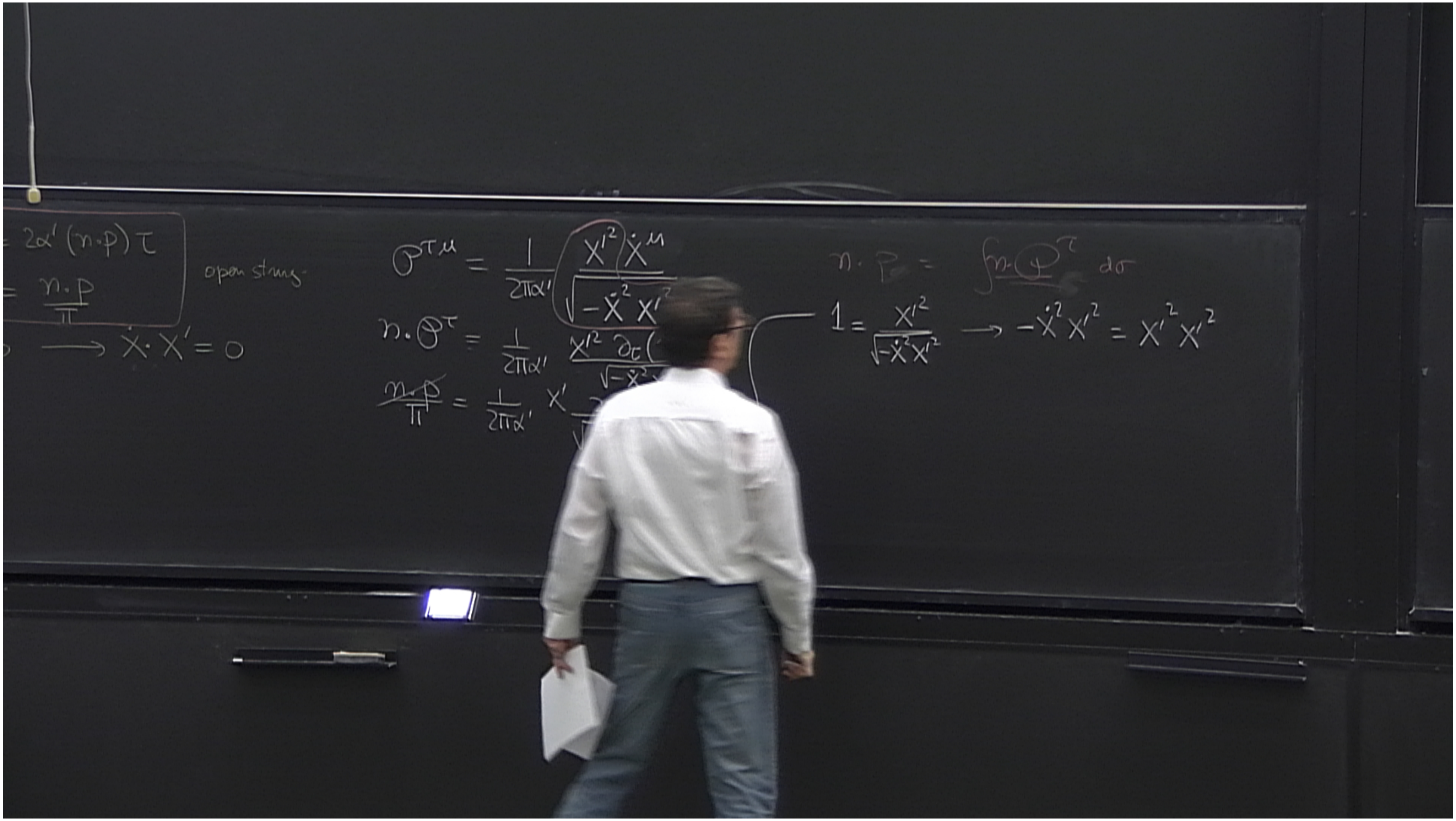
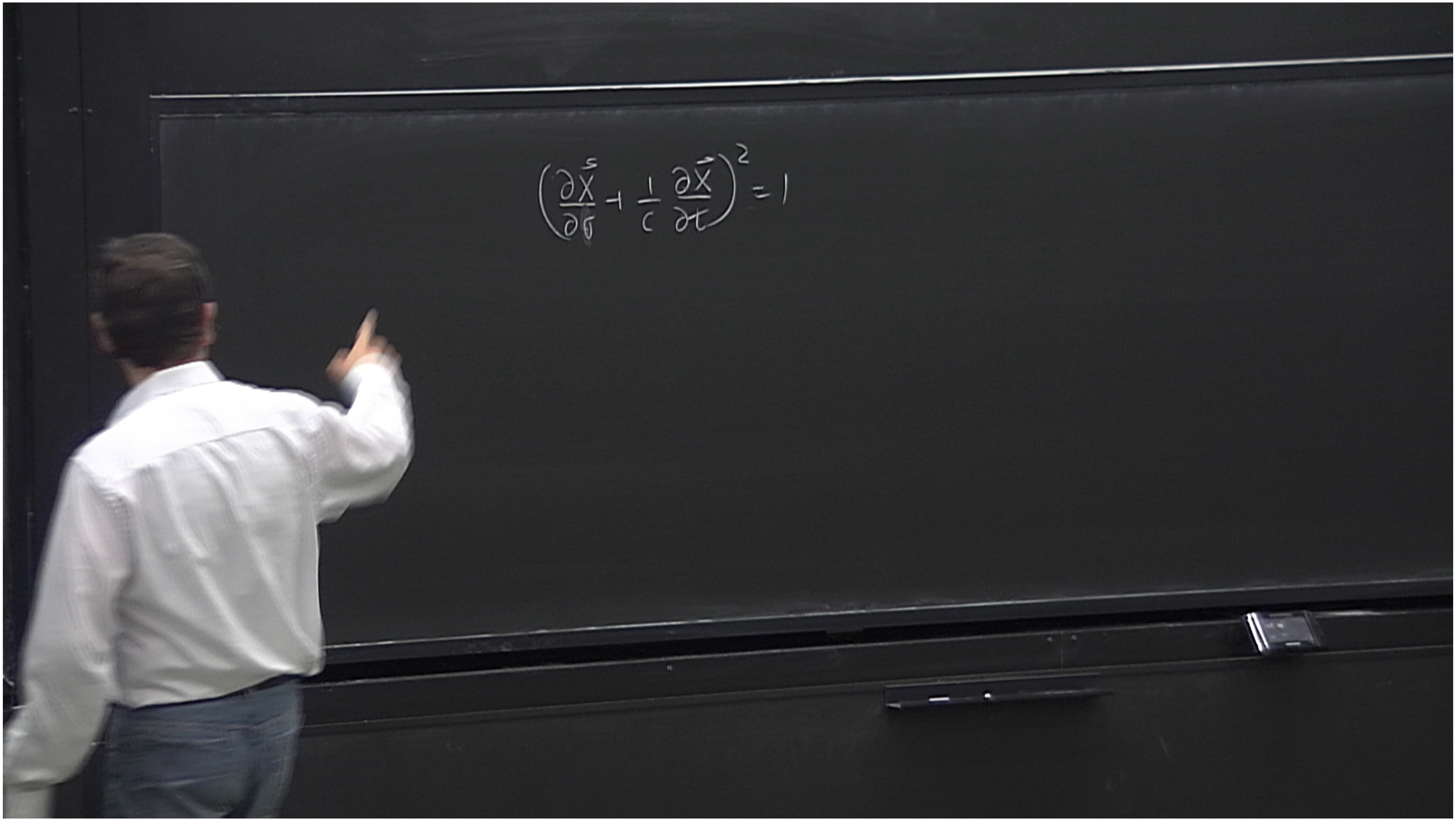
Title: 12/13 PSI - String Theory Review Lecture 7

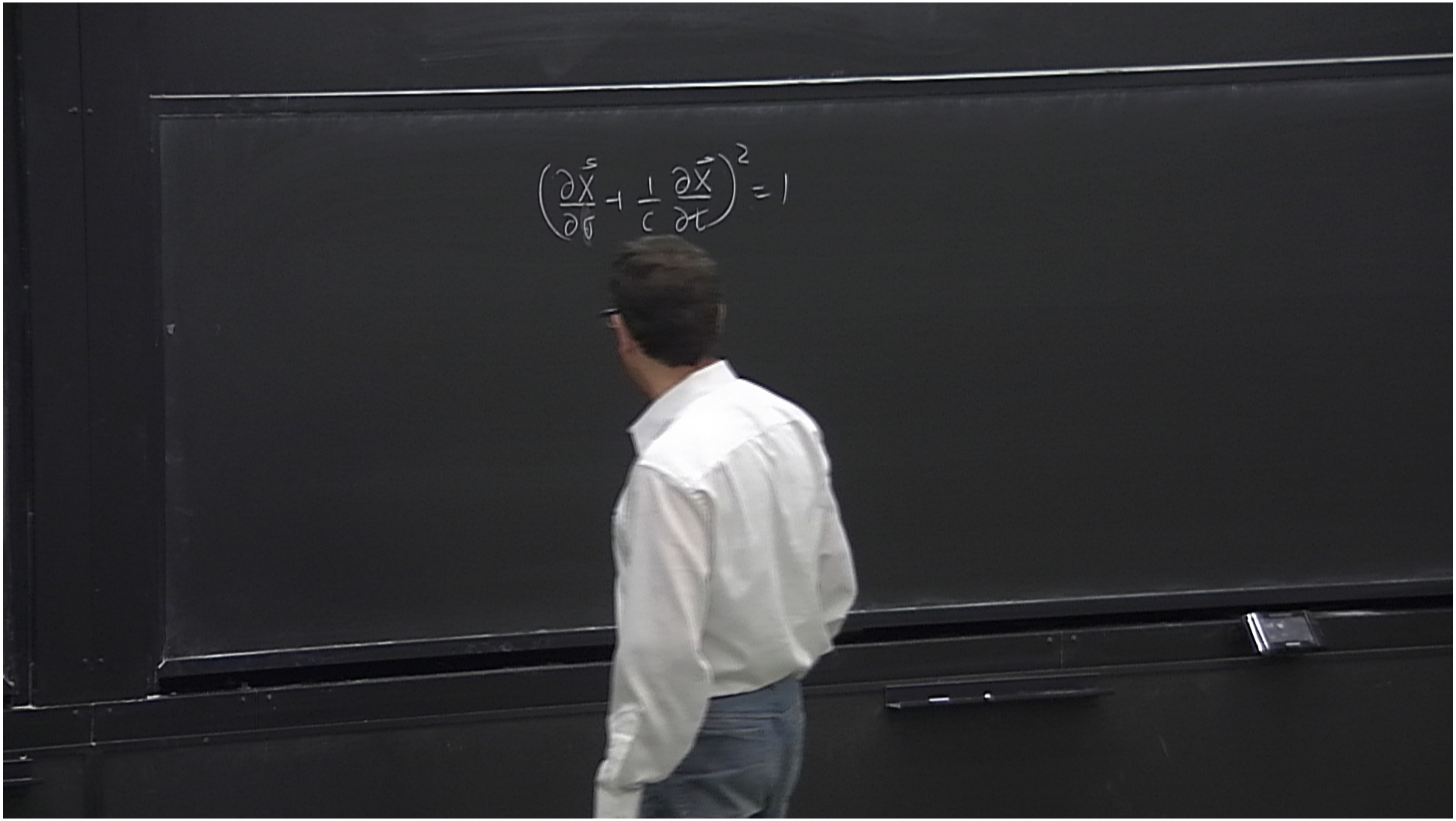
Date: Jan 15, 2013 10:15 AM

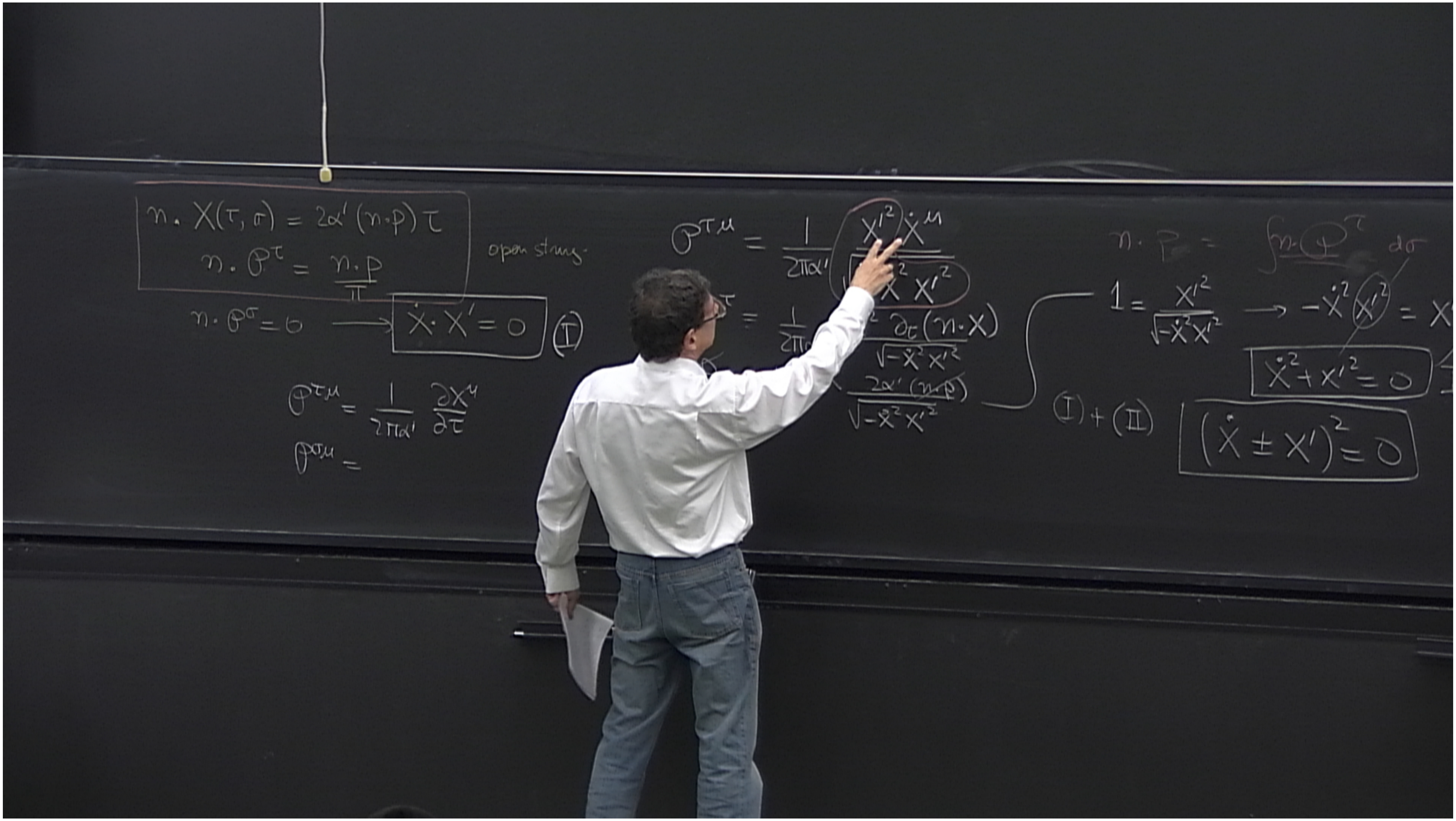
URL: http://pirsa.org/13010053

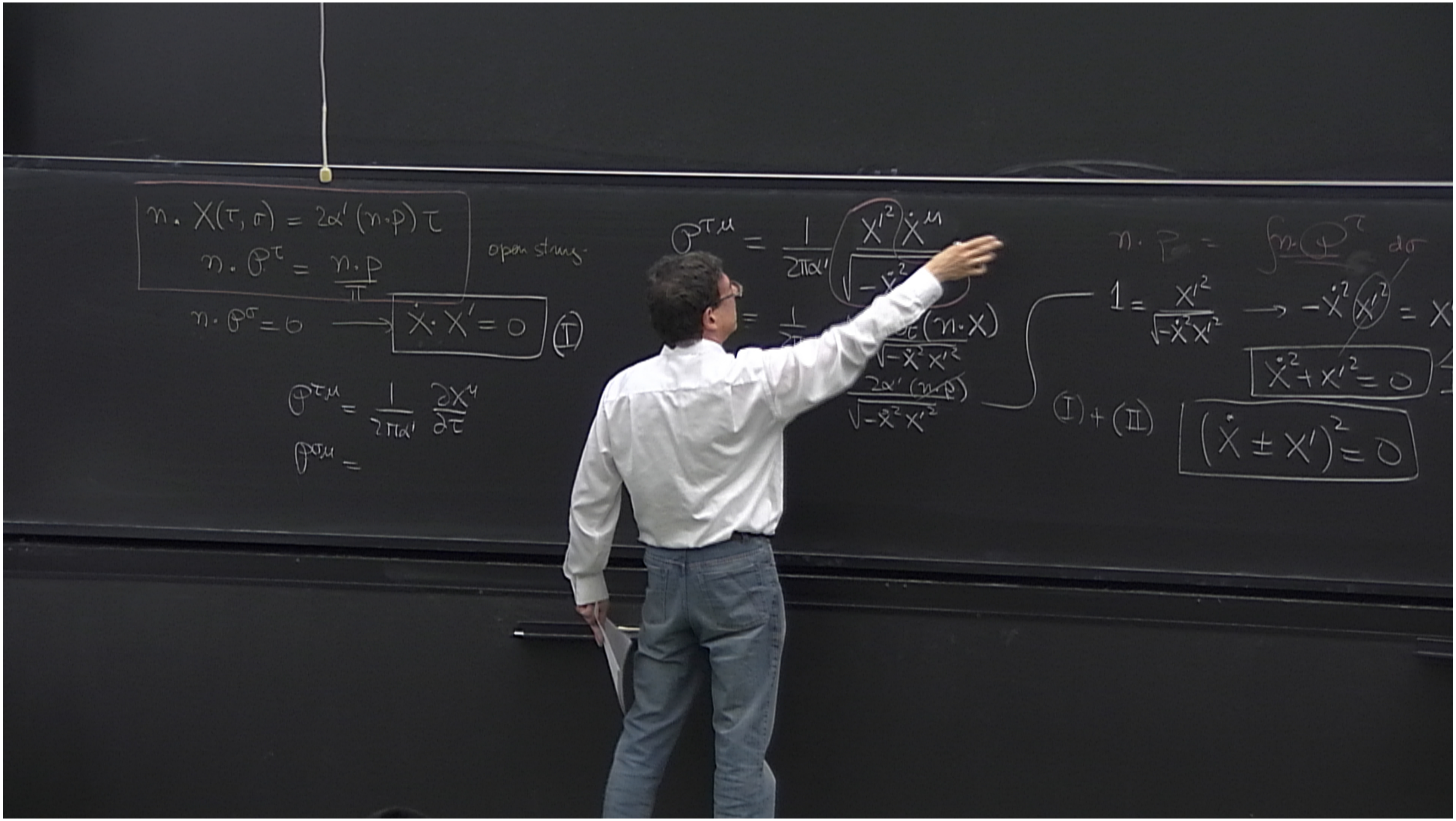
Abstract:


= 2α′ (n·p) τ
= n·p / π
open string
Ẋ · X′ = 0
P^{τμ} = 1/(2πα′)
n·P^τ = 1/(2πα′)
n·p / π = 1/(2πα′)
n · p = ∫ n·P^τ dσ
1 = X′² / √(−Ẋ² X′²) ⟶ −Ẋ² X′² = X′² X′²


$\left(\frac{\partial X^5}{\partial \sigma} + \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1$


$\left(\frac{\partial \vec{X}}{\partial \sigma} + \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1$





n · X(τ, σ) = 2α' (n·p) τ
n · P^τ = n·p / π
open string
n · P^σ = 0
Ẋ · X' = 0
(I)
P^τμ = 1/(2πα') ∂X^μ/∂τ
P^σμ =
(I) + (II)
Ẋ² + X'² = 0
(Ẋ ± X')² = 0

...on string.

$$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'} \frac{X'^2 \dot{X}^\mu}{\sqrt{-\dot{X}^2 X'^2}}$$

$$n\cdot\mathcal{P}^\tau = \frac{1}{2\pi\alpha'} \frac{X'^2\, \partial_\tau (n\cdot X)}{\sqrt{-\dot{X}^2 X'^2}}$$

$$\frac{n\cdot p}{\pi} = \frac{1}{2\pi\alpha'} X' \frac{2\alpha'(n\cdot p)}{\sqrt{-\dot{X}^2 X'^2}}$$

(I)

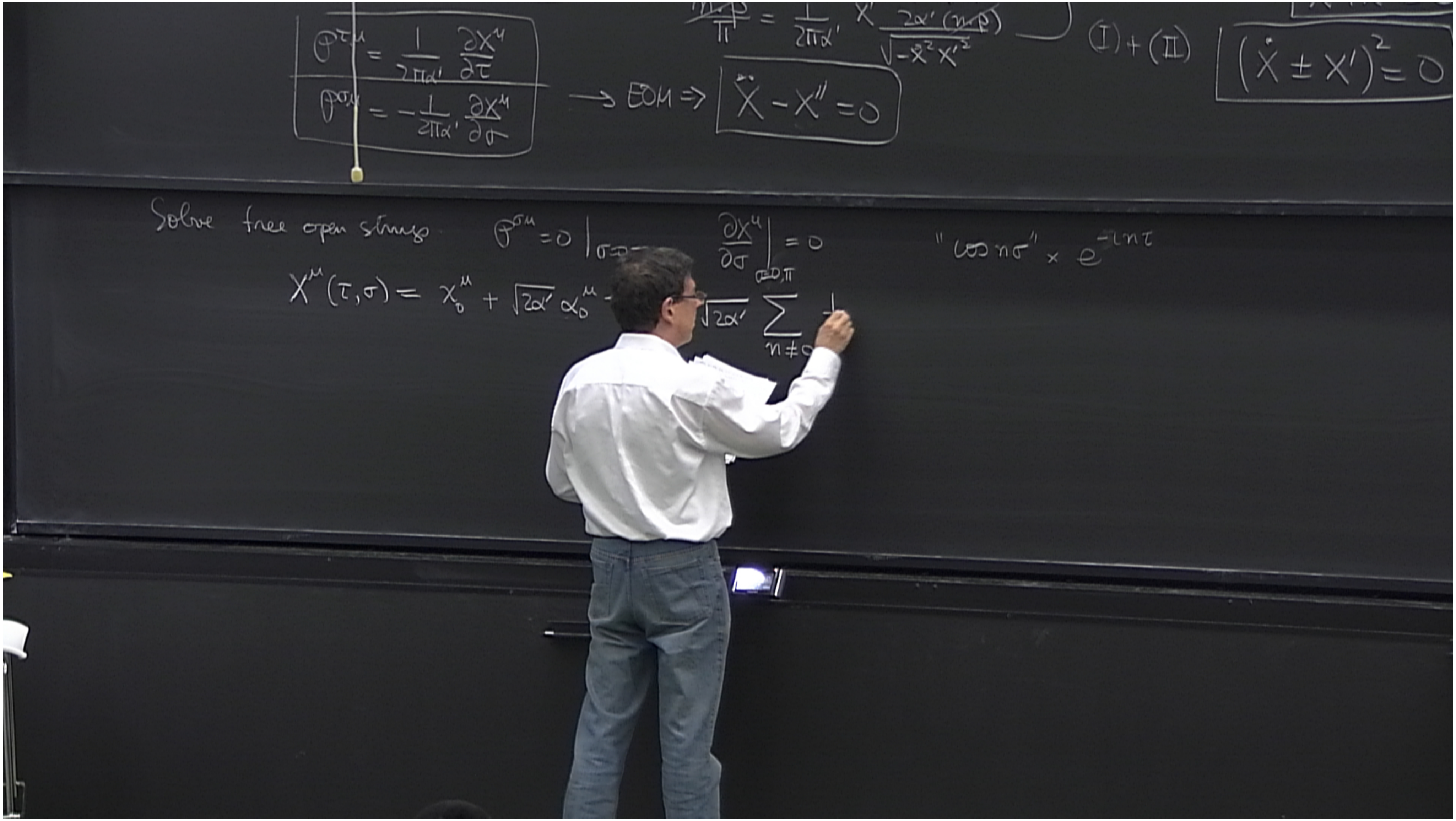
$$n\cdot p = \int n\cdot\mathcal{P}^\tau \, d\sigma$$

$$1 = \frac{X'^2}{\sqrt{-\dot{X}^2 X'^2}} \longrightarrow -\dot{X}^2 X'^2 = X$$

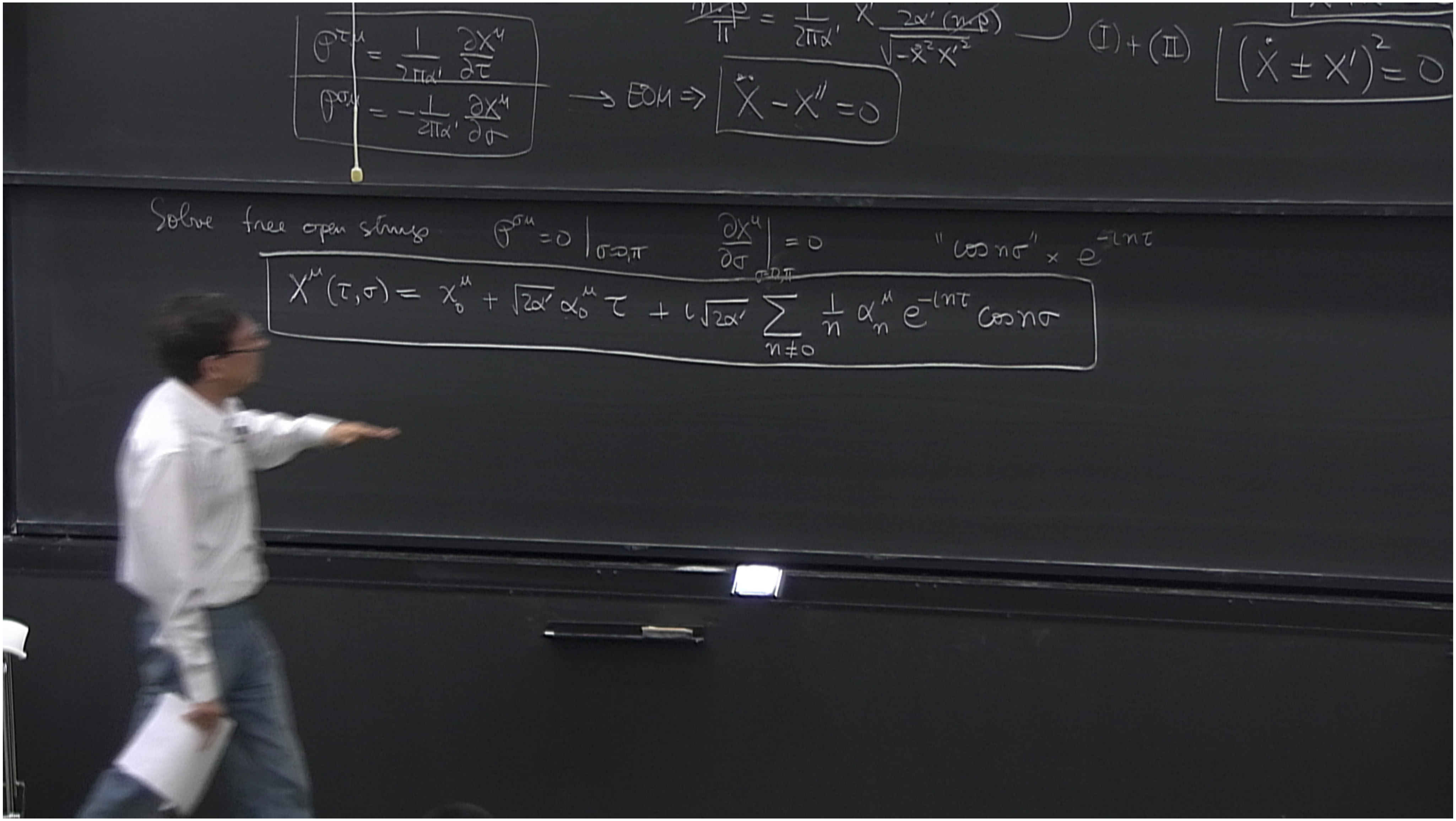
$$\dot{X}^2 + X'^2 = 0$$

(I) + (II)

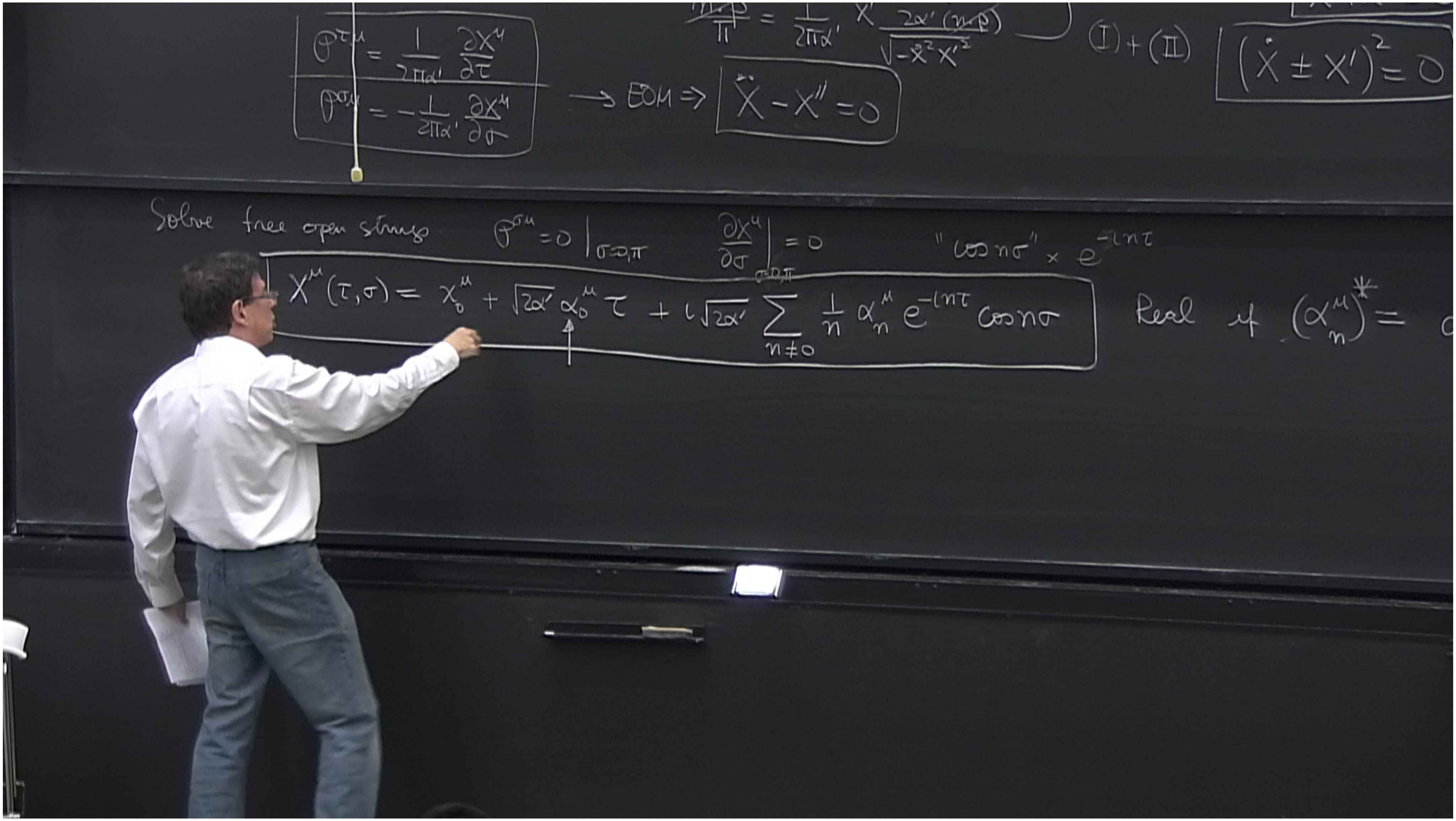
$$(\dot{X} \pm X')^2 = 0$$



EOM ⇒
Ẍ − X″ = 0
(I) + (II)
(Ẋ ± X′)² = 0
Solve free open strings
"cos nσ" × e^{−inτ}



$P^{\tau\mu} = \frac{1}{2\pi\alpha'} \frac{\partial X^\mu}{\partial \tau}$
$P^{\sigma\mu} = -\frac{1}{2\pi\alpha'} \frac{\partial X^\mu}{\partial \sigma}$
→ EOM ⇒ $\ddot{X} - X'' = 0$
(I) + (II)
$(\dot{X} \pm X')^2 = 0$
Solve free open strings
$P^{\sigma\mu} = 0 \big|_{\sigma=0,\pi}$
$\frac{\partial X^\mu}{\partial \sigma}\Big|_{\sigma=0,\pi} = 0$
"$\cos n\sigma$" × $e^{-in\tau}$
$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$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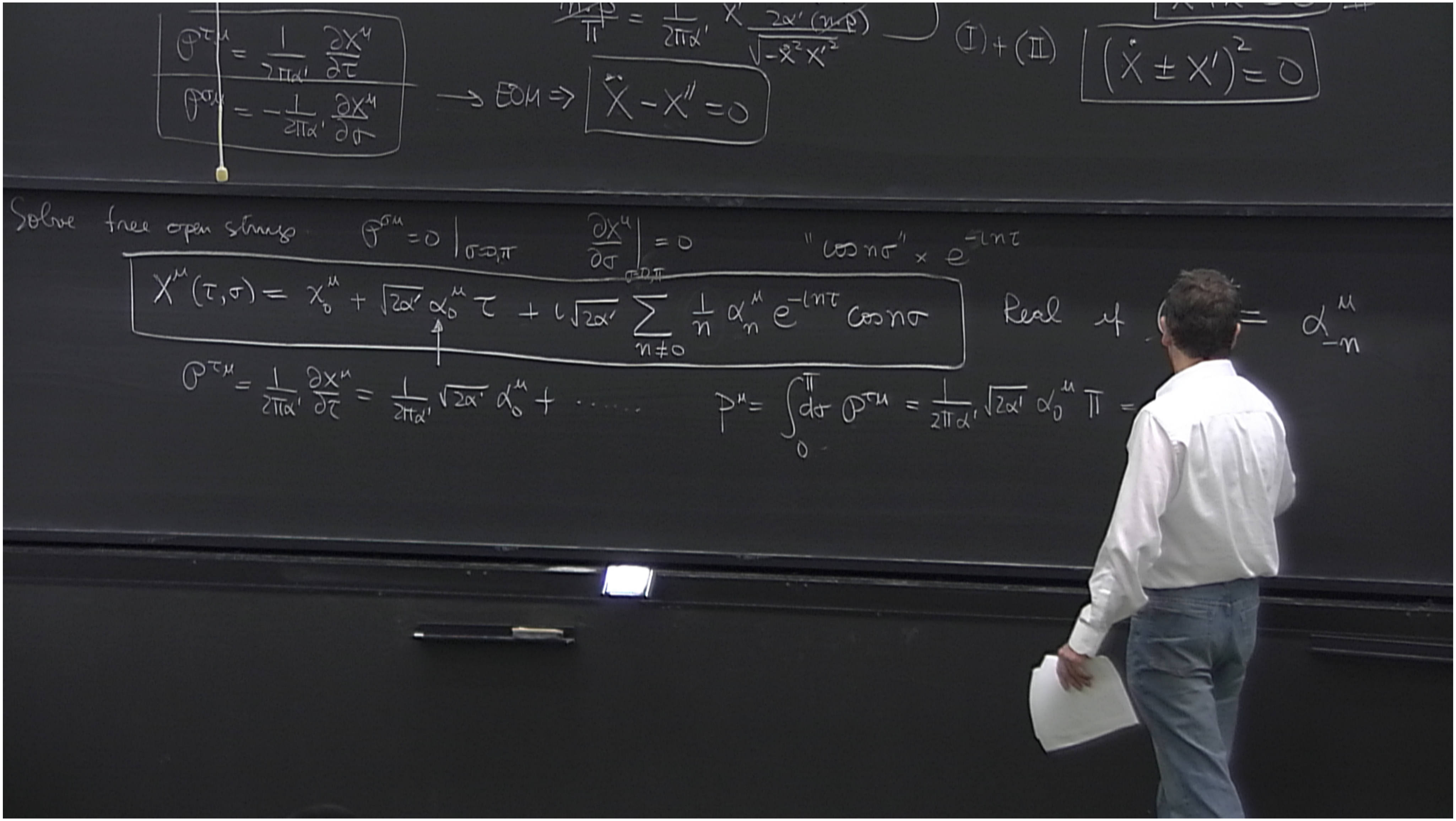
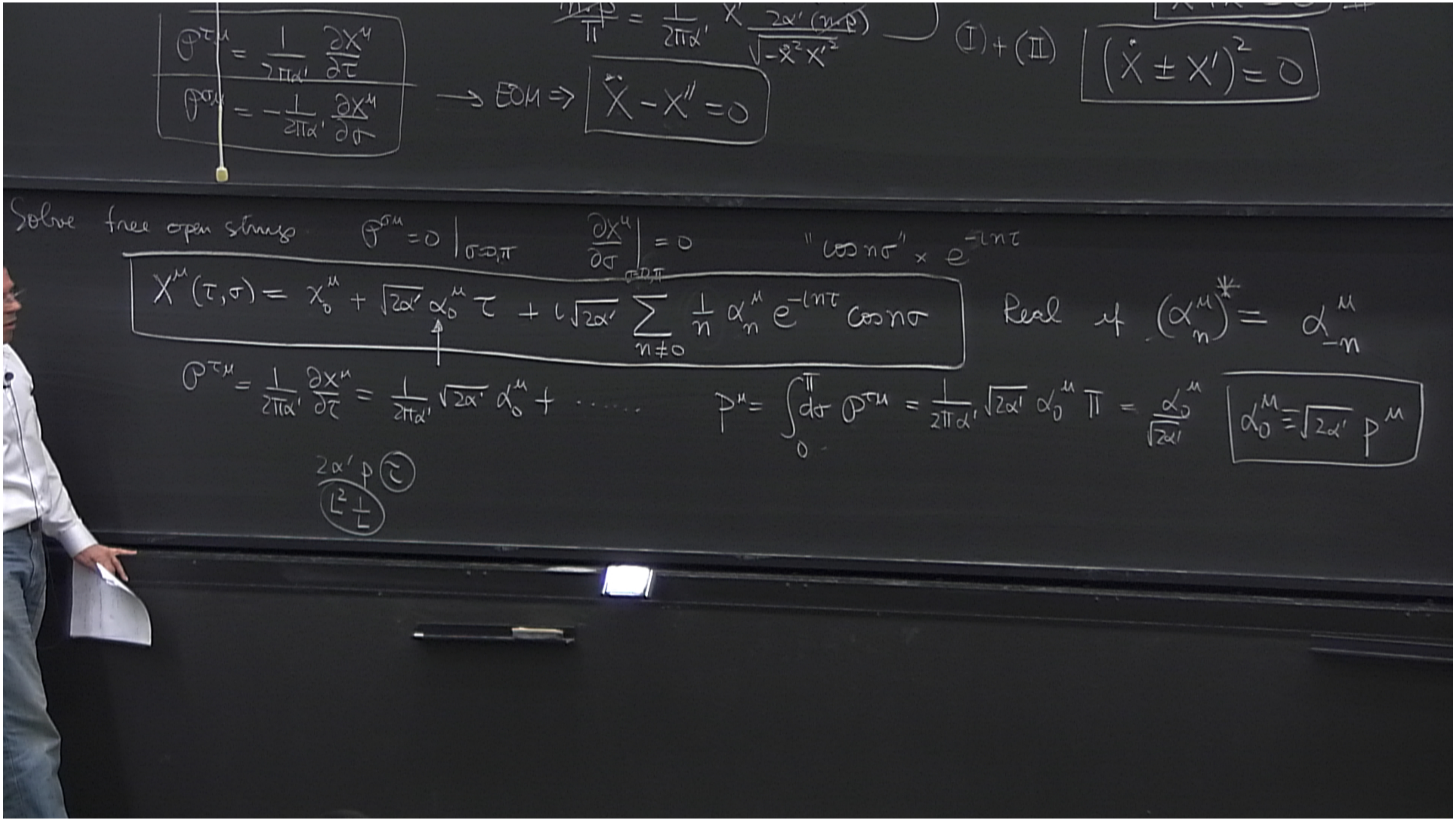
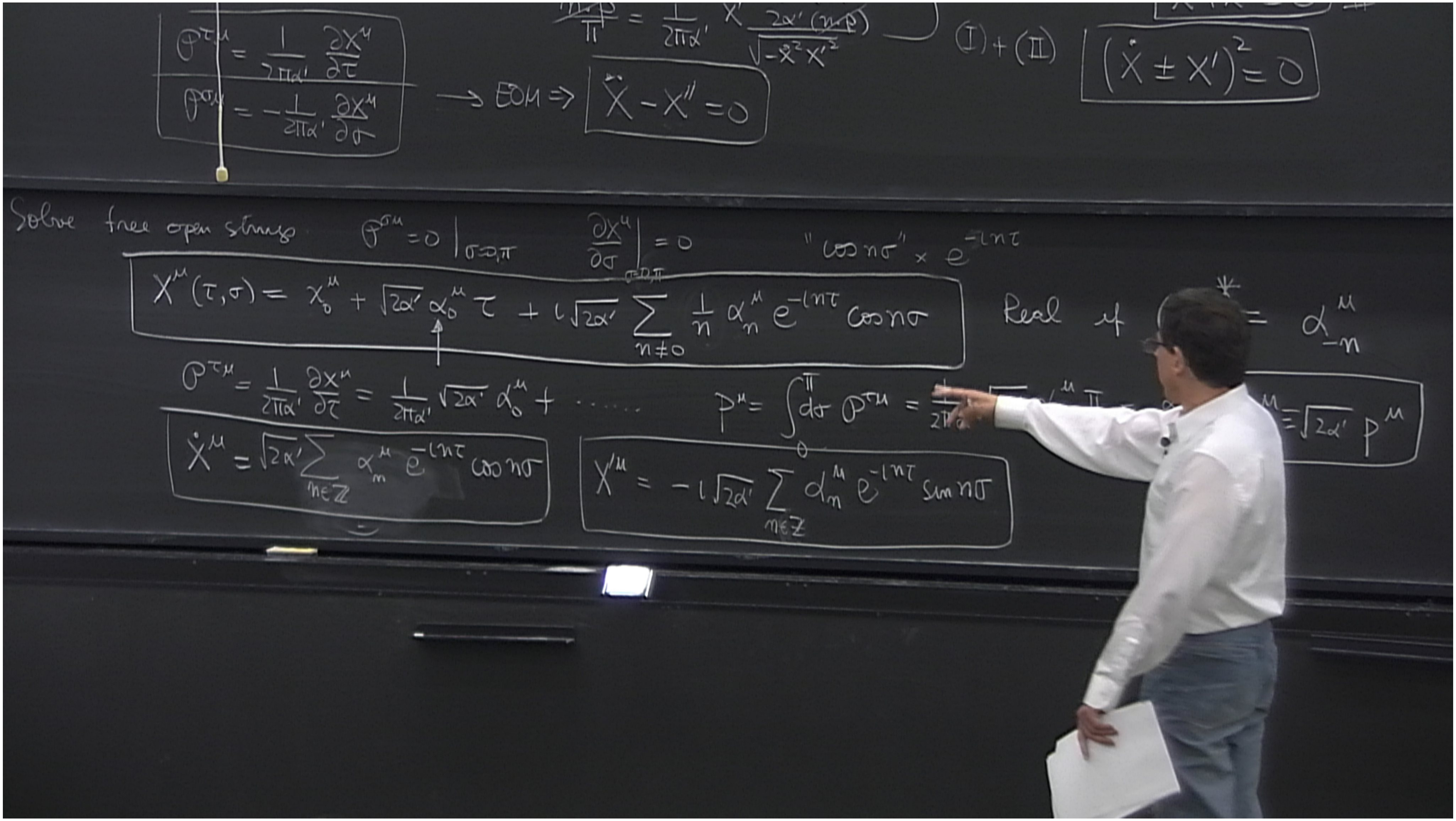
→ EOM ⇒ $\ddot{X} - X'' = 0$
(I) + (II)
$(\dot{X} \pm X')^2 = 0$
Solve free open string
$P^{\sigma\mu} = 0 \big|_{\sigma=0,\pi}$
$\frac{\partial X^\mu}{\partial \sigma}\Big|_{\sigma=0,\pi} = 0$
"$\cos n\sigma$" × $e^{-in\tau}$
$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$
Real if $(\alpha_n^\mu)^* =$


$P^{\tau\mu} = \frac{1}{2\pi\alpha'} \frac{\partial X^\mu}{\partial \tau}$
$P^{\sigma\mu} = -\frac{1}{2\pi\alpha'} \frac{\partial X^\mu}{\partial \sigma}$
→ EOM ⇒ $\ddot{X} - X'' = 0$
(I) + (II)
$(\dot{X} \pm X')^2 = 0$
Solve free open string $P^{\sigma\mu} = 0 \big|_{\sigma=0,\pi}$ $\frac{\partial X^\mu}{\partial \sigma}\Big|_{\sigma=0,\pi} = 0$ "$\cos n\sigma$" × $e^{-in\tau}$
$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$
Real if
$= \alpha_{-n}^\mu$
$P^{\tau\mu} = \frac{1}{2\pi\alpha'} \frac{\partial X^\mu}{\partial \tau} = \frac{1}{2\pi\alpha'} \sqrt{2\alpha'}\,\alpha_0^\mu + \ldots\ldots$
$p^\mu = \int_0^\pi d\sigma\, P^{\tau\mu} = \frac{1}{2\pi\alpha'} \sqrt{2\alpha'}\,\alpha_0^\mu \pi =$



$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\frac{\partial X^\mu}{\partial \tau}$
$\mathcal{P}^{\sigma\mu} = -\frac{1}{2\pi\alpha'}\frac{\partial X^\mu}{\partial \sigma}$
$\rightarrow$ EOM $\Rightarrow$ $\ddot{X} - X'' = 0$
(I) + (II)
$(\dot{X} \pm X')^2 = 0$
Solve free open strings
$\mathcal{P}^{\sigma\mu} = 0 \big|_{\sigma=0,\pi}$
$\frac{\partial X^\mu}{\partial \sigma}\Big|_{\sigma=0,\pi} = 0$
"$\cos n\sigma$" $\times e^{-in\tau}$
$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$
Real if $(\alpha_n^\mu)^* = \alpha_{-n}^\mu$
$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\frac{\partial X^\mu}{\partial \tau} = \frac{1}{2\pi\alpha'}\sqrt{2\alpha'}\,\alpha_0^\mu + \ldots\ldots$
$p^\mu = \int_0^\pi d\sigma\, \mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\sqrt{2\alpha'}\,\alpha_0^\mu \pi = \frac{\alpha_0^\mu}{\sqrt{2\alpha'}}$
$\alpha_0^\mu = \sqrt{2\alpha'}\, p^\mu$
$2\alpha' p\, \tau$
$L^2 \frac{1}{L}$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(I) + (II)
$(\dot{X} \pm X')^2 = 0$
Solve free open strings
$P^{\sigma\mu} = 0 \big|_{\sigma=0,\pi}$
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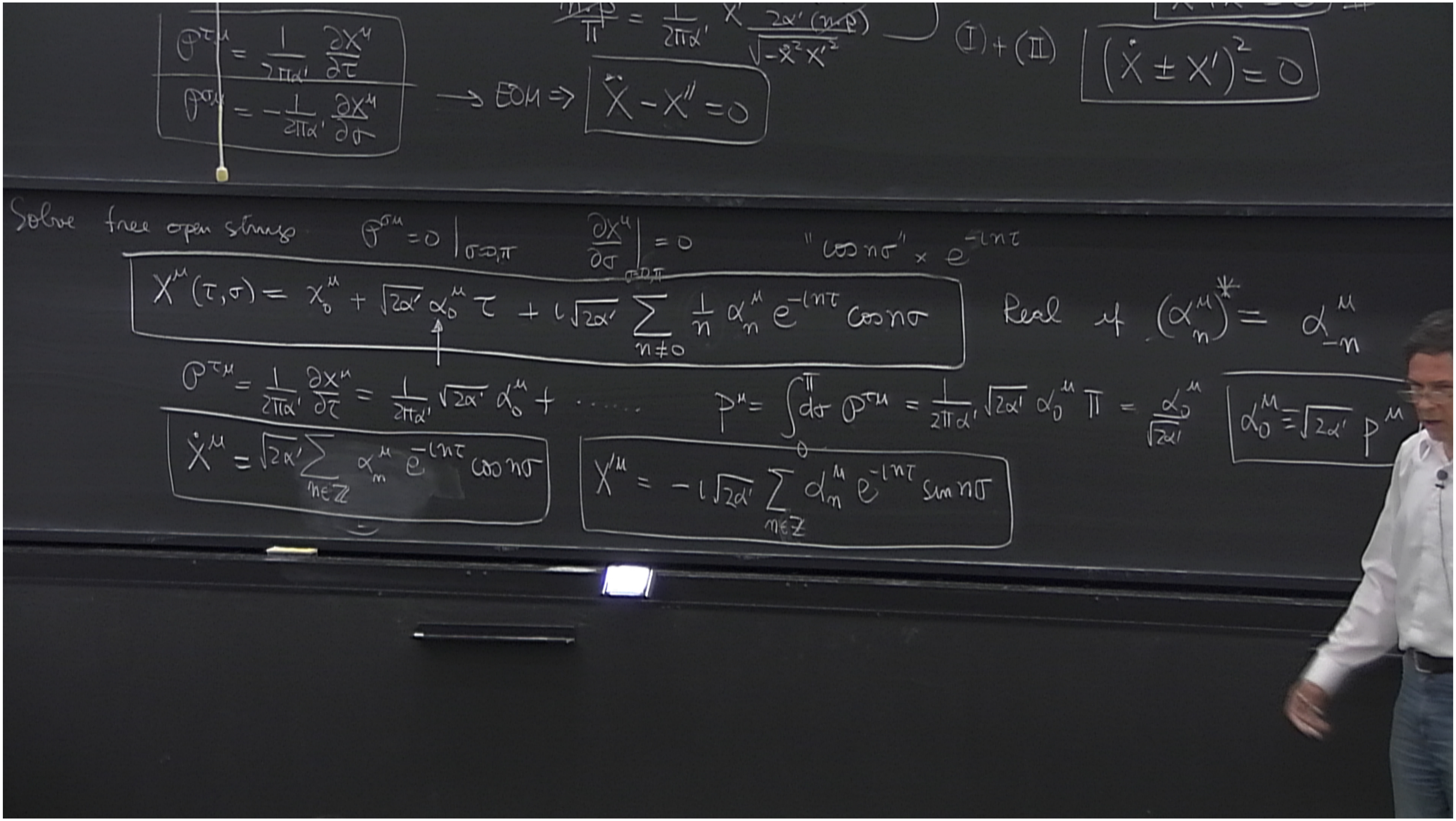
"$\cos n\sigma$" $\times\, e^{-in\tau}$
$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$
Real if
α_{-n}^μ
$P^{\tau\mu} = \frac{1}{2\pi\alpha'} \frac{\partial X^\mu}{\partial \tau} = \frac{1}{2\pi\alpha'} \sqrt{2\alpha'}\,\alpha_0^\mu + \ldots\ldots$
$p^\mu = \int_0^\pi d\sigma\, P^{\tau\mu}$
$= \sqrt{2\alpha'}\, p^\mu$
$\dot{X}^\mu = \sqrt{2\alpha'} \sum_{n\in\mathbb{Z}} \alpha_n^\mu e^{-in\tau} \cos n\sigma$
$X'^\mu = -i\sqrt{2\alpha'} \sum_{n\in\mathbb{Z}} \alpha_n^\mu e^{-in\tau} \sin n\sigma$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$P^{\sigma\mu} = -\frac{1}{2\pi\alpha'}\frac{\partial X^\mu}{\partial\sigma}$
$\rightarrow$ EOM $\Rightarrow$ $\ddot{X} - X'' = 0$
(I) + (II)
$(\dot{X} \pm X')^2 = 0$
Solve free open string
$P^{\sigma\mu} = 0 \big|_{\sigma=0,\pi}$
$\frac{\partial X^\mu}{\partial\sigma}\Big|_{\sigma=0,\pi} = 0$
"$\cos n\sigma$" $\times$ $e^{-in\tau}$
$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu\,\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-in\tau}\cos n\sigma$
Real if $(\alpha_n^\mu)^* = \alpha_{-n}^\mu$
$P^{\tau\mu} = \frac{1}{2\pi\alpha'}\frac{\partial X^\mu}{\partial\tau} = \frac{1}{2\pi\alpha'}\sqrt{2\alpha'}\,\alpha_0^\mu + \cdots$
$p^\mu = \int_0^\pi d\sigma\, P^{\tau\mu} = \frac{1}{2\pi\alpha'}\sqrt{2\alpha'}\,\alpha_0^\mu\,\pi = \frac{\alpha_0^\mu}{\sqrt{2\alpha'}}$
$\alpha_0^\mu = \sqrt{2\alpha'}\,p^\mu$
$\dot{X}^\mu = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}}\alpha_n^\mu e^{-in\tau}\cos n\sigma$
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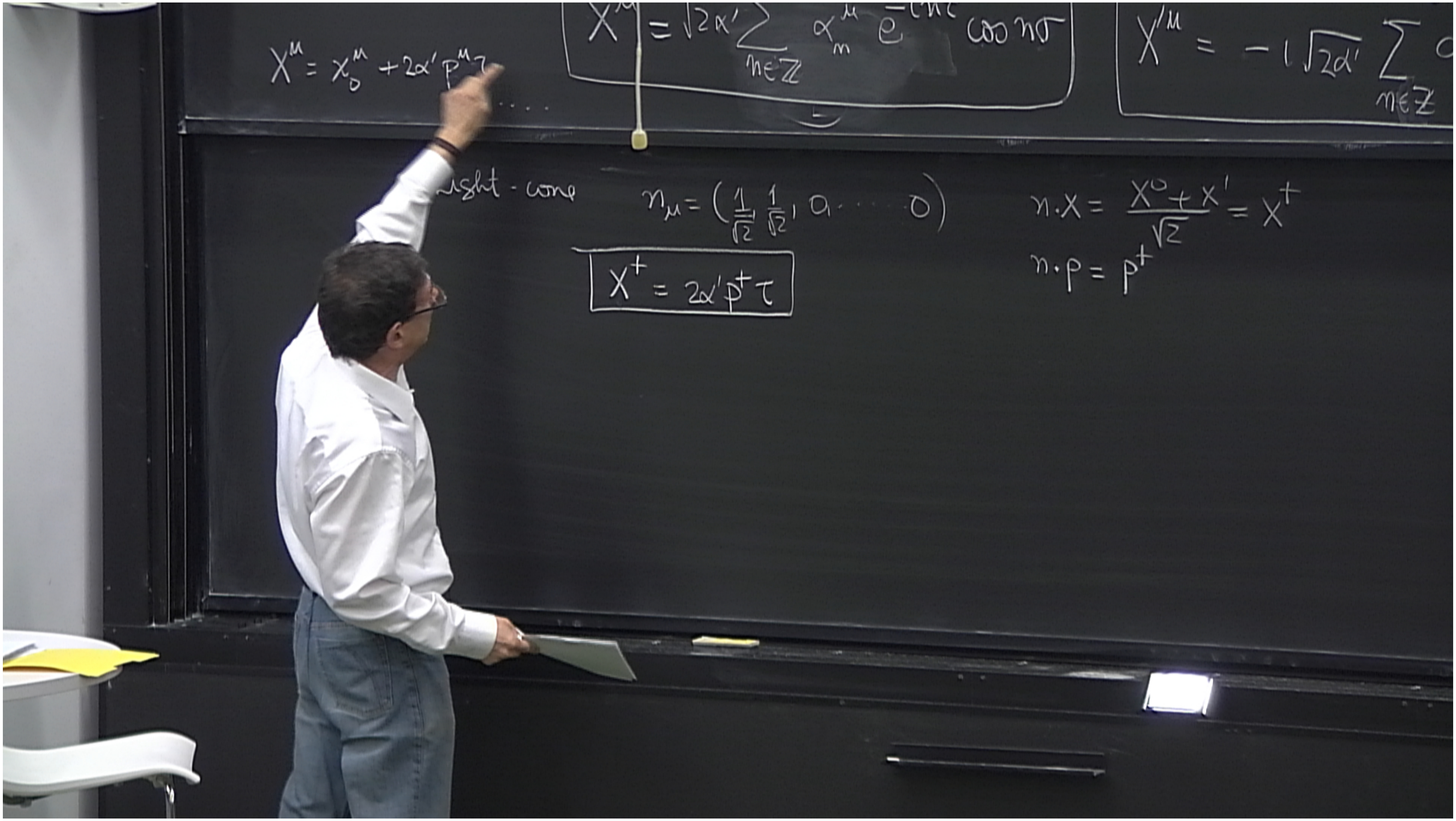
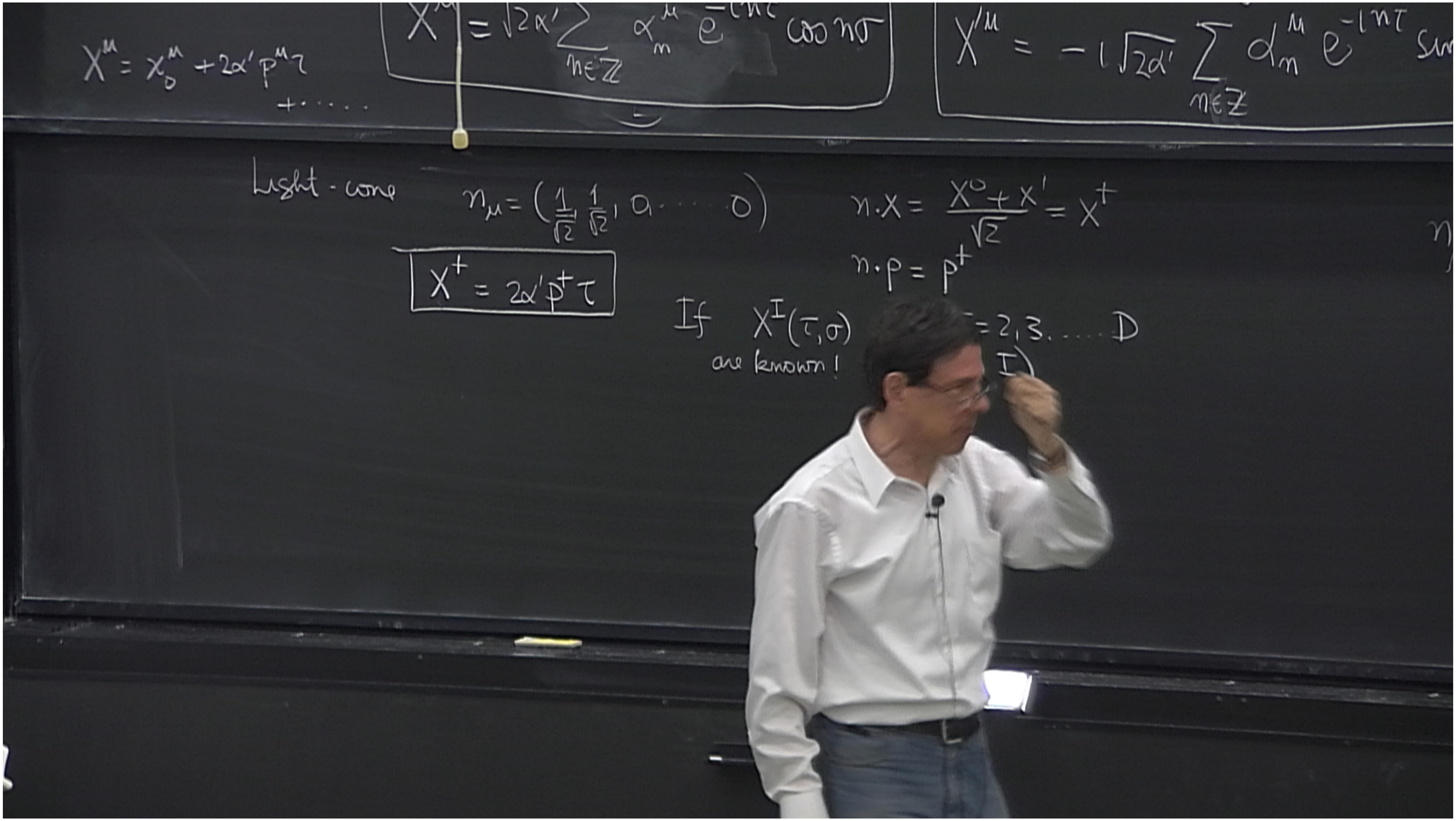

X^μ = x_0^μ + 2α' p^μ τ
n_μ = (1/√2, 1/√2, 0, …, 0)
n·X = (X^0 + X^1)/√2 = X^+
n·p = p^+
X^+ = 2α' p^+ τ


Light-cone
If
are known!

$$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\frac{\partial X^\mu}{\partial \tau} = \frac{1}{2\pi\alpha'}\sqrt{2\alpha'}\,\alpha_0^\mu + \cdots\cdots$$

$$p^\mu = \int_0^\pi d\sigma\, \mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\sqrt{2\alpha'}\,\alpha_0^\mu\,\pi = \frac{\alpha_0^\mu}{\sqrt{2\alpha'}}$$

$$\boxed{\alpha_0^\mu = \sqrt{2\alpha'}\,p^\mu}$$

$$\boxed{\dot{X}^\mu = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} \alpha_n^\mu\, e^{-in\tau}\cos n\sigma}$$

$$\boxed{X'^\mu = -i\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} \alpha_n^\mu\, e^{-in\tau}\sin n\sigma}$$

$$\boxed{\dot{X}^\mu \pm X'^\mu = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} \alpha_n^\mu\, e^{-in(\tau\pm\sigma)}}$$

$p^\mu \tau$

$+ \cdots\cdots$

light-cone $\quad n_\mu = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \cdots 0\right)$

$$n\cdot X = \frac{X^0 + X^1}{\sqrt{2}} = X^+$$

$$n\cdot p = p^+$$

$$n_\mu X^\mu = n_0 X^0 + n_1 X^1 + \cdots$$

$$\boxed{X^+ = 2\alpha' p^+ \tau}$$

If $X^I(\tau,\sigma)$ are known! $\quad I = 2, 3, \ldots D$

$(+-, I)$

$$X^+(\tau,\sigma),\ X^-(\tau,\sigma),\ X^I(\tau,\sigma)$$

$$= -(a^0)^2 + \left((a^1)^2 + (a^2)^2 + \cdots\right)$$

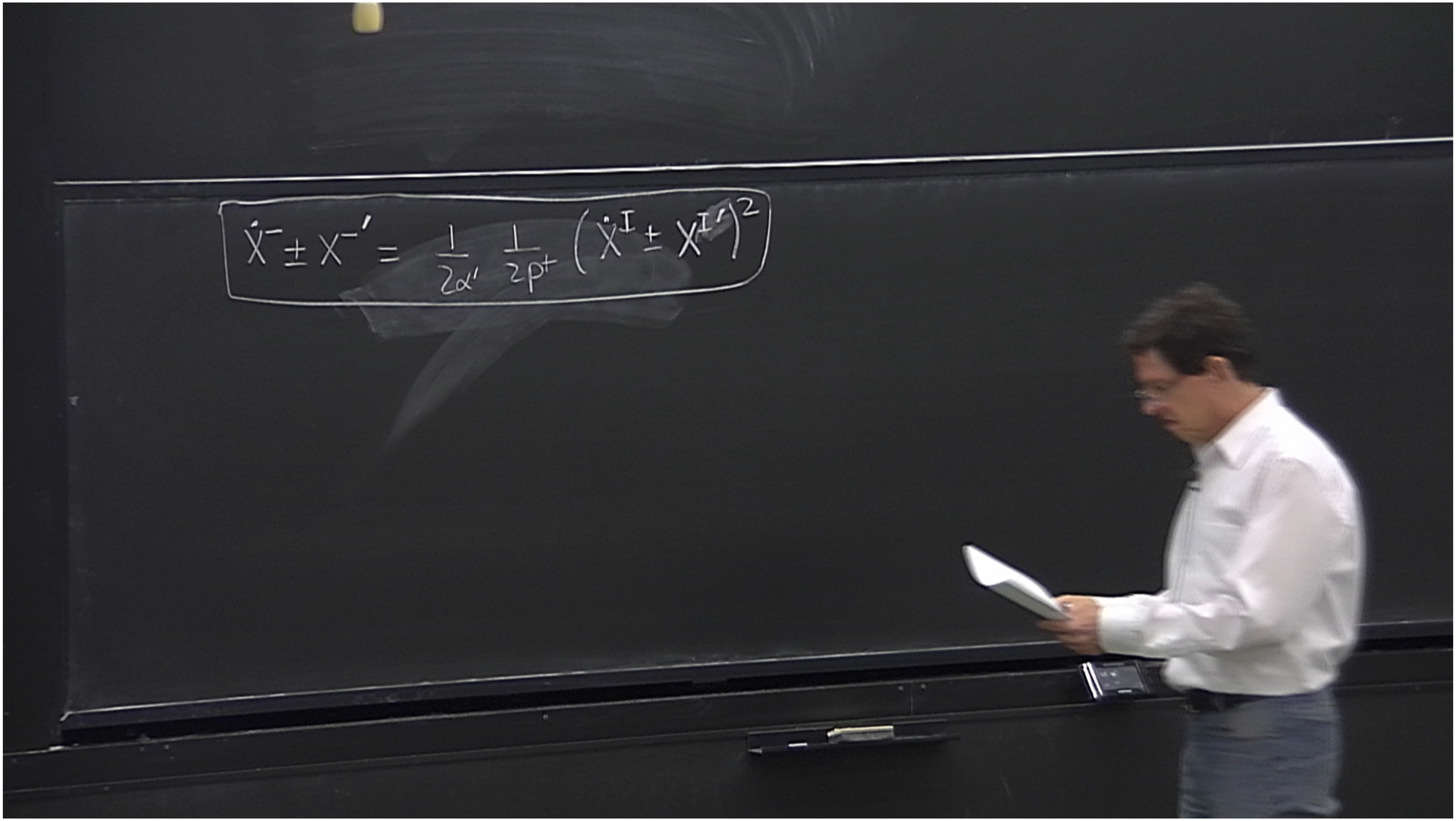
$$a\cdot a = -2a^+a^- + \vec{a}_T\cdot\vec{a}_T$$

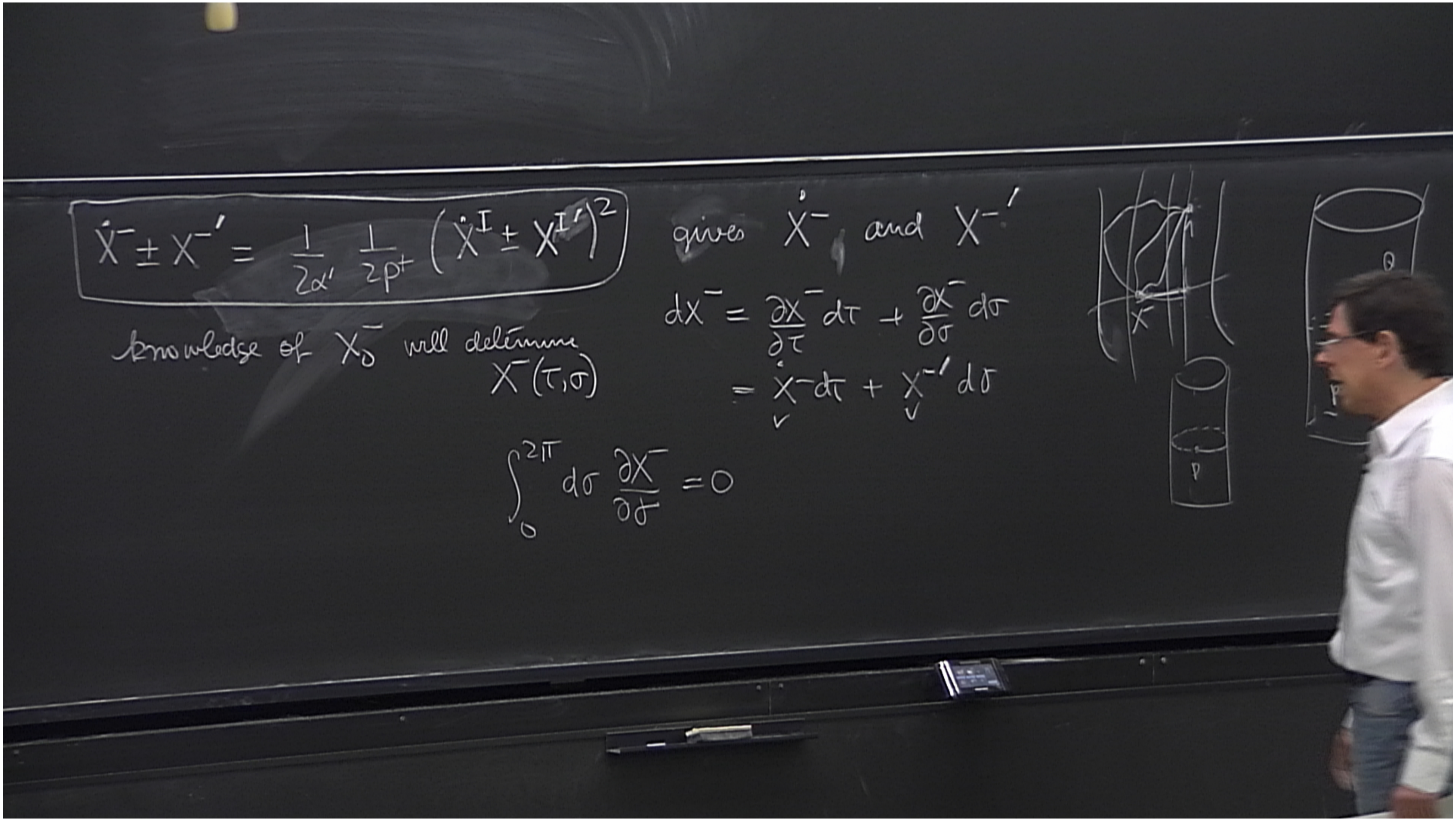
$$(\dot{X} \pm X')^2 = 0$$

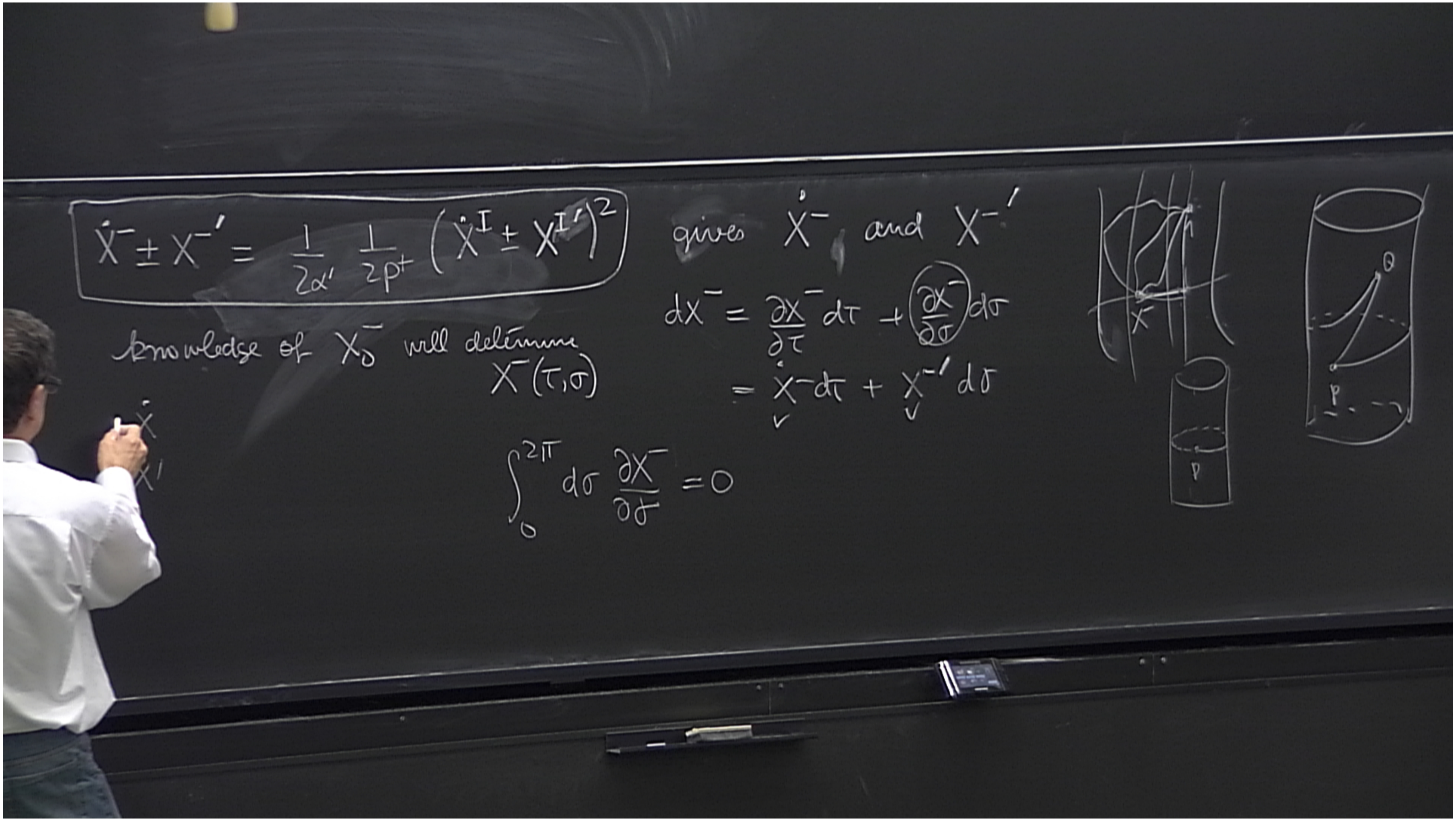
$$-2(\dot{X}^+ \pm X^{+\prime})(\dot{X}^- \pm X^{-\prime}) + (\dot{X}^I \pm X^{I\prime})(\dot{X}^I \pm X^{I\prime}) = 0$$

$$-2\cdot 2\alpha' p^+ (\dot{X}^- \pm X^{-\prime}) + \quad ''$$

$$\dot{X}^- \pm X^{-\prime} = \frac{1}{2\alpha'} \frac{1}{2p^+} \left(\dot{X}^I \pm X^{I\prime}\right)^2$$


$\dot{X}^- \pm X^{-\prime} = \frac{1}{2\alpha'}\frac{1}{2p^+}\left(\dot{X}^I \pm X^{I\prime}\right)^2$


$\dot{X}^- \pm X^{-\prime} = \frac{1}{2\alpha'}\frac{1}{2p^+}\left(\dot{X}^I \pm X^{I\prime}\right)^2$
gives $\dot{X}^-$ and $X^{-\prime}$
knowledge of X_0^- will determine $X^-(\tau,\sigma)$
$dX^- = \frac{\partial X^-}{\partial \tau} d\tau + \frac{\partial X^-}{\partial \sigma} d\sigma = \dot{X}^- d\tau + X^{-\prime} d\sigma$
$\int_0^{2\pi} d\sigma \frac{\partial X^-}{\partial \sigma} = 0$


$\dot{X}^- \pm X^{-\prime} = \frac{1}{2\alpha'} \frac{1}{2p^+} (\dot{X}^I \pm X^{I\prime})^2$
gives $\dot{X}^-$ and $X^{-\prime}$
knowledge of X_0^- will determine $X^-(\tau,\sigma)$
$dX^- = \frac{\partial X^-}{\partial \tau} d\tau + \frac{\partial X^-}{\partial \sigma} d\sigma$
$= \dot{X}^- d\tau + X^{-\prime} d\sigma$
$\int_0^{2\pi} d\sigma \frac{\partial X^-}{\partial \sigma} = 0$
$\dot{X}$
X'
X^-
P
Q
p

$$\dot X^- \pm X^{-\prime} = \frac{1}{2\alpha'}\frac{1}{2p^+}\left(\dot X^I \pm X^{I\prime}\right)^2$$

gives $\dot X^-$ and $X^{-\prime}$

$$dX^- = \frac{\partial X^-}{\partial \tau}d\tau + \frac{\partial X^-}{\partial \sigma}d\sigma$$

$$= \dot X^- d\tau + X^{-\prime} d\sigma$$

knowledge of x_0^- will determine $X^-(\tau,\sigma)$

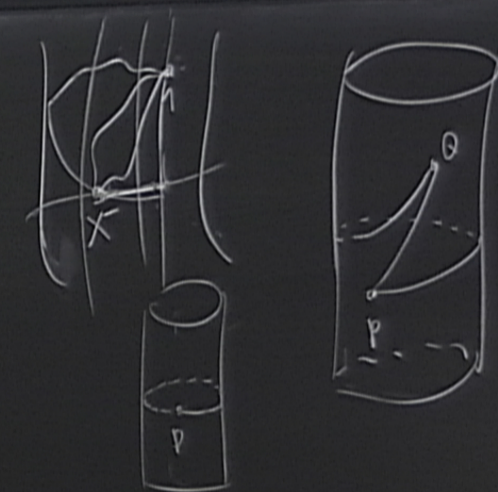
$\partial_\sigma \dot X$

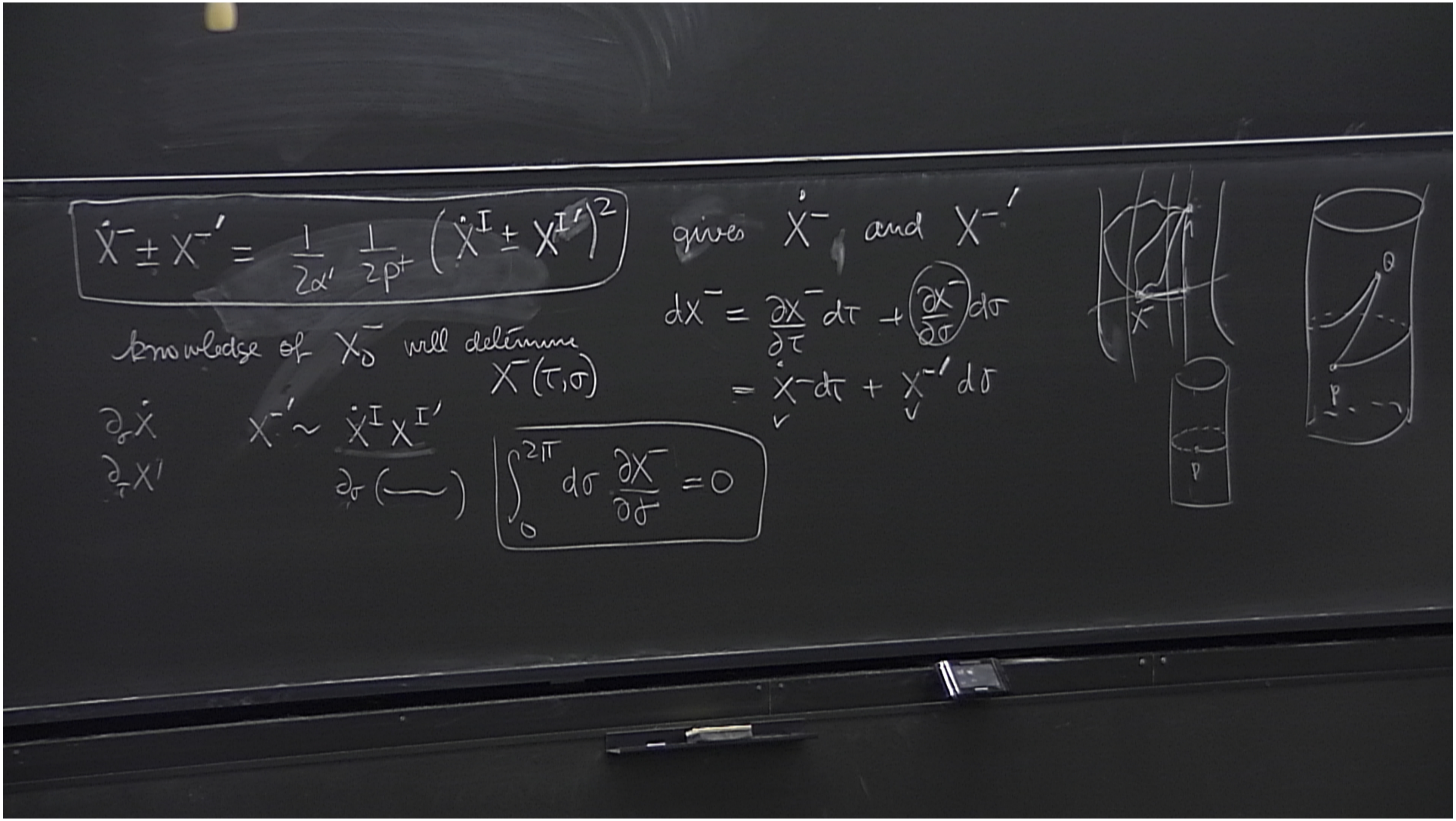
$\partial_\tau X'$

$X^{-\prime} \sim \dot X^I X^{I\prime}$

$\partial_\sigma(\text{——})$

$$\int_0^{2\pi} d\sigma \frac{\partial X^-}{\partial \sigma} = 0$$




Ẋ⁻ ± X⁻′ = 1/(2α′) 1/(2p⁺) (Ẋᴵ ± Xᴵ′)²
gives Ẋ⁻ and X⁻′
dX⁻ = ∂X⁻/∂τ dτ + ∂X⁻/∂σ dσ
= Ẋ⁻ dτ + X⁻′ dσ
knowledge of X₀⁻ will determine
X⁻(τ,σ)
X⁻′ ~ ẊᴵXᴵ′
∫₀^{2π} dσ ∂X⁻/∂σ = 0
Q
P

$$\dot X^- \pm X^{-\prime} = \frac{1}{2\alpha'}\frac{1}{2p^+}\left(\dot X^I \pm X^{I\prime}\right)^2$$

gives $\dot X^-$ and $X^{-\prime}$

$$dX^- = \frac{\partial X^-}{\partial \tau} d\tau + \frac{\partial X^-}{\partial \sigma} d\sigma = \dot X^- d\tau + X^{-\prime} d\sigma$$

knowledge of X_0^- will determine $X^-(\tau,\sigma)$

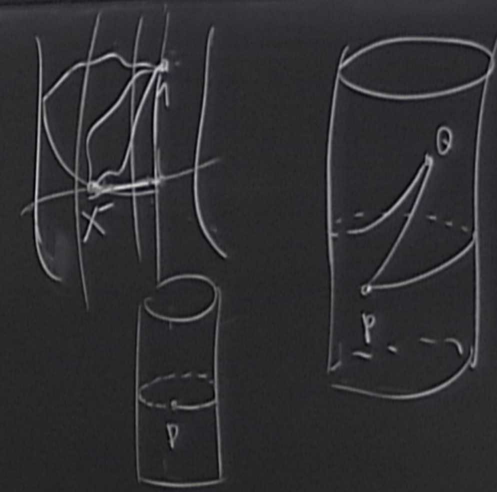
$\partial_\sigma \dot X$

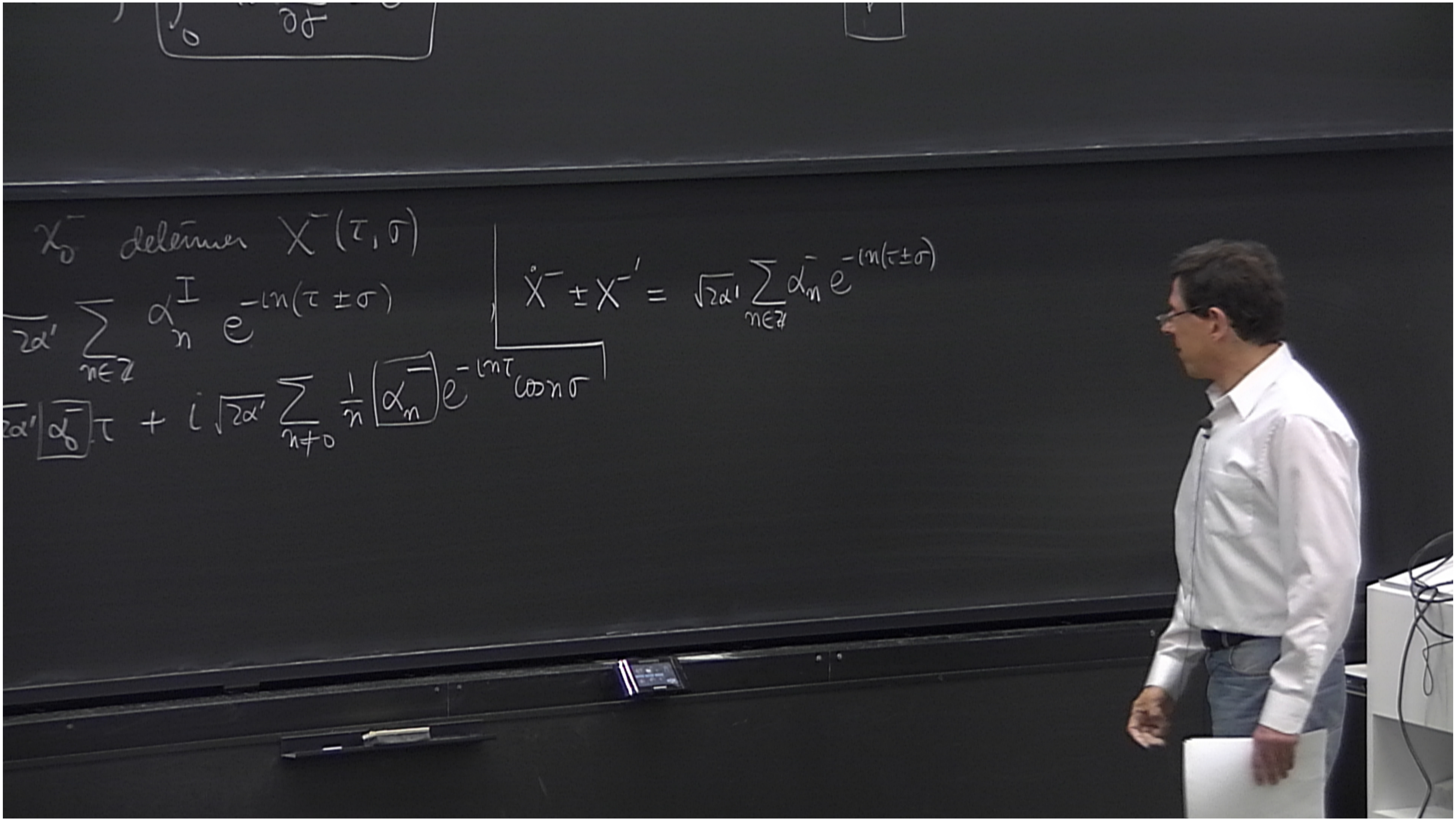
$\partial_\tau X'$

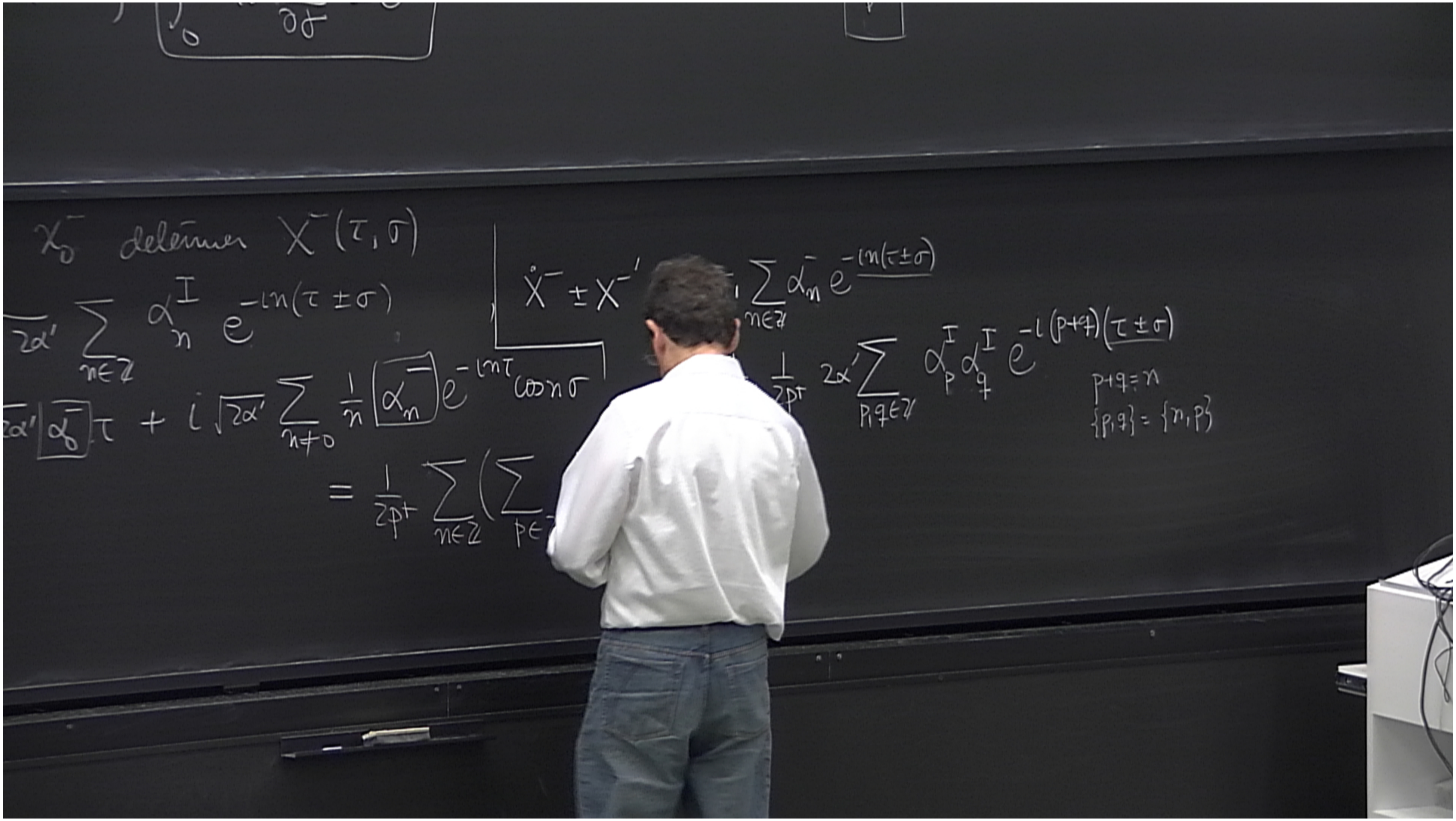
$X^{-\prime} \sim \dot X^I X^{I\prime}$

$\partial_\sigma(\text{——})$

$$\int_0^{2\pi} d\sigma \frac{\partial X^-}{\partial \sigma} = 0$$




x_0^- determines $X^-(\tau,\sigma)$
$\dot{X}^- \pm X^{-\prime} = \sqrt{2\alpha'} \sum_{n\in\mathbb{Z}} \alpha_n^- e^{-in(\tau\pm\sigma)}$

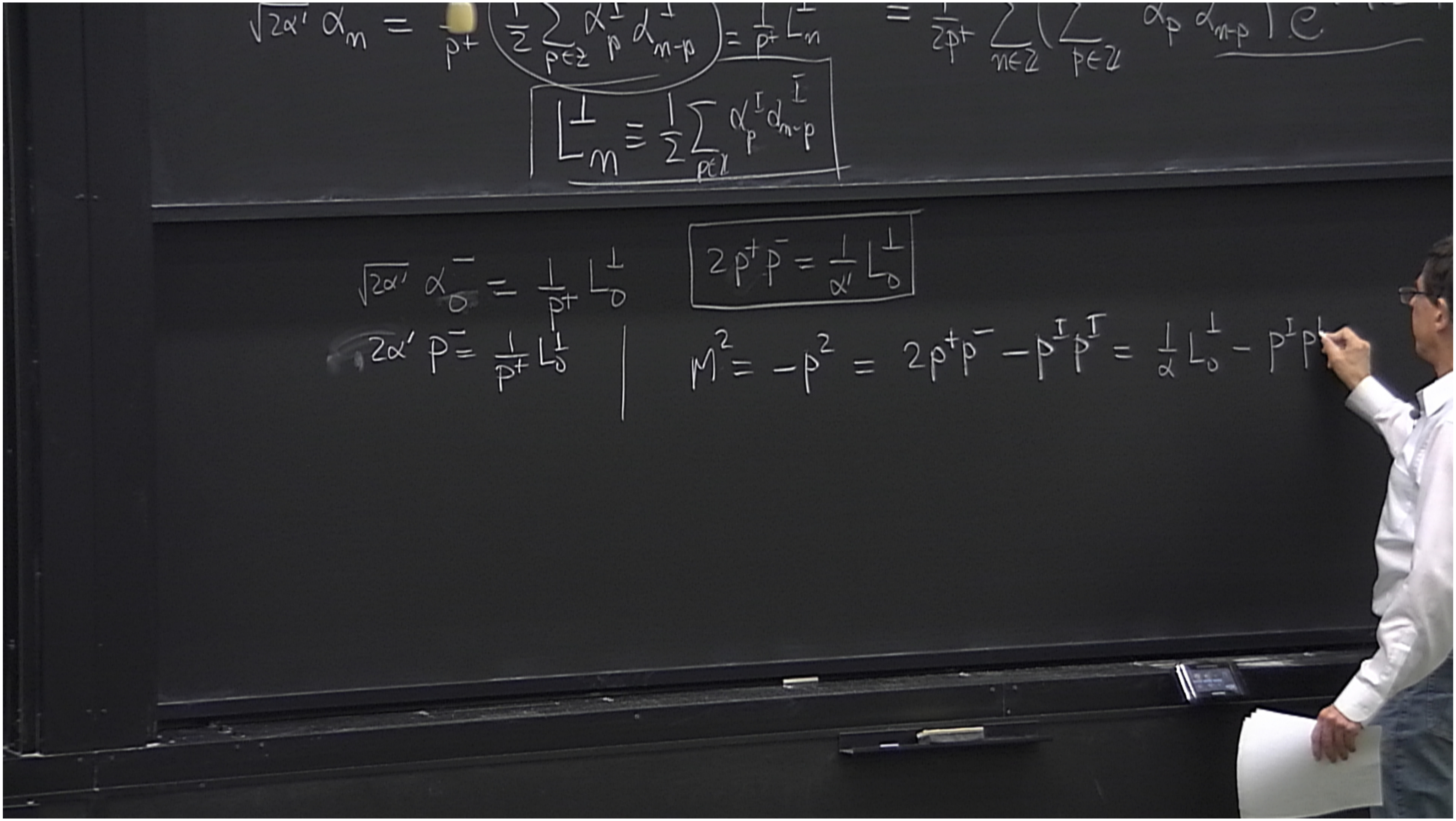

x0- determines X-(τ,σ)
p+q=n
{p,q}={n,p}

$$\sqrt{2\alpha'}\,\alpha^-_n = \frac{1}{p^+}\left(\frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha^I_p\,\alpha^I_{n-p}\right) = \frac{1}{p^+}L^\perp_n \qquad = \frac{1}{2p^+}\sum_{n\in\mathbb{Z}}\Big(\sum_{p\in\mathbb{Z}}\alpha_p\,\alpha_{n-p}\Big)e$$

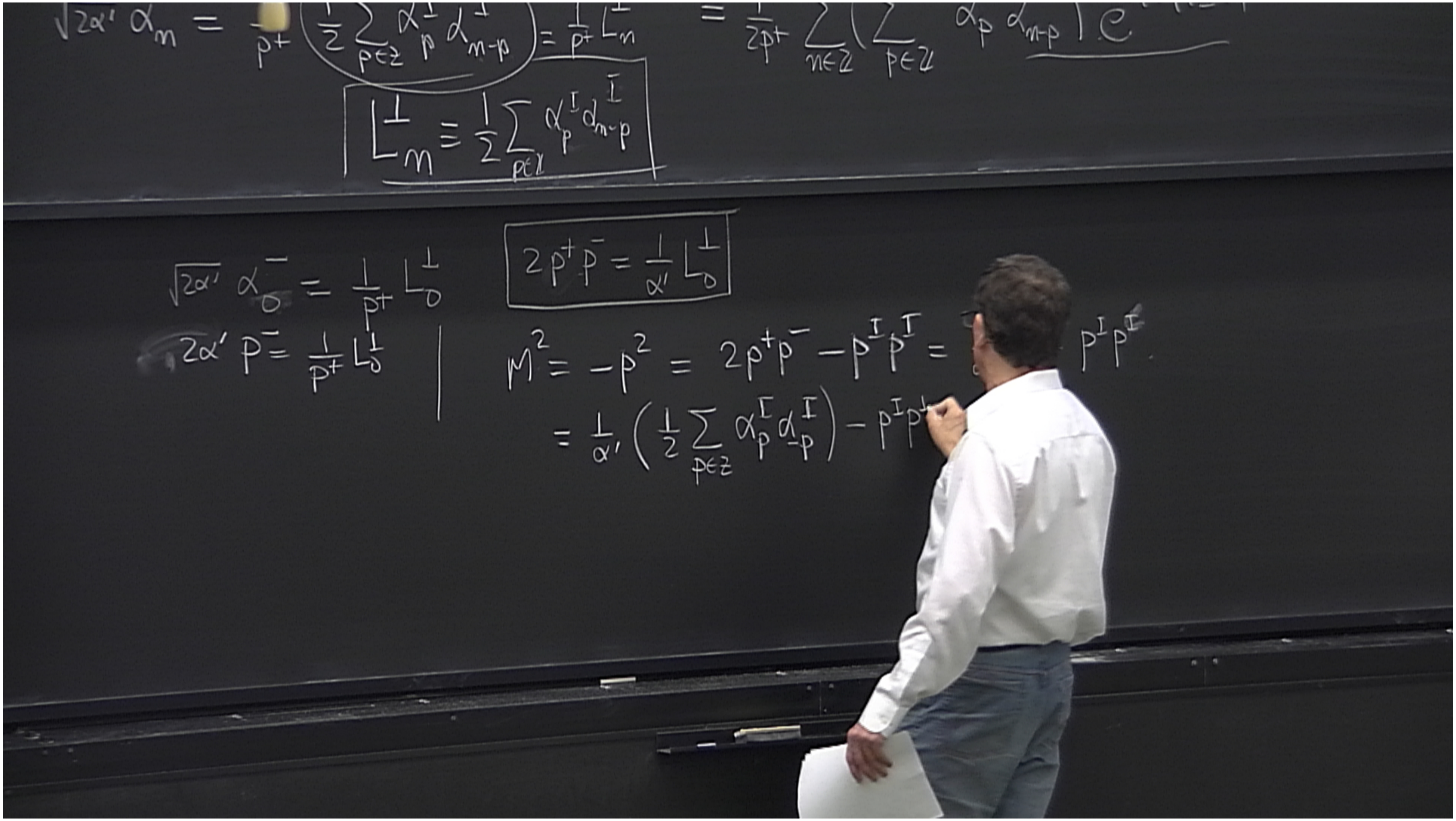
$$\boxed{L^\perp_n \equiv \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha^I_p\,\alpha^I_{n-p}}$$

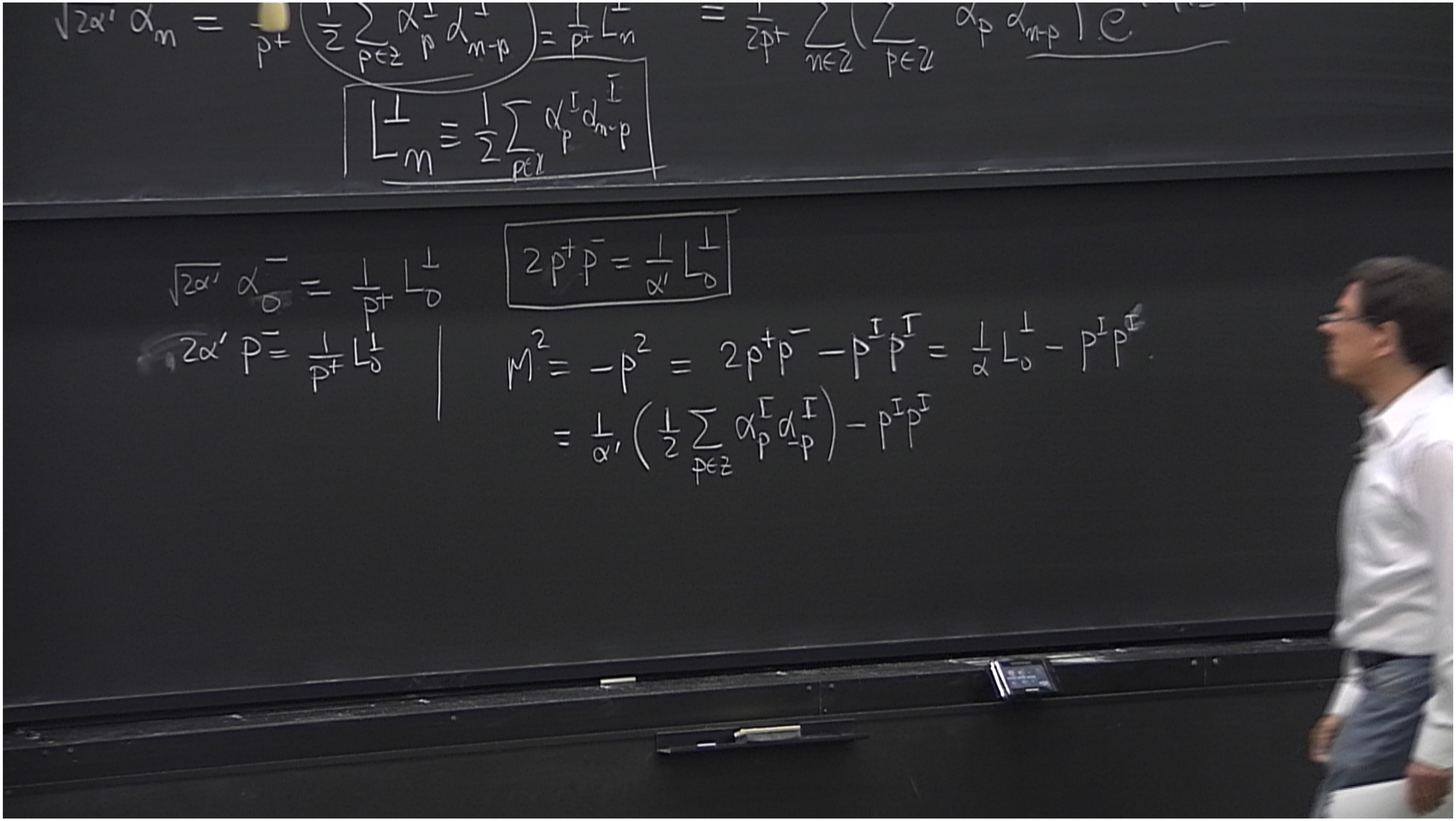
$$\sqrt{2\alpha'}\,\alpha^-_0 = \frac{1}{p^+}L^\perp_0 \qquad \boxed{2p^+p^- = \frac{1}{\alpha'}L^\perp_0}$$

$$2\alpha'\,p^- = \frac{1}{p^+}L^\perp_0$$



$\sqrt{2\alpha'}\,\alpha_n^- = \frac{1}{p^+}\,\frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_p^I \alpha_{n-p}^I = \frac{1}{p^+} L_n^\perp$
$L_n^\perp \equiv \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_p^I \alpha_{n-p}^I$
$\sqrt{2\alpha'}\,\alpha_0^- = \frac{1}{p^+} L_0^\perp$
$2p^+p^- = \frac{1}{\alpha'} L_0^\perp$
$2\alpha' p^- = \frac{1}{p^+} L_0^\perp$
$M^2 = -p^2 = 2p^+p^- - p^I p^I = \frac{1}{\alpha} L_0^\perp - p^I p^I$


$L^{\perp}_{n} \equiv \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha^{I}_{p}\alpha^{I}_{n-p}$
$\sqrt{2\alpha'}\,\alpha^{-}_{0} = \frac{1}{p^{+}} L^{\perp}_{0}$
$2\alpha' p^{-} = \frac{1}{p^{+}} L^{\perp}_{0}$
$2p^{+}p^{-} = \frac{1}{\alpha'} L^{\perp}_{0}$
$M^{2} = -p^{2} = 2p^{+}p^{-} - p^{I}p^{I}$
$= \frac{1}{\alpha'}\left(\frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha^{I}_{p}\alpha^{I}_{-p}\right) - p^{I}p^{I}$


√2α' α_n = 1/p+ (1/2 Σ_{p∈Z} α^⊥_p α^⊥_{n-p}) = 1/p+ L^⊥_n = 1/2p+ Σ_{n∈Z} (Σ_{p∈Z} α_p α_{n-p}) e
L^⊥_n ≡ 1/2 Σ_{p∈Z} α^I_p α^I_{n-p}
√2α' α^-_0 = 1/p+ L^⊥_0
2p^+ p^- = 1/α' L^⊥_0
2α' p^- = 1/p+ L^⊥_0
M^2 = -p^2 = 2p^+ p^- - p^I p^I = 1/α' L^⊥_0 - p^I p^I
= 1/α' (1/2 Σ_{p∈Z} α^I_p α^I_{-p}) - p^I p^I

$$\sqrt{2\alpha'}\,\alpha_n^- = \frac{1}{p^+}\left(\frac{1}{2}\sum_{p\in \mathbb{Z}}\alpha_p^I\alpha_{n-p}^I\right) = \frac{1}{p^+}L_n^\perp \qquad = \frac{1}{2p^+}\sum_{n\in\mathbb{Z}}\Big(\sum_{p\in\mathbb{Z}}\alpha_p\,\alpha_{n-p}\Big)e^{\cdots}$$

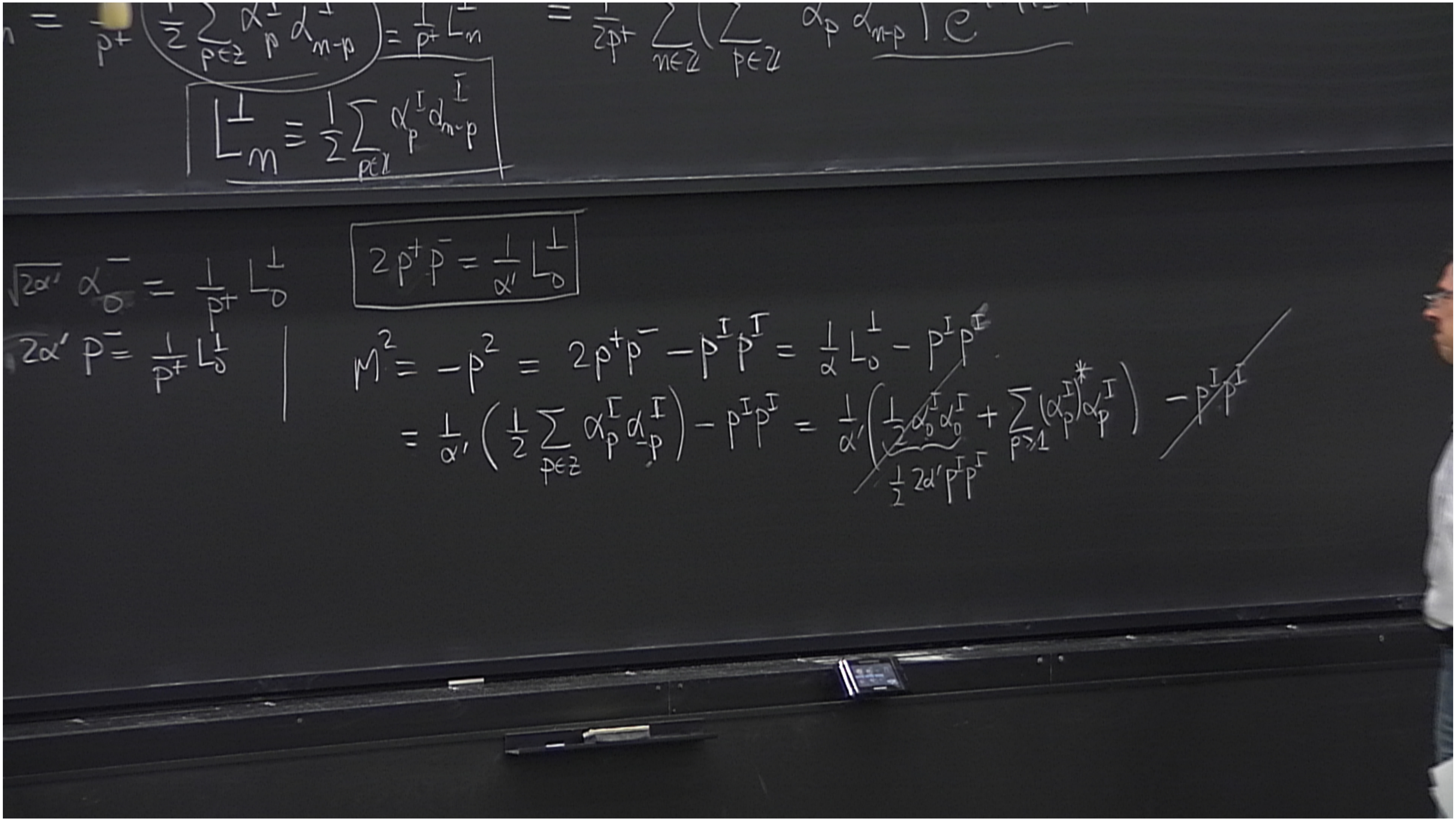
$$\boxed{L_n^\perp \equiv \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_p^I\alpha_{n-p}^I}$$

$$\sqrt{2\alpha'}\,\alpha_0^- = \frac{1}{p^+}L_0^\perp \qquad \boxed{2p^+p^- = \frac{1}{\alpha'}L_0^\perp}$$

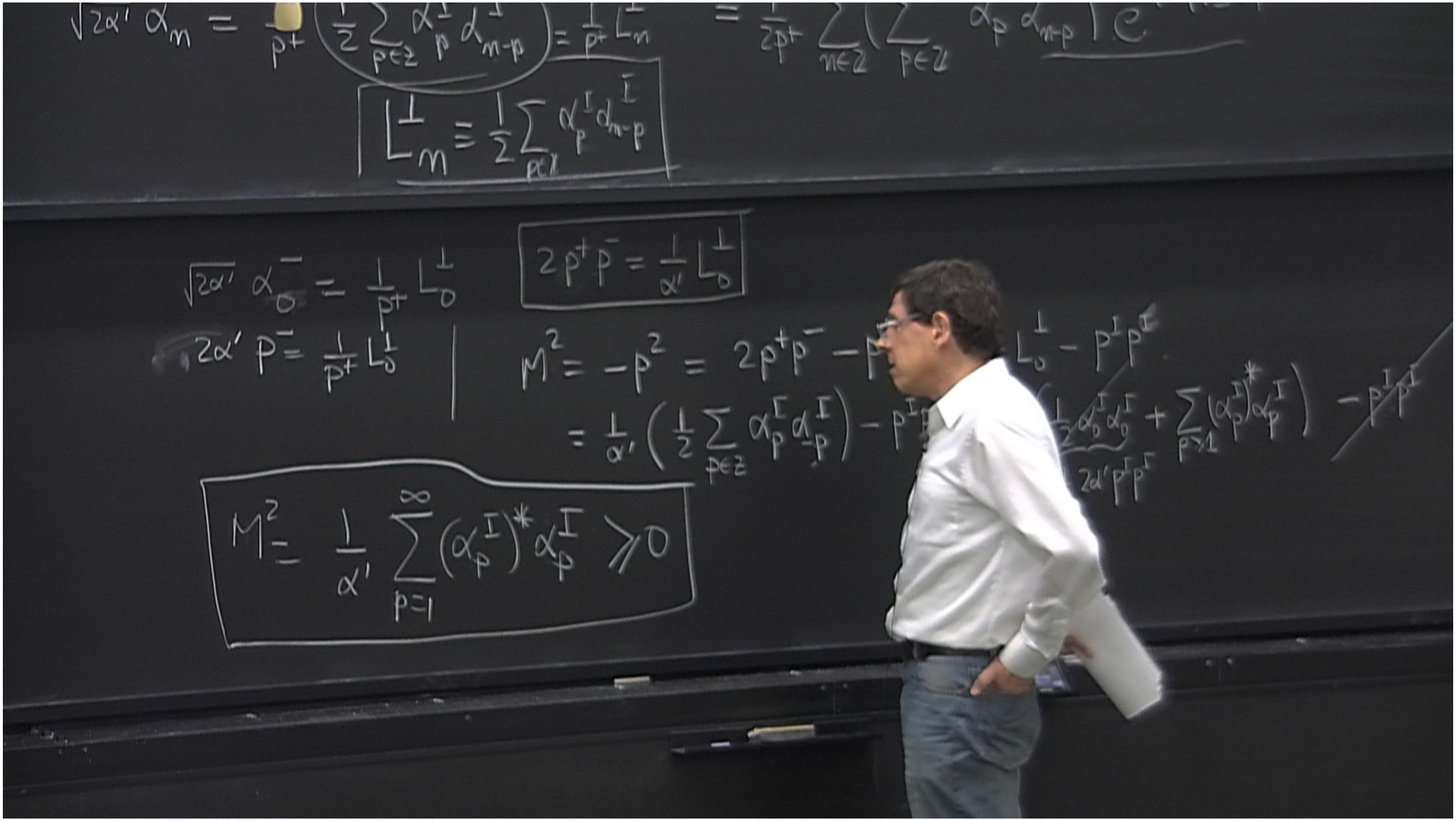
$$2\alpha'\,p^- = \frac{1}{p^+}L_0^\perp$$

$$M^2 = -p^2 = 2p^+p^- - p^Ip^I = \frac{1}{\alpha'}L_0^\perp - p^Ip^I$$

$$= \frac{1}{\alpha'}\left(\frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_p^I\alpha_{-p}^I\right) - p^Ip^I = \frac{1}{\alpha'}\left(\frac{1}{2}\alpha_0^I\alpha_0^I + \sum_{p\geq 1}(\alpha_p^I)^*\alpha_p^I\right.$$



L⊥m ≡ ½ Σ_{p∈Z} α_p^I α_{n−p}^I
2p⁺p⁻ = (1/α′) L⊥₀
√(2α′) α⁻₀ = (1/p⁺) L⊥₀
2α′ p⁻ = (1/p⁺) L⊥₀
M² = −p² = 2p⁺p⁻ − p^I p^I = (1/α′) L⊥₀ − p^I p^I
= (1/α′) (½ Σ_{p∈Z} α_p^I α_{−p}^I) − p^I p^I = (1/α′) (½ α₀^I α₀^I + Σ_{p≥1} (α_p^I)* α_p^I) − p^I p^I
½ 2α′ p^I p^I


√2α' α_n^- = 1/p^+ (1/2 Σ_{p∈Z} α_p^I α_{n-p}^I) = 1/p^+ L_n^⊥
L_n^⊥ ≡ 1/2 Σ_{p∈Z} α_p^I α_{n-p}^I
√2α' α_0^- = 1/p^+ L_0^⊥
2α' p^- = 1/p^+ L_0^⊥
2p^+p^- = 1/α' L_0^⊥
M^2 = -p^2 = 2p^+p^- - p^I p^I
= 1/α' (1/2 Σ_{p∈Z} α_p^I α_{-p}^I) - p^I p^I
2α' p^I p^I
M^2 = 1/α' Σ_{p=1}^∞ (α_p^I)^* α_p^I ≥ 0