

Title: 12/13 PSI - String Theory Review Lecture 4

Date: Jan 10, 2013 10:15 AM

URL: <http://pirsa.org/13010046>

Abstract:

$$S = -\frac{I_0}{c} \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$\dot{X}^m = \frac{\partial X^m}{\partial \tau} \quad X'^m = \frac{\partial X^m}{\partial \sigma}$$

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$$[X] = L \quad [S] = [\hbar] = \frac{ML^2}{T}$$

$$\dot{X}^m = \frac{\partial X^m}{\partial \tau} \quad X'^m = \frac{\partial X^m}{\partial \sigma}$$

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$$[X] = L \quad [S] = [\hbar] = \frac{ML^2}{T}$$

σ, τ units drop out,

$$[S] = \frac{[I_0]}{[c]} L^2$$

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$$\dot{X}^m = \frac{\partial X^m}{\partial \tau} \quad X'^m = \frac{\partial X^m}{\partial \sigma}$$

$$[X] = L$$

$$[S] = [\hbar] = \frac{ML^2}{T} = \frac{M}{T} L^2$$

σ, τ units drop out,

$$[S] = \left(\frac{[T_0]}{[c]} \right) L^2$$

$$\frac{[T_0]}{[c]} = \frac{M \cancel{[L]}^2}{T} \frac{1}{\cancel{[L]}}$$

$$S = - \frac{I_0}{c} \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$\dot{X}^m = \frac{\partial X^m}{\partial \tau}$$

$$X'^m = \frac{\partial X^m}{\partial \sigma}$$

$$[X] = L$$

$$[S] = [\hbar] = \frac{ML^2}{T} = \frac{M}{T} L^2$$

σ, τ units drop out,

$$[S] = \left(\frac{[I_0]}{[c]} \right) L^2$$

$$= \frac{M}{T} L^2 \checkmark$$

$$\frac{[I_0]}{[c]} = \frac{M [\cancel{L^2}]}{T} \frac{1}{[\cancel{L}]} = \frac{M}{T} L$$

$$S = -\frac{T_0}{c} \int_{\tau_0}^{\tau_1} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$\dot{X}^\mu = \frac{\partial X^\mu}{\partial \tau} \quad X'^\mu = \frac{\partial X^\mu}{\partial \sigma}$$

$$[X] = L$$

$$T_0 = \frac{ML^2}{T} = \frac{M}{T} L^2$$

$$[S] = \left(\frac{[T_0]}{[c]} \right) L^2$$

$$= \frac{M}{T} L^2 \checkmark$$

σ, τ units drop out

$$-ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \underbrace{\eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^i} \frac{\partial X^\nu}{\partial \xi^j}}_{\delta_{ij}(\xi)} d\xi^i d\xi^j$$

$$S = -\frac{T_0}{c} \int d\tau \int d\sigma \sqrt{\text{induced metric}}$$

$$S = -\frac{T_0}{c} \int_{\tau_0}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$\dot{X}^\mu = \frac{\partial X^\mu}{\partial \tau} \quad X'^\mu = \frac{\partial X^\mu}{\partial \sigma}$$

$$[X] = L$$

$$[S] = [h] = \frac{ML^2}{T} = \frac{M}{T} L^2$$

σ, τ units drop out,

$$[S] = \left(\frac{[T_0]}{[c]} \right) L^2 = \frac{M}{T} L^2 \checkmark$$

$$\frac{[T_0]}{[c]} = \frac{M [\cancel{\sigma}]}{T} \frac{L}{[\cancel{\sigma}]} = \frac{M}{T} L$$

$$-ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \underbrace{\eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^i} \frac{\partial X^\nu}{\partial \xi^j}}_{\text{induced metric } \gamma_{ij}(\xi)} d\xi^i d\xi^j \quad \xi^i = (\tau, \sigma)$$

$$S = -\frac{T_0}{c} \int d\tau \int d\sigma \sqrt{\gamma_{12}^2 - \gamma_{11} \gamma_{22}} = -\frac{T_0}{c} \int d\tau d\sigma \sqrt{-\det \gamma}$$

$$S = -\frac{T_0}{c} \int_{\tau_0}^{\tau_1} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$[X] = L$$

$$[S] = [\hbar] = \frac{ML^2}{T} = M$$

σ, τ units drop out,

$$[S] = \left(\frac{[T_0]}{[c]} \right) L^2 = \frac{M}{T} L^2 \checkmark$$

$$\frac{[T_0]}{[c]} = \frac{M [\cancel{\sigma}]^2}{[T]} \frac{1}{[\cancel{\sigma}]} = \frac{M}{T}$$

$$\dot{X}^\mu = \frac{\partial X^\mu}{\partial \tau} \quad X'^\mu = \frac{\partial X^\mu}{\partial \sigma}$$

$$\int \sqrt{-g} d^4x$$

$$-ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \underbrace{\eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^i} \frac{\partial X^\nu}{\partial \xi^j}}_{\text{induced metric } \gamma_{ij}(\xi)} d\xi^i d\xi^j \quad \xi^i = (\tau, \sigma)$$

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$$S = -\frac{T_0}{c} \int_{\tau_0}^{\tau_1} d\tau \int_0^{\sigma_1} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$[X] = L$$

$$[S] = [\hbar] = \frac{ML^2}{T} = \frac{M}{T} L^2$$

σ, τ units drop out,

$$[S] = \left(\frac{[T_0]}{[c]} \right) L^2 = \frac{M}{T} L^2 \checkmark$$

$$\frac{[T_0]}{[c]} = \frac{M [X']}{T} \frac{L}{[X]}$$

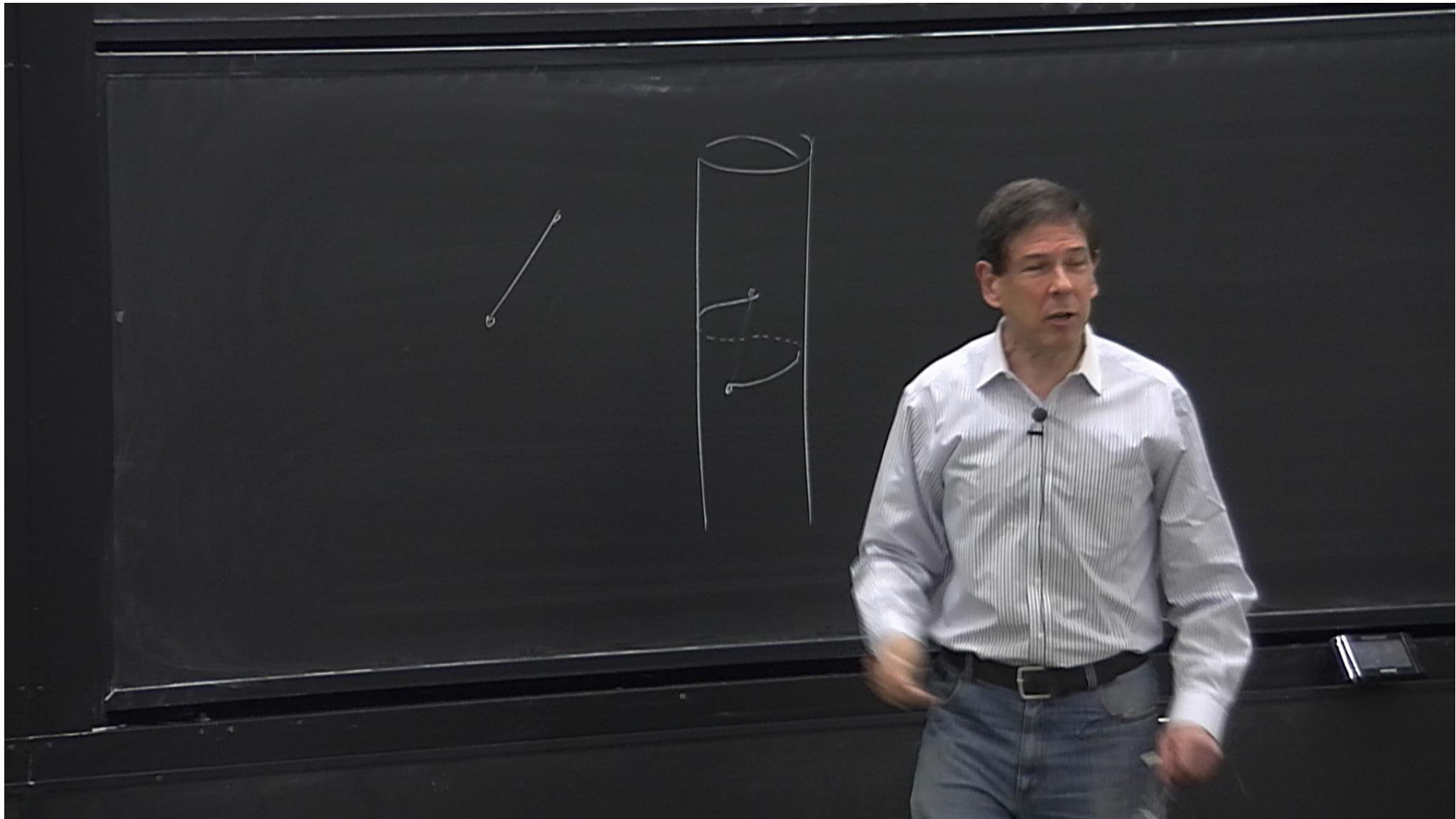
$$\dot{X}^m = \frac{\partial X^m}{\partial \tau}$$

$$X'^m = \frac{\partial X^m}{\partial \sigma}$$

$$\int \sqrt{-g} d^4 x$$

$$-ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \underbrace{\eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^i} \frac{\partial X^\nu}{\partial \xi^j}}_{\text{induced metric } \gamma_{ij}(\xi)} d\xi^i d\xi^j \quad \xi^i = (\tau, \sigma)$$

$$S = -\frac{T_0}{c} \int d\tau \int d\sigma \sqrt{\gamma_{12}^2 - \gamma_{11} \gamma_{22}} = -\frac{T_0}{c} \int d\tau d\sigma \sqrt{-\det \gamma}$$



σ, τ units drop out

$$[S] = \left(\frac{[T_0]}{[C]} \right)^2 = \frac{M [x^2]}{T} \frac{1}{[C]} = \frac{M}{T} L^2 \checkmark$$

induced metric

$$S = -\frac{T_0}{c} \int d\tau \int d\sigma \sqrt{\gamma_{12}^2 - \gamma_{11} \gamma_{22}} = -\frac{T_0}{c} \int d\tau d\sigma \sqrt{-\det \gamma}$$

$\gamma_{ij}(\xi) \quad \xi = (\tau, \sigma)$

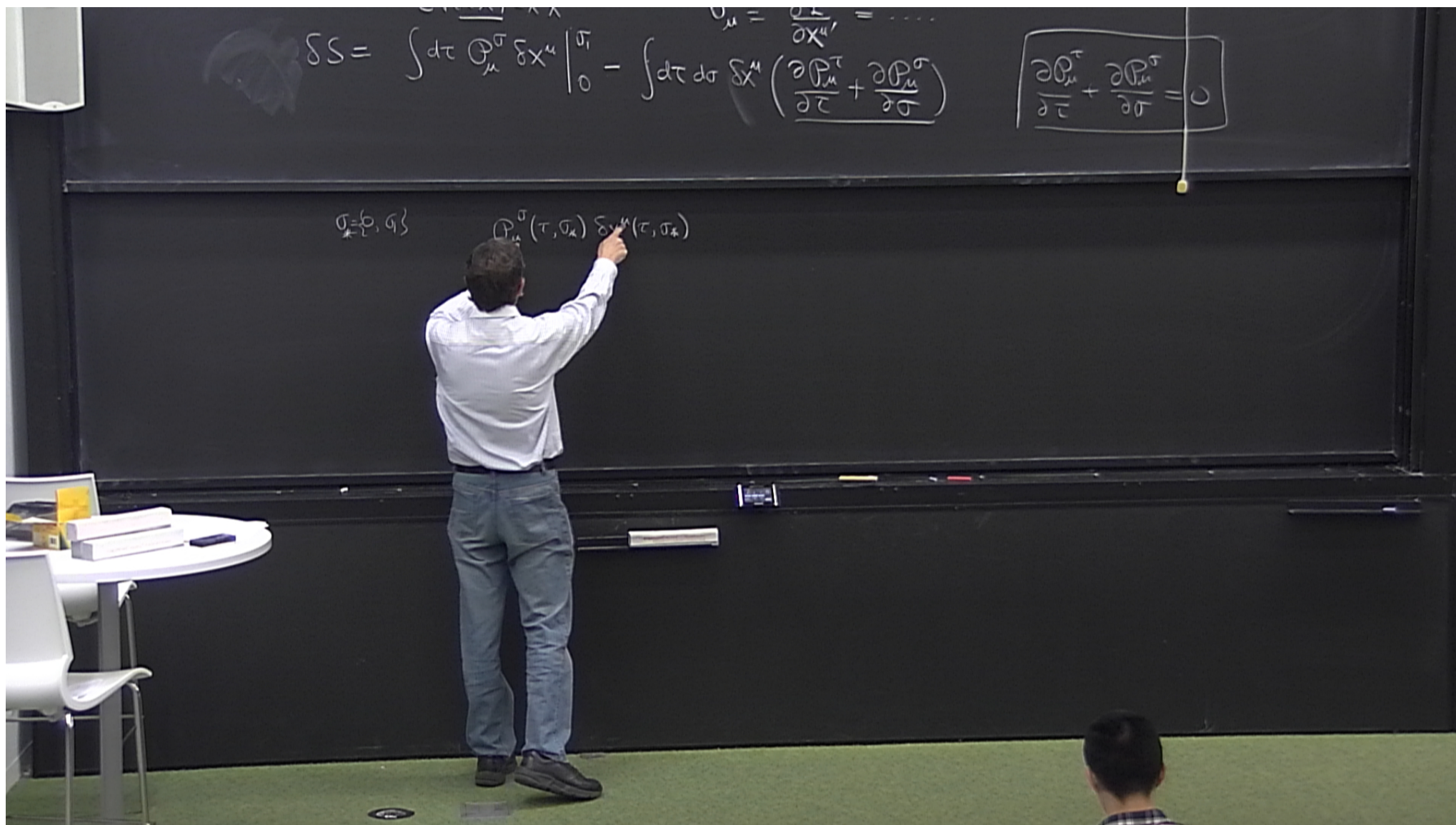
$$S = -\frac{T_0}{c} \int d\tau \int d\sigma \mathcal{L}(\dot{X}, X')$$

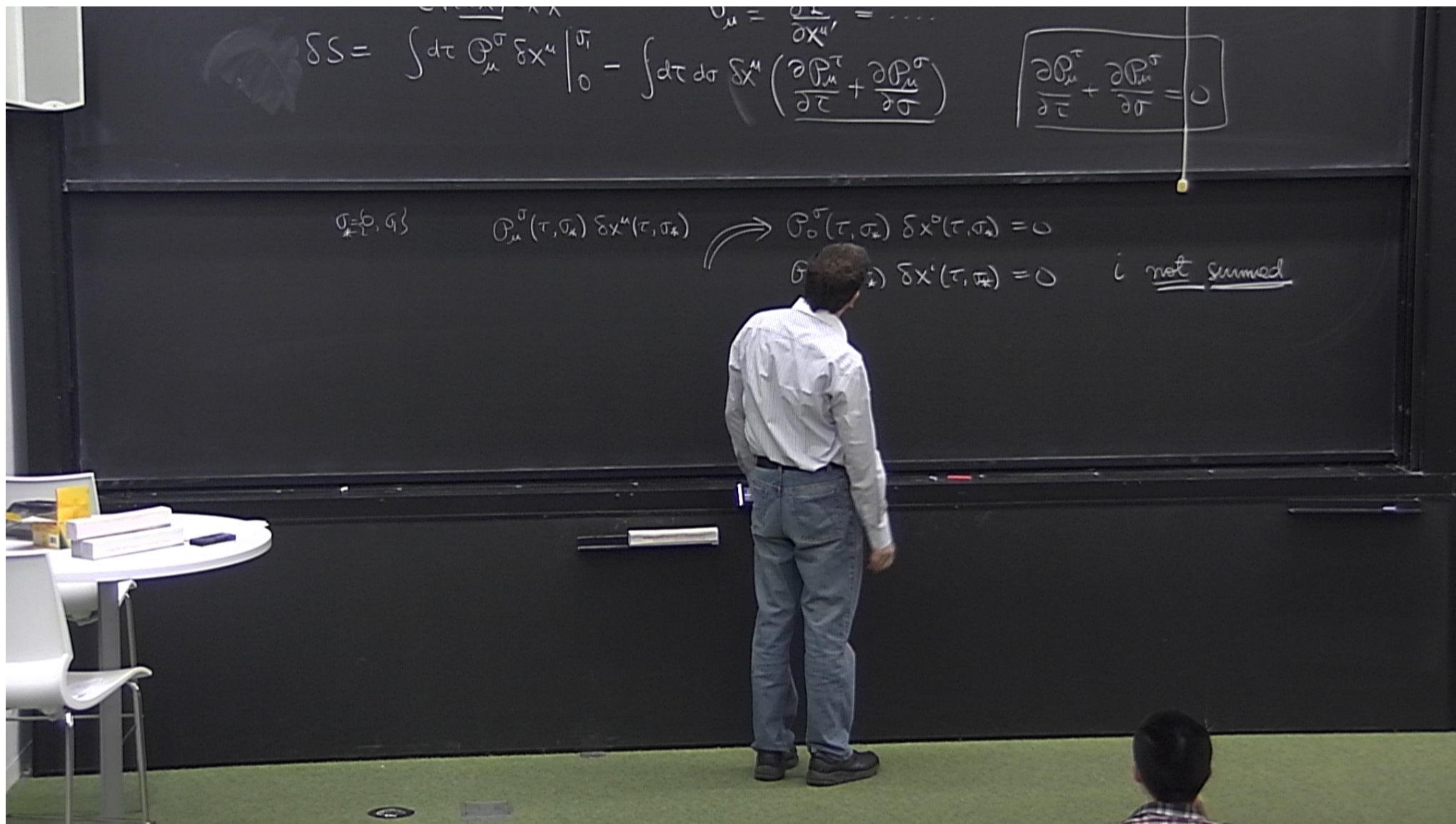
$$-\frac{T_0}{c} \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

$$\mathcal{P}_\mu^\tau = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = \frac{(\dot{X} \cdot X') X'_\mu - \dot{X}^2 X'_\mu}{\sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}}$$

$$\mathcal{P}_\mu^\sigma = \frac{\partial \mathcal{L}}{\partial X'^\mu} = \dots$$

$$\delta S = \int d\tau \mathcal{P}_\mu^\sigma \delta X^\mu \Big|_0^{\sigma_1} - \int d\tau d\sigma \delta X^\mu \left(\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} \right)$$





$$\delta S = \int d\tau \mathcal{P}_\mu^\sigma \delta x^\mu \Big|_0^{\tau_f} - \int d\tau d\sigma \delta x^\mu \left(\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} \right)$$

$$\boxed{\frac{\partial \mathcal{P}_\mu^\tau}{\partial \tau} + \frac{\partial \mathcal{P}_\mu^\sigma}{\partial \sigma} = 0}$$

$$\sigma_* = \{0, q\}$$

$$\mathcal{P}_\mu^\sigma(\tau, \sigma_*) \delta x^\mu(\tau, \sigma_*)$$

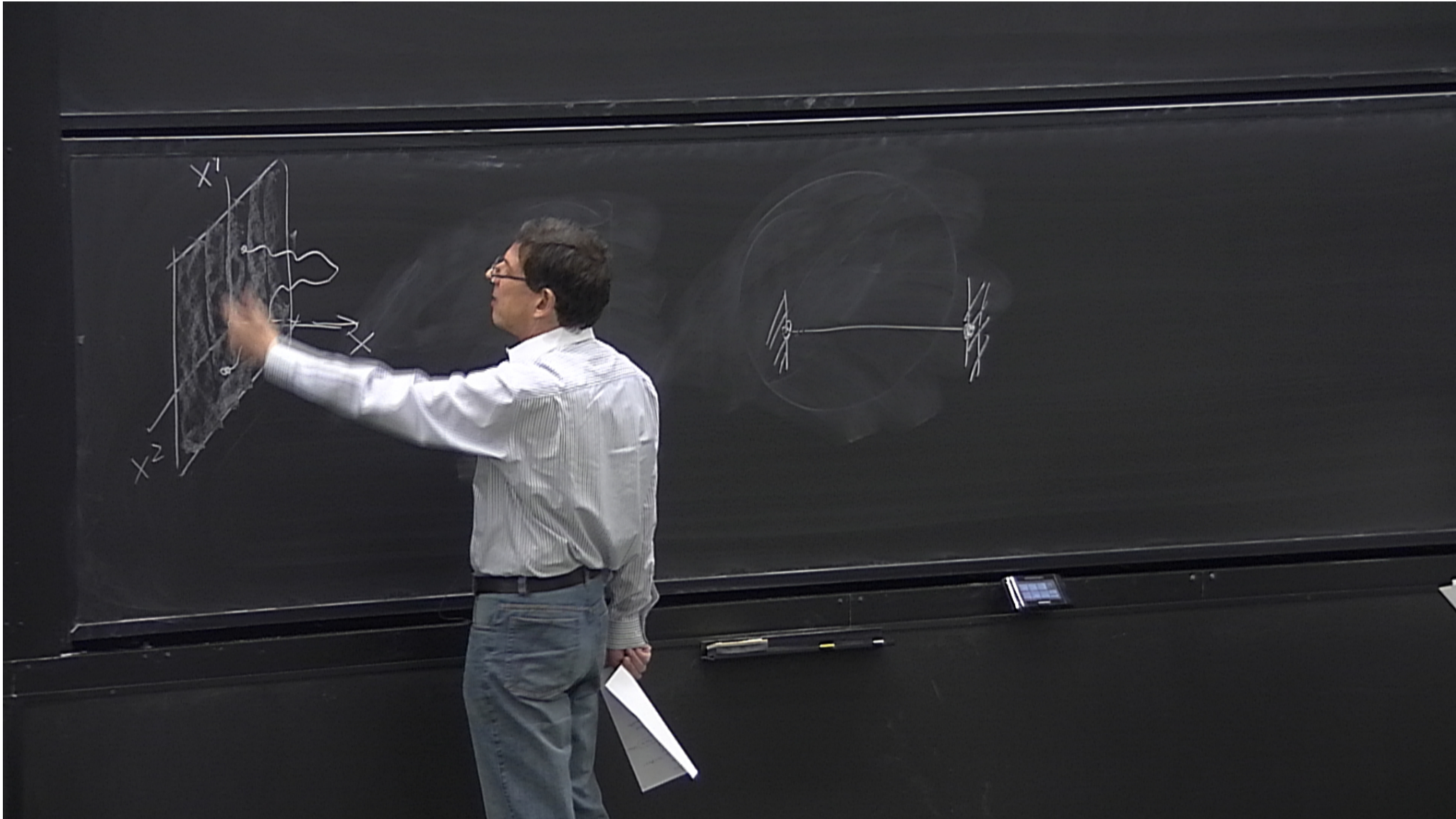
$$\Rightarrow \mathcal{P}_0^\sigma(\tau, \sigma_*) \delta x^0(\tau, \sigma_*) = 0$$

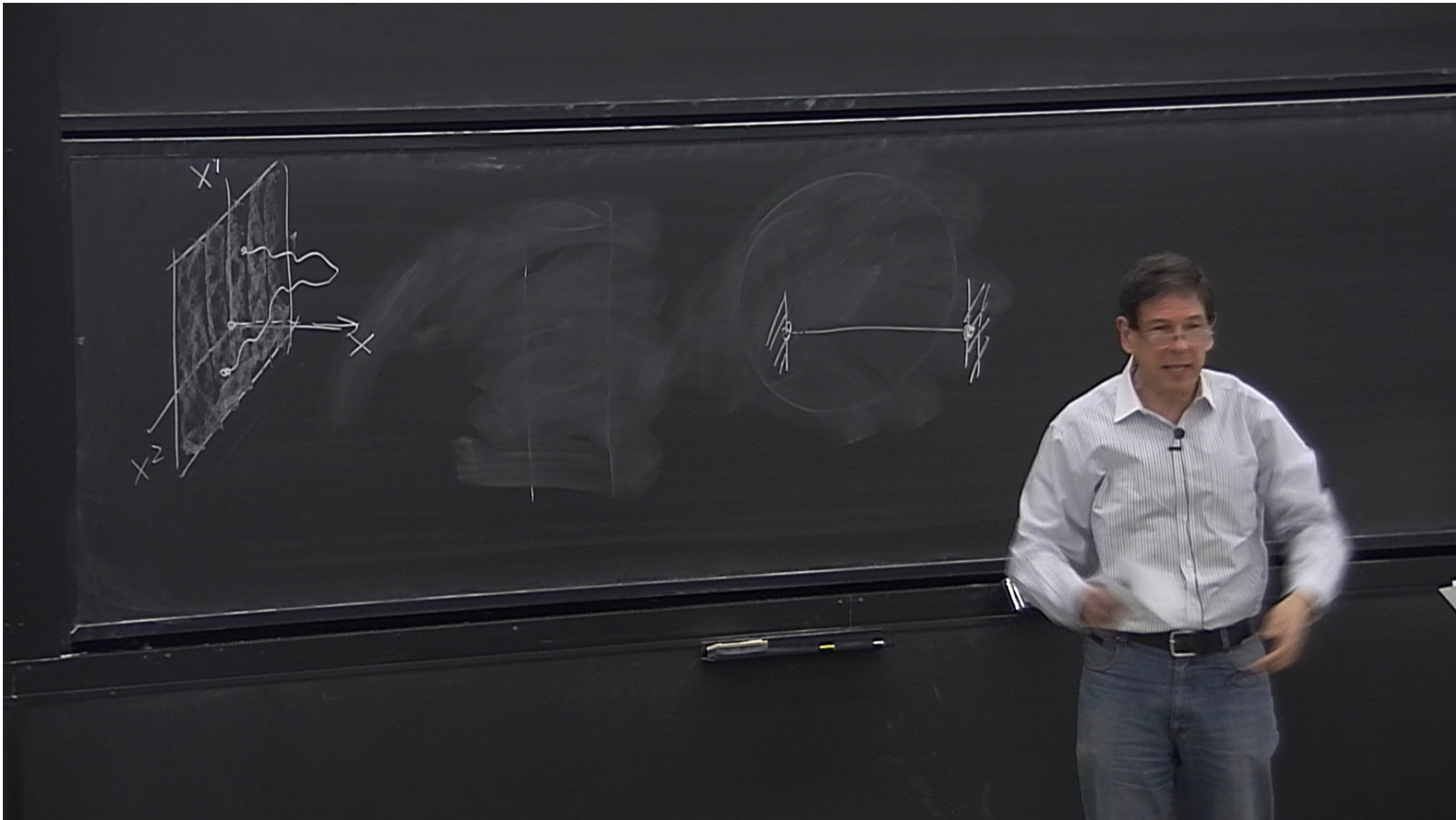
$$\mathcal{P}_i^\sigma(\tau, \sigma_*) \delta x^i(\tau, \sigma_*) = 0 \quad i \text{ not summed}$$

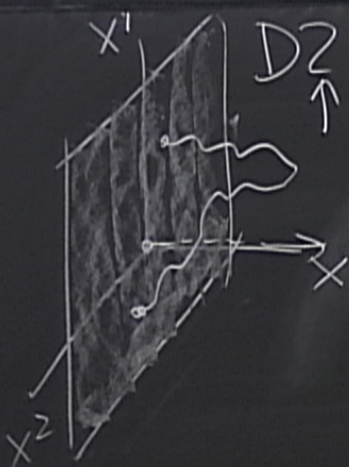
$$\boxed{\mathcal{P}_0^\sigma(\tau, \sigma_*) = 0}$$

$$\text{this} \rightarrow \mathcal{P}_i^\sigma(\tau, \sigma_*) = 0$$

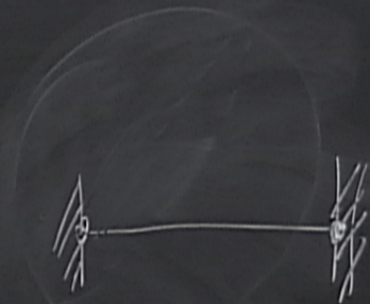
$$\Rightarrow \delta x^i(\tau, \sigma_*) = 0 \Rightarrow x^i(\tau, \sigma_*) = \text{constant} = \frac{\partial x^i}{\partial \tau}(\tau, \sigma_*) = 0$$

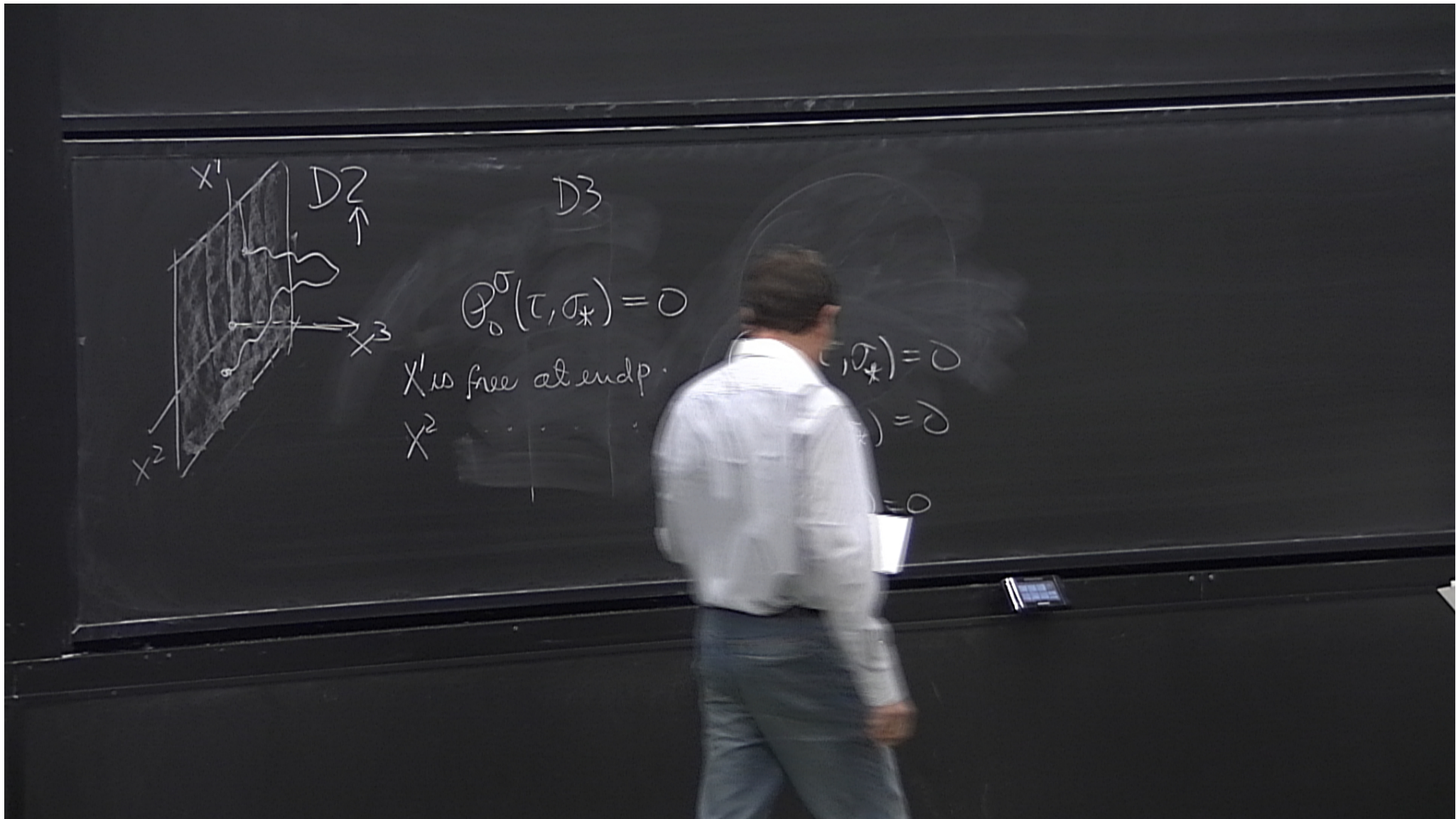


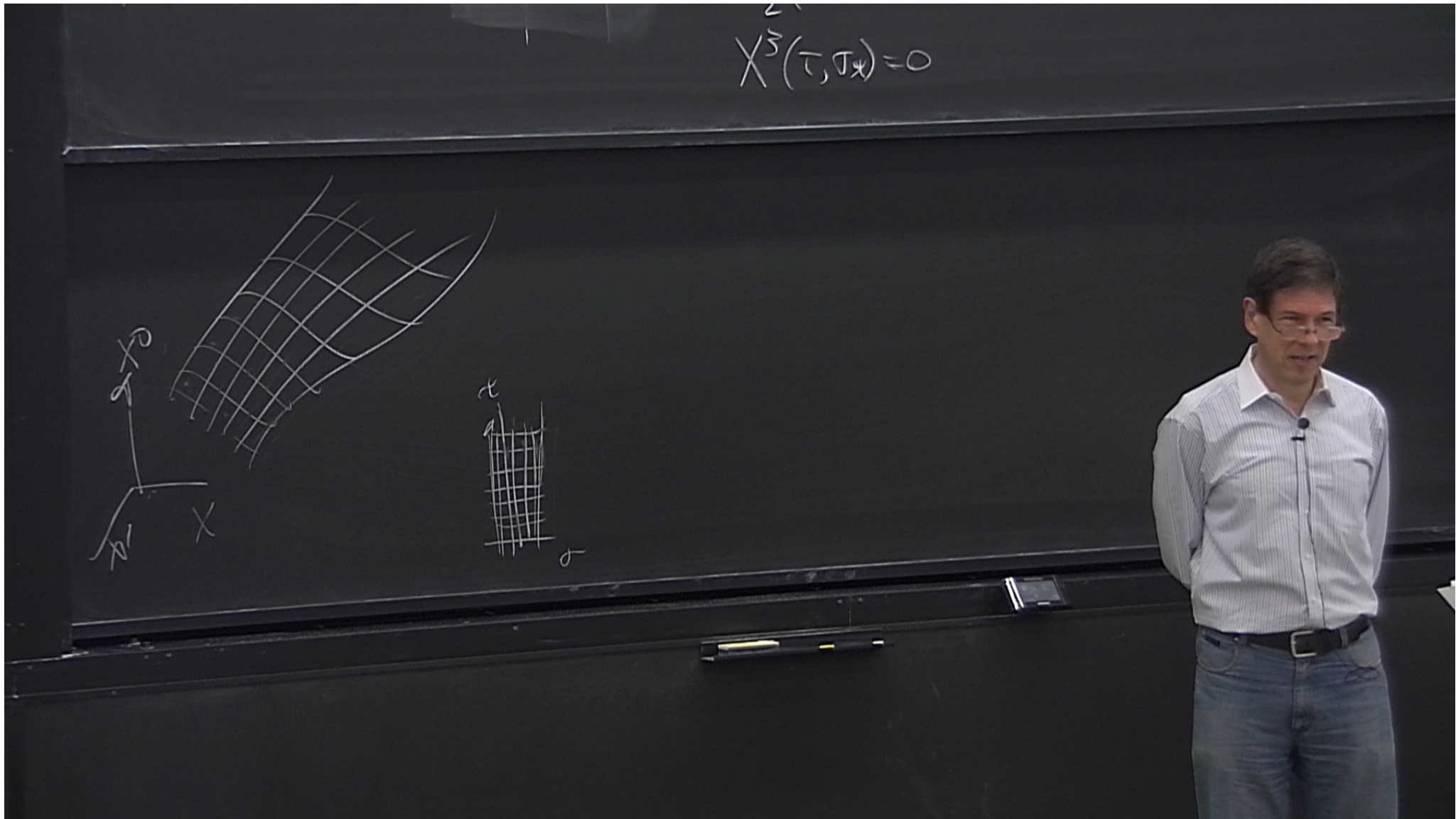




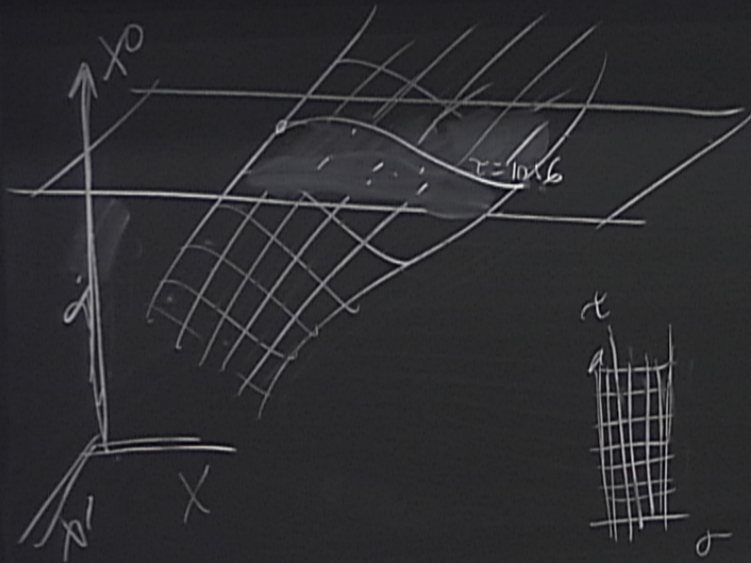
$D3$







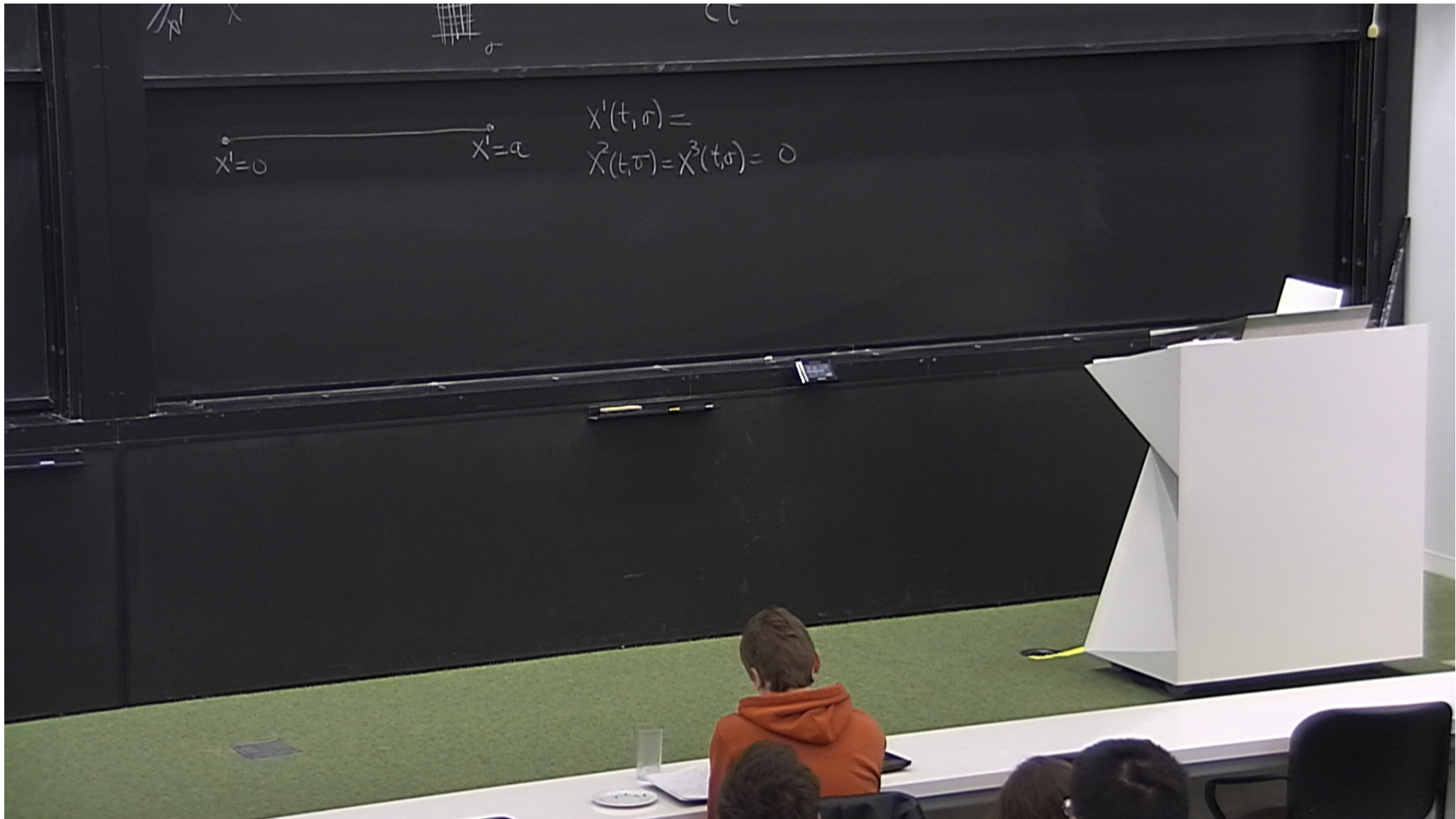
$$\vec{X}^3(\tau, \tau_\psi) = 0$$

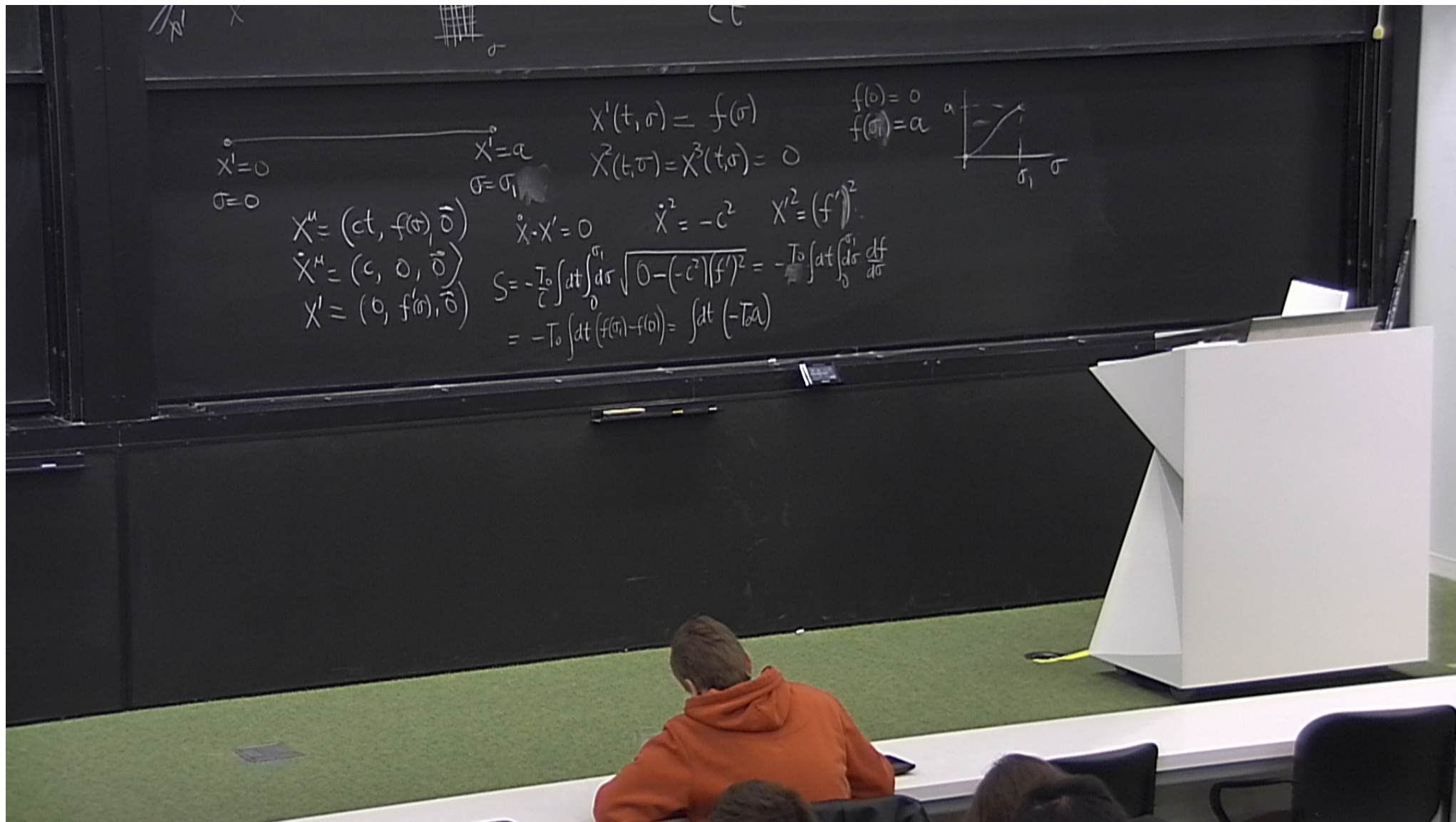


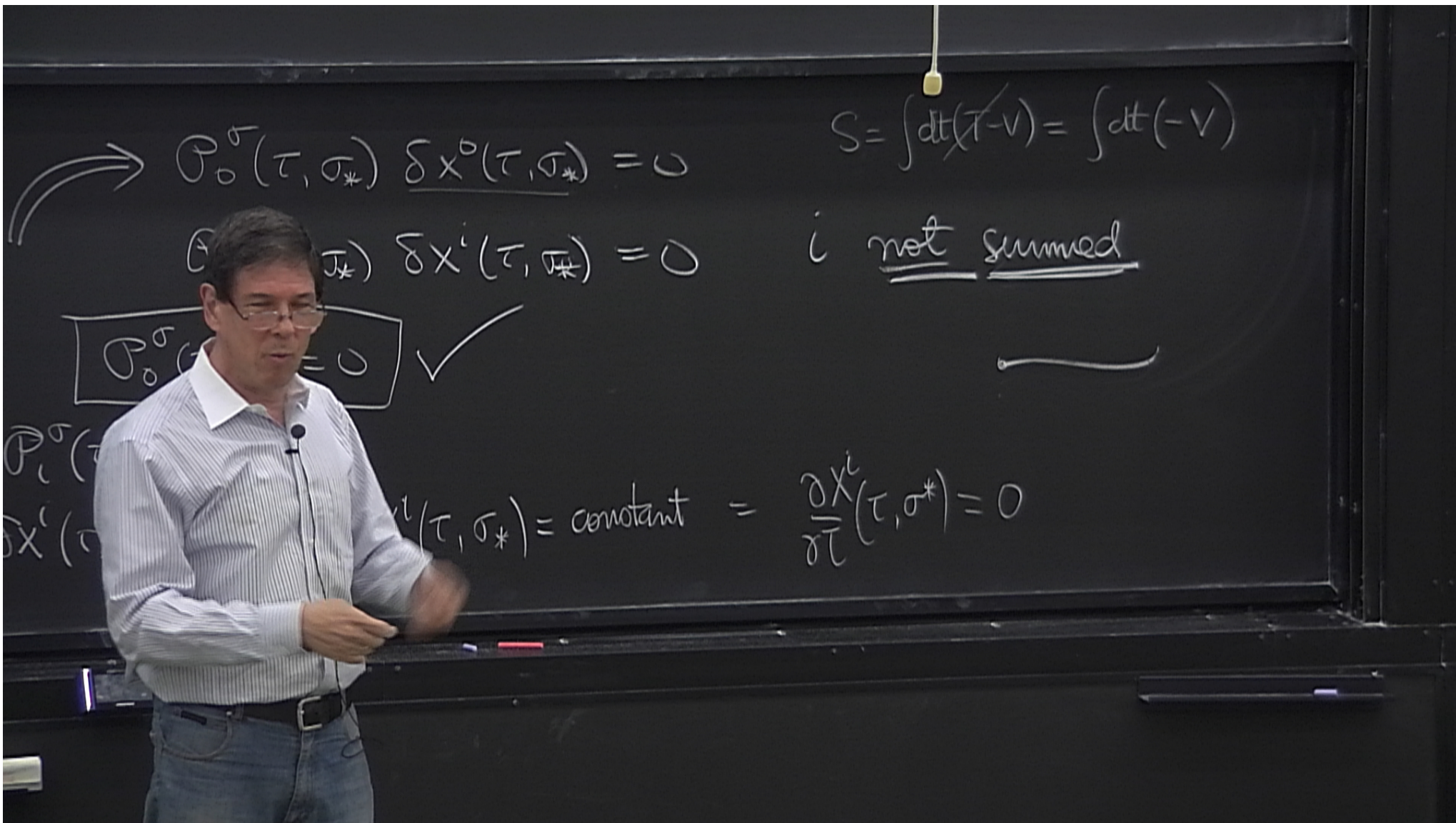
Static gauge

Set $\tau = t$

$$\boxed{X^0(\tau, \sigma) = c\tau}$$







$$\Rightarrow P_0^\sigma(\tau, \sigma_*) \delta x^0(\tau, \sigma_*) = 0$$

$$P_L^\sigma(\tau, \sigma_*) \delta x^i(\tau, \sigma_*) = 0$$

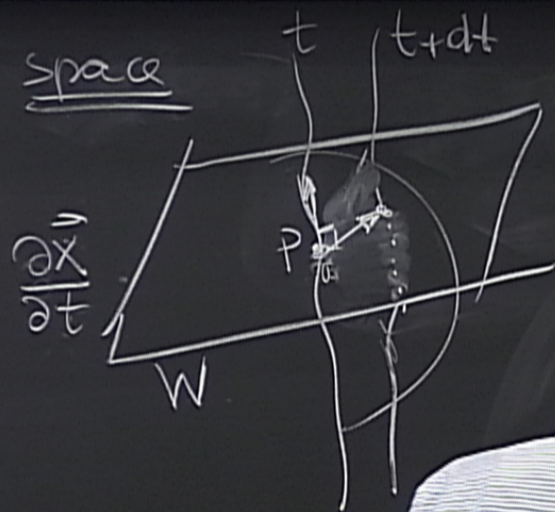
$$\boxed{P_0^\sigma(\tau, \sigma_*) = 0} \quad \checkmark$$

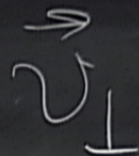
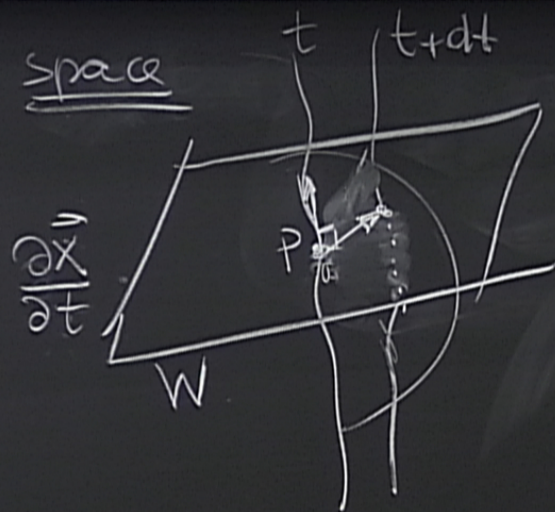
$$P_L^\sigma(\tau, \sigma_*) = 0$$

$$\delta x^i(\tau, \sigma_*) = 0 \Rightarrow x^i(\tau, \sigma_*) = \text{constant} = \frac{\partial x^i}{\partial \tau}(\tau, \sigma_*) = 0$$

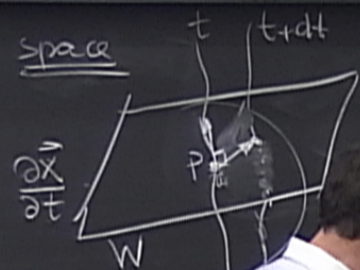
$$S = \int dt (\cancel{T} - V) = \int dt (-V)$$

i not summed





transverse velocity!



$$\vec{v}_\perp$$

transverse velocities!

$T \leftrightarrow S$ length along the string

