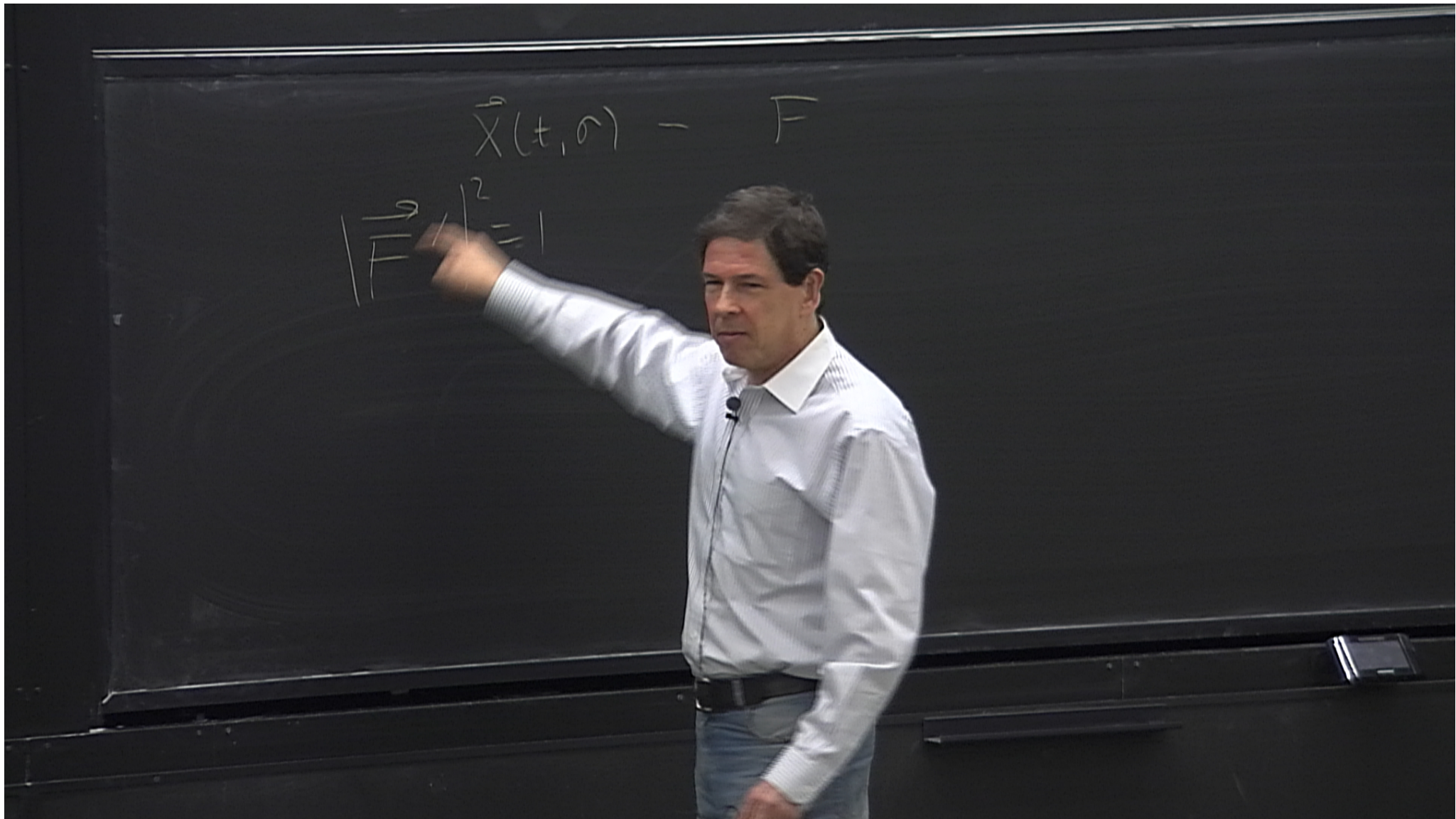


Title: 12/13 PSI - String Theory Review Lecture 6

Date: Jan 14, 2013 10:15 AM

URL: <http://pirsa.org/13010052>

Abstract:



What are :

$$\Theta^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial x'^\mu}$$

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What are :

$$\Theta^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial x'^\mu}$$

What are:

$$\begin{aligned} \mathcal{P}^{\mu} &= \frac{\partial \mathcal{L}}{\partial \dot{x}^{\mu}} \\ \mathcal{P}^{\mu\nu} &= \frac{\partial \mathcal{L}}{\partial \dot{x}^{\mu} \dot{x}^{\nu}} \end{aligned}$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(x)$, coordinates x^{μ} $\mu=0, \dots, D$
and symmetries $\delta\phi^a$

What are:

$$\begin{aligned} \mathcal{P}^{\mu} &= \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} \\ \mathcal{P}^{\mu} &= \end{aligned}$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^{α} $\alpha=0, \dots, D$
and symmetries $\delta \phi^a$ (leading to $\delta \mathcal{L}=0$) with parameters ϵ^i
we have currents j^{α}_i that are conserved:

What are:

$$\begin{aligned} \mathcal{P}^{\mu\nu} &= \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} \\ \mathcal{P}^{\mu\alpha} &= \frac{\partial \mathcal{L}}{\partial \dot{x}^\alpha} \end{aligned}$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^α $\alpha=0, \dots, D$
and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters ϵ^i
we have currents j_i^a that are conserved:

$$\epsilon^i j_i^a(\xi) = \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi^a)} \delta\phi^a$$

What are:

$$\mathcal{P}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$\mathcal{P}^{0\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^α $\alpha=0, \dots, D$
 and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters ϵ^i
 we have currents j_i^α that are conserved:

$$\epsilon^i j_i^\alpha(\xi) \equiv \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi^a)} \delta\phi^a \quad \frac{\partial j_i^\alpha}{\partial \xi^\alpha} = 0$$

What are:

$$\mathcal{P}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$$

$$\mathcal{P}^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^α $\alpha=0, \dots, D$
 and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters ϵ^i
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What are:

$$\begin{aligned} \mathcal{P}^{\mu\nu} &= \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} \\ \mathcal{P}^{\sigma\mu} &= \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} \end{aligned}$$

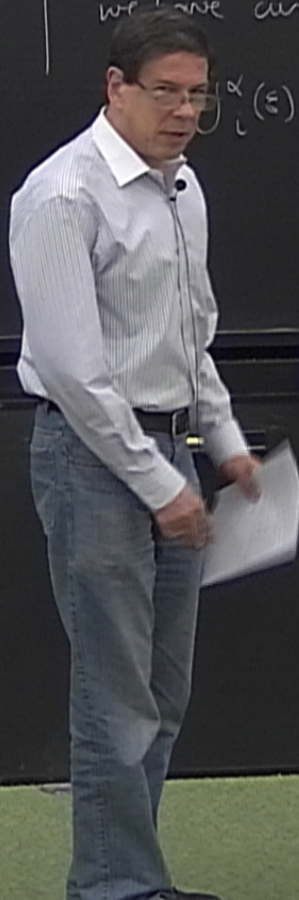
Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^α $\alpha=0, \dots, D$
 and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters $\underline{\epsilon}^i$
 we have currents J_i^α that are conserved:

$$J_i^\alpha(\xi) \equiv \frac{\partial \mathcal{L}}{\partial (2\alpha\phi^a)} \delta\phi^a$$

$$\frac{\partial J_i^\alpha}{\partial \xi^\alpha} = 0$$

$$Q_i = \int d\xi^1 \dots d\xi^D J_i^0(\xi)$$

$\downarrow$
 $\frac{dQ_i}{d\xi^0} = 0$ with good BC.



What are:

$$\Theta^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial X'^\mu}$$

$$\mathcal{L}(\dot{X}, X')$$

$$\xi^\alpha = \tau, \sigma$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^α $\alpha=0, 1$
 and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters ϵ^i
 we have currents J_i^α that are conserved:

$$\epsilon^i J_i^\alpha(\xi) \equiv \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi^a)} \delta \phi^a$$

$$\frac{\partial J_i^\alpha}{\partial \xi^\alpha} = 0$$

$$Q_i = \int d\xi^0 J_i^0(\xi)$$

What are:

$$\Theta^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial X'^\mu}$$

$$\mathcal{L}(\dot{X}, X')$$

$$\xi^\alpha = \tau, \sigma$$

$$\phi^a = X^\mu$$

$$\delta X^\mu = \epsilon^\mu \text{ constant}$$

Lagrangian density $\mathcal{L}$, fields $\phi^a(\xi)$, coordinates ξ^α $\alpha=0, 1$
 and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters ϵ^i
 we have currents J_i^a that are conserved:

$$\epsilon^i J_i^a(\xi) \equiv \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi^a)} \delta \phi^a$$

$$J_i^0 = 0$$

$$Q_i = \int d\xi^1 d\xi^0 J_i^0(\xi)$$

$$\downarrow$$

$$\frac{dQ_i}{d\xi^0} = 0 \text{ with good BC}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} \dot{X}^\sigma - \mathcal{L}$$

$$\mathcal{L}(\dot{X}, X') \quad (\xi^\alpha = \tau, \sigma)$$

$$\phi^a = X^\mu \quad \delta X^\mu = \epsilon^\mu \text{ constant}$$

and symmetries $\delta \phi^a$ (leading to $\delta \mathcal{L} = 0$) with parameters ϵ^α
we have currents J_i^α that are conserved:

$$\epsilon^\alpha J_i^\alpha(\xi) \equiv \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi^a)} \delta \phi^a$$

$$\epsilon^\mu J_\mu^\alpha$$

$$\frac{\partial J_i^\alpha}{\partial \xi^\alpha} = 0$$

$$Q_i = \int d\xi^1 d\xi^2 J_i^0(\xi)$$

$$\downarrow$$

$$\frac{dQ_i}{d\xi^0} = 0 \text{ with}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial X'^{\mu}}$$

$$\mathcal{L}(\dot{X}, X') \quad (\xi^\alpha = \tau, \sigma)$$

$$\phi^a = X^\mu \quad \delta X^\mu = \epsilon^\mu \text{ constant}$$

and symmetries $\delta \phi^a$ (leading to $\delta \mathcal{L} = 0$) with parameters ϵ^α
we have currents J_i^α that are conserved:

$$\epsilon^\alpha \int_i^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi^a)} \delta \phi^a \quad \frac{\partial J_i^\alpha}{\partial \xi^\alpha} = 0 \quad Q_i = \int d\xi^1 d\xi^2 J_i^0(\xi)$$

$$\epsilon^\mu \int_\mu^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)} \delta X^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)} \epsilon^\mu \quad \int_\mu^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)}$$

$$\frac{dQ_i}{d\xi^0} = 0 \text{ with good BC}$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$\mathcal{L}(\dot{X}, X') \quad (\xi^\alpha = \tau, \sigma)$$

$$\phi^a = X^\mu$$

$$\delta X^\mu = \epsilon^\mu \text{ constant}$$

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$$\epsilon^\alpha J_i^\alpha(\xi) \equiv \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi^a)} \delta \phi^a$$

$$\frac{\partial J_i^\alpha}{\partial \xi^\alpha} = 0$$

$$Q_i = \int ds^1 ds^D J_i^0(\xi)$$

$$\frac{dQ_i}{ds^0} = 0 \text{ with good BC}$$

$$\epsilon^\mu J_\mu^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)} \delta X^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)} \epsilon^\mu$$

$$J_\mu^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)} \quad J_\mu^0 = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^T$$

$$\Theta^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial X'^{\mu}} X'^{\sigma}$$

and symmetries $\delta\phi^a$ (leading to $\delta\mathcal{L}=0$) with parameters ϵ^i
we have currents J_i^a that are conserved:

$$\epsilon^i J_i^a(\xi) \equiv \frac{\partial \mathcal{L}}{\partial(\partial_a \phi^a)} \delta\phi^a \quad \frac{\partial J_i^a}{\partial \xi^a} = 0 \quad Q_i = \int ds^1 ds^D J_i^0(\xi)$$

$$\mathcal{L}(\dot{X}, X') \quad (\xi^\alpha = \tau, \sigma)$$

$$\phi^a = X^\mu \quad \delta X^\mu = \epsilon^\mu \text{ constant}$$

$$J_\mu^\perp = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\sigma$$

$$\epsilon^\mu J_\mu^a = \frac{\partial \mathcal{L}}{\partial(\partial_a X^\mu)} \delta X^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_a X^\mu)} \epsilon^\mu \quad J_\mu^a = \frac{\partial \mathcal{L}}{\partial(\partial_a X^\mu)} \quad J_\mu^0 = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\sigma$$

$\frac{dQ_i}{ds^0} = 0$ with good BC.

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\sigma$$

$$J_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

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$$J_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$\partial_0 J_\mu^0 + \partial_1 J_\mu^1 = 0$$

$$\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = 0 \quad \checkmark \quad \text{ECM}$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

$$Q_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$P_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

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$$\partial_0 J_\mu^0 + \partial_1 J_\mu^1 = 0$$

$$\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = 0 \quad \checkmark \text{ ECM}$$

$$Q_\mu \rightarrow P_\mu(\tau) = \int_0^\tau d\sigma \mathcal{P}_\mu^\tau(\tau, \sigma)$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

$$Q_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$P_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

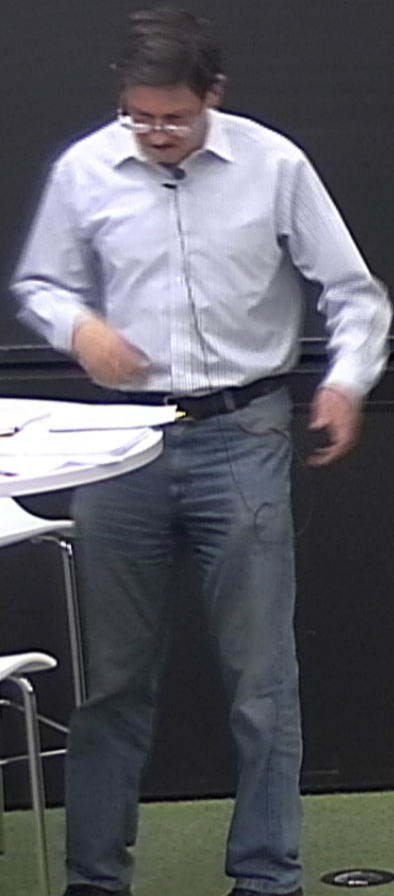
$$P_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu$$

$$\partial_0 J_\mu^0 + \partial_1 J_\mu^1 = 0$$

$$\frac{\partial \Phi_\mu^\tau}{\partial \tau} + \frac{\partial \Phi_\mu^\sigma}{\partial \sigma} = 0 \quad \checkmark \text{ EOM}$$

$$Q_\mu \rightarrow P_\mu(\tau) = \int_0^{\sigma_1} d\sigma \Phi_\mu^\tau(\tau, \sigma)$$

$$\frac{dP_\mu(\tau)}{d\tau} = \int_0^{\sigma_1} d\sigma \frac{\partial \Phi_\mu^\tau}{\partial \tau} = - \int_0^{\sigma_1} d\sigma \frac{\partial \Phi_\mu^\sigma}{\partial \sigma} = - [\Phi_\mu^\sigma]_0^{\sigma_1}$$



$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

$$\mathcal{L} = \mathcal{L}(X^\mu, \dot{X}^\mu)$$

$$\mathcal{L} = \mathcal{L}(X^\mu, \dot{X}^\mu)$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

$$\partial_0 J_\mu^0 + \partial_1 J_\mu^1 = 0$$

$$\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = 0 \quad \checkmark \text{ EOM}$$

$$Q_\mu \rightarrow P_\mu(\tau) = \int_0^{\sigma_1} d\sigma \mathcal{P}_\mu^\tau(\tau, \sigma)$$

$$\frac{dP_\mu(\tau)}{d\tau} = \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} = - \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = - [\mathcal{P}_\mu^\sigma]_0^{\sigma_1}$$

$$\mathcal{P}_\mu^\sigma(\sigma_1) = 0 \quad \checkmark$$

$$\frac{dP_\mu}{d\tau} = 0$$



$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$\mathcal{L} = \frac{1}{2} \dot{X}^{\mu} \dot{X}_{\mu}$$

$$\mathcal{L} = \frac{1}{2} \dot{X}^{\mu} \dot{X}_{\mu}$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$\partial_0 J_{\mu}^0 + \partial_1 J_{\mu}^1 = 0$$

$$\frac{\partial \mathcal{P}_{\mu}^{\tau}}{\partial \tau} + \frac{\partial \mathcal{P}_{\mu}^{\sigma}}{\partial \sigma} = 0 \quad \checkmark \quad \text{EOM}$$

$$Q_{\mu} \rightarrow P_{\mu}(\tau) = \int_0^{\tau} d\sigma \mathcal{P}_{\mu}^{\tau}(\tau, \sigma)$$

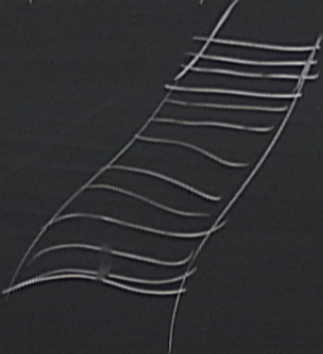
$$\frac{dP_{\mu}(\tau)}{d\tau} = \int_0^{\tau_1} d\sigma \frac{\partial \mathcal{P}_{\mu}^{\tau}}{\partial \tau} = - \int_0^{\tau_1} d\sigma \frac{\partial \mathcal{P}_{\mu}^{\sigma}}{\partial \sigma} = - \mathcal{P}_{\mu}^{\sigma} \Big|_0^{\tau_1}$$

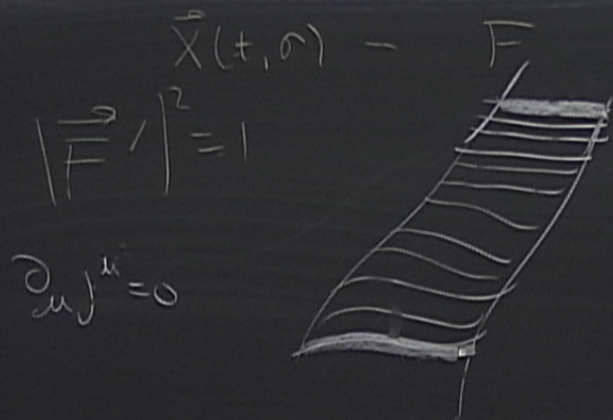
$$\mathcal{P}_{\mu}^{\sigma}(\sigma_1) = 0 \quad \checkmark$$

$$\frac{dP_{\mu}}{d\tau} = 0$$

$$\vec{X}(t, \sigma) = F$$

$$|\vec{F}|^2 = 1$$

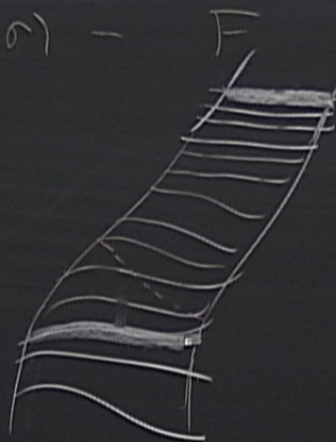
$$\partial_\mu J^\mu = 0$$




$$\vec{X}(t, \sigma) = F$$

$$|\vec{F}|^2 = 1$$

$$\partial_\mu J^\mu = 0$$



$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$\mathcal{L} = \frac{1}{2} \dot{X}^{\mu} \dot{X}_{\mu}$$

$$\mathcal{L} = \frac{1}{2} \dot{X}^{\mu} \dot{X}_{\mu}$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$\partial_0 J_{\mu}^0 + \partial_1 J_{\mu}^1 = 0$$

$$\frac{\partial \mathcal{P}_{\mu}^{\tau}}{\partial \tau} + \frac{\partial \mathcal{P}_{\mu}^{\sigma}}{\partial \sigma} = 0 \quad \checkmark \text{ EOM}$$

$$Q_{\mu} \rightarrow P_{\mu}(\tau) = \int_0^{\sigma_1} d\sigma \mathcal{P}_{\mu}^{\tau}(\tau, \sigma)$$

$$\frac{dP_{\mu}(\tau)}{d\tau} = \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_{\mu}^{\tau}}{\partial \tau} = - \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_{\mu}^{\sigma}}{\partial \sigma} = - \mathcal{P}_{\mu}^{\sigma} \Big|_0^{\sigma_1}$$

$$\mathcal{P}_{\mu}^{\sigma}(\sigma_1) = 0 \quad \checkmark$$

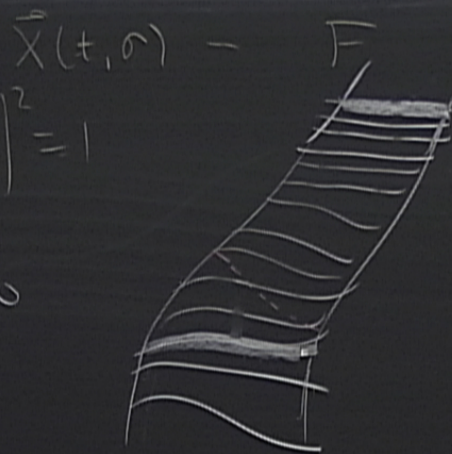
$$\frac{dP_{\mu}}{d\tau} = 0$$

P_{μ} is conserved

Momentum of the string

$$|\vec{F}|^2 = 1$$

$$\partial_\mu J^\mu = 0$$



$$\phi \dots \phi$$

$$\delta A_\mu = \partial_\mu \epsilon$$

$$\delta A_\mu = U^\dagger \partial_\mu U$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

$$\mathcal{L} = \mathcal{L}(X^\mu, \dot{X}^\mu)$$

$$\mathcal{L} = \mathcal{L}(X^\mu, \dot{X}^\mu)$$

$$J_\mu^\pm = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = P_\mu^\pm$$

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$$\partial_0 J_\mu^0 + \partial_1 J_\mu^1 = 0$$

$$\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = 0 \quad \checkmark \text{ EOM}$$

$$Q_\mu \rightarrow P_\mu(\tau) = \int_0^{\sigma_1} d\sigma \mathcal{P}_\mu^\tau(\tau, \sigma)$$

$$\frac{dP_\mu(\tau)}{d\tau} = \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} = - \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = - [\mathcal{P}_\mu^\sigma]_0^{\sigma_1}$$

$$\mathcal{P}_\mu^\sigma(\sigma_1) = 0 \quad \checkmark$$

$$\frac{dP_\mu}{d\tau} = 0$$

P_μ is conserved

Momentum of LP

Lorentz invariance

$$\delta X^\mu = \epsilon^{\mu\nu} X_\nu$$

$$\delta(\eta_{\mu\nu} X^\mu X^\nu) = 0$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$\mathcal{L} = \frac{1}{2} \dot{X}^{\mu} \dot{X}_{\mu}$$

$$\mathcal{L} = \frac{1}{2} \dot{X}^{\mu} \dot{X}_{\mu}$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$J_{\mu}^{\pm} = \frac{\partial \mathcal{L}}{\partial \dot{X}^{\mu}} = P_{\mu}^{\pm}$$

$$\partial_0 J_{\mu}^0 + \partial_1 J_{\mu}^1 = 0$$

$$\frac{\partial \mathcal{P}_{\mu}^{\tau}}{\partial \tau} + \frac{\partial \mathcal{P}_{\mu}^{\sigma}}{\partial \sigma} = 0 \quad \checkmark \quad \text{EOM}$$

$$Q_{\mu} \rightarrow P_{\mu}(\tau) = \int_0^{\sigma_1} d\sigma \mathcal{P}_{\mu}^{\tau}(\tau, \sigma)$$

$$\frac{dP_{\mu}(\tau)}{d\tau} = \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_{\mu}^{\tau}}{\partial \tau} = - \int_0^{\sigma_1} d\sigma \frac{\partial \mathcal{P}_{\mu}^{\sigma}}{\partial \sigma} = - \mathcal{P}_{\mu}^{\sigma} \Big|_0^{\sigma_1}$$

$$\mathcal{P}_{\mu}^{\sigma}(\sigma_1) = 0 \quad \checkmark$$

$$\frac{dP_{\mu}}{d\tau} = 0$$

P_{μ} is conserved

Momentum of the string

Lorentz invariance

$$\delta X^{\mu} = \epsilon^{\mu\nu} X_{\nu}$$

$$\delta(\eta_{\mu\nu} X^{\mu} X^{\nu}) = 0$$

$$\eta_{\mu\nu} \delta X^{\mu} X^{\nu} = 0$$

$$\eta_{\mu\nu} (\epsilon^{\mu\rho} X_{\rho}) X^{\nu} =$$

$$\partial_0 j_\mu^0 + \partial_1 j_\mu^1 = 0$$

$$\frac{\partial \Phi_\mu^\tau}{\partial \tau} + \frac{\partial \Phi_\mu^\sigma}{\partial \sigma} = 0 \quad \checkmark \text{ EOM}$$

$$Q_\mu \rightarrow \boxed{P_\mu(\tau) = \int_0^{\sigma_1} d\sigma \Phi_\mu^\tau(\tau, \sigma)}$$

$$\frac{dP_\mu(\tau)}{d\tau} = \int_0^{\sigma_1} d\sigma \frac{\partial \Phi_\mu^\tau}{\partial \tau} = - \int_0^{\sigma_1} d\sigma \frac{\partial \Phi_\mu^\sigma}{\partial \sigma} = - [\Phi_\mu^\sigma]_0^{\sigma_1}$$

Lorentz invariance

$$\delta X^\mu = \epsilon^{\mu\nu} X_\nu$$

$$\frac{dP_\mu}{d\tau} = 0$$

P_μ is conserved

Momentum of the string

$$X^\mu X^\nu = 0$$

$$\eta_{\mu\nu} \delta X^\mu X^\nu = 0$$

$$\eta_{\mu\nu} (\epsilon^{\mu\rho} X_\rho) X^\nu = \epsilon^{\mu\rho} X_\rho X_\mu = 0$$

Lorentz invariance

$$\delta x^\mu = \epsilon^{\mu\nu} x_\nu$$

$$\psi P_\mu^\sigma(\sigma) = 0 \quad \checkmark$$

$$\frac{dP_\mu}{d\tau} = 0$$

P_μ is conserved

Momentum of the string

$$\delta(\eta_{\mu\nu} x^\mu x^\nu) = 0$$

$$\eta_{\mu\nu} \delta x^\mu x^\nu = 0$$

$$\eta_{\mu\nu} (\epsilon^{\mu\rho} x_\rho) x^\nu = \epsilon^{\mu\rho} x_\rho x_\mu = 0 \quad \forall x$$

$$\delta(\eta_{\mu\nu} X^\mu X^\nu) = 0 \quad \eta_{\mu\nu} \delta X^\mu X^\nu = 0 \quad \eta_{\mu\nu} (\epsilon^{\mu\rho} X_\rho) X^\nu = \epsilon^{\mu\rho} X_\rho X^\mu = 0 \quad \forall X$$

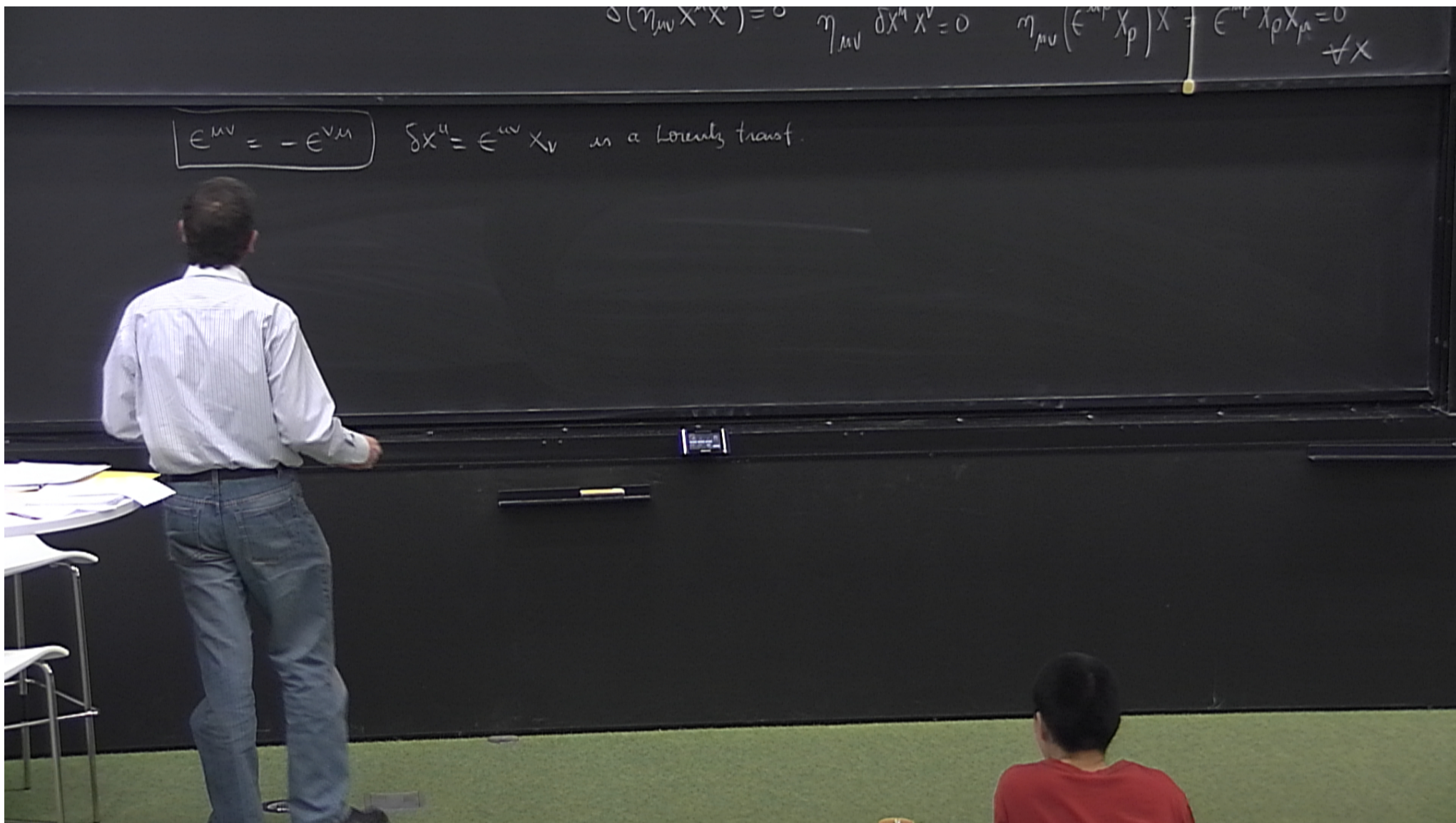
$$\boxed{\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}} \quad \delta X^\mu = \epsilon^{\mu\nu} X_\nu \text{ in a Lorentz transf.}$$



$$\delta(\eta_{\mu\nu} X^\mu X^\nu) = 0 \quad \eta_{\mu\nu} \delta X^\mu X^\nu = 0 \quad \eta_{\mu\nu} (\epsilon^{\mu\rho} X_\rho) X^\nu = \epsilon^{\mu\rho} X_\rho X^\nu = 0 \quad \forall X$$

$$\boxed{\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}} \quad \delta X^\mu = \epsilon^{\mu\nu} X_\nu \text{ in a Lorentz transf.}$$





Lorentz invariance

$$\delta X^\mu = \epsilon^{\mu\nu} X_\nu$$

$$\frac{dP_\mu}{d\tau} = 0$$

P_μ is conserved

Momentum of the string

$$\delta(\eta_{\mu\nu} X^\mu X^\nu) = 0$$

$$\eta_{\mu\nu} \delta X^\mu X^\nu = 0$$

$$\eta_{\mu\nu} (\epsilon^{\mu\rho} X_\rho) X^\nu = \epsilon^{\mu\rho} X_\rho X_\mu = 0 \quad \forall X$$

$$\boxed{\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}} \quad \delta X^\mu = \epsilon^{\mu\nu} X_\nu \text{ is a Lorentz transf.}$$

$$J_{\mu\nu}^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha X^\mu)} \epsilon^{\mu\nu} X_\nu$$

Lorentz invariance

$$\delta x^\mu = \epsilon^{\mu\nu} x_\nu$$

$$\frac{dP_\mu}{d\tau} = 0$$

P_μ is conserved

Momentum of the string

$$\delta(\eta_{\mu\nu} x^\mu x^\nu) = 0$$

$$\eta_{\mu\nu} \delta x^\mu x^\nu = 0$$

$$\eta_{\mu\nu} (\epsilon^{\mu\rho} x_\rho) x^\nu = \epsilon^{\mu\rho} x_\rho x_\mu = 0 \quad \forall x$$

$$\boxed{\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}}$$

$\delta x^\mu = \epsilon^{\mu\nu} x_\nu$ is a Lorentz transf.

$$\epsilon^{\mu\nu} \int_{\mu\nu}^\alpha = \frac{\partial \mathcal{L}}{\partial (\partial_\alpha x^\mu)} \epsilon^{\mu\nu} x_\nu$$

Lorentz invariance

$$\delta X^\mu = \epsilon^{\mu\nu} X_\nu$$

$$\frac{dP_\mu}{d\tau} = 0$$

P_μ is conserved

Momentum of the string

$$\delta(\eta_{\mu\nu} X^\mu X^\nu) = 0$$

$$\eta_{\mu\nu} \delta X^\mu X^\nu = 0$$

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$\forall X$

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$$\epsilon^{\mu\nu}$$

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$$M_{\mu\nu}^\alpha \equiv X_\mu P_\nu^\alpha - X_\nu P_\mu^\alpha$$

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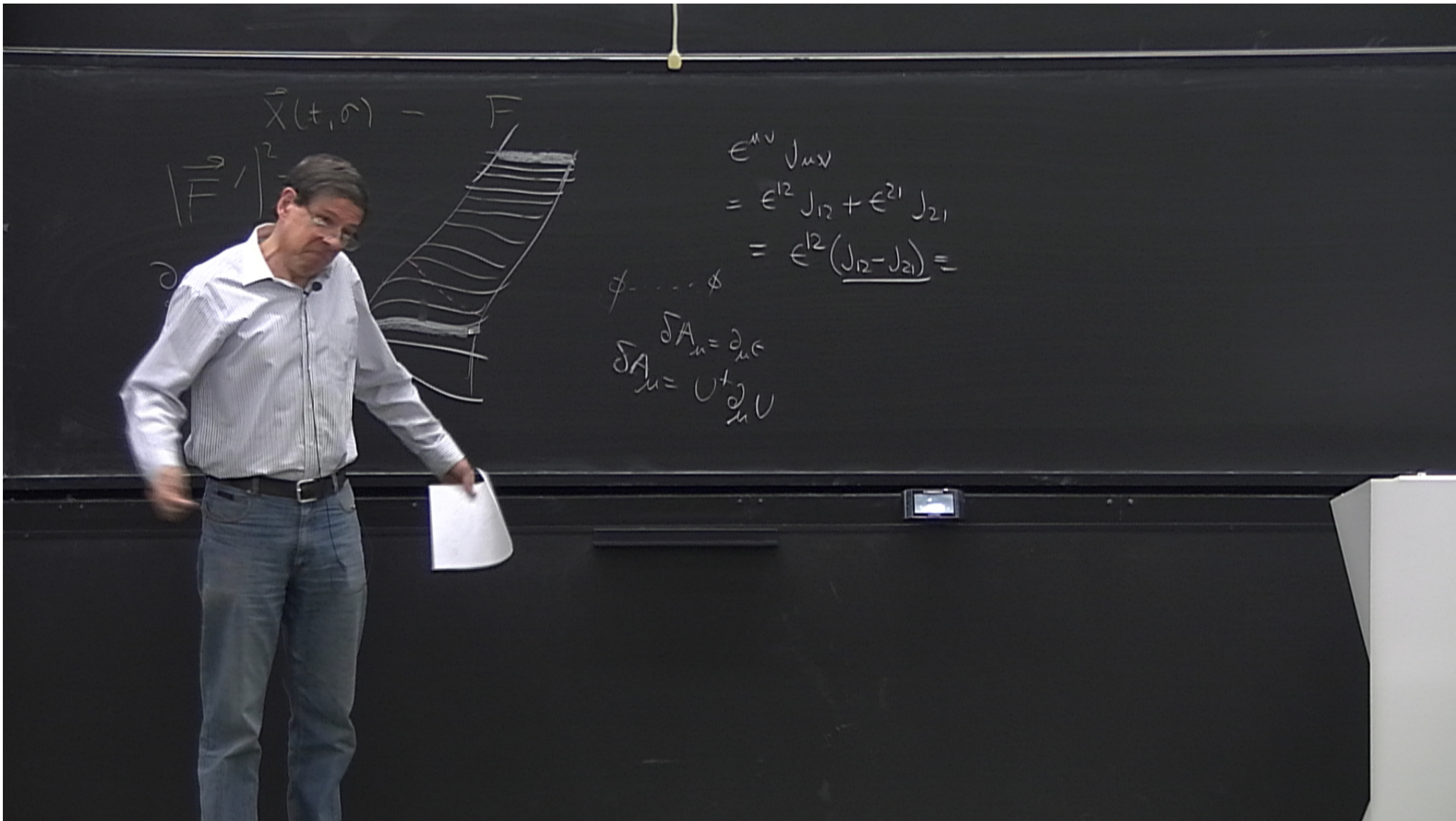
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$$M_{\mu\nu} = \int d\sigma M_{\mu\nu}^0(\tau, \sigma) = \int_0^{\sigma_1} (X_\mu P_\nu^\tau - X_\nu P_\mu^\tau) d\sigma$$

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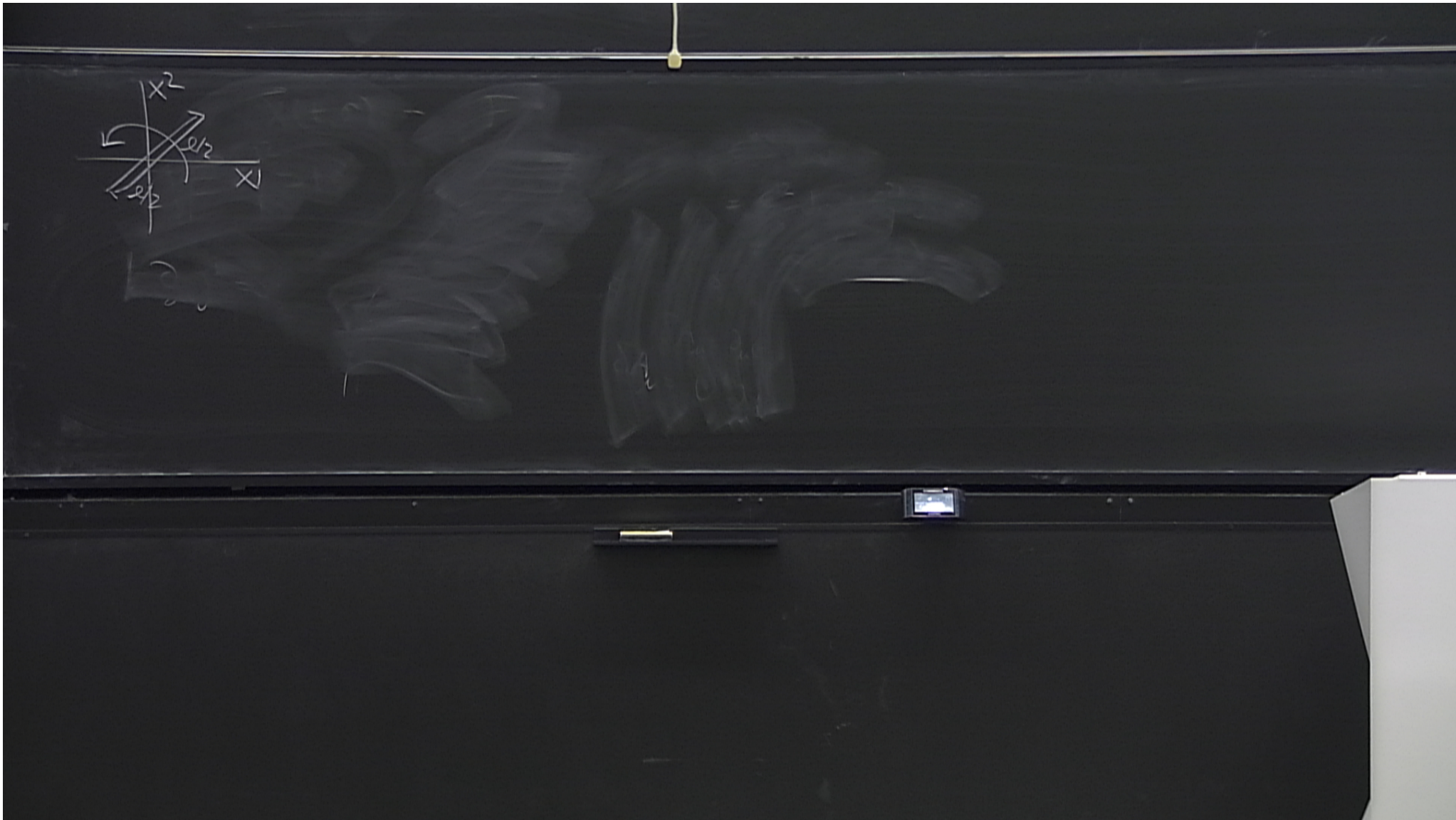
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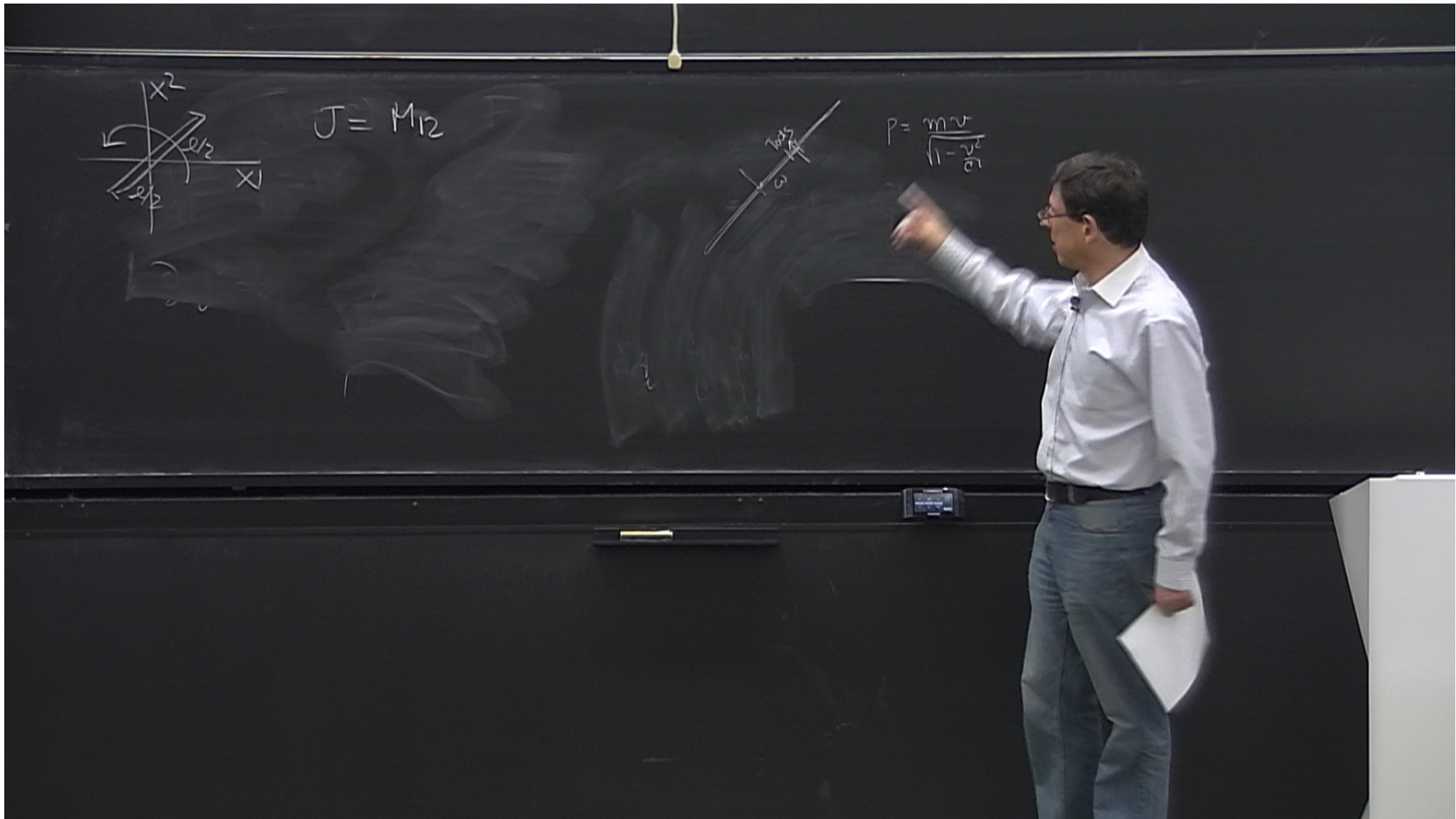
$$\boxed{M_{\mu\nu}^\alpha = X_\mu P_\nu^\alpha - X_\nu P_\mu^\alpha}$$

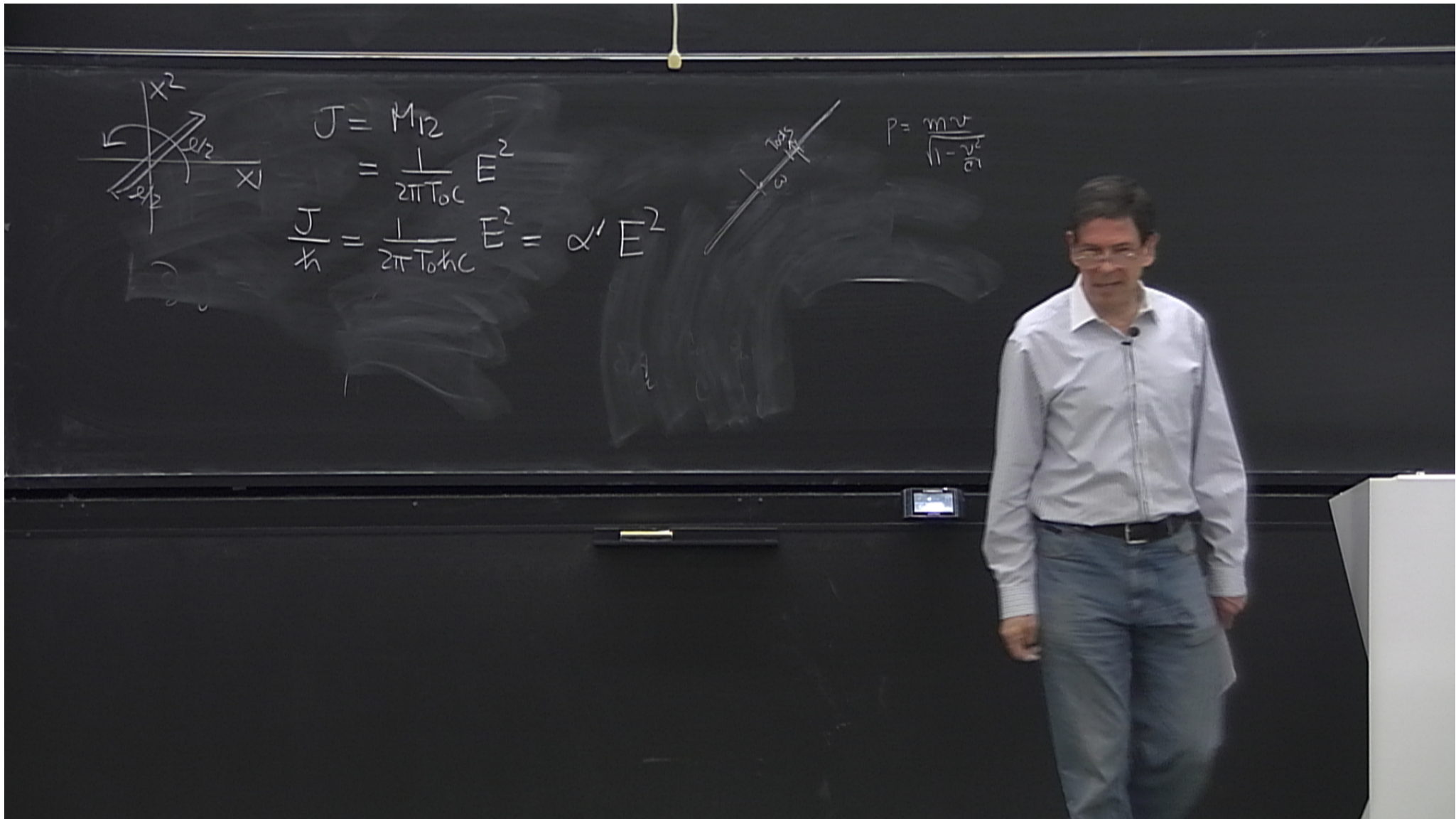
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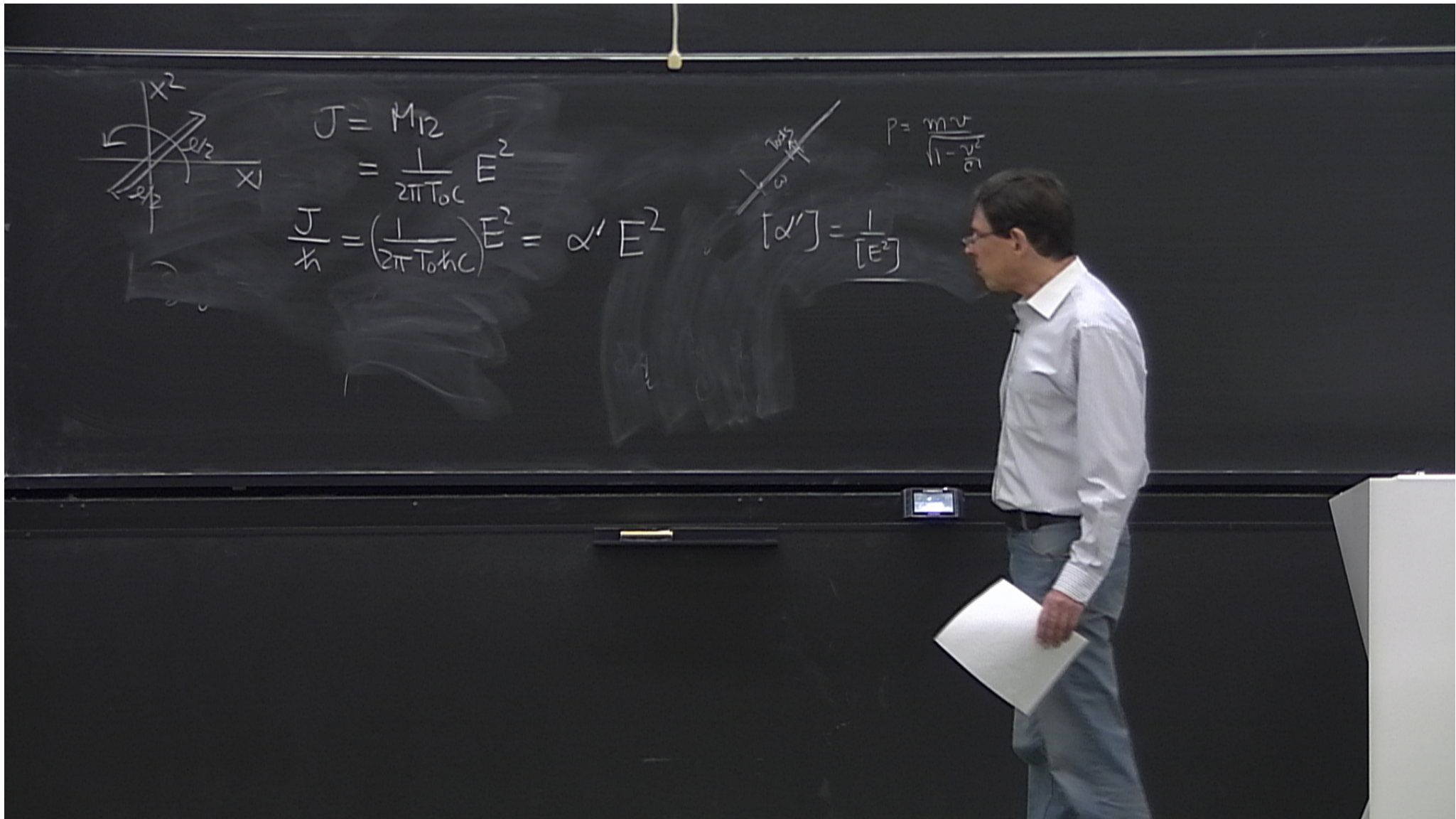
$$M_{ij} = \epsilon_{ijk} \underline{J_k}$$

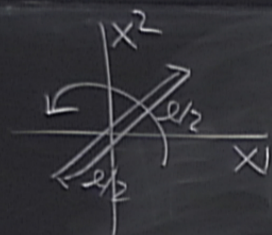
$$M_{12} = J_3$$











$$J = M_{12} = \frac{1}{2\pi T_0 c} E^2$$

$$\frac{J}{\hbar} = \left(\frac{1}{2\pi T_0 \hbar c} \right) E^2 = \alpha' E^2$$



$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

$$[\alpha'] = \frac{1}{[E^2]}$$

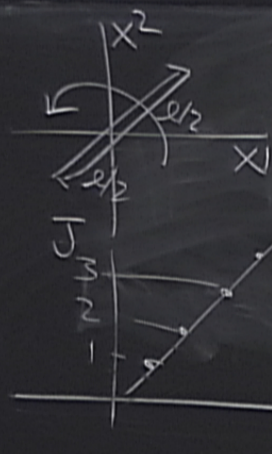


$$J \sim \sqrt{E}$$

$$J = I\omega$$

$$E = \frac{1}{2} I \omega^2$$

$$\omega \sim \sqrt{E}$$



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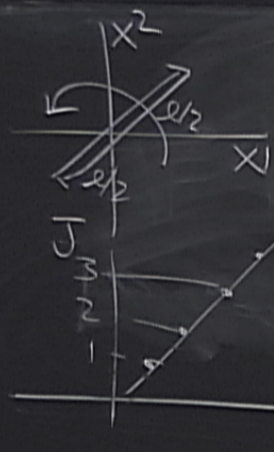


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$$S_{NG} = -\frac{I_0}{c} \int \dots$$

$$S_{NG} = -\frac{1}{2\pi \alpha'} \int \dots$$



$$[\alpha'] = \frac{1}{[E^2]}$$

$$\hbar = c = 1$$

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$



$$[\alpha'] \sim L^2$$

$$\sqrt{\alpha'} \equiv \ell_s$$

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$$M_{12} = J_3$$

$$M_{\mu\nu} = \int d\sigma M_{\mu\nu}^0(\tau, \sigma) = \int_0^{\sigma_1} (X_\mu P_\nu^\tau - X_\nu P_\mu^\tau) d\sigma$$

$$M_{0i} = \int d\sigma (t P_i^\tau - X_i P_0^\tau)$$

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$$\mathcal{P}_\mu^\tau = P_\mu$$

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Gauges

$$\eta_{\mu} X^{\mu}(\tau, \sigma) = \lambda \tau$$

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η_{μ} is a ^{constant} vector

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Demand that the components of P_{μ} in $n \cdot p$ are conserved.

$$\eta \cdot \mathcal{P}^{\sigma} \Big|_{\sigma_*} = 0$$

Gauges

$$\boxed{n_\mu X^\mu(\tau, \sigma) = \lambda \tau}$$

n_μ is a ^{constant} vector
timelike or null

$$n_\mu = (1, 0, \dots, 0)$$

Demand that the components of P_μ in $n \cdot p$ are conserved.

$$n \cdot \mathcal{P}^\sigma \Big|_{\sigma_*} = 0$$

Write

$$\boxed{n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau}$$

Gauges

$$\eta_{\mu} X^{\mu}(\tau, \sigma) = \lambda \tau$$

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no units

σ , no units either

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Demand that the components of P_{μ} in $n \cdot p$ are conserved.

$$n \cdot \dot{X}^{\sigma} \Big|_{\sigma_*} = 0$$

Write

$$n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau$$

no units
 σ , no units either

Require $n \cdot \dot{X}^{\tau}$

Gauges

$$\boxed{n_\mu X^\mu(\tau, \sigma) = \lambda \tau}$$

n_μ is a vector
timelike or null

$$n_\mu = (1, 0, \dots, 0)$$

Demand that the components of P_μ in $n \cdot p$ are conserved.

$$n \cdot \dot{X}^\sigma \Big|_{\tau_*} = 0$$

Write

$$\boxed{n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau}$$

→ no

alter



Require $n \cdot \dot{X}^\tau = \text{constant over the string (constant along } \sigma^+)$

$$n \cdot \left. \frac{\partial \sigma}{\partial \tau} \right|_{\tau_*} = 0$$

Write

$$n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau$$

σ , no units either



Require

$n \cdot \dot{\sigma} = \text{constant over the string (constant along } \sigma) \checkmark$

Parameterize the string $(0, \pi)$

$$n \cdot \mathcal{P}^\sigma|_{\mathcal{T}_*} = 0$$

Write $n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau$ σ , no units either

Require $n \cdot \mathcal{P}^\tau = \text{constant over the string (constant along } \sigma) \checkmark$
 Parameterize the string $(0, \pi)$

$$\int_0^\pi d\sigma (n \cdot \mathcal{P}^\tau) = n \cdot p$$

$$n \cdot \mathcal{P}^\tau = \frac{n \cdot p}{\pi}$$

$$\pi n \cdot \mathcal{P}^\tau = n \cdot p$$

$$n \cdot \Phi^{\sigma} \Big|_{\Sigma_*} = 0$$

Write

$$n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau \quad \sigma, \text{ no units either}$$



Require

$n \cdot \Phi^{\tau} = \text{constant over the string}$

(constant along σ) ✓

Parameterize the string $(0, \pi)$

$$\int_0^{\pi} d\sigma (n \cdot \Phi^{\tau}) = n \cdot p$$

$$\boxed{n \cdot \Phi^{\tau} = \frac{n \cdot p}{\pi}}$$

From EDM

$$\frac{\partial \Phi^{\tau}}{\partial \tau} + \frac{\partial \Phi^{\sigma}}{\partial \sigma} = 0$$

$$\pi n \cdot \Phi^{\tau} = n \cdot p$$

$$\frac{\partial (n \cdot \Phi^{\tau})}{\partial \tau} + \boxed{\frac{\partial (n \cdot \Phi^{\sigma})}{\partial \sigma}} = 0$$

$$n \cdot \dot{X}^\sigma \Big|_{\tau_*} = 0$$

Write $n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau$ σ , no units either



Require

$n \cdot \dot{P}^\tau = \text{constant over the string}$ (constant along σ) ✓

Parameterize the string $(0, \pi)$

$$\int_0^\pi d\sigma (n \cdot \dot{P}^\tau) = n \cdot p$$

$$n \cdot \dot{P}^\tau = \frac{n \cdot p}{\pi}$$

$$\pi n \cdot \dot{P}^\tau = n \cdot p$$

From EDM

$$\frac{\partial P_\mu^\tau}{\partial \tau} + \frac{\partial P_\mu^\sigma}{\partial \sigma} = 0$$

$$\frac{\partial (n \cdot \dot{P}^\tau)}{\partial \tau} + \frac{\partial (n \cdot \dot{P}^\sigma)}{\partial \sigma} = 0$$

$$n \cdot P_\sigma \equiv 0 \rightarrow -\frac{1}{2\pi\alpha'} \left(\frac{(\dot{X} \cdot X') \partial_\tau (n \cdot X) - \dot{X}^2 \partial_\sigma (n \cdot X)}{\sqrt{\dots}} \right)$$

$$n \cdot \Phi^0 \Big|_{\Sigma_*} = 0$$

Write $n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau$ σ , no units either



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$n \cdot \Phi^\tau = \text{constant over the string}$ (constant along σ) ✓

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$$n \cdot \dot{X}^\sigma \Big|_{\tau_*} = 0$$

Write $n \cdot X(\tau, \sigma) = 2\alpha' (n \cdot p) \tau$ σ , no units either

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 Parameterize the string $(0, \pi)$
 $\int_0^\pi d\sigma (n \cdot \dot{P}^\tau) = n \cdot p$

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From EDM

$$\frac{\partial \dot{P}_\mu^\tau}{\partial \tau} + \frac{\partial \dot{P}_\mu^\sigma}{\partial \sigma} = 0$$

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$$\frac{\partial \dot{X}^\mu}{\partial \tau} \frac{\partial \dot{X}^\nu}{\partial \sigma}$$

$$+ \frac{\partial \dot{X}^\mu}{\partial \tau} \frac{\partial \dot{X}^\nu}{\partial \sigma} = 0$$

$$n \cdot P_\sigma \equiv 0 \rightarrow -\frac{1}{2\pi\alpha'} \left(\frac{(\dot{X} \cdot X') \partial_\tau (n \cdot X) - \dot{X}^2 \partial_\sigma (n \cdot X)}{\sqrt{\dots}} \right)$$

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Parameterizing the string $(0, \pi)$

$$\int_0^\pi d\sigma (n \cdot \mathcal{P}^\tau) = n \cdot p$$

$$\boxed{n \cdot \mathcal{P}^\tau = \frac{n \cdot p}{\pi}}$$

$$\pi n \cdot \mathcal{P}^\tau = n \cdot p$$

From EOM

$$\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = 0$$

$$\frac{\partial (n \cdot \mathcal{P}^\tau)}{\partial \tau} + \boxed{\frac{\partial (n \cdot \mathcal{P}^\sigma)}{\partial \sigma}} = 0$$

$$\rightarrow \boxed{\dot{X}^2 + X'^2 = 0}$$

$$\frac{\partial \dot{X}^\mu}{\partial \tau} \frac{\partial \dot{X}^\mu}{\partial \sigma} = 0$$

$$+ \frac{\partial \dot{X}^\mu}{\partial \tau} \frac{\partial \dot{X}^\mu}{\partial \sigma} = 0$$

$$n \cdot \mathcal{P}_\sigma = 0 \rightarrow$$

$$\boxed{\dot{X} \cdot X' = 0}$$

$$-\frac{1}{2\pi\alpha'} \left(\frac{(\dot{X} \cdot X') \partial_\tau (n \cdot X) - \dot{X}^2 \partial_\sigma (n \cdot X)}{\sqrt{\dots}} \right)$$

$$\boxed{(\dot{X} \pm X')^2 = 0}$$