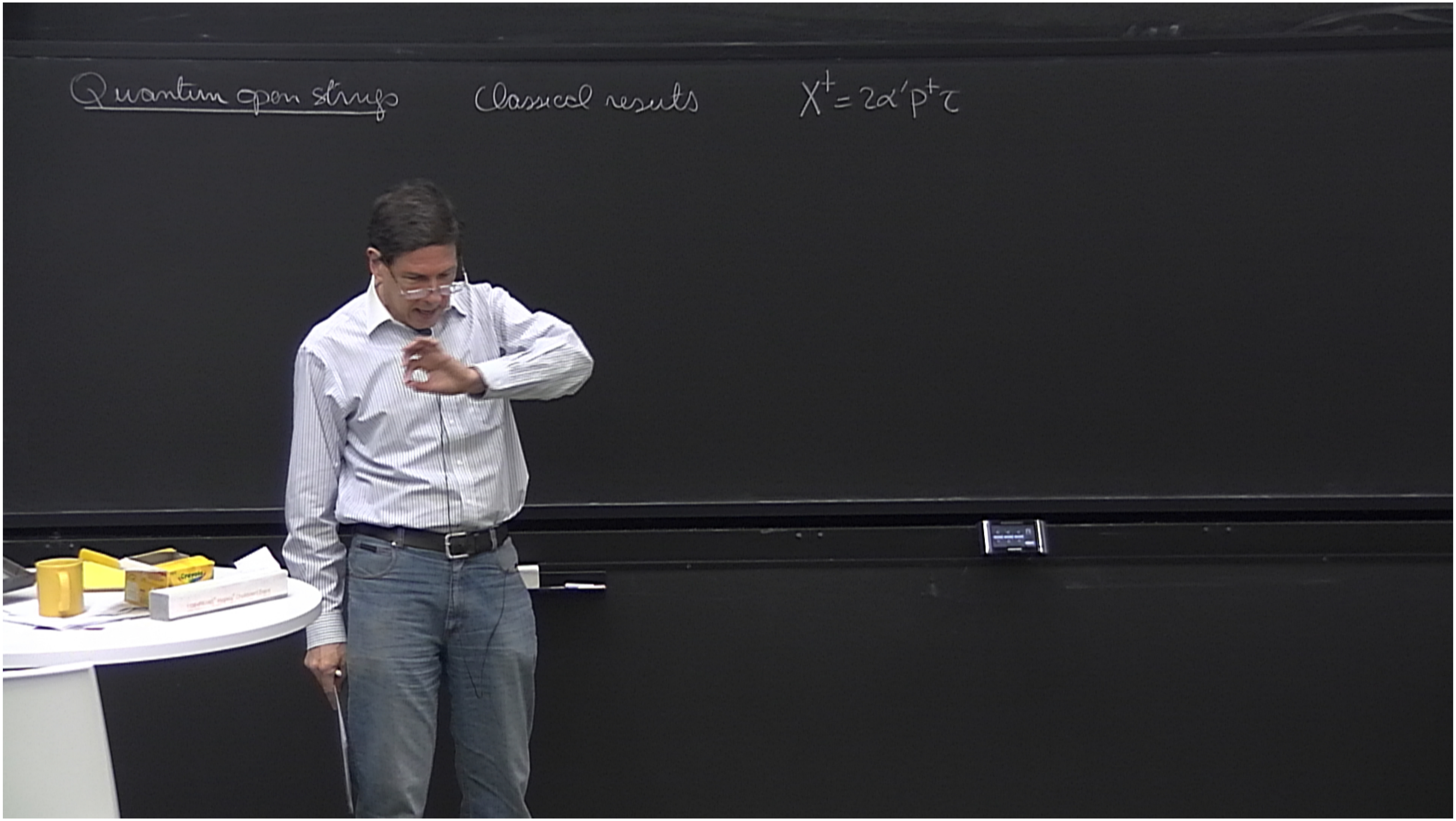
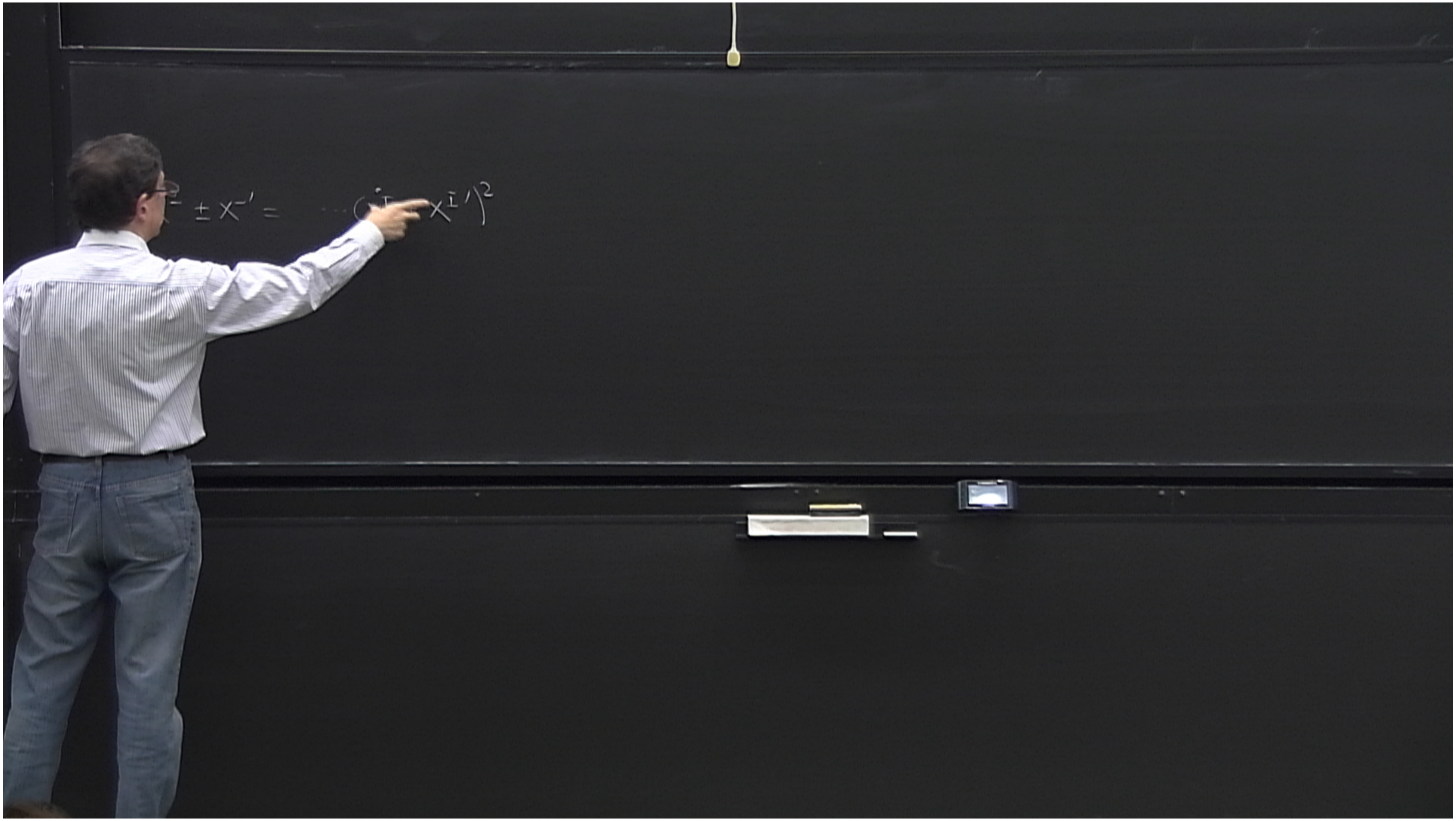
Title: 12/13 PSI - String Theory Review Lecture 10

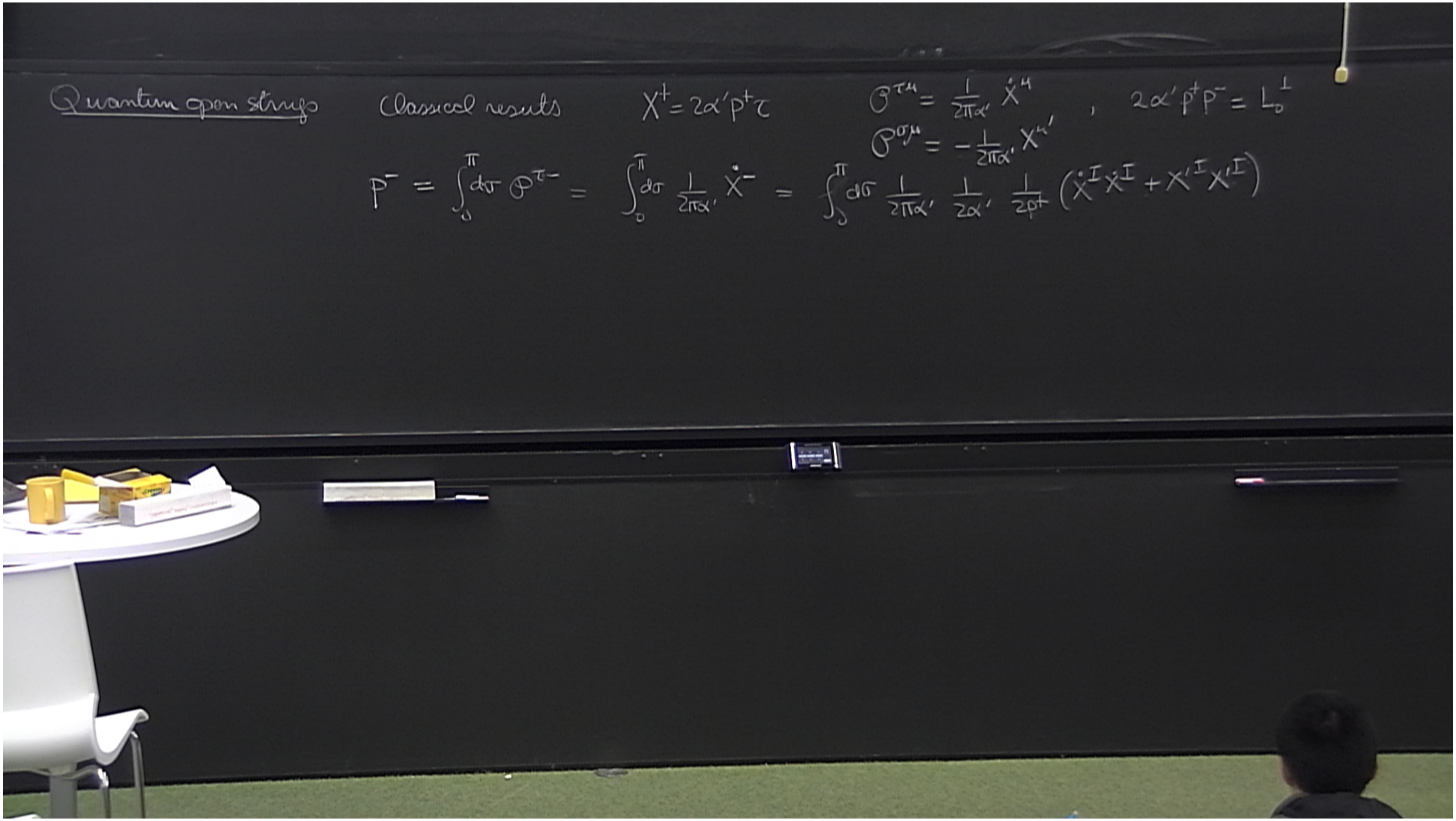
Date: Jan 18, 2013 10:15 AM

URL: http://pirsa.org/13010058

Abstract:


Quantum open strings
Classical results
$X^+ = 2\alpha' p^+ \tau$




Quantum open strings
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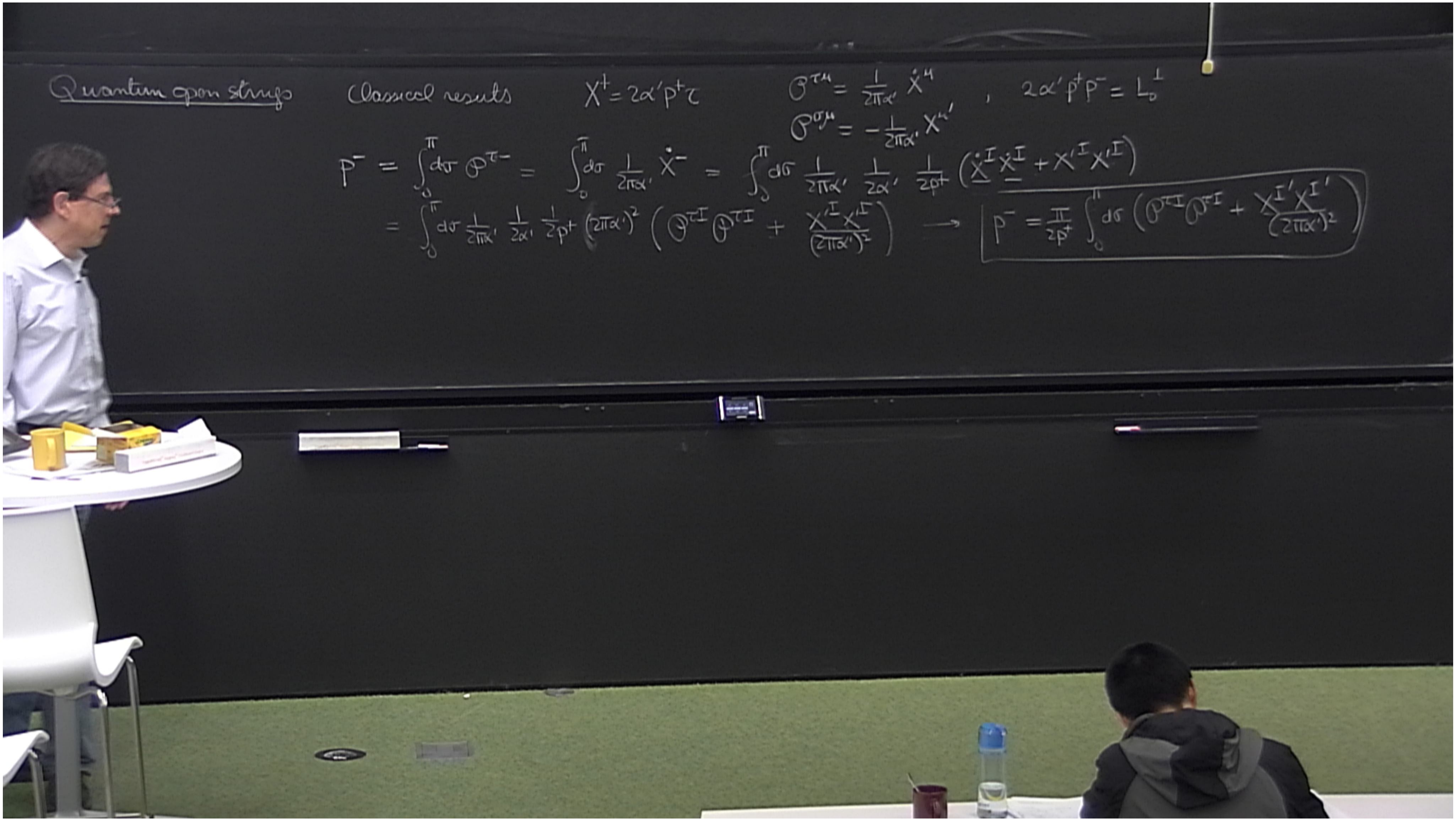
$X^+ = 2\alpha' p^+ \tau$
$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu$, $2\alpha' p^+ p^- = L_0^\perp$
$\mathcal{P}^{\sigma\mu} = -\frac{1}{2\pi\alpha'} X^{\mu\prime}$
$p^- = \int_0^\pi d\sigma\, \mathcal{P}^{\tau -} = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'}\dot{X}^- = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'}\frac{1}{2\alpha'}\frac{1}{2p^+}\left(\dot{X}^I \dot{X}^I + X'^I X'^I\right)$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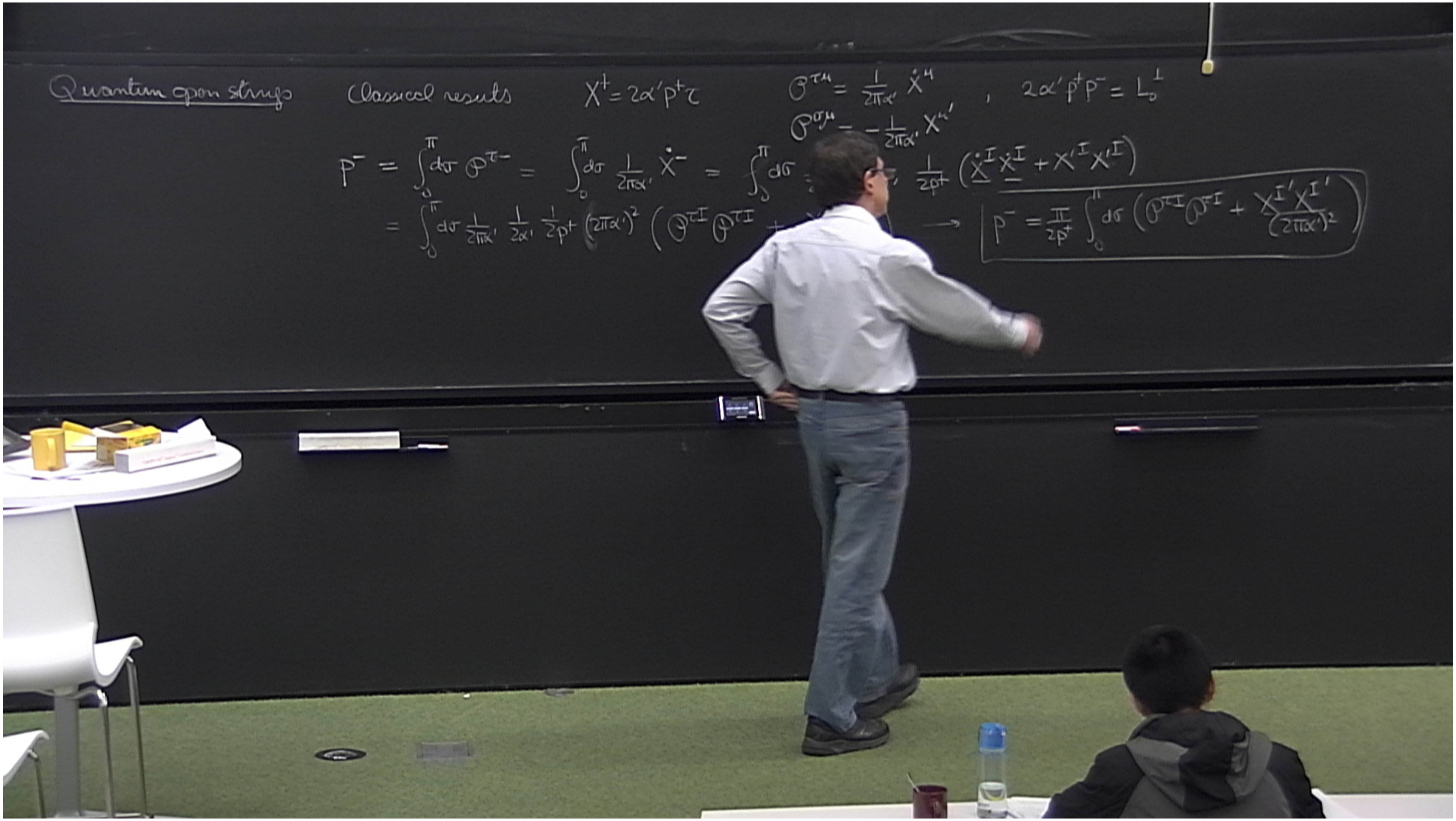
$p^- = \int_0^\pi d\sigma\, \mathcal{P}^{\tau -} = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \dot{X}^- = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} \left(\dot{X}^I \dot{X}^I + X'^I X'^I \right)$
$= \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} (2\pi\alpha')^2 \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X'^I X'^I}{(2\pi\alpha')^2} \right) \longrightarrow p^- = \frac{\pi}{2p^+} \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X^{I\prime} X^{I\prime}}{(2\pi\alpha')^2} \right)$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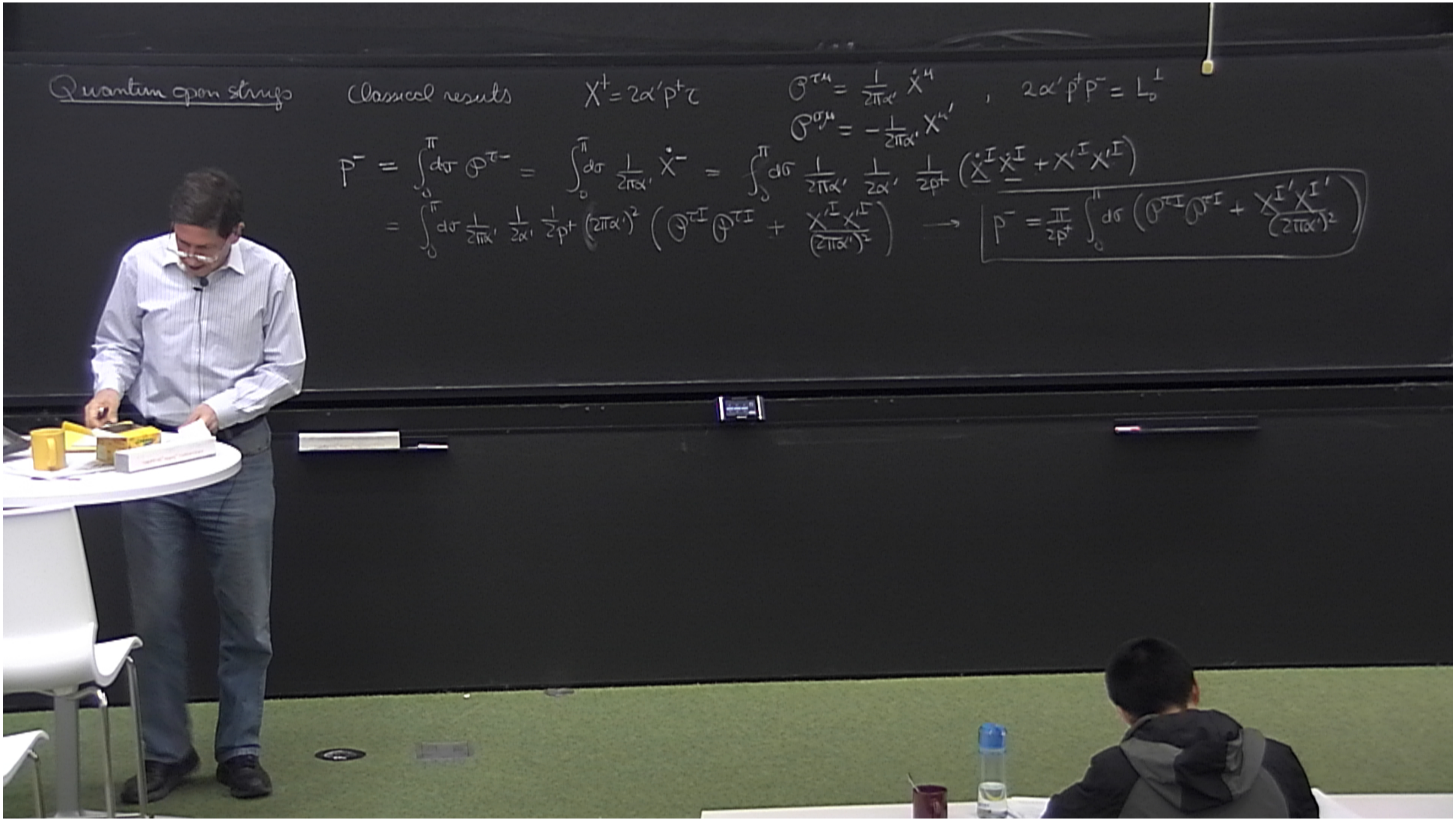
$p^- = \int_0^\pi d\sigma\, \mathcal{P}^{\tau -} = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \dot{X}^- =$
$\frac{1}{2p^+} (\dot{X}^I \dot{X}^I + X'^I X'^I)$
$= \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} (2\pi\alpha')^2 (\mathcal{P}^{\tau I} \mathcal{P}^{\tau I}$
$p^- = \frac{\pi}{2p^+} \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X^{I\prime} X^{I\prime}}{(2\pi\alpha')^2} \right)$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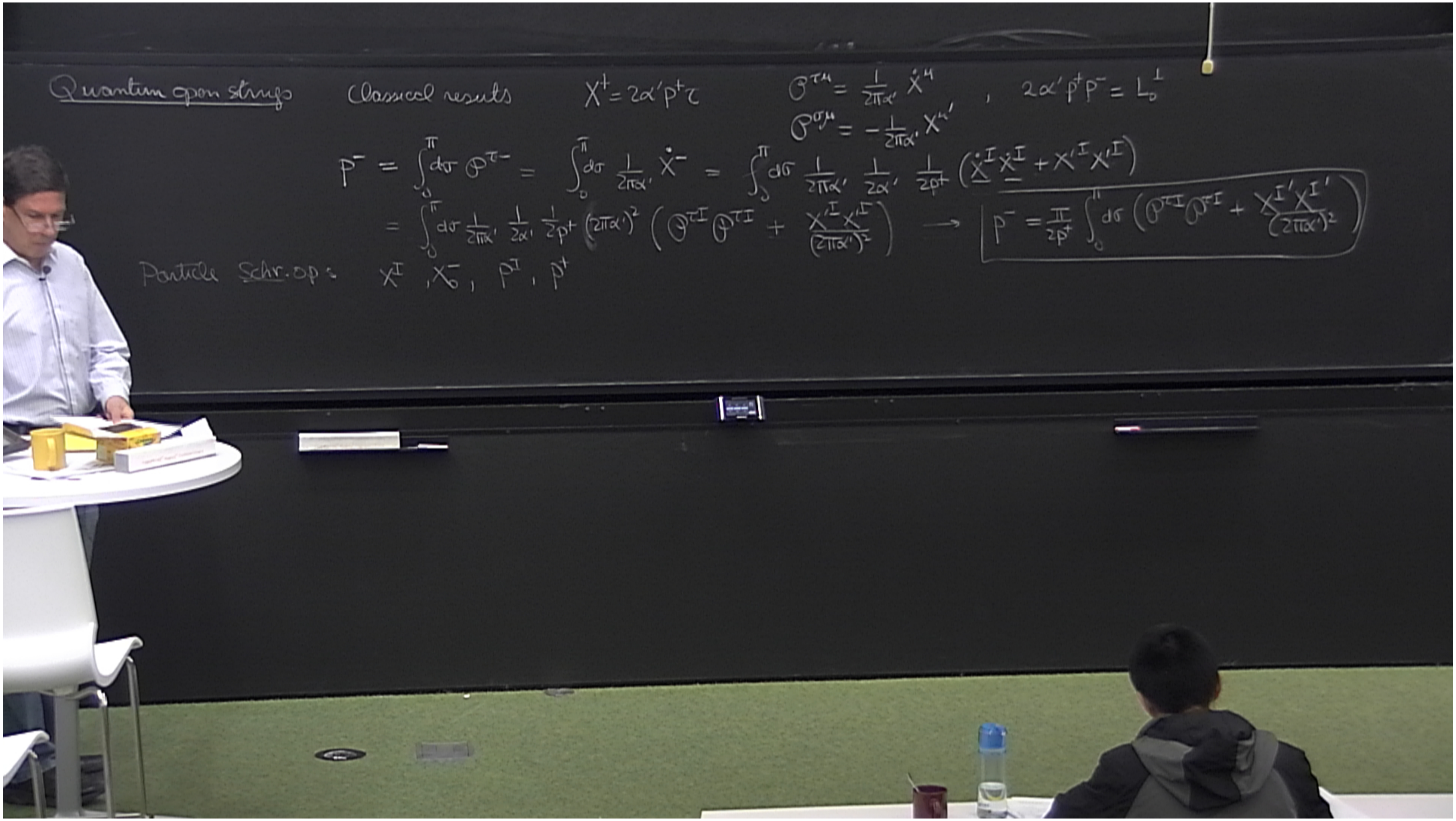
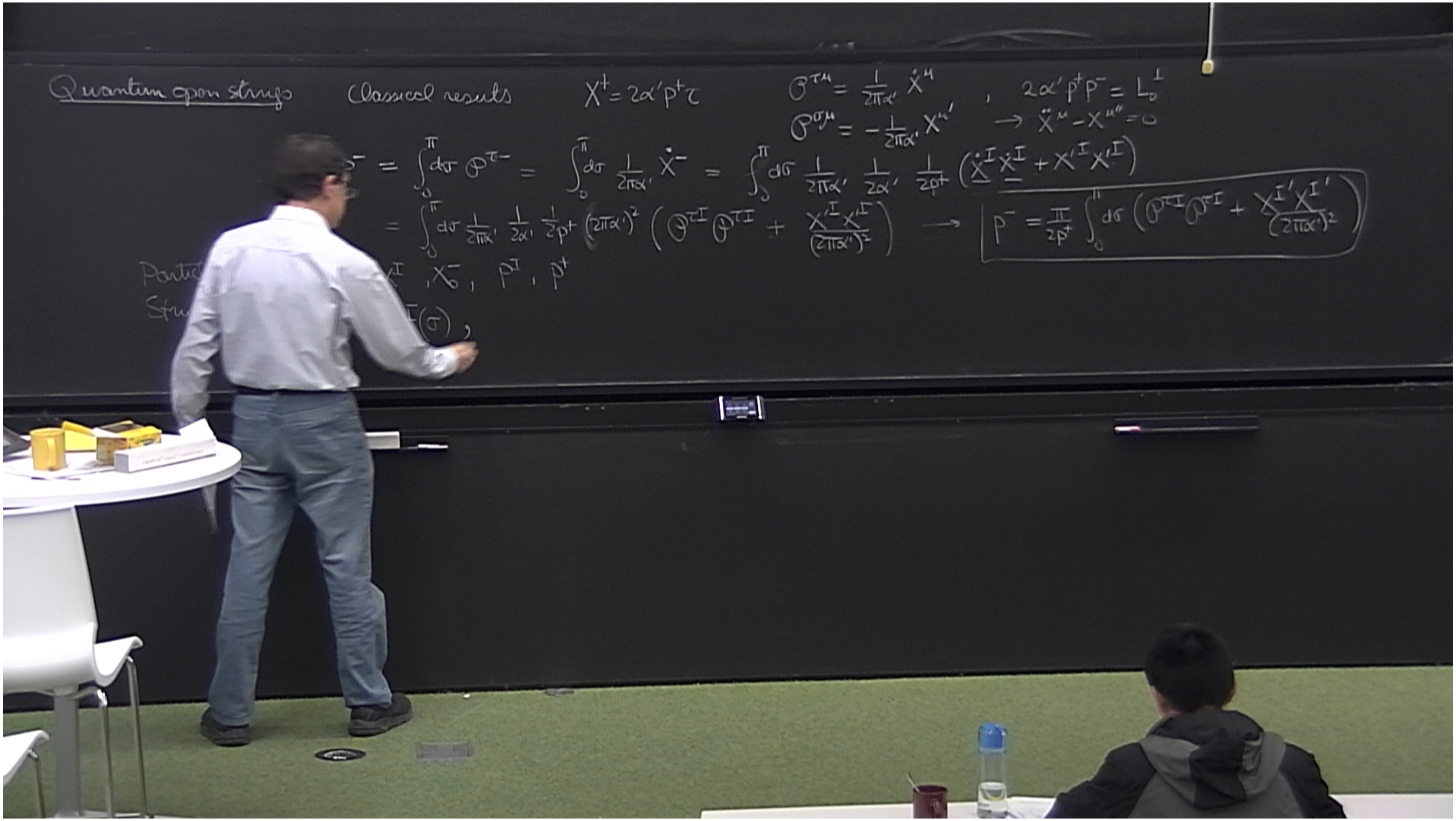
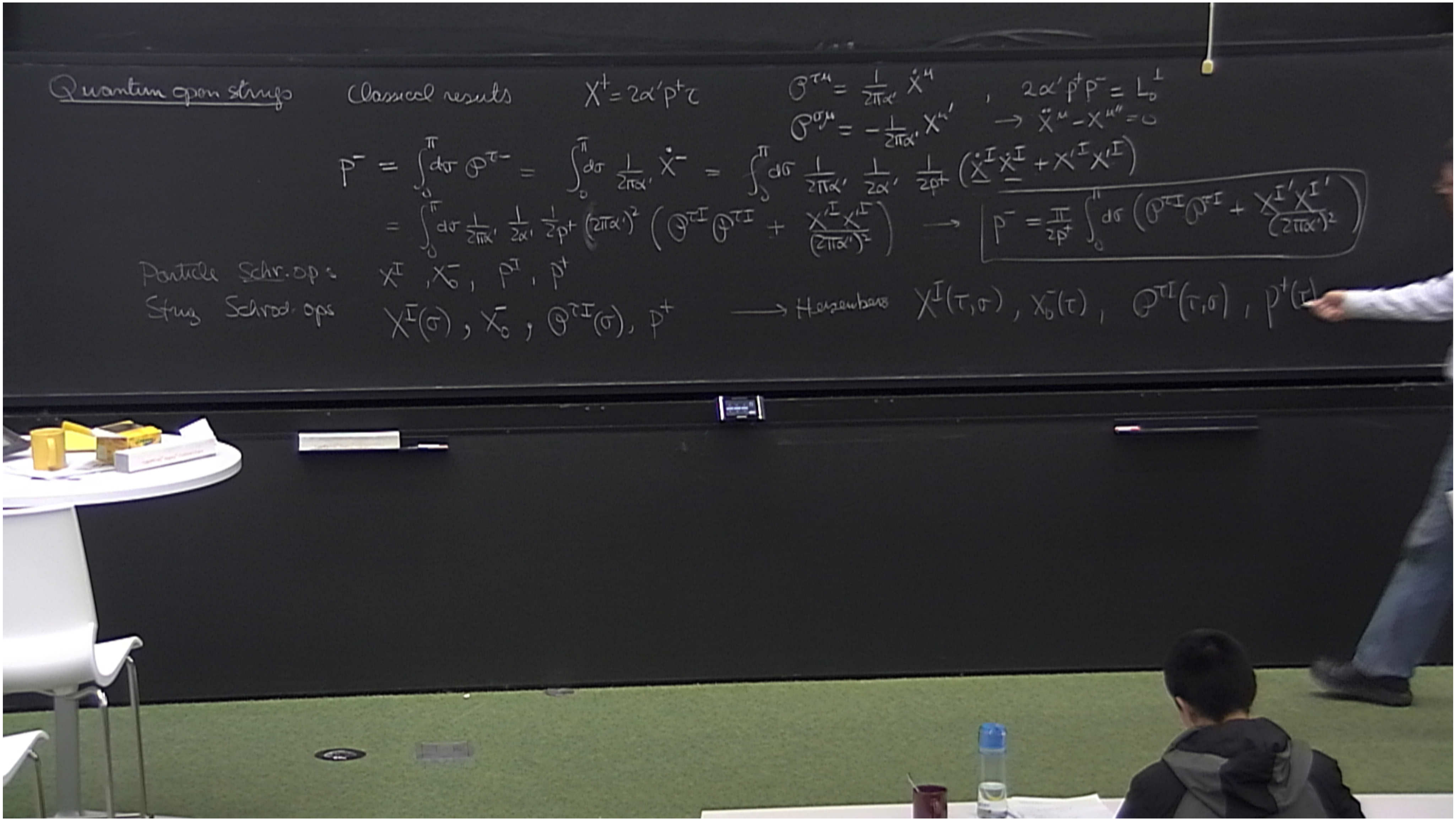
$= \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} (2\pi\alpha')^2 \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X'^I X'^I}{(2\pi\alpha')^2} \right) \longrightarrow p^- = \frac{\pi}{2p^+} \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X^{I'} X^{I'}}{(2\pi\alpha')^2} \right)$


Quantum open strings
Classical results
X^+ = 2α'p^+τ
P^{τμ} = 1/(2πα') Ẋ^μ , 2α'p^+p^- = L_0^⊥
P^{σμ} = − 1/(2πα') X^{μ'}
p^- = ∫_0^π dσ P^{τ-} = ∫_0^π dσ 1/(2πα') Ẋ^- = ∫_0^π dσ 1/(2πα') 1/(2α') 1/(2p^+) (Ẋ^I Ẋ^I + X'^I X'^I)
= ∫_0^π dσ 1/(2πα') 1/(2α') 1/(2p^+) (2πα')^2 (P^{τI} P^{τI} + X'^I X'^I/(2πα')^2)
p^- = π/(2p^+) ∫_0^π dσ (P^{τI} P^{τI} + X^{I'} X^{I'}/(2πα')^2)
Particle Schr. op: x^I, x_0^-, p^I, p^+


Quantum open strings
Classical results
X+ = 2α'p+τ
Pτμ = 1/2πα' Ẋμ
Pσμ = − 1/2πα' X'μ
2α'p+p− = L0⊥
p− = π/2p+ ∫0π dσ (PτI PτI + X'I X'I/(2πα')2)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Classical results
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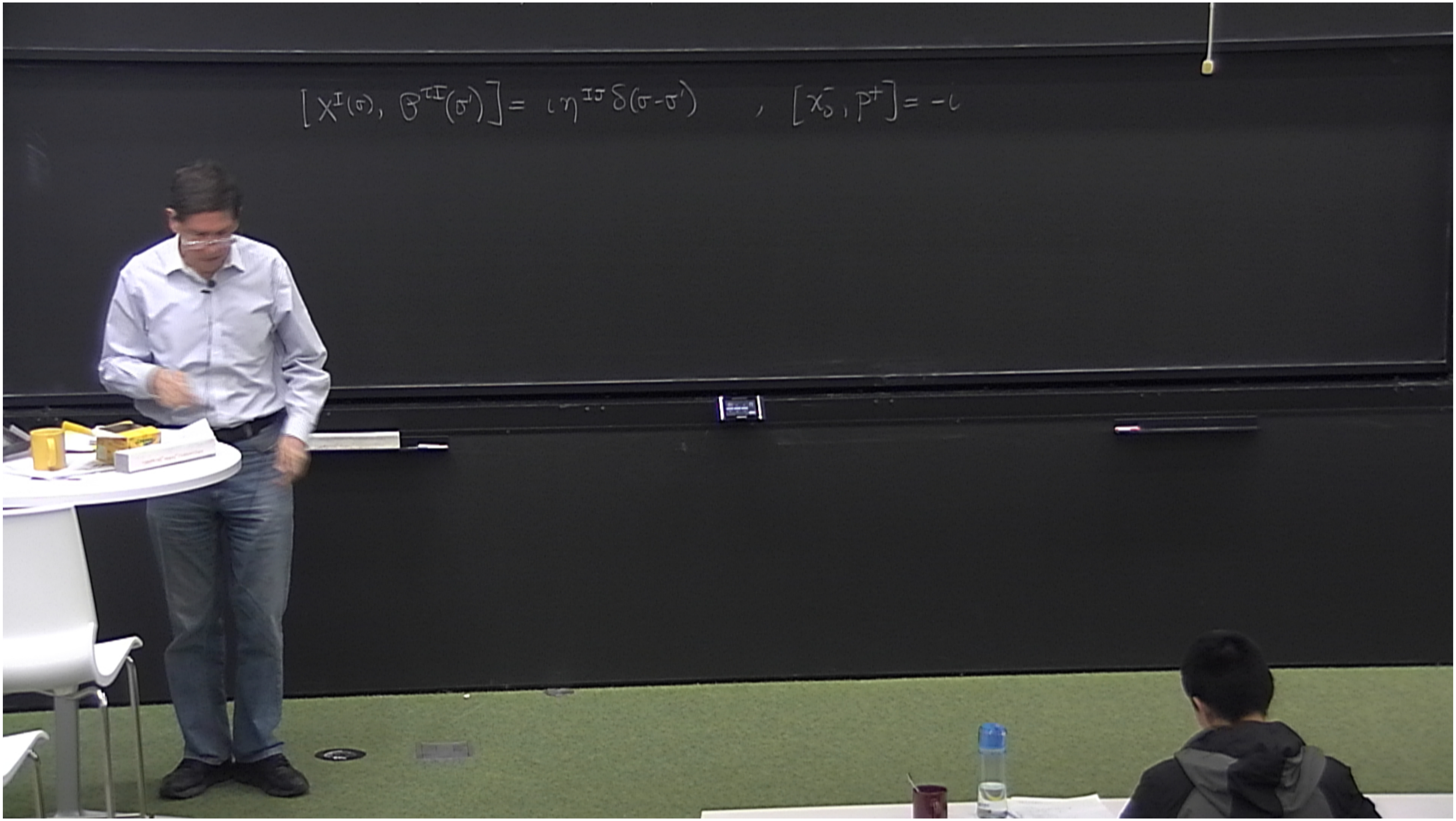
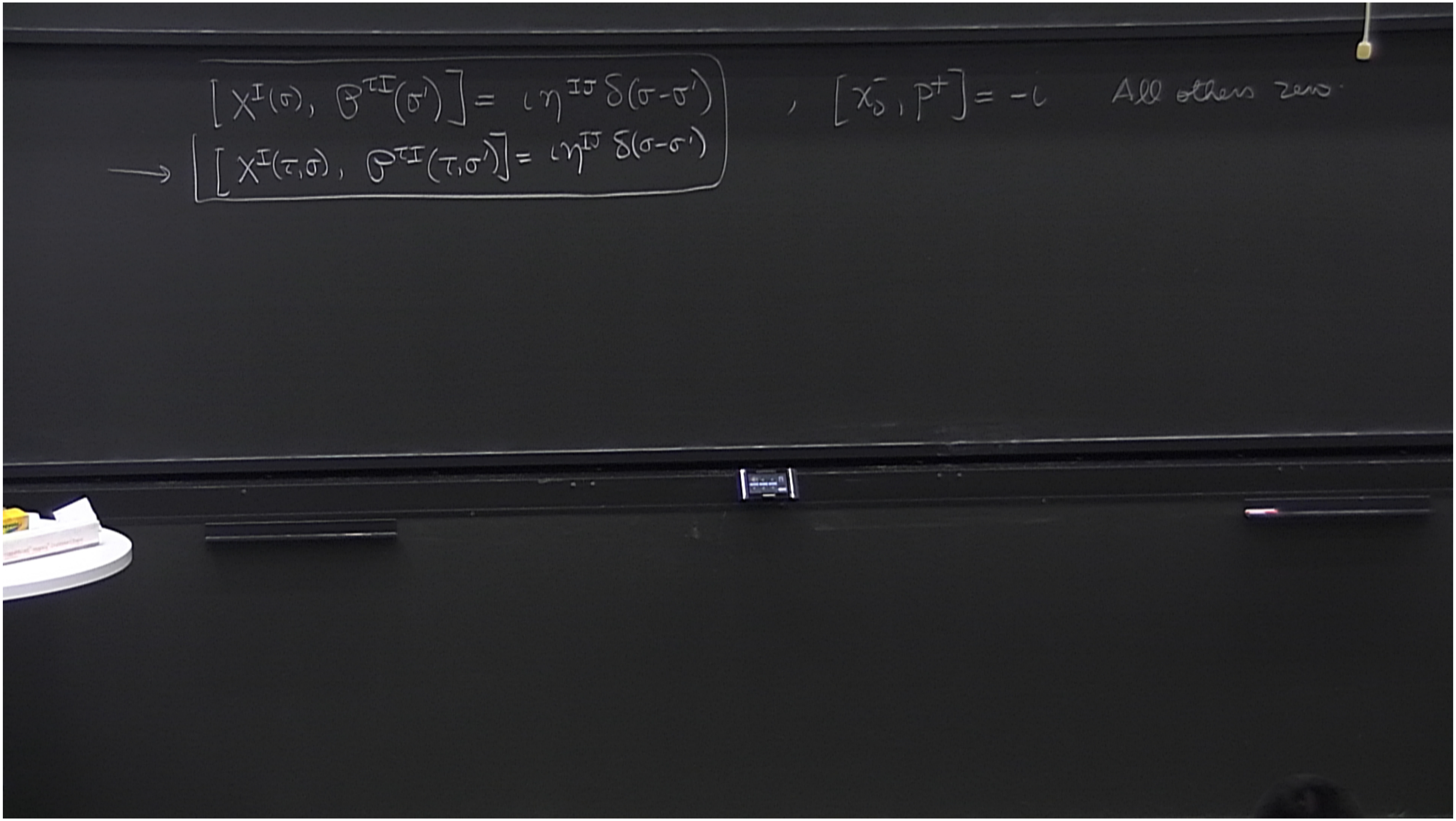
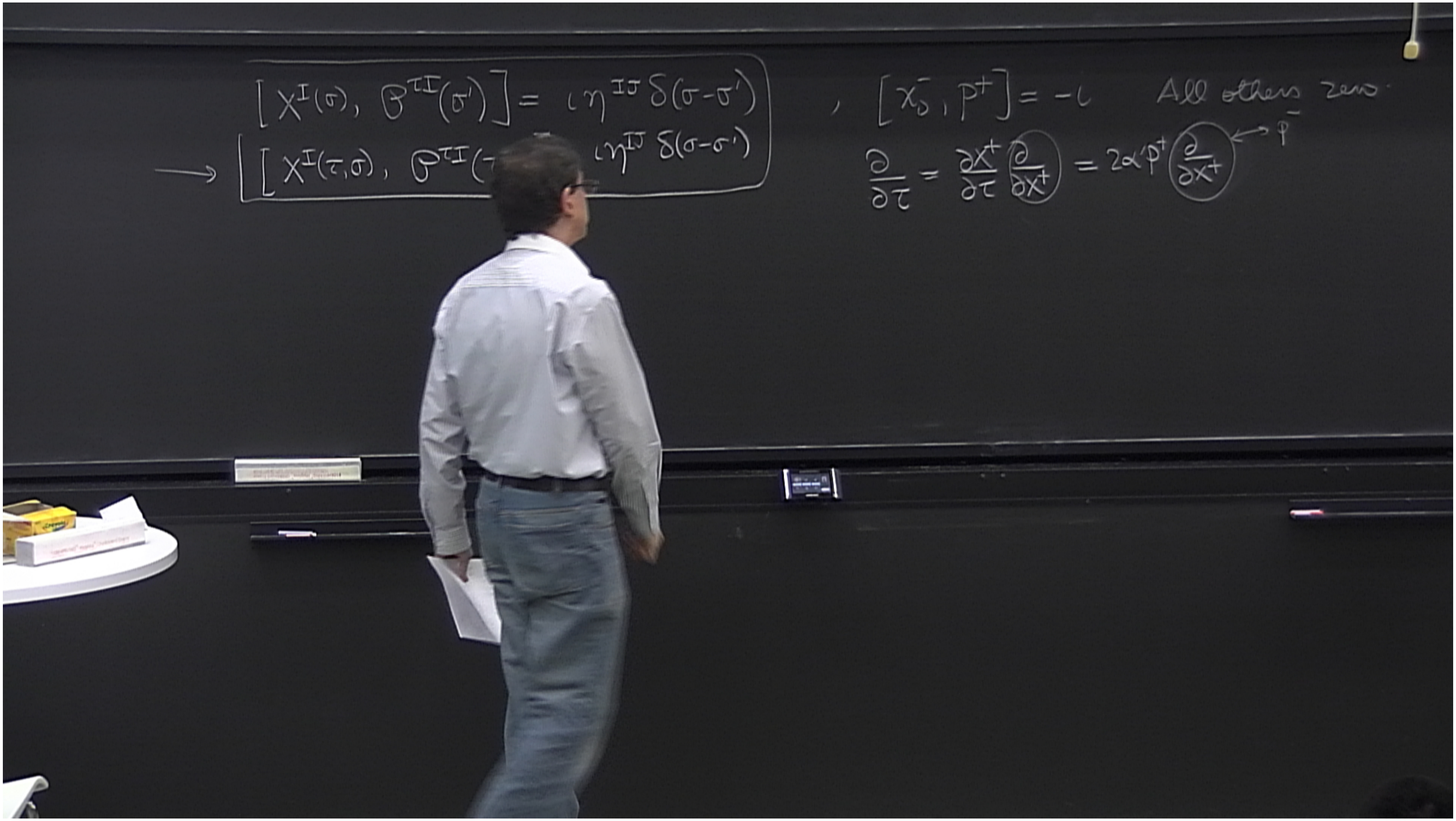
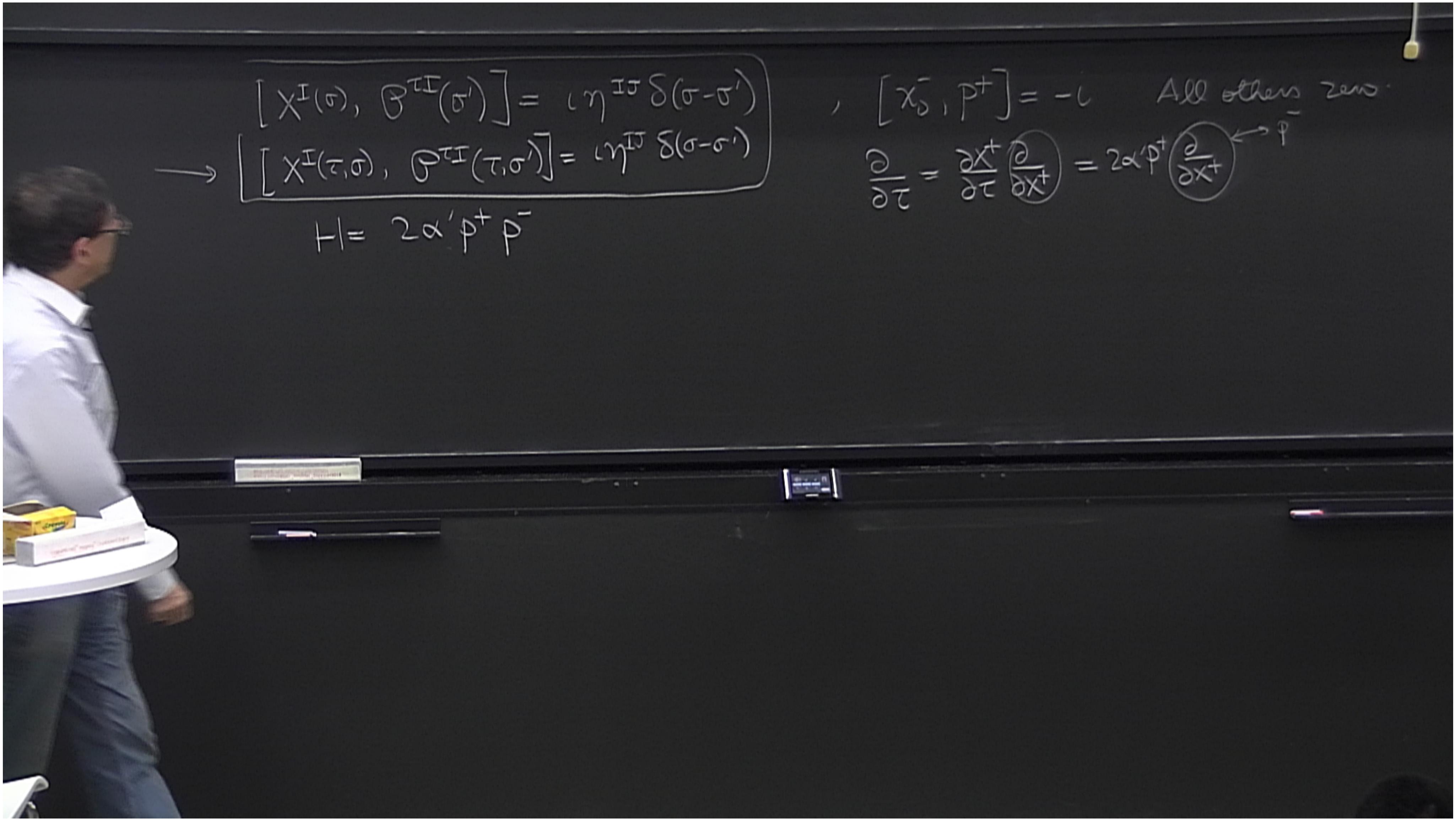
$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'} \dot{X}^\mu$, $2\alpha' p^+ p^- = L_0^\perp$
$\mathcal{P}^{\sigma\mu} = -\frac{1}{2\pi\alpha'} X^{\mu\prime}$ $\rightarrow \ddot{X}^\mu - X^{\mu\prime\prime} = 0$
$p^- = \int_0^\pi d\sigma\, \mathcal{P}^{\tau -} = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \dot{X}^- = \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} (\dot{X}^I \dot{X}^I + X'^I X'^I)$
$= \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} (2\pi\alpha')^2 \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X'^I X'^I}{(2\pi\alpha')^2}\right) \longrightarrow p^- = \frac{\pi}{2p^+} \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I} + \frac{X^{I\prime} X^{I\prime}}{(2\pi\alpha')^2}\right)$
Particle Schr. ops: x^I, x_0^-, p^I, p^+
String Schrod. ops $X^I(\sigma), x_0^-, \mathcal{P}^{\tau I}(\sigma), p^+$ $\longrightarrow$ Heisenberg $X^I(\tau,\sigma), x_0^-(\tau), \mathcal{P}^{\tau I}(\tau,\sigma), p^+(\tau)$


[X^I(σ), P^τI(σ')] = iη^IJ δ(σ-σ') , [x_0^-, p^+] = -i


$[X^I(\sigma), \mathcal{P}^{\tau I}(\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$, $[x_0^-, p^+] = -i$ All others zero.
$\rightarrow [X^I(\tau,\sigma), \mathcal{P}^{\tau I}(\tau,\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$

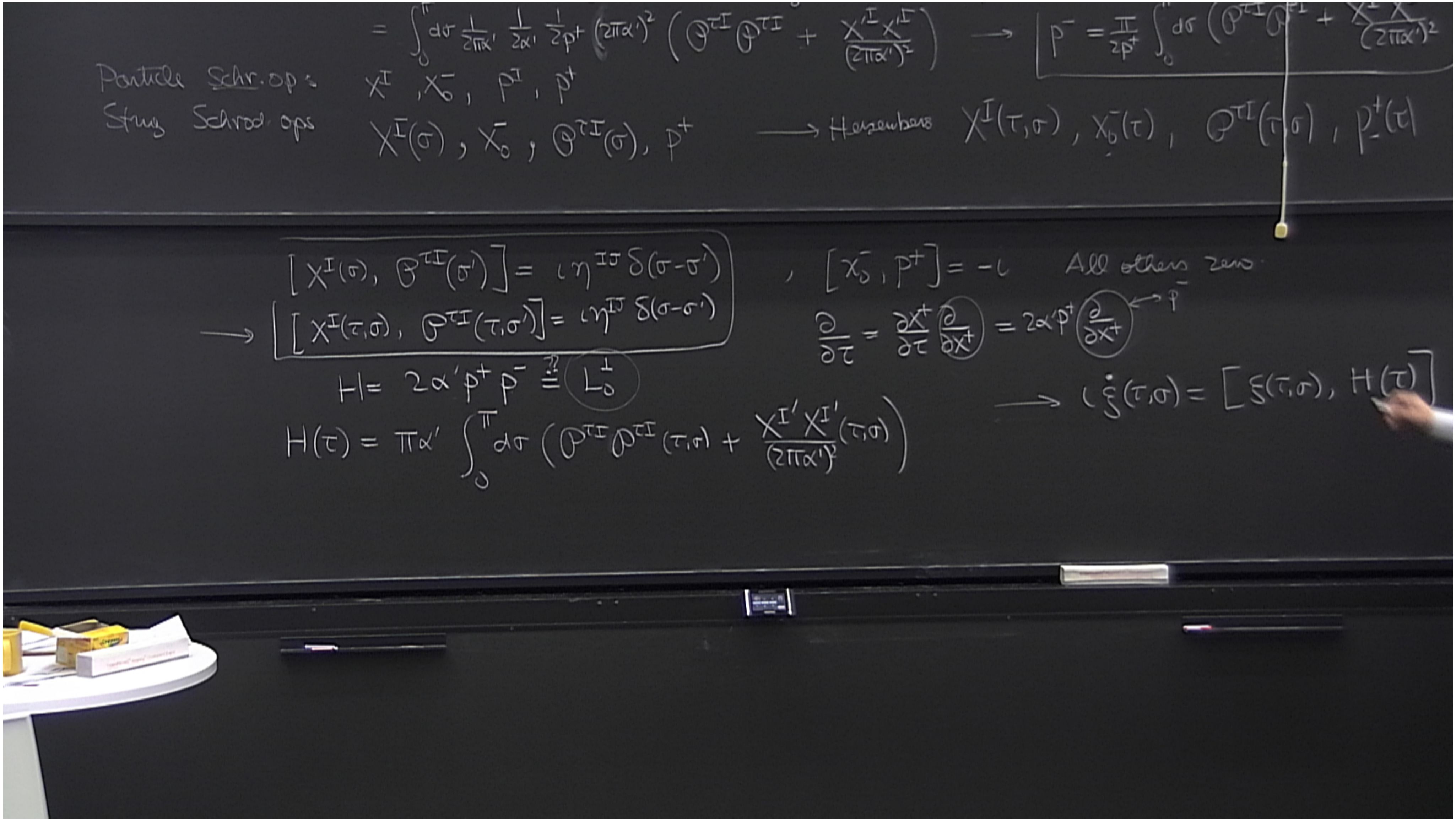


[X^I(σ), P^τI(σ')] = iη^IJ δ(σ−σ')
, [x_0^-, p^+] = −i   All others zero
ηIJ δ(σ−σ')
∂/∂τ = ∂x^+/∂τ ∂/∂x^+ = 2α'p^+ ∂/∂x^+

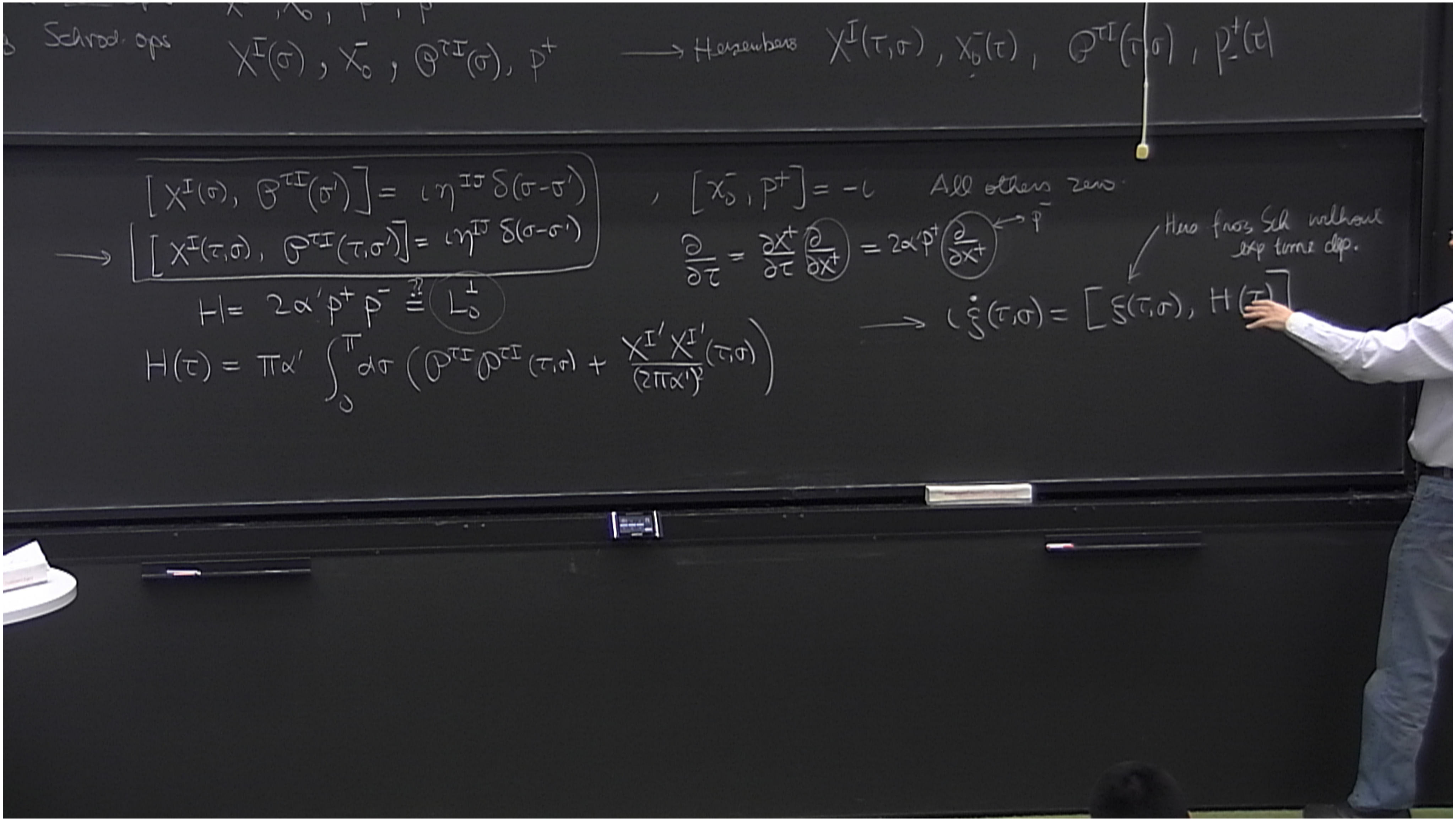

$[X^I(\sigma), \mathcal{P}^{\tau I}(\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$
$[X^I(\tau,\sigma), \mathcal{P}^{\tau I}(\tau,\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$
$H = 2\alpha' p^+ p^-$
$[x_0^-, p^+] = -i$
All others zero.
$\frac{\partial}{\partial\tau} = \frac{\partial x^+}{\partial\tau}\frac{\partial}{\partial x^+} = 2\alpha' p^+ \frac{\partial}{\partial x^+}$
p^-

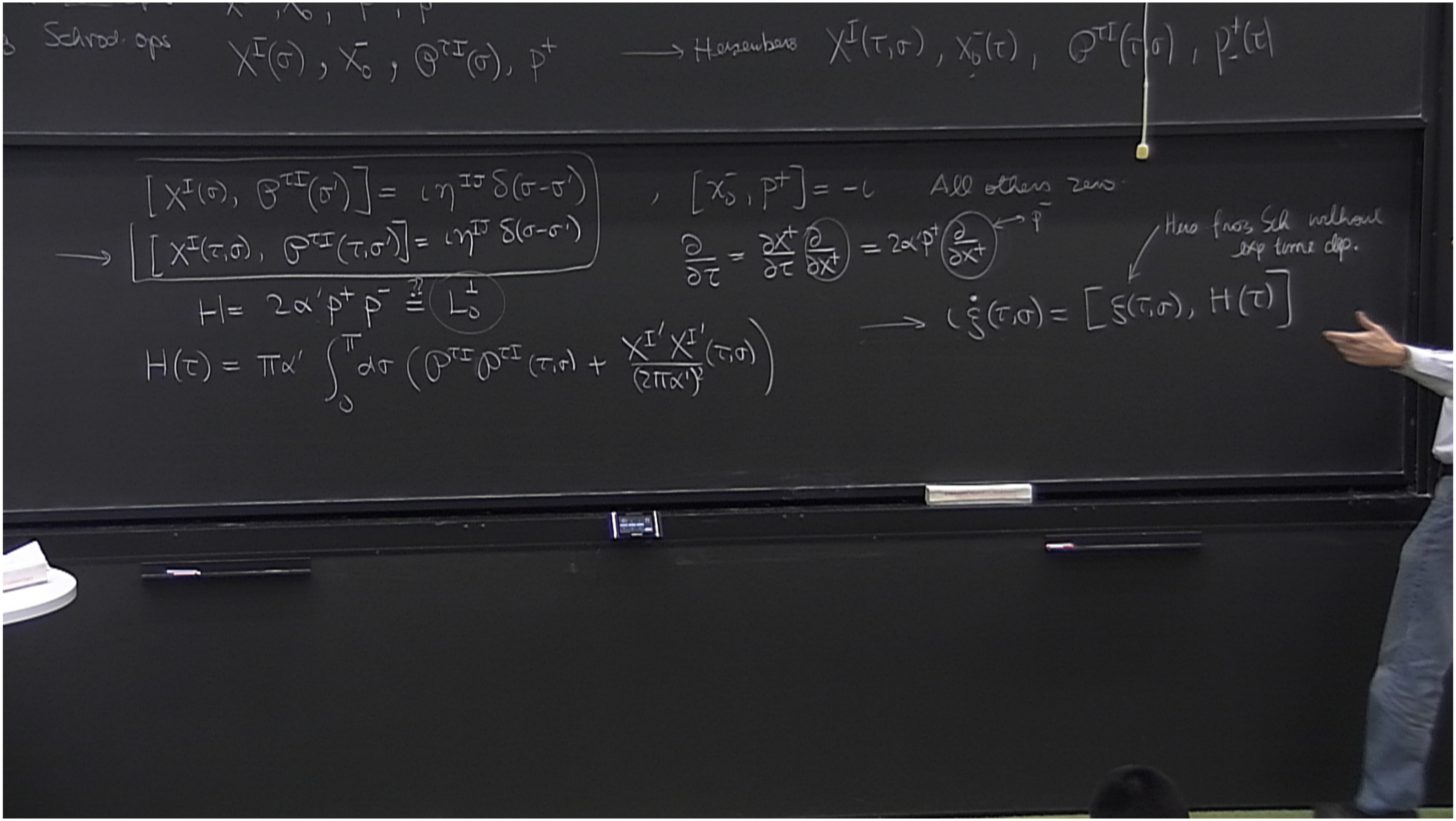
$$\mathring{X}^- \pm X^{-\prime} = \cdots (\mathring{X}^I \pm X^{I\prime})^2$$

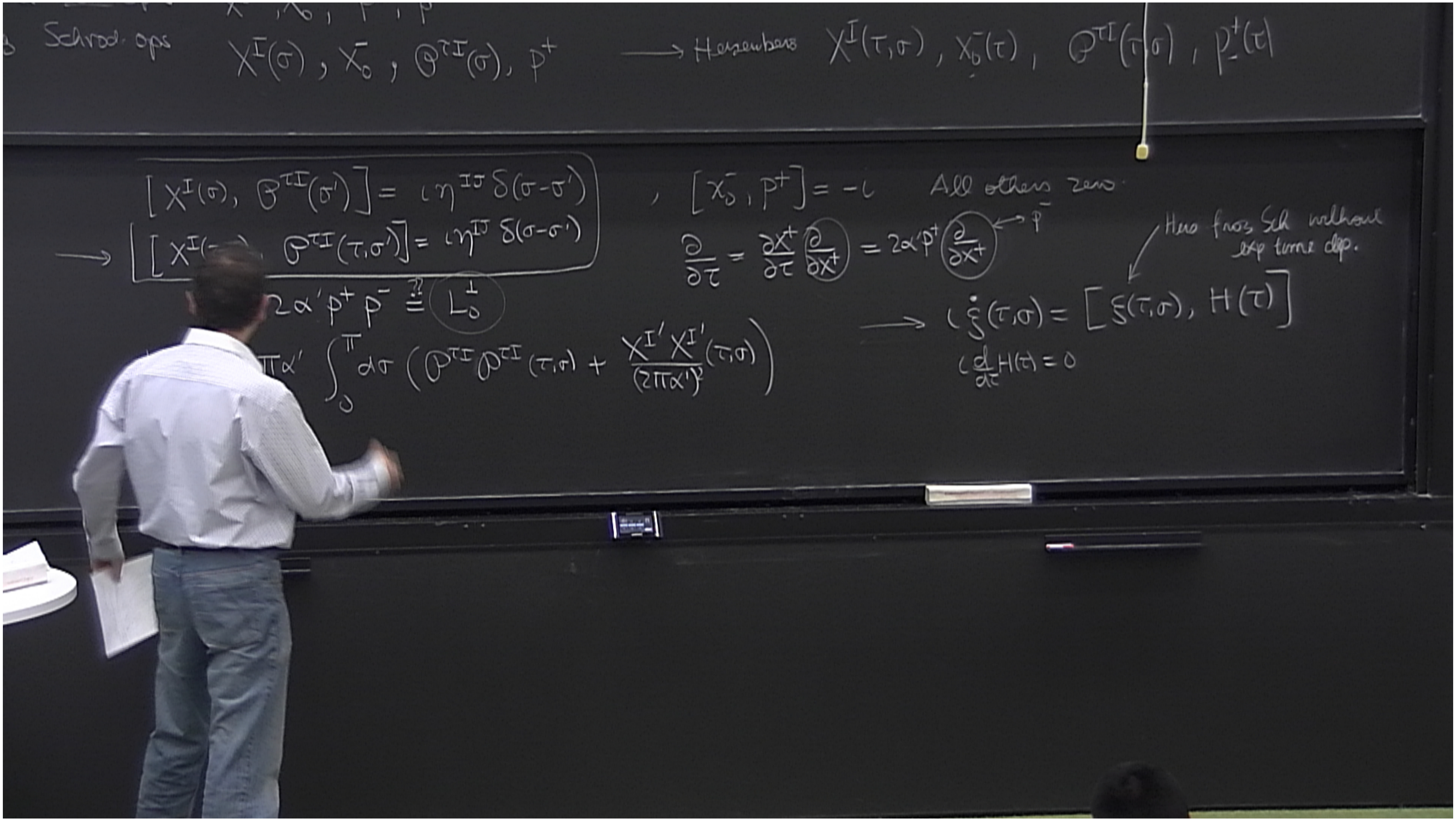
$$L_n^+ = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha^I_{n-p}\,\alpha^I_p$$

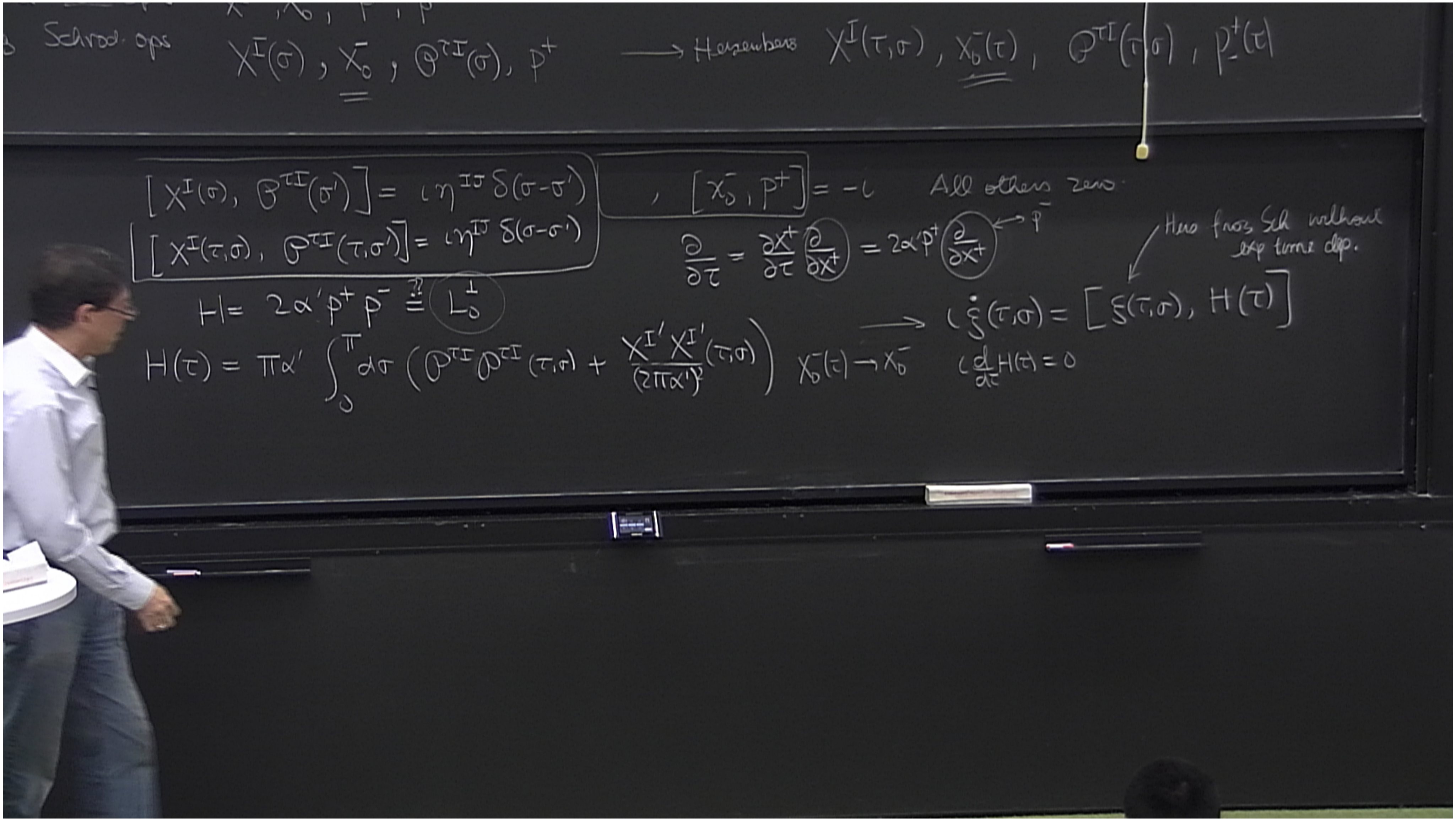


$= \int_0^\pi d\sigma \frac{1}{2\pi\alpha'} \frac{1}{2\alpha'} \frac{1}{2p^+} (2\pi\alpha')^2 \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I} + \frac{X^{I'}X^{I'}}{(2\pi\alpha')^2} \right) \rightarrow p^- = \frac{\pi}{2p^+} \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I} + \frac{X^{I'}X^{I'}}{(2\pi\alpha')^2} \right)$
Particle Schr. ops: X^I, x_0^-, p^I, p^+
String Schrod. ops $X^I(\sigma), x_0^-, \mathcal{P}^{\tau I}(\sigma), p^+ \longrightarrow$ Heisenberg $X^I(\tau,\sigma), x_0^-(\tau), \mathcal{P}^{\tau I}(\tau,\sigma), p^+(\tau)$
$[X^I(\sigma), \mathcal{P}^{\tau J}(\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$, $[x_0^-, p^+] = -i$ All others zero.
$\rightarrow [X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$
$\frac{\partial}{\partial\tau} = \frac{\partial x^+}{\partial\tau}\frac{\partial}{\partial x^+} = 2\alpha' p^+ \frac{\partial}{\partial x^+} \leftrightarrow p^-$
$H = 2\alpha' p^+ p^- \overset{?}{=} L_0^\perp$
$\rightarrow i\dot{\xi}(\tau,\sigma) = [\xi(\tau,\sigma), H(\tau)]$
$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I'}X^{I'}}{(2\pi\alpha')^2}(\tau,\sigma) \right)$


Schröd. ops $X^I(\sigma)$, x_0^-, $\mathcal{P}^{\tau I}(\sigma)$, p^+ ⟶ Heisenberg $X^I(\tau,\sigma)$, $x_0^-(\tau)$, $\mathcal{P}^{\tau I}(\tau,\sigma)$, $p^+(\tau)$
$[X^I(\sigma), \mathcal{P}^{\tau J}(\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$, $[x_0^-, p^+] = -i$ All others zero.
⟶ $[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$
$\frac{\partial}{\partial\tau} = \frac{\partial x^+}{\partial\tau}\frac{\partial}{\partial x^+} = 2\alpha' p^+ \frac{\partial}{\partial x^+}$ ⟷ p^-
Heis fnos Sch without exp time dep.
$H = 2\alpha' p^+ p^- \overset{?}{=} L_0^\perp$
⟶ $i\dot{\xi}(\tau,\sigma) = [\xi(\tau,\sigma), H(\tau)]$
$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I\prime}X^{I\prime}}{(2\pi\alpha')^2}(\tau,\sigma)\right)$


Schrod. ops $X^I(\sigma)$, x_0^-, $\mathcal{P}^{\tau I}(\sigma)$, p^+ → Heisenberg $X^I(\tau,\sigma)$, $x_0^-(\tau)$, $\mathcal{P}^{\tau I}(\tau,\sigma)$, $p^+(\tau)$
$[X^I(\sigma), \mathcal{P}^{\tau J}(\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$, $[x_0^-, p^+] = -i$ All others zero.
→ $[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$
$\frac{\partial}{\partial\tau} = \frac{\partial x^+}{\partial\tau}\frac{\partial}{\partial x^+} = 2\alpha' p^+ \frac{\partial}{\partial x^+}$ ↔ p^-
Here from Sch without exp time dep.
$H = 2\alpha' p^+ p^- \overset{??}{=} L_0^\perp$
→ $i\dot{\xi}(\tau,\sigma) = [\xi(\tau,\sigma), H(\tau)]$
$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I'}X^{I'}}{(2\pi\alpha')^2}(\tau,\sigma)\right)$


Schrod. ops $X^I(\sigma)$, x_0^-, $\mathcal{P}^{\tau I}(\sigma)$, p^+ → Heisenberg $X^I(\tau,\sigma)$, $x_0^-(\tau)$, $\mathcal{P}^{\tau I}(\tau,\sigma)$, $p^+(\tau)$
$[X^I(\sigma), \mathcal{P}^{\tau I}(\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$, $[x_0^-, p^+] = -i$ All others zero.
$\frac{\partial}{\partial\tau} = \frac{\partial x^+}{\partial\tau}\frac{\partial}{\partial x^+} = 2\alpha' p^+ \frac{\partial}{\partial x^+}$
Heis from Sch without exp time dep.
$i\dot{\xi}(\tau,\sigma) = [\xi(\tau,\sigma), H(\tau)]$
$i\frac{d}{d\tau}H(\tau) = 0$
$2\alpha' p^+ p^- \overset{?}{=} L_0^\perp$
$\int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I\prime}X^{I\prime}}{(2\pi\alpha')^2}(\tau,\sigma)\right)$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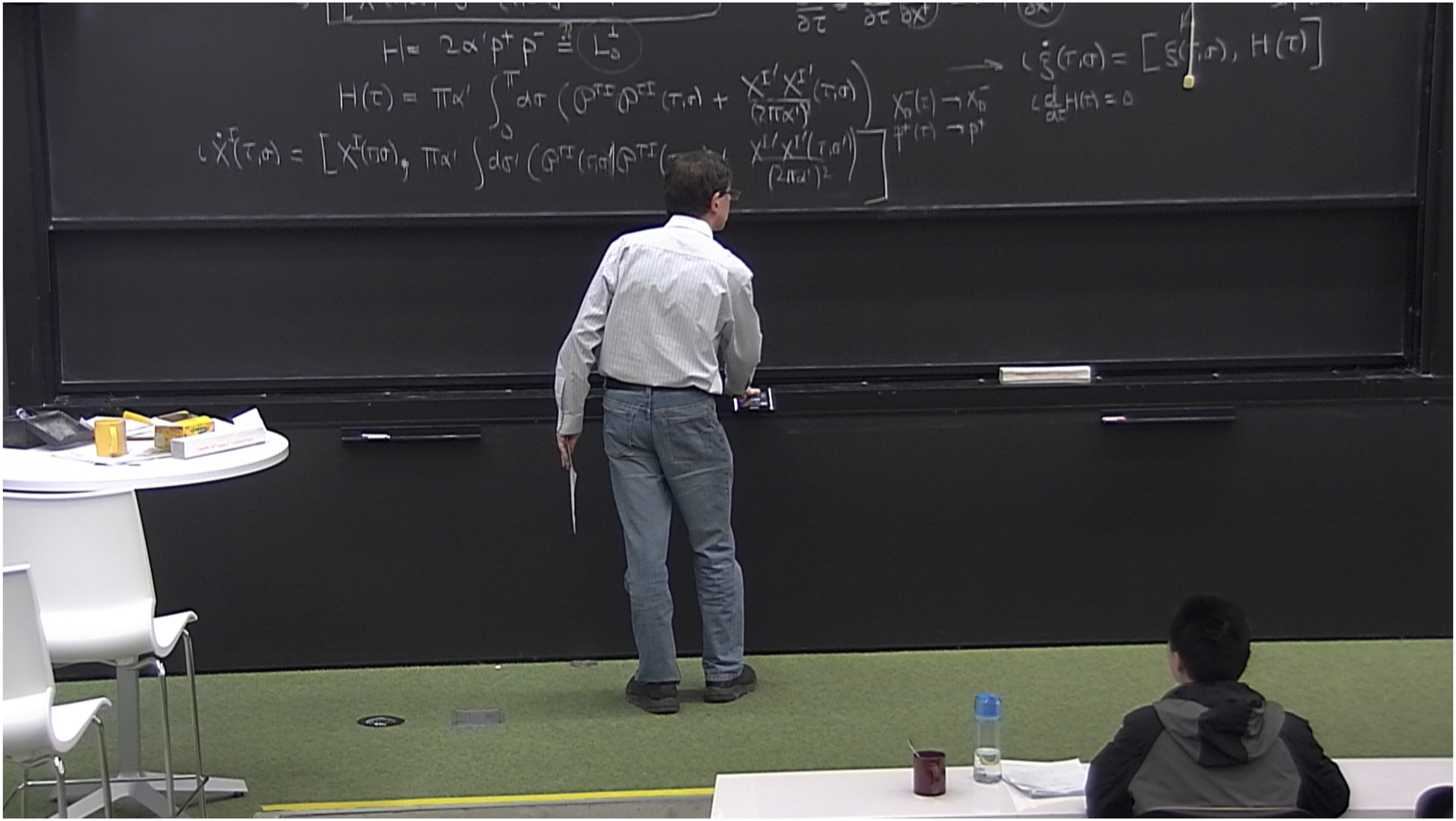
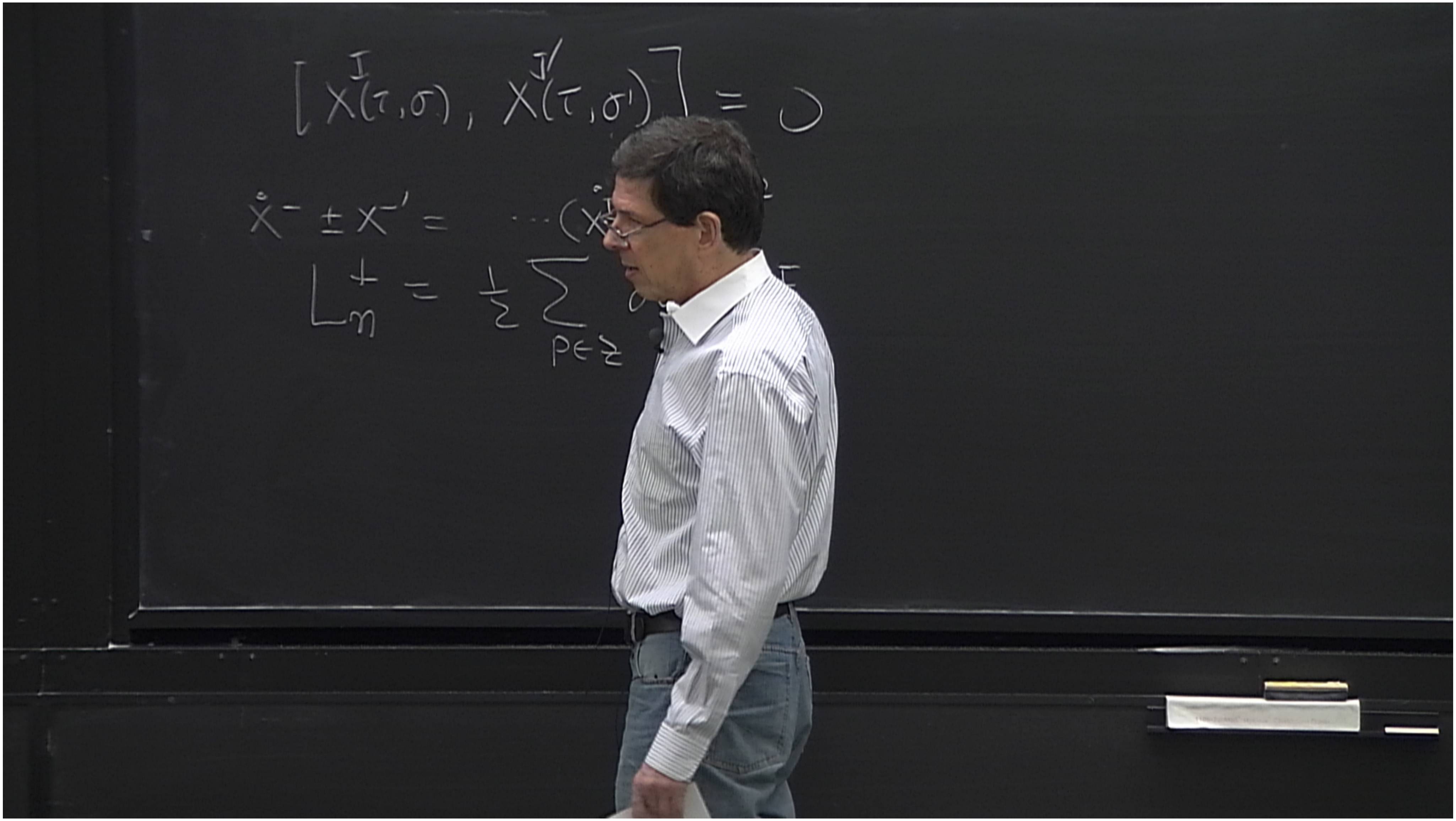
$[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma')] = i\eta^{IJ}\delta(\sigma-\sigma')$
$\frac{\partial}{\partial\tau} = \frac{\partial x^+}{\partial\tau}\frac{\partial}{\partial x^+} = 2\alpha' p^+ \frac{\partial}{\partial x^+}$ ↔ p^-
Heis from Sch without exp time dep.
$H = 2\alpha' p^+ p^- \overset{??}{=} L_0^\perp$
→ $i\dot{\xi}(\tau,\sigma) = [\xi(\tau,\sigma), H(\tau)]$
$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I\prime}X^{I\prime}}{(2\pi\alpha')^2}(\tau,\sigma)\right)$ $x_0^-(\tau) \to x_0^-$ $i\frac{d}{d\tau}H(\tau) = 0$

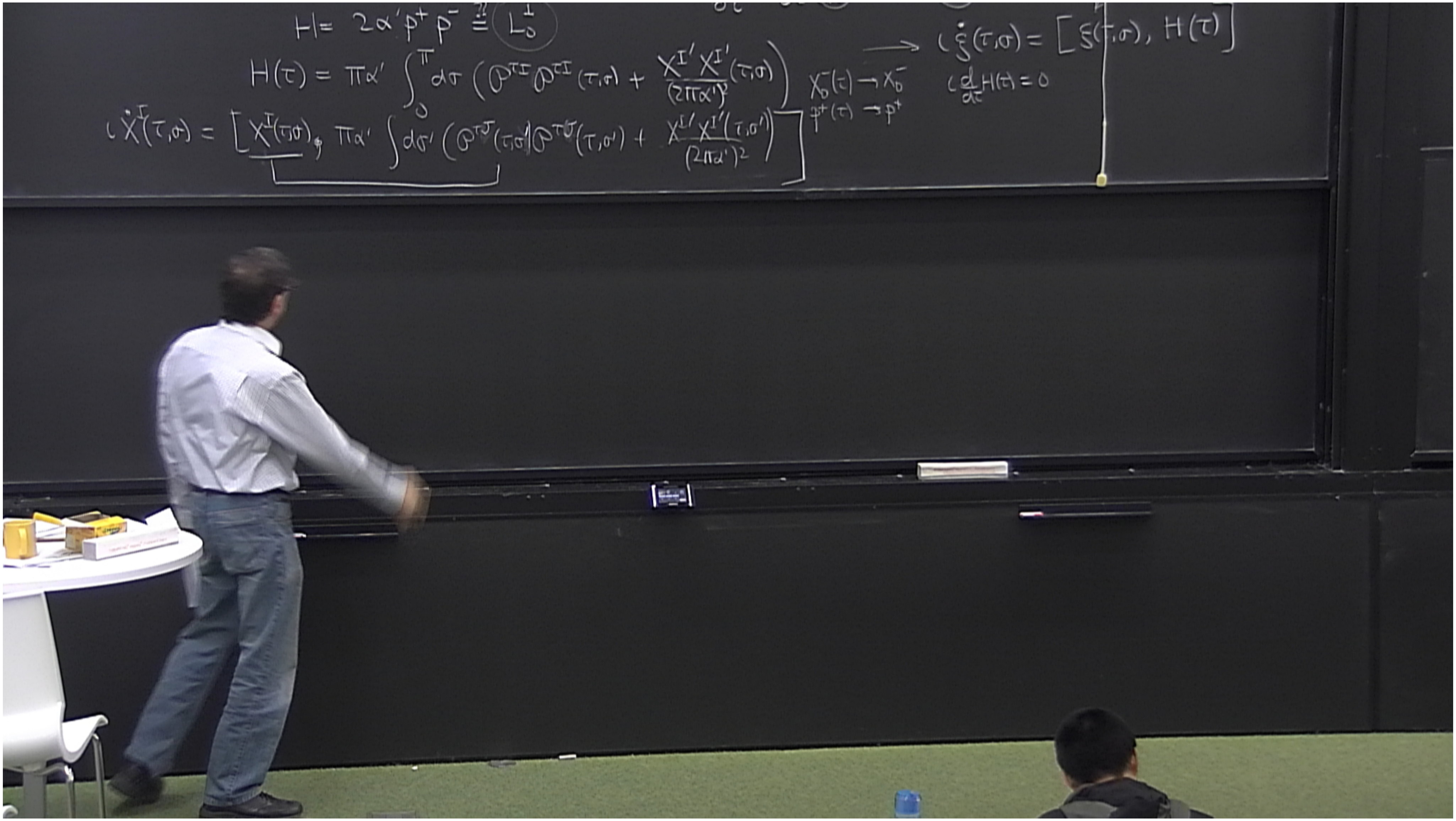

H = 2α'p⁺p⁻ = L₀⊥
H(τ) = πα' ∫₀^π dσ (𝒫^τI 𝒫^τI (τ,σ) + X^I' X^I' / (2πα')² (τ,σ))
i ξ̇(τ,σ) = [ξ(τ,σ), H(τ)]
i d/dτ H(τ) = 0

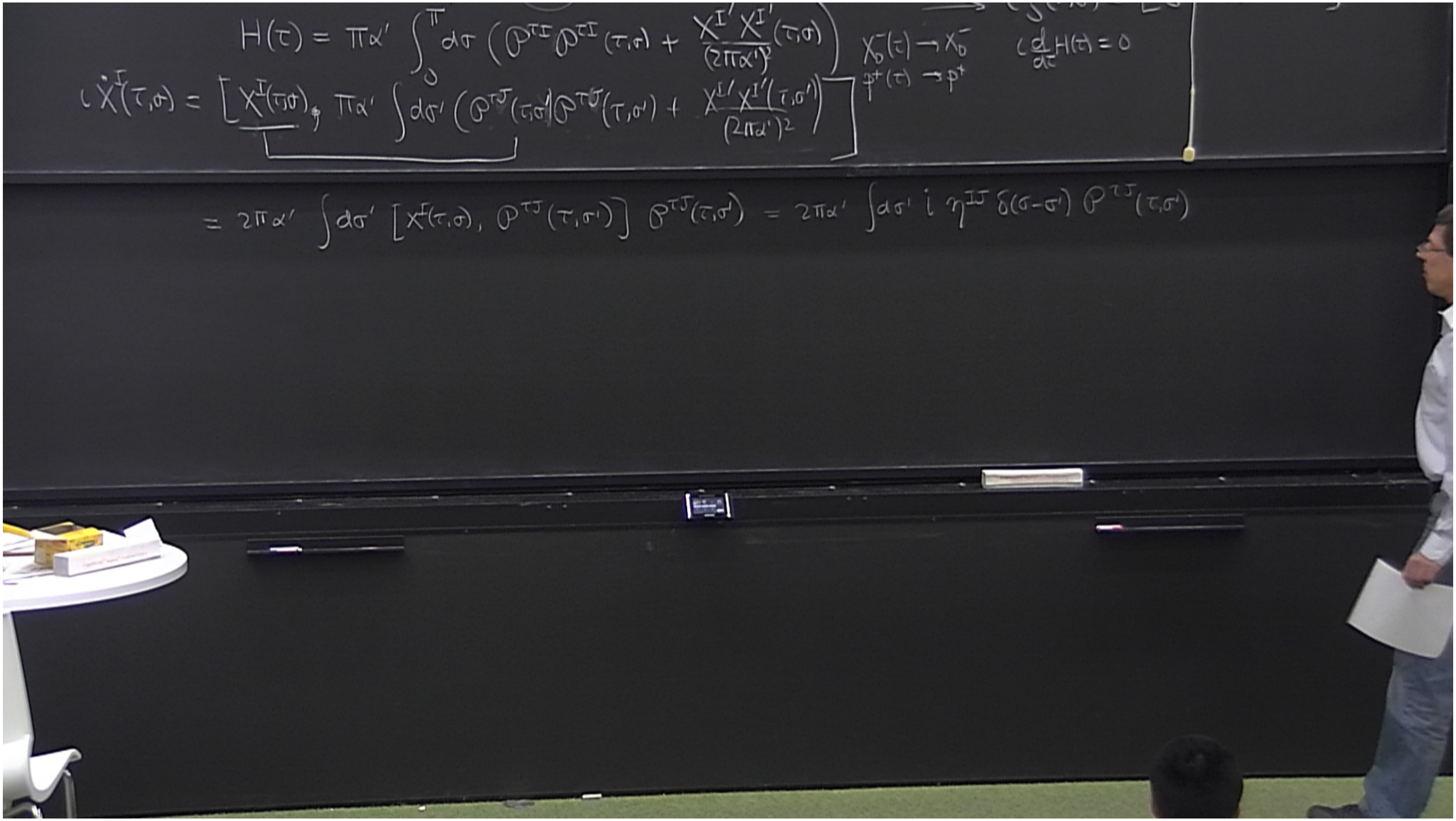

[X^I(τ,σ), X^J'(τ,σ')] = 0
Ẋ⁻ ± X⁻' = ···(Ẋ
L_n^+ = ½ Σ_{p∈Z}

$$[X^I(\tau,\sigma),\ X^{J}(\tau,\sigma')] = 0$$

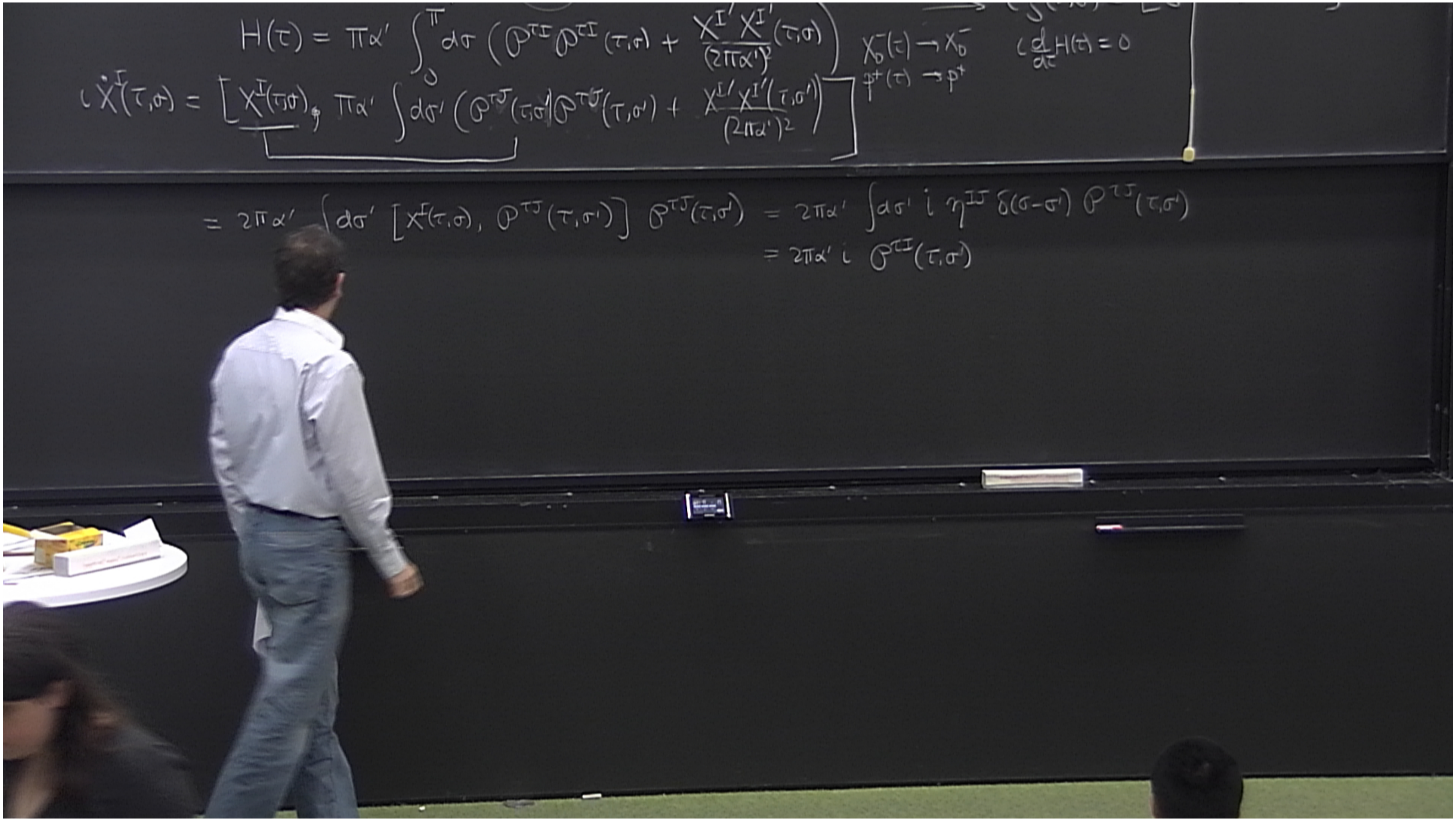
$$\dot{X}^- \pm X^{-\prime} = \cdots (\dot{X}^I \pm X^{I\prime})^2$$

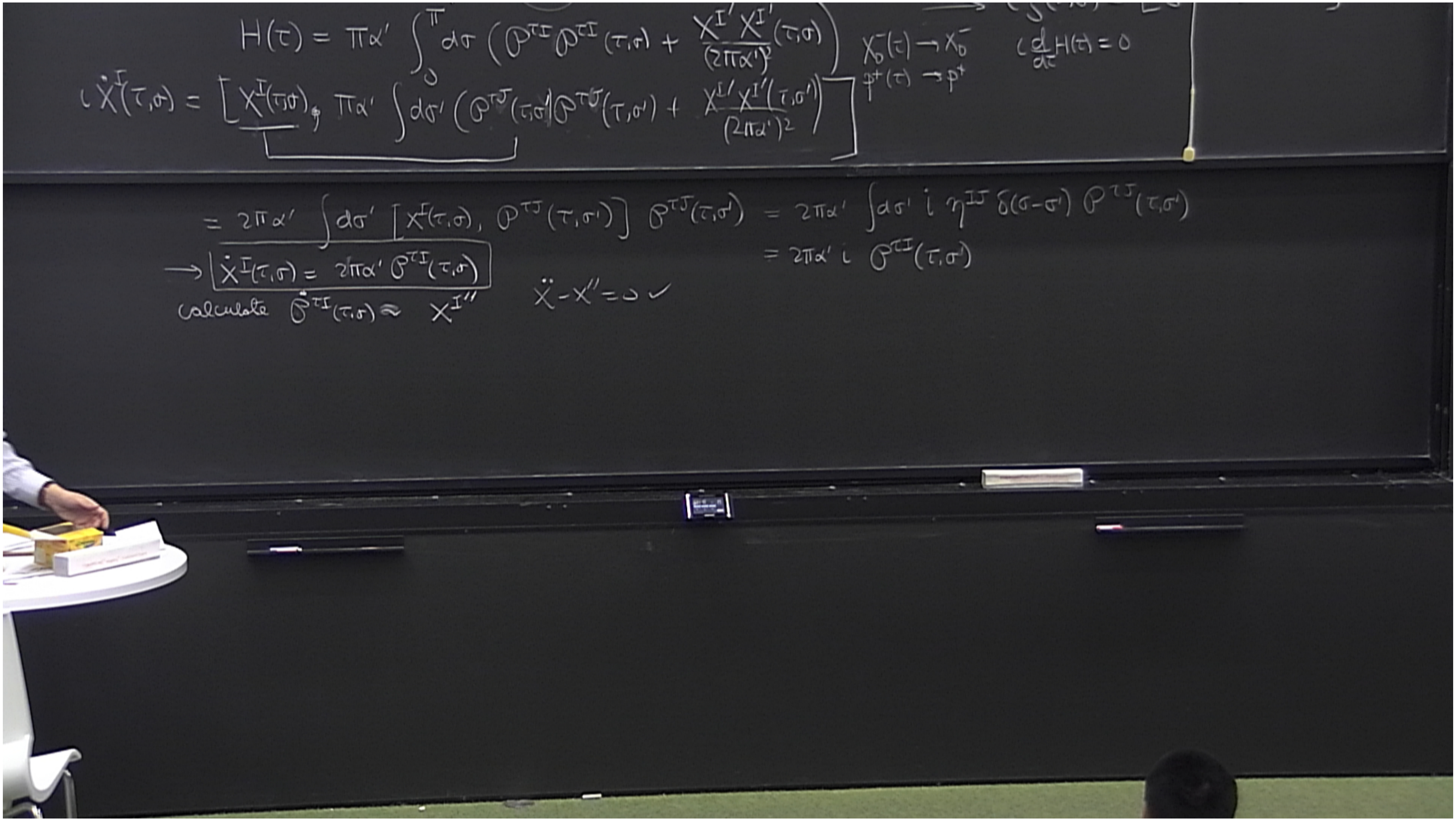
$$L_n^+ = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha^I_{n-p}\,\alpha^I_p$$

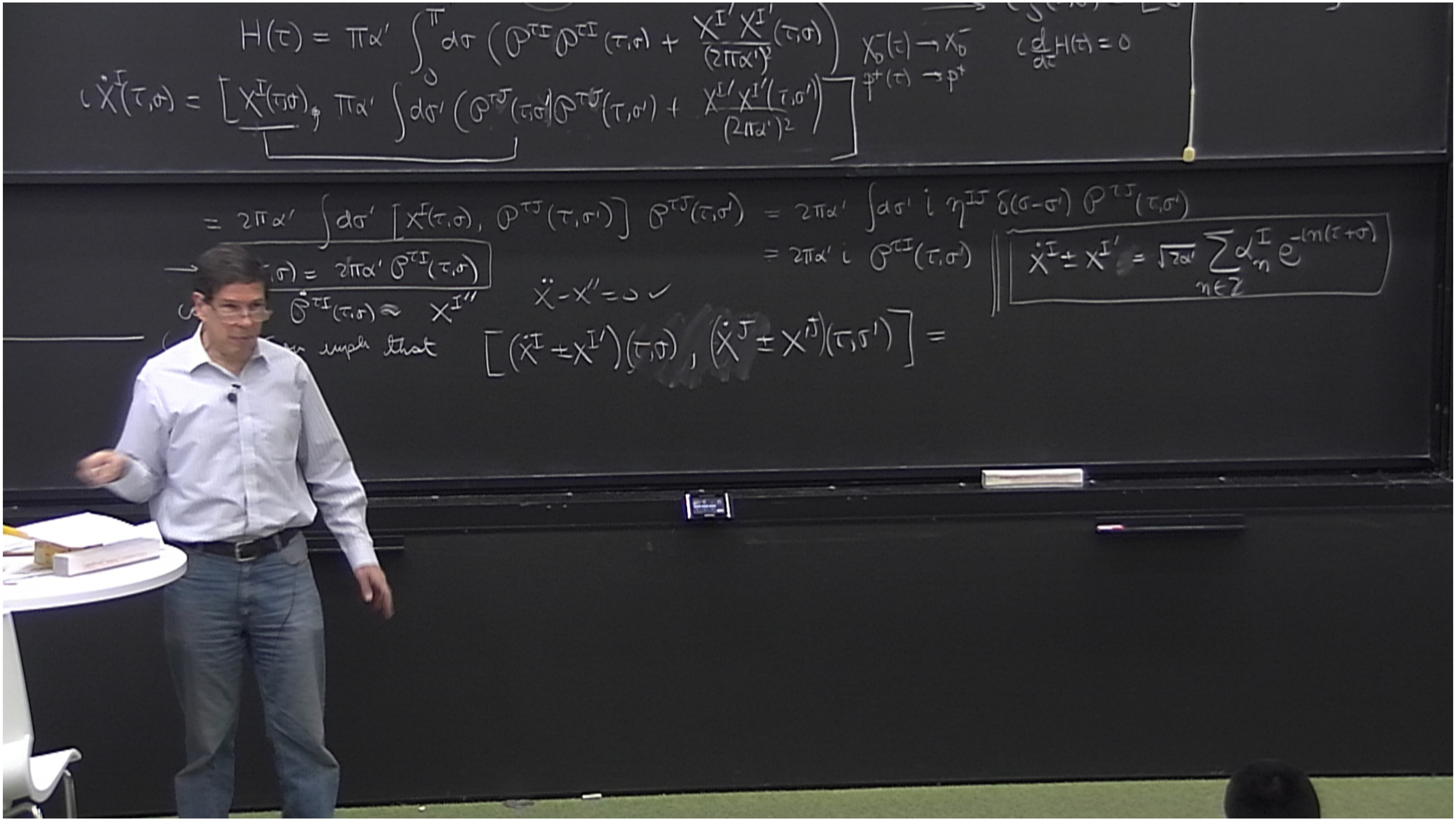

$H = 2\alpha' p^+ p^- \overset{??}{=} L_0^\perp$
$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I'} X^{I'}}{(2\pi\alpha')^2}(\tau,\sigma) \right)$
$i \dot{X}^I(\tau,\sigma) = \left[X^I(\tau,\sigma), \pi\alpha' \int d\sigma' \left(\mathcal{P}^{\tau J}(\tau,\sigma') \mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{J'} X^{J'}(\tau,\sigma')}{(2\pi\alpha')^2} \right) \right]$
$X_0^-(\tau) \to x_0^-$
$p^+(\tau) \to p^+$
$i \dot{\xi}(\tau,\sigma) = [\xi(\tau,\sigma), H(\tau)]$
$i \frac{d}{d\tau} H(\tau) = 0$

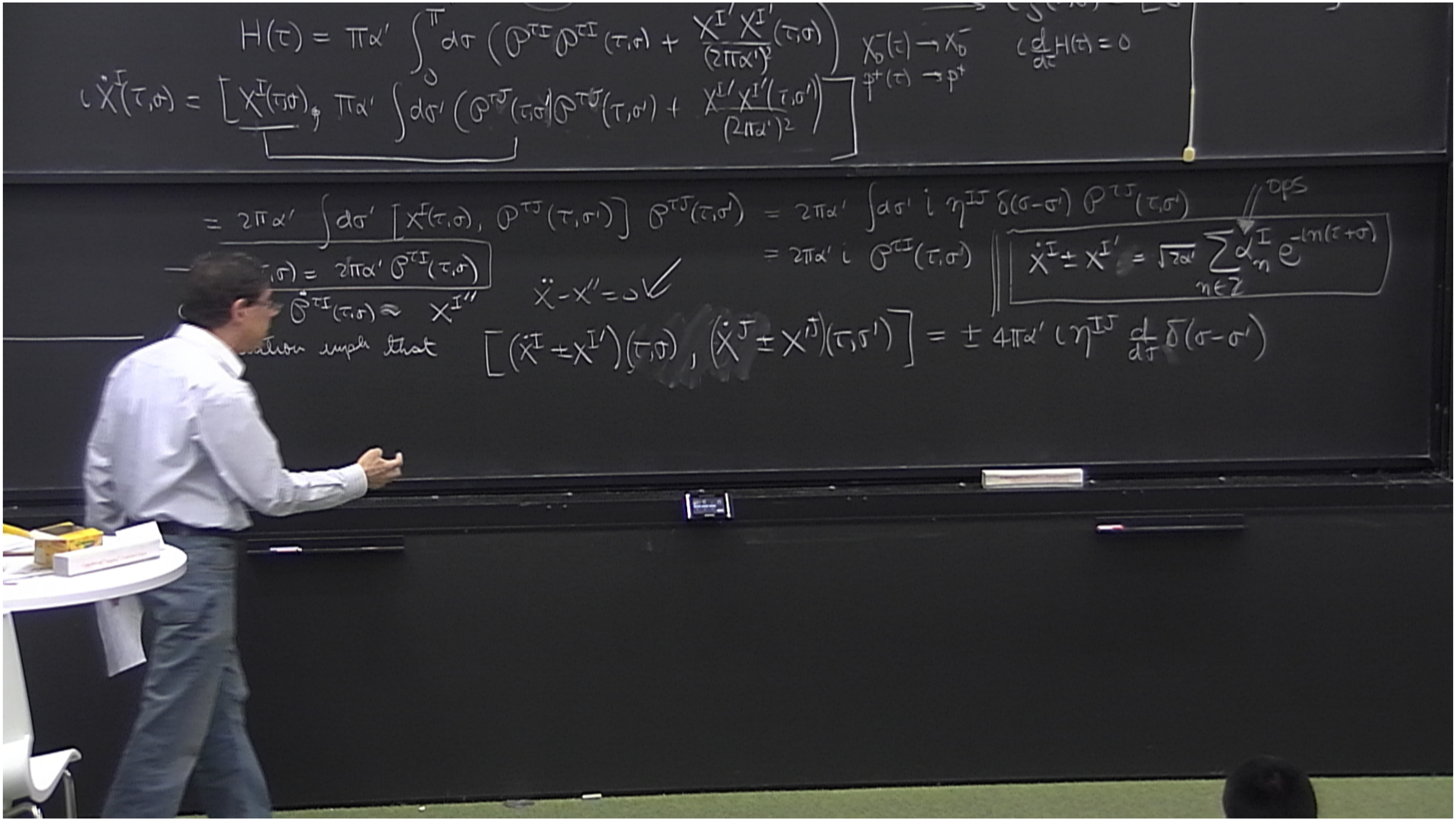


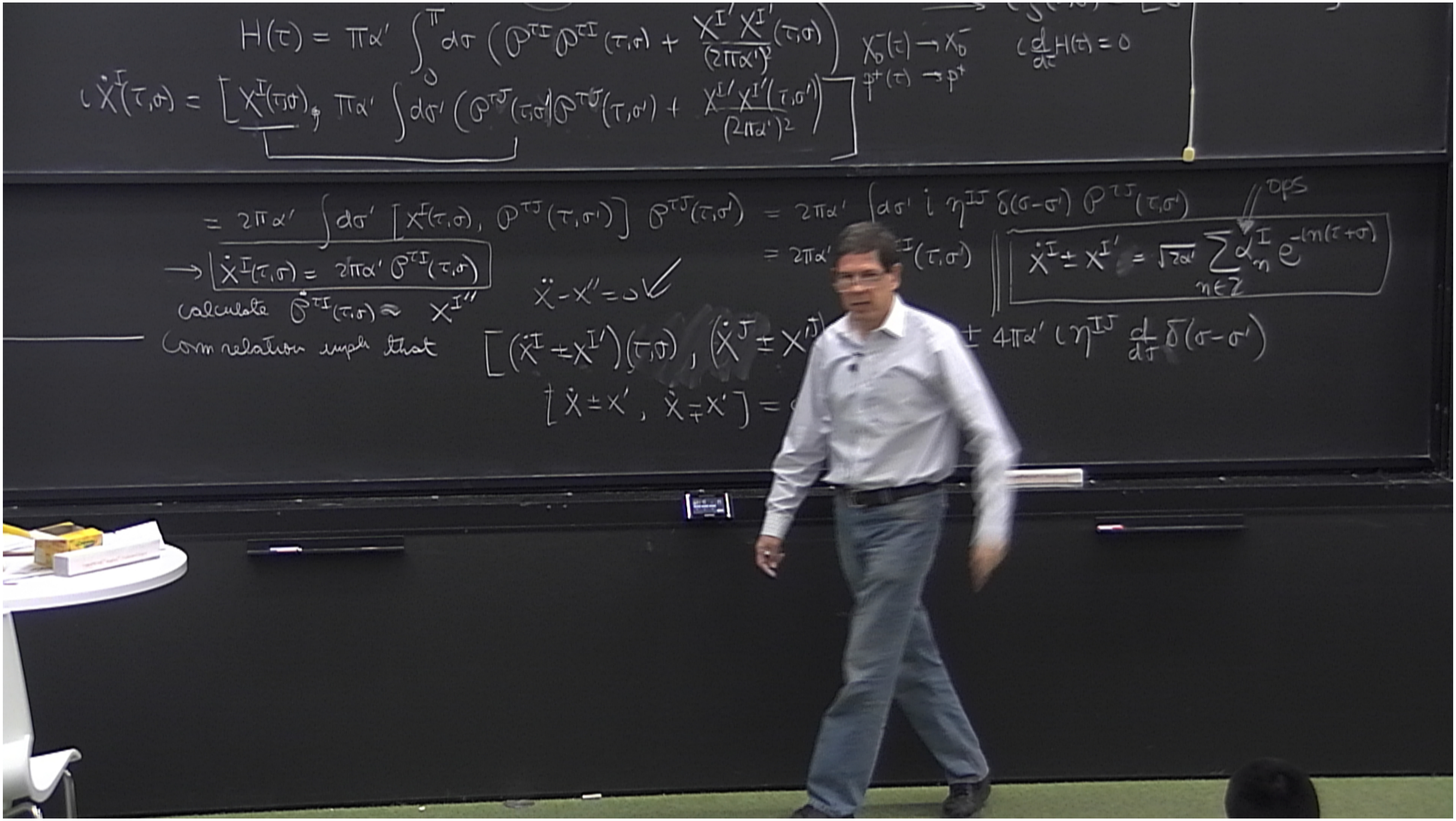
$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I'} X^{I'}}{(2\pi\alpha')^2}(\tau,\sigma) \right)$
$X_0^-(\tau) \to x_0^-$
$i \frac{d}{d\tau} H(\tau) = 0$
$p^+(\tau) \to p^+$
$i \dot{X}^I(\tau,\sigma) = \left[X^I(\tau,\sigma), \pi\alpha' \int d\sigma' \left(\mathcal{P}^{\tau J}(\tau,\sigma') \mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{J'} X^{J'}(\tau,\sigma')}{(2\pi\alpha')^2} \right) \right]$
$= 2\pi\alpha' \int d\sigma' \left[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma') \right] \mathcal{P}^{\tau J}(\tau,\sigma') = 2\pi\alpha' \int d\sigma' \, i \, \eta^{IJ} \delta(\sigma-\sigma') \mathcal{P}^{\tau J}(\tau,\sigma')$

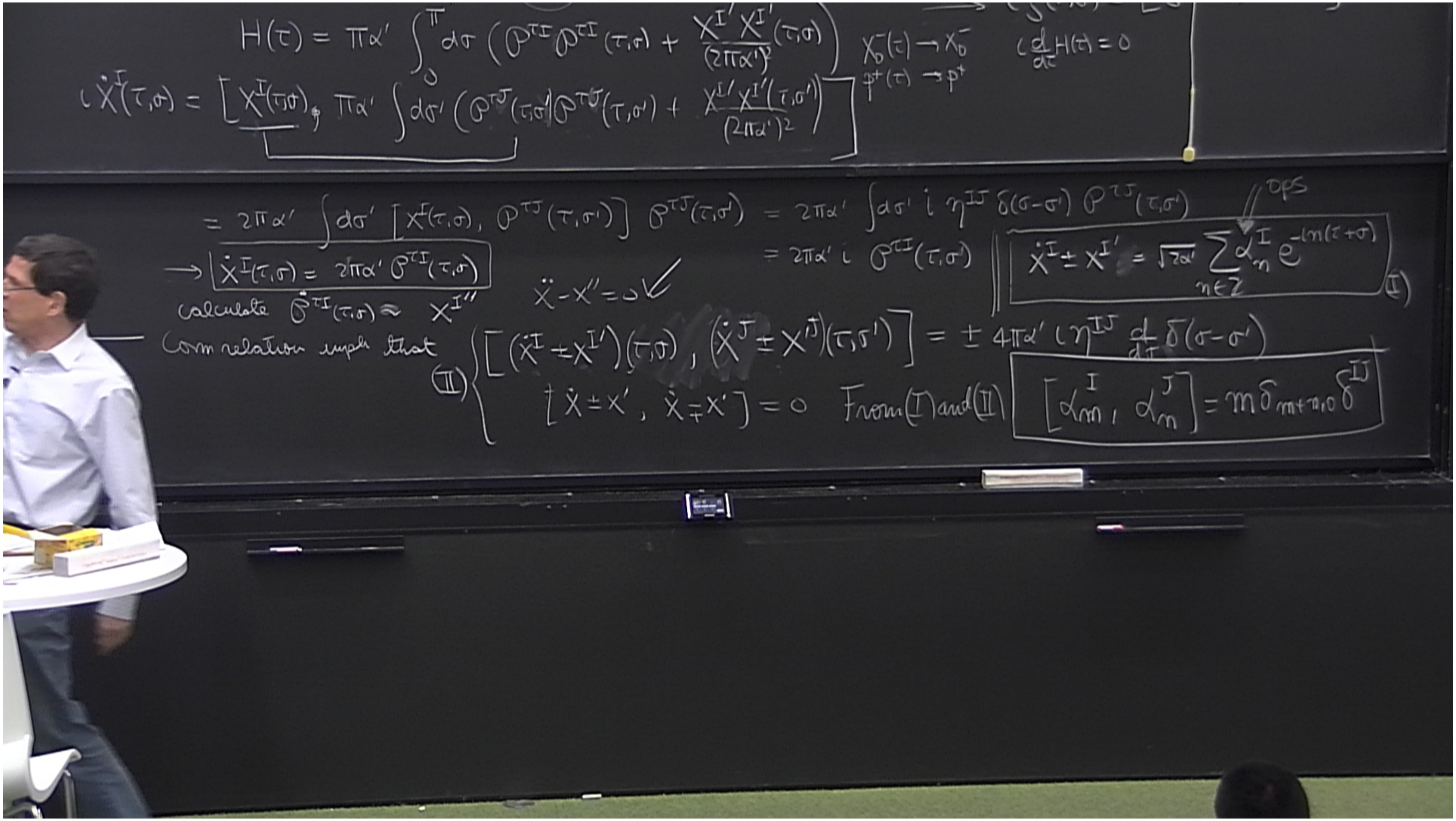

H(τ) = πα' ∫_0^π dσ (P^{τI} P^{τI}(τ,σ) + X^{I'} X^{I'}/(2πα')^2 (τ,σ))
X_0^-(τ) → x_0^-
p^+(τ) → p^+
i d/dτ H(τ) = 0
i Ẋ^I(τ,σ) = [X^I(τ,σ), πα' ∫dσ' (P^{τJ}(τ,σ') P^{τJ}(τ,σ') + X^{J'} X^{J'}(τ,σ')/(2πα')^2)]
= 2πα' ∫dσ' [X^I(τ,σ), P^{τJ}(τ,σ')] P^{τJ}(τ,σ') = 2πα' ∫dσ' i η^{IJ} δ(σ-σ') P^{τJ}(τ,σ')
= 2πα' i P^{τI}(τ,σ)


$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I} \mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I'} X^{I'}}{(2\pi\alpha')^2}(\tau,\sigma) \right)$
$X_0^-(\tau) \to X_0^-$
$p^+(\tau) \to p^+$
$(\frac{d}{d\tau} H(\tau) = 0$
$i\dot{X}^I(\tau,\sigma) = \left[X^I(\tau,\sigma), \pi\alpha' \int d\sigma' \left(\mathcal{P}^{\tau J}(\tau,\sigma') \mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{J'} X^{J'}(\tau,\sigma')}{(2\pi\alpha')^2} \right) \right]$
$= 2\pi\alpha' \int d\sigma' \left[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma') \right] \mathcal{P}^{\tau J}(\tau,\sigma') = 2\pi\alpha' \int d\sigma' \, i \, \eta^{IJ} \delta(\sigma-\sigma') \mathcal{P}^{\tau J}(\tau,\sigma')$
$= 2\pi\alpha' i \, \mathcal{P}^{\tau I}(\tau,\sigma)$
$\rightarrow \dot{X}^I(\tau,\sigma) = 2\pi\alpha' \mathcal{P}^{\tau I}(\tau,\sigma)$
calculate $\dot{\mathcal{P}}^{\tau I}(\tau,\sigma) \approx X^{I''}$
$\ddot{X} - X'' = 0$ ✓

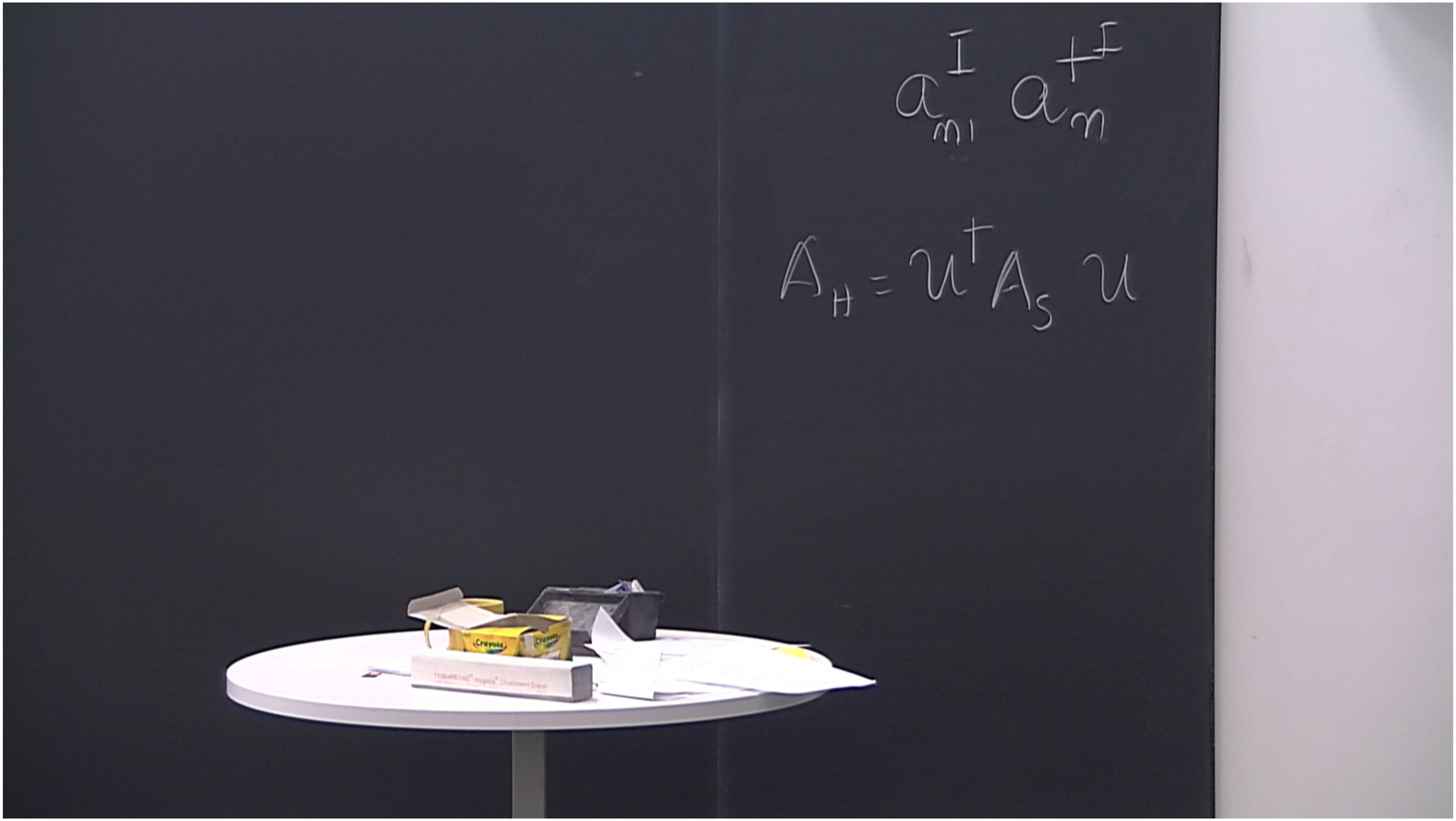

H(τ) = πα' ∫₀^π dσ (P^τI P^τI (τ,σ) + X^I' X^I' / (2πα')² (τ,σ))
X₀⁻(τ) → X₀⁻
i d/dτ H(τ) = 0
p⁺(τ) → p⁺
i Ẋ^I(τ,σ) = [X^I(τ,σ), πα' ∫dσ' (P^τJ(τ,σ') P^τJ(τ,σ') + X^J' X^J'(τ,σ') / (2πα')²)]
= 2πα' ∫dσ' [X^I(τ,σ), P^τJ(τ,σ')] P^τJ(τ,σ') = 2πα' ∫dσ' i η^IJ δ(σ−σ') P^τJ(τ,σ')
= 2πα' i P^τI(τ,σ)
Ẋ^I ± X^I' = √(2α') Σ_{n∈Z} α_n^I e^{−in(τ+σ)}
Ẍ − X'' = 0 ✓
[(Ẋ^I ± X^I')(τ,σ), (Ẋ^J ± X^J')(τ,σ')] =

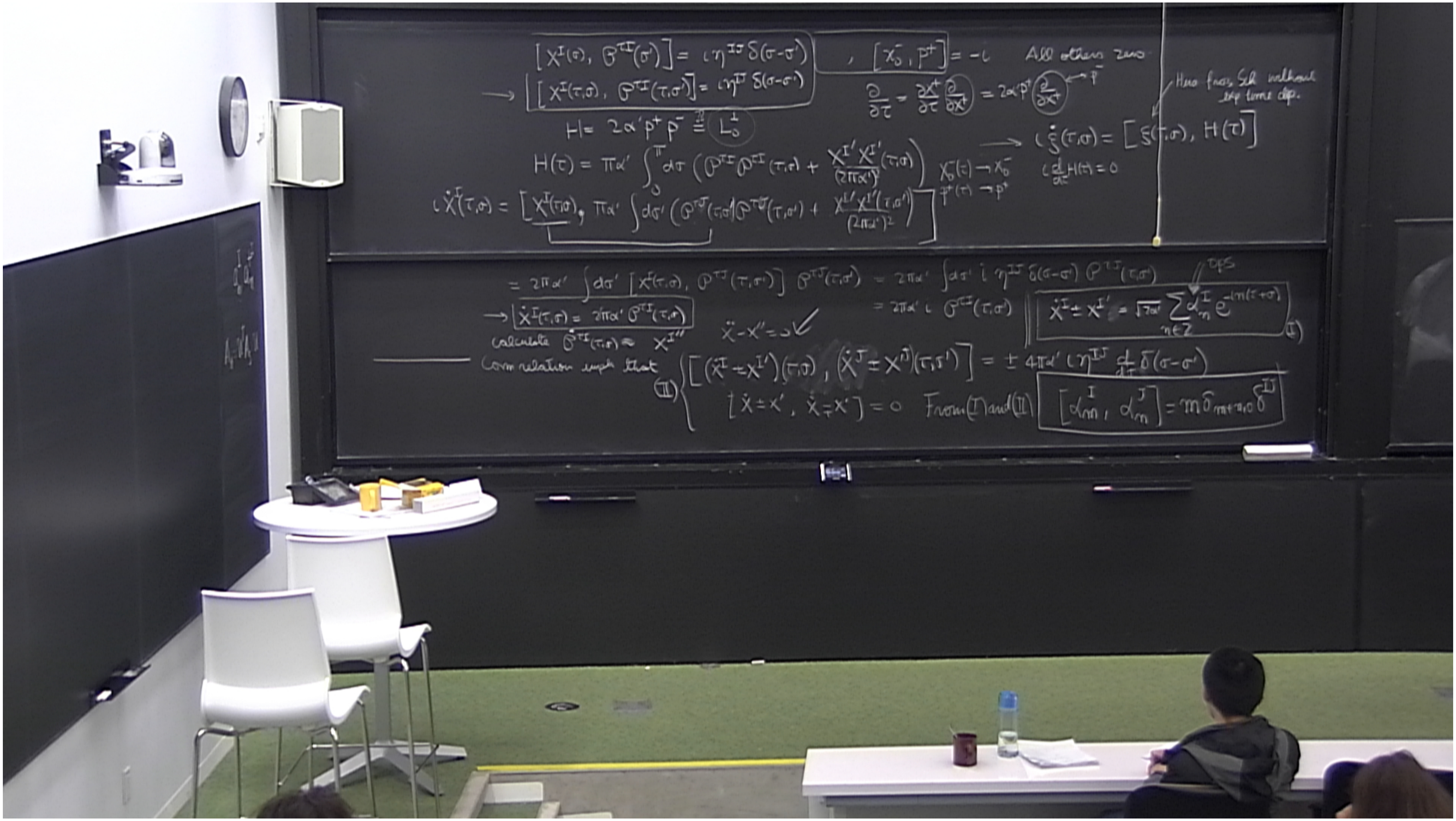

$H(\tau) = \pi\alpha' \int_0^\pi d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I'}X^{I'}}{(2\pi\alpha')^2}(\tau,\sigma) \right)$
$x_0^-(\tau) \to x_0^-$
$p^+(\tau) \to p^+$
$i\frac{d}{d\tau}H(\tau) = 0$
$i\dot{X}^I(\tau,\sigma) = \left[X^I(\tau,\sigma), \pi\alpha' \int d\sigma' \left(\mathcal{P}^{\tau J}(\tau,\sigma')\mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{J'}X^{J'}(\tau,\sigma')}{(2\pi\alpha')^2} \right) \right]$
$= 2\pi\alpha' \int d\sigma' \left[X^I(\tau,\sigma), \mathcal{P}^{\tau J}(\tau,\sigma') \right] \mathcal{P}^{\tau J}(\tau,\sigma') = 2\pi\alpha' \int d\sigma' \, i\eta^{IJ}\delta(\sigma-\sigma')\mathcal{P}^{\tau J}(\tau,\sigma')$
$= 2\pi\alpha' \, i \, \mathcal{P}^{\tau I}(\tau,\sigma)$
ops
$\dot{X}^I \pm X^{I'} = \sqrt{2\alpha'} \sum_{n\in\mathbb{Z}} \alpha_n^I e^{-in(\tau\pm\sigma)}$
$= 2\pi\alpha' \mathcal{P}^{\tau I}(\tau,\sigma)$
$\ddot{X} - X'' = 0$
$\dot{\mathcal{P}}^{\tau I}(\tau,\sigma) \approx X^{I''}$
implies that
$\left[(\dot{X}^I \pm X^{I'})(\tau,\sigma), (\dot{X}^J \pm X^{J'})(\tau,\sigma') \right] = \pm 4\pi\alpha' \, i\eta^{IJ} \frac{d}{d\sigma}\delta(\sigma-\sigma')$

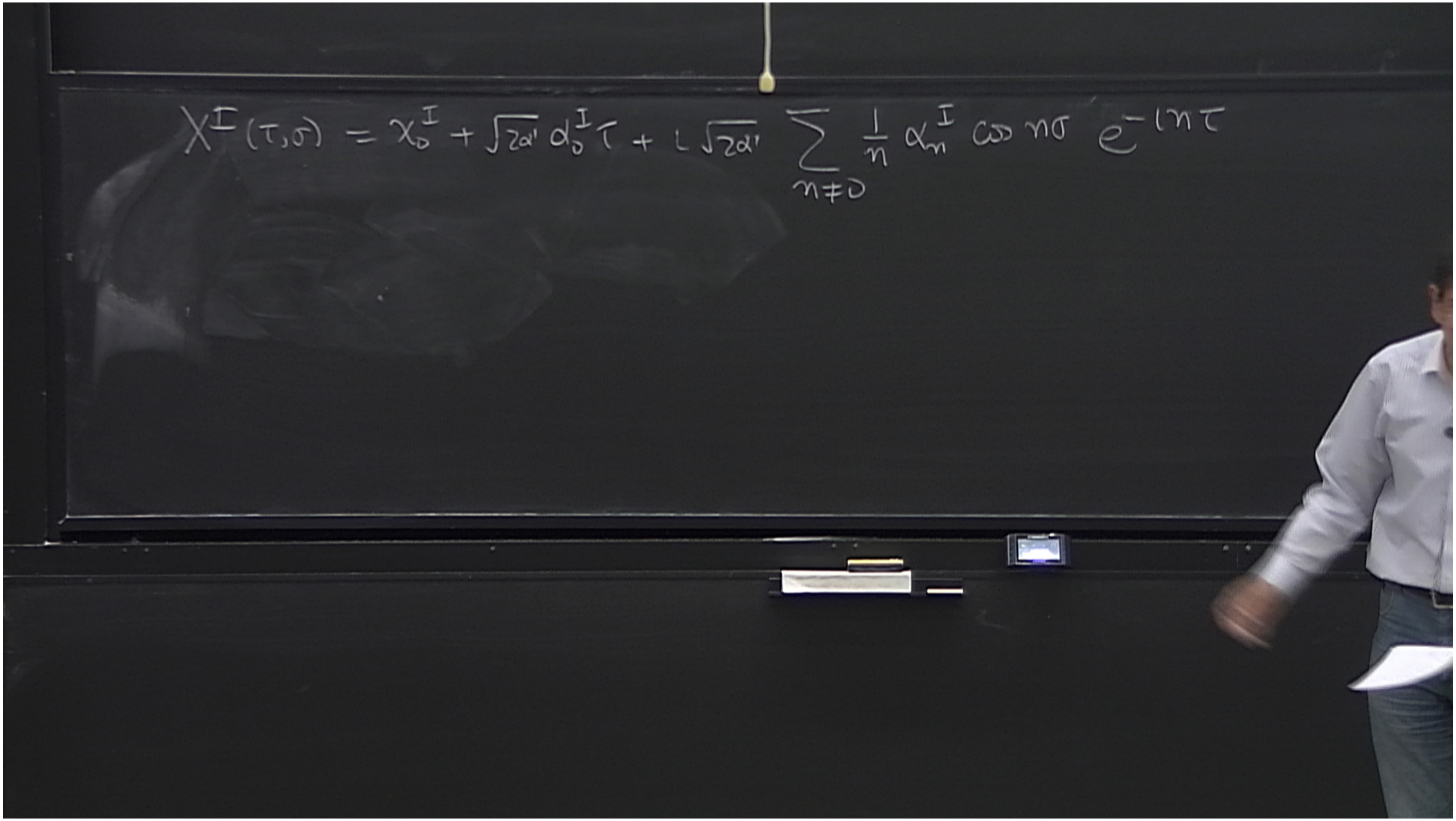

H(τ) = πα' ∫₀^π dσ (P^τI P^τI (τ,σ) + X^I' X^I' / (2πα')² (τ,σ))
X₀⁻(τ) → x₀⁻
p⁺(τ) → p⁺
i d/dτ H(τ) = 0
= 2πα' ∫dσ' [X^I(τ,σ), P^τJ(τ,σ')] P^τJ(τ,σ')
→ Ẋ^I(τ,σ) = 2πα' P^τI(τ,σ)
calculate
Ẍ − X'' = 0
ops
Ẋ^I ± X^I' = √(2α') Σ_{n∈Z} α_n^I e^{−in(τ+σ)}
Comm relation imply that



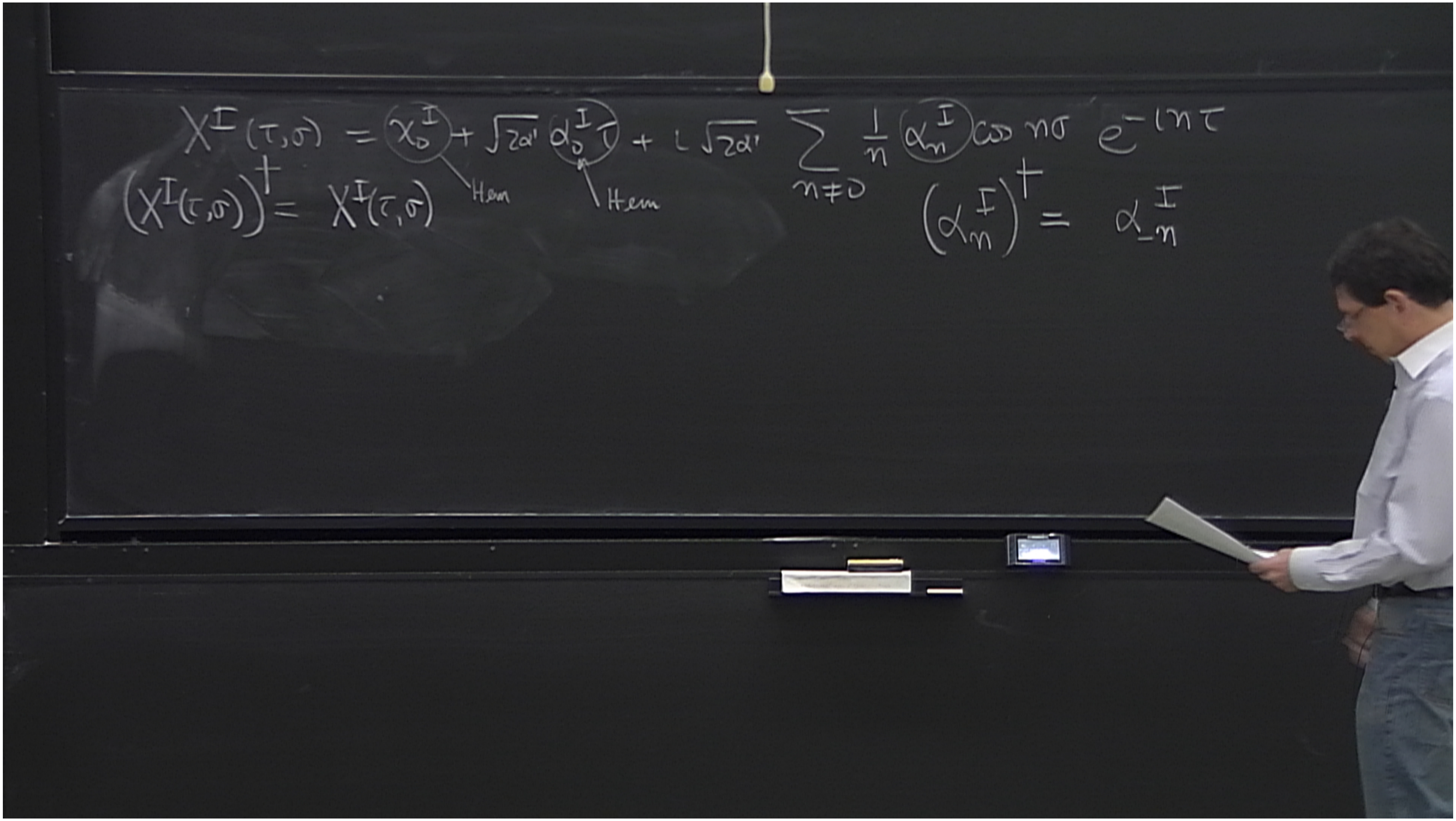
H(τ) = πα' ∫_0^π dσ (P^{τI} P^{τI}(τ,σ) + X^{I'} X^{I'}/(2πα')^2 (τ,σ))
X_0^-(τ) → x_0^-
p^+(τ) → p^+
i d/dτ H(τ) = 0
i Ẋ^I(τ,σ) = [X^I(τ,σ), πα' ∫dσ' (P^{τJ}(τ,σ') P^{τJ}(τ,σ') + X^{I'} X^{I'}(τ,σ')/(2πα')^2)]
= 2πα' ∫dσ' [X^I(τ,σ), P^{τJ}(τ,σ')] P^{τJ}(τ,σ') = 2πα' ∫dσ' i η^{IJ} δ(σ−σ') P^{τJ}(τ,σ')
= 2πα' i P^{τI}(τ,σ)
→ Ẋ^I(τ,σ) = 2πα' P^{τI}(τ,σ)
calculate Ṗ^{τI}(τ,σ) ≈ X^{I''}
Ẍ − X'' = 0 ✓
ops
Ẋ^I ± X^{I'} = √(2α') Σ_{n∈Z} α_n^I e^{−in(τ±σ)} (I)
Comm relation imply that
(II) [(Ẋ^I ± X^{I'})(τ,σ), (Ẋ^J ± X^{J'})(τ,σ')] = ± 4πα' i η^{IJ} d/dσ δ(σ−σ')
[Ẋ ± X', Ẋ ∓ X'] = 0
From (I) and (II)
[α_m^I, α_n^J] = m δ_{m+n,0} δ^{IJ}

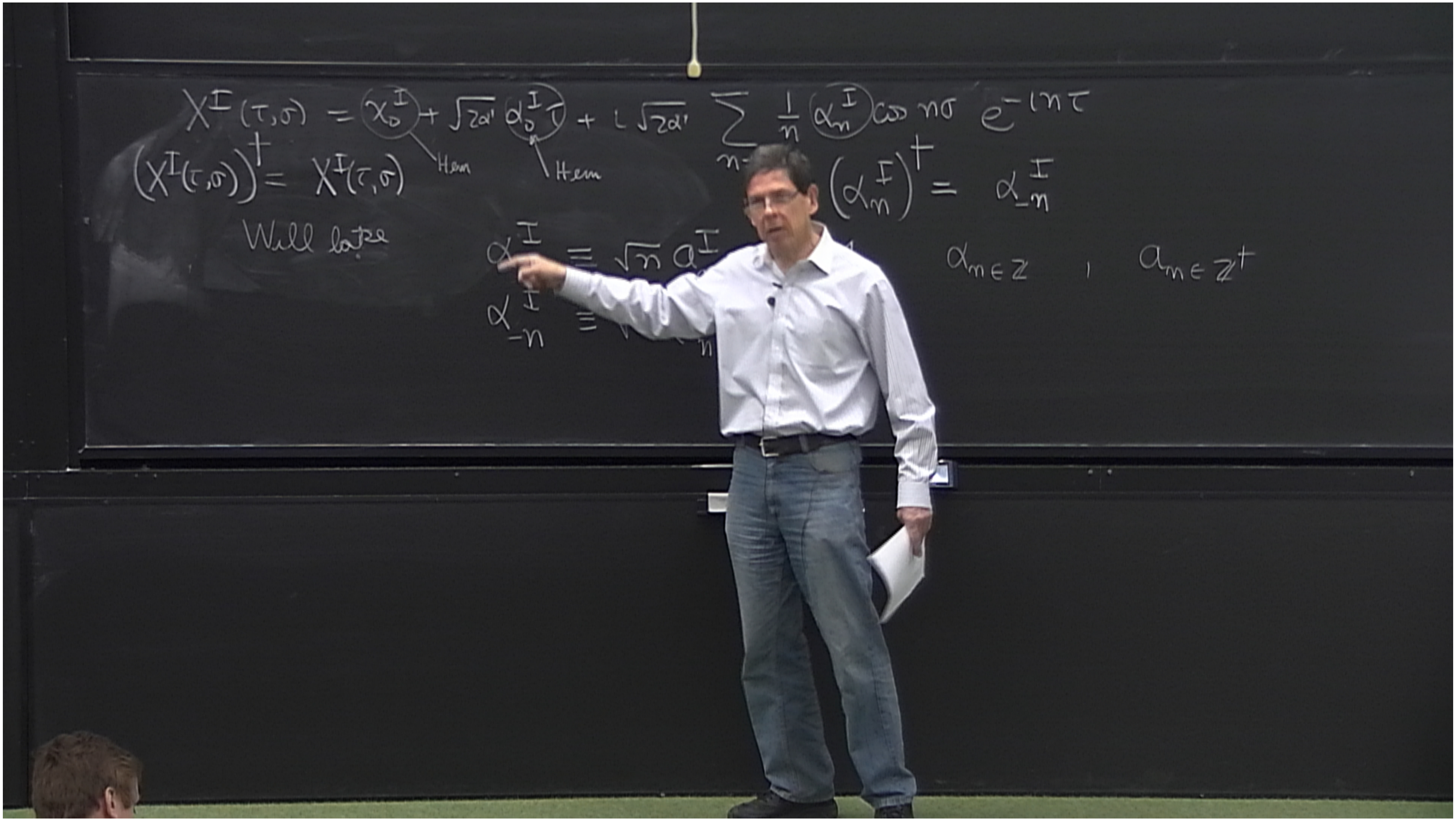

$a_{m'}^{I} a_{n}^{\dagger I}$
$A_H = U^\dagger A_S U$

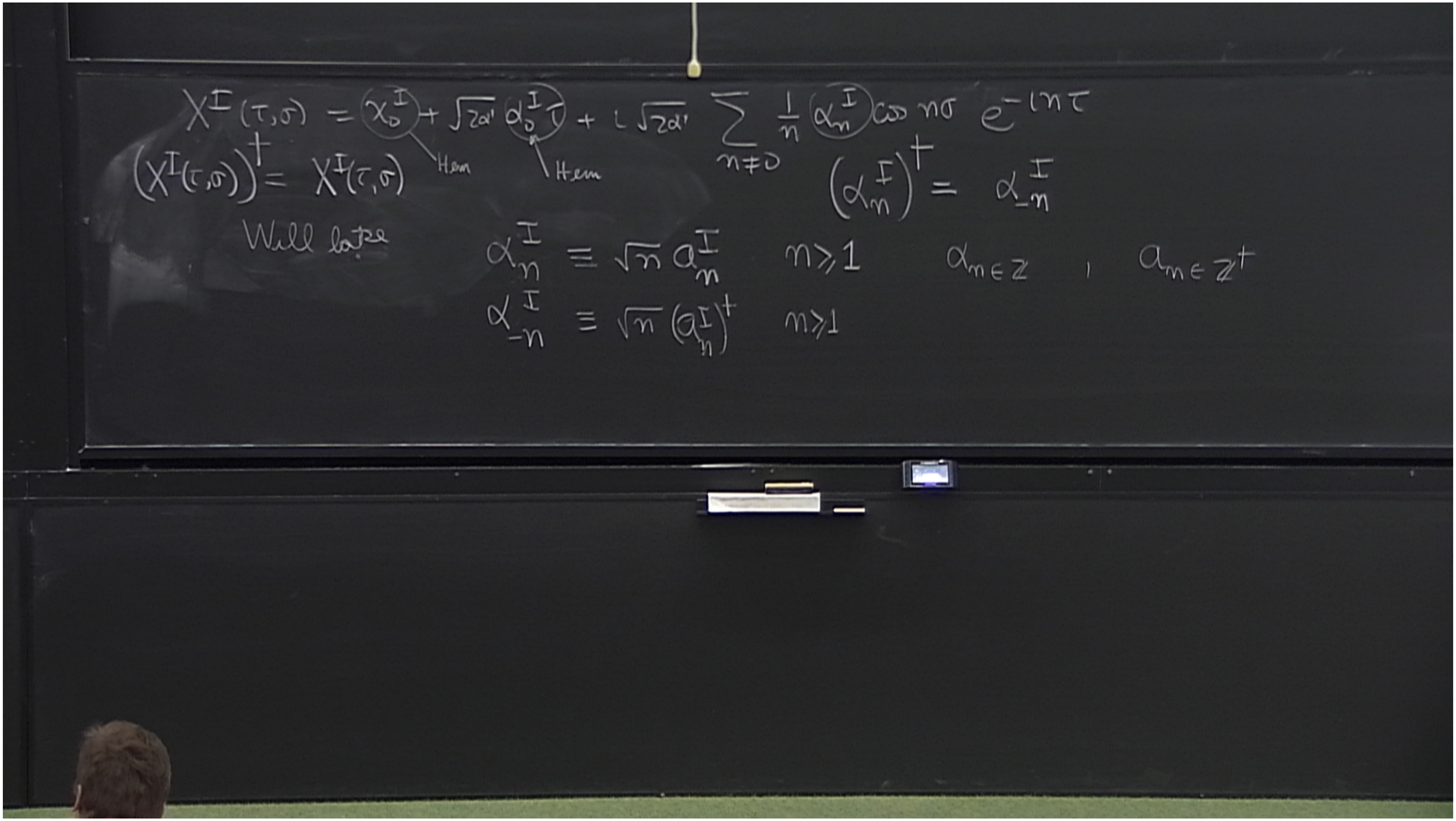

[X^I(σ), P^{τJ}(σ')] = iη^{IJ} δ(σ-σ')
[x_0^-, p^+] = -i
All others zero
[X^I(τ,σ), P^{τJ}(τ,σ')] = iη^{IJ} δ(σ-σ')
H = 2α'p^+p^- = L_0^⊥
H(τ) = πα' ∫_0^π dσ (P^{τI}P^{τI}(τ,σ) + X^{I'}X^{I'}(τ,σ)/(2πα')^2)
x_0^-(τ) → x_0^-
p^+(τ) → p^+
i dξ/dτ(τ,σ) = [ξ(τ,σ), H(τ)]
Heis. from Sch. without exp. time dep.
= 2πα' ∫dσ' [X^I(τ,σ), P^{τJ}(τ,σ')] P^{τJ}(τ,σ')
Ẋ^I(τ,σ) = 2πα' P^{τI}(τ,σ)
Ẍ - X'' = 0
Calculate
Ẋ^I ± X^{I'} = √(2α') Σ_{n∈Z} α_n^I e^{-in(τ±σ)}
Comm. relation imply that
[(Ẋ^I ± X^{I'})(τ,σ), (Ẋ^J ± X^{J'})(τ,σ')] = ± 4πα' iη^{IJ} d/dσ δ(σ-σ')
[Ẋ ± X', Ẋ ∓ X'] = 0
From (I) and (II)
[α_m^I, α_n^J] = m δ_{m+n,0} η^{IJ}


$X^I(\tau,\sigma) = x_0^I + \sqrt{2\alpha'}\,\alpha_0^I\,\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^I \cos n\sigma\, e^{-in\tau}$

$$X^I(\tau,\sigma) = x_0^I + \sqrt{2\alpha'}\,\alpha_0^I\,\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\,\alpha_n^I \cos n\sigma\, e^{-in\tau}$$


$X^I(\tau,\sigma) = x_0^I + \sqrt{2\alpha'}\,\alpha_0^I\,\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^I \cos n\sigma\, e^{-in\tau}$
Herm
Herm
$(X^I(\tau,\sigma))^\dagger = X^I(\tau,\sigma)$
$(\alpha_n^I)^\dagger = \alpha_{-n}^I$


Hem
Hem
Will later


$X^I(\tau,\sigma) = x_0^I + \sqrt{2\alpha'}\,\alpha_0^I\,\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^I \cos n\sigma\, e^{-in\tau}$
Herm
Herm
$(X^I(\tau,\sigma))^\dagger = X^I(\tau,\sigma)$
$(\alpha_n^I)^\dagger = \alpha_{-n}^I$
Will later
$\alpha_n^I \equiv \sqrt{n}\, a_n^I \quad n\geq 1 \qquad \alpha_{n\in\mathbb{Z}} \;,\; a_{n\in\mathbb{Z}^+}$
$\alpha_{-n}^I \equiv \sqrt{n}\,(a_n^I)^\dagger \quad n\geq 1$

$$H(\tau) = \pi\alpha' \int_0 d\sigma \left(\mathcal{P}^{\tau I}\mathcal{P}^{\tau I}(\tau,\sigma) + \frac{X^{I\prime} X^{I\prime}}{(2\pi\alpha')^2}(\tau,\sigma) \right)$$

$X_0^-(\tau) \to x_0^-$ $\quad i\frac{d}{d\tau}H(\tau) = 0$

$p^+(\tau) \to p^+$

$$\dot{X}^I(\tau,\sigma) = \left[X^I(\tau,\sigma),\ \pi\alpha' \int d\sigma' \left(\mathcal{P}^{\tau J}(\tau,\sigma')\mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{J\prime} X^{J\prime}(\tau,\sigma')}{(2\pi\alpha')^2} \right) \right]$$

$$= 2\pi\alpha' \int d\sigma' \left[X^I(\tau,\sigma),\ \mathcal{P}^{\tau J}(\tau,\sigma') \right] \mathcal{P}^{\tau J}(\tau,\sigma') = 2\pi\alpha' \int d\sigma'\, i\, \eta^{IJ} \delta(\sigma-\sigma') \mathcal{P}^{\tau J}(\tau,\sigma')$$

$$= 2\pi\alpha'\, i\, \mathcal{P}^{\tau I}(\tau,\sigma)$$

$\longrightarrow$ $\boxed{\dot{X}^I(\tau,\sigma) = 2\pi\alpha' \mathcal{P}^{\tau I}(\tau,\sigma)}$

calculate $\dot{\mathcal{P}}^{\tau I}(\tau,\sigma) \approx X^{I\prime\prime}$ $\qquad \ddot{X} - X'' = 0$ ✓

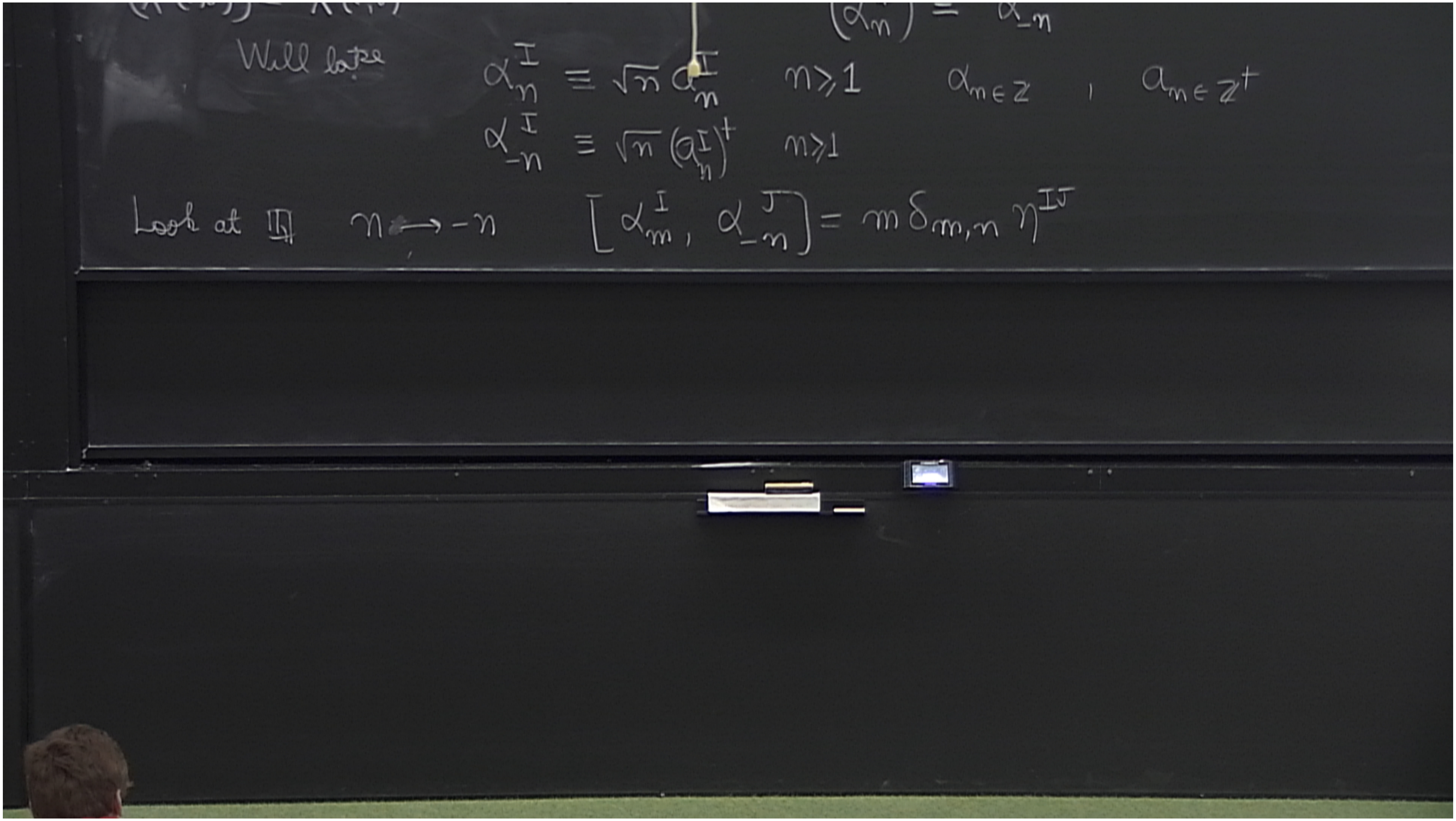
ops

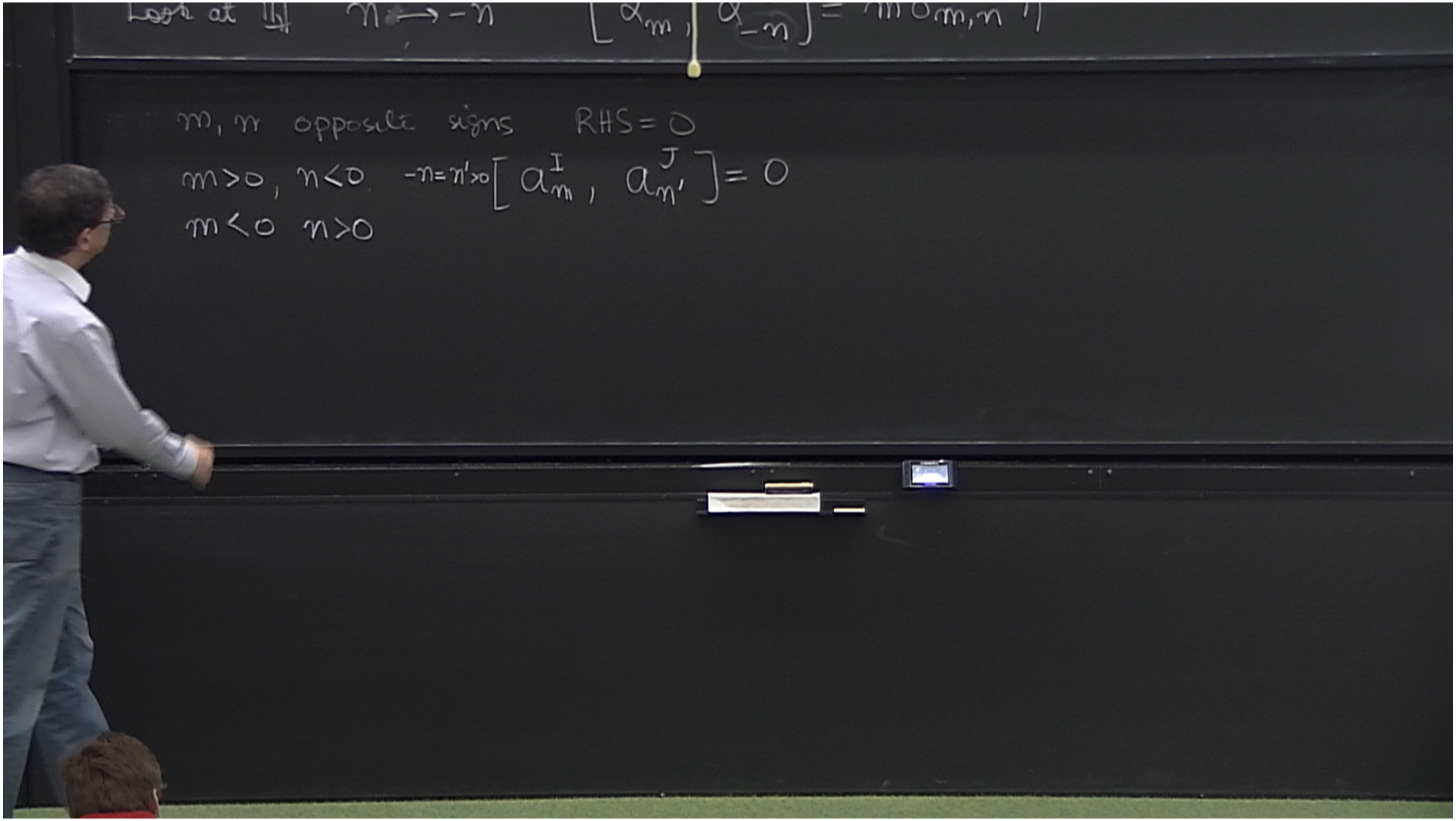
$$\boxed{\dot{X}^I \pm X^{I\prime} = \sqrt{2\alpha'} \sum_{n\in\mathbb{Z}} \alpha_n^I e^{-in(\tau\pm\sigma)}} \quad (\mathrm{I})$$

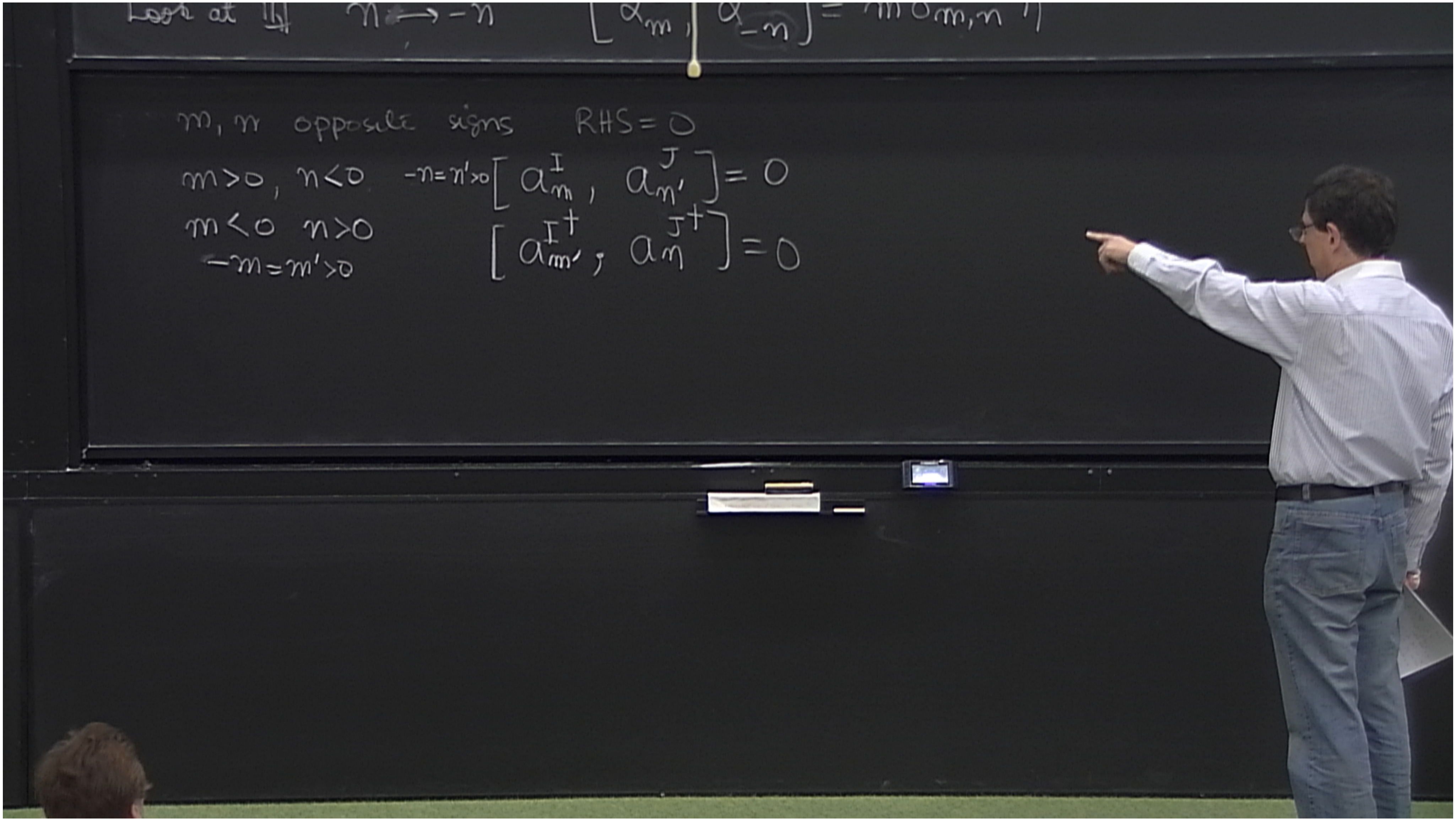
Comm relation imply that

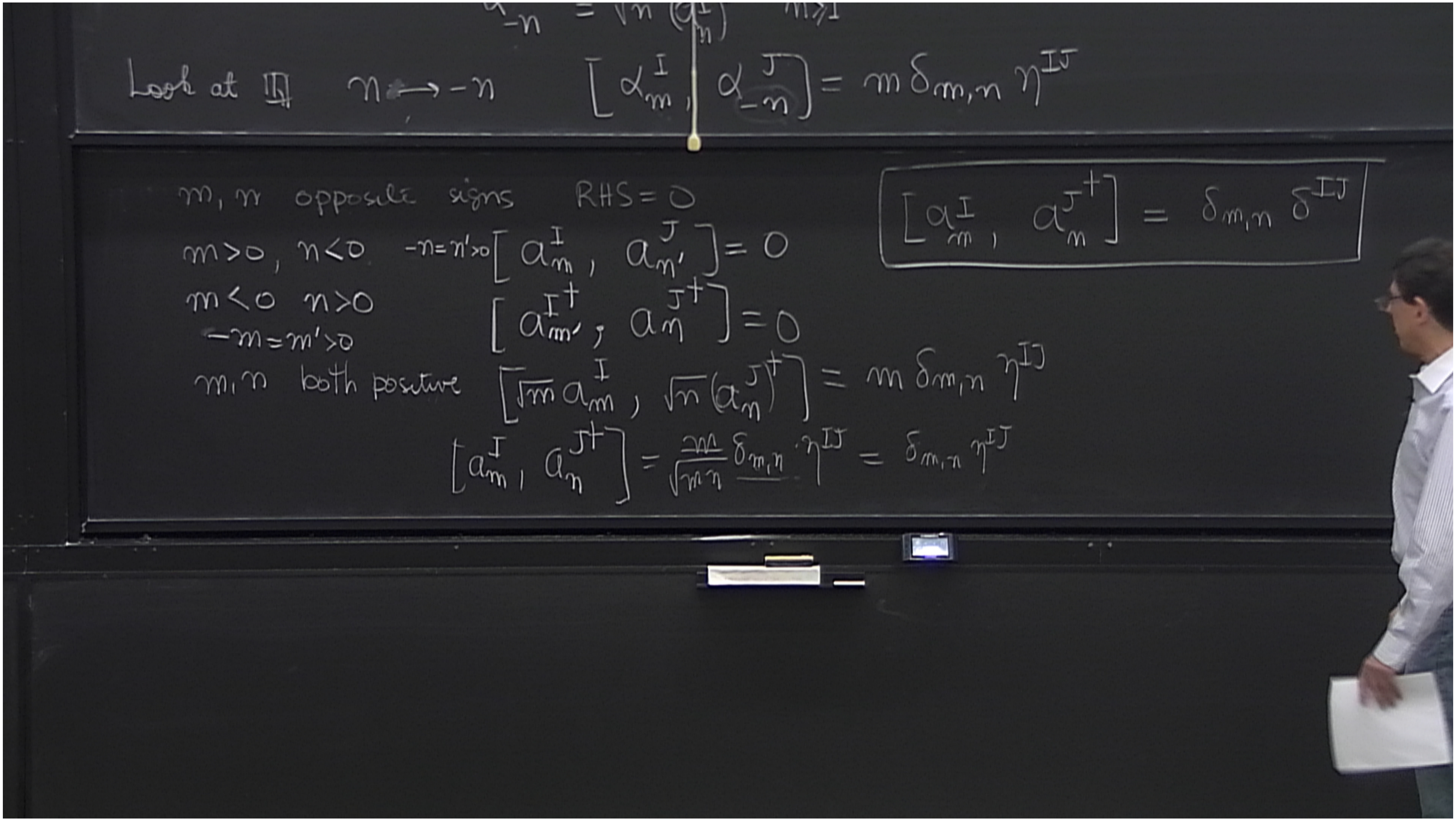
(II) $\begin{cases} \left[(\dot{X}^I \pm X^{I\prime})(\tau,\sigma),\ (\dot{X}^J \pm X^{J\prime})(\tau,\sigma') \right] = \pm 4\pi\alpha'\, i\, \eta^{IJ} \frac{d}{d\sigma}\delta(\sigma-\sigma') \quad (\mathrm{III}) \\ \left[\dot{X} \pm X',\ \dot{X} \mp X' \right] = 0 \end{cases}$

From (I) and (II) $\quad \boxed{\left[\alpha_m^I,\ \alpha_n^J \right] = m\,\delta_{m+n,0}\,\delta^{IJ}}$


Will later
$\alpha^I_n \equiv \sqrt{n}\, a^I_n \quad n \geq 1 \qquad \alpha_{n \in \mathbb{Z}} \,,\; a_{n \in \mathbb{Z}^+}$
$\alpha^I_{-n} \equiv \sqrt{n}\, (a^I_n)^\dagger \quad n \geq 1$
Look at $n \longrightarrow -n$
$[\alpha^I_m, \alpha^J_{-n}] = m\, \delta_{m,n}\, \eta^{IJ}$


m, n opposite signs RHS = 0
m>0, n<0 -n=n'>0 [a^I_m , a^J_n'] = 0
m<0 n>0


n → −n
m, n opposite signs RHS = 0
m>0, n<0 −n=n'>0 $[a^I_m, a^J_{n'}] = 0$
m<0 n>0 $[a^{I\dagger}_{m'}, a^{J\dagger}_n] = 0$
−m=m'>0


Look at III   n ⟶ −n   [α^I_m, α^J_{−n}] = m δ_{m,n} η^{IJ}
m, n opposite signs   RHS = 0
[a^I_m, a^{J†}_n] = δ_{m,n} δ^{IJ}
m > 0, n < 0   −n = n' > 0   [a^I_m, a^J_{n'}] = 0
m < 0 n > 0   [a^{I†}_{m'}, a^{J†}_n] = 0
−m = m' > 0
m, n both positive   [√m a^I_m, √n (a^J_n)^†] = m δ_{m,n} η^{IJ}
[a^I_m, a^{J†}_n] = m/√(mn) δ_{m,n} η^{IJ} = δ_{m,n} η^{IJ}

$\alpha_{-n} = \sqrt{n}\,(a^I_n)^{\dagger} \quad n \geq 1$

Look at III $\quad n \to -n \quad [\alpha^I_m, \alpha^J_{-n}] = m\,\delta_{m,n}\,\eta^{IJ}$

m, n opposite signs RHS $= 0$

$m > 0,\ n < 0 \quad -n = n' > 0 \quad [a^I_m, a^J_{n'}] = 0$

$m < 0 \quad n > 0 \quad [a^{I\dagger}_{m'}, a^{J\dagger}_n] = 0$

$-m = m' > 0$

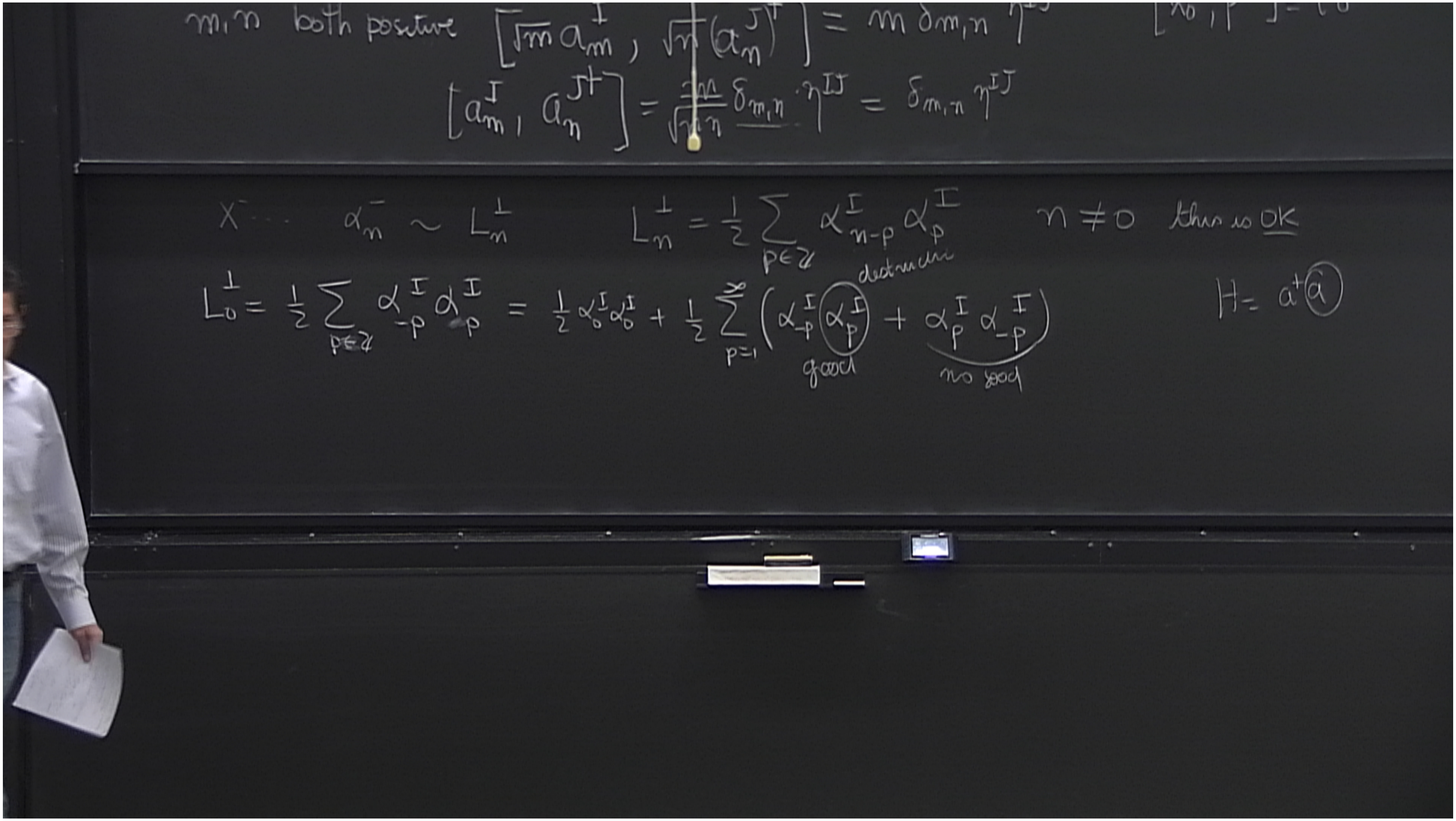
m, n both positive $\quad [\sqrt{m}\,a^I_m, \sqrt{n}\,(a^J_n)^{\dagger}] = m\,\delta_{m,n}\,\eta^{IJ}$

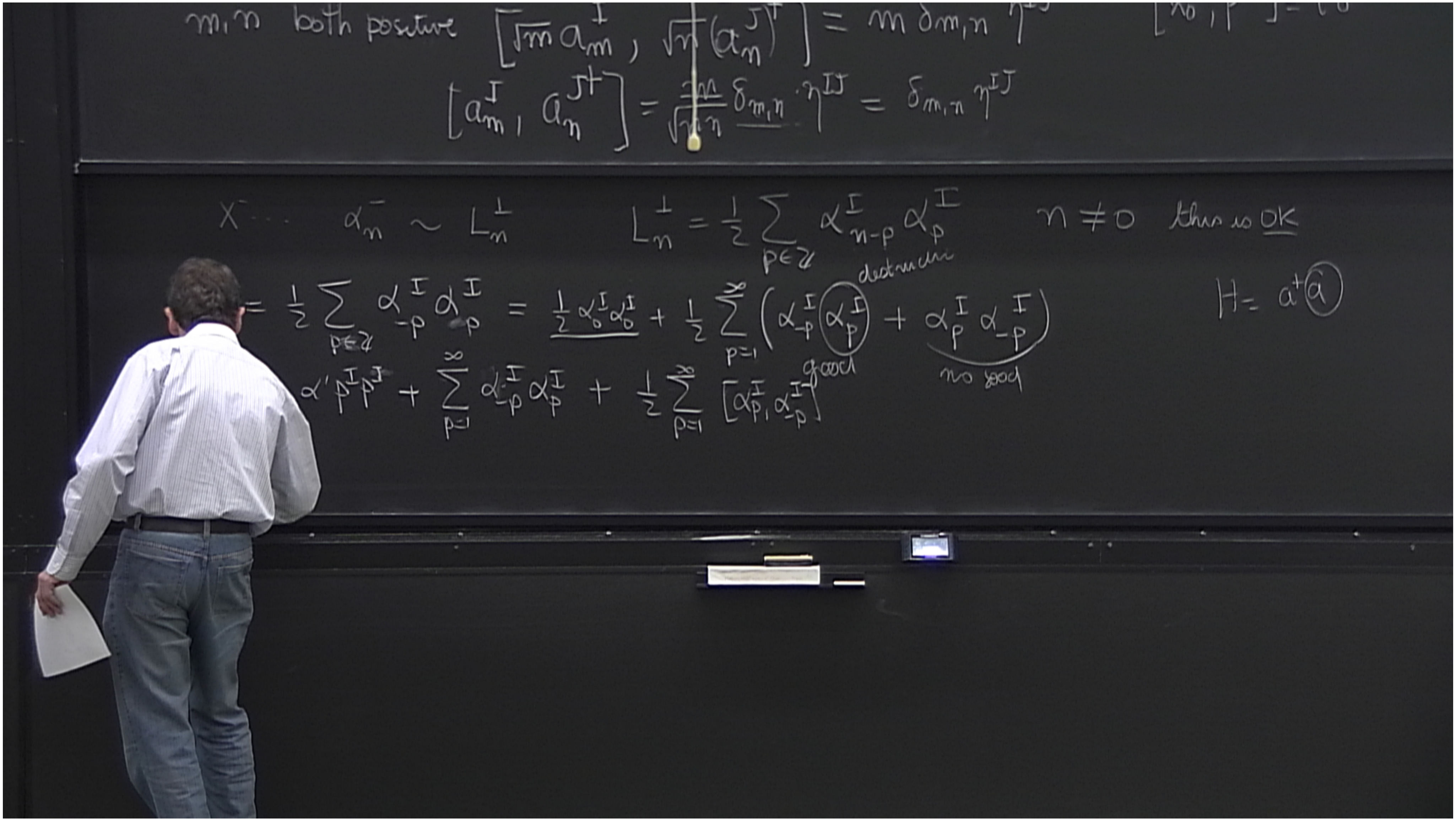
$$[a^I_m, a^{J\dagger}_n] = \frac{m}{\sqrt{mn}}\,\delta_{m,n}\,\eta^{IJ} = \delta_{m,n}\,\eta^{IJ}$$

$$[a^I_m, a^{J\dagger}_n] = \delta_{m,n}\,\delta^{IJ}$$

$\alpha^I_0 = \sqrt{2\alpha'}\,p^I$ ✓

$[x^I_0, p^J] = i\,\delta^{IJ}$


m, n both positive $[\sqrt{m}\, a_m^I, \sqrt{n}(a_n^J)^\dagger] = m\, \delta_{m,n}\, \eta^{IJ}$
$[a_m^I, a_n^{J\dagger}] = \frac{m}{\sqrt{m n}} \delta_{m,n} \eta^{IJ} = \delta_{m,n} \eta^{IJ}$
$X^- \ldots \quad \alpha_n^- \sim L_n^\perp \quad L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{n-p}^I \alpha_p^I \quad n \neq 0$ this is OK
$L_0^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{-p}^I \alpha_p^I = \frac{1}{2}\alpha_0^I\alpha_0^I + \frac{1}{2}\sum_{p=1}^{\infty}(\alpha_{-p}^I \alpha_p^I + \alpha_p^I \alpha_{-p}^I)$
destruction
good
no good
$H = a^\dagger a$


m, n both positive
this is OK
good
no good

m, n both positive $\left[\sqrt{m}\, a_m^I, \sqrt{n}\,(a_n^J)^\dagger\right] = m\,\delta_{m,n}\,\eta^{IJ}$

$$\left[a_m^I, a_n^{J\dagger}\right] = \frac{m}{\sqrt{m n}}\,\delta_{m,n}\,\eta^{IJ} = \delta_{m,n}\,\eta^{IJ}$$

$X^- \dots \quad \alpha_n^- \sim L_n^\perp \qquad L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{n-p}^I \alpha_p^I \qquad n \neq 0$ this is OK

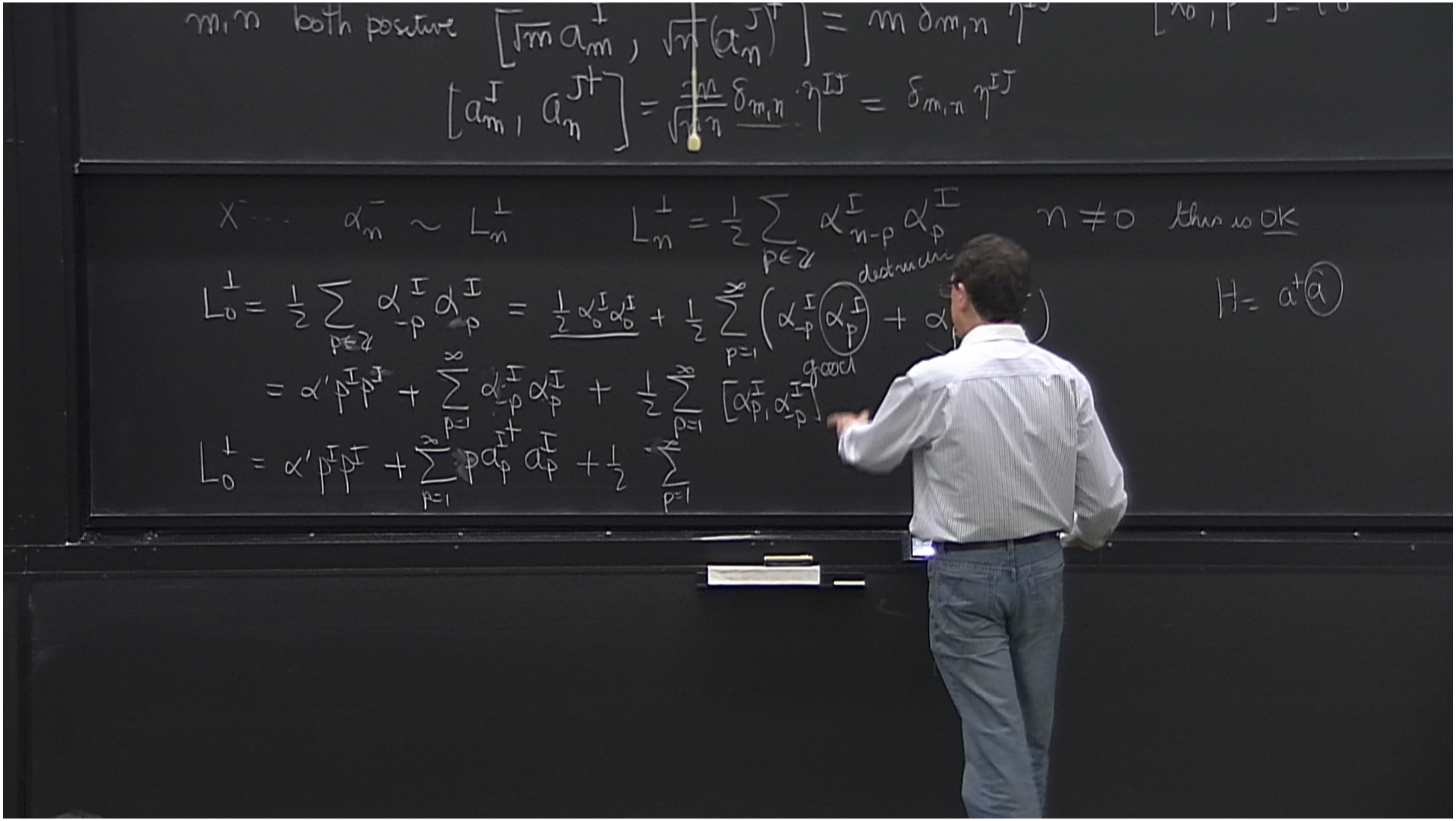
$$L_0^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{-p}^I \alpha_p^I = \frac{1}{2}\alpha_0^I\alpha_0^I + \frac{1}{2}\sum_{p=1}^{\infty}\left(\alpha_{-p}^I \alpha_p^I + \alpha_p^I \alpha_{-p}^I\right)$$

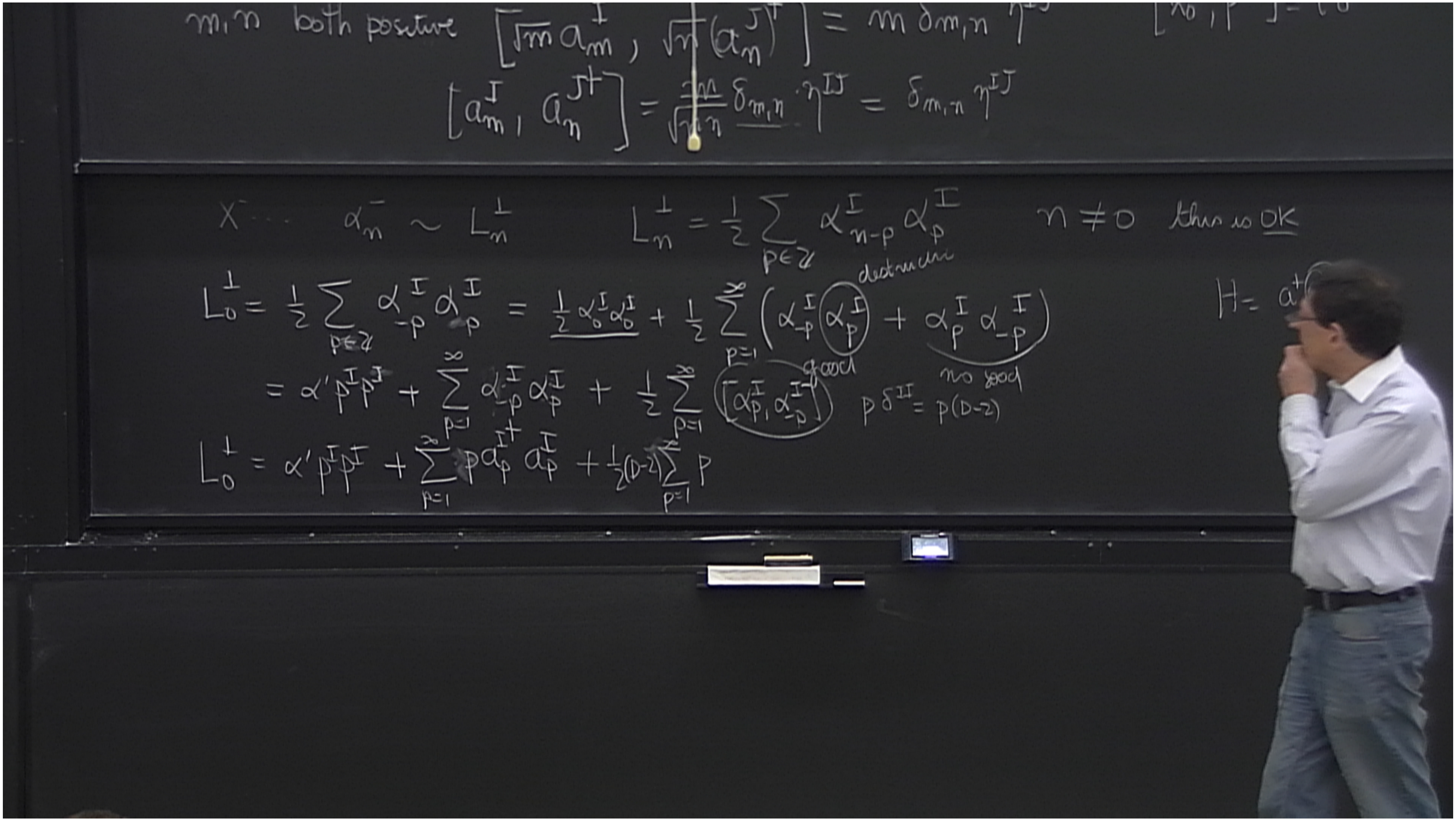
destruction; good; no good

$H = a^\dagger a$

$$= \alpha' p^I p^I + \sum_{p=1}^{\infty} \alpha_{-p}^I \alpha_p^I + \frac{1}{2}\sum_{p=1}^{\infty}\left[\alpha_p^I, \alpha_{-p}^I\right]$$

$$L_0^\perp = \alpha' p^I p^I +$$


m, n both positive [√m a_m^I , √n (a_n^J)†] = m δ_m,n η^IJ
[a_m^I , a_n^J†] = m/√(mn) δ_m,n η^IJ = δ_m,n η^IJ
X^- ... α_n^- ~ L_n^⊥
L_n^⊥ = 1/2 Σ_{p∈Z} α^I_{n-p} α^I_p
n ≠ 0 this is OK
destruction
H = a† a
L_0^⊥ = 1/2 Σ_{p∈Z} α^I_{-p} α^I_p = 1/2 α_0^I α_0^I + 1/2 Σ_{p=1}^∞ (α^I_{-p} α^I_p + α
good
= α' p^I p^I + Σ_{p=1}^∞ α^I_{-p} α^I_p + 1/2 Σ_{p=1}^∞ [α^I_p , α^I_{-p}]
L_0^⊥ = α' p^I p^I + Σ_{p=1}^∞ p a_p^I† a_p^I + 1/2 Σ_{p=1}


m, n both positive
this is OK
destructive
good
no good
H = a†

m, n both positive $\left[\sqrt{m}\, a_m^I , \sqrt{n}\, (a_n^J)^\dagger\right] = m\, \delta_{m,n}\, \eta^{IJ}$

$$\left[a_m^I , a_n^{J\dagger}\right] = \frac{m}{\sqrt{m}\sqrt{n}}\, \delta_{m,n}\, \eta^{IJ} = \delta_{m,n}\, \eta^{IJ}$$

$X^- \ldots \quad \alpha_n^- \sim L_n^\perp \qquad L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{n-p}^I \alpha_p^I \qquad n \neq 0$ this is OK

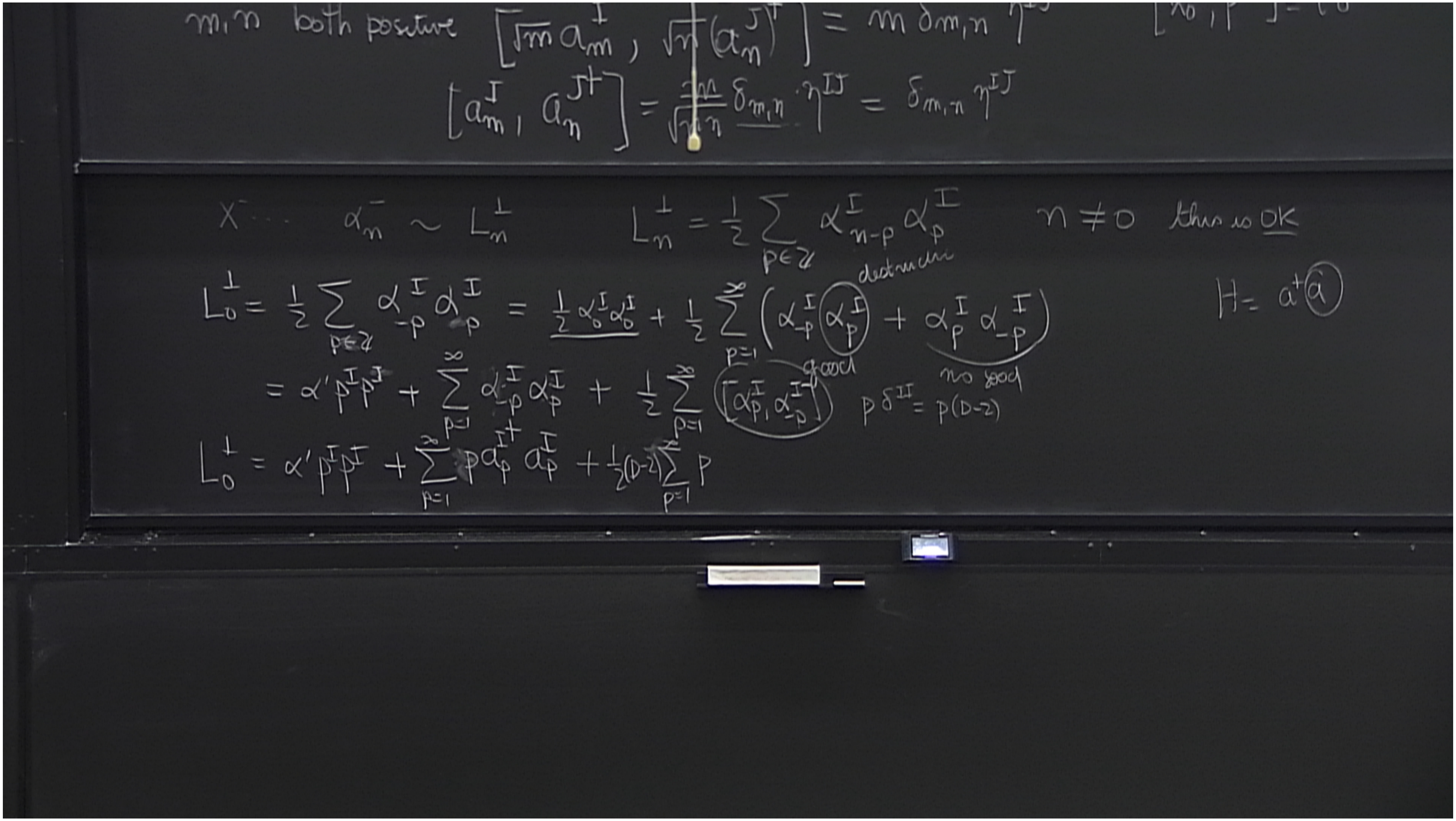
$$L_0^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{-p}^I \alpha_p^I = \frac{1}{2}\alpha_0^I\alpha_0^I + \frac{1}{2}\sum_{p=1}^{\infty}\left(\alpha_{-p}^I \alpha_p^I + \alpha_p^I \alpha_{-p}^I\right)$$

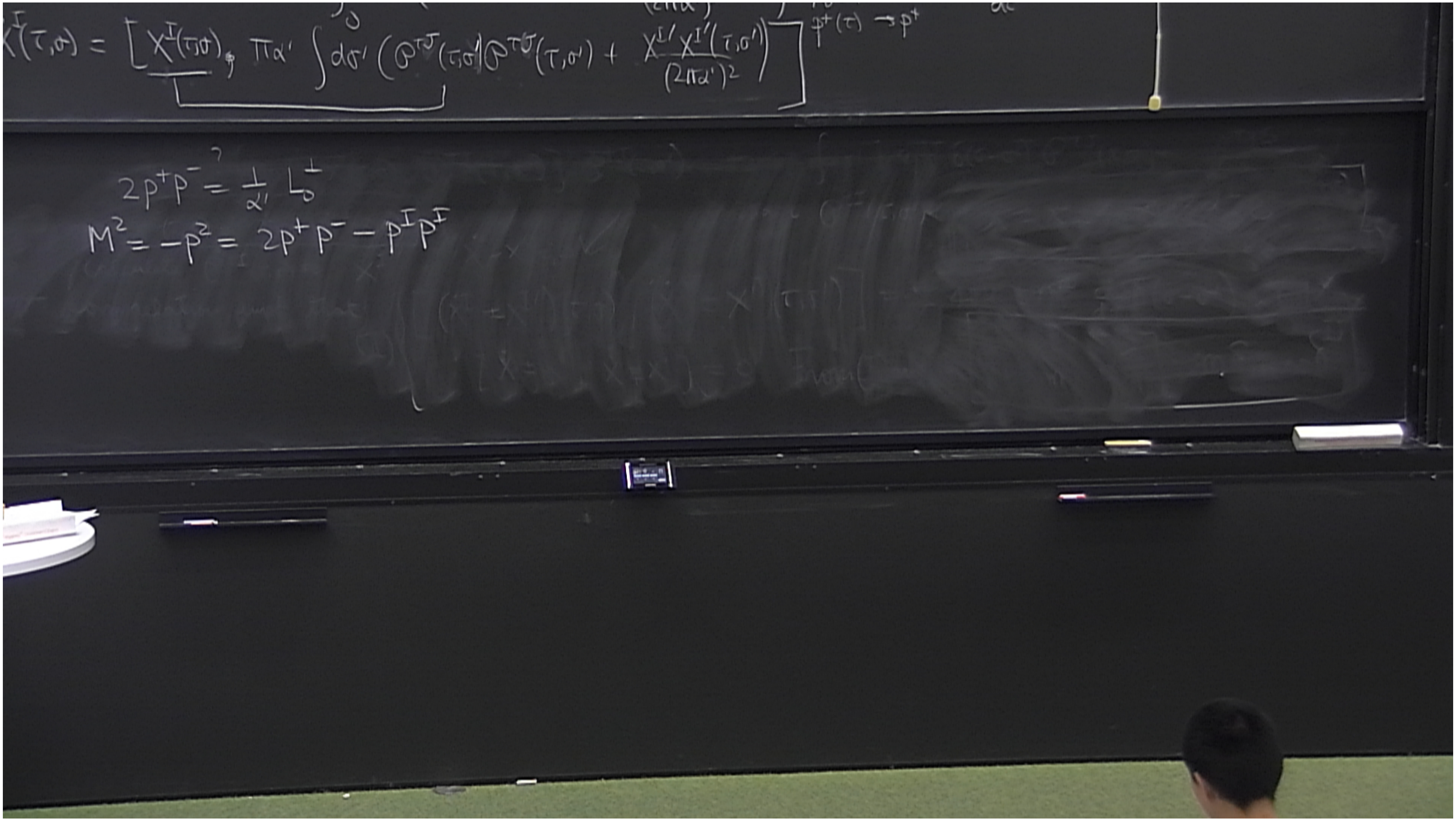
destruction; good; no good

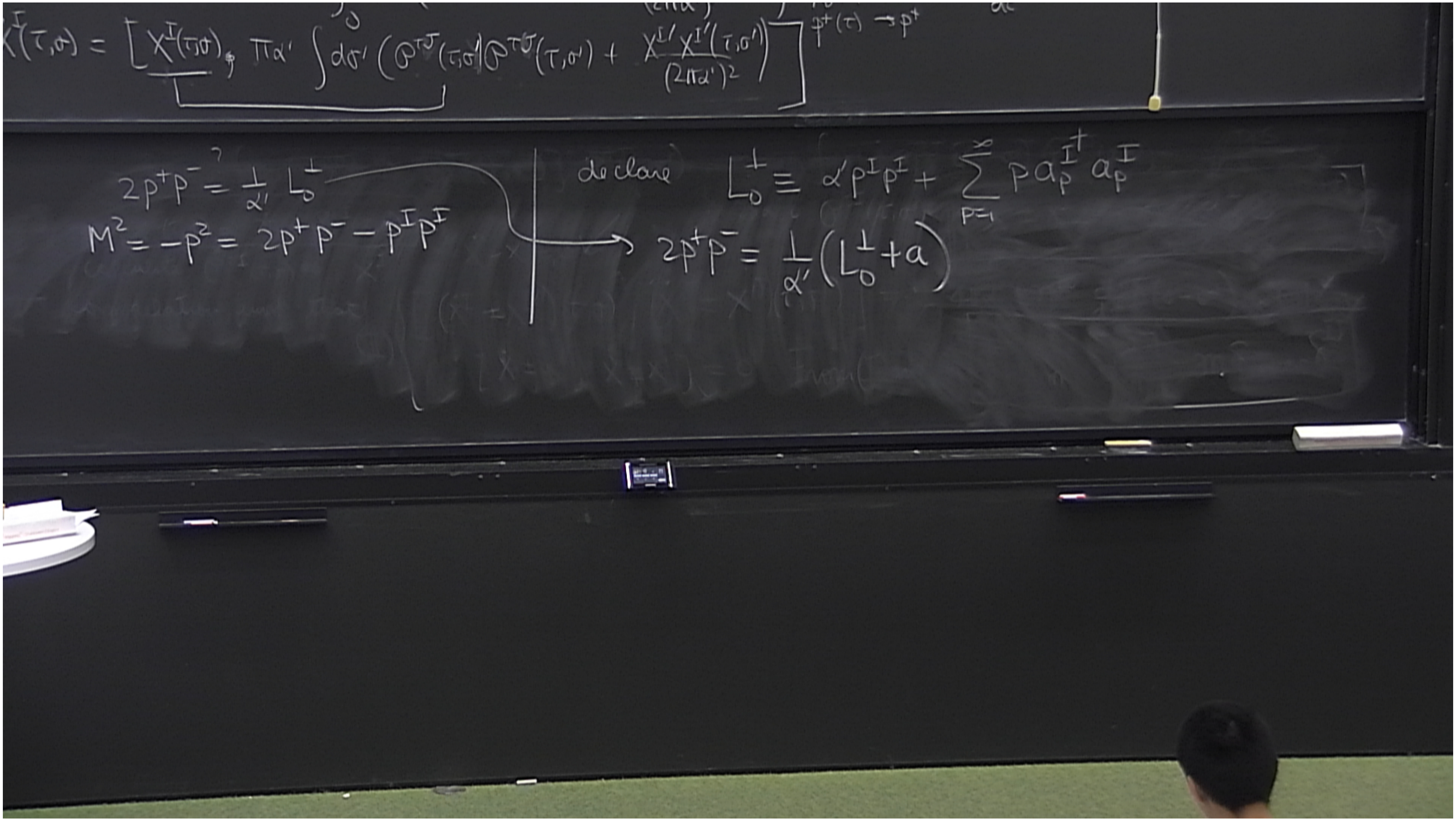
$$= \alpha' p^I p^I + \sum_{p=1}^{\infty} \alpha_{-p}^I \alpha_p^I + \frac{1}{2}\sum_{p=1}^{\infty} \left[\alpha_p^I , \alpha_{-p}^I\right] \qquad p\,\delta^{II} = p(D-2)$$

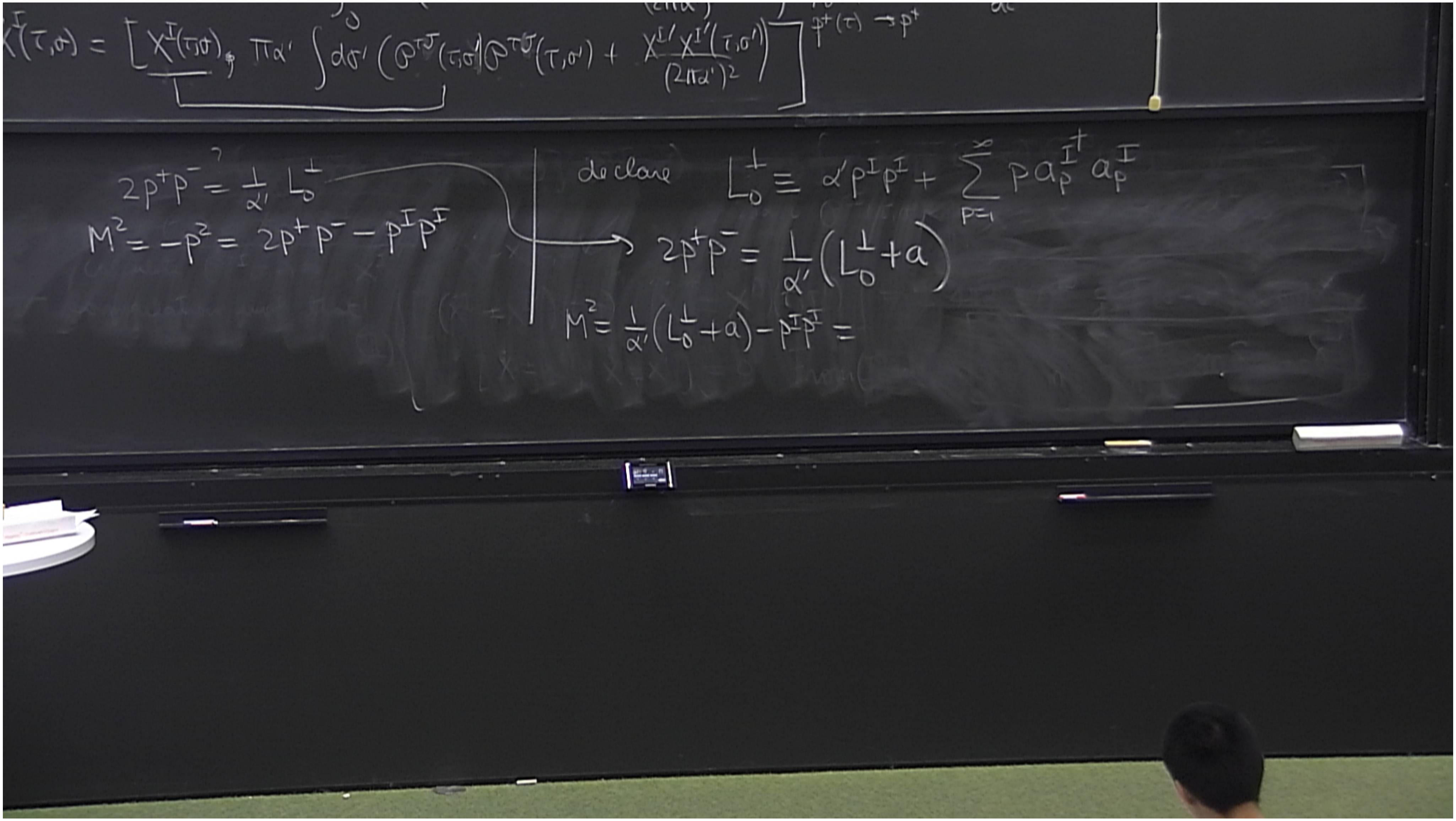
$$L_0^\perp = \alpha' p^I p^I + \sum_{p=1}^{\infty} p\, a_p^{I\dagger} a_p^I + \frac{1}{2}(D-2)\sum_{p=1}^{\infty} p$$

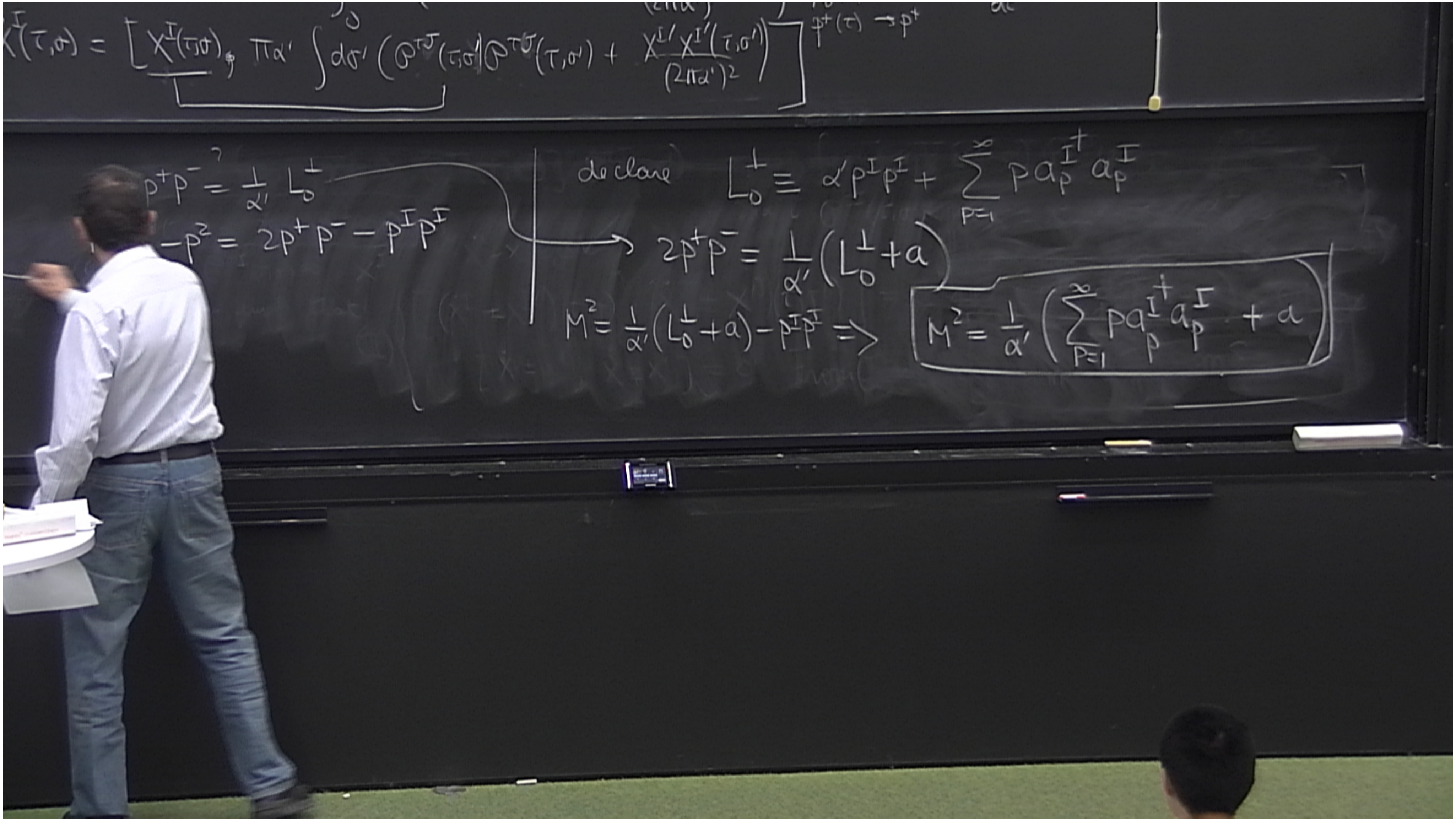
$H = a^\dagger a$

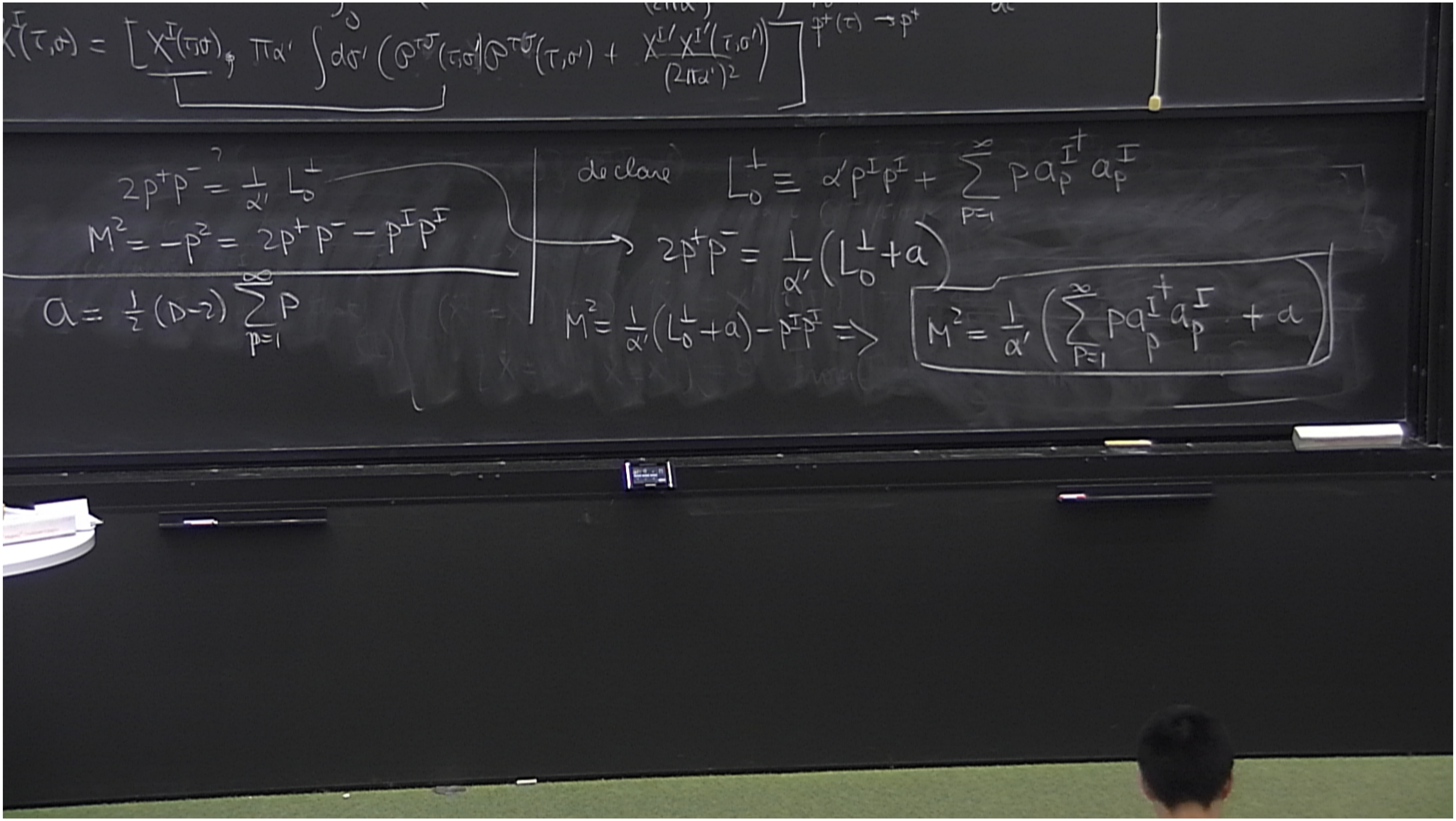

m, n both positive $[\sqrt{m}\, a_m^I, \sqrt{n}\, (a_n^J)^\dagger] = m\, \delta_{m,n}\, \eta^{IJ}$
$[a_m^I, a_n^{J\dagger}] = \frac{m}{\sqrt{mn}} \delta_{m,n} \eta^{IJ} = \delta_{m,n} \eta^{IJ}$
$X^- \cdots \quad \alpha_n^- \sim L_n^\perp \qquad L_n^\perp = \frac{1}{2} \sum_{p\in\mathbb{Z}} \alpha^I_{n-p} \alpha^I_p$
$n \neq 0$ this is OK
destruction
$L_0^\perp = \frac{1}{2} \sum_{p\in\mathbb{Z}} \alpha^I_{-p} \alpha^I_p = \frac{1}{2} \alpha_0^I \alpha_0^I + \frac{1}{2} \sum_{p=1}^\infty \left(\alpha^I_{-p} \alpha^I_p + \alpha^I_p \alpha^I_{-p} \right)$
$H = a^\dagger a$
good
no good
$= \alpha' p^I p^I + \sum_{p=1}^\infty \alpha^I_{-p} \alpha^I_p + \frac{1}{2} \sum_{p=1}^\infty [\alpha^I_p, \alpha^I_{-p}]$
$p\, \delta^{II} = p(D-2)$
$L_0^\perp = \alpha' p^I p^I + \sum_{p=1}^\infty p\, a_p^{I\dagger} a_p^I + \frac{1}{2}(D-2) \sum_{p=1}^\infty p$


X^I(τ,σ) = [X^I(τ,σ), πα' ∫dσ' (P^τJ(τ,σ)P^τJ(τ,σ') + X^I' X^I'(τ,σ') / (2πα')^2)]
p^+(τ) → p^+
2p^+p^- =? 1/α' L_0^⊥
M^2 = −p^2 = 2p^+p^- − p^I p^I

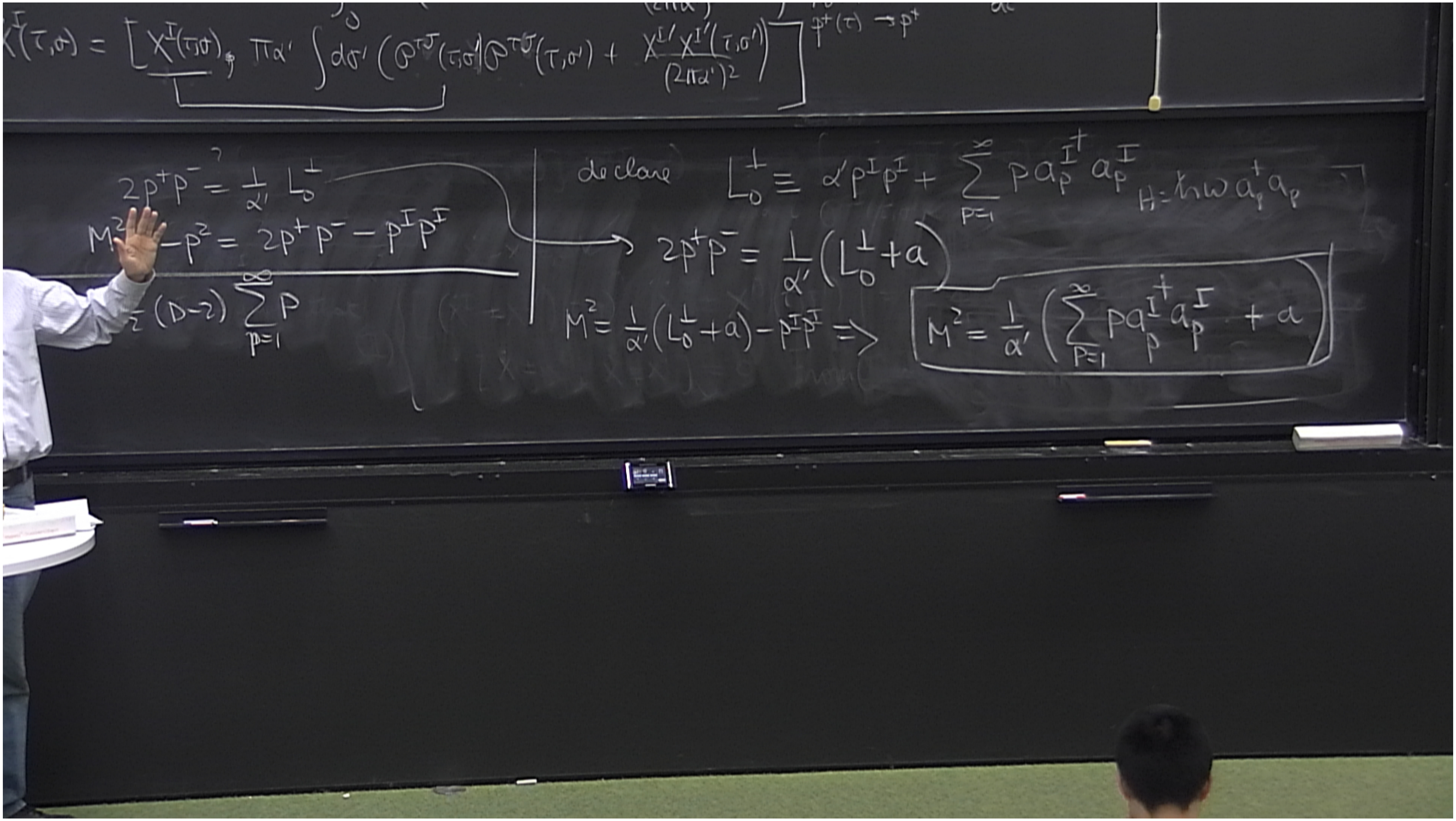

$\dot{X}^I(\tau,\sigma) = \left[X^I(\tau,\sigma), \pi\alpha' \int d\sigma' \left(\mathcal{P}^{\tau J}(\tau,\sigma')\mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{J\prime} X^{J\prime}(\tau,\sigma')}{(2\pi\alpha')^2} \right) \right]$
$p^+(\tau) \to p^+$
$2p^+p^- \stackrel{?}{=} \frac{1}{\alpha'} L_0^\perp$
$M^2 = -p^2 = 2p^+p^- - p^Ip^I$
declare $L_0^\perp \equiv \alpha' p^I p^I + \sum_{p=1}^{\infty} p\, a_p^{I\dagger} a_p^I$
$2p^+p^- = \frac{1}{\alpha'}\left(L_0^\perp + a\right)$

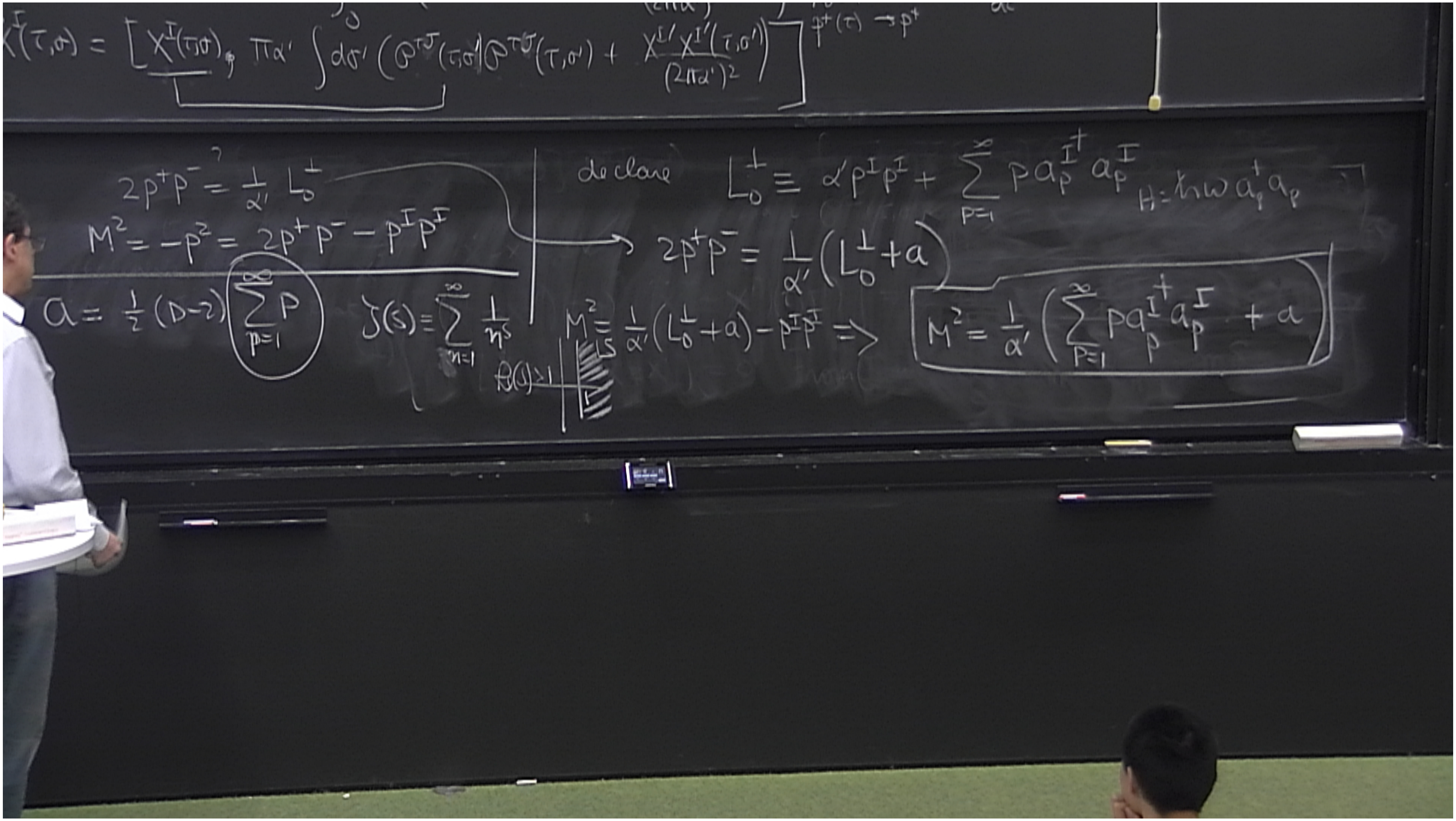

$2p^+p^- = \frac{1}{\alpha'} L_0^\perp$
$M^2 = -p^2 = 2p^+p^- - p^I p^I$
declare $L_0^\perp \equiv \alpha' p^I p^I + \sum_{p=1}^\infty p\, a_p^{I\dagger} a_p^I$
$2p^+p^- = \frac{1}{\alpha'}(L_0^\perp + a)$
$M^2 = \frac{1}{\alpha'}(L_0^\perp + a) - p^I p^I =$

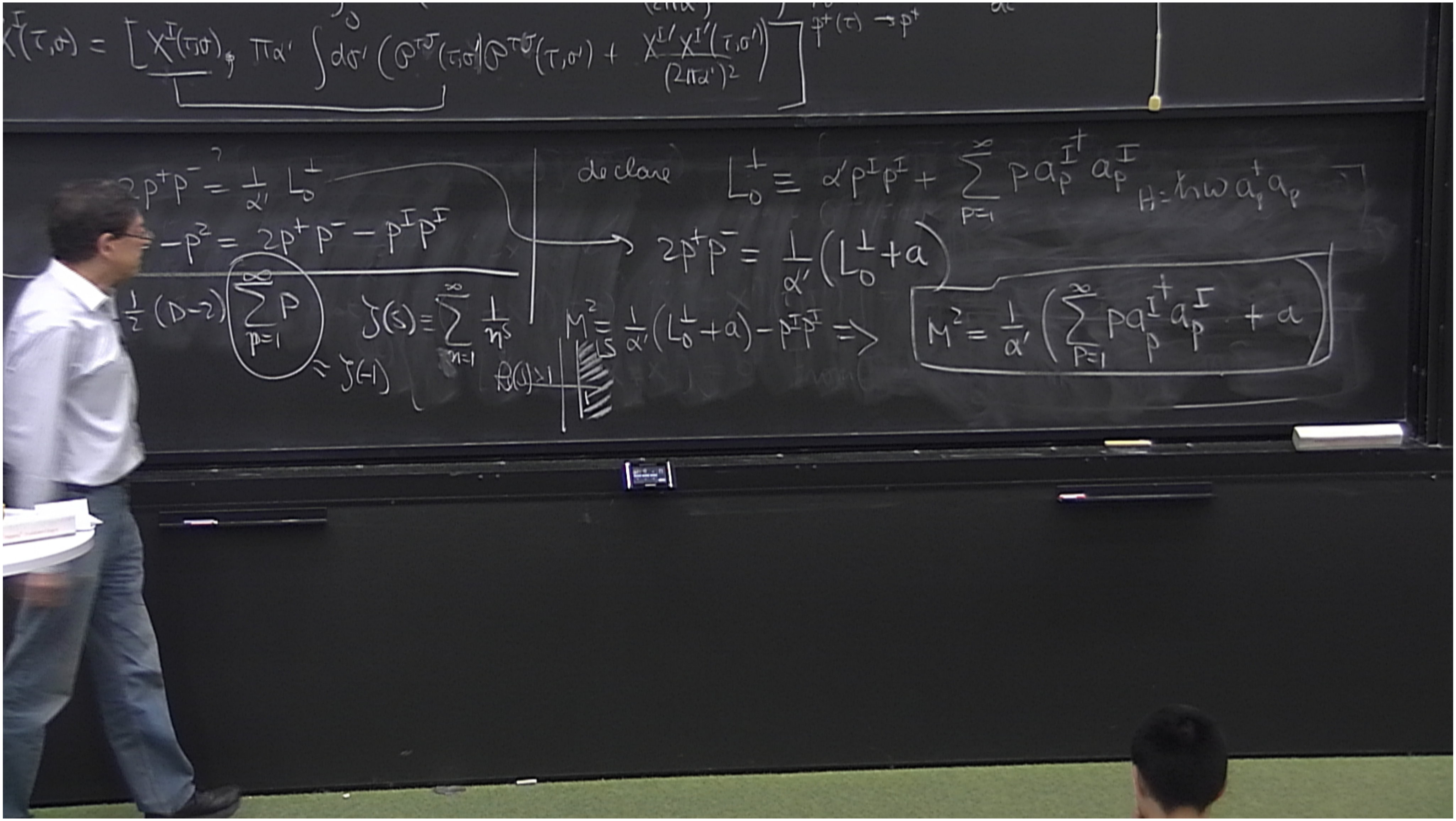

$p^+ p^- \overset{?}{=} \frac{1}{\alpha'} L_0^\perp$
$-p^2 = 2p^+p^- - p^I p^I$
declare
$L_0^\perp \equiv \alpha' p^I p^I + \sum_{p=1}^{\infty} p\, a_p^{I\dagger} a_p^I$
$2p^+p^- = \frac{1}{\alpha'}(L_0^\perp + a)$
$M^2 = \frac{1}{\alpha'}(L_0^\perp + a) - p^I p^I \Rightarrow$
$M^2 = \frac{1}{\alpha'}\left(\sum_{p=1}^{\infty} p\, a_p^{I\dagger} a_p^I + a\right)$
$p^+(\tau) \to p^+$

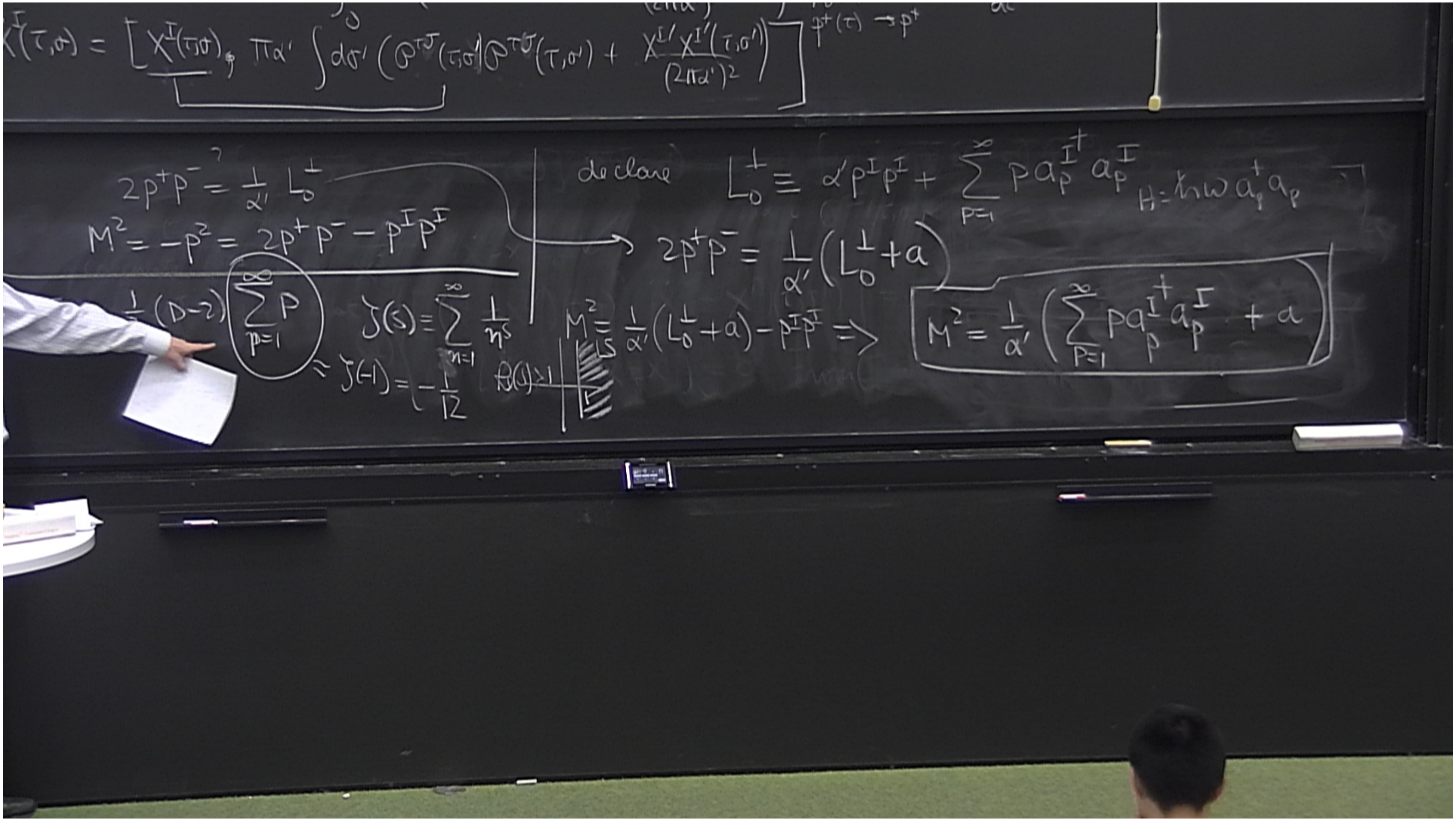


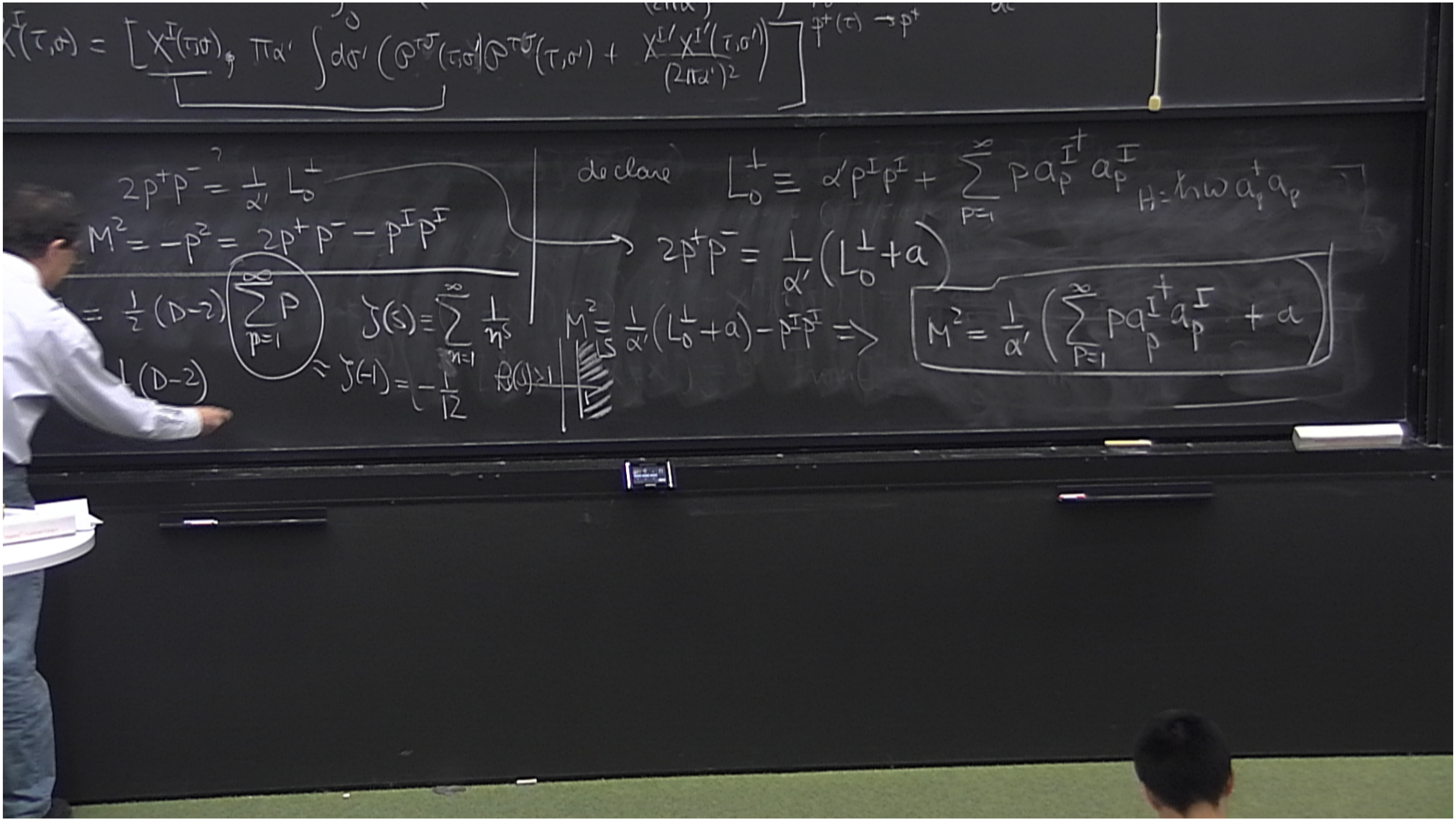
$X^I(\tau,\sigma) = [X^I(\tau,\sigma), \pi\alpha' \int d\sigma' (\mathcal{P}^{\tau J}(\tau,\sigma')\mathcal{P}^{\tau J}(\tau,\sigma') + \frac{X^{I'} X^{I'}(\tau,\sigma')}{(2\pi\alpha')^2})]$
$p^+(\tau) \to p^+$
$2p^+p^- \overset{?}{=} \frac{1}{\alpha'} L_0^\perp$
$M^2 = -p^2 = 2p^+p^- - p^I p^I$
$a = \frac{1}{2}(D-2)\sum_{p=1}^{\infty} p$
declare $L_0^\perp \equiv \alpha' p^I p^I + \sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I$
$\to 2p^+p^- = \frac{1}{\alpha'}(L_0^\perp + a)$
$M^2 = \frac{1}{\alpha'}(L_0^\perp + a) - p^I p^I \Rightarrow$
$M^2 = \frac{1}{\alpha'}\left(\sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I + a\right)$

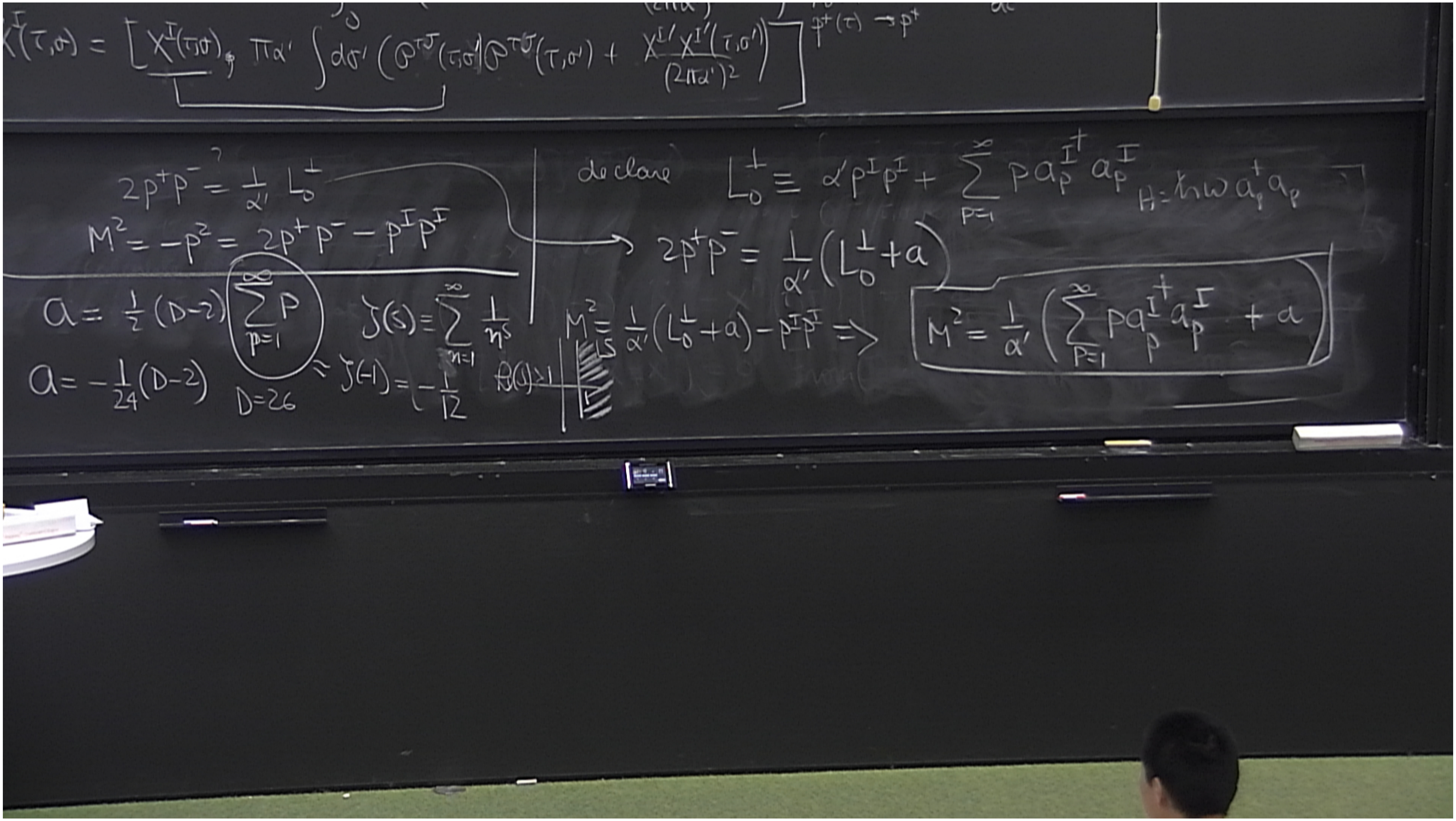

declare

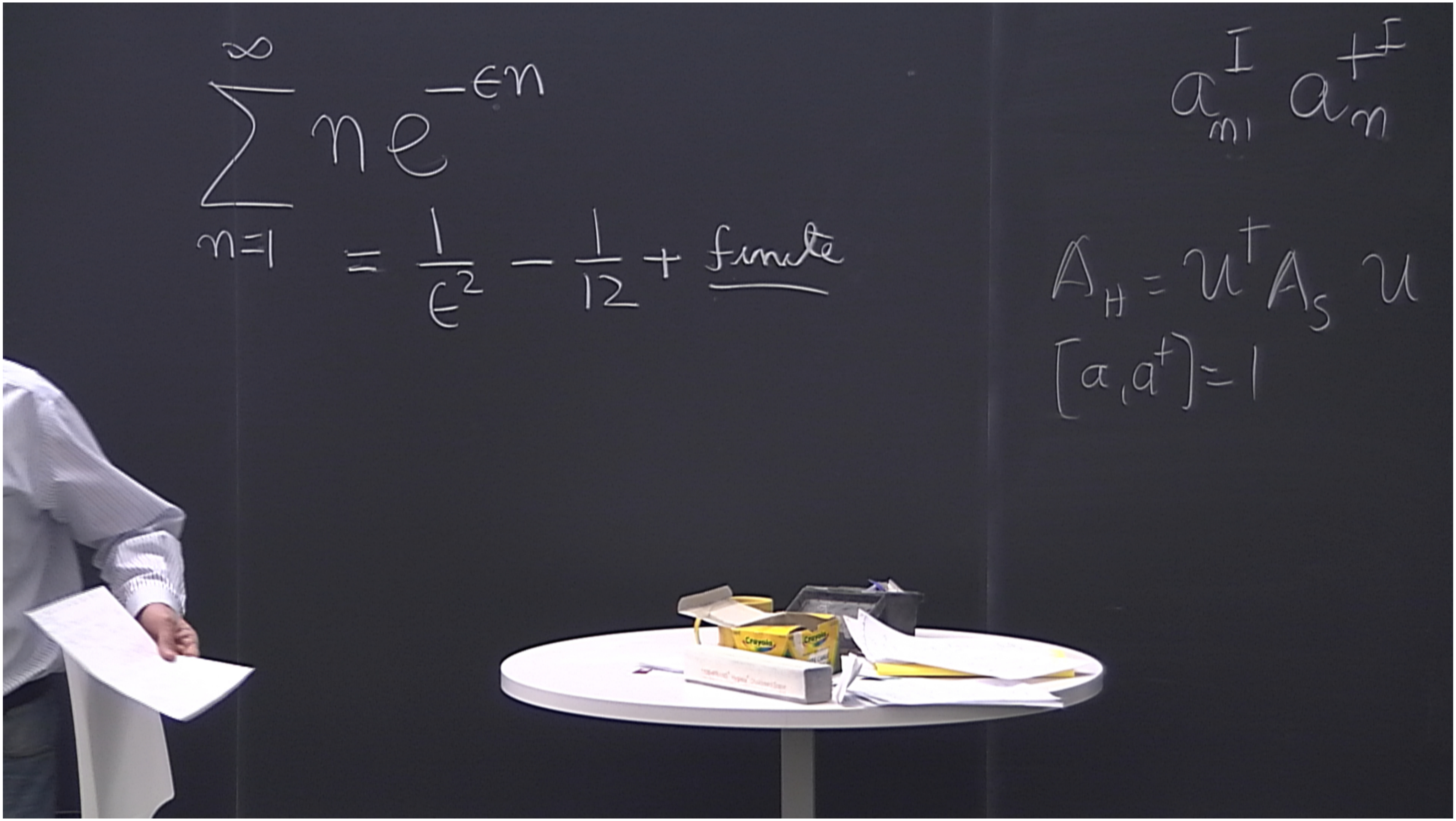

declare
$2p^+p^- = \frac{1}{\alpha'} L_0^\perp$
$M^2 = -p^2 = 2p^+p^- - p^Ip^I$
$L_0^\perp \equiv \alpha' p^I p^I + \sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I$
$H = \hbar\omega a_p^\dagger a_p$
$2p^+p^- = \frac{1}{\alpha'}(L_0^\perp + a)$
$a = \frac{1}{2}(D-2)\sum_{p=1}^{\infty} p$
$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$
$M^2 = \frac{1}{\alpha'}(L_0^\perp + a) - p^Ip^I \Rightarrow$
$M^2 = \frac{1}{\alpha'}\left(\sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I + a\right)$




declare
2p+p− = 1/α' L0⊥
M² = −p² = 2p+p− − pIpI
L0⊥ ≡ α'pIpI + Σ p=1 ∞ p apI† apI
H = ħω ap† ap
2p+p− = 1/α' (L0⊥ + a)
1/2 (D−2) Σ p=1 ∞ p
ζ(s) = Σ n=1 ∞ 1/n^s
ζ(−1) = −1/12
M² = 1/α' (L0⊥ + a) − pIpI ⇒
M² = 1/α' (Σ p=1 ∞ p apI† apI + a)


declare
ζ(-1) = -1/12


$2p^+p^- \overset{?}{=} \frac{1}{\alpha'} L_0^\perp$
$M^2 = -p^2 = 2p^+p^- - p^I p^I$
declare $L_0^\perp \equiv \alpha' p^I p^I + \sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I$
$H = \hbar\omega a_p^\dagger a_p$
$2p^+p^- = \frac{1}{\alpha'}(L_0^\perp + a)$
$a = \frac{1}{2}(D-2)\sum_{p=1}^{\infty} p$
$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$
$M^2 = \frac{1}{\alpha'}(L_0^\perp + a) - p^I p^I \Rightarrow$
$M^2 = \frac{1}{\alpha'}\left(\sum_{p=1}^{\infty} p a_p^{I\dagger} a_p^I + a\right)$
$a = -\frac{1}{24}(D-2)$
$D=26$
$\zeta(-1) = -\frac{1}{12}$



$\sum_{n=1}^{\infty} n e^{-\epsilon n} = \frac{1}{\epsilon^2} - \frac{1}{12} + \underline{\text{finite}}$
$a^I_{m} a^{+I}_{n}$
$A_H = u^+ A_S u$
$[a, a^+] = 1$