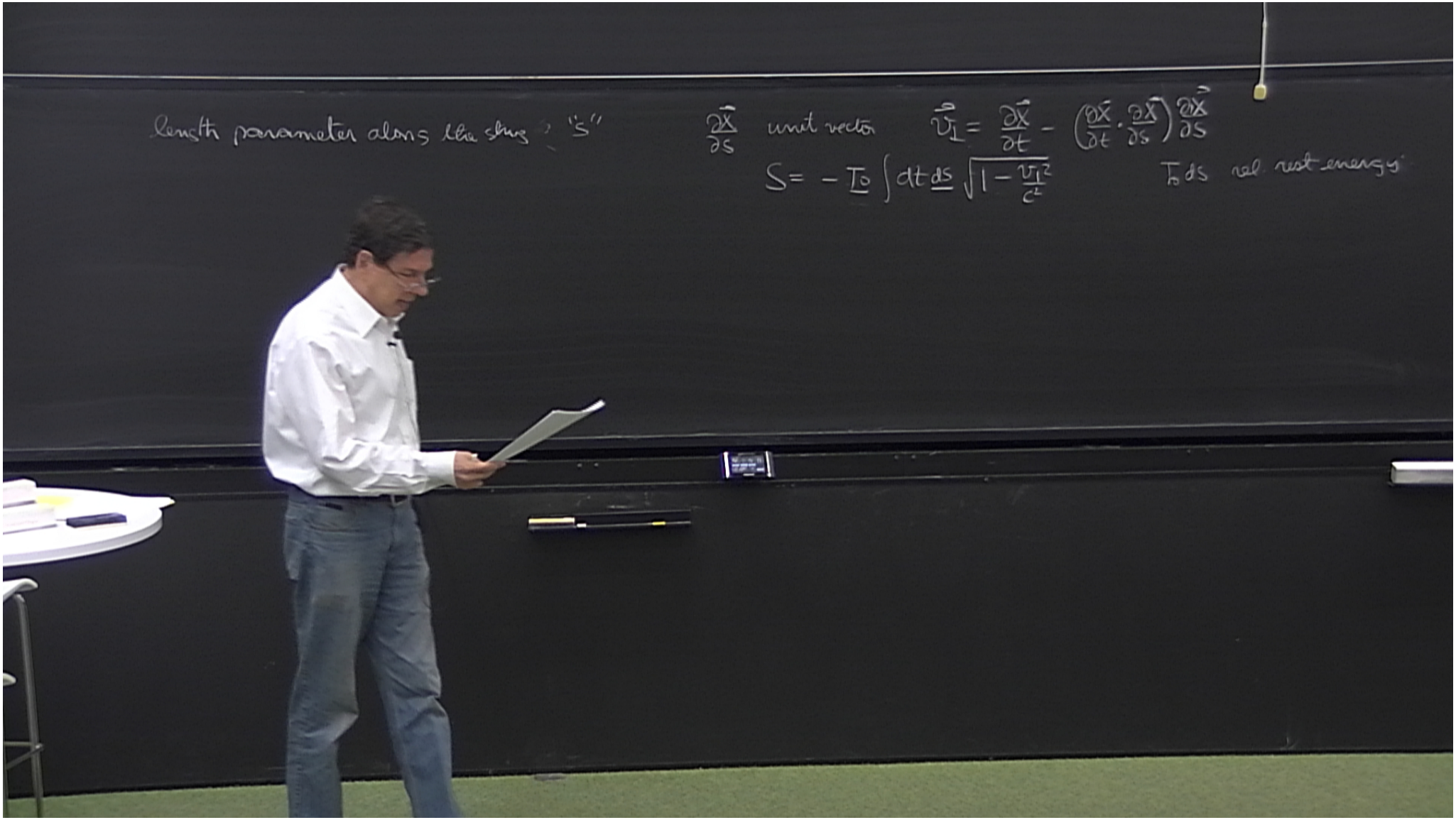
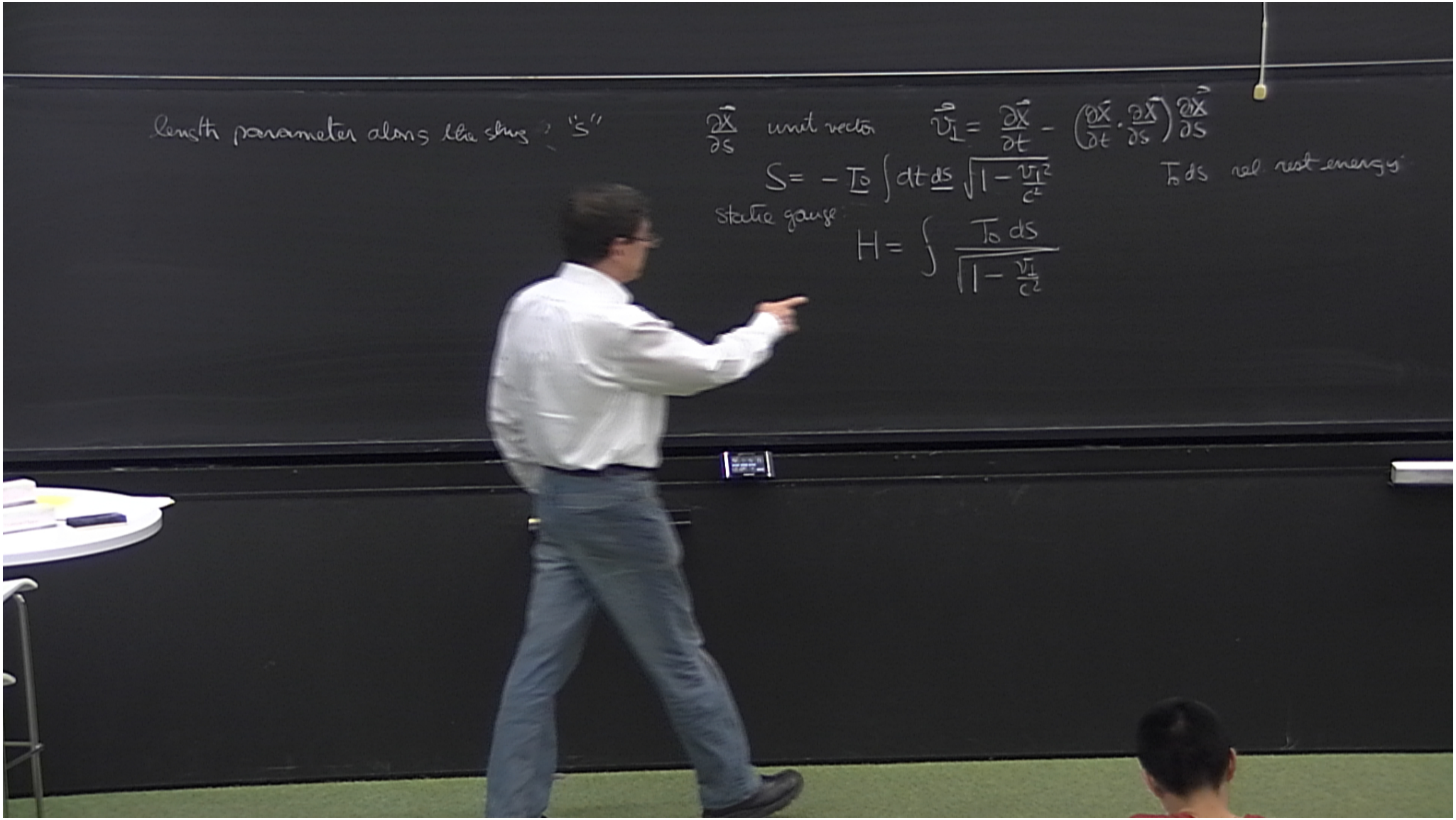
Title: 12/13 PSI - String Theory Review Lecture 5

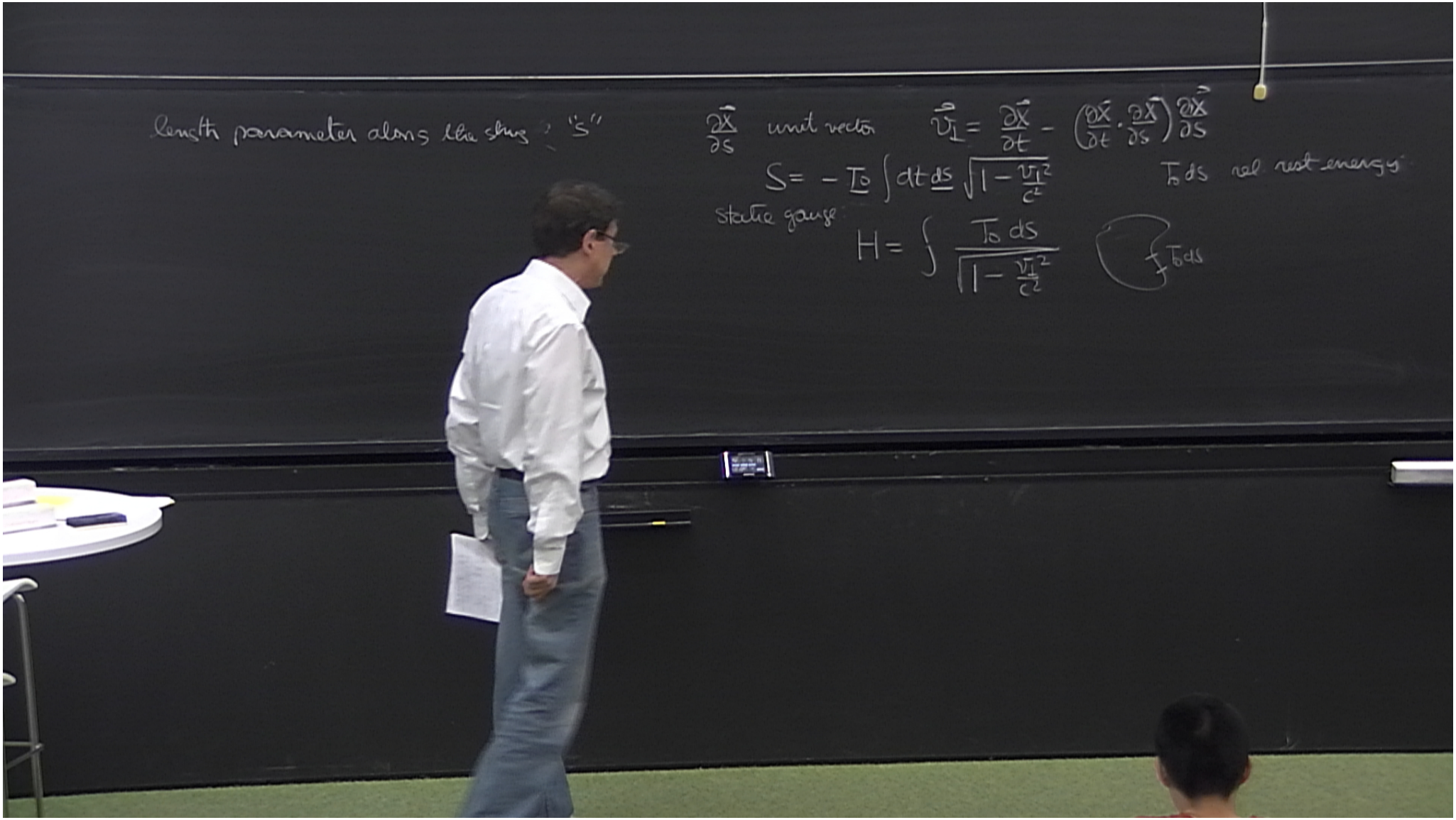
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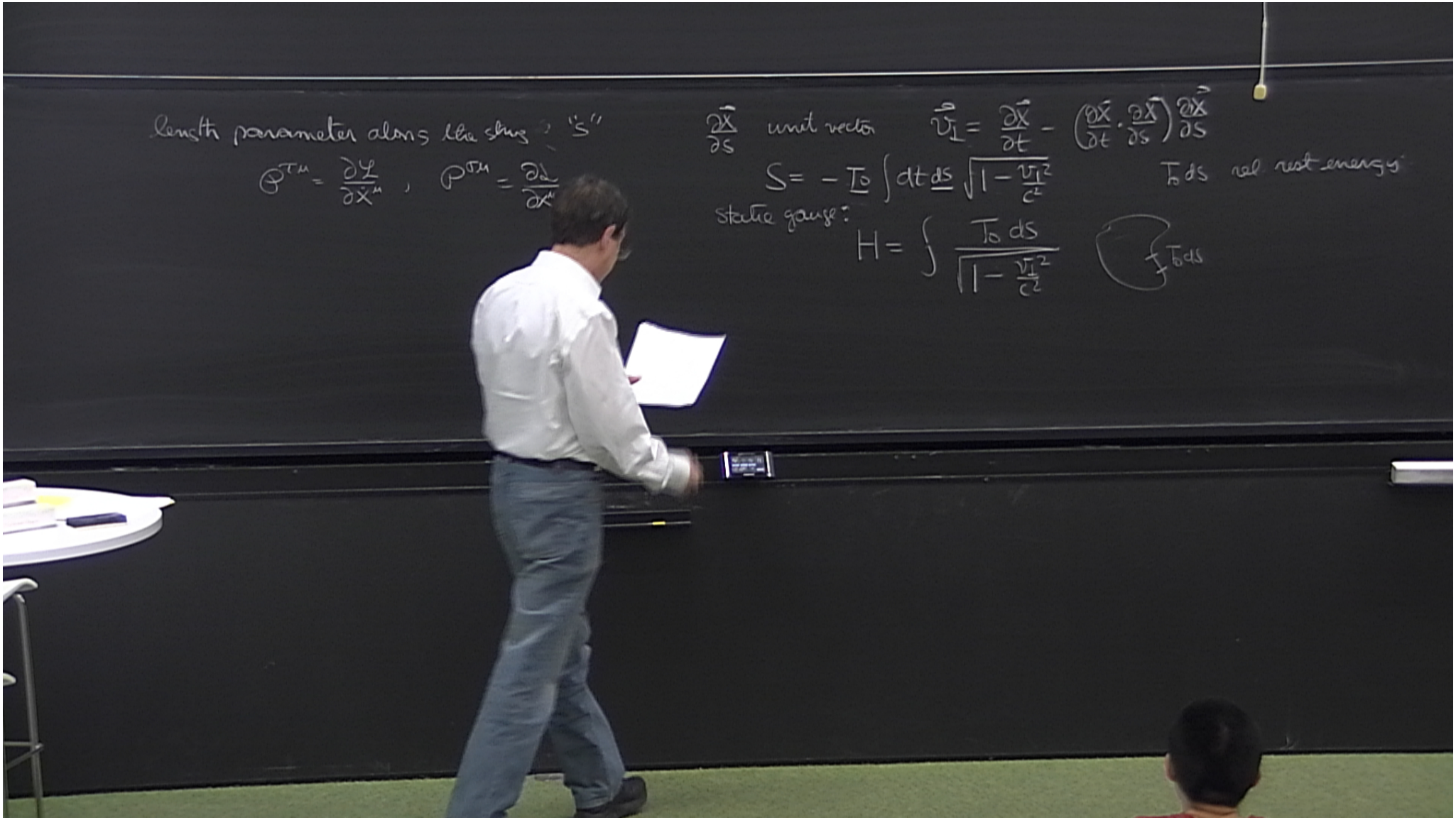
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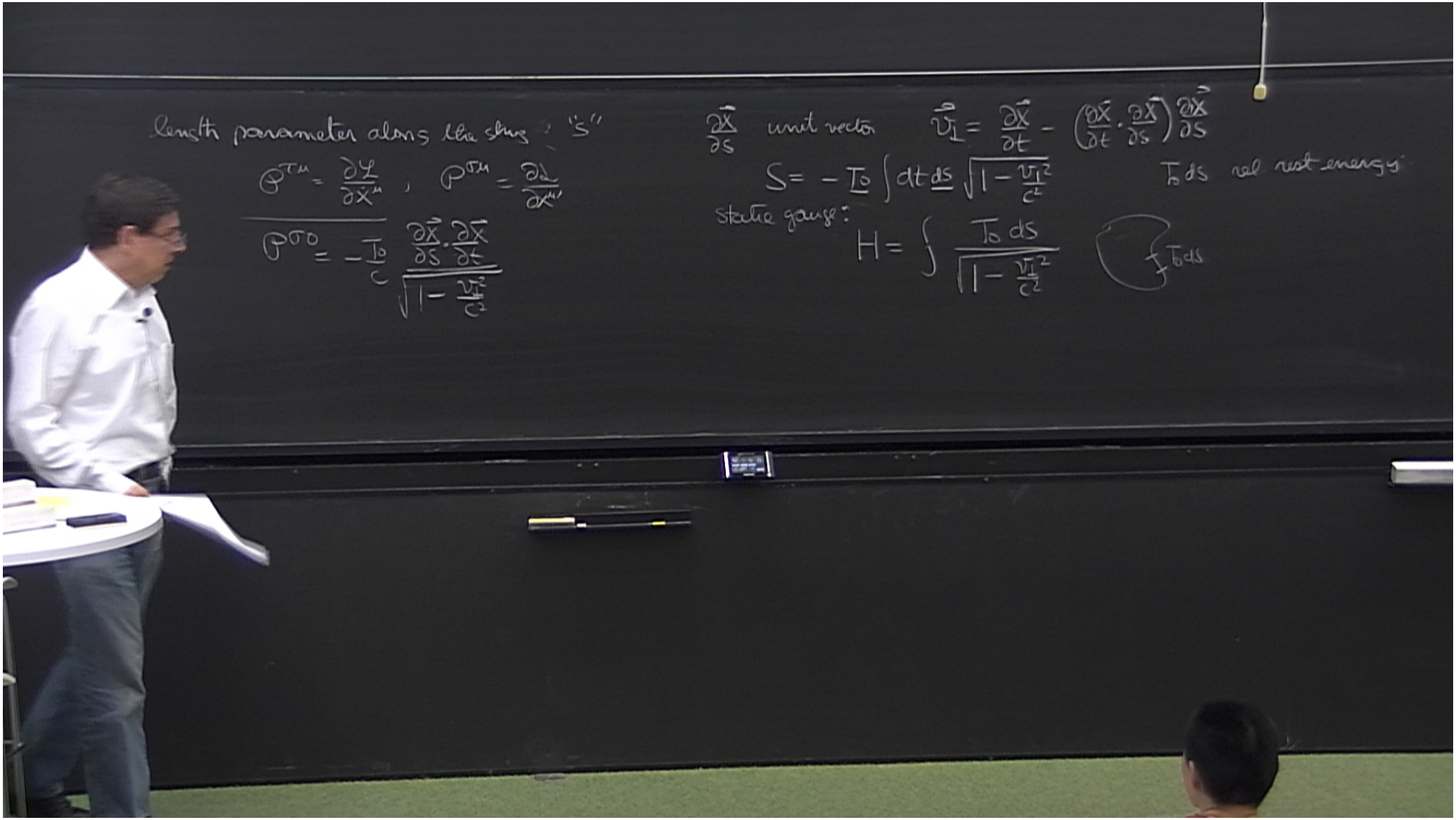
Abstract:

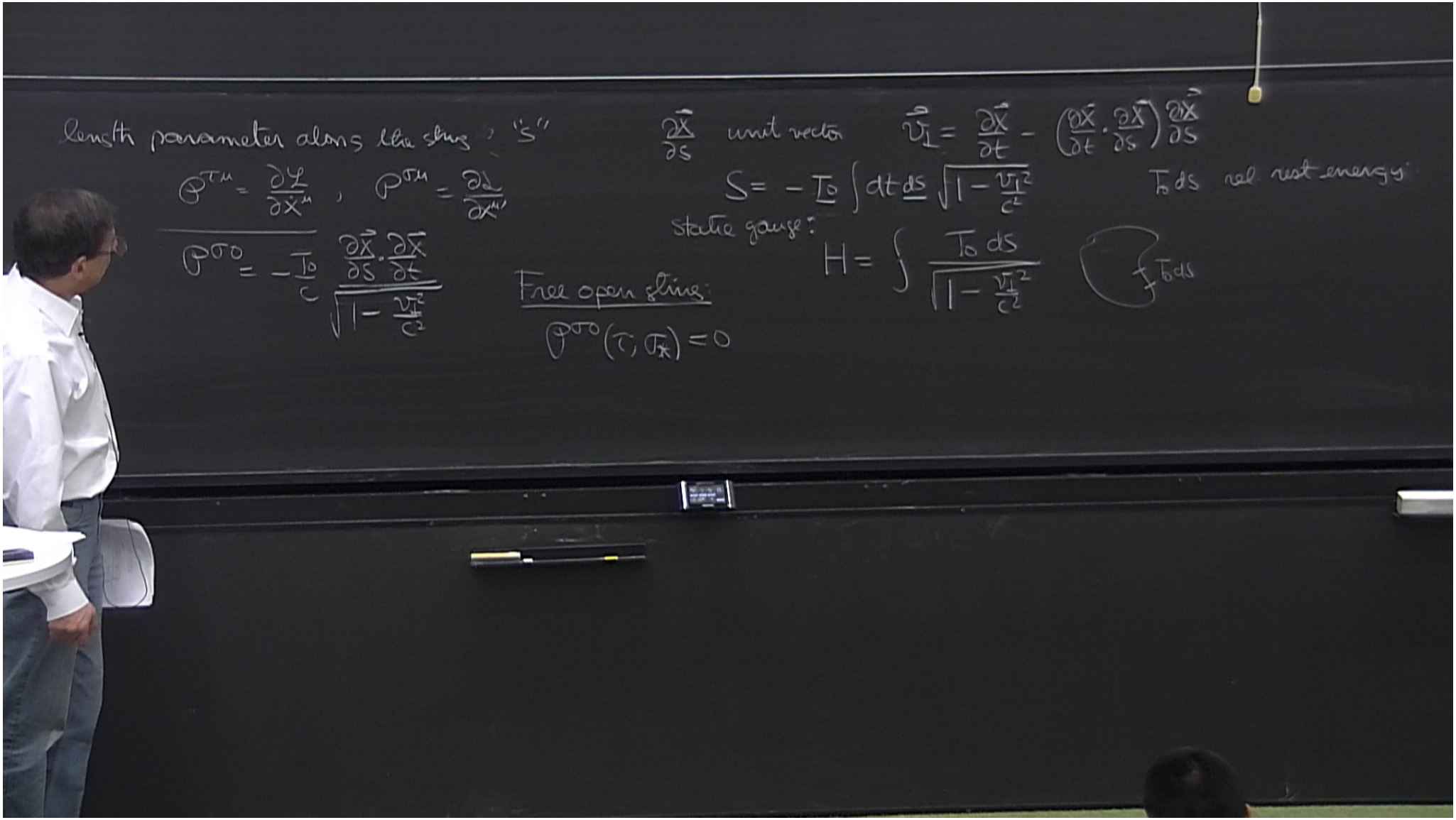

length parameter along the string : "s"
$\frac{\partial \vec{x}}{\partial s}$ unit vector
$\vec{v}_\perp = \frac{\partial \vec{x}}{\partial t} - \left(\frac{\partial \vec{x}}{\partial t} \cdot \frac{\partial \vec{x}}{\partial s}\right) \frac{\partial \vec{x}}{\partial s}$
$S = -T_0 \int dt\, ds \sqrt{1 - \frac{v_\perp^2}{c^2}}$
$T_0 ds$ rel. rest energy


length parameter along the string : "s"
$\frac{\partial \vec{x}}{\partial s}$ unit vector
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$S = -T_0 \int dt\, ds \sqrt{1 - \frac{v_\perp^2}{c^2}}$
$T_0 ds$ rel. rest energy
static gauge
$H = \int \frac{T_0\, ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$


length parameter along the string : "s"
$\frac{\partial \vec{x}}{\partial s}$ unit vector
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static gauge
$H = \int \frac{T_0\, ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
$\int T_0 ds$


length parameter along the string : "s"
$\Theta^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$, $P^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial x'^\mu}$
$\frac{\partial \vec{x}}{\partial s}$ unit vector
$\vec{v}_\perp = \frac{\partial \vec{x}}{\partial t} - \left(\frac{\partial \vec{x}}{\partial t} \cdot \frac{\partial \vec{x}}{\partial s}\right) \frac{\partial \vec{x}}{\partial s}$
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static gauge:
$H = \int \frac{T_0\, ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
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$\frac{\partial \vec{x}}{\partial s}$ unit vector
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$S = -T_0 \int dt\, ds \sqrt{1-\frac{v_\perp^2}{c^2}}$
$T_0 ds$ rest energy
static gauge:
$H = \int \frac{T_0\, ds}{\sqrt{1-\frac{v_\perp^2}{c^2}}}$
$\int T_0 ds$
$P^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$, $P^{\sigma\mu} = \frac{\partial \mathcal{L}}{\partial x'^\mu}$
$P^{\sigma 0} = -\frac{T_0}{c}\frac{\frac{\partial \vec{x}}{\partial s}\cdot\frac{\partial \vec{x}}{\partial t}}{\sqrt{1-\frac{v_\perp^2}{c^2}}}$


Length parameter along the string : "s"
$\frac{\partial \vec{X}}{\partial s}$ unit vector
$\vec{v}_\perp = \frac{\partial \vec{X}}{\partial t} - \left(\frac{\partial \vec{X}}{\partial t}\cdot\frac{\partial \vec{X}}{\partial s}\right)\frac{\partial \vec{X}}{\partial s}$
$\mathcal{P}^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$, $\mathcal{P}^{s\mu} = \frac{\partial \mathcal{L}}{\partial X'^\mu}$
$S = -T_0 \int dt\, ds \sqrt{1 - \frac{v_\perp^2}{c^2}}$
$T_0 ds$ rel. rest energy
$\mathcal{P}^{\tau 0} = -\frac{T_0}{c} \frac{\frac{\partial \vec{X}}{\partial s}\cdot\frac{\partial \vec{X}}{\partial t}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
static gauge:
$H = \int \frac{T_0\, ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
$\int T_0 ds$
Free open string:
$\mathcal{P}^{\tau 0}(\tau, \sigma_*) = 0$

Length parameter along the string : "s"

$\frac{\partial \vec{X}}{\partial s}$ unit vector $\quad \vec{v}_\perp = \frac{\partial \vec{X}}{\partial t} - \left(\frac{\partial \vec{X}}{\partial t}\cdot\frac{\partial \vec{X}}{\partial s}\right)\frac{\partial \vec{X}}{\partial s}$

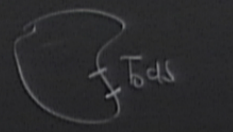
$\mathcal{P}^{\tau\mu} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}$, $\mathcal{P}^{s\mu} = \frac{\partial \mathcal{L}}{\partial X'^\mu}$

$S = -T_0 \int dt\, ds \sqrt{1 - \frac{v_\perp^2}{c^2}}$ $\quad T_0 ds$ rel. rest energy

$\mathcal{P}^{\sigma 0} = -\frac{T_0}{c} \frac{\frac{\partial \vec{X}}{\partial s}\cdot\frac{\partial \vec{X}}{\partial t}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$

static gauge:

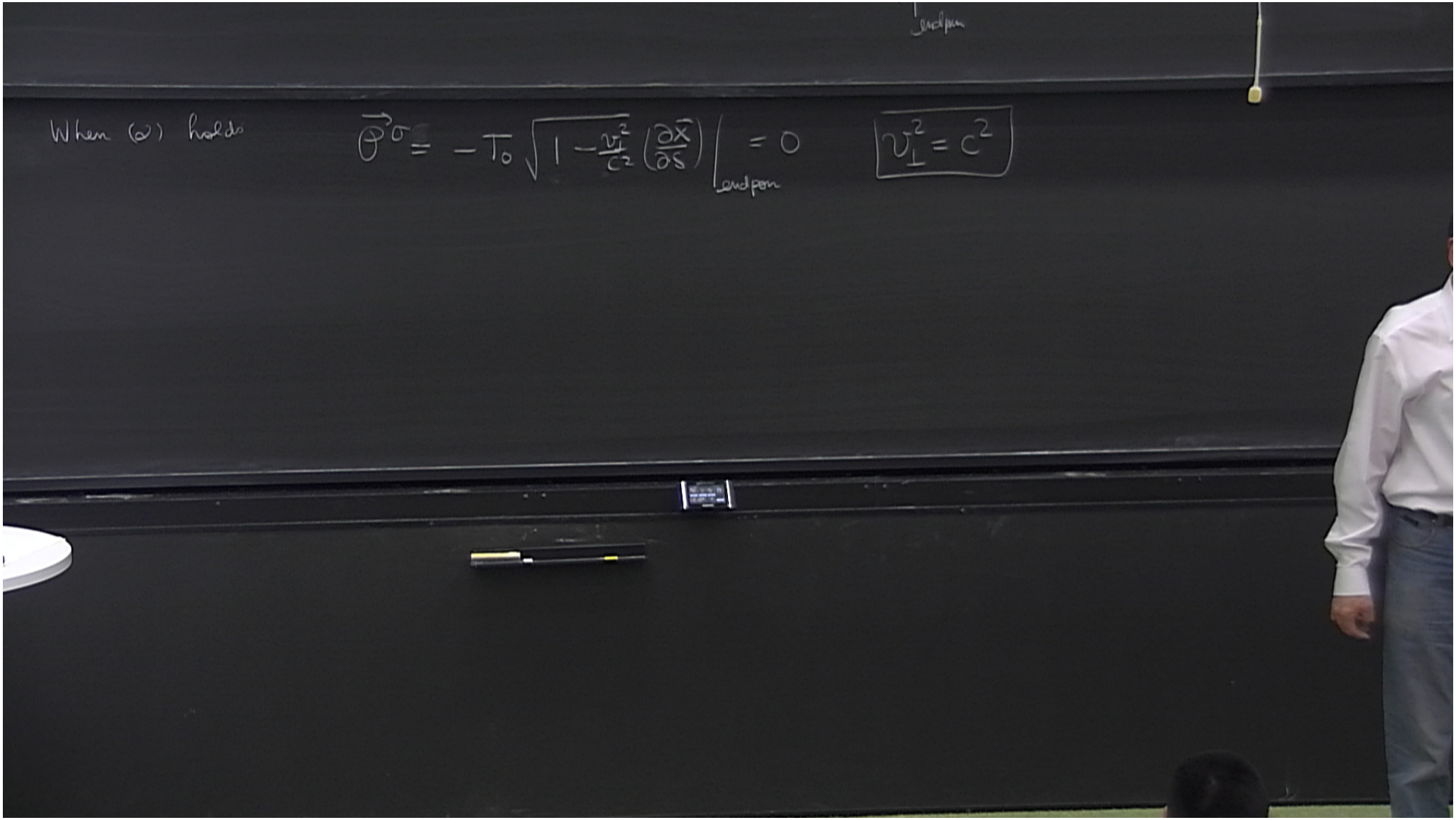
$H = \int \frac{T_0\, ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$

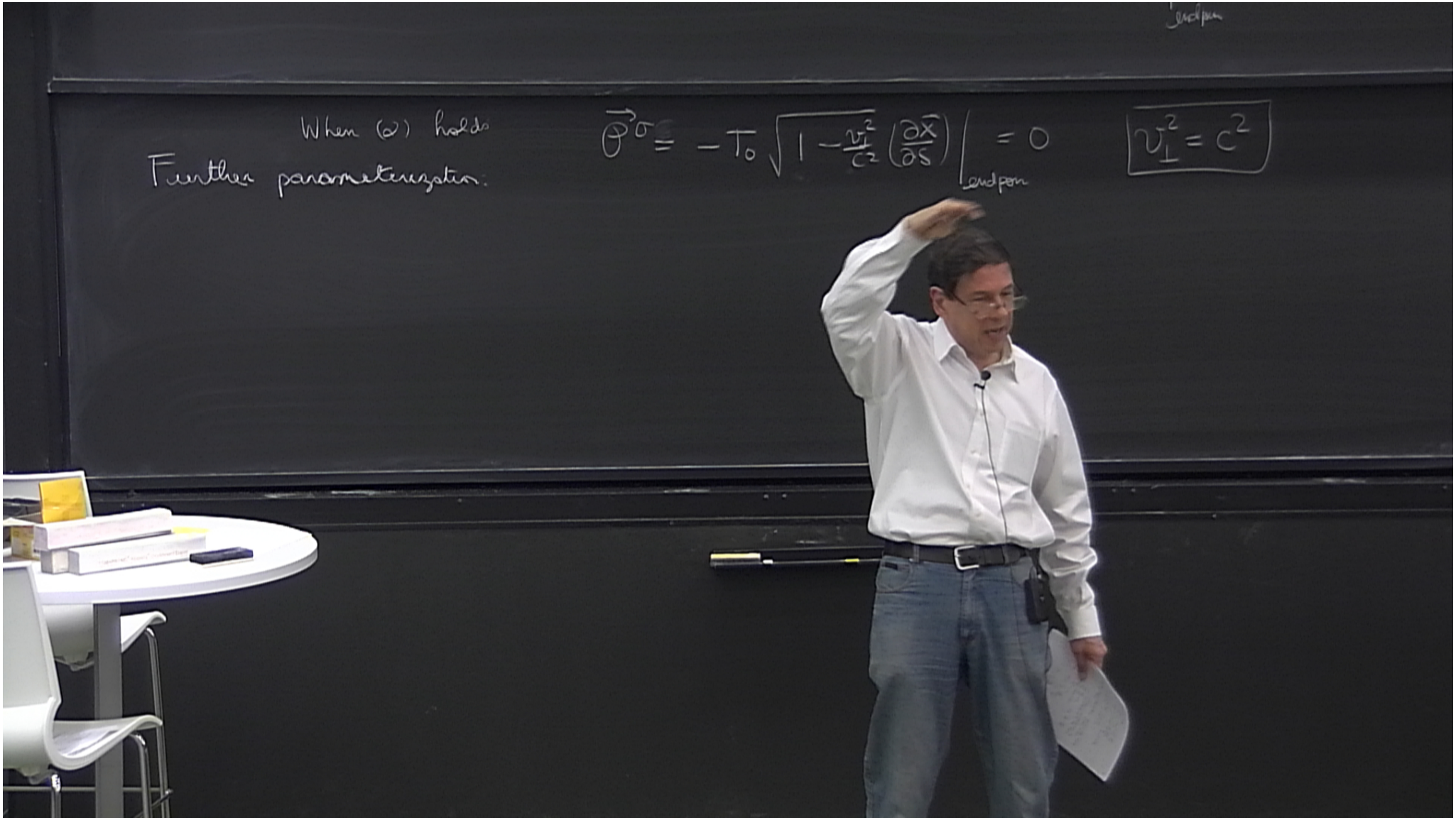


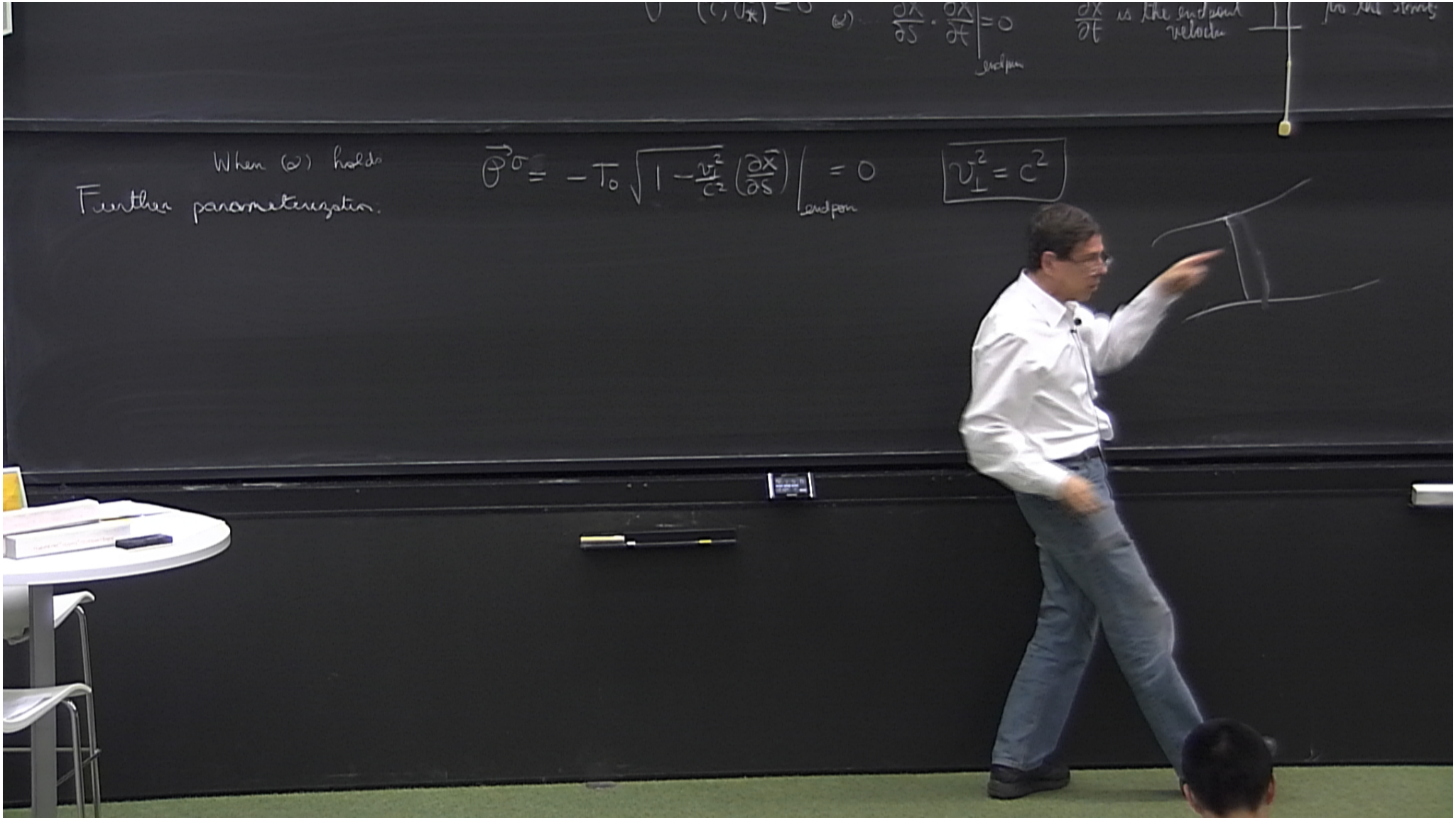
Free open string:

$\mathcal{P}^{\sigma 0}(\tau, \sigma_*) = 0$

$\frac{\partial \vec{X}}{\partial s}\cdot\frac{\partial \vec{X}}{\partial t}\Big|_{\text{endpoint}} = 0$ $\quad \frac{\partial \vec{X}}{\partial t}$ is the endpoint velocity $\perp$ to the string


When (a) holds
$\vec{\nabla}\sigma = -T_0\sqrt{1-\frac{v_\perp^2}{c^2}}\left(\frac{\partial \vec{x}}{\partial s}\right)\Big|_{end\,pen} = 0$
$v_\perp^2 = c^2$


When (2) holds
Further parameterization:
$\vec{\partial}^0 = -T_0 \sqrt{1 - \frac{v_\perp^2}{c^2}} \left(\frac{\partial \vec{x}}{\partial s}\right)\Big|_{\text{endpoints}} = 0$
$v_\perp^2 = c^2$


When (ω) holds
Further parameterization.
$\vec{\sigma}^{\sigma} = -T_0 \sqrt{1 - \frac{v^2}{c^2}} \left(\frac{\partial \vec{x}}{\partial s}\right)\Big|_{\text{endpoint}} = 0$
$v_\perp^2 = c^2$
$\frac{\partial x}{\partial s} \cdot \frac{\partial x}{\partial t}\Big|_{\text{endpoint}} = 0$
$\frac{\partial x}{\partial t}$ is the endpoint velocity

When (*) holds

$$\vec{\theta}^{\sigma} = -T_0 \sqrt{1 - \frac{v_\perp^2}{c^2}} \left(\frac{\partial \vec{x}}{\partial s}\right) \Big|_{\text{endpoints}} = 0$$

$$v_\perp^2 = c^2$$

Further parameterization:

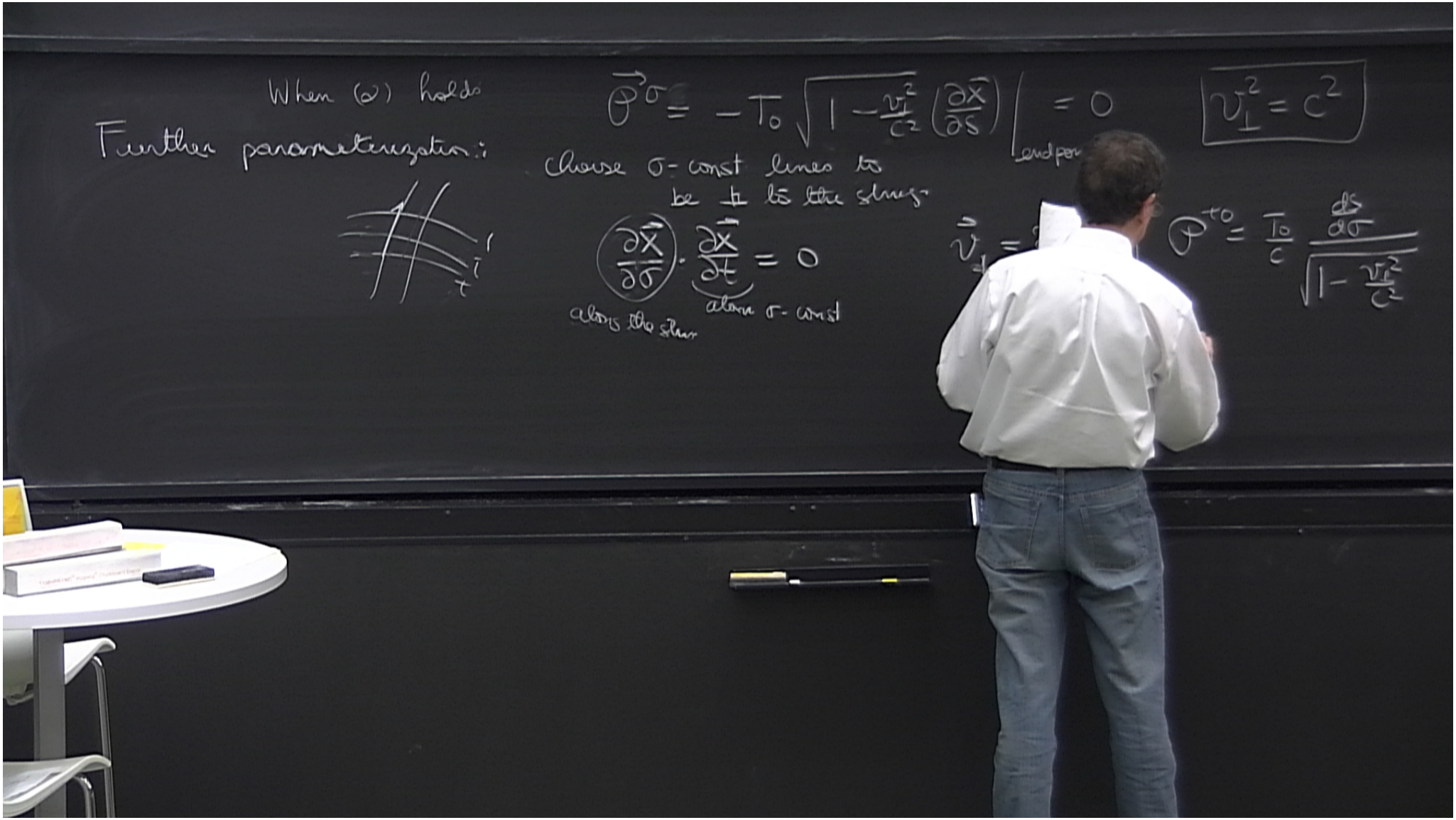


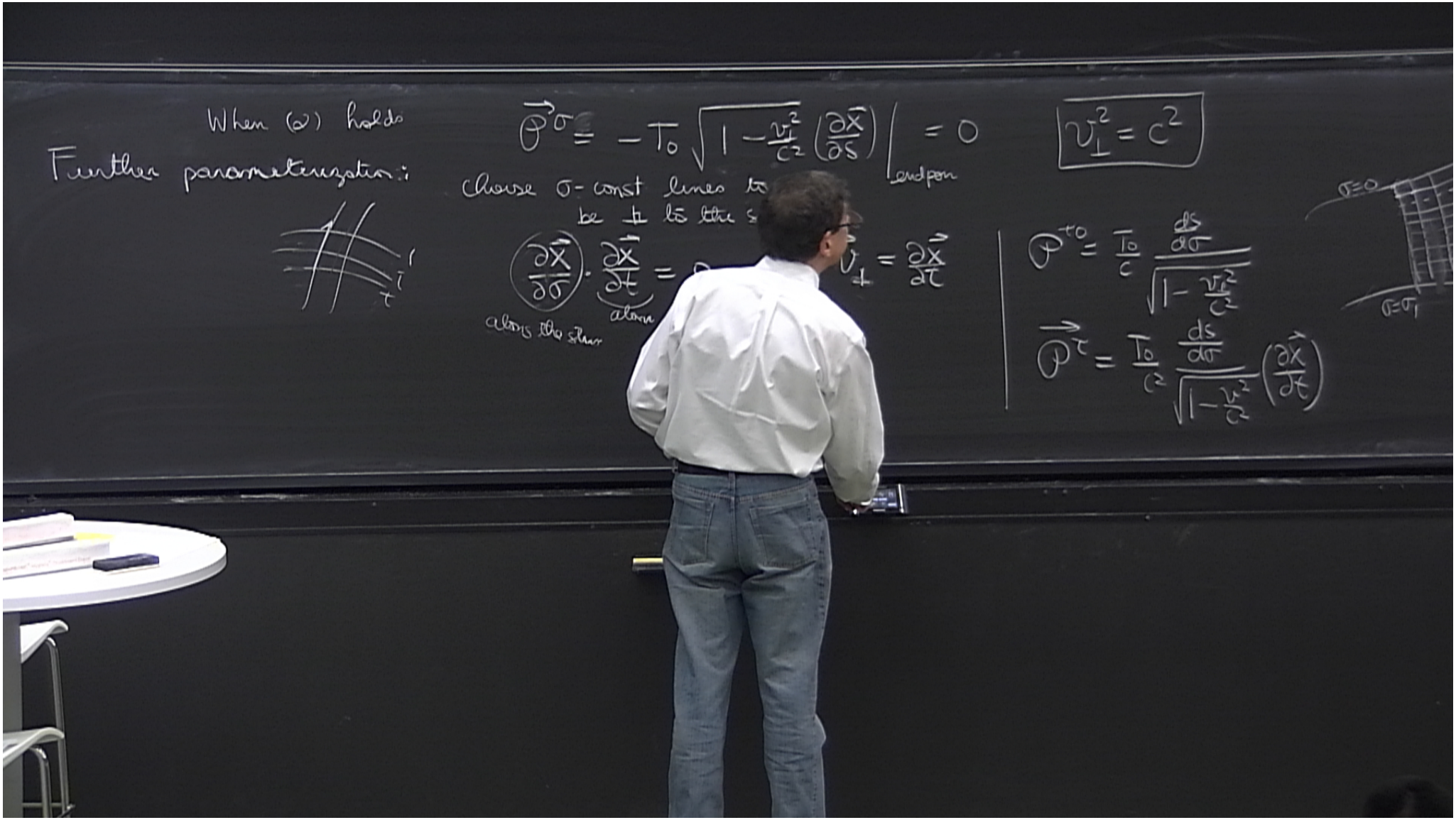
Choose σ-const lines to be $\perp$ to the slices.

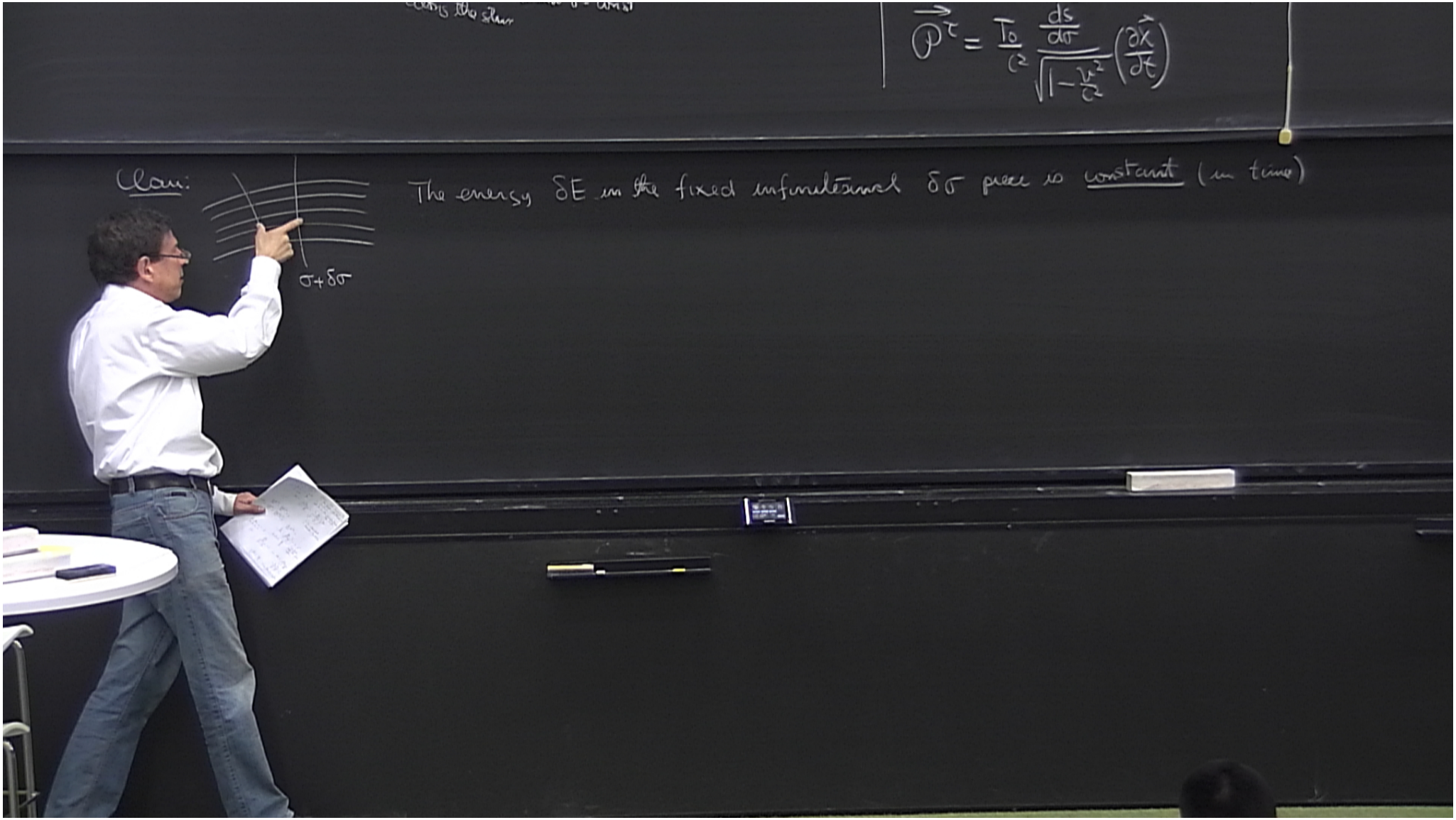
$$\left(\frac{\partial \vec{x}}{\partial \sigma}\right) \cdot \left(\frac{\partial \vec{x}}{\partial t}\right) = 0$$

$\frac{\partial \vec{x}}{\partial \sigma}$: across the slice

$\frac{\partial \vec{x}}{\partial t}$: along σ-const


When (*) holds
Further parameterization:
$\vec{P}^0 = -T_0 \sqrt{1 - \frac{v_\perp^2}{c^2}} \left(\frac{\partial \vec{x}}{\partial s}\right)\Big|_{\text{endpo}} = 0$
$v_\perp^2 = c^2$
Choose σ-const lines to be $\perp$ to the slices
$\left(\frac{\partial \vec{x}}{\partial \sigma}\right) \cdot \left(\frac{\partial \vec{x}}{\partial t}\right) = 0$
along the slice
along σ-const
$P^{\tau 0} = \frac{T_0}{c} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$


When (2) holds
Further parameterization:
$\vec{\mathcal{P}}^{\sigma} = -T_0 \sqrt{1 - \frac{v^2}{c^2}} \left(\frac{\partial \vec{x}}{\partial s}\right) \Big|_{\text{endpoints}} = 0$
$v_\perp^2 = c^2$
Choose σ - const lines to
be ⊥ to the
$\left(\frac{\partial \vec{x}}{\partial \sigma}\right) \cdot \left(\frac{\partial \vec{x}}{\partial \tau}\right) = 0$
along
along the shar
$v_\perp = \frac{\partial \vec{x}}{\partial \tau}$
$\mathcal{P}^{\tau 0} = \frac{T_0}{c} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
$\vec{\mathcal{P}}^{\tau} = \frac{T_0}{c^2} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v^2}{c^2}}} \left(\frac{\partial \vec{x}}{\partial \tau}\right)$
σ=0
σ=σ1


$\vec{p}^{\tau} = \frac{T_0}{c^2} \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} \left(\frac{\partial \vec{x}}{\partial t}\right)$
Claim:
The energy δE in the fixed infinitesimal $\delta\sigma$ piece is constant (in time)
$\sigma + \delta\sigma$

$$\left(\frac{dx}{dt}\right)^2 = \frac{T_0}{\rho}\frac{d\sigma}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{dx}{dt}\right)$$

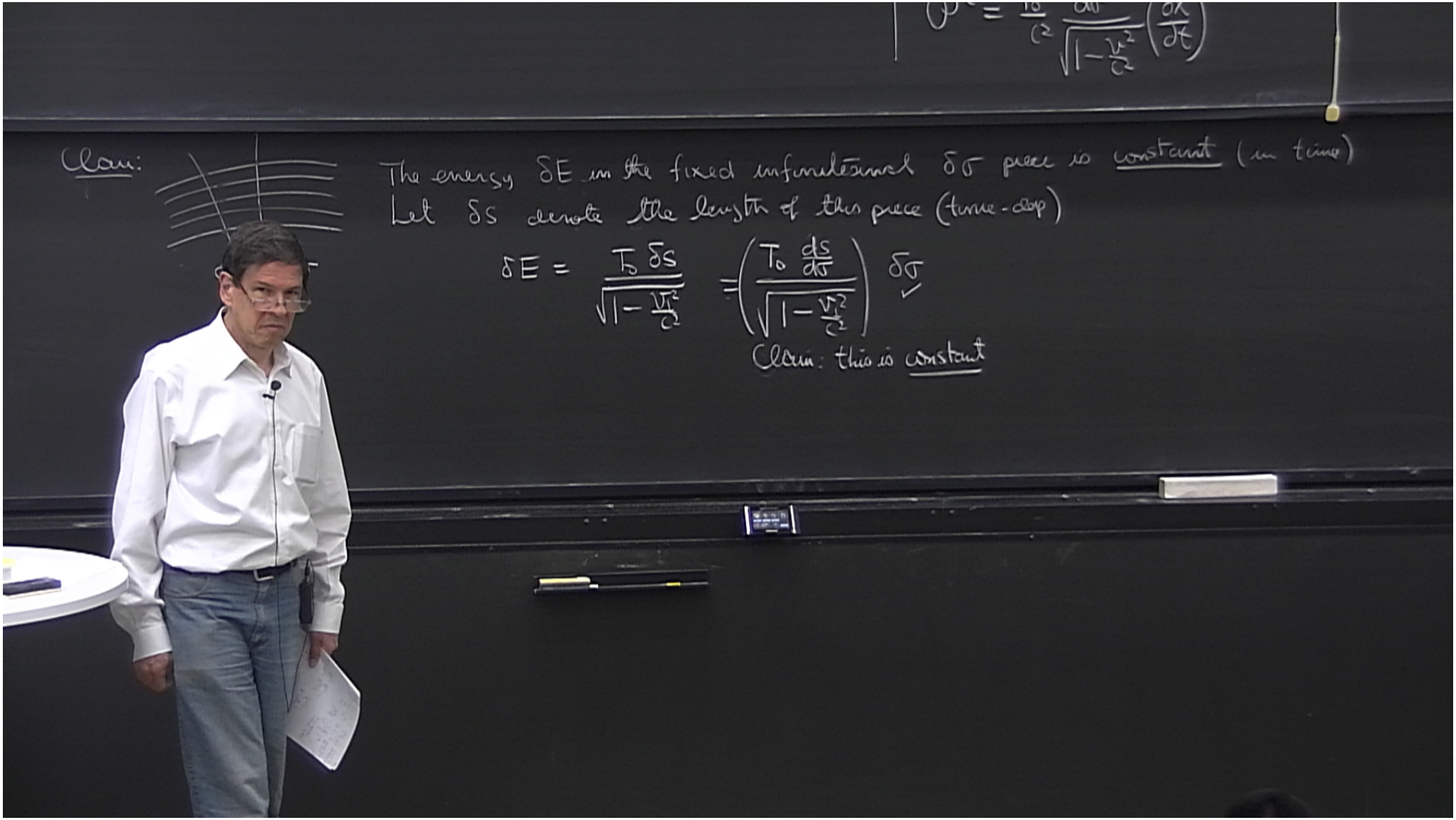
Claim:

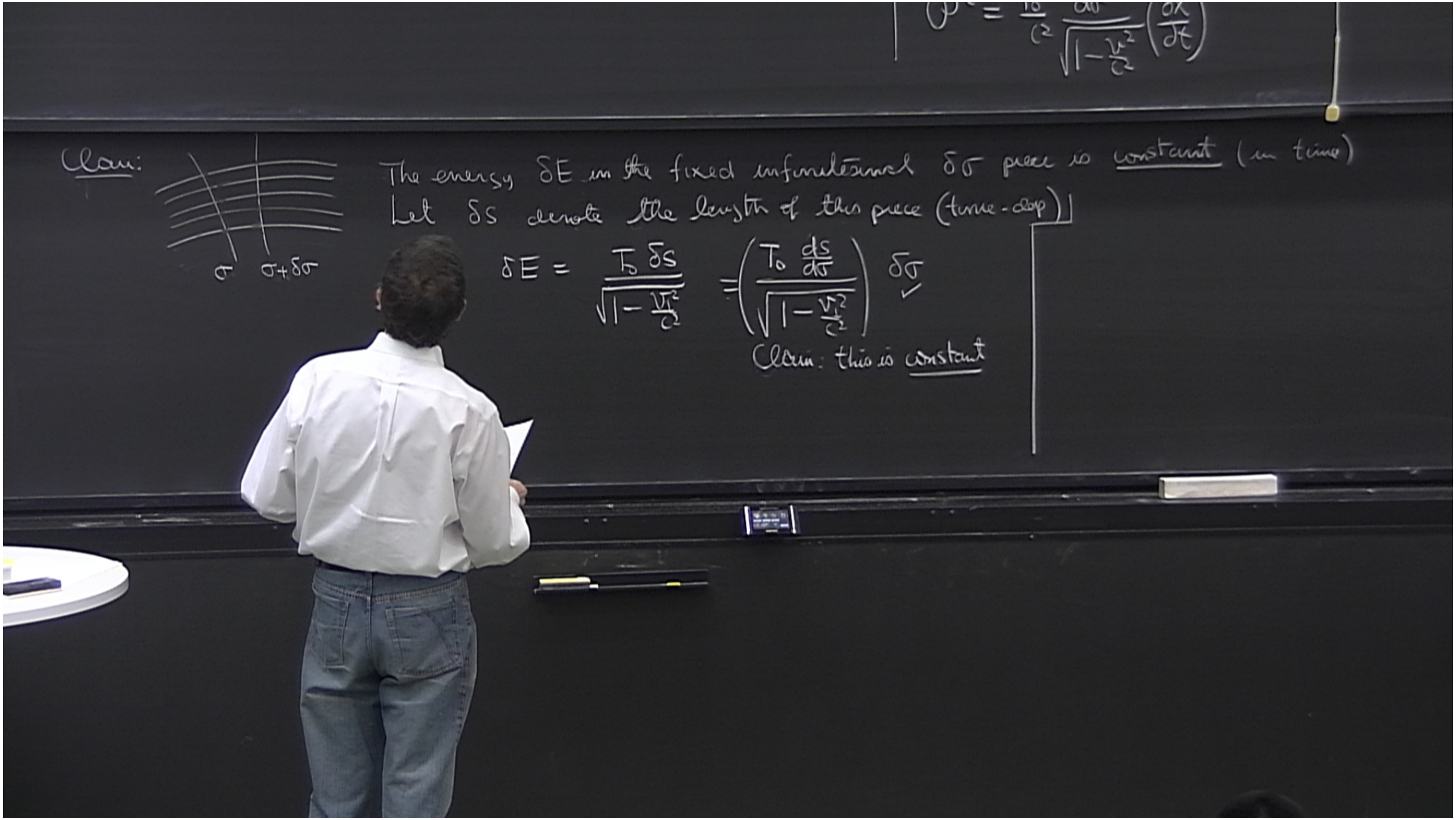


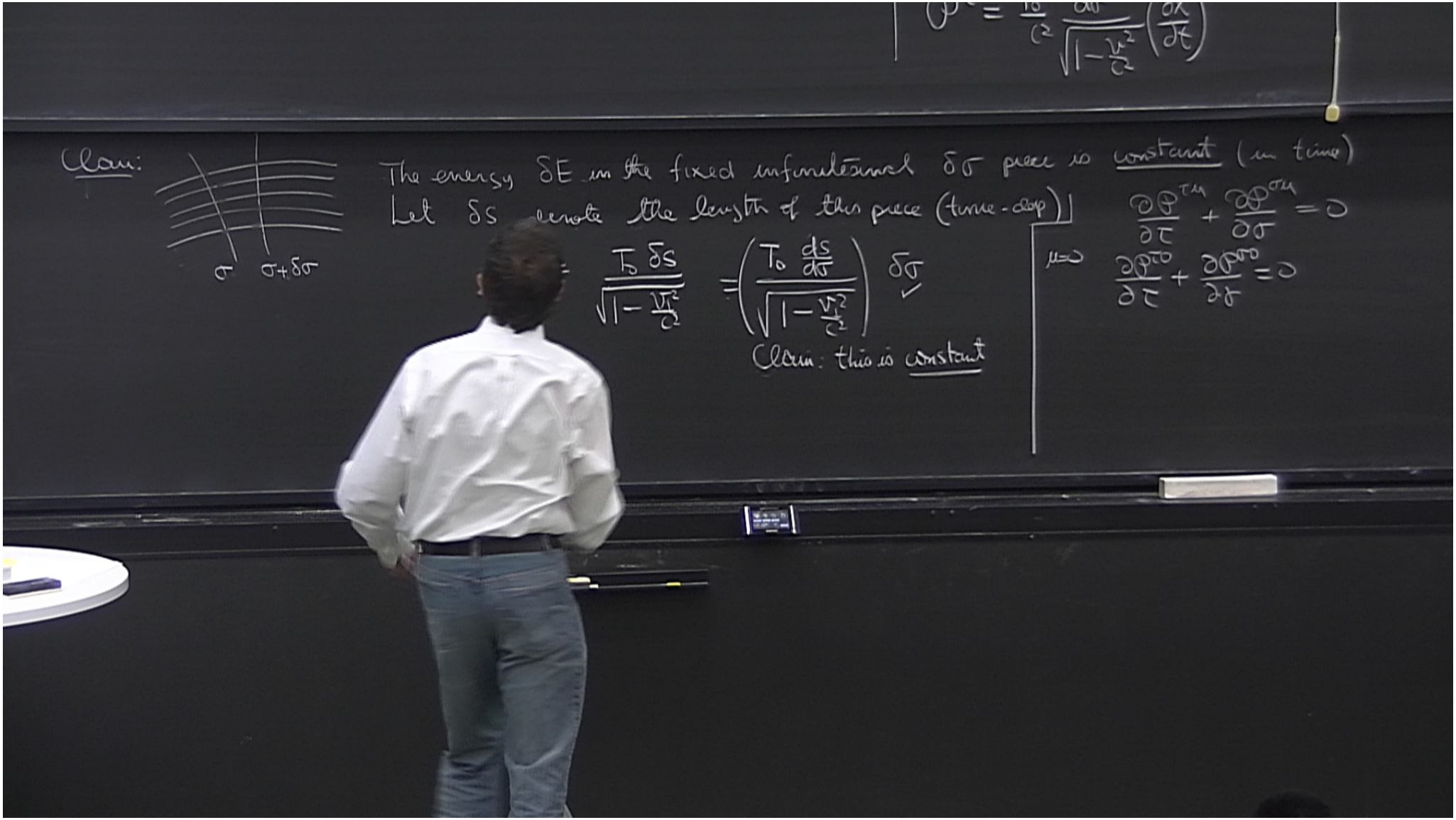
The energy δE in the fixed infinitesimal $\delta\sigma$ piece is constant (in time)

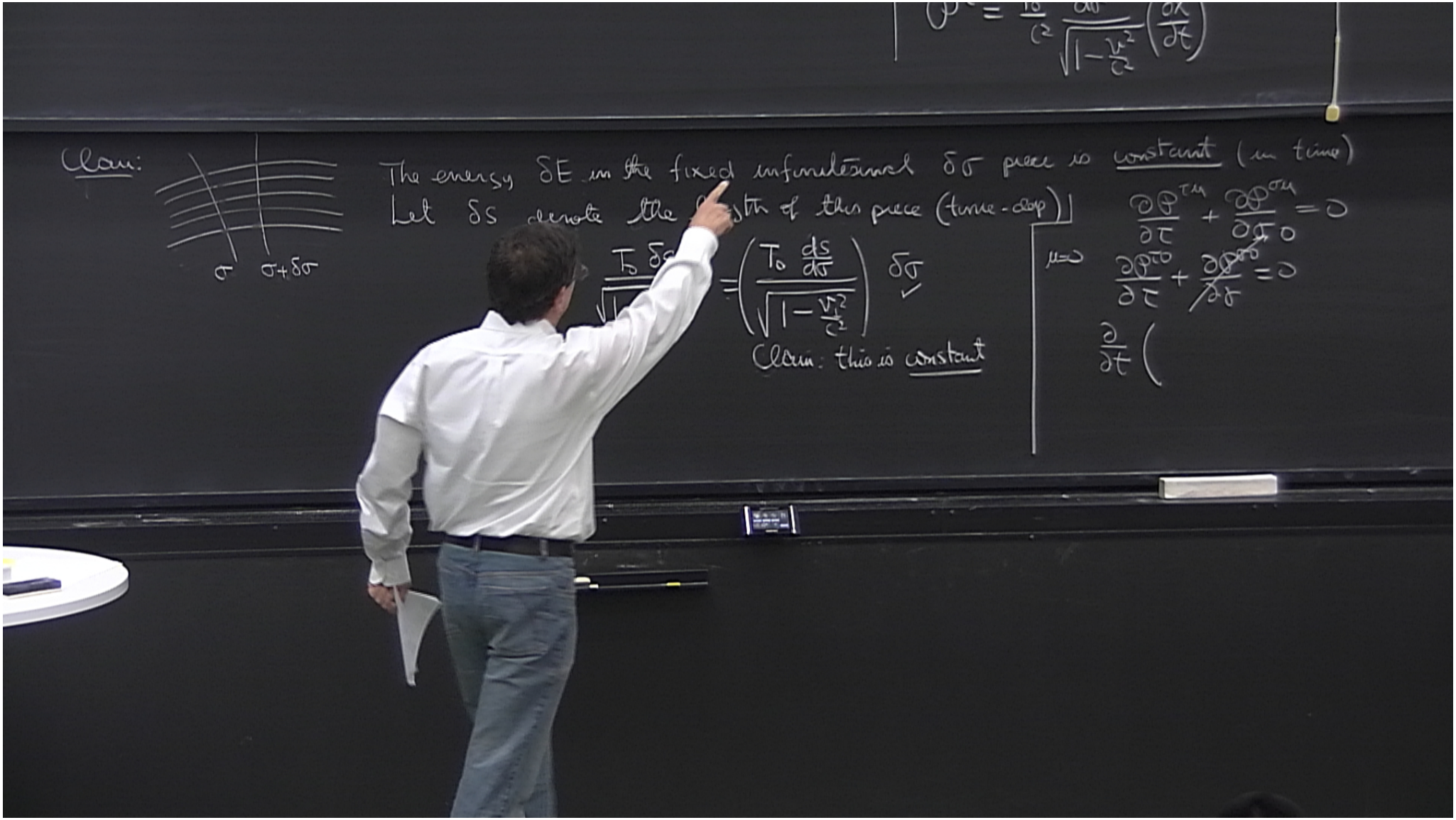
Let δs denote the length of this piece (time-dep)

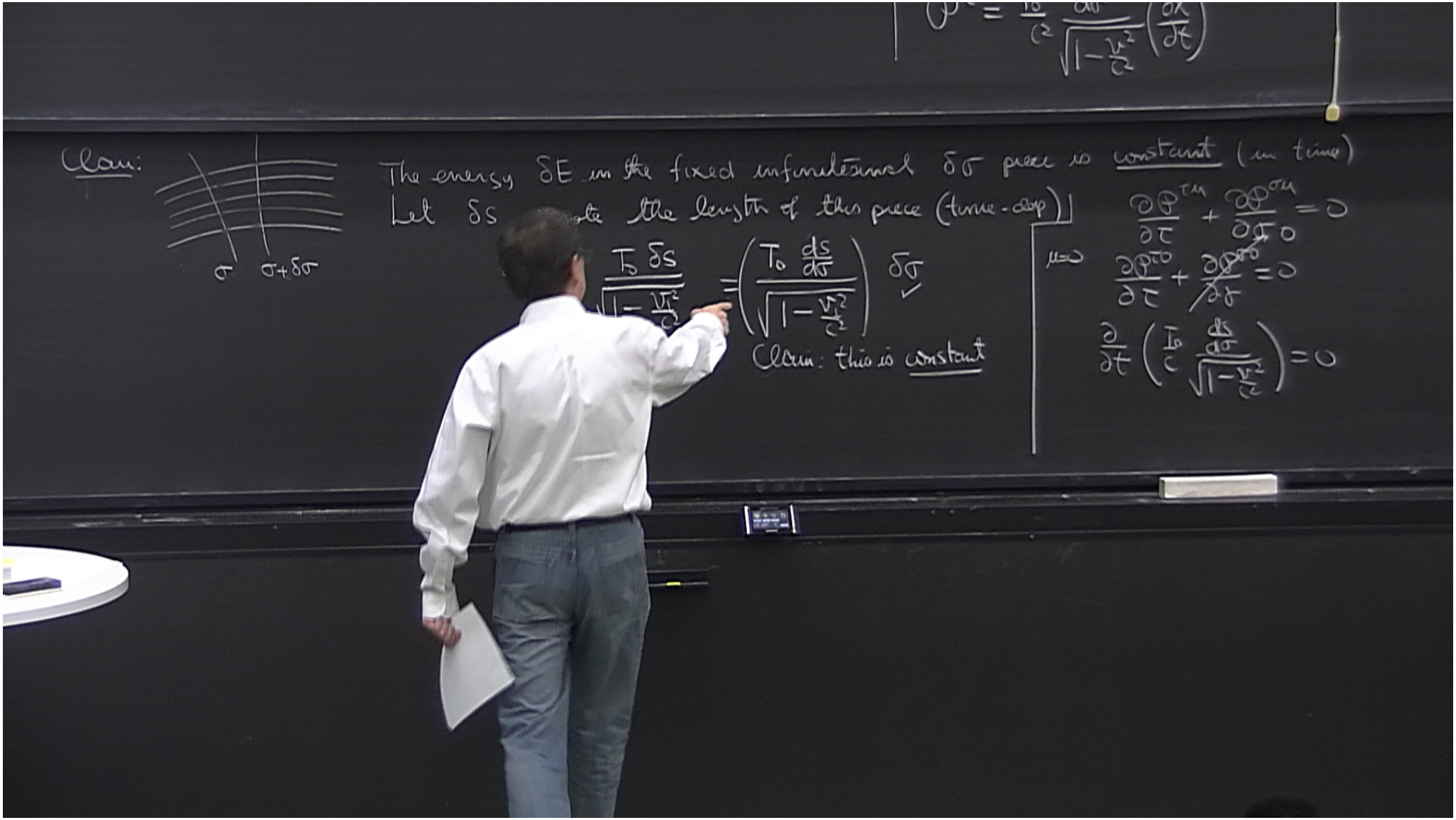
$$\delta E = \frac{T_0\,\delta s}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{T_0\,\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\,\delta\sigma$$

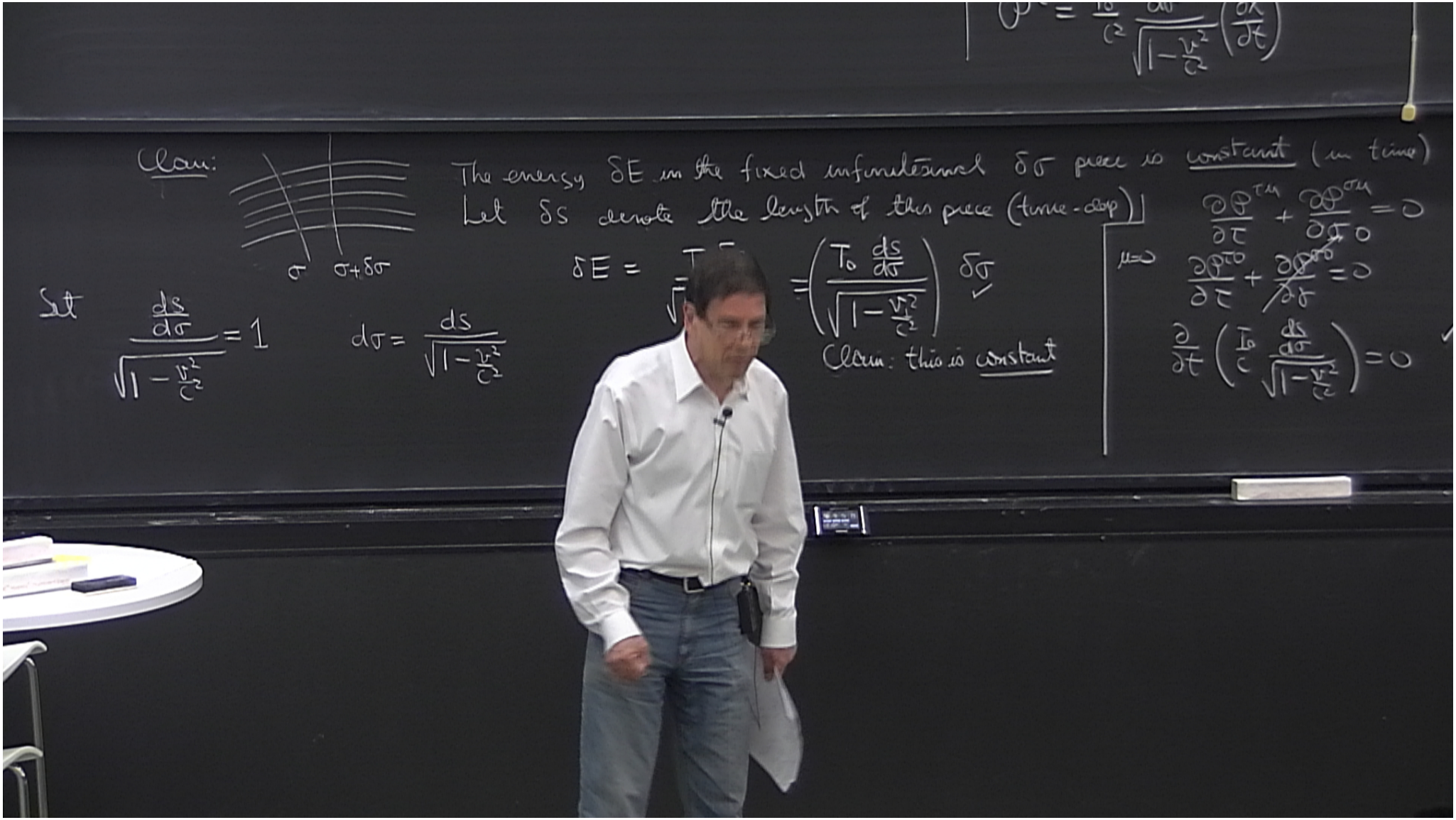

Claim:
The energy δE in the fixed infinitesimal $\delta\sigma$ piece is constant (in time)
Let δs denote the length of this piece (time-slice)
$\delta E = \frac{T_0 \delta s}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{T_0 \frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\right)\delta\sigma$
Claim: this is constant

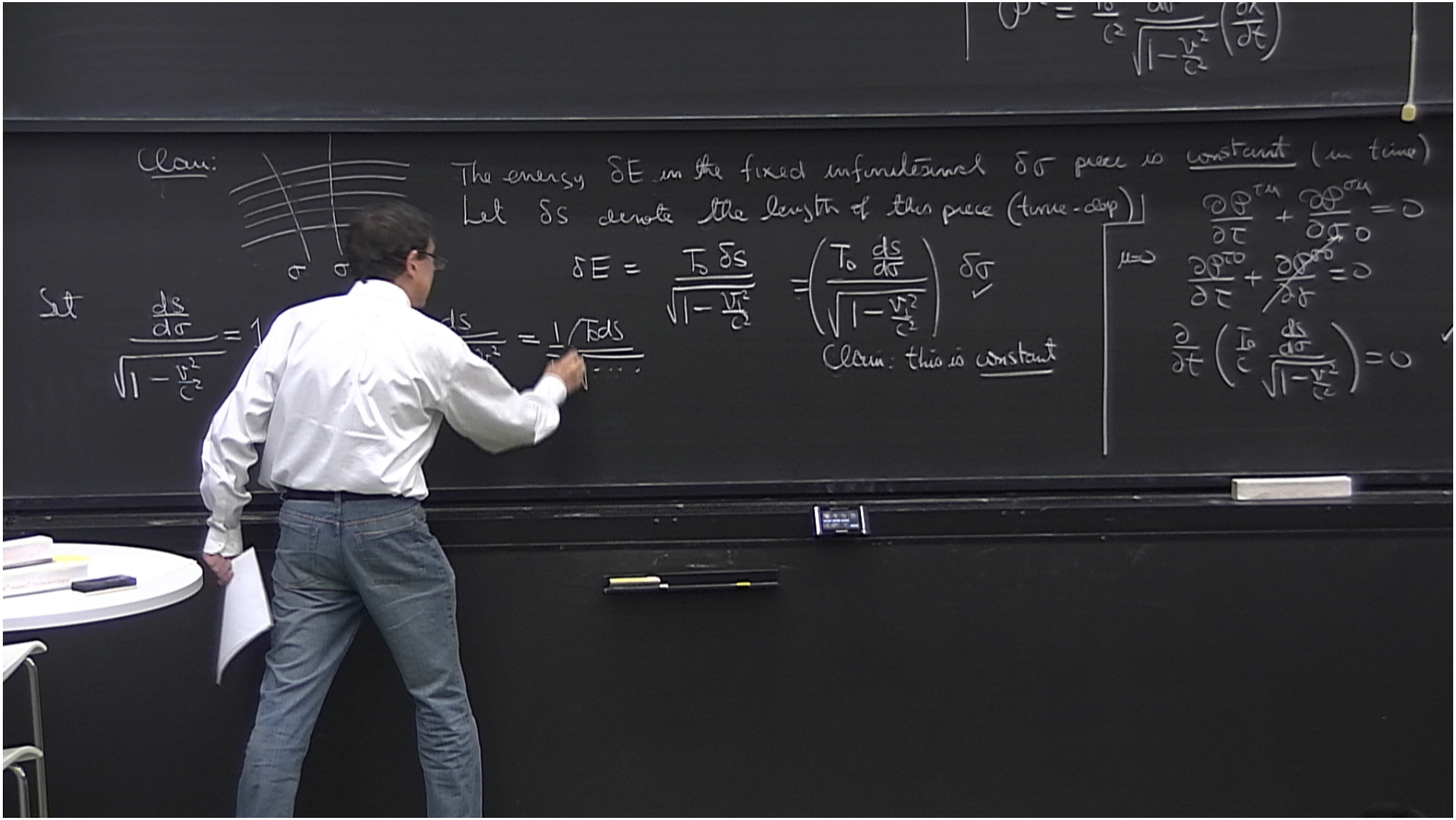

Claim:
The energy δE in the fixed infinitesimal δσ piece is constant (in time)
Let δs denote the length of this piece (time-dep)
σ
σ+δσ
δE = T₀ δs / √(1 − v²/c²) = (T₀ ds/dσ / √(1 − v²/c²)) δσ
Claim: this is constant

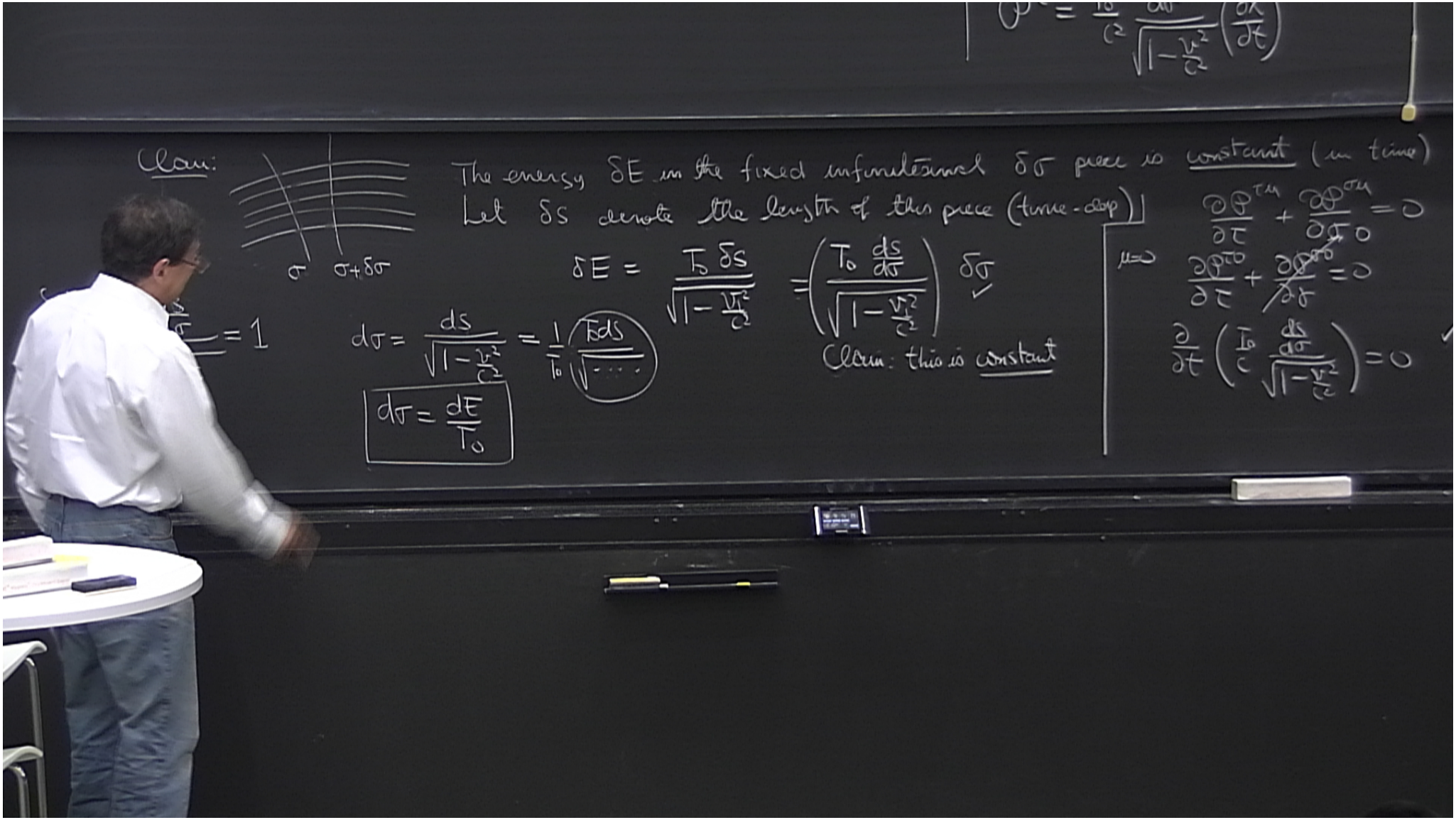

Claim:
The energy δE in the fixed infinitesimal δσ piece is constant (in time)
Let δs denote the length of this piece (time-dep)
σ
σ+δσ
Claim: this is constant
μ=0

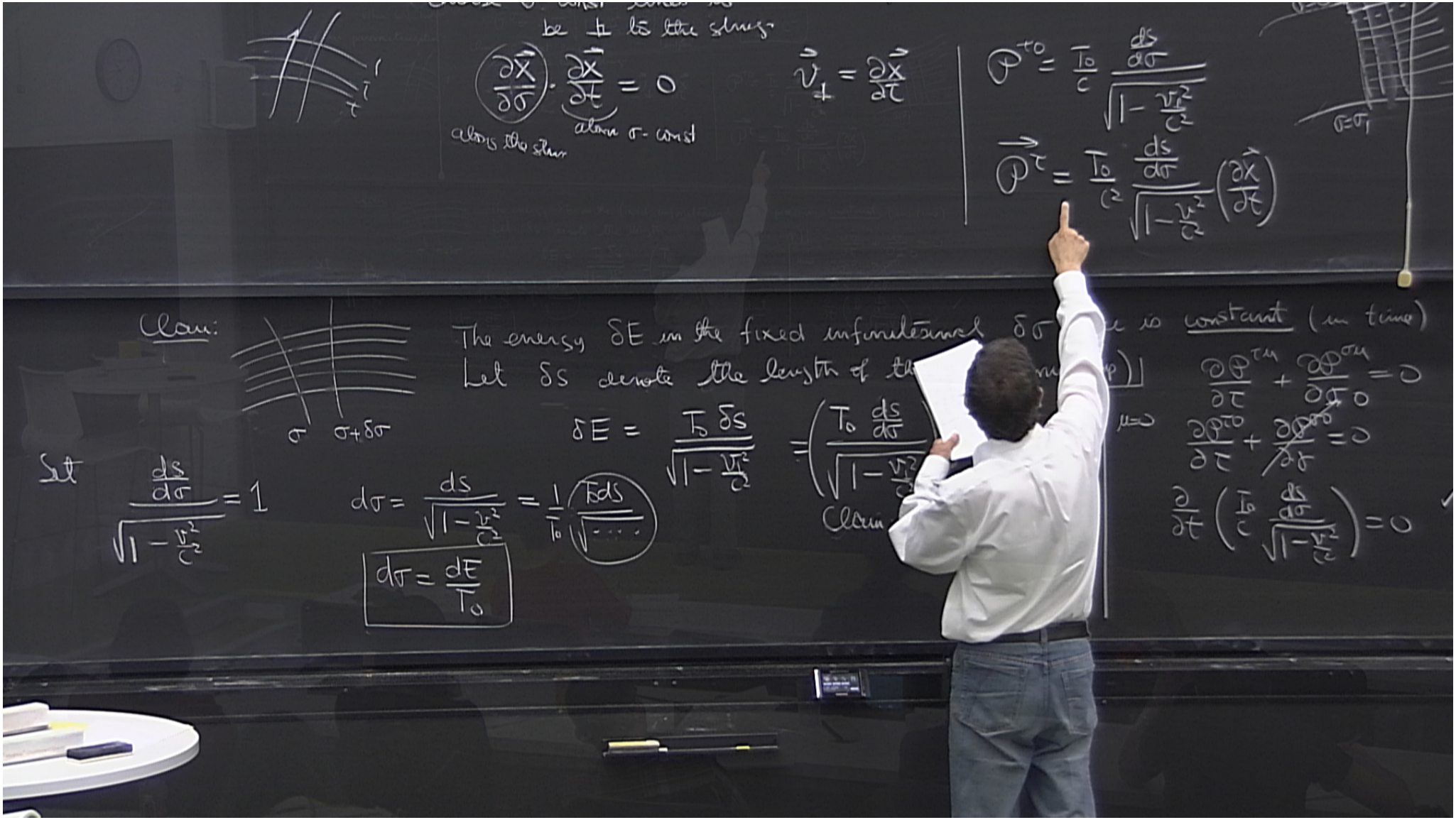

Claim:
The energy δE in the fixed infinitesimal δσ piece is constant (in time)
Let δs denote the
Claim: this is constant
σ
σ+δσ

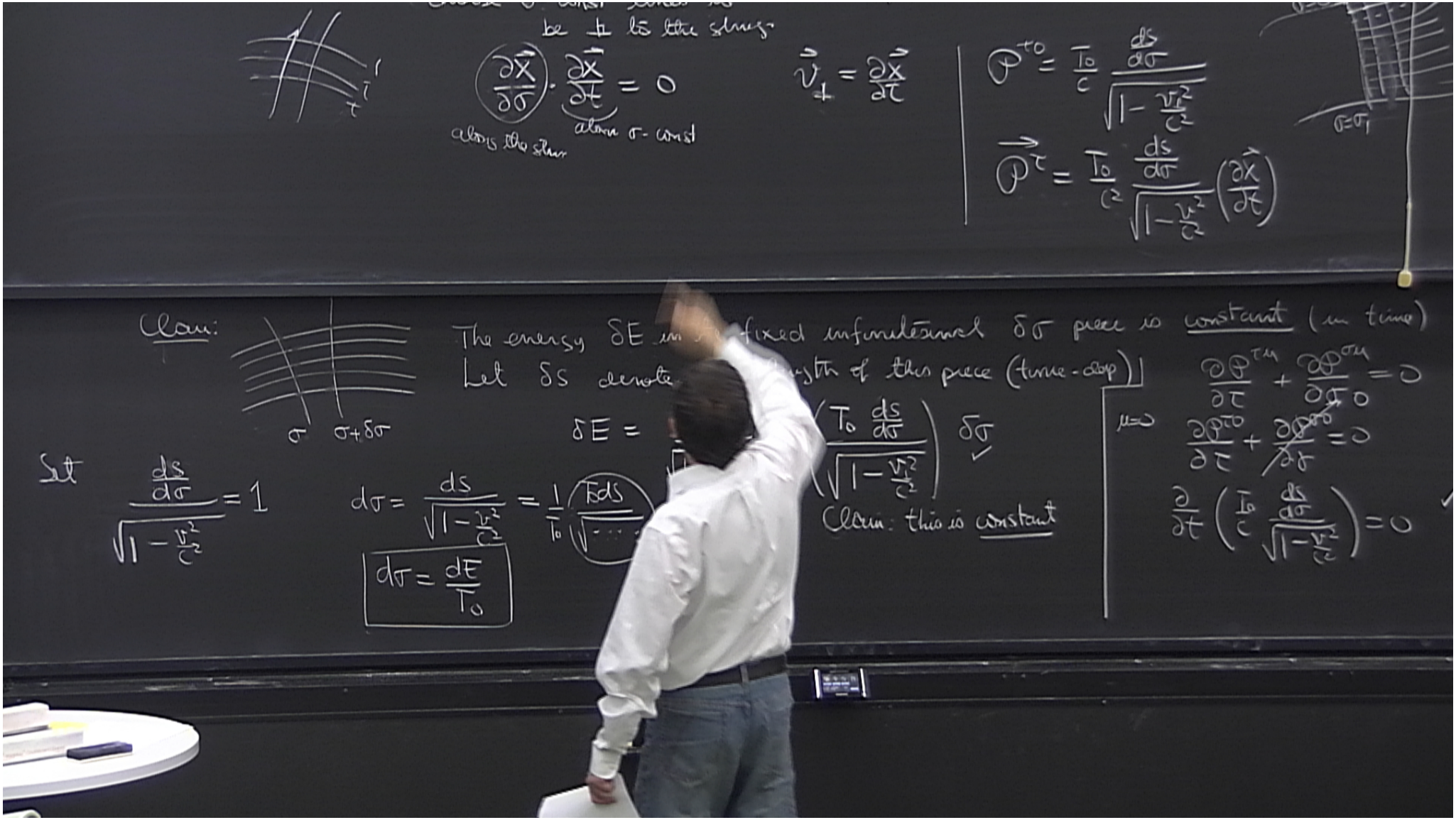

Claim:
The energy δE in the fixed infinitesimal δσ piece is constant (in time)
Let δs
Claim: this is constant
μ=0

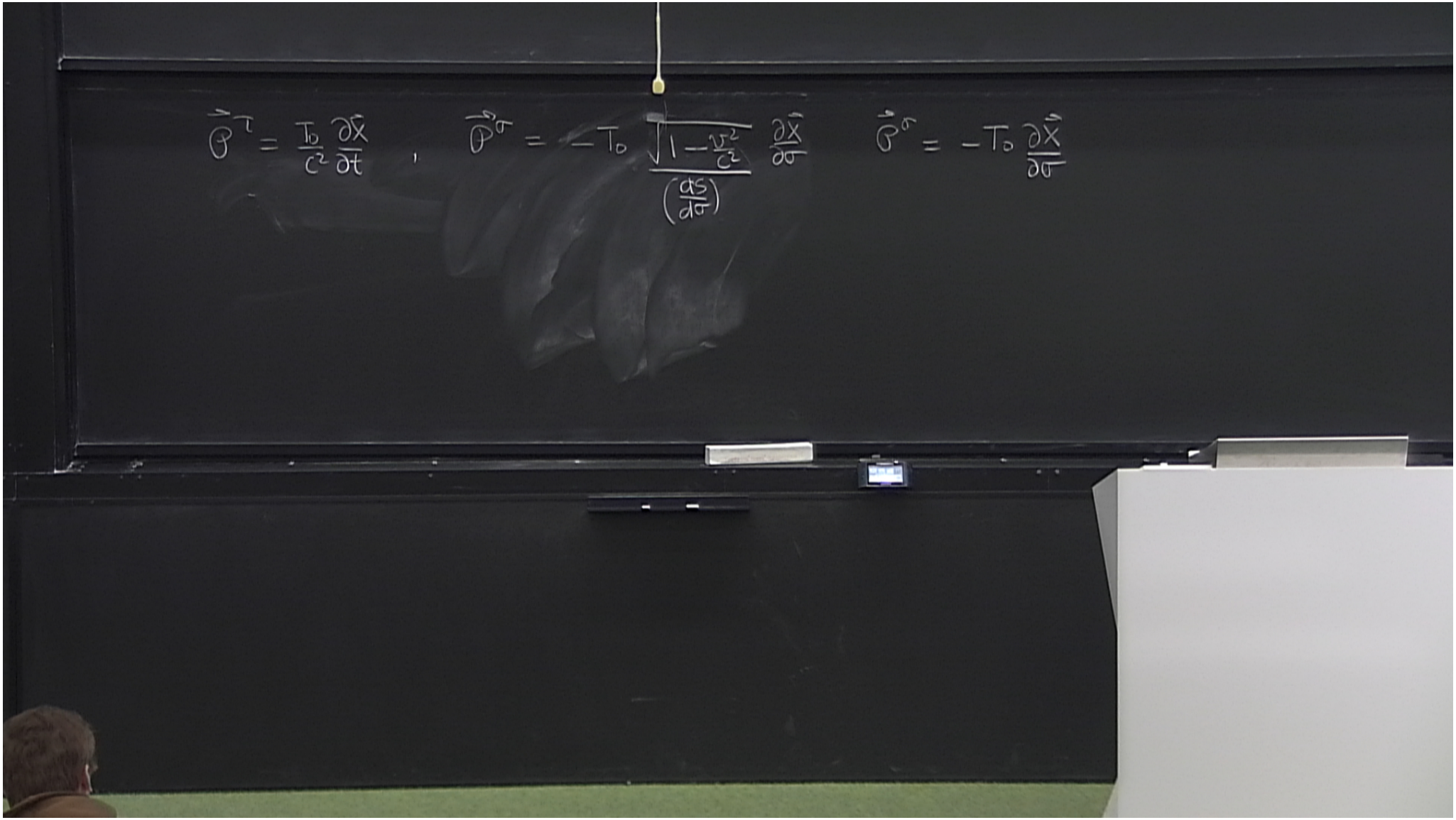

Claim:
The energy δE in the fixed infinitesimal $\delta\sigma$ piece is constant (in time)
Let δs denote the length of this piece (time-slice)
σ
$\sigma+\delta\sigma$
$\delta E =$
Claim: this is constant
Set
$\frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$d\sigma = \frac{ds}{\sqrt{1-\frac{v^2}{c^2}}}$
$\mu=0$
$\frac{\partial}{\partial t}\left(\frac{T_0}{c}\frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\right)=0$

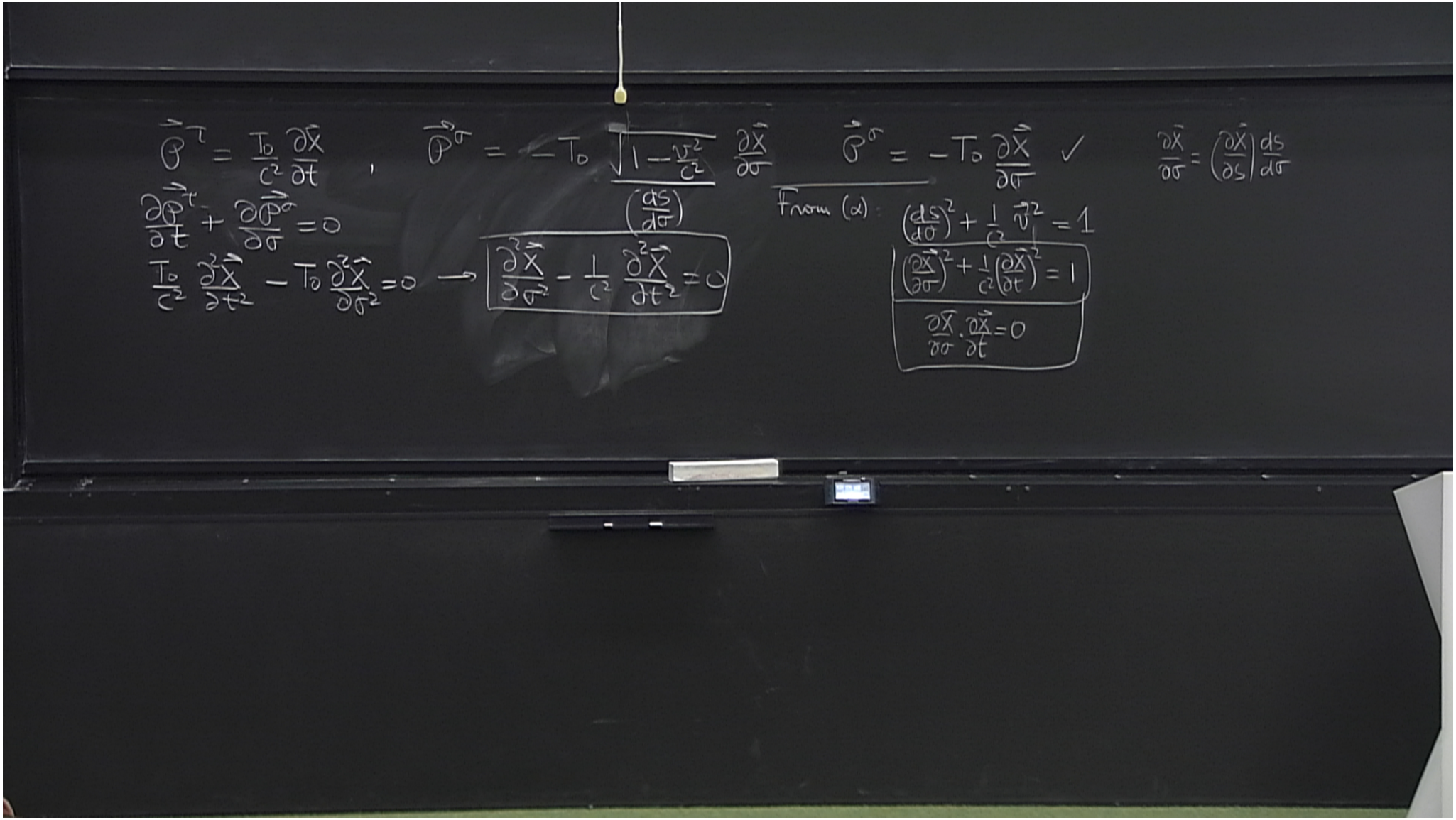

Claim:
σ σ
The energy δE in the fixed infinitesimal δσ piece is constant (in time)
Let δs denote the length of this piece (time-slice)
$\delta E = \frac{T_0 \delta s}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{T_0 \frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\right)\delta\sigma$
Claim: this is constant
Set
$\frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$\frac{ds}{\sqrt{\quad}} = \frac{1}{\sqrt{\quad}} T_0 ds$
$\frac{\partial \mathcal{P}^{\tau\mu}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma\mu}}{\partial \sigma} = 0$
$\mu = 0$
$\frac{\partial \mathcal{P}^{\tau 0}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma 0}}{\partial \sigma} = 0$
$\frac{\partial}{\partial \tau}\left(\frac{T_0 \frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = 0$

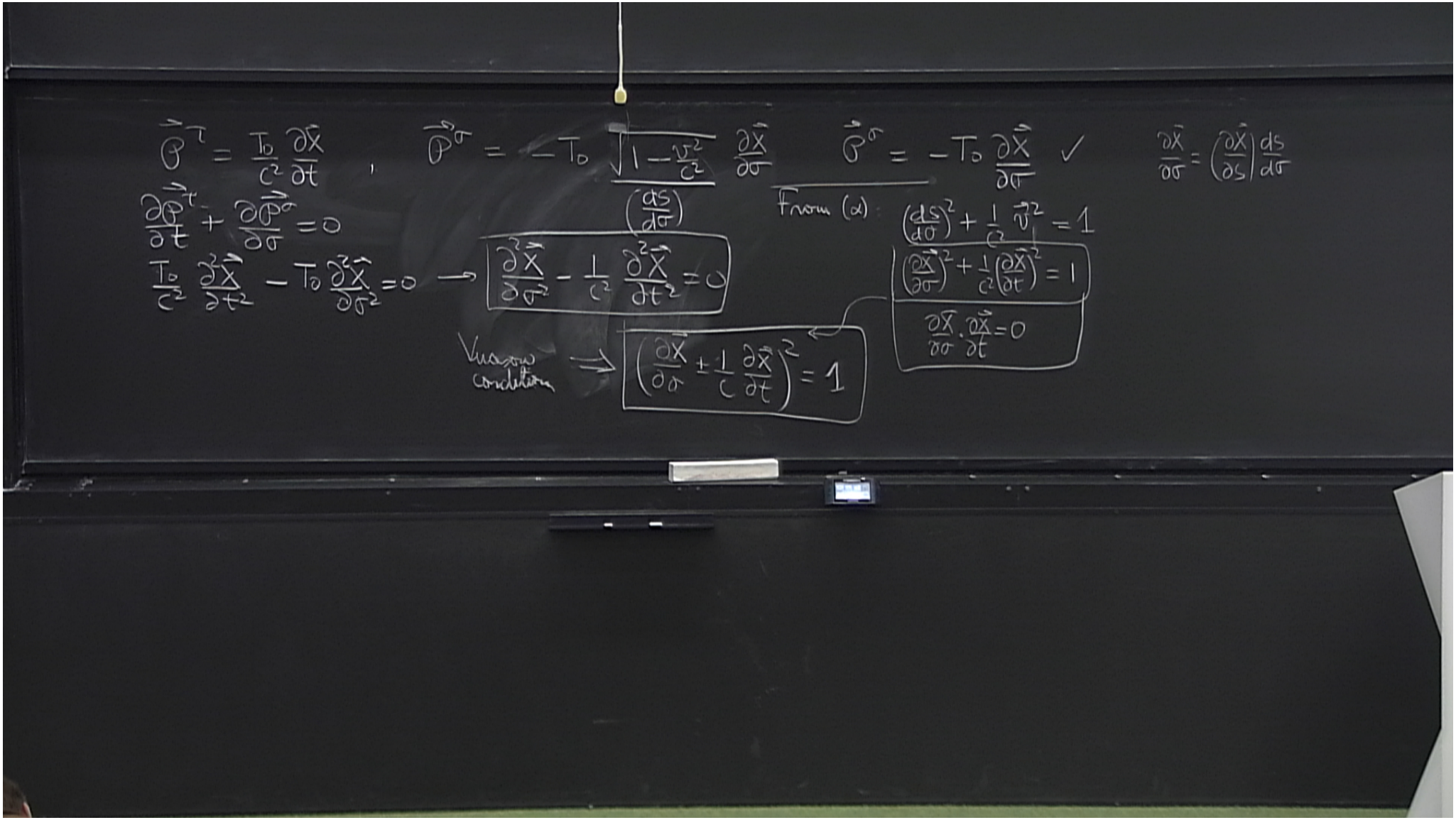

Claim:
The energy δE in the fixed infinitesimal $\delta\sigma$ piece is constant (in time)
Let δs denote the length of this piece (time-slice)
σ
$\sigma+\delta\sigma$
$\delta E = \frac{T_0\,\delta s}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{T_0\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\right)\delta\sigma$
$d\sigma = \frac{ds}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{T_0}\left(\frac{T_0\,ds}{\sqrt{\cdots}}\right)$
$d\sigma = \frac{dE}{T_0}$
Claim: this is constant
$\mu=0$
$\frac{\partial}{\partial t}\left(\frac{T_0}{c}\frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}}\right)=0$

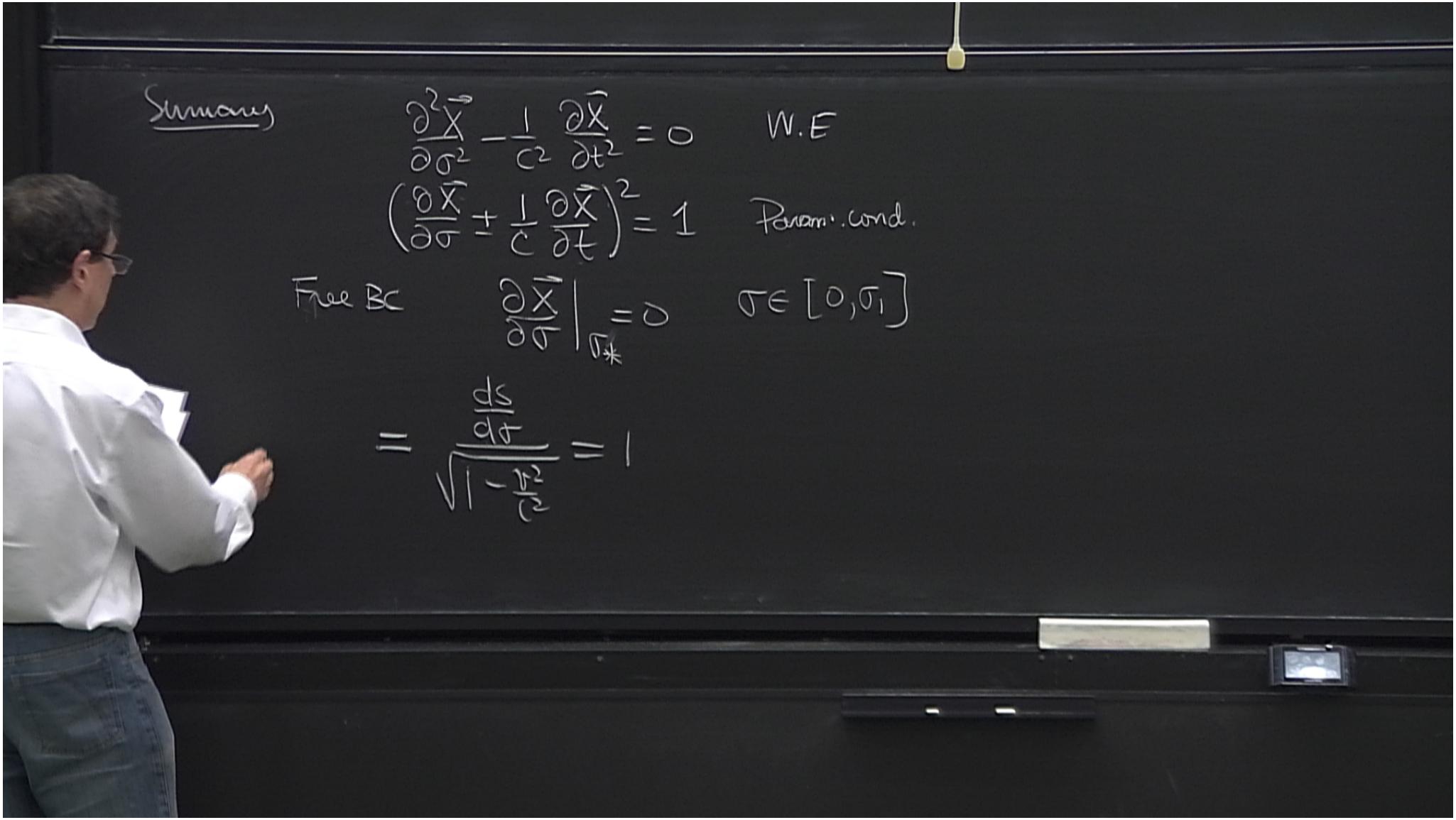

be ⊥ to the strings
$\frac{\partial \vec{x}}{\partial \sigma} \cdot \frac{\partial \vec{x}}{\partial \tau} = 0$
along the string
along σ - const
$\vec{v}_\perp = \frac{\partial \vec{x}}{\partial \tau}$
$\mathcal{P}^{\tau 0} = \frac{T_0}{c} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
$\vec{\mathcal{P}}^{\tau} = \frac{T_0}{c^2} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} \left(\frac{\partial \vec{x}}{\partial \tau}\right)$
$\sigma = \sigma_1$
Claim:
σ $\sigma + \delta\sigma$
The energy δE in the fixed infinitesimal $\delta\sigma$ [illegible] is constant (in time)
Let δs denote the length of th[illegible]
$\delta E = \frac{T_0 \delta s}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} = \left(\frac{T_0 \frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}\right)$
Claim
$\mu = 0$
$\frac{\partial \mathcal{P}^{\tau\mu}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma\mu}}{\partial \sigma} = 0$
$\frac{\partial \mathcal{P}^{\tau 0}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma 0}}{\partial \sigma} = 0$
$\frac{\partial}{\partial t}\left(\frac{T_0}{c} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}\right) = 0$
Set $\frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} = 1$
$d\sigma = \frac{ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} = \frac{1}{T_0} \left(T_0 ds \sqrt{\cdots}\right)$
$d\sigma = \frac{dE}{T_0}$


be ⊥ to the strings
$\frac{\partial \vec{x}}{\partial \sigma} \cdot \frac{\partial \vec{x}}{\partial \tau} = 0$
along σ - const
along the string
$\vec{v}_\perp = \frac{\partial \vec{x}}{\partial \tau}$
$\mathcal{P}^{\tau 0} = \frac{T_0}{c} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}$
$\vec{\mathcal{P}}^{\tau} = \frac{T_0}{c^2} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} \left(\frac{\partial \vec{x}}{\partial \tau}\right)$
$\sigma = \sigma_1$
Claim:
σ $\sigma + \delta\sigma$
The energy δE in a fixed infinitesimal $\delta\sigma$ piece is constant (in time)
Let δs denote the length of this piece (time-dep)
$\delta E = \left(\frac{T_0 \frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}\right) \delta\sigma$
Claim: this is constant
$\mu = 0$
$\frac{\partial \mathcal{P}^{\tau\tau}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma\tau}}{\partial \sigma} = 0$
$\frac{\partial \mathcal{P}^{\tau 0}}{\partial \tau} + \frac{\partial \mathcal{P}^{\sigma 0}}{\partial \sigma} = 0$
$\frac{\partial}{\partial t}\left(\frac{T_0}{c} \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}}\right) = 0$
Set $\frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} = 1$
$d\sigma = \frac{ds}{\sqrt{1 - \frac{v_\perp^2}{c^2}}} = \frac{1}{T_0} \left(T_0 ds \sqrt{\cdots}\right)$
$d\sigma = \frac{dE}{T_0}$


$\vec{\theta}^{\tau} = \frac{T_0}{c^2}\frac{\partial \vec{x}}{\partial t}$, $\vec{\theta}^{\sigma} = -T_0 \frac{\sqrt{1-\frac{v^2}{c^2}}}{\left(\frac{ds}{d\sigma}\right)}\frac{\partial \vec{x}}{\partial \sigma}$ $\vec{\theta}^{\sigma} = -T_0 \frac{\partial \vec{x}}{\partial \sigma}$


$\vec{P}^\tau = \frac{T_0}{c^2}\frac{\partial \vec{x}}{\partial t}$, $\vec{P}^\sigma = -T_0 \frac{\sqrt{1-\frac{v^2}{c^2}}}{\left(\frac{ds}{d\sigma}\right)}\frac{\partial \vec{x}}{\partial \sigma}$
$\vec{P}^\sigma = -T_0 \frac{\partial \vec{x}}{\partial \sigma}$ ✓
$\frac{\partial \vec{x}}{\partial \sigma} = \left(\frac{\partial \vec{x}}{\partial s}\right)\frac{ds}{d\sigma}$
From (α)
$\frac{\partial \vec{P}^\tau}{\partial t} + \frac{\partial \vec{P}^\sigma}{\partial \sigma} = 0$
$\frac{T_0}{c^2}\frac{\partial^2 \vec{x}}{\partial t^2} - T_0 \frac{\partial^2 \vec{x}}{\partial \sigma^2} = 0 \rightarrow \boxed{\frac{\partial^2 \vec{x}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{x}}{\partial t^2} = 0}$
$\left(\frac{ds}{d\sigma}\right)^2 + \frac{1}{c^2}\vec{v}^2 = 1$
$\boxed{\left(\frac{\partial \vec{x}}{\partial \sigma}\right)^2 + \frac{1}{c^2}\left(\frac{\partial \vec{x}}{\partial t}\right)^2 = 1}$
$\boxed{\frac{\partial \vec{x}}{\partial \sigma}\cdot\frac{\partial \vec{x}}{\partial t} = 0}$


$\vec{\Theta}^\tau = \frac{T_0}{c^2}\frac{\partial \vec{x}}{\partial t}$
$\vec{\Theta}^\sigma = -T_0 \frac{\sqrt{1-\frac{v^2}{c^2}}}{\left(\frac{ds}{d\sigma}\right)} \frac{\partial \vec{x}}{\partial \sigma}$
From (α)
$\vec{\Theta}^\sigma = -T_0 \frac{\partial \vec{x}}{\partial \sigma}$ ✓
$\frac{\partial \vec{x}}{\partial \sigma} = \left(\frac{\partial \vec{x}}{\partial s}\right)\frac{ds}{d\sigma}$
$\frac{\partial \vec{\Theta}^\tau}{\partial t} + \frac{\partial \vec{\Theta}^\sigma}{\partial \sigma} = 0$
$\frac{T_0}{c^2}\frac{\partial^2 \vec{x}}{\partial t^2} - T_0 \frac{\partial^2 \vec{x}}{\partial \sigma^2} = 0 \rightarrow \boxed{\frac{\partial^2 \vec{x}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{x}}{\partial t^2} = 0}$
$\left(\frac{ds}{d\sigma}\right)^2 + \frac{1}{c^2}\vec{v}^2 = 1$
$\left(\frac{\partial \vec{x}}{\partial \sigma}\right)^2 + \frac{1}{c^2}\left(\frac{\partial \vec{x}}{\partial t}\right)^2 = 1$
$\frac{\partial \vec{x}}{\partial \sigma}\cdot\frac{\partial \vec{x}}{\partial t} = 0$
Vincolo di conduttore
$\Rightarrow \boxed{\left(\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t}\right)^2 = 1}$


Summary
$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial \vec{X}}{\partial t^2} = 0$ W.E
$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1$ Param. cond.
Free BC $\left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0$ $\sigma \in [0, \sigma_1]$
$= \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{\dot{r}^2}{c^2}}} = 1$

Summary

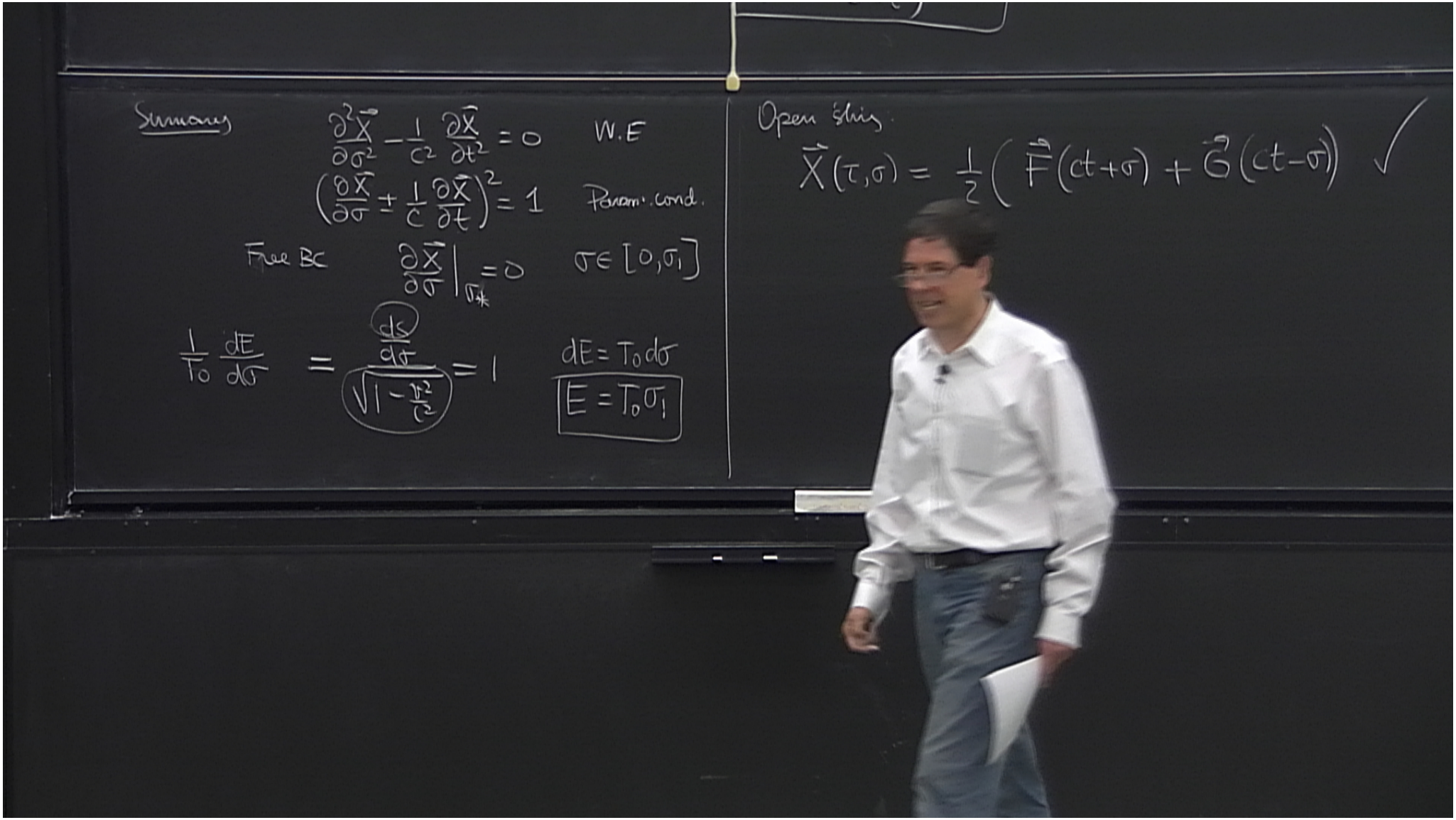
$$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial \vec{X}}{\partial t^2} = 0 \qquad \text{W.E}$$

$$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1 \qquad \text{Param. cond.}$$

Free BC $\qquad \left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0 \qquad \sigma \in [0, \sigma_1]$

$$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{v^2}{c^2}}} = 1 \qquad dE = T_0 \, d\sigma$$

$$\boxed{E = T_0 \sigma_1}$$


Summary
$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2} = 0$ W.E
$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1$ Param. cond.
Free BC $\left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0$ $\sigma \in [0, \sigma_1]$
$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$dE = T_0 d\sigma$
$E = T_0 \sigma_1$
Open string
$\vec{X}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{G}(ct-\sigma)\right)$ ✓

Summary

$$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2} = 0 \qquad \text{W.E}$$

$$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1 \qquad \text{Param. cond.}$$

Free BC $\qquad \left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0 \qquad \sigma \in [0, \sigma_1]$

$$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{\dot{r}^2}{c^2}}} = 1 \qquad dE = T_0 d\sigma$$

$$\boxed{E = T_0 \sigma_1}$$

Open string

$$\vec{X}(\tau, \sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{G}(ct-\sigma)\right) \checkmark$$

$$\frac{\partial \vec{X}}{\partial \sigma}(\tau, \sigma) = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{G}'(ct-\sigma)\right)$$

$$\frac{\partial \vec{X}}{\partial \sigma}(\tau, 0) = \frac{1}{2}\left(\vec{F}'(ct) - \vec{G}'(ct)\right) = 0$$

Summary

$$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2} = 0 \quad \text{W.E}$$

$$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1 \quad \text{Param. cond.}$$

Free BC $\quad \left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0 \quad \sigma \in [0, \sigma_1]$

$$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1 - \frac{\dot{r}^2}{c^2}}} = 1 \qquad dE = T_0 d\sigma$$

$$\boxed{E = T_0 \sigma_1}$$

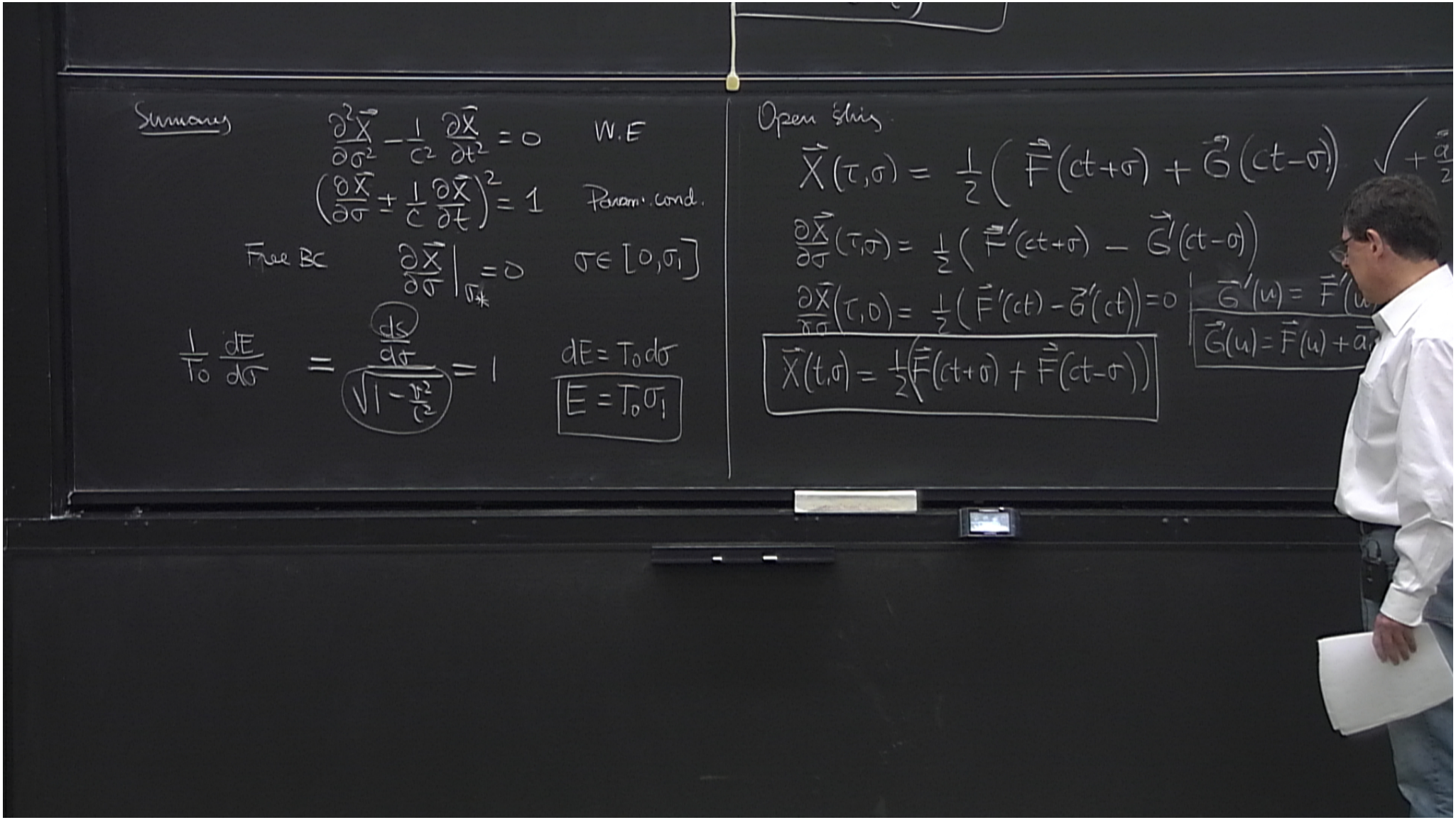
Open string

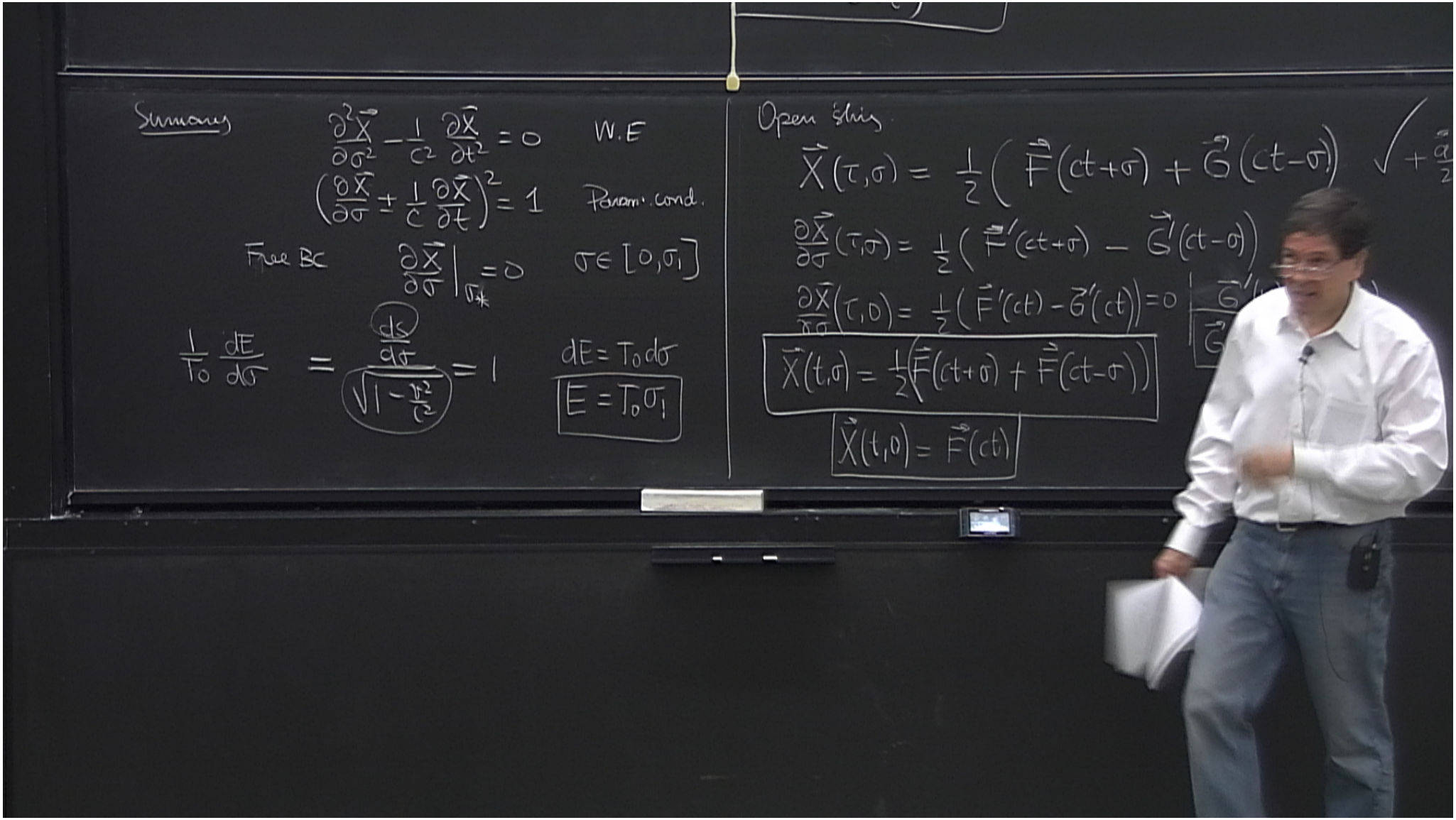
$$\vec{X}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}(c t + \sigma) + \vec{G}(c t - \sigma)\right) \checkmark$$

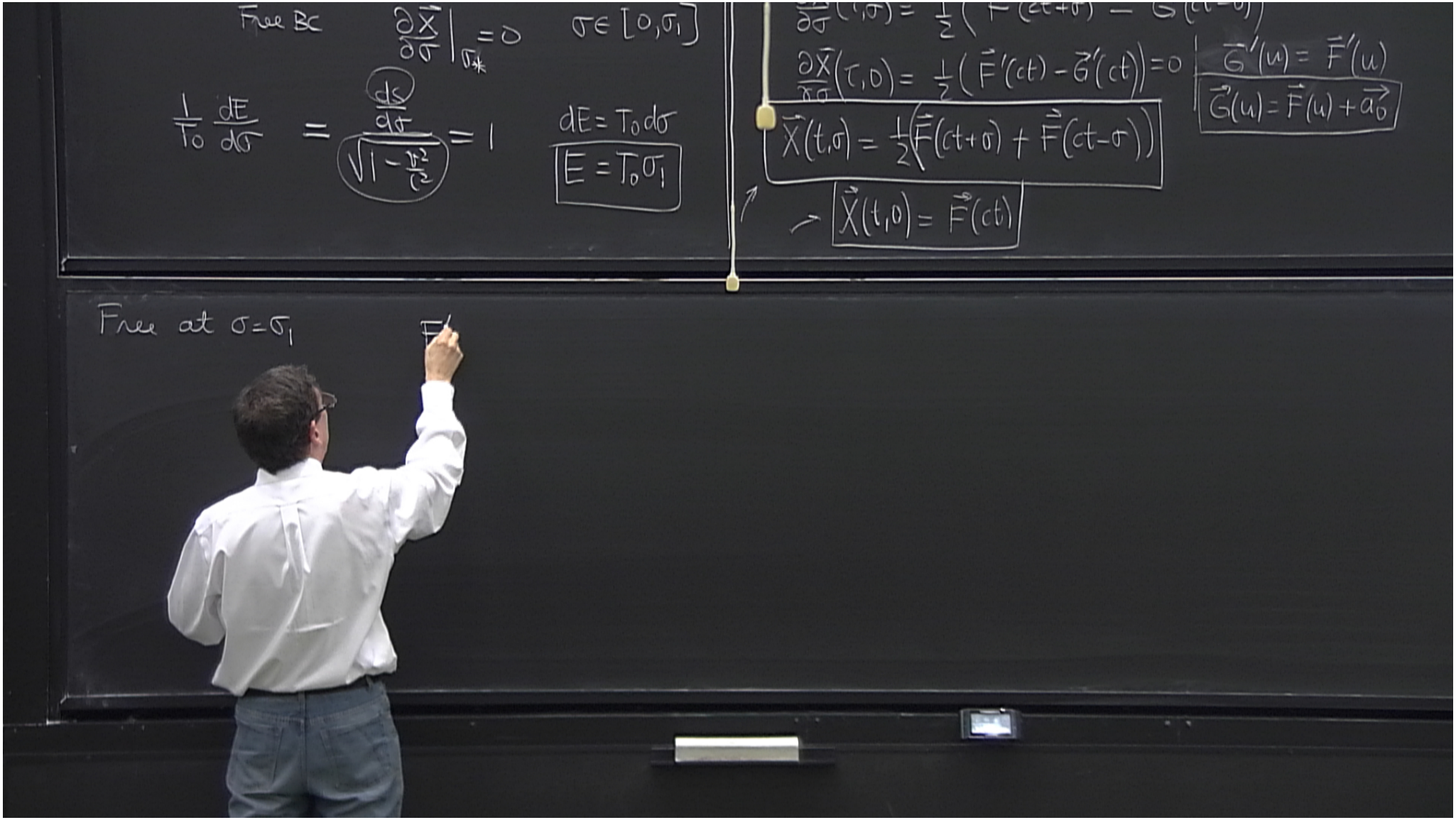
$$\frac{\partial \vec{X}}{\partial \sigma}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}'(c t + \sigma) - \vec{G}'(c t - \sigma)\right)$$

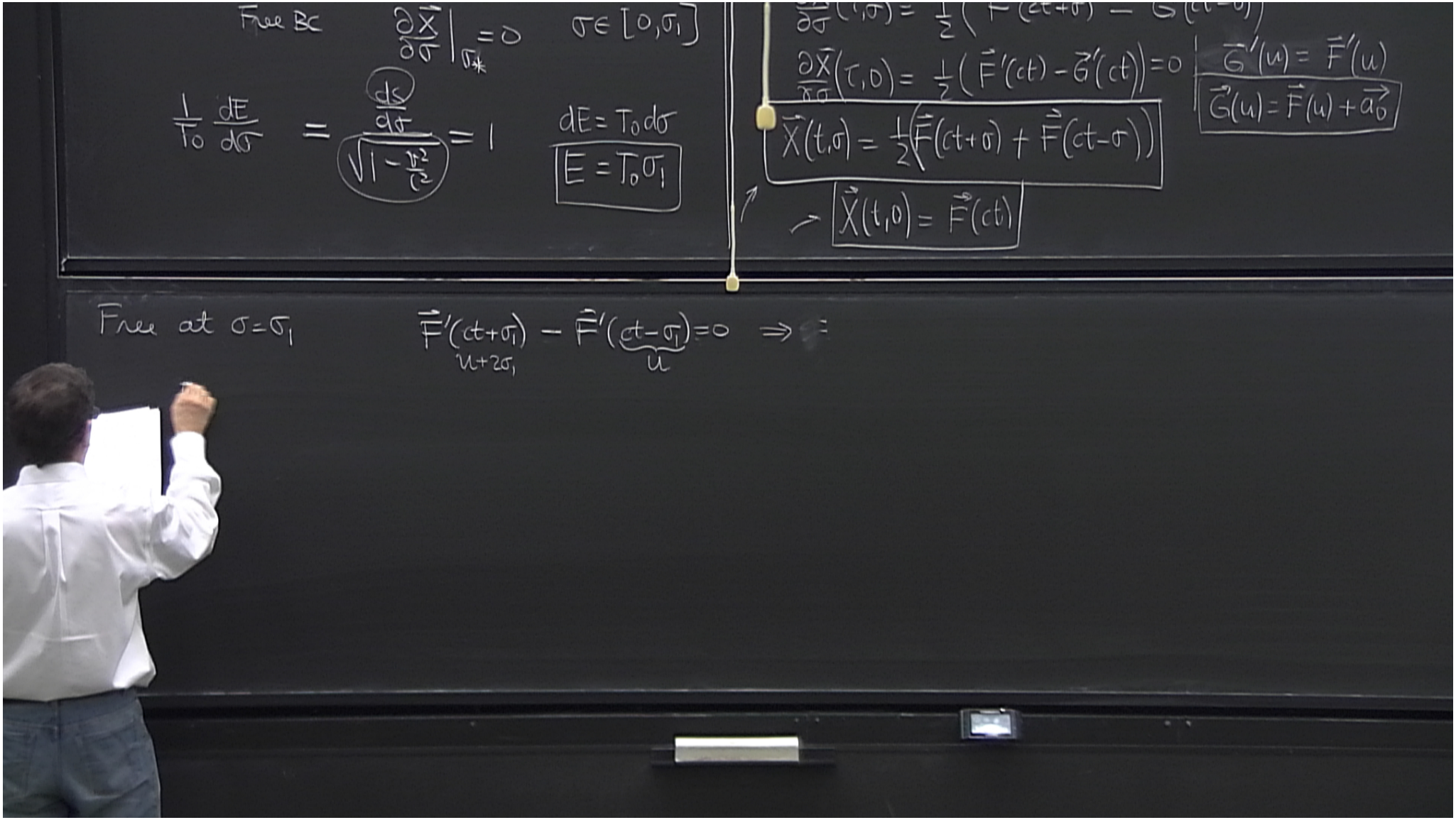
$$\frac{\partial \vec{X}}{\partial \sigma}(\tau,0) = \frac{1}{2}\left(\vec{F}'(c t) - \vec{G}'(c t)\right) = 0 \quad \vec{G}'(u) = \vec{F}'(u)$$

$$\boxed{\vec{G}(u) = \vec{F}(u) + \vec{a}_0}$$


Summary
$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2} = 0$ W.E
$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1$ Param. cond.
Free BC $\left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0$ $\sigma \in [0, \sigma_1]$
$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$ $dE = T_0 d\sigma$
$E = T_0 \sigma_1$
Open string
$\vec{X}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}(c\tau+\sigma) + \vec{G}(c\tau-\sigma)\right) + \frac{\vec{a}}{2}$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}'(c\tau+\sigma) - \vec{G}'(c\tau-\sigma)\right)$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau,0) = \frac{1}{2}\left(\vec{F}'(c\tau) - \vec{G}'(c\tau)\right) = 0$
$\vec{G}'(u) = \vec{F}'(u)$
$\vec{G}(u) = \vec{F}(u) + \vec{a}$
$\vec{X}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}(c\tau+\sigma) + \vec{F}(c\tau-\sigma)\right)$


Summary
$\frac{\partial^2 \vec{X}}{\partial \sigma^2} - \frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2} = 0$ W.E
$\left(\frac{\partial \vec{X}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{X}}{\partial t}\right)^2 = 1$ Param. cond.
Free BC $\left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0$ $\sigma \in [0, \sigma_1]$
$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$dE = T_0 d\sigma$
$E = T_0 \sigma_1$
Open string
$\vec{X}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{G}(ct-\sigma)\right)$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{G}'(ct-\sigma)\right)$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau,0) = \frac{1}{2}\left(\vec{F}'(ct) - \vec{G}'(ct)\right) = 0$
$\vec{X}(t,\sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{F}(ct-\sigma)\right)$
$\vec{X}(t,0) = \vec{F}(ct)$


Free BC
$\frac{\partial \vec{X}}{\partial \sigma}\Big|_{\sigma_*} = 0$
$\sigma \in [0, \sigma_1]$
$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$dE = T_0 d\sigma$
$E = T_0 \sigma_1$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau, 0) = \frac{1}{2}(\vec{F}'(c\tau) - \vec{G}'(c\tau)) = 0$
$\vec{G}'(u) = \vec{F}'(u)$
$\vec{G}(u) = \vec{F}(u) + \vec{a}_0$
$\vec{X}(t,\sigma) = \frac{1}{2}(\vec{F}(ct+\sigma) + \vec{F}(ct-\sigma))$
$\vec{X}(t,0) = \vec{F}(ct)$
Free at $\sigma = \sigma_1$


Free BC
$\left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0$
$\sigma \in [0, \sigma_1]$
$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$dE = T_0 d\sigma$
$E = T_0 \sigma_1$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau, 0) = \frac{1}{2}\left(\vec{F}'(c\tau) - \vec{G}'(c\tau)\right) = 0$
$\vec{G}'(u) = \vec{F}'(u)$
$\vec{G}(u) = \vec{F}(u) + \vec{a}_0$
$\vec{X}(t,\sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{F}(ct-\sigma)\right)$
$\vec{X}(t,0) = \vec{F}(ct)$
Free at $\sigma = \sigma_1$
$\vec{F}'(ct+\sigma_1) - \vec{F}'(ct-\sigma_1) = 0 \Rightarrow$
$u+2\sigma_1$
u

Free BC $\left.\frac{\partial \vec{X}}{\partial \sigma}\right|_{\sigma_*} = 0$ $\sigma \in [0, \sigma_1]$

$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{\frac{ds}{d\sigma}}{\sqrt{1-\frac{v^2}{c^2}}} = 1$ $dE = T_0 d\sigma$

$\boxed{E = T_0 \sigma_1}$

$\frac{\partial \vec{X}}{\partial \sigma}(\tau,\sigma) = \frac{1}{2}\left(\vec{F}'(\sigma+\tau) - \vec{G}'(\sigma-\tau)\right)$

$\frac{\partial \vec{X}}{\partial \sigma}(\tau,0) = \frac{1}{2}\left(\vec{F}'(c\tau) - \vec{G}'(c\tau)\right) = 0$ $\boxed{\vec{G}'(u) = \vec{F}'(u)}$

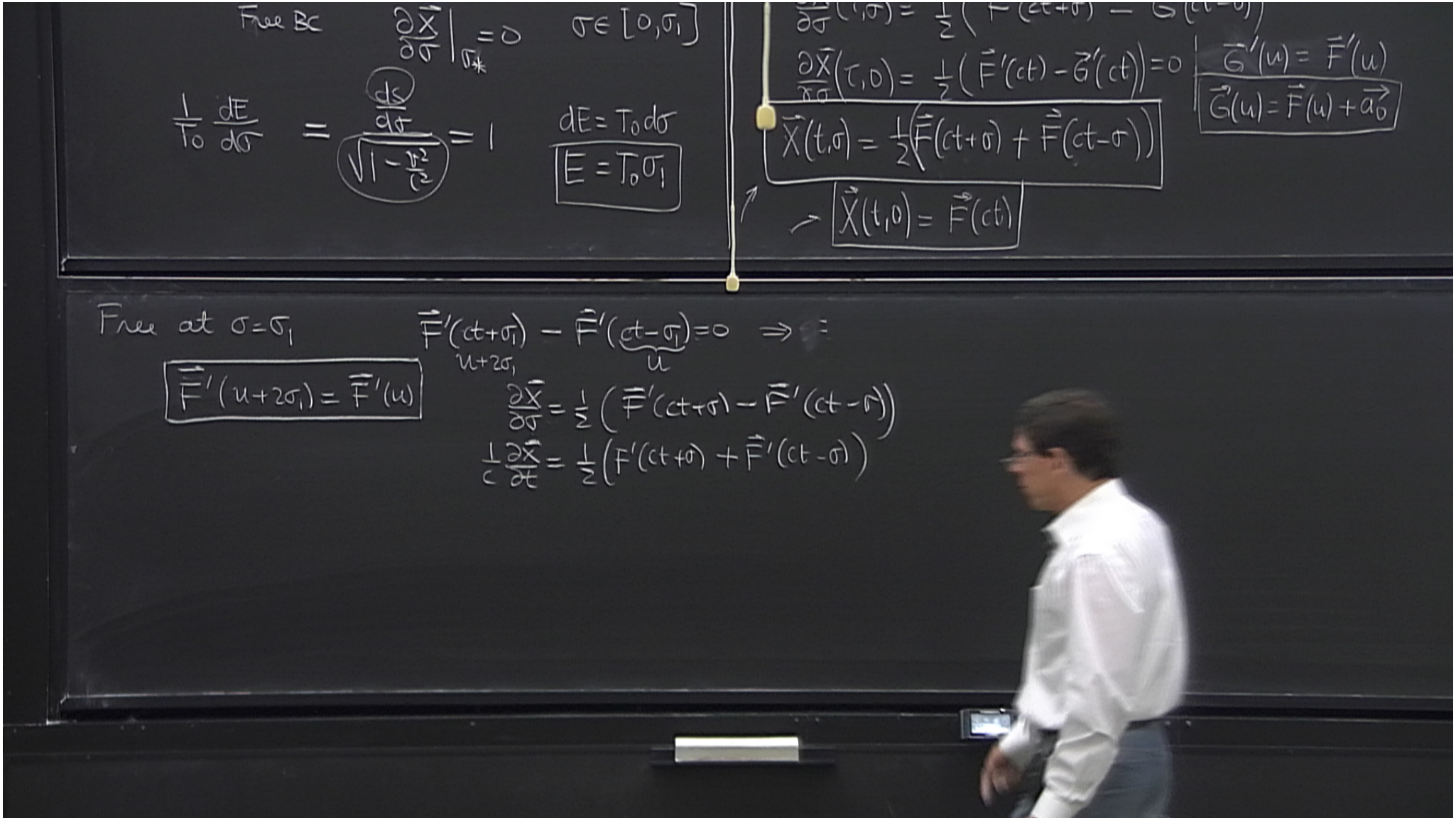
$\boxed{\vec{G}(u) = \vec{F}(u) + \vec{a}_0}$

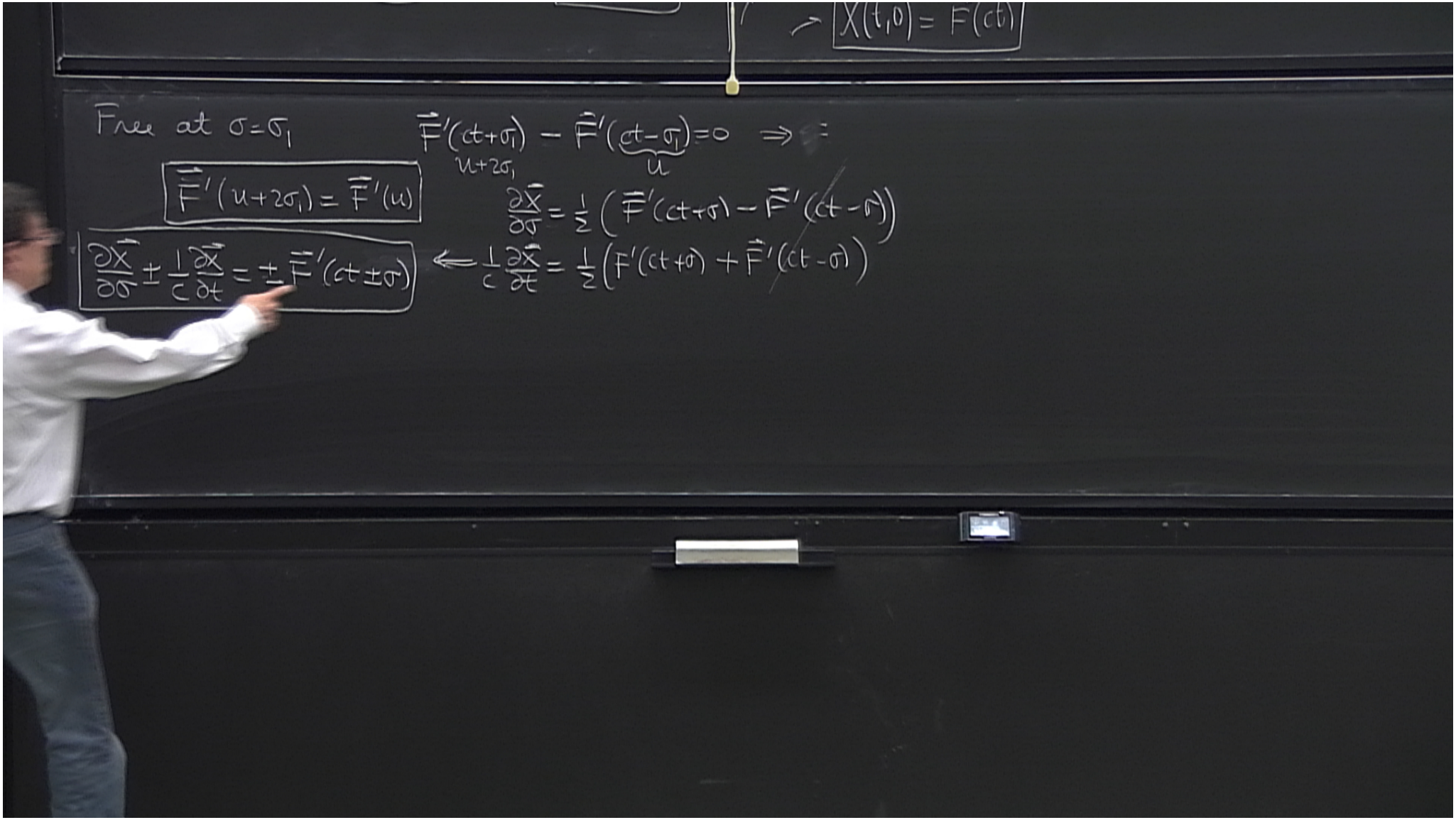
$\boxed{\vec{X}(t,\sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{F}(ct-\sigma)\right)}$

$\boxed{\vec{X}(t,0) = \vec{F}(ct)}$

Free at $\sigma = \sigma_1$ $\vec{F}'(\underbrace{ct+\sigma_1}_{u+2\sigma_1}) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$

$\boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$


Free BC
$\frac{\partial \vec{X}}{\partial \sigma}\Big|_{\sigma_*} = 0$
$\sigma \in [0, \sigma_1]$
$\frac{1}{T_0}\frac{dE}{d\sigma} = \frac{ds}{d\sigma} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1$
$dE = T_0 d\sigma$
$E = T_0 \sigma_1$
$\frac{\partial \vec{X}}{\partial \sigma}(\tau, 0) = \frac{1}{2}\left(\vec{F}'(c\tau) - \vec{G}'(c\tau)\right) = 0$
$\vec{G}'(u) = \vec{F}'(u)$
$\vec{G}(u) = \vec{F}(u) + \vec{a}_0$
$\vec{X}(t,\sigma) = \frac{1}{2}\left(\vec{F}(ct+\sigma) + \vec{F}(ct-\sigma)\right)$
$\vec{X}(t,0) = \vec{F}(ct)$
Free at $\sigma = \sigma_1$
$\vec{F}'(ct+\sigma_1) - \vec{F}'(ct-\sigma_1) = 0 \Rightarrow$
$u+2\sigma_1$
u
$\vec{F}'(u+2\sigma_1) = \vec{F}'(u)$
$\frac{\partial \vec{X}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$
$\frac{1}{c}\frac{\partial \vec{X}}{\partial t} = \frac{1}{2}\left(F'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$


$X(t,0) = F(ct)$
Free at $\sigma=\sigma_1$
$\vec{F}'(ct+\sigma_1) - \vec{F}'(ct-\sigma_1) = 0 \Rightarrow$
$u+2\sigma_1$ u
$\vec{F}'(u+2\sigma_1) = \vec{F}'(u)$
$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$
$\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \pm \vec{F}'(ct \pm \sigma)$
$\Leftarrow \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \frac{1}{2}\left(F'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$

$\rightarrow \boxed{\vec{X}(t,0) = \vec{F}(ct)}$

Free at $\sigma = \sigma_1$ $\quad \vec{F}'(\underbrace{ct+\sigma_1}_{u+2\sigma_1}) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$

$\boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$

$$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$$

$$\boxed{\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \pm \vec{F}'(ct \pm \sigma)} \Leftarrow \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$$

$\boxed{\vec{F}'(u) \text{ is a unit vector}}$

$\rightarrow \boxed{X(t,0) = F(ct)}$ Soln

Free at $\sigma = \sigma_1$ $\quad \vec{F}'(ct+\sigma_1) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$ $\quad u+2\sigma_1$

$* \boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$

$$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$$

$$\boxed{\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \pm \vec{F}'(ct \pm \sigma)} \Leftarrow \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \frac{1}{2}\left(F'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$$

$\boxed{\vec{F}'(u) \text{ is a unit vector}}$ $*$

$\rightarrow \boxed{X(t,0) = F(ct)}$ Soln

Free at $\sigma = \sigma_1$ $\quad \vec{F}'(ct+\sigma_1) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$ $\quad u+2\sigma_1$

$* \boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$

$$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$$

$$\boxed{\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \pm \vec{F}'(ct \pm \sigma)} \Leftarrow \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \frac{1}{2}\left(F'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$$

Circular string

$\boxed{\vec{F}'(u) \text{ is a unit vector}}$ $*$

$\rightarrow \boxed{X(t,0) = F(ct)}$ Soln

Free at $\sigma = \sigma_1$ $\quad \vec{F}'(ct+\sigma_1) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$ $\quad \vec{F}'(\underbrace{u+2\sigma_1}_{u+2\sigma_1}) = \vec{F}'(u)$

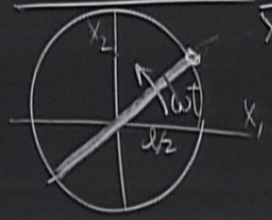
$$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$$

$$\frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \frac{1}{2}\left(F'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$$

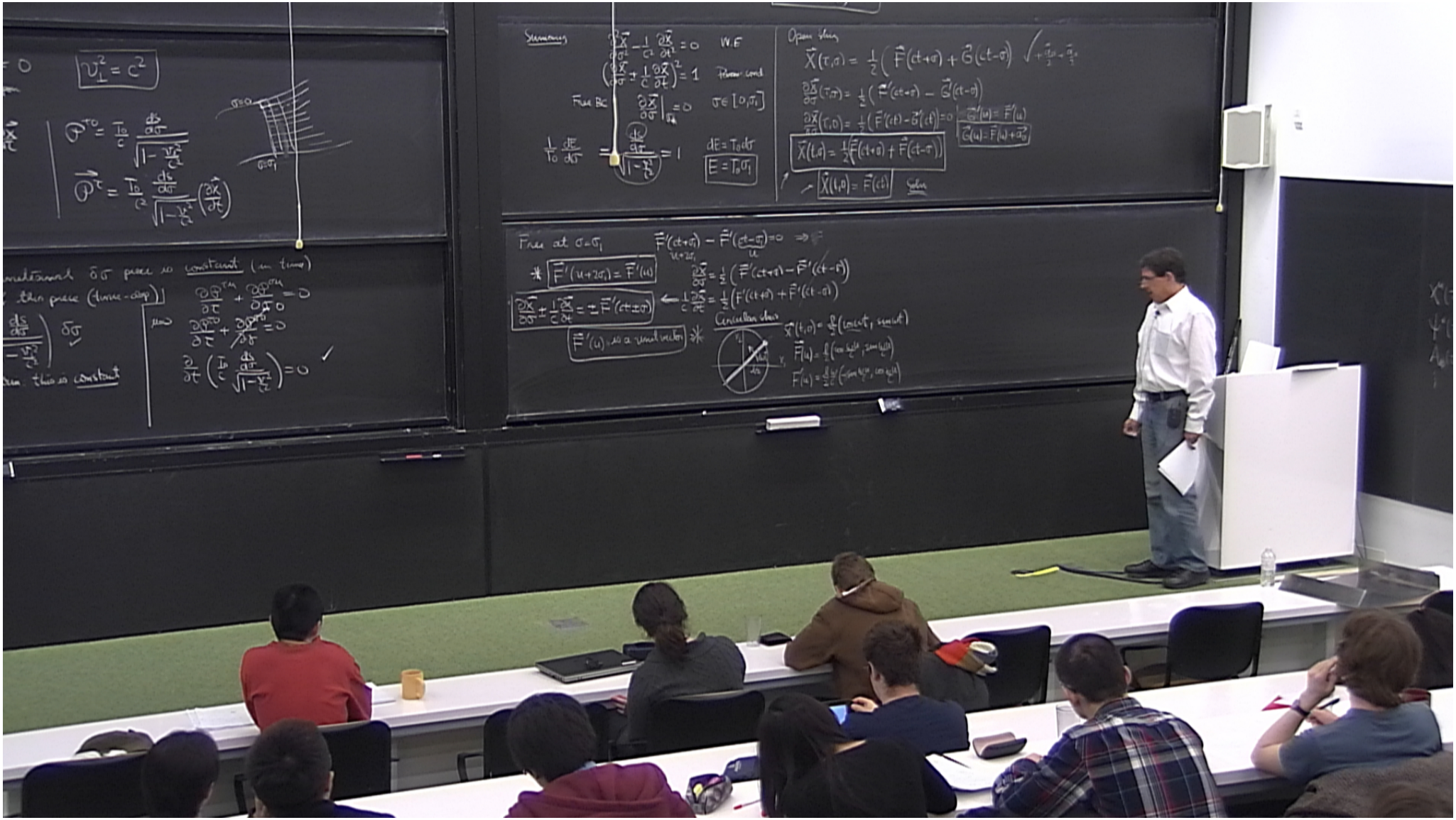
$$\boxed{\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \pm \vec{F}'(ct \pm \sigma)}$$

$\boxed{\vec{F}'(u) \text{ is a unit vector}}$ $\quad \ast\ast$

Circular string



$$\vec{X}(t,0) = \frac{\ell}{2}(\cos\omega t, \sin\omega t)$$


Summary
W.E
Free BC
dE = T₀dσ
E = T₀σ₁
Open string
Free at σ=σ₁
Circular string

$\Rightarrow X(t,0) = F(ct)$ Soln

Free at $\sigma = \sigma_1$ $\quad \vec{F}'(ct+\sigma_1) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$ $\quad u+2\sigma_1$

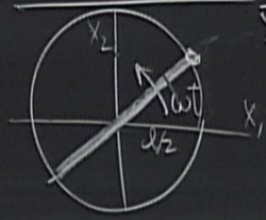
$* \boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$

$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$

$\boxed{\frac{\partial \vec{x}}{\partial \sigma} \pm \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \pm \vec{F}'(ct \pm \sigma)} \Leftarrow \frac{1}{c}\frac{\partial \vec{x}}{\partial t} = \frac{1}{2}\left(F'(ct+\sigma) + \vec{F}'(ct-\sigma)\right)$

$\boxed{\vec{F}'(u) \text{ is a unit vector}} *$

Circular string



$\vec{X}(t,0) = \frac{\ell}{2}(\cos\omega t, \sin\omega t)$

$\vec{F}(u) = \frac{\ell}{2}\left(\cos\frac{\omega u}{c}, \sin\frac{\omega u}{c}\right)$

$\vec{F}'(u) = \frac{\ell}{2}\frac{\omega}{c}\left(-\sin\frac{\omega u}{c}, \cos\frac{\omega u}{c}\right)$

$\Rightarrow X(t,0) = F(ct)$ Soln

Free at $\sigma = \sigma_1$ $\quad \vec{F}'(ct+\sigma_1) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$ $\quad \underbrace{ct+\sigma_1}_{u+2\sigma_1}$

$\frac{\omega}{c}\frac{\ell}{2} = 1$

$\omega\left(\frac{\ell}{2}\right) = c$ ✓

$* \boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$

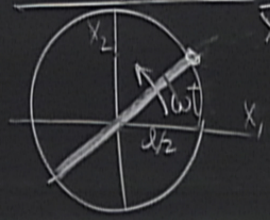
$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$

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"ℓ"

$\boxed{\vec{F}'(u) \text{ is a unit vector}} *$

Rotating open string



$\vec{X}(t,0) = \frac{\ell}{2}(\cos\omega t, \sin\omega t)$

$\vec{F}(u) = \frac{\ell}{2}\left(\cos\frac{\omega u}{c}, \sin\frac{\omega u}{c}\right)$

$\vec{F}'(u) = \frac{\ell}{2}\frac{\omega}{c}\left(-\sin\frac{\omega u}{c}, \cos\frac{\omega u}{c}\right)$

$\Rightarrow X(t,0) = F(ct)$ Soln

Free at $\sigma=\sigma_1$ $\quad \vec{F}'(ct+\sigma_1) - \vec{F}'(\underbrace{ct-\sigma_1}_{u}) = 0 \Rightarrow$ $\quad (u+2\sigma_1)$

$\frac{\omega}{c}\frac{\ell}{2} = 1$

$\omega\left(\frac{\ell}{2}\right) = c$ ✓

$**$

$\frac{\omega 2\sigma_1}{c} = 2\pi$

$\frac{\omega}{c} = \frac{\pi}{\sigma_1}$

$** \quad \boxed{\vec{F}'(u+2\sigma_1) = \vec{F}'(u)}$

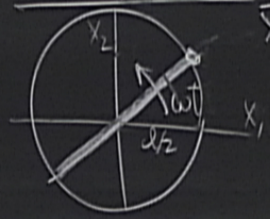
$\frac{\partial \vec{x}}{\partial \sigma} = \frac{1}{2}\left(\vec{F}'(ct+\sigma) - \vec{F}'(ct-\sigma)\right)$

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"ℓ"

$\vec{F}'(u)$ is a unit vector $*$

Rotating open string



$\vec{X}(t,0) = \frac{\ell}{2}(\cos\omega t, \sin\omega t)$

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