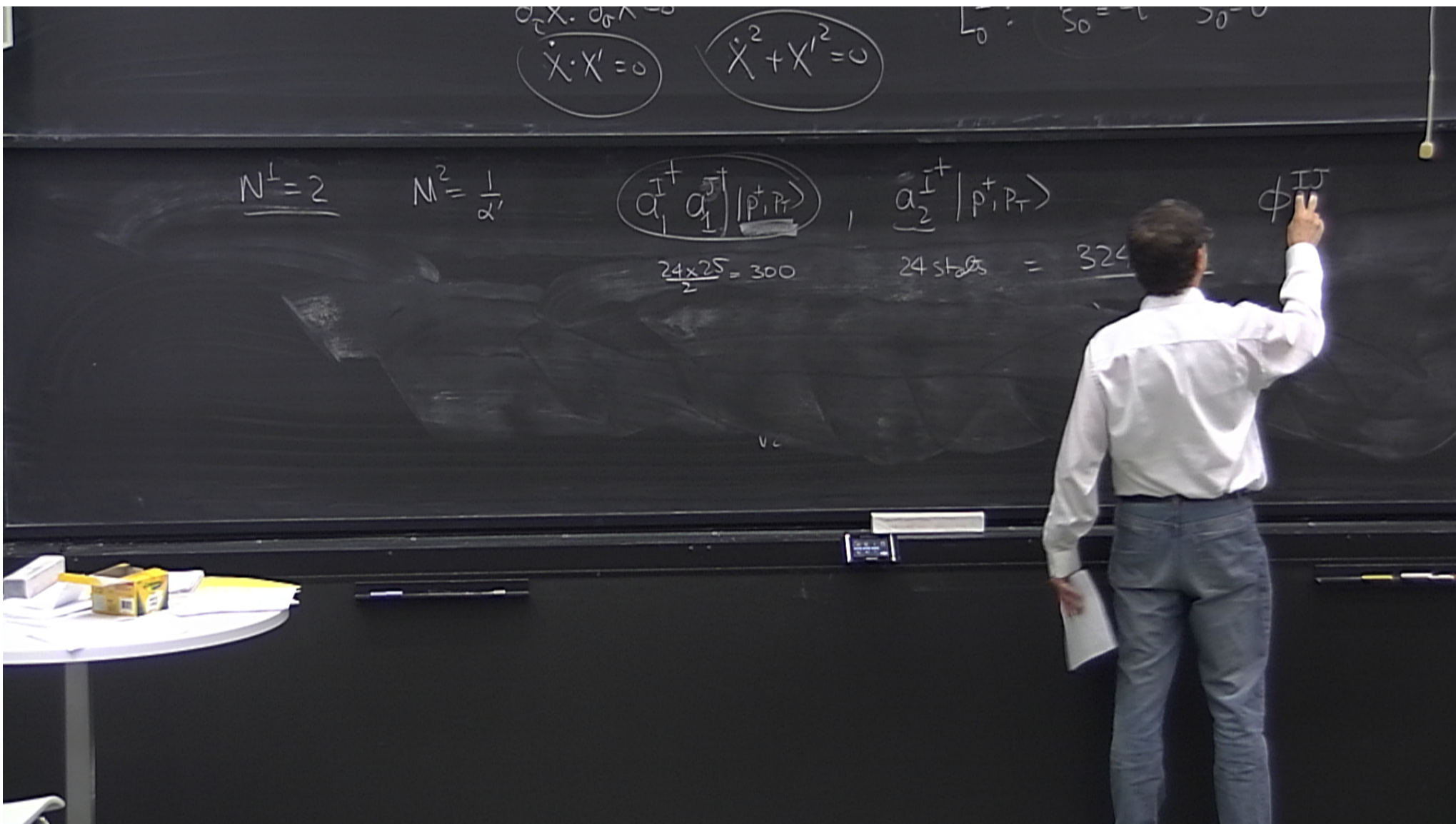
$$M^2=0$$

$$(a_1^{I\dagger}) |P^+, P_T\rangle$$

D=2 states

Maxwell field

$$a_{P^+, P_T}^{I\dagger} |\Omega\rangle$$



$$\partial_\mu X \cdot \partial_\mu X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_0 : S_0 = -1 \quad S_0 = 0$$

$$N^L = 2$$

$$M^2 = \frac{1}{\alpha'}$$

$$a_1^{\dagger} a_1^{\dagger} |p^+, p_T\rangle$$

$$a_2^{\dagger} |p^+, p_T\rangle$$

$$\phi^{IJ}$$

$$\frac{24 \times 25}{2} = 300$$

$$24 \text{ states} = 324 \text{ states}$$

$$|P^+, P_T\rangle \text{ raised by } a_m^{(25)^+} \quad \lambda_{m,I}$$

$$a_1^{(2)^+} a_1^{(3)^+} \dots a_1^{(25)^+}$$

$$a_2^{(2)^+} a_1^{(1)^+}$$

$$|\lambda\rangle = \prod_{m=1}^{\infty} \prod_{I=2}^{25} (a_m^{I+}) |P^+, P_T\rangle$$

$$N^L = 0$$

$$M^2 |P^+, P_T\rangle = \frac{1}{\alpha'} (-1 + N^L) |P^+, P_T\rangle = -\frac{1}{\alpha'} |P^+, P_T\rangle$$

$$M^2 = -\frac{1}{\alpha'}$$

tachyon

scalar field
E.

$$N^L$$

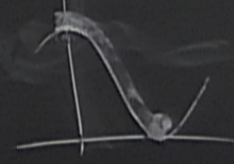
$$M^2 = 0$$

$$(a_1^{I+}) |P^+, P_T\rangle$$

D=2 states

Maxwell field

$$a_{P^+, P_T}^{I+} |\Omega\rangle$$



$$\partial_\tau X \cdot \partial_\sigma X = 0$$

$$\dot{X} \cdot X' = 0$$

$$\dot{X}^2 + X'^2 = 0$$

$$L_0 : S_0 = -1 \quad S_0 = 0$$

$$N^L = 2$$

$$M^2 = \frac{1}{\alpha'}$$

$$a_1^{I+} a_1^{J+} |p^+, p_T\rangle$$

$$a_2^{I+} |p^+, p_T\rangle$$

$$\phi^{IJ}$$

$$\frac{24 \times 25}{2} = 300$$

$$24 \text{ states} = 324 \text{ states}$$

$$\underline{\underline{\Omega}}$$

$$\Omega X(\tau, \sigma) \Omega = X(\tau, \pi - \sigma)$$

$$|P^+, P_T\rangle \text{ raised by } a_m^{(25)^+} \quad \lambda_{m,I}$$

$$a_1^{(2)^+} a_1^{(3)^+} \dots a_1^{(25)^+}$$

$$a_2^{(2)^+} a_2^{(1)^+}$$

$$|\lambda\rangle = \prod_{m=1}^{\infty} \prod_{I=2}^{25} (a_m^{I+}) |P^+, P_T\rangle$$

$$\boxed{N^L=0}$$

$$M^2 |P^+, P_T\rangle = \frac{1}{\alpha'} (-1 + N^L) |P^+, P_T\rangle = -\frac{1}{\alpha'} |P^+, P_T\rangle$$

$$M^2 = -\frac{1}{\alpha'}$$

tachyon

scalar field
E.

$$\underline{N^L=1}$$

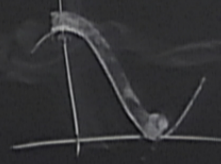
$$M^2=0$$

$$(a_1^{I+}) |P^+, P_T\rangle$$

D=2 states

Maxwell field

$$a_{P^+, P_T}^{I+} |\Omega\rangle$$



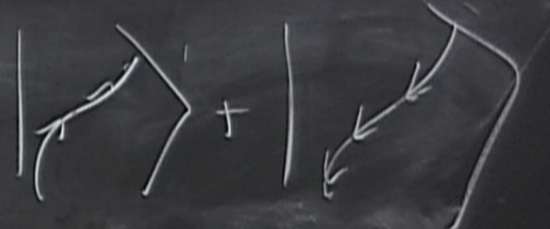
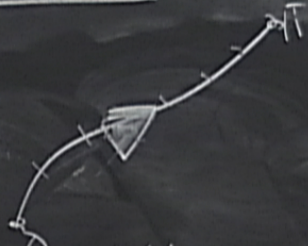
$$a_2^{I+} |p^+, p_T\rangle$$

$$\phi^{IJ}$$

$$24 \text{ states} = 324 \text{ states}$$

$$\underline{\underline{\Omega}}$$

$$\Omega X(\tau, \sigma) \Omega = X(\tau, \pi - \sigma)$$



Closed Strings:

$$X^M(\tau, \sigma) = X^M_L(\tau + \sigma) + X^M_R(\tau - \sigma)$$

$$\begin{aligned} \tau &= \tau + \sigma \\ \sigma &= \tau - \sigma \end{aligned}$$
$$X^M(\tau, \sigma) = X^M_L(\tau + \sigma) + X^M_R(\tau - \sigma)$$

Closed Strings:

$$\sigma \sim \sigma + 2\pi$$

$$X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma)$$

$$X^\mu(\tau, \sigma) = X_L^\mu(\tau + \sigma) + X_R^\mu(\tau - \sigma)$$

$$u = \tau + \sigma$$

$$v = \tau - \sigma$$

$$X^\mu(\tau, \sigma) = X_L^\mu(u) + X_R^\mu(v)$$

Closed Strings:

$$\sigma \sim \sigma + 2\pi$$

$$X^\mu(\tau, \sigma + 2\pi) = X^\mu(\tau, \sigma)$$

$$X^\mu(\tau, \sigma) = X_L^\mu(\tau + \sigma) + X_R^\mu(\tau - \sigma)$$

$$u = \tau + \sigma$$

$$v = \tau - \sigma$$

$$X^\mu(\tau, \sigma) = X_L^\mu(u) + X_R^\mu(v)$$

$$X^\mu = X_L(u + 2\pi) + X_R(v - 2\pi) = X_L(u) + X_R(v)$$

$$X_L(u + 2\pi) - X_L(u) = X_R(v) - X_R(v - 2\pi)$$

$$\partial_\sigma X(\tau, \sigma) \Omega = \partial_\sigma X(\tau, \tau - \sigma)$$

Closed Strings:

$$\sigma \sim \sigma + 2\pi$$

$$X^{\mu}(\tau, \sigma + 2\pi) = X^{\mu}(\tau, \sigma)$$

$$X^{\mu}(\tau, \sigma) = X_L^{\mu}(\tau + \sigma) + X_R^{\mu}(\tau - \sigma)$$

$$u = \tau + \sigma$$

$$v = \tau - \sigma$$

$$X^{\mu}(\tau, \sigma) = X_L^{\mu}(u) + X_R^{\mu}(v)$$

$$X^{\mu}(\tau, \sigma + 2\pi) = X_L^{\mu}(u + 2\pi) + X_R^{\mu}(v - 2\pi) = X_L^{\mu}(u) + X_R^{\mu}(v)$$

$$X_L(u + 2\pi) - X_L(u) = X_R(v) - X_R(v - 2\pi)$$

$$\Omega X(\tau, \sigma) \Omega = X(\tau, \tau - \sigma)$$