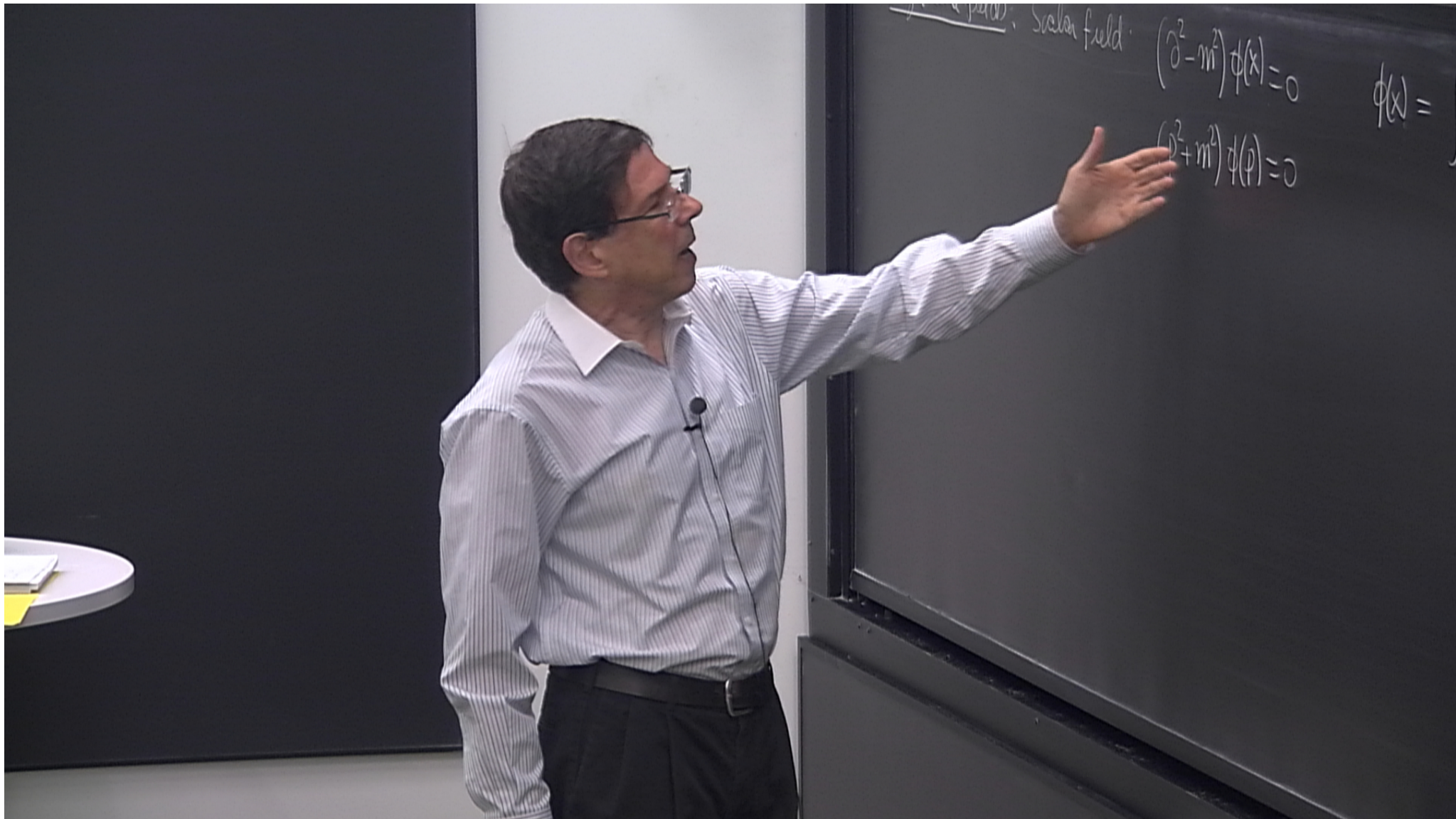


Title: 12/13 PSI - String Theory Review Lecture 8

Date: Jan 16, 2013 10:15 AM

URL: <http://pirsa.org/13010054>

Abstract:



Light cone fields: Scalar field

$$(\partial^2 - m^2) \phi(x) = 0$$

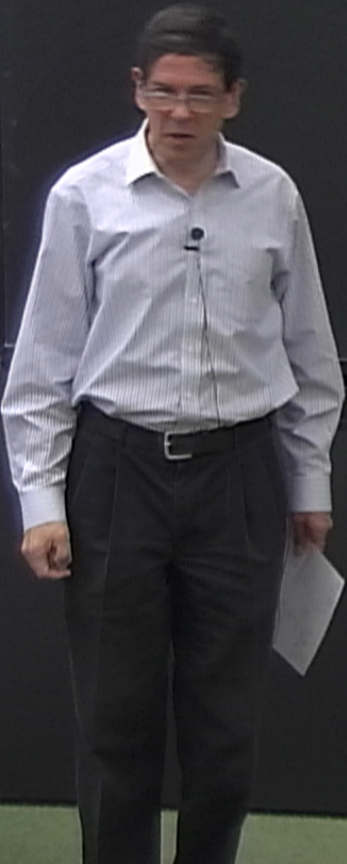
$$(p^2 + m^2) \phi(p) = 0$$

$$\phi(x) = \int \frac{d^D p}{(2\pi)^D} e^{i p \cdot x} \phi(p)$$

$\left\{ \begin{array}{l} \text{If } p^2 + m^2 \neq 0, \text{ then } \phi(p) = 0 \\ \text{If } p^2 + m^2 = 0, \text{ then } \phi(p) \text{ is arbitrary} \end{array} \right.$

$\phi(x)$ real

$$(\phi(p))^* = \phi(-p)$$



2: Scalar field · $(\partial^2 - m^2) \phi(x) = 0$ $\phi(x) = \int \frac{d^D p}{(2\pi)^D} e^{i p \cdot x} \phi(p)$ $\phi(x)$

$(p^2 + m^2) \phi(p) = 0$ $\left\{ \begin{array}{l} \text{If } p^2 + m^2 \neq 0, \text{ then } \phi(p) = 0 \\ \text{If } p^2 + m^2 = 0, \text{ then } \phi(p) \text{ is arbitrary} \end{array} \right.$ $\uparrow$ complex

$p^2 + m^2 = -E^2 + \vec{p}^2 + m^2 = 0$

$E_p = \sqrt{\vec{p}^2 + m^2}$

$$m^2) \phi(x) = 0$$

$$m^2) \phi(p) = 0$$

$\left\{ \begin{array}{l} \text{If } p^2 + m^2 \neq 0, \text{ then } \phi(p) = 0 \\ \text{If } p^2 + m^2 = 0, \text{ then } \phi(p) \text{ is arbitrary} \end{array} \right.$

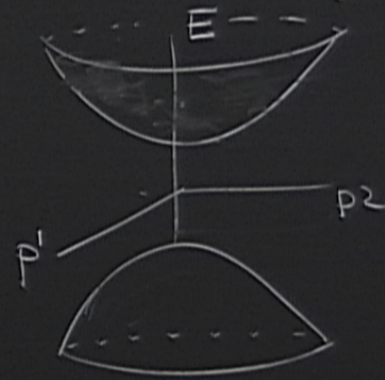
$$p^2 + m^2 = -E^2 + \vec{p}^2 + m^2 = 0$$

$$E_p = \sqrt{\vec{p}^2 + m^2}$$

$$\phi(x) = \int \frac{d^D p}{(2\pi)^D} e^{i p \cdot x} \phi(p)$$

↑ complex

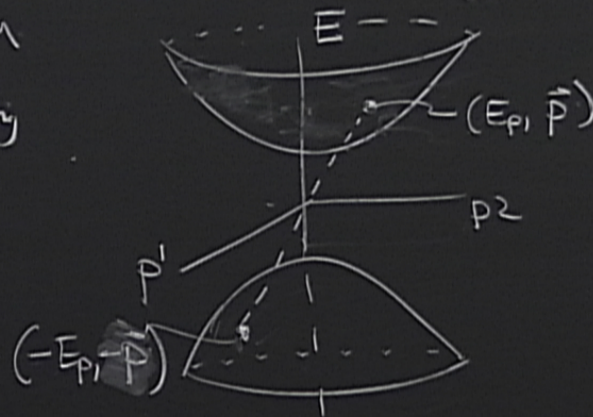
$$\phi(x) \text{ real} \quad (\phi(p))^* = \phi(-p)$$



$$= \int \frac{d^D p}{(2\pi)^D} e^{i p \cdot x} \phi(p)$$

$\neq 0$, $\phi(p) = 0$
 $\phi(p)$ is arbitrary

$$\phi(x) \text{ real} \quad (\phi(p))^* = \phi(-p)$$



$\phi(p)$ 2 real #s
 fixes the value
 of the scalar at two
 points, $p, -p$

Light Cone fields: Scalar field

$$(\partial^2 - m^2)\phi(x) = 0$$

$$\phi(x) = \int \frac{d^D p}{(2\pi)^D} e^{i p \cdot x} \phi(p)$$

Quantum field

$$(p^2 + m^2)\phi(p) = 0 \quad \begin{cases} \text{If } p^2 + m^2 \neq 0, \text{ then } \phi(p) = 0 \\ \text{If } p^2 + m^2 = 0, \text{ then } \phi(p) \text{ is arbit} \end{cases}$$

$$\phi(t, \vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}} e^{-iE_p t + i\vec{p} \cdot \vec{x}} + a_{\vec{p}}^\dagger e^{iE_p t - i\vec{p} \cdot \vec{x}} \right) \quad p^2 + m^2 = -E^2 + \vec{p}^2 + m^2 = 0$$

$$E_p = \sqrt{\vec{p}^2 + m^2}$$

$$[a_p, a_k^\dagger] = \delta_{p,k}$$

Particle states

$|\Omega\rangle$ vacuum

$a_{\vec{p}}^\dagger |\Omega\rangle$ 1-particle momentum $\vec{p}$

Scalar field has 1 D.O.F

$$a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger \dots a_{\vec{p}_n}^\dagger$$

$$\phi(x) = \int \frac{d^D p}{(2\pi)^D} e^{i p \cdot x} \phi(p)$$

↑ complex

$\left\{ \begin{array}{l} \text{If } p^2 + m^2 \neq 0, \text{ then } \phi(p) = 0 \\ \text{If } p^2 + m^2 = 0, \text{ then } \phi(p) \text{ is arbitrary} \end{array} \right.$

$$m^2 = -E^2 + \vec{p}^2 + m^2 = 0$$

$$E_p = \sqrt{\vec{p}^2 + m^2}$$

$|\Omega\rangle$ vacuum

$|\Omega\rangle$ 1-particle mom $\vec{p}$

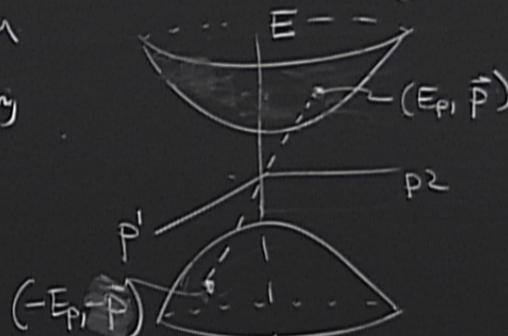
Scalar field
has 1 D.O.F

$$a_{\vec{p}_1}^\dagger a_{\vec{p}_2}^\dagger \dots a_{\vec{p}_n}^\dagger |\Omega\rangle$$

multiparticle $\rightarrow$ n-particles

$$\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n$$

$\phi(x)$ real $(\phi(p))^* = \phi(-p)$



$\phi(p)$ 2 real #s
fixes the value
of the scalar at two
points, $p, -p$

Light Cone fields: Scalar field

$$(\partial^2 - m^2) \phi(x) = 0$$

$$\phi(x) = \int \frac{d^4 p}{(2\pi)^4}$$

Quantum field

$$(p^2 + m^2) \phi(p) = 0 \quad \begin{cases} \text{If } p^2 + m^2 \neq 0, \text{ then} \\ \text{If } p^2 + m^2 = 0, \text{ then} \end{cases}$$

$$\phi(t, \vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}} e^{-iE_p t + i\vec{p} \cdot \vec{x}} + a_{\vec{p}}^\dagger e^{iE_p t - i\vec{p} \cdot \vec{x}} \right) \quad p^2 + m^2 = -E^2 + \vec{p}^2 + m^2 = 0$$

$E_p = \sqrt{\vec{p}^2 + m^2}$

$$[a_p, a_k^\dagger] = \delta_{p,k}$$

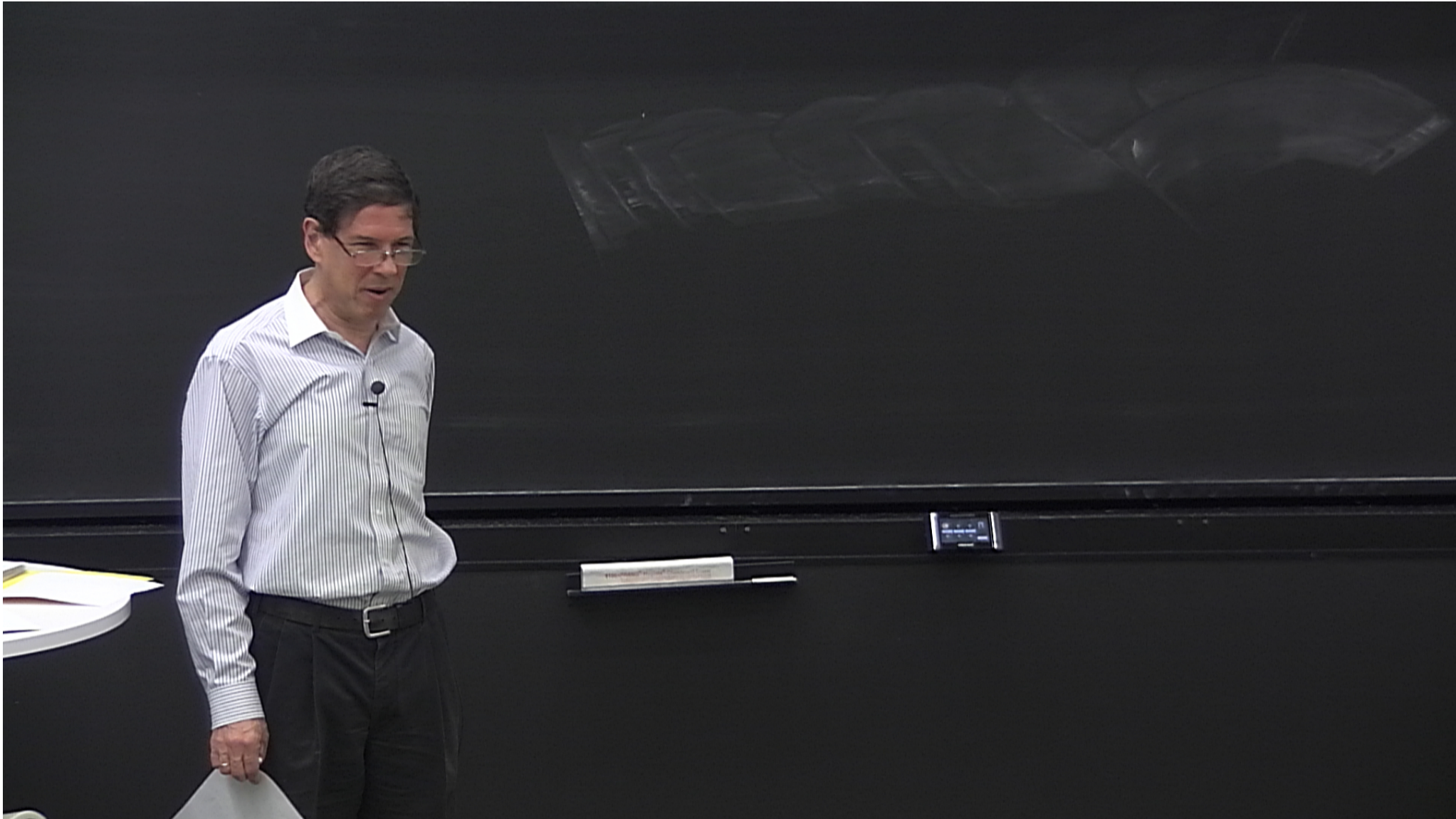
Particle states $|\Omega\rangle$ vacuum

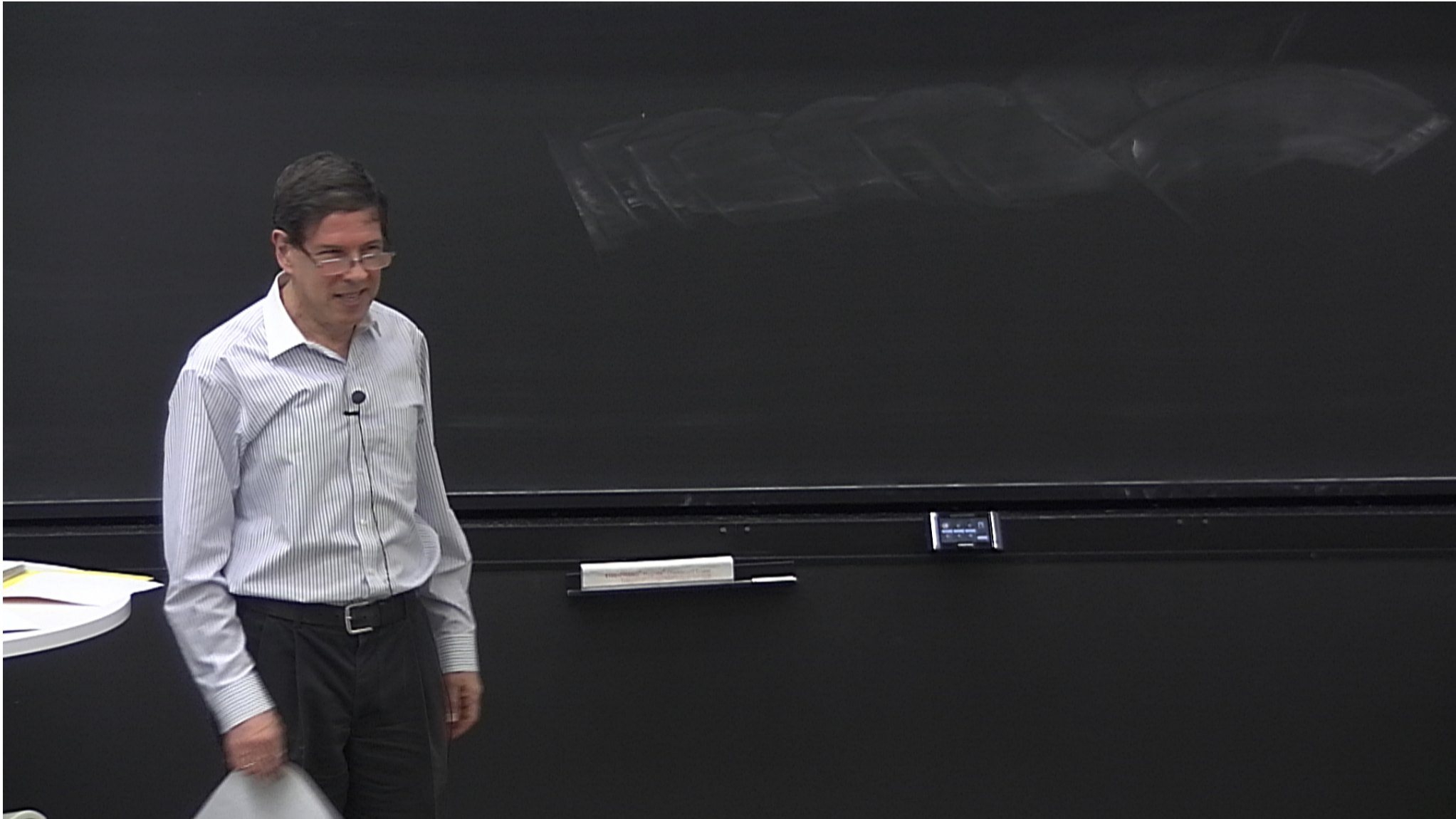
$$\hat{p} = \sum_{\vec{k}} \vec{k} a_{\vec{k}}^\dagger a_{\vec{k}}$$

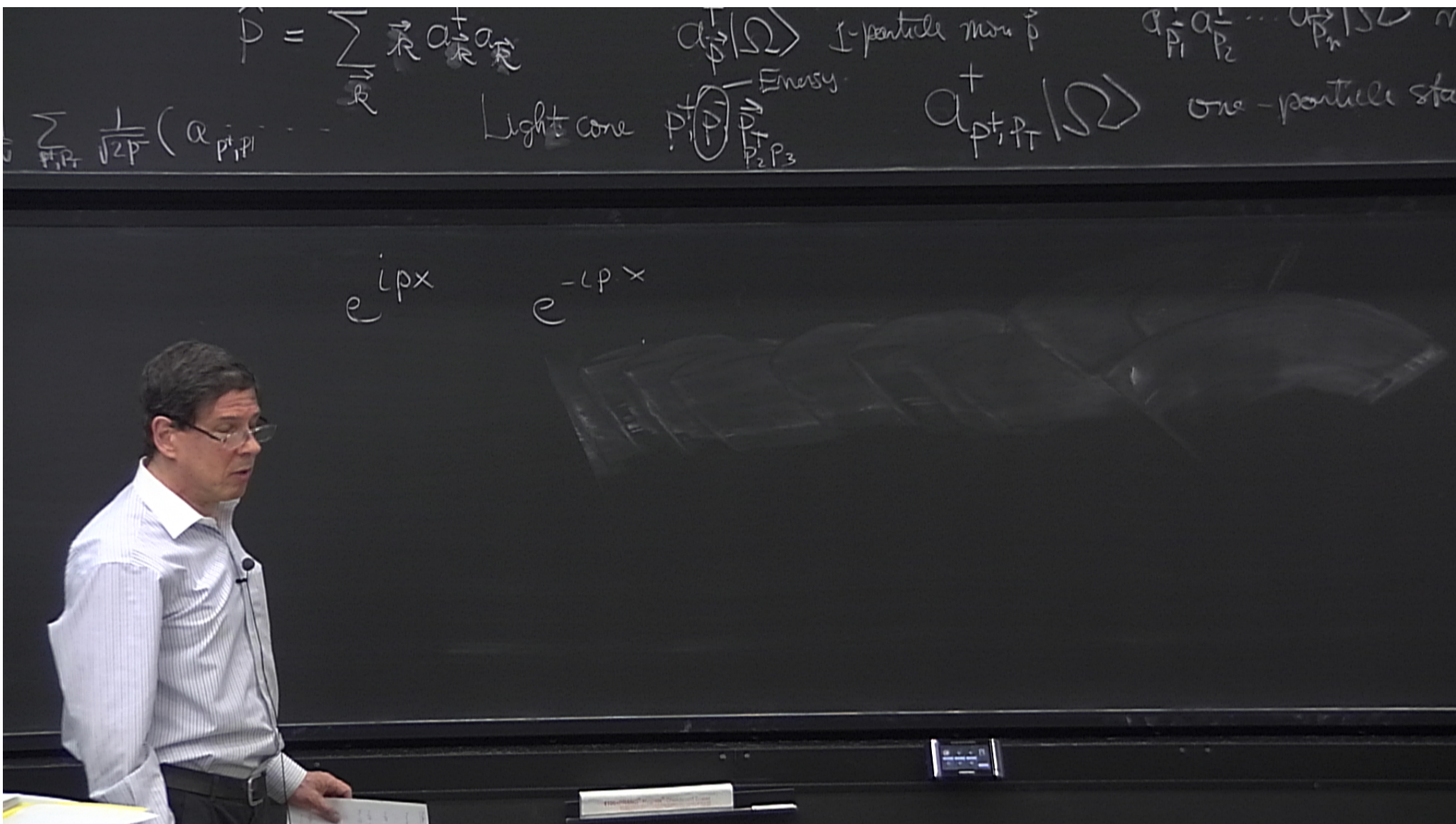
$a_{\vec{p}}^\dagger |\Omega\rangle$ 1-particle momentum $\vec{p}$

Light cone $p_+ = E + p_z$ Energy

$$a_{p_+^\dagger, p_+}^\dagger |\Omega\rangle$$







$$\vec{p} = \sum_{\vec{k}} \vec{k} a_{\vec{k}}^{\dagger} a_{\vec{k}}$$

$a_{\vec{p}}^{\dagger} |\Omega\rangle$ 1-particle momentum $\vec{p}$

$a_{\vec{p}_1}^{\dagger} a_{\vec{p}_2}^{\dagger} \dots a_{\vec{p}_n}^{\dagger} |\Omega\rangle$ n-particle state

$$\sum_{\vec{p}, \vec{p}_T} \frac{1}{\sqrt{2p}} (a_{\vec{p}_T, \vec{p}}^{\dagger} + a_{\vec{p}_T, \vec{p}})$$

Light cone $p^{\pm} = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$ Energy

$a_{p^+, p_T}^{\dagger} |\Omega\rangle$ one-particle state

Photon States

$$\partial^2 A^{\mu} - \partial^{\mu} (\partial \cdot A) = 0$$

$$\delta A_{\mu} = \partial_{\mu} \epsilon$$

$A(p)$

$$p^2 A^{\mu} - p^{\mu} (p \cdot A) = 0$$

$$\delta A_{\mu}(p) = i p_{\mu} \epsilon$$

Light cone gauge: $A^{\mu} = \{A^+, A^-, A^I\}$

$\delta A^+(p) = i p^+ \epsilon$, can set $A^+ = 0$

$\mu = +$

$$-p^+ (p \cdot A) = 0$$

$$\epsilon = -\frac{A^+}{p^+}$$

$$p \cdot A = 0 \rightarrow -p^+ A^- - \vec{p} \cdot \vec{A} + p^I A^I$$

$$A^- = \frac{1}{p^+} (\vec{p} \cdot \vec{A})$$

Photon States

$A(p)$

light cone

$$p(\phi) = 0$$

gauge

$$\partial^2 A^\mu - \partial^\mu (\partial \cdot A) = 0$$

$$p^2 A^\mu - p^\mu (p \cdot A) = 0$$

$$A^\mu = \{A^+, A^-, A^I\}$$

$$\mu = +$$

$$-p^+ (p \cdot A) = 0$$

$$p \cdot A = 0 \rightarrow -p^+ A^- - \cancel{p^- A^+} + p^I A^I$$

EOM

$$\rightarrow p^2 A^\mu = 0$$

$$p^2 A^+ = 0 \checkmark$$

$$p^2 A^I(p) = 0$$

D-2 massless DOF

$$\delta A_\mu = \partial_\mu \epsilon$$

$$\delta A_\mu(p) = i p_\mu \epsilon$$

$$\delta A^+(p) = i p^+ \epsilon$$

$$\epsilon = -\frac{A^+}{i p^+}$$

$$A^- = \frac{1}{p^+} (p^I A^I)$$

Photon States

$$\partial^2 A^\mu - \partial^\mu (\partial \cdot A) = 0$$

$$\delta A_\mu = \partial_\mu \epsilon$$

$$A(p)$$

$$p^2 A^\mu - p^\mu (p \cdot A) = 0$$

$$\delta A_\mu(p) = i p_\mu \epsilon$$

Light cone gauge: $A^\mu = \{A^+, A^-, A^I\}$

$$\delta A^+(p) = i p^+ \epsilon$$

$$(p^2 + m^2)\phi(p) = 0$$

$\mu = +$ $-p^+(p \cdot A) = 0$

$$\epsilon = -\frac{A^+}{i p^+}$$

$$p \cdot A = 0 \rightarrow -p^+ A^- - p^- A^+ + p^I A^I$$

$$A^- = \frac{1}{p^+} (p^I A^I)$$

EOM $\rightarrow p^2 A^\mu = 0$

$$\begin{aligned} p^2 A^+ &= 0 \checkmark \\ p^2 A^I(p) &= 0 \\ p^2 A^- &= 0 \checkmark \end{aligned}$$

D-2 massless DOF

$\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$ $\vec{R} \propto \frac{1}{R}$

Light cone $p_1^+ (p) \xrightarrow{\text{Energy}} p_2^+ p_3^+$ $a_{p_1^+, p_T}^+ |\Omega\rangle$ one-particle state

$$\partial^2 A^\mu - \partial^\mu (\partial \cdot A) = 0 \quad \delta A_\mu = \partial_\mu \epsilon$$

$$p^2 A^\mu - p^\mu (p \cdot A) = 0 \quad \delta A_\mu(p) = L p_\mu \epsilon$$

gauge: $A^\mu = \{A^+, A^-, A^I\}$ $\delta A^+(p) = L p^+ \epsilon$, can set $A^+ = 0$ light cone gauge.

$\mu = + \quad -p^+ (p \cdot A) = 0 \quad \epsilon = -\frac{A^+}{p^+}$

$p \cdot A = 0 \rightarrow -p^+ A^- - \cancel{p^- A^+} + p^I A^I$ $A^- = \frac{1}{p^+} (p^I A^I)$

EOM $\rightarrow p^2 A^\mu = 0$ $\begin{cases} p^2 A^+ = 0 \checkmark \\ p^2 A^-(p) = 0 \\ p^2 A^- = 0 \checkmark \end{cases}$ D-2 massless DOF

$a_{p_1^+, p_T}^I |\Omega\rangle$ one-photon states $\sum_I \sum_{p_T} a_{p_1^+, p_T}^I |\Omega\rangle$

h

EOM: $p^2 h^{\mu\nu} - p_\alpha (p^\mu h^{\nu\alpha} + p^\nu h^{\mu\alpha}) + p^\mu p^\nu h = 0$ $h = h^{\mu\nu} \eta_{\mu\nu}$

GAUGE INV: $\delta h^{\mu\nu} = i(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$

$h^{\mu\nu} = \left\{ h^{IJ}, \boxed{h^{-I}, h^{--}}, \boxed{h^{+I}, h^{+-}, h^{++}} \right\} \in^{\mu} \equiv 0$ L.C. gauge con

$\epsilon^\mu = \left\{ \epsilon^I, \epsilon^-, \epsilon^+ \right\}$

$\uparrow$
solved for

$$\text{EOM: } p^2 h^{\mu\nu} - p_\alpha (p^\mu h^{\nu\alpha} + p^\nu h^{\mu\alpha}) + p^\mu p^\nu h = 0 \quad h = h^{\mu\nu} \eta_{\mu\nu}$$

GAUGE INV:

$$\delta h^{\mu\nu} = i(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$$

$$h^{\mu\nu} = \left\{ h^{IJ}, \boxed{h^{-I}, h^{--}}, \boxed{h^{+I}, h^{+-}, h^{++}} \right\} \in^{\mu} \quad \text{L.C. gauge con}$$

$$\epsilon^\mu = \left\{ \epsilon^I, \epsilon^-, \epsilon^+ \right\}$$

↑
solved for

p^μ
 EOM $\rightarrow p^2 A^\mu = 0$
 $p^2 A^+ = 0 \checkmark$
 $p^2 A^-(p) = 0$
 $p^2 A^- = 0 \checkmark$
 D-2 massless DOF

ϵ, ϵ^ν

Gravity: $\underline{h_{\mu\nu}}$ ($g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$);

$$\mu=\nu=+ \quad +p^+p^+h=0$$

$$h = h^{\mu\nu} \eta_{\mu\nu} = 0$$

$$= -2h^{+-} + h^{II} = 0$$

$$h^{II} = 0$$

$$N(\text{dof}) = \text{sym traceless} \quad (D-2) \times (D-2) \text{ matrix} = \frac{1}{2}(D-2)(D-1) - 1 = \frac{1}{2}D(D-3)$$

EOM: $p^2 h^{\mu\nu} - p_\alpha (p^\mu h^{\nu\alpha} + p^\nu h^{\mu\alpha}) = 0$

GAUGE INV:

$$\delta h^{\mu\nu} = i(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$$

$$h^{\mu\nu} = \left\{ \begin{array}{c} \underline{h^{IJ}} \\ h^{-I}, h^{--} \\ h^{+I}, h^{++} \end{array} \right\}$$

$$\epsilon^\mu = \left\{ \begin{array}{c} \epsilon^I, \epsilon^{--} \\ \epsilon^{+I}, \epsilon^{++} \end{array} \right\}$$

solved for

$\epsilon = 0 \checkmark$
 $\epsilon(p) = 0$
 $\epsilon = 0 \checkmark$

D-2 massless DOF

ϵ, ϵ^ν

$\alpha_{p^+ \vec{p}_T} |\Omega\rangle$
 $\sum_I \xi_I \alpha_{p^+ \vec{p}_T}^I |\Omega\rangle$

$h_{\mu\nu} + h_{\mu\nu}$;

$h = 0$
 $h^{II} = 0$

$(D-2)(D-1) - 1 = \frac{1}{2}D(D-3)$

EOM: $p^2 h^{\mu\nu} - p_\alpha (p^\mu h^{\nu\alpha} + p^\nu h^{\mu\alpha}) + p^\mu p^\nu h = 0$

GAUGE INV: $\delta h^{\mu\nu} = i(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$

$h^{\mu\nu} = \left\{ \begin{array}{c} \underline{\underline{h^{II}}} \\ \boxed{h^{-I}, h^{-}} \end{array} , \begin{array}{c} \boxed{h^{+I}, h^{+-}, h^{++}} \end{array} \right\} \equiv 0 \text{ L.C. } \epsilon^\mu$

$\epsilon^\mu = \left\{ \begin{array}{c} \epsilon^I, \epsilon^-, \epsilon^+ \end{array} \right\}$

↑
Solved for

D=4	2
D=5	5
D=10	35

$$\begin{aligned} &= 0 \checkmark \\ &\boxed{p(p)=0} \\ &= 0 \checkmark \end{aligned} \quad D-2 \text{ massless DOF}$$

$$\epsilon, \epsilon^\nu$$

$$\alpha_{p^+ \vec{p}_T} |\Omega\rangle \quad \sum_I \xi_I \alpha_{p^+ \vec{p}_T}^I |\Omega\rangle$$

$$h_{\mu\nu} + h_{\nu\mu}) ; \quad \text{EOM: } p^2 h^{\mu\nu} - p_\alpha (p^\mu h^{\nu\alpha} + p^\nu h^{\mu\alpha}) + p^\mu p^\nu h = 0 \quad h = h^{\mu\nu} \eta_{\mu\nu}$$

GAUGE INV:

$$\delta h^{\mu\nu} = i(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$$

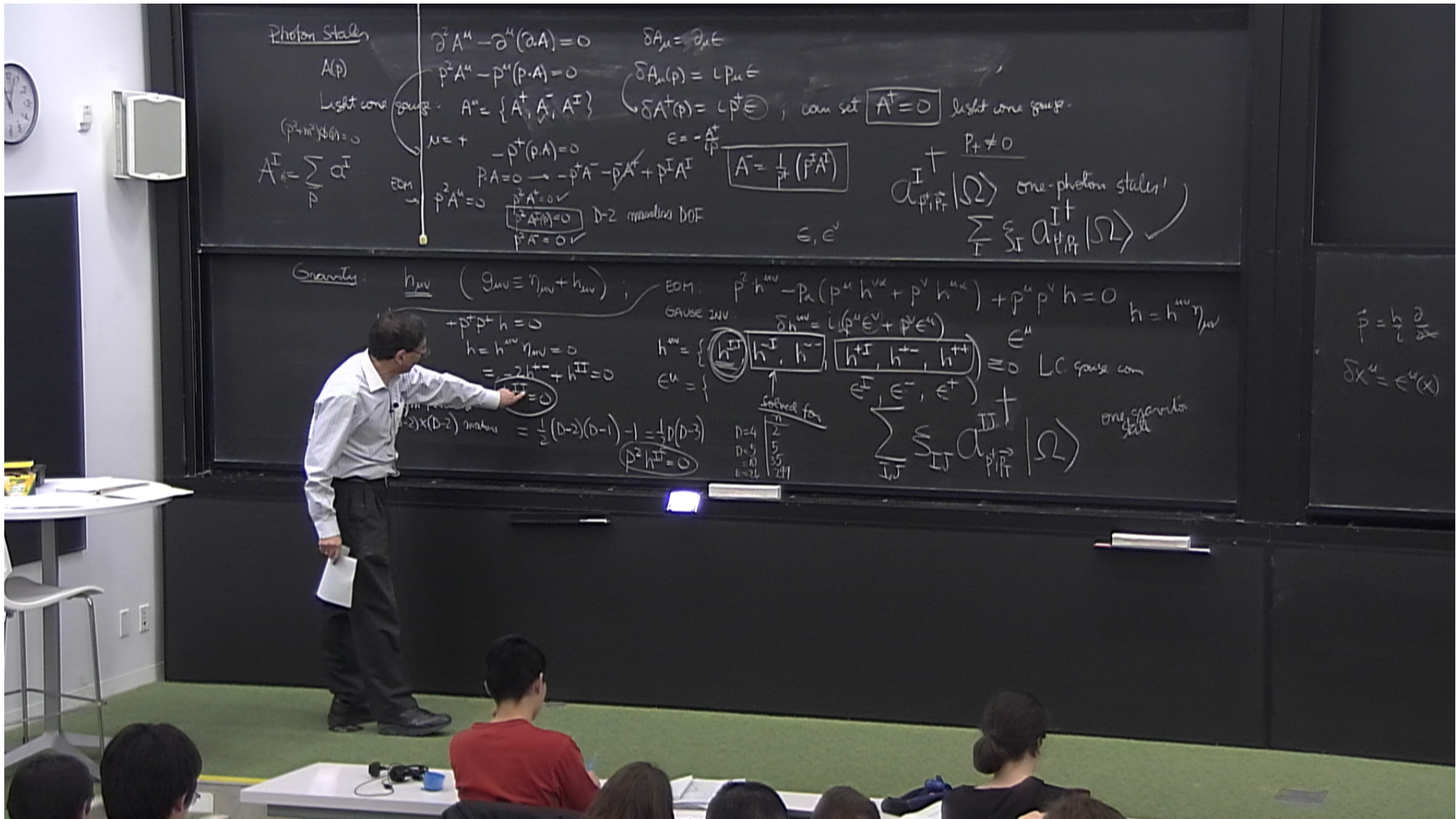
$$h^{\mu\nu} = \left\{ \begin{array}{c} \underline{\underline{h^{IJ}}} \\ \boxed{h^{-I}, h^{--}} \\ \boxed{h^{+I}, h^{+-}, h^{++}} \end{array} \right\} \equiv 0 \quad \text{L.C. gauge con}$$

$$\epsilon^\mu = \left\{ \begin{array}{c} \epsilon^I, \epsilon^-, \epsilon^+ \end{array} \right\}$$

Solved for

	n
$D=4$	2
$D=5$	5
$D=10$	35
$D=26$	299

$$\frac{1}{2}(D-2)(D-1) - 1 = \frac{1}{2}D(D-3)$$



Photon States

$$\begin{aligned}
 &A(p) \\
 &\text{Light cone gauge: } A^\mu = \{A^+, A^-, A^I\} \\
 &(p^2 + m^2)A = 0 \\
 &A^I = \sum_p \alpha^I_p \\
 &\partial^2 A^\mu - \partial^\mu (\partial \cdot A) = 0 \quad \delta A_\mu = \partial_\mu \epsilon \\
 &p^2 A^\mu - p^\mu (p \cdot A) = 0 \quad \delta A_\mu(p) = L p_\mu \epsilon \\
 &\delta A^+(p) = L p^+ \epsilon, \text{ can set } \boxed{A^+ = 0} \text{ light cone gauge} \\
 &\epsilon = -\frac{A^+}{p^+} \\
 &\boxed{A^- = \frac{1}{p^+} (p^I A^I)} \\
 &\epsilon, \epsilon^I \\
 &\alpha_{p^I p^+}^{I+} |\Omega\rangle \text{ one-photon states} \\
 &\sum_I \sum_{p^+} \alpha_{p^I p^+}^{I+} |\Omega\rangle
 \end{aligned}$$

Gravity

$$\begin{aligned}
 &h_{\mu\nu} \quad (g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}) \\
 &+ p^+ p^+ h = 0 \\
 &h = h^{\mu\nu} \eta_{\mu\nu} = 0 \\
 &= -2h^{+-} + h^{II} = 0 \\
 &\boxed{h^{+-} = 0} \\
 &(D-2) \times (D-2) \text{ matrix} = \frac{1}{2} (D-2)(D-1) - 1 = \frac{1}{2} D(D-3) \\
 &\boxed{p^2 h^{II} = 0} \\
 &\text{EOM: } p^2 h^{\mu\nu} - p^\mu (p^\nu h^{\alpha\alpha} + p^\alpha h^{\mu\nu}) + p^\mu p^\nu h = 0 \\
 &\text{GAUGE INV: } \delta h^{\mu\nu} = L(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu) \\
 &h^{\mu\nu} = \left\{ \begin{pmatrix} h^{++} & h^{+-} & h^{+I} \\ h^{+-} & h^{--} & h^{-I} \\ h^{+I} & h^{-I} & h^{IJ} \end{pmatrix} \right\} \epsilon^\mu \\
 &\epsilon^\mu = \left\{ \begin{pmatrix} \epsilon^+ \\ \epsilon^- \\ \epsilon^I \end{pmatrix} \right\} \\
 &\text{Solved for } \epsilon^\mu \\
 &D=4 \quad \begin{matrix} 2 \\ 5 \\ 10 \\ 14 \end{matrix} \\
 &D=5 \quad \begin{matrix} 2 \\ 5 \\ 10 \\ 14 \\ 19 \end{matrix} \\
 &D=6 \quad \begin{matrix} 2 \\ 5 \\ 10 \\ 14 \\ 19 \\ 24 \end{matrix} \\
 &\sum_{IJ} \sum_{p^+} \alpha_{p^I p^+}^{IJ+} |\Omega\rangle \text{ one graviton state}
 \end{aligned}$$

2 massless DOF

ϵ, ϵ^ν

$\alpha_{p_1, \vec{p}_T} | \Omega \rangle$

$$\sum_I \sum_J \alpha_{p_1, \vec{p}_T}^{IJ} | \Omega \rangle$$

EOM: $p^2 h^{\mu\nu} - p_\alpha (p^\mu h^{\nu\alpha} + p^\nu h^{\mu\alpha}) + p^\mu p^\nu h = 0$ $h = h^{\mu\nu} \eta_{\mu\nu}$

GAUGE INV: $\delta h^{\mu\nu} = i(p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$

$h^{\mu\nu} = \left\{ \begin{pmatrix} \underline{\underline{h^{IJ}}} & h^{-I} & h^{--} \\ h^{+I} & h^{+-} & h^{++} \end{pmatrix} \right\} \equiv 0$ L.C. gauge con

$\epsilon^\mu = \left\{ \begin{pmatrix} \epsilon^I & \epsilon^- & \epsilon^+ \end{pmatrix} \right\}$

Solved for

	n
$D=4$	2
$D=5$	5
$D=10$	35
$D=26$	299

$-1 = \frac{1}{2} D(D-3)$

$p^2 h^{IJ} = 0$

$\sum_{I,J} \sum \alpha_{p_1, \vec{p}_T}^{IJ} | \Omega \rangle$

one graviton still

$\sum_{II} = 0$

Point Particle Quantization

$$S = -m \int_{\tau_i}^{\tau_f} \sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} d\tau = \int d\tau L(\tau)$$

$$L = -m \sqrt{-\dot{X}^2}$$

$$p_\mu = \frac{\partial L}{\partial \dot{X}^\mu} = -m \frac{1}{2} \frac{-2 \cdot \dot{X}_\mu}{\sqrt{-\dot{X}^2}} = \frac{m \dot{X}_\mu}{\sqrt{-\dot{X}^2}}$$

$$\boxed{p^\mu = \frac{m \dot{X}^\mu}{\sqrt{-\dot{X}^2}}} \quad (I)$$

$$\boxed{p^\mu p_\mu = -m^2} \quad \text{Constraint}$$

L.C. condition

$$H = p_\mu \dot{X}^\mu - L = 0 \quad \text{no units}$$

$$X^+ = \frac{1}{m} p^+ \tau$$

Point Particle Quantization

$$S = -m \int_{\tau_i}^{\tau_f} \sqrt{-\eta_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} d\tau = \int d\tau L(\tau)$$

$$L = -m \sqrt{-\dot{X}^2}$$

$$p_\mu = \frac{\partial L}{\partial \dot{X}^\mu} = -m \frac{1}{2} \frac{-2 \cdot \dot{X}_\mu}{\sqrt{-\dot{X}^2}} = \frac{m \dot{X}_\mu}{\sqrt{-\dot{X}^2}}$$

$$\boxed{p^\mu = \frac{m \dot{X}^\mu}{\sqrt{-\dot{X}^2}}} \quad (I)$$



$$p^\mu = m^2 \dot{X}^\mu$$

$$\boxed{p^\mu p_\mu = -m^2}$$

Constant

l.c. condition

$$\text{Eq (I)} \quad p^\mu = \frac{m \dot{X}^\mu}{\sqrt{-\dot{X}^2}} = \frac{1}{m} \frac{p^\mu}{\sqrt{-\dot{X}^2}} \rightarrow \boxed{m \sqrt{-\dot{X}^2} = 1}$$

$$H = p_\mu \dot{X}^\mu - L = 0 \quad \text{no units}$$

$$\boxed{X^\mu = \frac{1}{m} p^\mu \tau}$$

$$p^\mu = m^2 \dot{x}^\mu \quad \dot{x}^\mu = \frac{1}{m^2} p^\mu$$

$$\text{Eq (I)} \quad p^\dagger = \frac{m \dot{x}^\dagger}{\sqrt{-\dot{x}^2}} = \frac{1}{m \sqrt{-\dot{x}^2}} p^\dagger \rightarrow \boxed{m \sqrt{-\dot{x}^2} = 1}$$

$$\frac{dx^-}{d\tau} = \frac{p^-}{m^2}$$

$$x^-(\tau) = x_0^- + \frac{p^-}{m^2} \tau$$

$$p^2 + m^2 = 0 \quad \boxed{p^- = \frac{1}{2p^+} (p_I p_I + m^2)}$$

$\downarrow$ $p^\mu = m \dot{x}^\mu$ $\dot{x}^\mu = \frac{1}{m} p^\mu$ $\text{Eq (I)} \quad p^t = \frac{m \dot{x}^t}{\sqrt{-\dot{x}^2}} = \frac{1}{m \sqrt{-\dot{x}^2}} p^t \rightarrow \boxed{m \sqrt{-\dot{x}^2} = 1}$

$\frac{dx^-}{d\tau} = \frac{p^-}{m^2}$ $x^-(\tau) = x_0^- + \frac{p^- \tau}{m^2}$
 $\frac{dx^I}{d\tau} = \frac{p^I}{m^2}$ $x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$

$p^2 + m^2 = 0 \quad \boxed{\bar{p} = \frac{1}{2p^+} (P_I P_I + m^2)}$

$$\boxed{p^\mu = m^2 \dot{x}^\mu}$$

$$\dot{x}^\mu = \frac{1}{m^2} p^\mu$$

$$\text{Eq (I)} \quad p^+ = \frac{m \dot{x}^+}{\sqrt{-\dot{x}^2}} = \frac{1}{m} \frac{p^+}{\sqrt{-\dot{x}^2}} \rightarrow \boxed{m \sqrt{-\dot{x}^2} = 1}$$

$$\frac{dx^-}{d\tau} = \frac{p^-}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2}$$

$$x^-(\tau) = x_0^- + \frac{p^- \tau}{m^2}$$

$$x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$p^2 + m^2 = 0$$

$$\boxed{p^- = \frac{1}{2p^+} (p_I p^I + m^2)}$$

Dynamical variables $\underline{p^+, p^I, x_0^-, x_0^I}$

$$p^\mu p_\mu = -m^2$$

$$\boxed{p^\mu = m^2 \dot{x}^\mu} \quad \dot{x}^\mu = \frac{1}{m^2} p^\mu$$

Eq (I) $p^+ = \frac{m \dot{x}^+}{\sqrt{-\dot{x}^2}} = \frac{1}{m} \frac{p^+}{\sqrt{-\dot{x}^2}}$

$$\frac{dx^-}{d\tau} = \frac{p^-}{m^2} \quad x^-(\tau) = x_0^- + \frac{p^- \tau}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2} \quad x^I(\tau) = x_0^I + \frac{p^I \tau}{m^2}$$

$$p^2 + m^2 = 0$$

Dynamical variables p^+

Quantum theory Schrödinger ops
 x^I, x_0^I, p^I, p^+

$$p^\mu p_\mu = -m^2 \quad \text{Constraint}$$

$$p^\mu = m^2 \dot{x}^\mu \quad \dot{x}^\mu = \frac{1}{m^2} p^\mu$$

$$\text{Eq (I)} \quad p^+ = \frac{m \dot{x}^+}{\sqrt{-\dot{x}^2}} = \frac{1}{m} \frac{p^+}{\sqrt{-\dot{x}^2}} \rightarrow m \sqrt{-\dot{x}^2} = 1$$

$$\frac{dx^-}{d\tau} = \frac{p^-}{m^2}$$

$$\frac{dx^I}{d\tau} = \frac{p^I}{m^2}$$

$$x^-(\tau) = x_0^- + \frac{p^-}{m^2} \tau$$

$$x^I(\tau) = x_0^I + \frac{p^I}{m^2} \tau$$

$$p^2 + m^2 = 0$$

$$p^- = \frac{1}{2p^+} (p_I p^I + m^2)$$

Dynamical variables

$$p^+, p^I, x_0^-, x_0^I$$

Quantum theory
Schrödinger eqs
 x^I, x_0^-, p^I, p^+

$$X^-(\tau) = X_0^- + \frac{P^- \tau}{m^2}$$

$$X^I(\tau) = X_0^I + \frac{P^I \tau}{m^2}$$

Schrodinger ops

$$X^I, X_0^-, P^I, P^+$$

$$X^+(\tau) \equiv \frac{P^+ \tau}{m^2}, \quad X^-(\tau) \equiv X_0^- + \frac{P^- \tau}{m^2}, \quad P^- = \frac{1}{2P^+} (P^I P^I + m^2)$$

$$P^2 + m^2 = 0$$

$$P^- = \frac{1}{2P^+} (P^I P^I + m^2)$$

Dynamical variables

$$P^+, P^I, X_0^-, X^I$$

$$[X^I, P^J] = i \delta^{IJ}, \quad [X_0^-, P^+] = -i \quad \text{all others zero!}$$

χ_0^-, p^I, p^+

$$[X^I, p^I] = i \delta^{IJ}, \quad [X_0^-, p^+] = -i \quad \text{all others zero!}$$

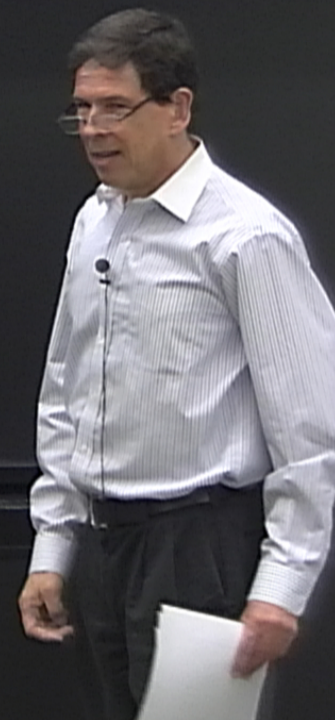
$$X^+(\tau) \equiv \frac{p^+ \tau}{m^2}, \quad X^-(\tau) \equiv X_0^- + \frac{p^- \tau}{m^2}, \quad p^- = \frac{1}{2p^+} (p^I p^I + m^2)$$

$$[X^+(\tau), p^-]$$

$$[X^\mu, p^\nu] = i \eta^{\mu\nu}$$

$$[X^-, p^+] = i \eta^{-+} = -i$$

$$[X^+, p^-] = -i$$



χ_0^-, p^I, p^+

$$[X^I, p^I] = i \delta^{IJ}, \quad [X_0^-, p^+] = -i \quad \text{all others zero!}$$

$$X^+(\tau) \equiv \frac{p^+ \tau}{m^2}, \quad X^-(\tau) \equiv X_0^- + \frac{p^- \tau}{m^2}, \quad p^- \equiv \frac{1}{2p^+} (p^I p^I + m^2)$$

$$[X^+(\tau), p^-] = \left[\frac{p^+ \tau}{m^2}, \frac{1}{2p^+} (p^I p^I + m^2) \right] = 0$$

$$[X^\mu, p^\nu] = i \eta^{\mu\nu}$$

$$[X^-, p^+] = i \eta^{-+} = -i$$

$$[X^+, p^-] = -i$$

$a_{\vec{p}}^\dagger a_{\vec{p}}^\dagger | \Omega \rangle$ multiparticle $\rightarrow n$ -particle $\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n$
 $a_{\vec{p}}^\dagger | \Omega \rangle$ one-particle state
 $\partial_\mu \epsilon = L p_\mu \epsilon$
 $\epsilon = L \vec{p}^\dagger \epsilon$, can set $A^+ = 0$ (not one gauge)
 $A^- = \frac{1}{p^+} (F^+ A^-)$
 $\alpha_{\vec{p}, \vec{p}^\dagger}^\dagger | \Omega \rangle$ one-photon states
 $\sum_I \xi_I \alpha_{\vec{p}, \vec{p}^\dagger}^\dagger | \Omega \rangle$
 $\epsilon, \epsilon^\dagger$
 $p^2 h^{\mu\nu} - p_\mu (p^\mu h^{\nu\mu} + p^\nu h^{\mu\mu}) + p^\mu p^\nu h = 0$ $h = h^{\mu\nu} \eta_{\mu\nu}$
 $\delta h^{\mu\nu} = L (p^\mu \epsilon^\nu + p^\nu \epsilon^\mu)$
 $\{ \begin{pmatrix} h^{+-} \\ h^{+I} \end{pmatrix}, \begin{pmatrix} h^{--} \\ h^{-I} \end{pmatrix} \} \equiv 0$ LC gauge con
 $\epsilon^I, \epsilon^-, \epsilon^+$
 $\sum_{IJ} \xi_{IJ} \alpha_{\vec{p}, \vec{p}^\dagger}^{IJ} | \Omega \rangle$ one-photon state
 $\xi_{IJ} = 0$

D=4	2
D=3	5
D=2	35
D=1	241

$\vec{p}^\mu = m^2 \dot{x}^\mu$ $\dot{x}^\mu = \frac{1}{m^2} p^\mu$
 $\frac{dx^-}{d\tau} = \frac{p^-}{m^2}$ $x^-(\tau) = x_0^- + \frac{p^-}{m^2} \tau$
 $\frac{dx^I}{d\tau} = \frac{p^I}{m^2}$ $x^I(\tau) = x_0^I + \frac{p^I}{m^2} \tau$
 Quantum theory Schrödinger ops
 x^I, x_0^I, p^I, p^+
 $x^I(\tau) = \frac{p^I}{m^2} \tau$, $x^I(\tau) = x_0^I + \frac{p^I}{m^2} \tau$ $p^- = \frac{1}{2p^+} (p^I p^I + m^2)$
 Heisenberg ops $x^I(\tau), x_0^I(\tau), p^I(\tau), p^+(\tau)$
 invert H

Eq (1) $p^+ = \frac{m \dot{x}^+}{\sqrt{-\dot{x}^2}} = \frac{1}{m} \frac{p^+}{\sqrt{-\dot{x}^2}} \rightarrow \boxed{m \sqrt{-\dot{x}^2} = 1}$
 $\vec{p}^2 + m^2 = 0$ $\boxed{p^- = \frac{1}{2p^+} (p^I p^I + m^2)}$
 Dynamical variables p^+, p^I, x_0^I, x^I
 $[x^I, p^J] = i \delta^{IJ}$, $[x_0^I, p^+] = -i$ all others zero!
 $[x^I(\tau), p^-] = \left[\frac{p^I}{m^2}, \frac{1}{2p^+} (p^J p^J + m^2) \right]$

