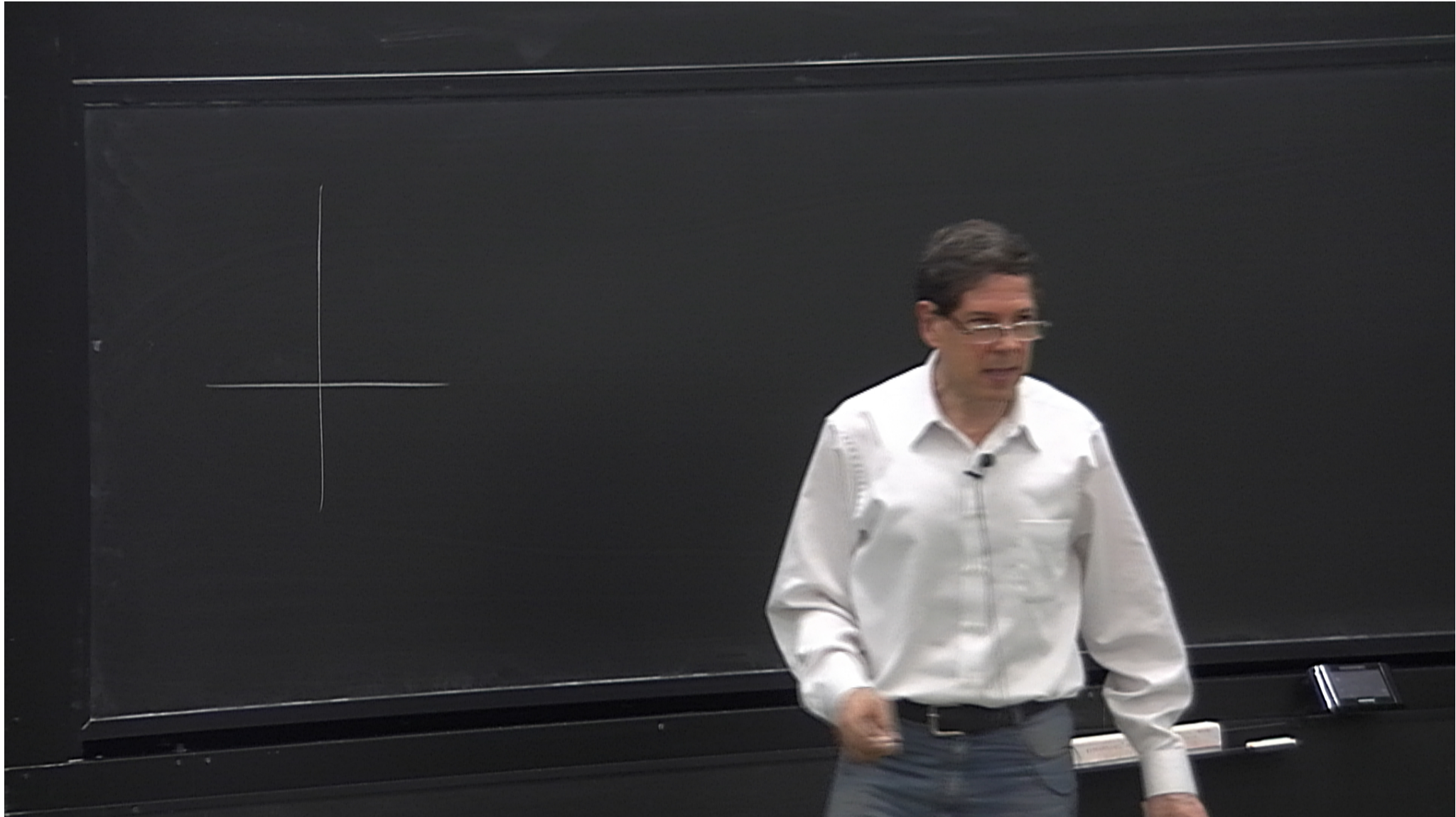


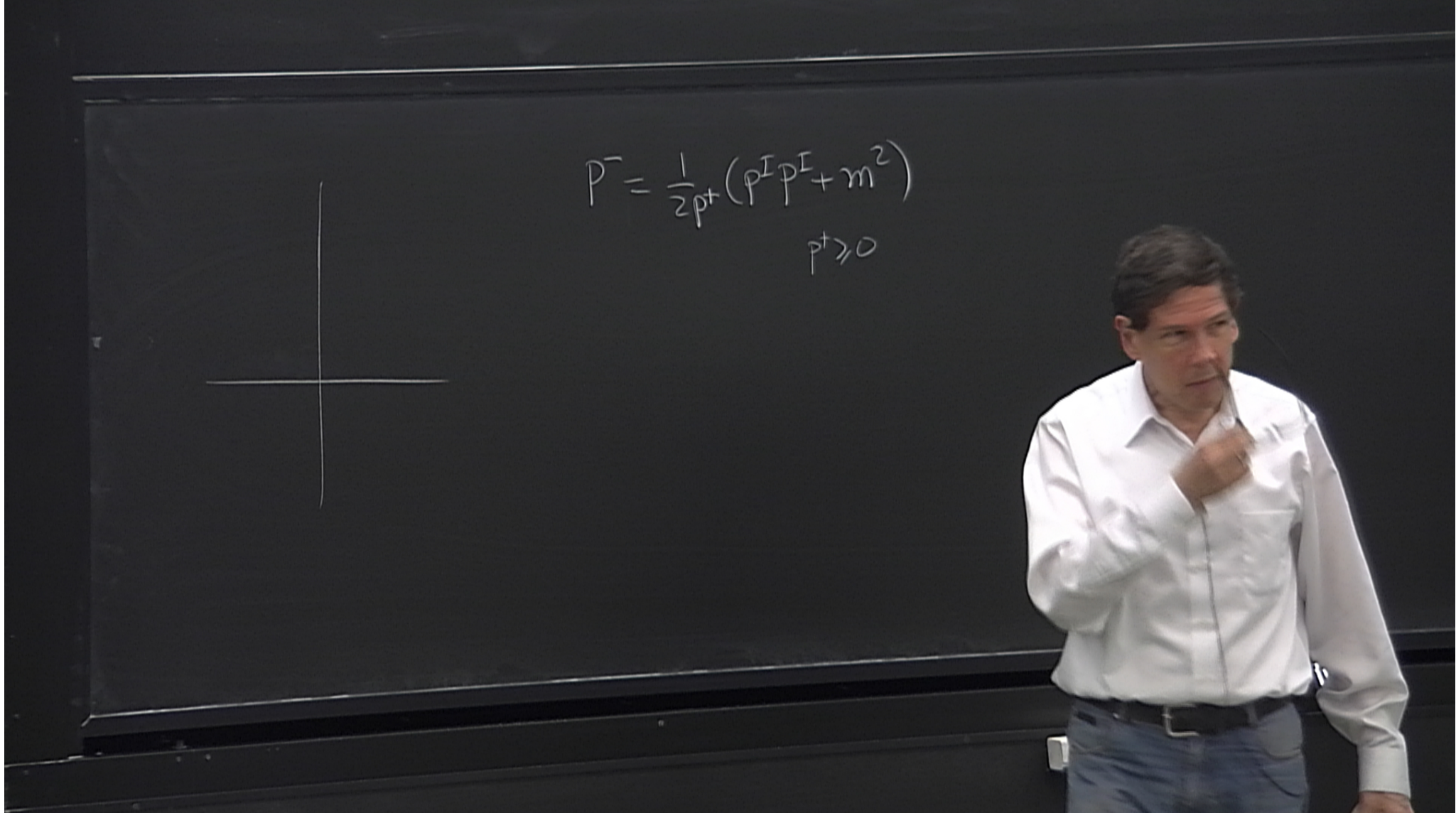
Title: 12/13 PSI - String Theory Review Lecture 9

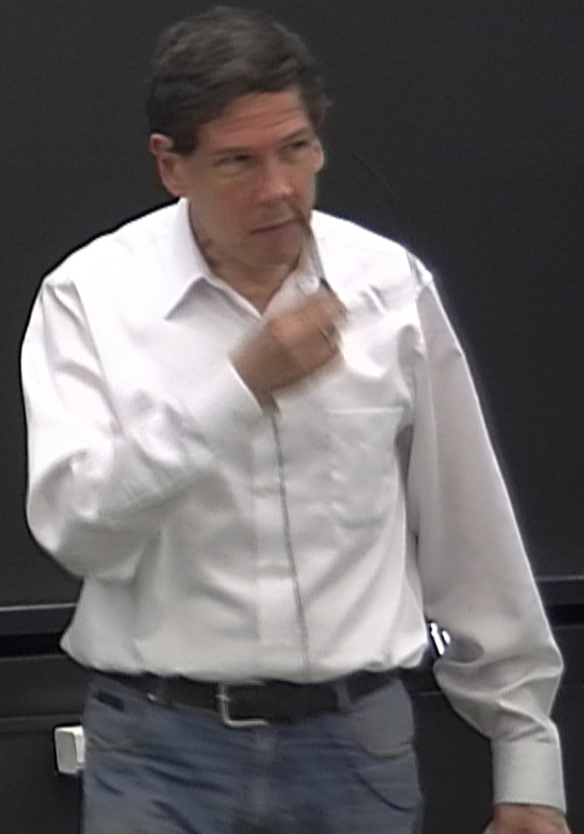
Date: Jan 17, 2013 10:15 AM

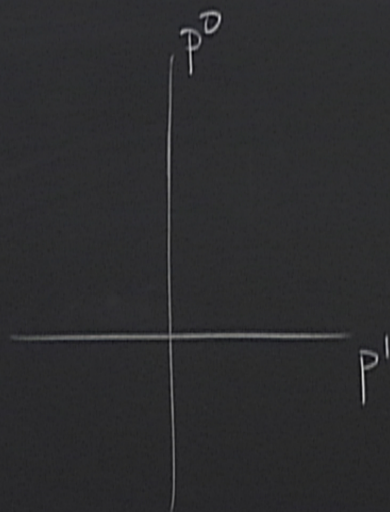
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Abstract:

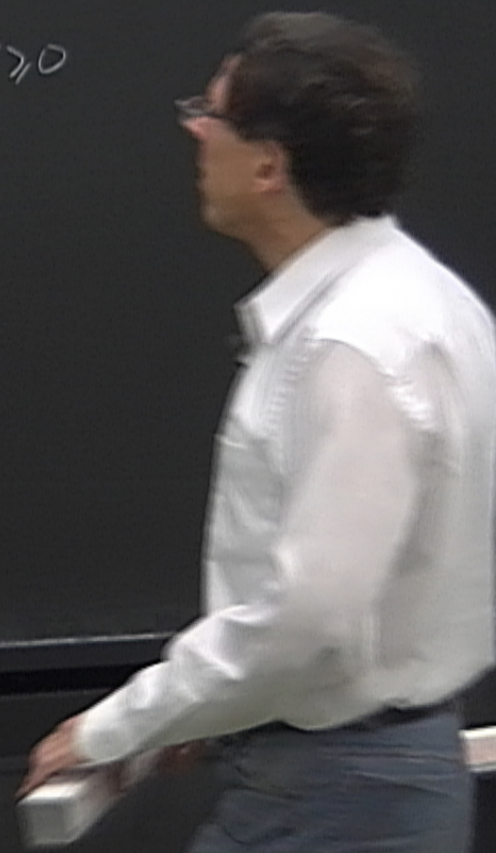


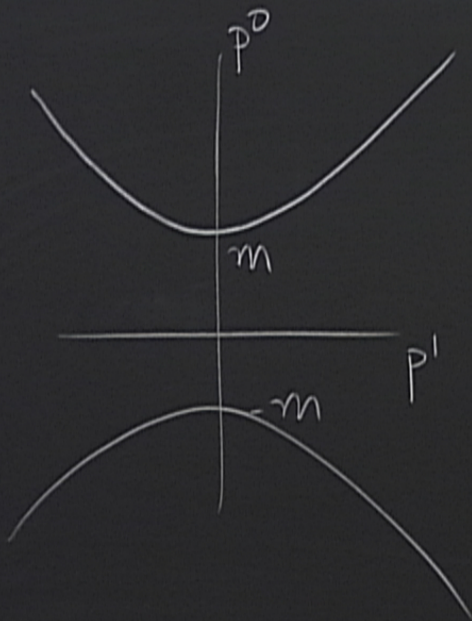
A chalkboard with a coordinate system on the left and a physics equation on the right. The coordinate system consists of a vertical line and a horizontal line intersecting at the origin. The equation is
$$p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$
 with $p^+ \geq 0$ written below it.
$$p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$
$$p^+ \geq 0$$





$$p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$
$$p^+ \geq 0$$



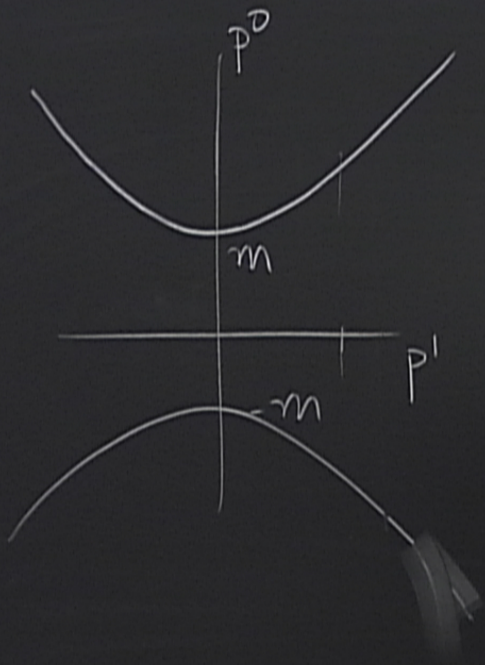


$$p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$

$$p^2 + m^2 = 0 \quad p^+ \geq 0$$

$$-(p^0)^2 + (p^1)^2 = -m^2$$

$$p^0^2 - p^1^2 = m^2$$

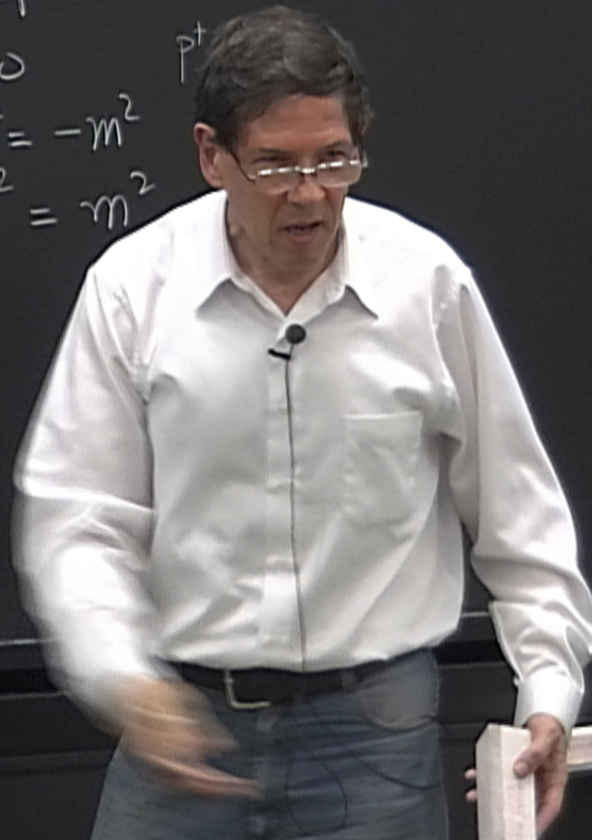


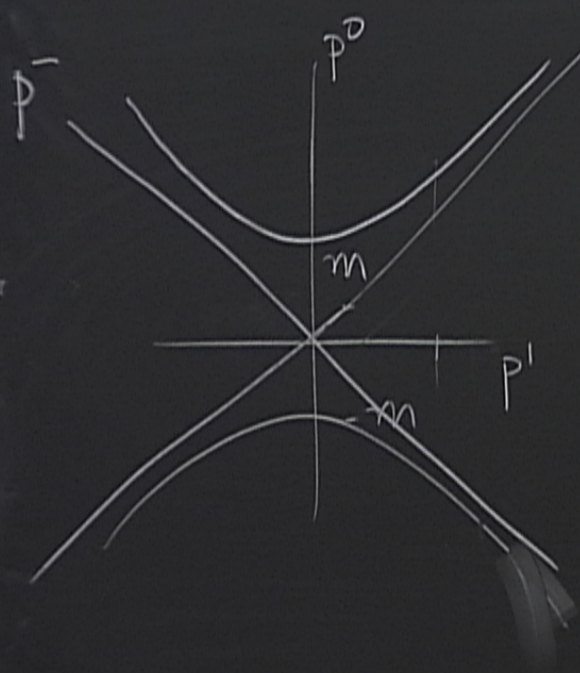
$$p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$

$$p^2 + m^2 = 0$$

$$-(p^0)^2 + (p^1)^2 = -m^2$$

$$p^{0^2} - p^{1^2} = m^2$$



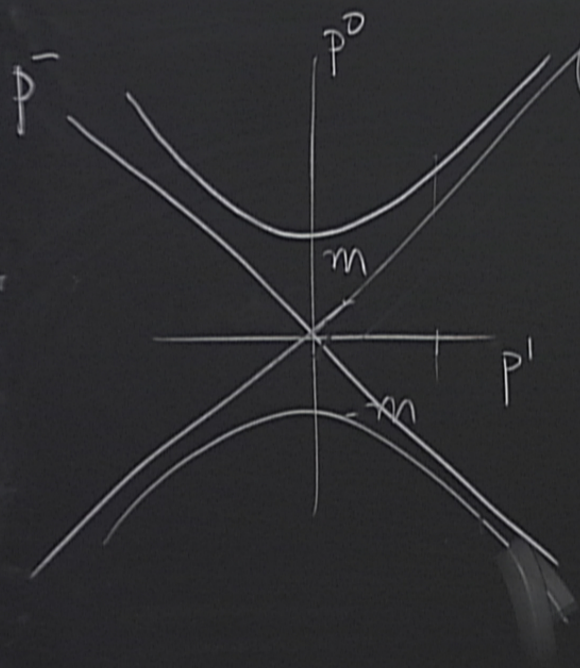


$$p^- = \frac{1}{2p^+}(p^I p^I + m^2)$$

$$p^2 + m^2 = 0 \quad p^+ \geq 0$$

$$-(p^0)^2 + (p^1)^2 = -m^2$$

$$p^{02} - p^{12} = m^2$$



$$\textcircled{p^+} \quad p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$

$$p^2 + m^2 = 0 \quad p^+ \geq 0$$

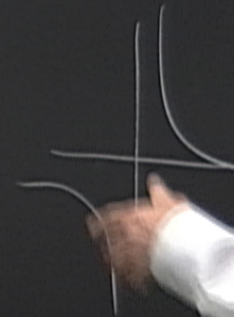
$$-(p^0)^2 + (p^1)^2 = -m^2$$

$$p^{02} - p^{12} = m^2$$

$$-2p^+ p^- = -m^2$$

$$p^+ p^- = \frac{m^2}{2}$$

$$xy = c$$



Sch. ops

x^I, x_0^-, p^I, p^+

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+]$$

Sch. ops X^I, X_0, P^I, P^+

$$[X^I, P^J] = i\delta^{IJ}$$

$$[X_0, P^+] = -i$$

Def. ops $\chi^+(\vec{r}) = \frac{P^+ \vec{r}}{m^2}$

χ^-

Sch. ops

$$x^I, x^J, p^I, p^J$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x^I, p^I] = -i$$

$$[\bar{\chi}_0, p^+] = -i$$

Def. ops

$$\chi^+(\tau) = \frac{p^+ \tau}{m^2}$$

$$\chi^-(\tau) = \chi_0 + \frac{p^- \tau}{m^2}$$

$$p^- = \frac{1}{2\alpha'}$$

$$[x_0, p^+] = -i$$

Def ops

$$x^+(\tau) \equiv \frac{p^+ \tau}{m^2}$$

$$x^-(\tau) \equiv x_0 + \frac{p^- \tau}{m^2}$$

$$p^- \equiv \frac{1}{2p^+} (p^I p^I + m^2)$$

Sch. ops
Heur. opt

$$x^I, x_0^-, p^I, p^+$$

$$x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+] = i$$

Sch. ops
Heur. ops

$$X^I, X_0^-, p^I, p^+$$

$$[X^I, p^J] = i\delta^{IJ}$$

$$[X_0^-, p^+] = -i$$

$$X^I(\tau), X_0^-(\tau), p^I(\tau), p^+(\tau)$$

Hamiltonian:

$$\frac{\partial}{\partial X^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial X^+}{\partial \tau} \frac{\partial}{\partial X^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial X^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2}$$

$$[x_0, p^+] = -i$$

Def ops
Sch ops

$$x^+(\tau) \equiv \frac{p^+ \tau}{m^2}$$

$$x^-(\tau) \equiv x_0 + \frac{p^- \tau}{m^2}$$

$$p^- \equiv \frac{1}{2p^+} (p^T p^T + m^2)$$

$$\frac{\partial}{\partial x^+}$$

Sch. ops
Heur. ops

$$x^I, x_0^-, p^I, p^+ \\ x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

$$[x^I, p^J] = i\delta^{IJ} \quad [x_0^-, p^+]$$

Hamiltonian:

$$\frac{\partial}{\partial x^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial x^+}{\partial \tau} \frac{\partial}{\partial x^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial x^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2} = \frac{1}{2m} (p^I(\tau) p^I(\tau) + m^2)$$

$$[x_0, p^+] = -i$$

Def ops
Sch ops

$$x^+(\tau) \equiv \frac{p^+ \tau}{m^2}$$

$$x^-(\tau) \equiv x_0 + \frac{p^- \tau}{m^2}$$

$$p^- \equiv \frac{1}{2p^+} (p^T p^T + m^2)$$

$$\rightarrow p^-(\tau) = \frac{1}{2p^+(\tau)} (p^T(\tau) \dots)$$

$$\left(\frac{\partial}{\partial x^+} + m^2 \right)$$

Sch. ops

$$x^I, x_0^-, p^I, p^+$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+] = -i$$

Heus. ops

$$x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

Hamiltonian:

$$\frac{\partial}{\partial x^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial x^+}{\partial \tau} \frac{\partial}{\partial x^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial x^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2} = \frac{1}{2m^2} (p^I(\tau) p^I(\tau) + m^2) \rightarrow$$

$$[X_0, P^+] = -i$$

Def ops
Sch ops

$$X^+(\tau) \equiv \frac{P^+ \tau}{m^2}$$

$$X^-(\tau) \equiv X_0 + \frac{P^- \tau}{m^2}$$

$$P^- \equiv \frac{1}{2P^+} (P^I P^I + m^2) \rightarrow \bar{P}(\tau) = \frac{1}{2P^+(\tau)} (P^I(\tau) P^I(\tau) + m^2)$$

$$\left(\frac{\partial}{\partial X^+} + m^2 \right) \rightarrow i \frac{d\xi(\tau)}{d\tau} = [\xi(\tau), H(\tau)]$$

Schr. op $\xi \rightarrow \xi(\tau)$

$$[X_0, P^+] = -i$$

Def ops
Sch ops

$$X^+(\tau) \equiv \frac{P^+ \tau}{m^2}$$

$$X^-(\tau) \equiv X_0 + \frac{P^- \tau}{m^2}$$

$$P^- \equiv \frac{1}{2p^+} (P^I P^I + m^2)$$

$$\rightarrow P^-(\tau) = \frac{1}{2p^+(\tau)} (P^I(\tau) P^I(\tau) + m^2)$$

Schr. op $\xi \rightarrow \xi(\tau)$

$$\left(\frac{d\xi(\tau)}{d\tau} = [\xi(\tau), H(\tau)] + i \frac{\partial \xi}{\partial \tau} \right) \leftarrow \text{explicit } \tau \text{ in } \xi_{\text{SCHRODINGER}}$$

Sch. ops
Heur. ops

$$x^I, x_0^-, p^I, p^+$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+]$$

$$x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

Ha

$$\frac{\partial}{\partial x^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial x^+}{\partial \tau} \frac{\partial}{\partial x^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial x^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2} = \frac{1}{2m^2} (p^I(\tau) p^I(\tau) + m^2)$$

$$\left\{ \begin{matrix} p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{matrix} \right\} =$$

Sch. ops
Heur. ops

$$x^I, x_0^-, p^I, p^+$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+]$$

$$x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

Hamiltonian:

$$\frac{\partial}{\partial x^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial x^+}{\partial \tau} \frac{\partial}{\partial x^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial x^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2} = \frac{1}{2m^2} (p^I(\tau) p^I(\tau) + m^2)$$

$$\left(\frac{d}{d\tau} \begin{Bmatrix} p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{Bmatrix} \right) = \begin{bmatrix} -p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{bmatrix}, H(\tau)$$

Sch. ops

Heur. ops

Hamilton

$$x^I, x_0^-, p^I, p^+$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+]$$

$$x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

$$\frac{\partial}{\partial x^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial x^+}{\partial \tau} \frac{\partial}{\partial x^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial x^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2} = \frac{1}{2m^2} (p^I(\tau) p^I(\tau) + m^2)$$

$$\frac{d}{d\tau} \begin{Bmatrix} p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{Bmatrix} = \begin{bmatrix} p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{bmatrix}, H(\tau)$$

$$x^I, x_0^-, p^I, p^+$$

$$[x^I, p^J] = i\delta^{IJ}$$

$$[x_0^-, p^+] = -i$$

Def. of
Sch. o

$$x^I(\tau), x_0^-(\tau), p^I(\tau), p^+(\tau)$$

$$\frac{\partial}{\partial x^+} \leftrightarrow p^-$$

$$\frac{\partial}{\partial \tau} = \frac{\partial x^+}{\partial \tau} \frac{\partial}{\partial x^+} = \left(\frac{p^+}{m^2} \right) \left(\frac{\partial}{\partial x^+} \right)$$

$$H(\tau) \stackrel{?}{=} \frac{p^+(\tau) p^-(\tau)}{m^2} = \frac{1}{2m^2} (p^I(\tau) p^I(\tau) + m^2) \rightarrow \mathcal{L} \frac{d\xi(\tau)}{d\tau}$$

$$\left(\frac{d}{d\tau} \begin{Bmatrix} p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{Bmatrix} \right) = \begin{bmatrix} p^+(\tau) \\ p^I(\tau) \\ x_0^-(\tau) \end{bmatrix}, H(\tau) = 0$$

$$\begin{aligned} p^+(\tau) &\rightarrow p^+ \\ p^I(\tau) &\rightarrow p^I \\ x_0^-(\tau) &\rightarrow x_0^- \end{aligned}$$

$$[X_0, P^+] = -i$$

Def ops
Sch ops

$$X^+(\tau) \equiv \frac{P^+ \tau}{m^2}$$

$$X^-(\tau) \equiv X_0 + \frac{P^- \tau}{m^2}$$

$$P^- \equiv \frac{1}{2P^+} (P^I P^I + m^2)$$

$$\rightarrow P^-(\tau) = \frac{1}{2P^+(\tau)} (P^I(\tau) \dots)$$

Schr. op $\xi \rightarrow \xi(\tau)$

$$i \frac{d\xi(\tau)}{d\tau} = [\xi(\tau), H(\tau)]$$

$$+ i \frac{\partial \xi}{\partial \tau}$$

explicit τ in $\xi_{\text{SCHRODINGER}}$

$$\left(\frac{\partial}{\partial X^+} + m^2 \right)$$

$$P^+(\tau) \rightarrow P^+$$

$$P^I(\tau) \rightarrow P^I$$

$$X_0(\tau) \rightarrow X_0^-$$

$$[X_0, P^+] = -i$$

Def ops
Sch ops

$$X^+(\tau) \equiv \frac{P^+ \tau}{m^2}$$

$$X^-(\tau) \equiv X_0 + \frac{P^- \tau}{m^2}$$

$$P^- \equiv \frac{1}{2P^+} (P^I P^I + m^2) \rightarrow P^-(\tau) = \frac{1}{2P^+(\tau)} (P^I(\tau) \dots)$$

$$\left(\frac{\partial}{\partial X^+} + m^2 \right)$$

$\rightarrow$

$$i \frac{d\xi(\tau)}{d\tau} = [\xi(\tau), H(\tau)] + i \frac{\partial \xi}{\partial \tau}$$

Schr. op $\xi \rightarrow \xi(\tau)$

$\leftarrow$ explicit τ in $\xi_{\text{SCHRODINGER}}$

$$\begin{aligned} P^+(\tau) &\rightarrow P^+ \\ P^I(\tau) &\rightarrow P^I \\ X_0^-(\tau) &\rightarrow X_0^- \end{aligned}$$

$$H(\tau) \Rightarrow H = \frac{1}{2m^2} (P^I P^I + m^2)$$

$$i \frac{dx^I}{d\tau} =$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (\vec{p}^T \vec{p} + m^2) \right] =$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (p^J p^J + m^2) \right] = \frac{p^I}{m^2}$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (p^J p^J + m^2) \right] = \frac{p^I}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \quad x^I(\tau)$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2}$$

$$X^I(\tau) = X_0^I + \frac{p^I \tau}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (\vec{p}^T \vec{p}^T + m^2) \right] = \frac{p^I}{m^2}$$

$$i \frac{dx^+}{d\tau} =$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (p^J p^J + m^2) \right] = \frac{p^I}{m^2}$$

$$i \frac{dx^+}{d\tau} = i \frac{\partial x^+}{\partial \tau} + [x^+(\tau), H]_0$$

$$= \frac{p^+}{m^2} + \left[\frac{p^+}{m^2}, H \right]_0$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau)$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (\vec{p}^J \vec{p}^J + m^2) \right] = \frac{p^I}{m^2}$$

$$i \frac{dx^+}{d\tau} = i \frac{\partial x^+}{\partial \tau} + [x^+(\tau), H] \\ = \frac{p^+}{m^2} + \left[\frac{p^+}{m^2}, H \right] \checkmark$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau)$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (p^J p^J + m^2) \right] = \frac{p^I}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau)$$

$$i \frac{dx^+}{d\tau} = x^+ + [x^+(\tau), H] + \left[\frac{p^+}{m^2}, H \right] \checkmark$$

States:

$$x_0^I, \dots, p^I, \quad x_0^- \leftrightarrow p^+$$

$$i \frac{dx^I}{d\tau} = \left[x^I(\tau), \frac{1}{2m^2} (\vec{p}^T \vec{p}^T + m^2) \right] = \frac{p^I}{m^2}$$

$$i \frac{dx^+}{d\tau} = i \frac{\partial x^+}{\partial \tau} + [x^+(\tau), H]$$

$$= \frac{p^+}{m^2} + \left[\frac{p^+}{m^2}, H \right]$$

States:

$$x_0^I, \underline{p^I}, \quad x_0^- \leftrightarrow \underline{p^+}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0 \leftrightarrow \underline{p^+} \quad |\underline{p^+}, \vec{p}_T\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$\underline{p^I}, \quad x_0^- \leftrightarrow \underline{p^+}$$

$$|p^+, \vec{p}_T\rangle$$

$$\hat{p}^+ |p^+, \vec{p}_T\rangle = p^+ |p^+, \vec{p}_T\rangle$$

$$\hat{\vec{p}}_T |p^+, \vec{p}_T\rangle = \vec{p}_T |p^+, \vec{p}_T\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0^- \leftrightarrow \underline{p^+}$$

$$\underline{|\vec{p}^+, \vec{p}_T\rangle}$$

$$\hat{p}^+ |\vec{p}^+, \vec{p}_T\rangle = p^+ |\vec{p}^+, \vec{p}_T\rangle$$

$$\hat{\vec{p}}_T |\vec{p}^+, \vec{p}_T\rangle = \vec{p}_T |\vec{p}^+, \vec{p}_T\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0^- \leftrightarrow \underline{p^+}$$

$$\underline{|\vec{p}^+, \vec{p}_T\rangle}$$

$$\hat{p}_- |\vec{p}^+, \vec{p}_T\rangle =$$

$$\hat{p}^+ |\vec{p}^+, \vec{p}_T\rangle = p^+ |\vec{p}^+, \vec{p}_T\rangle$$

$$\hat{\vec{p}}_T |\vec{p}^+, \vec{p}_T\rangle = \vec{p}_T |\vec{p}^+, \vec{p}_T\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0^I \leftrightarrow \underline{p^I}$$

$$\underline{|\vec{p}^+, \vec{p}_T\rangle}$$

$$\hat{p}^+ |\vec{p}^+, \vec{p}_T\rangle = p^+ |\vec{p}^+, \vec{p}_T\rangle$$

$$\hat{\vec{p}}_T |\vec{p}^+, \vec{p}_T\rangle = \vec{p}_T |\vec{p}^+, \vec{p}_T\rangle$$

$$\hat{p}_- |\vec{p}^+, \vec{p}_T\rangle = \frac{1}{2p^+} (\vec{p}_T^2 + m^2) |\vec{p}^+, \vec{p}_T\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0 \leftrightarrow \underline{p^+}$$

$$\underline{|\vec{p}^+, \vec{p}_T\rangle}$$

$$\underline{p_+ > 0}$$

$$\hat{p}^+ |\vec{p}^+, \vec{p}_T\rangle = p^+ |\vec{p}^+, \vec{p}_T\rangle$$

$$\hat{\vec{p}}_T |\vec{p}^+, \vec{p}_T\rangle = \vec{p}_T |\vec{p}^+, \vec{p}_T\rangle$$

$$\hat{p}_- |\vec{p}^+, \vec{p}_T\rangle = \frac{1}{2p^+} (\vec{p}_T^2 + m^2) |\vec{p}^+, \vec{p}_T\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0 \leftrightarrow \underline{p^+}$$

$$a_{\underline{p^+}, \underline{p_T}}^+ |\Omega\rangle$$

$$|\underline{p^+}, \underline{p_T}\rangle$$

$$\underline{p_T} > 0$$

$$\hat{p}_- |\underline{p^+}, \underline{p_T}\rangle = \frac{1}{2p^+} (\vec{p_T} \cdot \vec{p_T} + m^2) |\underline{p^+}, \underline{p_T}\rangle$$

$$\hat{p}^+ |\underline{p^+}, \underline{p_T}\rangle = p^+ |\underline{p^+}, \underline{p_T}\rangle$$

$$\hat{\vec{p}}_T |\underline{p^+}, \underline{p_T}\rangle = \vec{p_T} |\underline{p^+}, \underline{p_T}\rangle$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \Rightarrow x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$[x_0^I, p^J] = i \delta^{IJ}$$

$$x_0^I, \underline{p^I}, \quad x_0 \leftrightarrow \underline{p^+}$$

$$a_{p^+, \vec{p}_T}^+ |\Omega\rangle$$

↑ Q states of the
scalar field theory

Q states rel. point
particle

$$|\underline{p^+}, \vec{p}_T\rangle$$

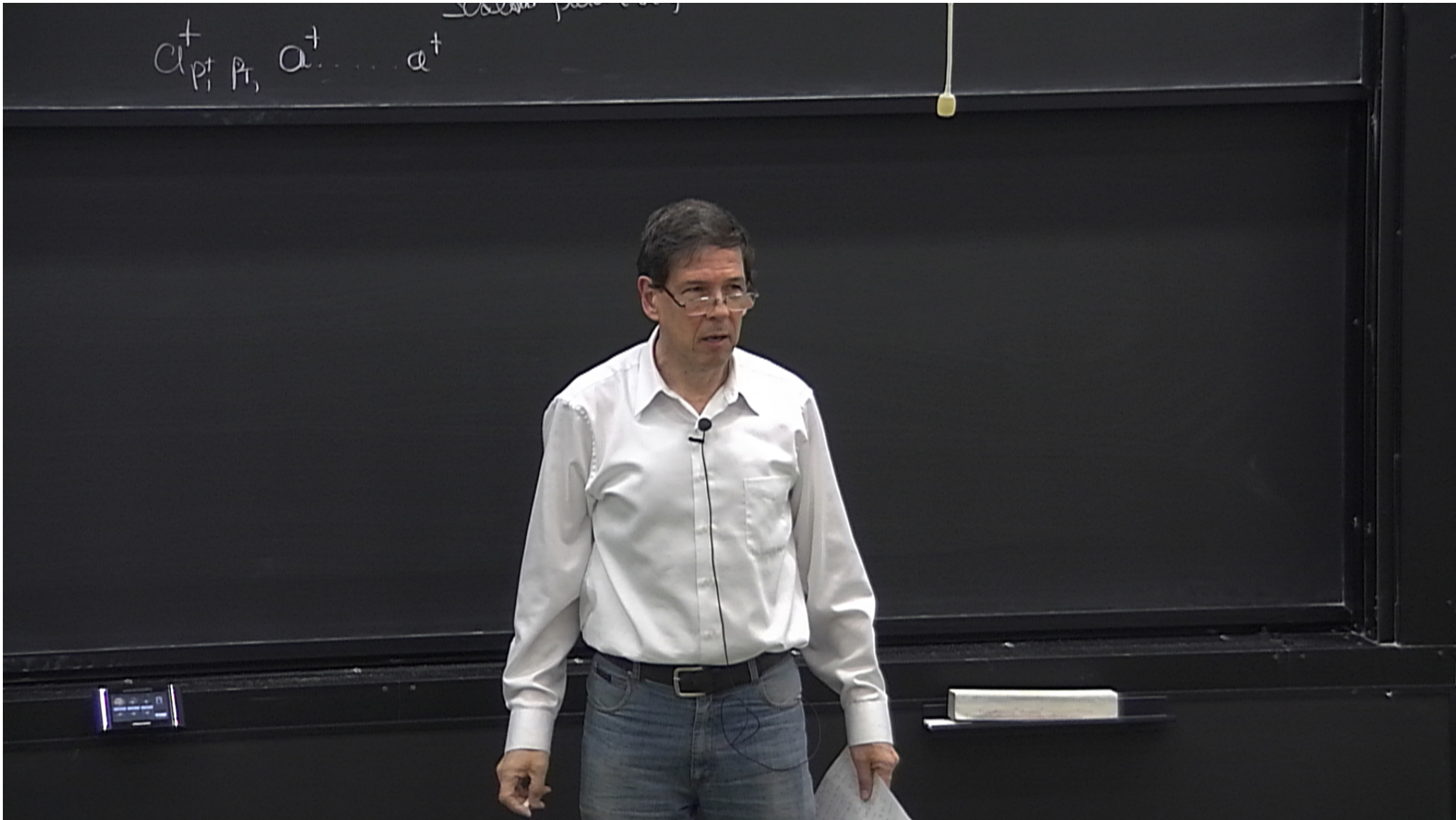
$p_+ > 0$

$$\hat{p}_- |\underline{p^+}, p_T\rangle = \frac{1}{2p^+} (\vec{p}_T^2 + m^2) |\underline{p^+}, p_T\rangle$$

$$\hat{p}^+ |\underline{p^+}, \vec{p}_T\rangle = p^+ |\underline{p^+}, \vec{p}_T\rangle$$

$$\hat{\vec{p}}_T |\underline{p^+}, \vec{p}_T\rangle = \vec{p}_T |\underline{p^+}, \vec{p}_T\rangle$$

$a^\dagger p_i^\dagger p_i, a^\dagger \dots a^\dagger$



Pont partice

Schr. eqn

$|\psi, \tau\rangle$

ψ^+
 ψ^-

$$a_{p_T}^+ a^+ \dots a^+$$

$$\int d p_T^+ d \bar{p}_T^- \psi(\tau, p_T^+, \bar{p}_T^-) |p_T^+, p_T^-\rangle$$

$$a_{p_1^+ p_1}^+ a^+ \dots a^+$$

Schr. eqn)

$$|\psi, \tau\rangle = \int d p^+ d \bar{p}_+ \psi(\tau, p^+, \bar{p}_+) |p^+, p_+ \rangle$$

$$a_{p_1^+ p_1}^+, a^+$$

Pont partice

Schr. eqn)

$$|\psi, \tau\rangle = \int d p^+ d \bar{p}_+ \psi(\tau, p^+, \bar{p}_+) |p^+, \bar{p}_+\rangle$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle$$

$$a_{p_1^+ p_1}^+, a^+$$

Pont partice

Schr. eqn)

$$|\psi, \tau\rangle = \int d p^+ d \bar{p}_+ \psi(\tau, p^+, \bar{p}_+) |p^+, \bar{p}_+\rangle$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle$$

$$a^\dagger = \frac{1}{\sqrt{2m}} \left(\frac{\partial \phi}{\partial t} + i m \phi \right)$$

Stable

$$X_0 \leftrightarrow \underline{p}^\dagger$$

$$a_{\underline{p}^\dagger, \underline{p}_T}^\dagger |\Omega\rangle$$

Q starts of the
Scalar field theory

$$a_{\underline{p}^\dagger, \underline{p}_T}^\dagger, a^\dagger, a^\dagger$$

$$|\underline{p}^\dagger, \underline{p}_T\rangle$$

$$\underline{p}^\dagger |\underline{p}^\dagger, \underline{p}_T\rangle = \underline{p}^\dagger |\underline{p}^\dagger, \underline{p}_T\rangle$$

$$\underline{p}_T |\underline{p}^\dagger, \underline{p}_T\rangle = \underline{p}_T |\underline{p}^\dagger, \underline{p}_T\rangle$$

$$\hat{p}^\dagger |\underline{p}^\dagger, \underline{p}_T\rangle = \frac{1}{2m} (\underline{p}_T^2 + m^2) |\underline{p}^\dagger, \underline{p}_T\rangle$$

Pont practice

Schr. eqn

$$|\psi, \tau\rangle = \int d\underline{p}^\dagger d\underline{p}_T \psi(\tau, \underline{p}^\dagger, \underline{p}_T) |\underline{p}^\dagger, \underline{p}_T\rangle$$

Scalar field eqn

$$(\partial^2 - m^2)\phi = 0$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle$$

$$i \frac{\partial}{\partial \tau} \psi(\tau, \underline{p}^\dagger, \underline{p}_T) = \frac{1}{2m} (\underline{p}_T^2 + m^2) \psi(\tau, \underline{p}^\dagger, \underline{p}_T)$$

Pont partice

Schr. eqn

$$|\psi, \tau\rangle = \int d\vec{p}^+ d\vec{p}_\tau \psi(\tau)$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle \rightarrow$$

Scalar field

$$(\partial^2 - m^2)\phi = 0$$

$$+ \frac{\partial}{\partial x}$$

$$a_{p_1^+ p_T}^+, a^+ \dots a^+$$

$$p^+ d\vec{p}_T \psi(\tau, p_1^+, \vec{p}_T) |p_1^+, p_T\rangle$$

$$\rightarrow \left[\frac{\partial}{\partial \tau} \psi(\tau, p_1^+, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p_1^+, p_T) \right]$$

$$a^\dagger = \frac{1}{\sqrt{m}} \left(\frac{\partial \phi}{\partial x} + i m \phi \right)$$

Stable

$$x_0 \leftrightarrow p_0$$

$$a_{p_1, p_T}^\dagger |Q\rangle$$

Q starts as the
Scalar field theory

$$a_{p_1, p_T}^\dagger, a^\dagger, a^\dagger$$

$$|p_1, p_T\rangle$$

$$p_1 |p_1, p_T\rangle = p_1 |p_1, p_T\rangle$$

$$p_T |p_1, p_T\rangle = p_T |p_1, p_T\rangle$$

$$\hat{p}_T |p_1, p_T\rangle = \frac{1}{2 p_T} (\vec{p}_T \vec{p}_T + m^2) |p_1, p_T\rangle$$

Pont partice

Schr. eqn.

$$|\psi, \tau\rangle = \int d p^\dagger d \bar{p}_T \psi(\tau, p^\dagger, \bar{p}_T) |p^\dagger, p_T\rangle$$

Scalar field

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle$$

$$(\partial^2 - m^2) \phi = 0$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial x^\dagger} - m^2 \right) \phi(x^\dagger, x, x^\dagger) = 0$$

$$i \frac{\partial}{\partial \tau} \psi(\tau, p^\dagger, \bar{p}_T) = \frac{1}{2 m^2} (p_T^2 + m^2) \psi(\tau, p^\dagger, p_T)$$

Pont partice

Schr. eqn

$$|\psi, \tau\rangle = \int d p^+ d \bar{p}_\tau \psi(\tau)$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle \rightarrow$$

Scalar field eqn

$$(\partial^2 - m^2) \phi = 0$$

$$\left(-2 \frac{\partial}{\partial x^+} \frac{\partial}{\partial x^-} + \frac{\partial}{\partial x^+} \frac{\partial}{\partial x^+} - m^2 \right) \phi(x^+, x^-, x^\perp) = 0$$

$$a_{p_1^+ p_T}^+, a^+ \dots a^+$$

$$|\psi, \tau\rangle = \int dp^+ d\vec{p}_T \psi(\tau, p^+, \vec{p}_T) |p^+, \vec{p}_T\rangle$$

$$H|\psi, \tau\rangle \rightarrow \left[i \frac{\partial}{\partial \tau} \psi(\tau, p^+, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p^+, \vec{p}_T) \right]$$

$$(x_1^+, x_1^-, x_1^T) = 0$$

$$\phi(x) = \int dp^+ d\vec{p}_T e^{-i(x^+ p^+ + i\vec{x}_T \cdot \vec{p}_T)} \phi(x^+, p^+, \vec{p}_T)$$

$$a_{p_1^+ p_T}^+, a^+ \dots a^+$$

$$|\psi, \tau\rangle = \int dp^+ d\vec{p}_T \psi(\tau, p^+, \vec{p}_T) |p^+, \vec{p}_T\rangle$$

$$H|\psi, \tau\rangle \rightarrow \left[i \frac{\partial}{\partial \tau} \psi(\tau, p^+, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p^+, \vec{p}_T) \right]$$

$$(x_1^+, x_1^-, x_1^T) = 0$$

$$\phi(x) = \int dp^+ d\vec{p}_T e^{-i(x^+ p^+ + i\vec{x}_T \cdot \vec{p}_T)} \phi(x_1^+, p_1^+, \vec{p}_T)$$

$$a^\dagger = \frac{1}{\sqrt{m}} \left(\frac{\partial \phi}{\partial x} + i \left[\frac{p^2}{m^2}, H \right] \right)^\dagger$$

Stable

$$x_0 \leftrightarrow p^0, \quad x_0 \leftrightarrow p^0$$

$$a_{p^0, p_T}^\dagger |Q\rangle$$

Q starts of the
Scalar field theory

$$a_{p^0, p_T}^\dagger, a^\dagger, a^\dagger$$

$$|p^0, p_T\rangle$$

$$p^0 |p^0, p_T\rangle = p^0 |p^0, p_T\rangle$$

$$p_T |p^0, p_T\rangle = p_T |p^0, p_T\rangle$$

$$\hat{p}^0 |p^0, p_T\rangle = \frac{1}{2p^0} (\vec{p}_T^2 + m^2) |p^0, p_T\rangle$$

Pont partice

Schr. eqn

$$|\psi, \tau\rangle = \int d p^0 d \vec{p}_T \psi(\tau, p^0, \vec{p}_T) |p^0, \vec{p}_T\rangle$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle$$

$$\left(i \frac{\partial}{\partial \tau} \psi(\tau, p^0, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p^0, \vec{p}_T) \right)$$

Scalar field eqn

$$(\partial^2 - m^2) \phi = 0$$

$$\left(-2 \frac{\partial}{\partial x^0} \frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^0} \frac{\partial}{\partial x^0} - m^2 \right) \phi(x^0, x^1, x^2) = 0$$

$$\left(-2 \frac{\partial}{\partial x^0} (-\frac{1}{2} p^0) - (p_T^2 + m^2) \right) \phi(x^0, p^0, \vec{p}_T)$$

$$\phi(x) = \int d p^0 d \vec{p}_T e^{-i x^0 p^0 + i \vec{x}_T \cdot \vec{p}_T} \phi(x^0, p^0, \vec{p}_T)$$

$$a_{p_1^+ p_1}^+, a^+ \dots a^+$$

$$|\psi, \tau\rangle = \int dp^+ d\vec{p}_T \psi(\tau, p^+, \vec{p}_T) |p^+, \vec{p}_T\rangle$$

$$H|\psi, \tau\rangle \rightarrow \left[i \frac{\partial}{\partial \tau} \psi(\tau, p^+, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p^+, \vec{p}_T) \right]$$

$$(x^+, x^-, x^T) = 0$$

$$\phi(x) = \int dp^+ d\vec{p}_T e^{-ix^- p^+ + i\vec{x}_T \cdot \vec{p}_T} \phi(x^+, p^+, \vec{p}_T)$$

$$(x^+, p^+, \vec{p}_T) = 0$$

$$x^+ = \frac{p^+}{m^2}$$

$$\Rightarrow \left(i \frac{\partial}{\partial \tau} - \frac{1}{2m} (\vec{p}_T^2 + m^2) \right) \phi(\tau, p^+, \vec{p}_T) = 0$$

Pont partice

Schr. eqn

$$|\psi, \tau\rangle = \int d p^+ d \bar{p}_T \psi(\tau)$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle \rightarrow$$

Scalar field eqn

$$(\partial^2 - m^2) \phi = 0$$

$$\left(-2 \frac{\partial}{\partial x^+} \frac{\partial}{\partial x^-} + \frac{\partial}{\partial x^+} \frac{\partial}{\partial x^+} - m^2 \right) \phi(x^+, x^-, x^T) = 0$$

$$\left(-2 \frac{\partial}{\partial x^+} (-\frac{1}{2} p^+) - (\bar{p}_T^2 + m^2) \right) \phi(x^+, p^+, p_T) = 0$$

$$x^+ = \frac{p^+ \tau}{m^2}$$

$$a_{p_1^+ p_T}^+ \dots a^+$$

$$p^+ d\vec{p}_T \psi(\tau, p^+, \vec{p}_T) |p^+, p_T\rangle$$

$$\rightarrow \left[i \frac{\partial}{\partial \tau} \psi(\tau, p^+, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p^+, p_T) \right]$$

$$\phi(x) = \int dp^+ dp_T e^{-i x^- p^+ + i \vec{x}_T \cdot \vec{p}_T} \phi(x^+, p^+, \vec{p}_T)$$

$$x^+ = \frac{p^+ \tau}{m^2} \Rightarrow \left[\left(i \frac{\partial}{\partial \tau} - \frac{1}{2m} (\vec{p}_T^2 + m^2) \right) \phi(\tau, p^+, \vec{p}_T) = 0 \right]$$

$$a_{p_1^+ p_T}^+, a^+ \dots a^+$$

$$p^+ d\vec{p}_T \psi(\tau, p^+, \vec{p}_T) |p^+, p_T\rangle$$

$$\rightarrow \left[i \frac{\partial}{\partial \tau} \psi(\tau, p^+, \vec{p}_T) = \frac{1}{2m^2} (p_T^2 + m^2) \psi(\tau, p^+, p_T) \right]$$

$$\phi(x) = \int dp^+ dp_T e^{-i(x^- p^+ + i \vec{x}_T \cdot \vec{p}_T)} \phi(x^+, p^+, \vec{p}_T)$$

$$x^+ = \frac{p^+ \tau}{m^2} \Rightarrow \left[\left(i \frac{\partial}{\partial \tau} - \frac{1}{2m^2} (\vec{p}_T^2 + m^2) \right) \phi(\tau, p^+, \vec{p}_T) = 0 \right]$$

Pont partice

Schr. eqn

$$|\psi, \tau\rangle = \int d p^+ d \bar{p}_T \Psi(\tau)$$

$$i \frac{\partial}{\partial \tau} |\psi, \tau\rangle = H |\psi, \tau\rangle \rightarrow$$

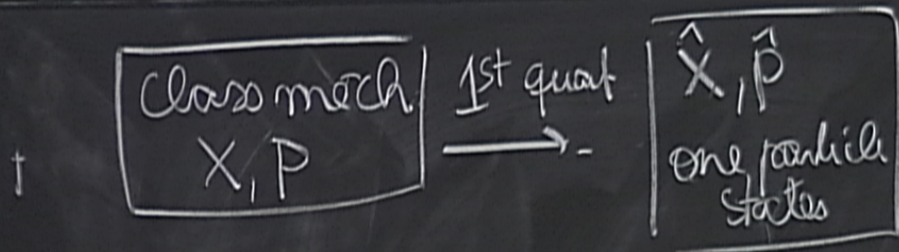
Scalar field eqn

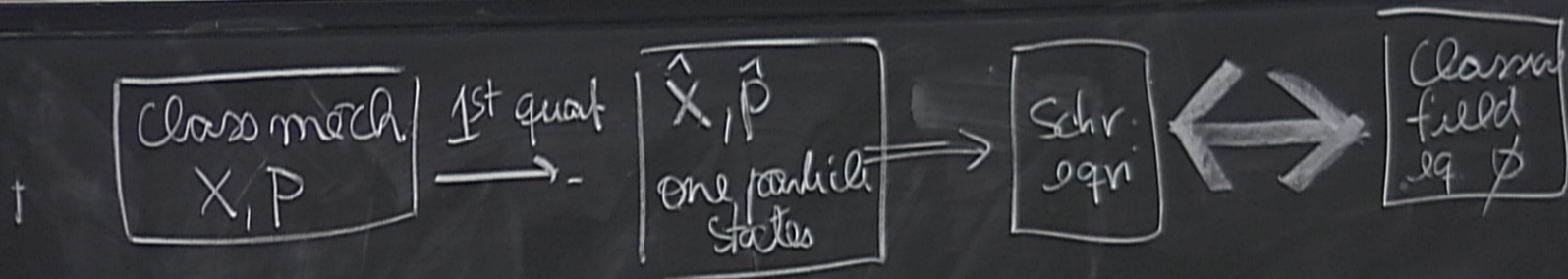
$$(\partial^2 - m^2) \phi = 0$$

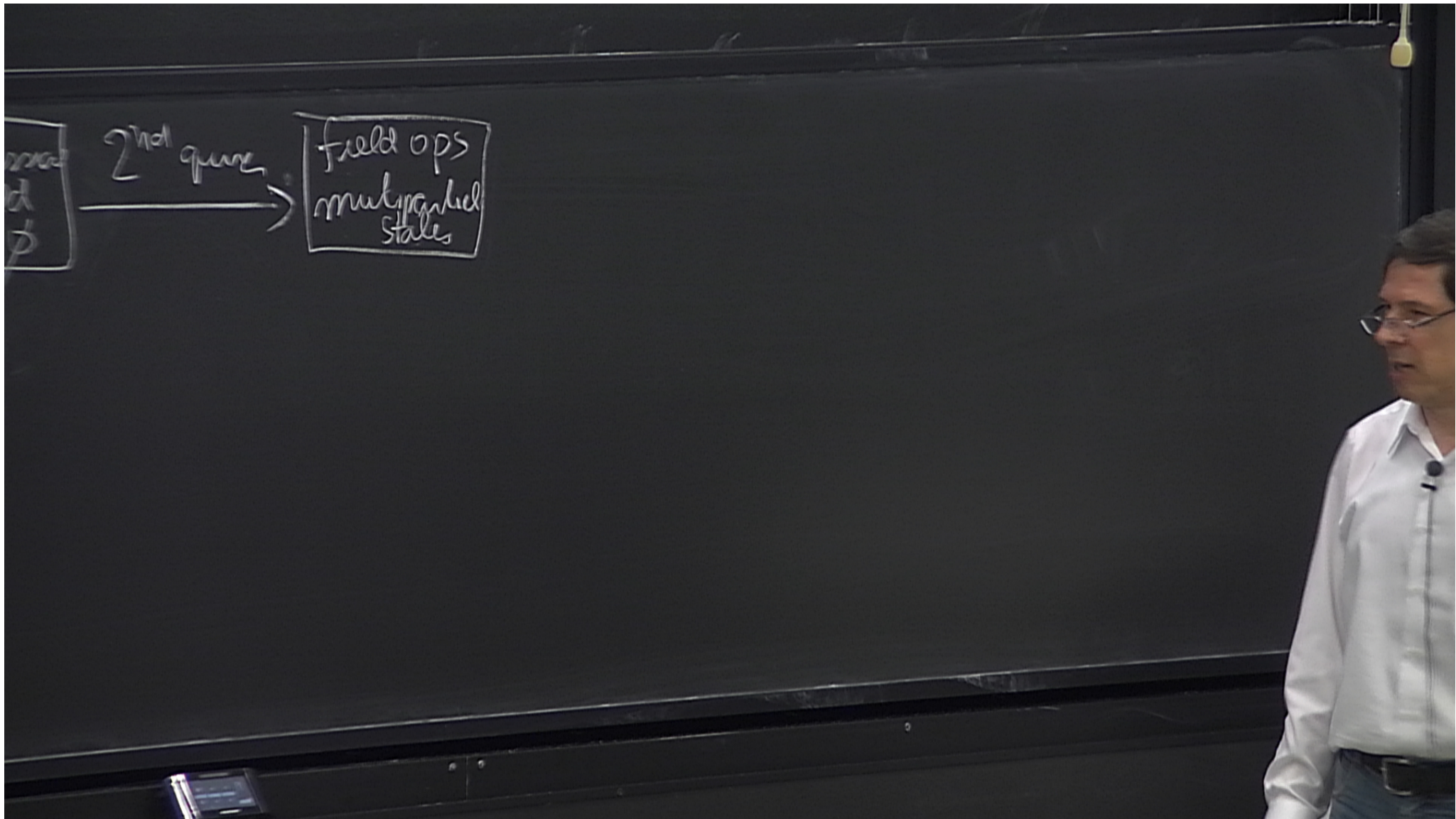
$$\left(-2 \frac{\partial}{\partial x^+} \frac{\partial}{\partial x^-} + \frac{\partial}{\partial x^+} \frac{\partial}{\partial x^+} - m^2 \right) \phi(x^+, x^-, x^T) = 0$$

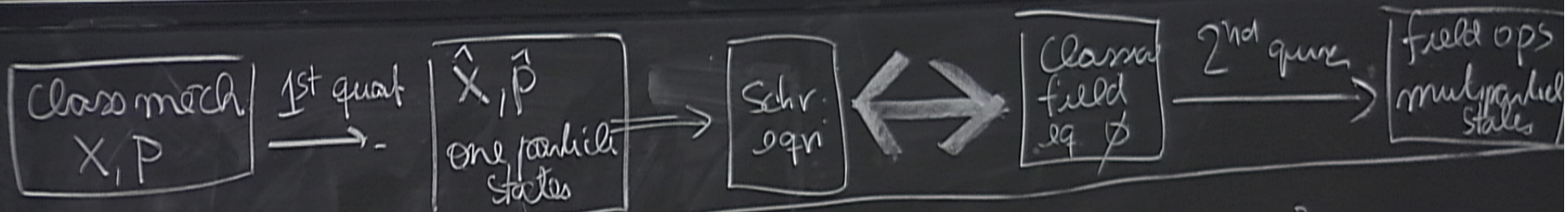
$$\left(-2 \frac{\partial}{\partial x^+} (-\frac{1}{2} p^+) - (\bar{p}_T^2 + m^2) \right) \phi(x^+, p^+, p_T) = 0$$

$$x^+ = \frac{p^+ \tau}{m^2}$$

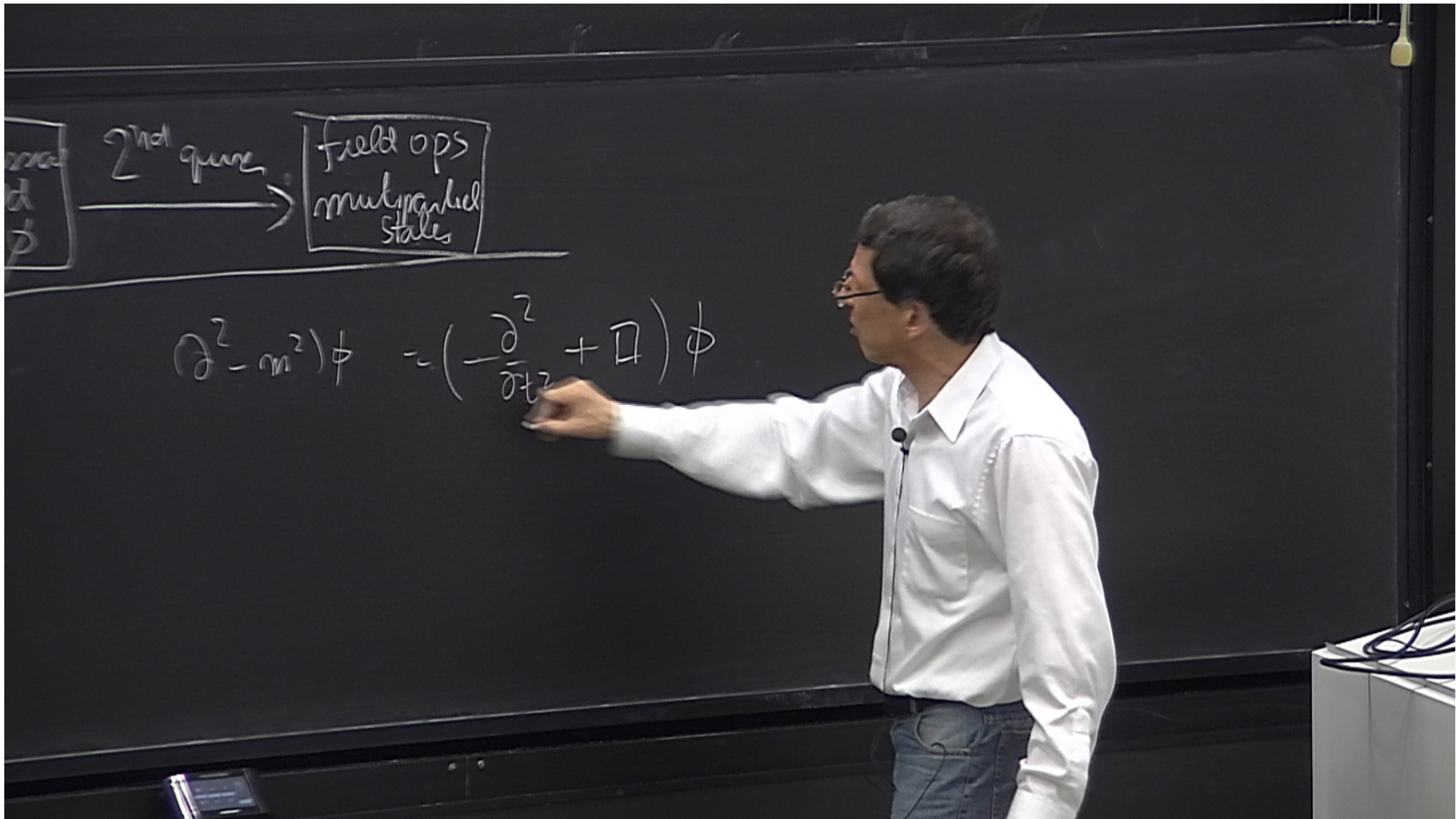


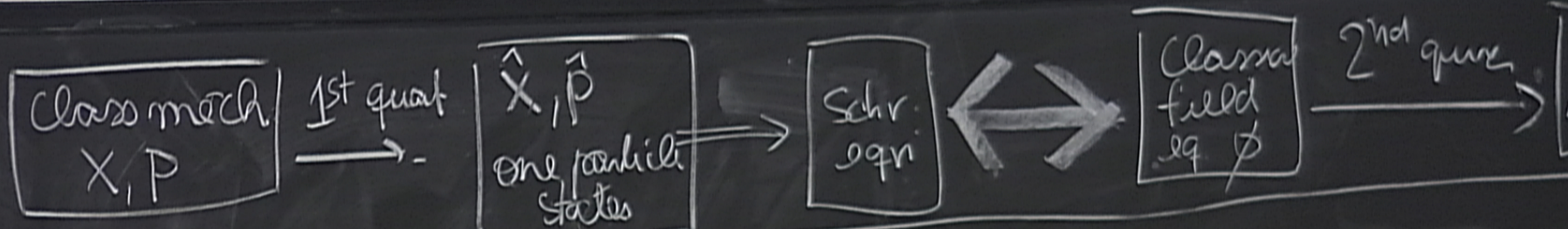




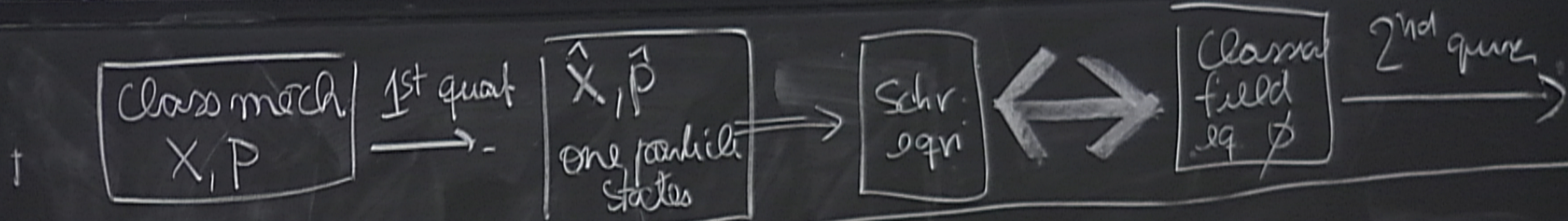


$$(\partial^2 - m^2)\phi =$$

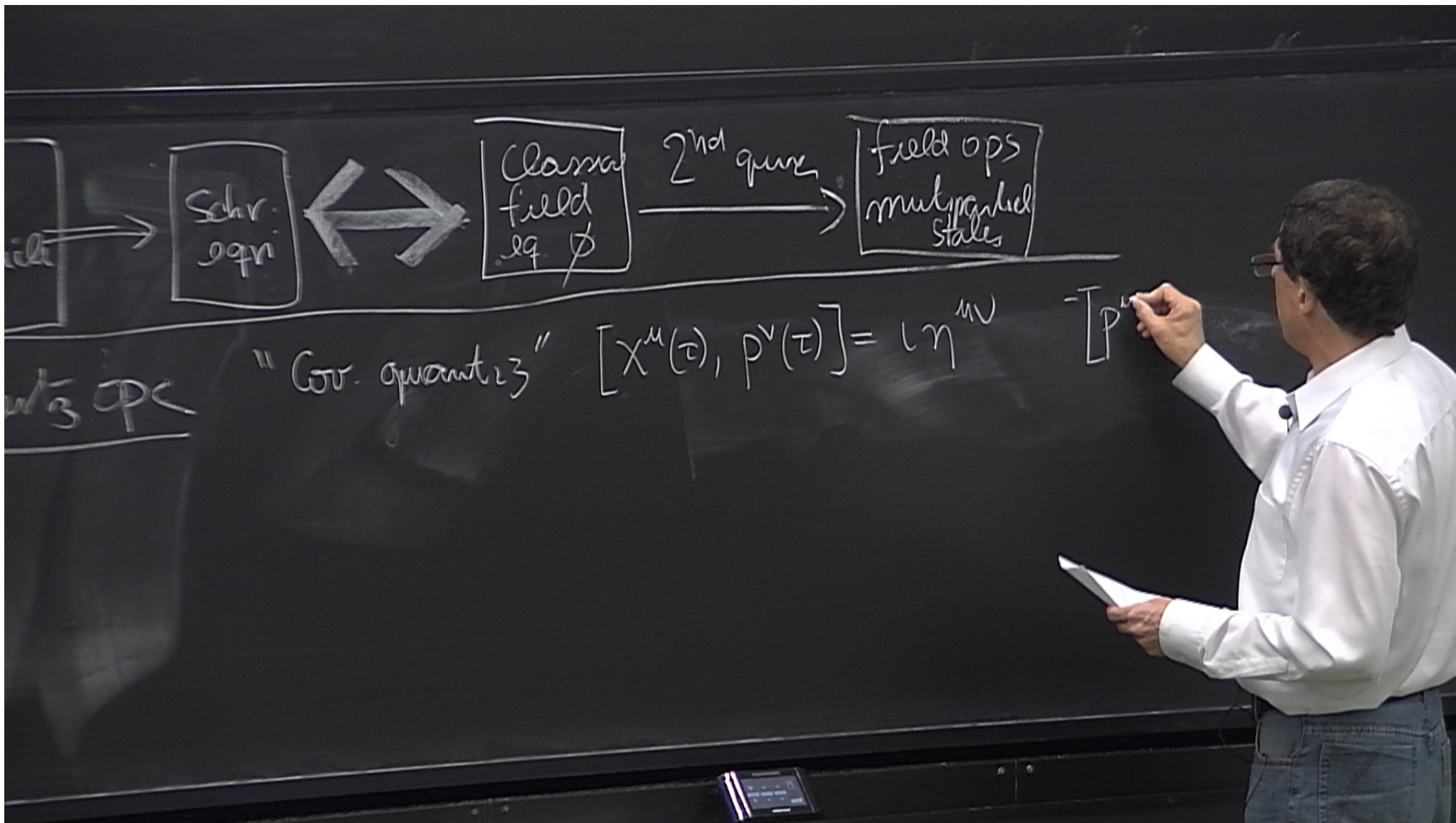




Lighcone. Mom and Lorentz op.



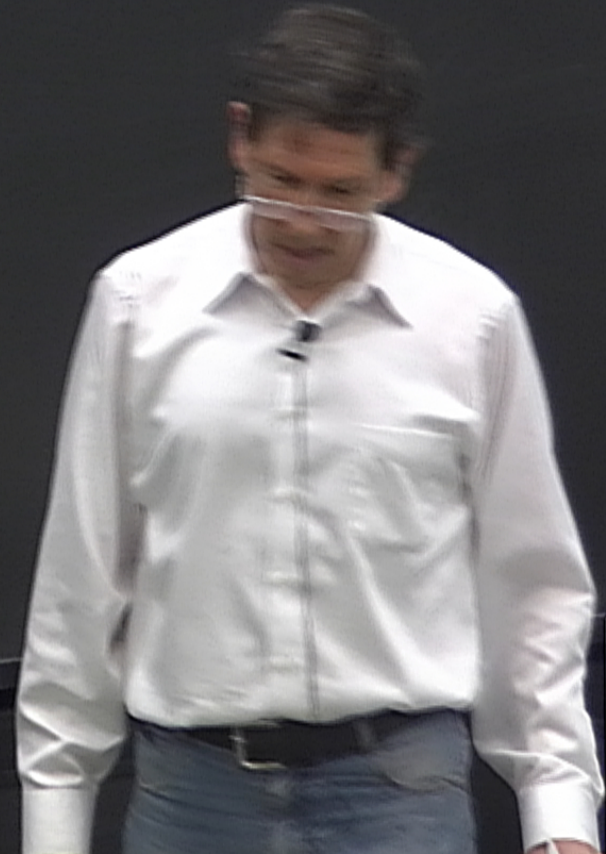
Ligheone. Mom and Lorentz opc "Cor. quantiz"

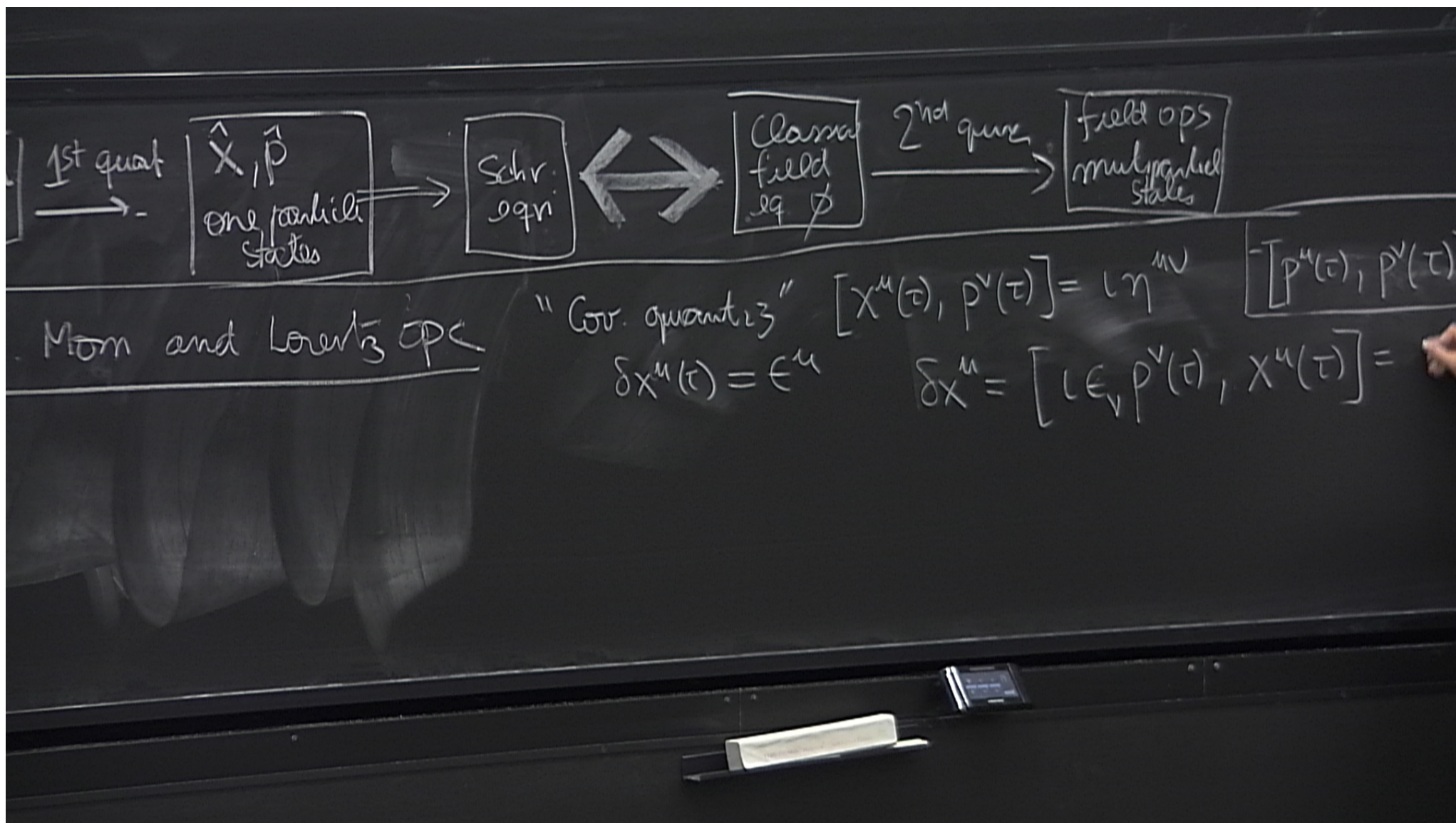


2nd quant
field ops
multiparticle
states

$$[x^\mu(\tau), p^\nu(\tau)] = i\eta^{\mu\nu}$$

$$[p^\mu(\tau), p^\nu(\tau)] = 0$$

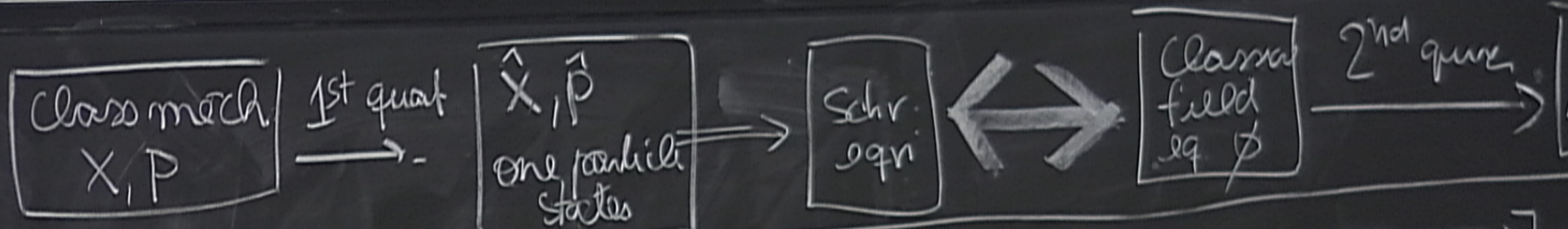




2nd quere $\rightarrow$ field ops
multiplicated
states

$$[X^\mu(\tau), p^\nu(\tau)] = i\eta^{\mu\nu} \quad [p^\mu(\tau), p^\nu(\tau)] = 0$$

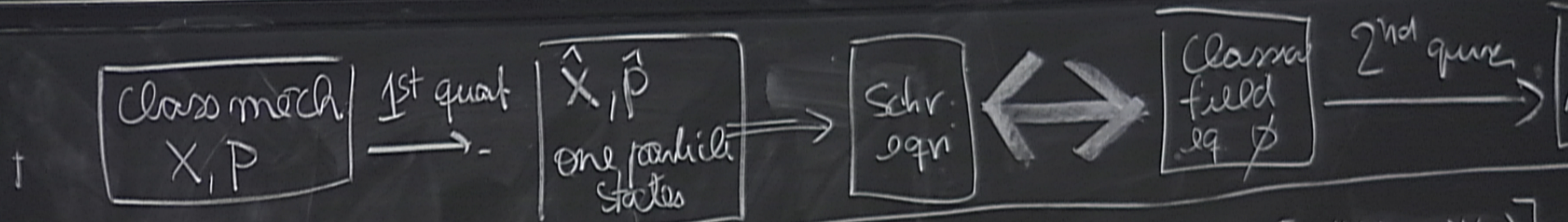
$$\delta X^\mu = [L\epsilon_\nu p^\nu(\tau), X^\mu(\tau)] = i\epsilon_\nu (-i\eta^{\mu\nu}) = \epsilon^\mu \checkmark$$



Lighcone Mom and Lorentz op

Lighcone $p^+, p^-, p^\perp$

"Con. quantiz" $[X^\mu(\tau), p^\nu(\tau)] =$
 $\delta X^\mu(\tau) = \epsilon^\mu$ $\delta X^\mu = [L$



Lighcone Mom and Lorentz op

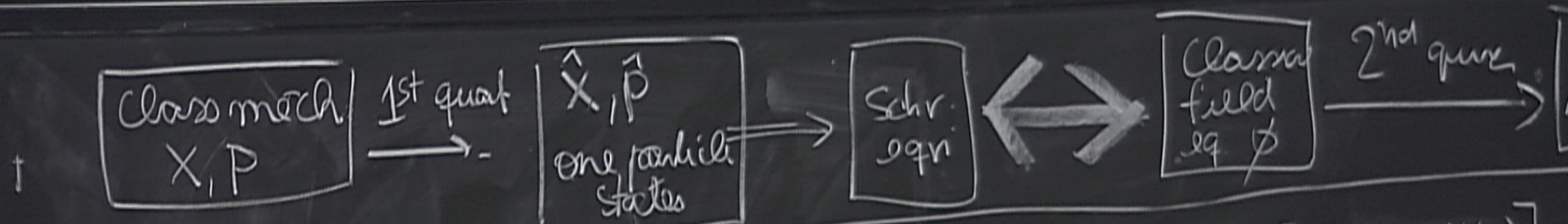
Lighcone $p^+, p^-, p^\perp$, they commute.

$$\delta X^M = ?$$

"Con. quantiz" $[X^\mu(\tau), p^\nu(\tau)] =$

$$\delta X^\mu(\tau) = \epsilon^\mu$$

$$\delta X^\mu = [L$$



Lightcone Mom and Lorentz ops

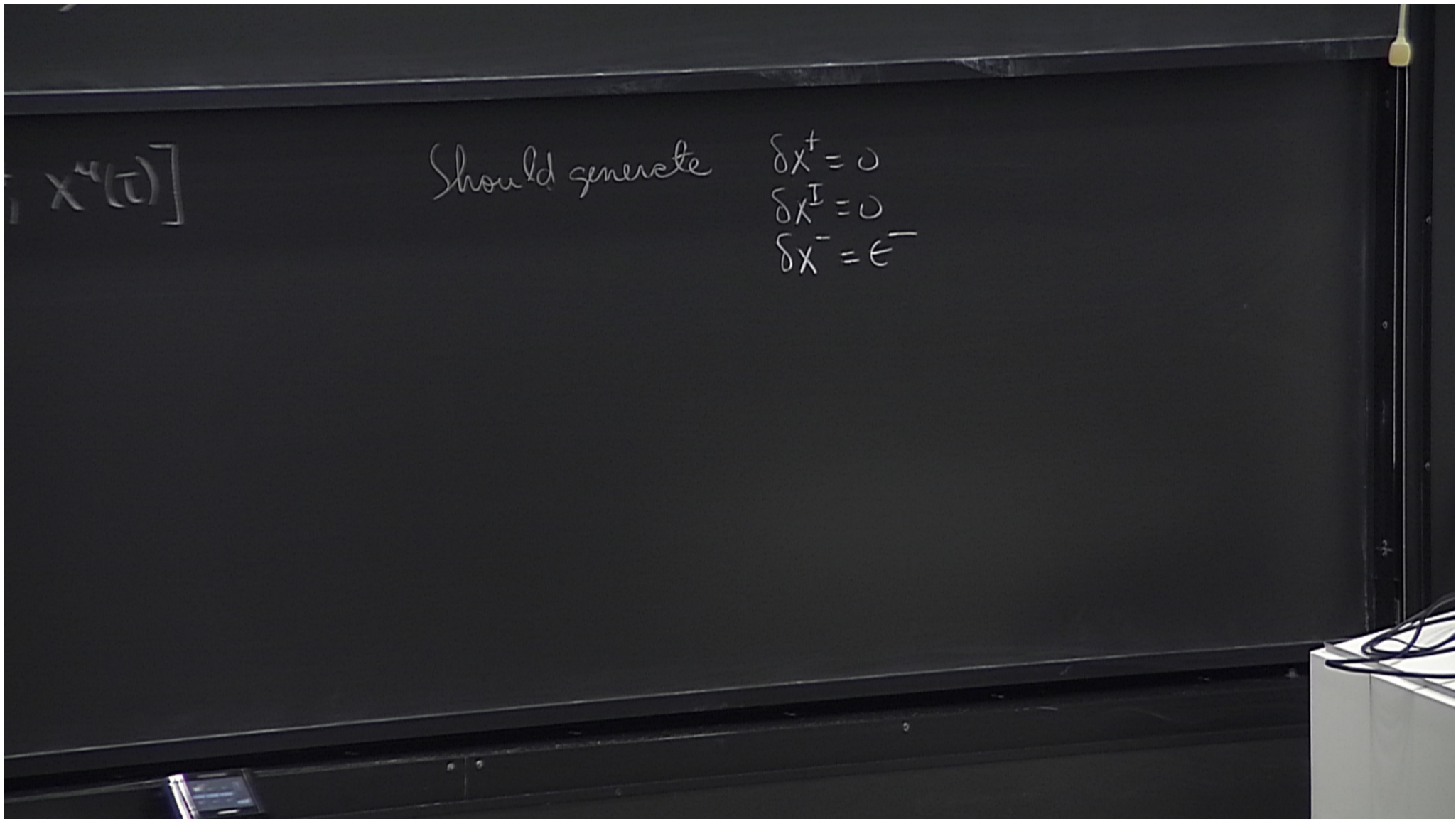
"Con. quantiz" $[X^\mu(\tau), P^\nu(\tau)] =$
 $\delta X^\mu(\tau) = \epsilon^\mu$

$$\delta X^\mu = [L$$

Lightcone $p^+, p^-, p^\perp$ they commute.

$$\delta X^\mu = ? \left[-\epsilon^+ p^- - \epsilon^- p^+ + i \epsilon^\perp p^\perp, X^\mu(\tau) \right]$$

Case ① Only $\epsilon^- \neq 0$, $\delta x^\mu = -i\epsilon^- [p_1^+, x^\mu(\tau)]$



Case ① Only $\epsilon^- \neq 0$.

$$\delta x^\mu = -i\epsilon^- \left[p_1^+ x^\mu(\tau) \right]$$

$$\delta x^+ = -i\epsilon^- \left[\frac{p_1^+ \tau}{m^2} \right]$$

Case ① Only $\epsilon^- \neq 0$,

$$\delta x^\mu = -i\epsilon^- [p^+, x^\mu(\tau)]$$

$$\delta x^+ = -i\epsilon^- [p^+, \frac{p^+ \tau}{m^2}] = 0$$

$$\delta x^I = -i\epsilon^- [p^+, x^I]$$

Case ① Only $\epsilon^- \neq 0$.

$$\delta x^\mu = -i\epsilon^- [p_1^+, x^\mu(\tau)]$$

$$\delta x^+ = -i\epsilon^- [p_1^+, \frac{p_1^+ \tau}{m^2}] = 0$$

$$\delta x^I = -i\epsilon^- [p_1^+, x_0^I + \frac{p_1^I \tau}{m^2}] = 0$$

$$\delta x^- = -i\epsilon^- [p_1^+, x_0^- + \frac{p_1^- \tau}{m^2}]$$

Case ① Only $\epsilon^- \neq 0$.

$$\delta x^\mu = -i\epsilon^- [p^+, x^\mu(\tau)]$$

$$\delta x^+ = -i\epsilon^- [p^+, \frac{p^+ \tau}{m^2}] = 0$$

$$\delta x^I = -i\epsilon^- [p^+, x_0^I + \frac{p^I \tau}{m^2}] = 0$$

$$\delta x^- = -i\epsilon^- [p^+, x_0^- + \frac{p^- \tau}{m^2}] = -i\epsilon^- (i) = \epsilon^-$$

Show

Case 1 Only $\epsilon^- \neq 0$.

$$\delta x^\mu = -i\epsilon^- [p_1^+, x^\mu(\tau)]$$

$$\delta x^+ = -i\epsilon^- [p_1^+, \frac{p_1^-}{m^2}] = 0$$

$$\delta x^I = -i\epsilon^- [p_1^+, x_0^I + \frac{p_1^I}{m^2}] = 0$$

$$\delta x^- = -i\epsilon^- [p_1^+, x_0^- + \frac{p_1^-}{m^2}] = -i\epsilon^- (i) = \epsilon^-$$

Case 2 Only $\epsilon^+ \neq 0$

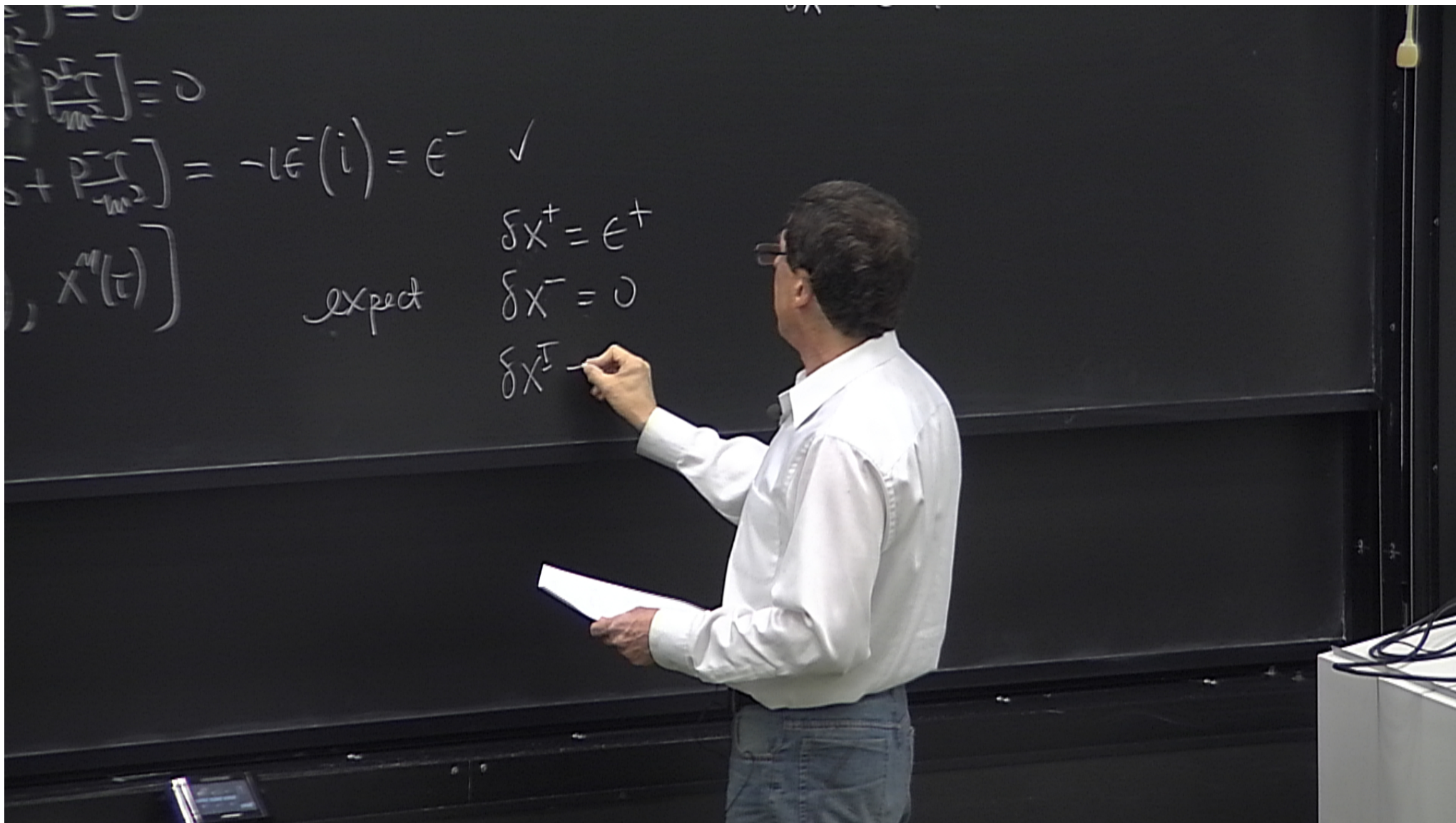
case 2 Only $\epsilon^+ \neq 0$

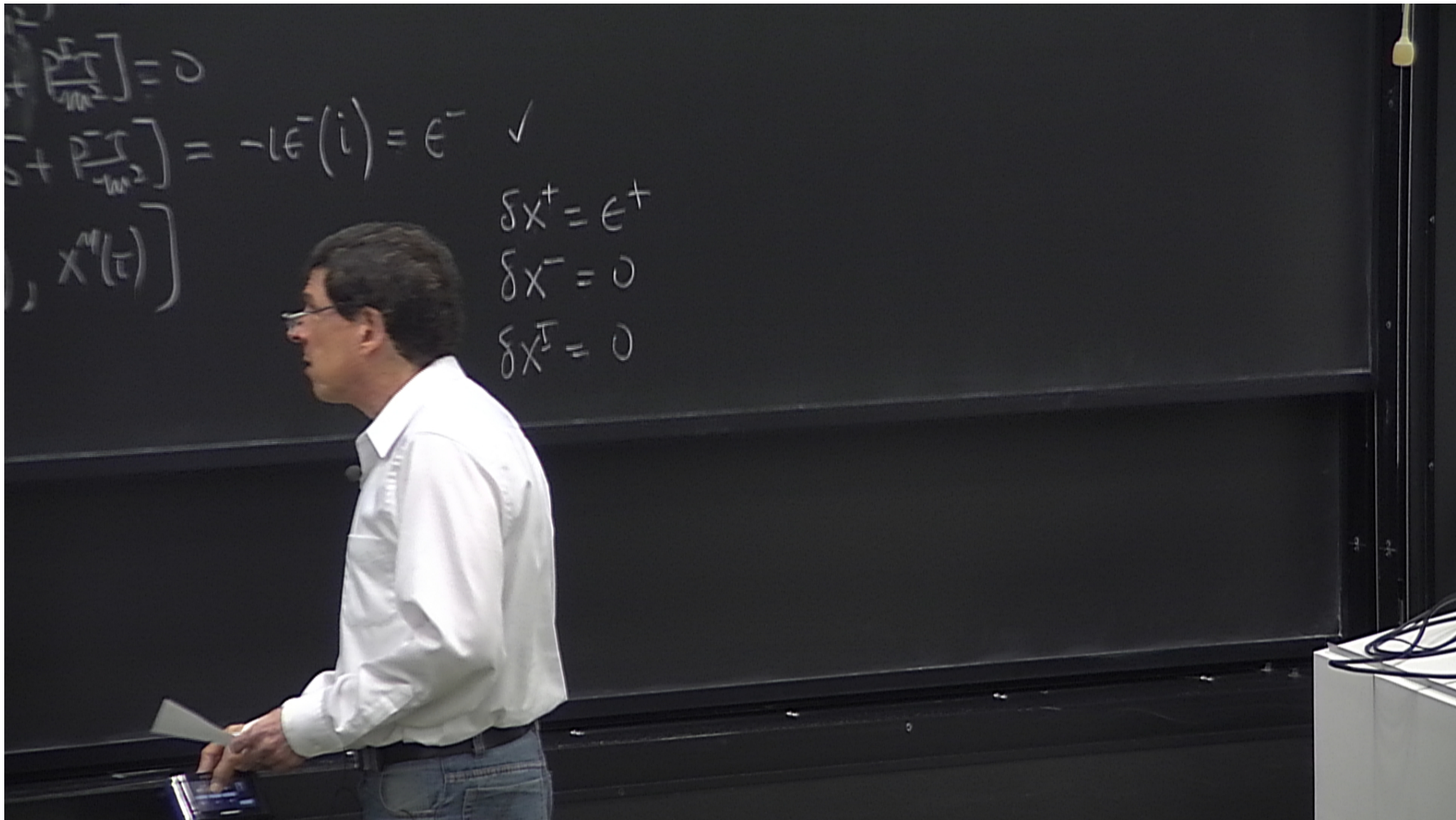
$$\delta X^I = -\epsilon^- \left[p^+, X_0^I + \frac{p^I \tau}{m^2} \right] = 0$$

$$\delta X^- = -\epsilon^- \left[p^+, X_0^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^- (i) = \epsilon^-$$

$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^\mu(\tau) \right]$$

ex





Case 2 Only $\epsilon^+ \neq 0$

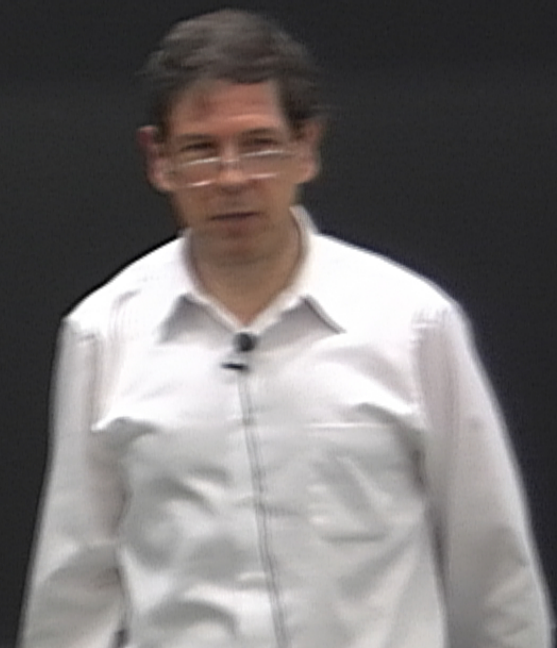
$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^\mu(\tau) \right]$$

$$\delta X^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right]$$

Case 2 Only $\epsilon^+ \neq 0$

$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^\perp p^\perp + m^2), X^\mu(\tau) \right]$$

$$\delta X^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$



Case 2 Only $\epsilon^+ \neq 0$

$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^\mu(\tau) \right]$$

$$\delta X^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta X^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^- + \frac{p^- \tau}{m^2} \right]$$

Case 2 Only $\epsilon^+ \neq 0$

$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^\mu(\tau) \right]$$

$$\delta X^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta X^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^- + \cancel{\frac{p^- \tau}{m^2}} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

Case 2 Only $\epsilon^+ \neq 0$

$$\delta x^+ = -\epsilon^+ \left[p^+, \frac{p^+_{\perp}}{m^2} \right] = 0$$

$$\delta x^I = -\epsilon^+ \left[p^+, x^I_0 + \frac{p^I_{\perp}}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[p^+, x^-_0 + \frac{p^-_{\perp}}{m^2} \right] = -\epsilon^+ (i) = \epsilon^- \quad \checkmark$$

$$\delta x^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I_{\perp} p^I_{\perp} + m^2), x^\mu(t) \right]$$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+_{\perp}}{m^2} \right] = 0$$

expect

$$\begin{aligned} \delta x^+ &= \epsilon^+ \quad \times \\ \delta x^- &= 0 \quad \times \\ \delta x^I &= 0 \end{aligned}$$

$$\begin{aligned} \delta x^+ &= 0 \\ \delta x^- &= \epsilon^- \end{aligned}$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I_{\perp} p^I_{\perp} + m^2), x^-_0 + \frac{p^-_{\perp}}{m^2} \right] = -\epsilon^+ \frac{p^-_{\perp}}{p^+}$$

Case 2 Only $\epsilon^+ \neq 0$

$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^\mu(\tau) \right]$$

$$\delta X^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta X^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta X^I = -\epsilon^+ \frac{p^I}{p^+}}$$

Case 2 Only $\epsilon^+ \neq 0$

$$\delta x^+ = -\epsilon^+ \left[p^+, \frac{p^+_{\perp}}{m^2} \right] = 0$$

$$\delta x^I = -\epsilon^+ \left[p^+, x^I_0 + \frac{p^I_{\perp}}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[p^+, x^-_0 + \frac{p^-_{\perp}}{m^2} \right] = -\epsilon^+ (i) = \epsilon^- \quad \checkmark$$

$$\delta x'' = -\epsilon^+ \left[\frac{1}{2p^+} (p^I_{\perp} p^I_{\perp} + m^2), x''(t) \right]$$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+_{\perp}}{m^2} \right] = 0$$

expect

$$\delta x^+ = \epsilon^+ \quad \times$$

$$\delta x^- = 0 \quad \times$$

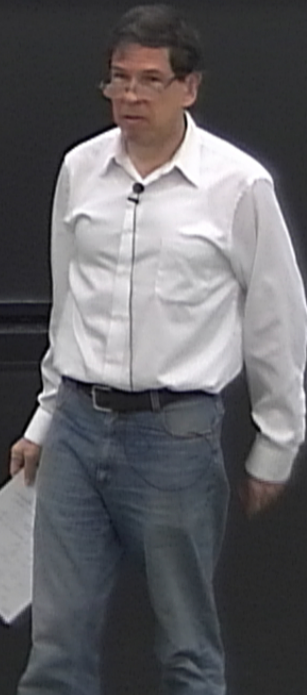
$$\delta x^I = 0 \quad \times$$

$$\delta x^+ = 0$$

$$\delta x^- = \epsilon^-$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I_{\perp} p^I_{\perp} + m^2), x^-_0 + \frac{p^-_{\perp}}{m^2} \right] = -\epsilon^+ \frac{p^-_{\perp}}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I_{\perp}}{p^+}}$$



Case 2 Only $\epsilon^+ \neq 0$

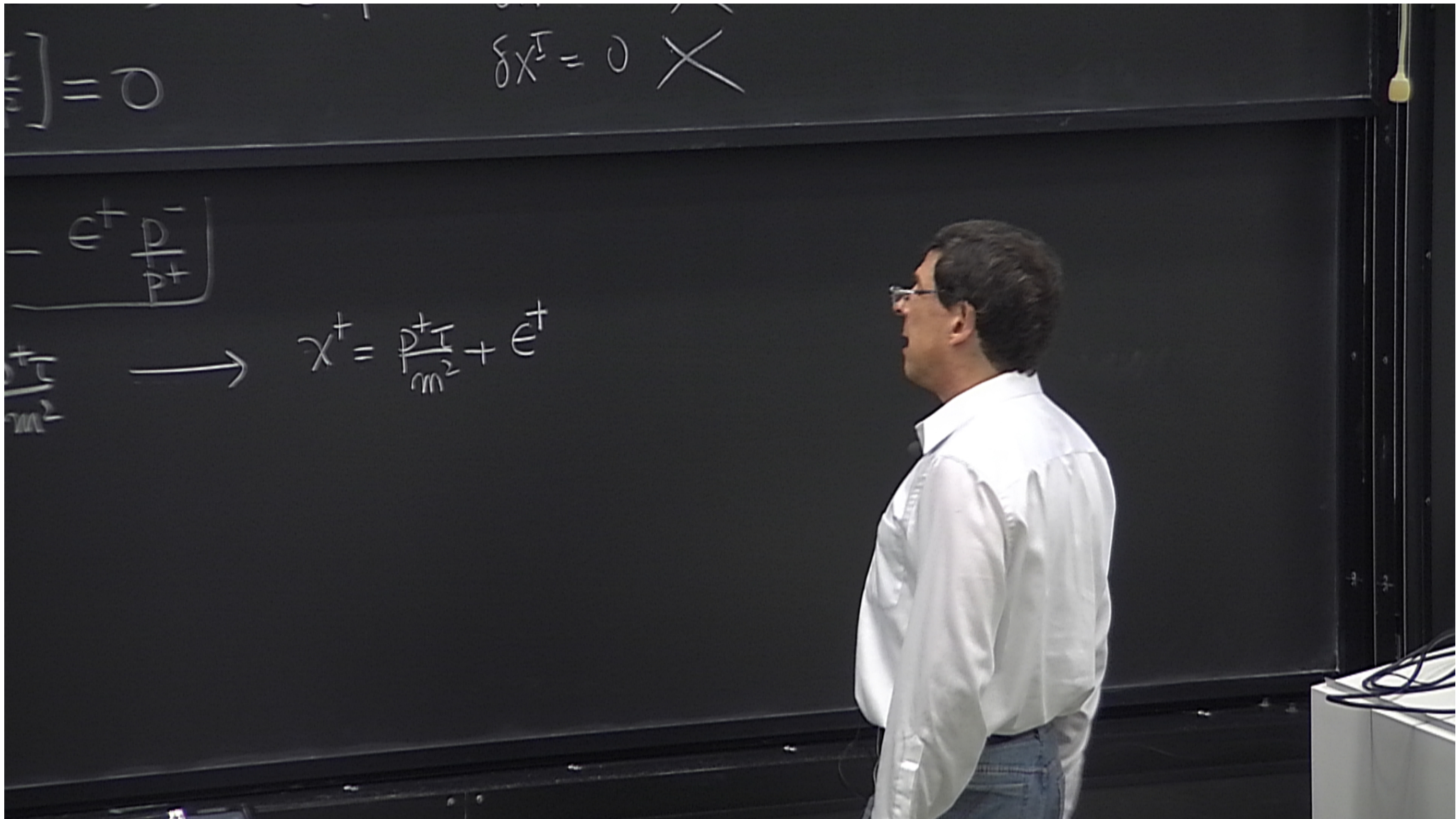
$$\delta X^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^\mu(\tau) \right]$$

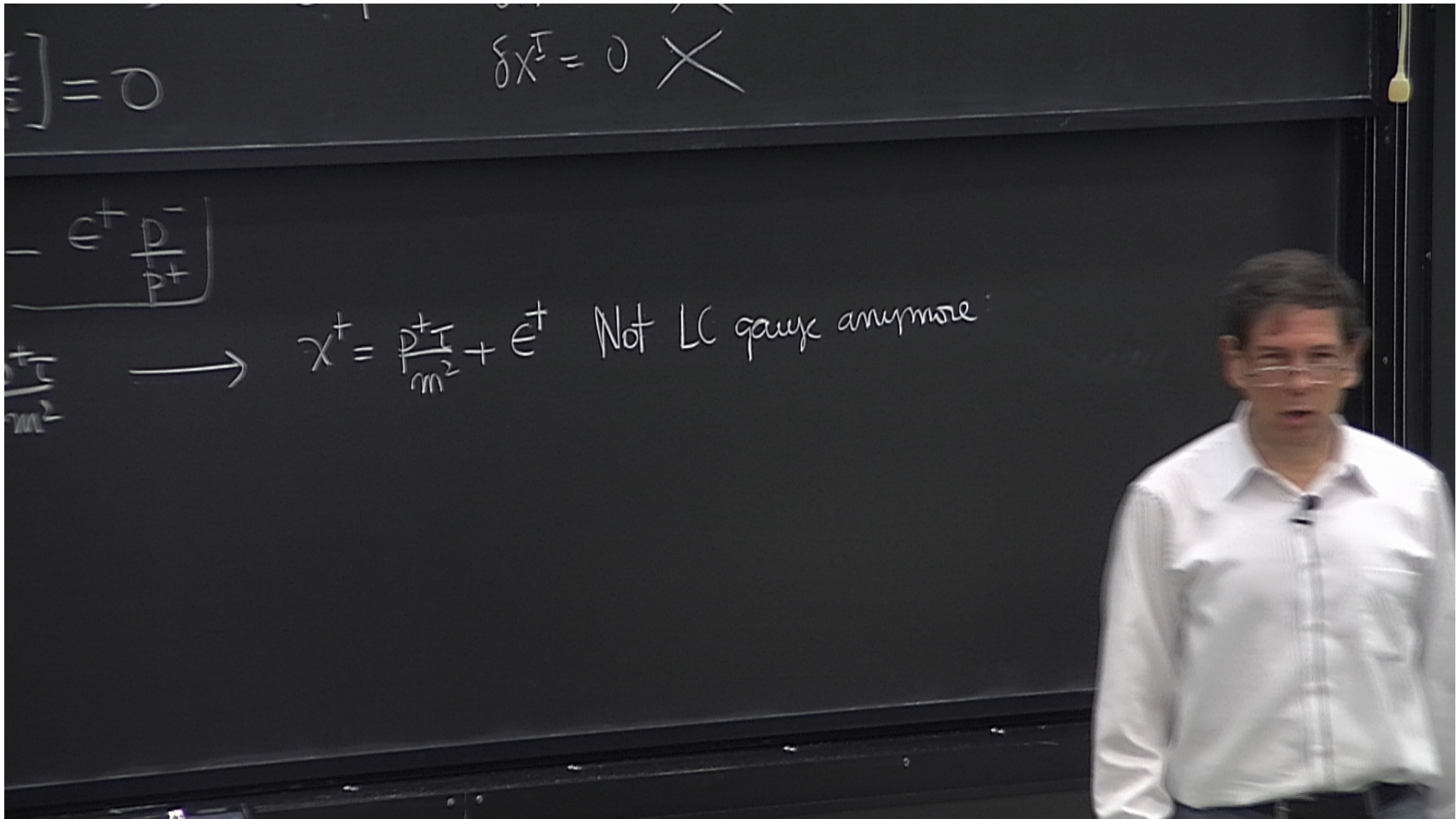
$$\delta X^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta X^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), X^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta X^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge - $X^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow$





$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge

$$\frac{p^+ \tau}{m^2} \rightarrow$$

$$x^\mu(\tau) \rightarrow x^\mu(\tau + \lambda(\tau))$$

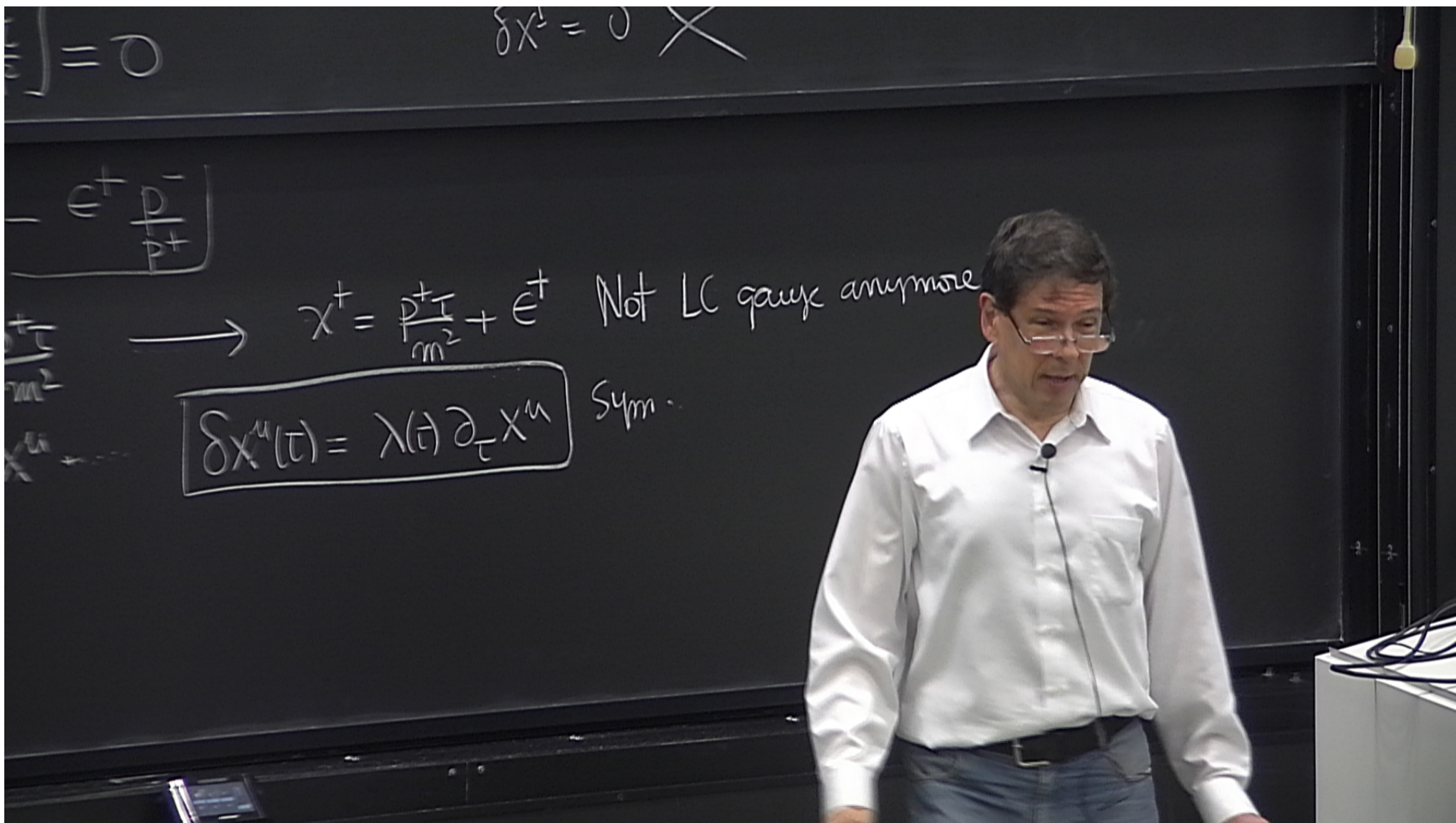
$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge - $x^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow \tau$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu + \dots \quad \delta x^\mu(\tau)$$



$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x_0^- + \frac{p^+ \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

$$\underline{L.C} \text{ gauge} - x^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow ?$$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu + \dots \quad \boxed{\delta x^\mu(\tau)}$$

$p^- \rightarrow \text{trans. + reparametrization}$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^- + \frac{p^+ \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge - $x^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow \tau$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu + \dots \quad \boxed{\delta x^\mu(\tau)}$$

$p^- \rightarrow$ trans. + reparametrization $x^+ \rightarrow x^+ + \epsilon^+$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^- + \frac{p^+ \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge - $x^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow ?$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu + \dots \quad \boxed{\delta x^\mu(\tau)}$$

$p^- \rightarrow$ trans. + reparametrization

$$x^+ \longrightarrow x^+ + \underbrace{\epsilon^+ + \lambda \partial_\tau x^+}$$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge - $x^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow ?$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu + \dots \quad \boxed{\delta x^\mu(\tau)}$$

$p^- \rightarrow$ trans. + reparametrization

$$x^+ \longrightarrow x^+ + \underbrace{\epsilon^+ + \lambda \partial_\tau x^+}_0$$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+ \tau}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^- + \frac{p^- \tau}{m^2} \right] = -\epsilon^+ \frac{p^-}{p^+}$$

$$\boxed{\delta x^I = -\epsilon^+ \frac{p^I}{p^+}}$$

L.C gauge - $x^+ \equiv \frac{p^+ \tau}{m^2} \longrightarrow \tau$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu + \dots \quad \boxed{\delta x^\mu(\tau)}$$

$p^- \rightarrow$ trans. + reparametrization

$$x^+ \longrightarrow x^+ + \underbrace{\epsilon^+ + \lambda \partial_\tau x^+}_0 \quad \epsilon^+ +$$

$$\epsilon^-] = 0 \quad \delta X^+ = 0 \quad \times$$

$$\left[\begin{array}{c} \epsilon^+ \\ \frac{p^-}{p^+} \end{array} \right]$$

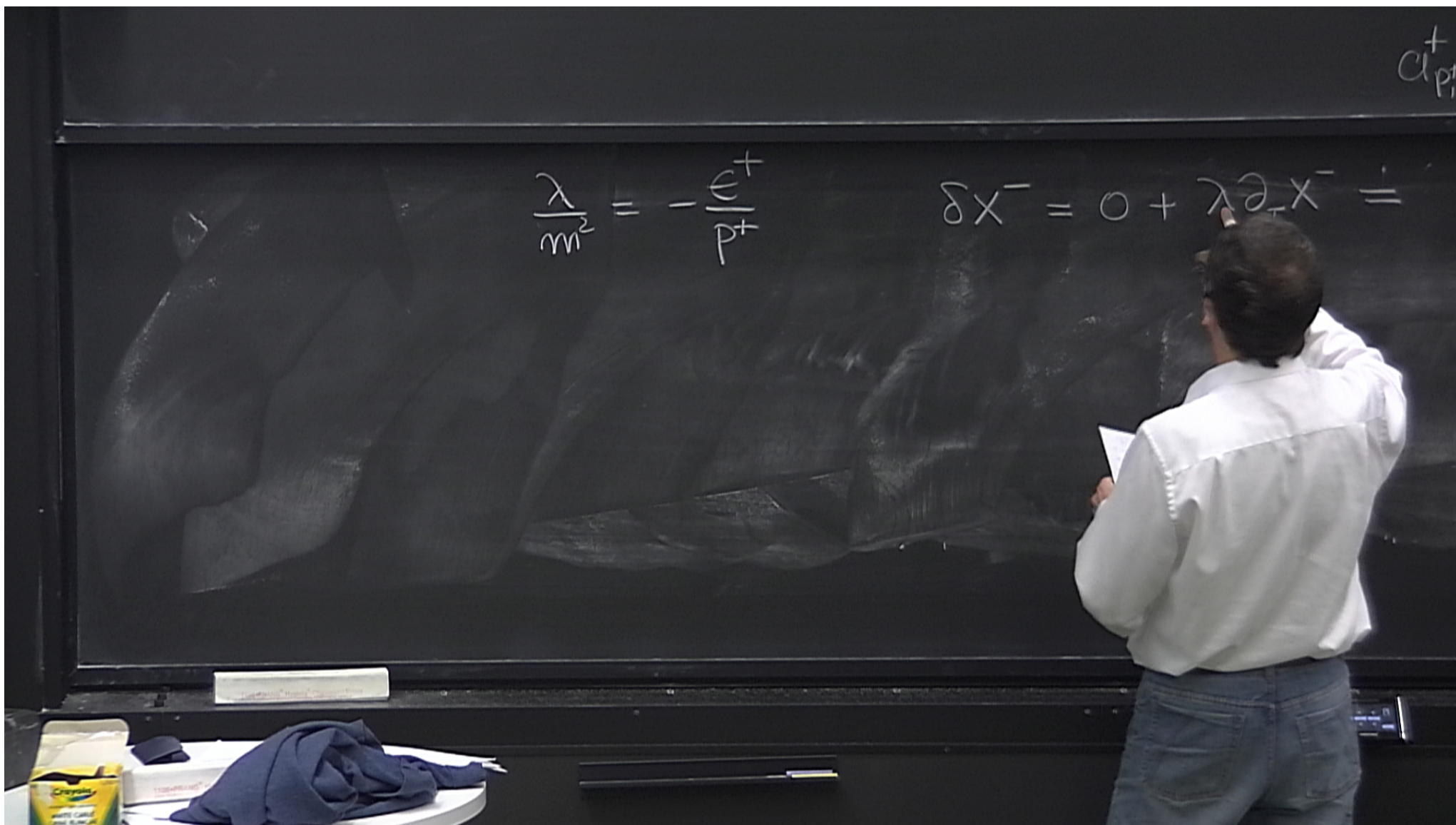
$$\rightarrow X^+ = \frac{p^+ \tau}{m^2} + \epsilon^+ \quad \text{Not LC gauge anymore}$$

$$\boxed{\delta X^u(\tau) = \lambda(\tau) \partial_\tau X^u} \quad \text{Sym.}$$

$$\underbrace{\lambda \partial_\tau X^+}_0 \quad \epsilon^+ + \lambda \frac{p^+}{m^2} = 0$$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

α^+
 p^+



$$a_{p_1}^+, a_{p_1}, a^+, \dots, a^+$$

$$\partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+}$$

$$a_{p^+ p^-}^+, a^+ \dots a^+$$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

$$\begin{aligned} \delta X^- &= 0 + \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+} \\ \delta X^I &= 0 + \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+} \end{aligned}$$

$a_{p^+}^\dagger, p^+, a^+ \dots a^+$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

$$\begin{aligned}\delta X^- &= 0 + \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+} \\ \delta X^I &= 0 + \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+}\end{aligned}$$

$$a_{p^+}^\dagger, p^+, a^+ \dots a^+$$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

$$\begin{aligned} \delta X^- &= 0 + \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+} \\ \delta X^I &= 0 + \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+} \end{aligned}$$

$$\delta X^P(\tau) = e^{P_V} \chi_V(\tau)$$

$$a_{p^+}^\dagger, p^+, a^\dagger, \dots, a^\dagger$$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

$$\delta X^- = 0 + \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+}$$

$$\delta X^I = 0 + \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+}$$

$$x(\tau) = e^{p^+ \tau} \chi_V(\tau)$$

$$M^{\mu\nu} = x^\mu(\tau) p^\nu(\tau) - x^\nu(\tau) p^\mu(\tau)$$

$$\alpha_{p^+}^\dagger, p^+, a^\dagger, a$$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

$$\delta X^- = 0 + \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+}$$

$$\delta X^I = 0 + \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+}$$

$$\delta X^P(\tau) = \epsilon^{P\nu} X_\nu(\tau)$$

$$M^{\mu\nu} = X^\mu(\tau) p^\nu(\tau) - X^\nu(\tau) p^\mu(\tau)$$

$$\delta X^P = \left[-\frac{i}{2} \epsilon_{\mu\nu} M^{\mu\nu}, X^P(\tau) \right]$$

$$a_{p^+}^\dagger, p^+, a^+ \dots a^+$$

$$\frac{\lambda}{m^2} = -\frac{e^+}{p^+}$$

$$\delta X^- = 0 + \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+}$$

$$\delta X^I = 0 + \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+}$$

$$\delta X^p(\tau) = \delta x^p(\tau)$$

$$M^{\mu\nu} = x^\mu(\tau) p^\nu(\tau) - x^\nu(\tau) p^\mu(\tau)$$

$$[M^{\mu\nu}, x^p(\tau)]$$

$$[M^{\mu\nu}, M^{\rho\sigma}]$$

$$a_{p_1^+ p_1}^+, a^+ \dots a^+$$

$$\partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+}$$

$$\partial_\tau X^\perp = \lambda \frac{p^\perp}{m^2} = -e^+ \frac{p^\perp}{p^+}$$

$$x^\mu(\tau) p^\nu(\tau) - x^\nu(\tau) p^\mu(\tau)$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\mu\rho} M^{\nu\sigma} - \eta^{\mu\sigma} M^{\nu\rho} + \dots) \text{ 3 more terms}$$

Case 2 Only $\epsilon^+ \neq 0$

P^-

$$\delta x^+ = -\epsilon^+ \left[p^+, \frac{p^+_{\tau}}{m^2} \right] = 0$$

$$\delta x^I = -\epsilon^+ \left[p^+, x^I_0 + \frac{p^I_{\tau}}{m^2} \right] = 0$$

$$\delta x^- = -\epsilon^+ \left[p^+, x^-_0 + \frac{p^-_{\tau}}{m^2} \right] = -\epsilon^+(i) = \epsilon^- \quad \checkmark$$

$$\delta x^\mu = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^\mu(\tau) \right]$$

$$\delta x^+ = -\epsilon^+ \left[\dots, \frac{p^+_{\tau}}{m^2} \right] = 0$$

expect

$$\delta x^+ = \epsilon^+ \quad \times$$

$$\delta x^- = 0 \quad \times$$

$$\delta x^I = 0 \quad \times$$

$$\delta x^+ = 0$$

$$\delta x^- = \epsilon^-$$

$$\delta x^- = -\epsilon^+ \left[\frac{1}{2p^+} (p^I p^I + m^2), x^-_0 + \frac{p^-_{\tau}}{m^2} \right] = -\epsilon^+ \frac{p^-_{\tau}}{p^+}$$

$$\left[\delta x^I = -\epsilon^+ \frac{p^I_{\tau}}{p^+} \right] \checkmark \text{ L.C gauge - } x^+ \equiv \frac{p^+_{\tau}}{m^2} \longrightarrow x^+ = \frac{p^+_{\tau}}{m^2} + \epsilon^+ \text{ Not LC gauge anymore}$$

$$x^\mu(\tau) \longrightarrow x^\mu(\tau + \lambda(\tau)) = x^\mu(\tau) + \lambda(\tau) \partial_\tau x^\mu \quad \boxed{\delta x^\mu(\tau) = \lambda(\tau) \partial_\tau x^\mu} \text{ Sym.}$$

$p^- \rightarrow$ trans + reparametrization

$$x^+ \longrightarrow x^+ + \underbrace{\epsilon^+ + \lambda \partial_\tau x^+}_0 \quad \epsilon^+ + \lambda \frac{p^+_{\tau}}{m^2} = 0$$

$$a_{p_1}^+, p_1, a^+, a^+$$

$$+ \lambda \partial_\tau X^- = \lambda \frac{p^-}{m^2} = -e^+ \frac{p^-}{p^+}$$

$$+ \lambda \partial_\tau X^I = \lambda \frac{p^I}{m^2} = -e^+ \frac{p^I}{p^+}$$

$$= \underline{X^\mu(\tau) p^\nu(\tau) - X^\nu(\tau) p^\mu(\tau)}$$

Manifestly Hermitian

$$[M^{-I}, M^{-J}] = 0$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\mu\rho} M^{\nu\sigma} - \eta^{\mu\sigma} M^{\nu\rho} + \dots)$$

3 more terms



$$M^{-I} = \chi^-(\tau) p^I - \chi^I(\tau) p^-$$

$$M^{-I} \quad \tau \quad - \quad , I$$

$$X^-(\tau) p^I - X^I(\tau) p^-$$

$$\left(X_0^- + \frac{p^- \tau}{m^2} \right) p^I - \left(X_0^I + \frac{p^I \tau}{m^2} \right) p^-$$

$$X_0^- p^I$$

C, p_i, p_t, a, c

$$\begin{aligned}
 M^{-I} &= \chi^-(\tau) p^I - \chi^I(\tau) p^- \\
 &= \left(\chi_0^- + \cancel{\frac{p^- \tau}{m^2}} \right) p^I - \left(\chi_0^I + \cancel{\frac{p^I \tau}{m^2}} \right) p^- \\
 &= \chi_0^- p^I - \chi_0^I p^-
 \end{aligned}$$

$$\begin{aligned}
 M^{-I} &\stackrel{?}{=} \chi^-(\tau) p^I - \chi^I(\tau) p^- \\
 &\stackrel{?}{=} \left(\chi_0^- + \cancel{\frac{p^- \tau}{m^2}} \right) p^I - \left(\chi_0^I + \cancel{\frac{p^I \tau}{m^2}} \right) p^- \\
 &\stackrel{?}{=} \chi_0^- p^I - \chi_0^I p^-
 \end{aligned}$$

$c, p_i, p_t, a, \dots, c$

$$\begin{aligned}
 M^{-I} &\stackrel{?}{=} \chi^-(\tau) p^I - \chi^I(\tau) p^- \\
 &\stackrel{?}{=} \left(\chi_0^- + \cancel{\frac{p^- \tau}{m^2}} \right) p^I - \left(\chi_0^I + \cancel{\frac{p^I \tau}{m^2}} \right) p^- \\
 &\stackrel{?}{=} \chi_0^- p^I - \chi_0^I p^-
 \end{aligned}$$

$$M^{-I} = \chi_0^- p^I - \frac{1}{2} (\chi_0^I p^- + p^- \chi_0^I)$$

c, p_1, p_2, a, d

$$\begin{aligned}
 -I &\stackrel{?}{=} \chi^-(\tau) p^I - \chi^I(\tau) p^- \\
 &\stackrel{?}{=} \left(\chi_0^- + \cancel{\frac{p^- \tau}{m^2}} \right) p^I - \left(\chi_0^I + \cancel{\frac{p^I \tau}{m^2}} \right) p^- \\
 &\stackrel{?}{=} \chi_0^- p^I - \chi_0^I p^-
 \end{aligned}$$

$$M^{-I} = \chi_0^- p^I - \frac{1}{2} (\chi_0^I p^- + p^- \chi_0^I)$$

$$(M^{-I}, M^{-J}) = 0$$

$\mathcal{M} \cdot D = 26$

α, p_i, p_i, a

$$\begin{aligned}
 -I &\stackrel{?}{=} \chi^-(\tau) p^I - \chi^I(\tau) p^- \\
 &\stackrel{?}{=} \left(\chi_0^- + \cancel{\frac{p^- \tau}{m^2}} \right) p^I - \left(\chi_0^I + \cancel{\frac{p^I \tau}{m^2}} \right) p^- \\
 &\stackrel{?}{=} \chi_0^- p^I - \chi_0^I p^-
 \end{aligned}$$

$$M^{-I} = \chi_0^- p^I - \frac{1}{2} (\chi_0^I p^- + p^- \chi_0^I)$$

$$(M^{-I}, M^{-J}) = 0$$

if $D=26$.

$\alpha_{-1}^I, \alpha_{-1}^J, \alpha$