

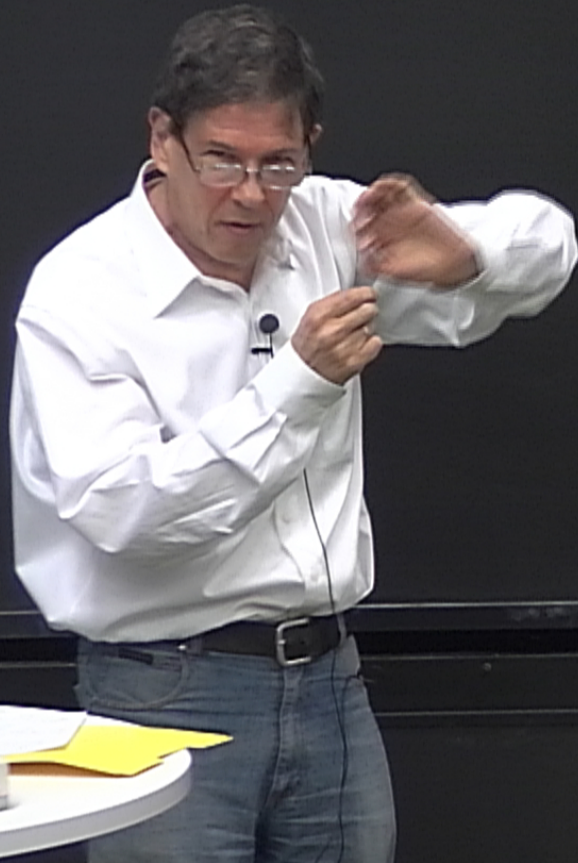
Title: 12/13 PSI - String Theory Review Lecture 3

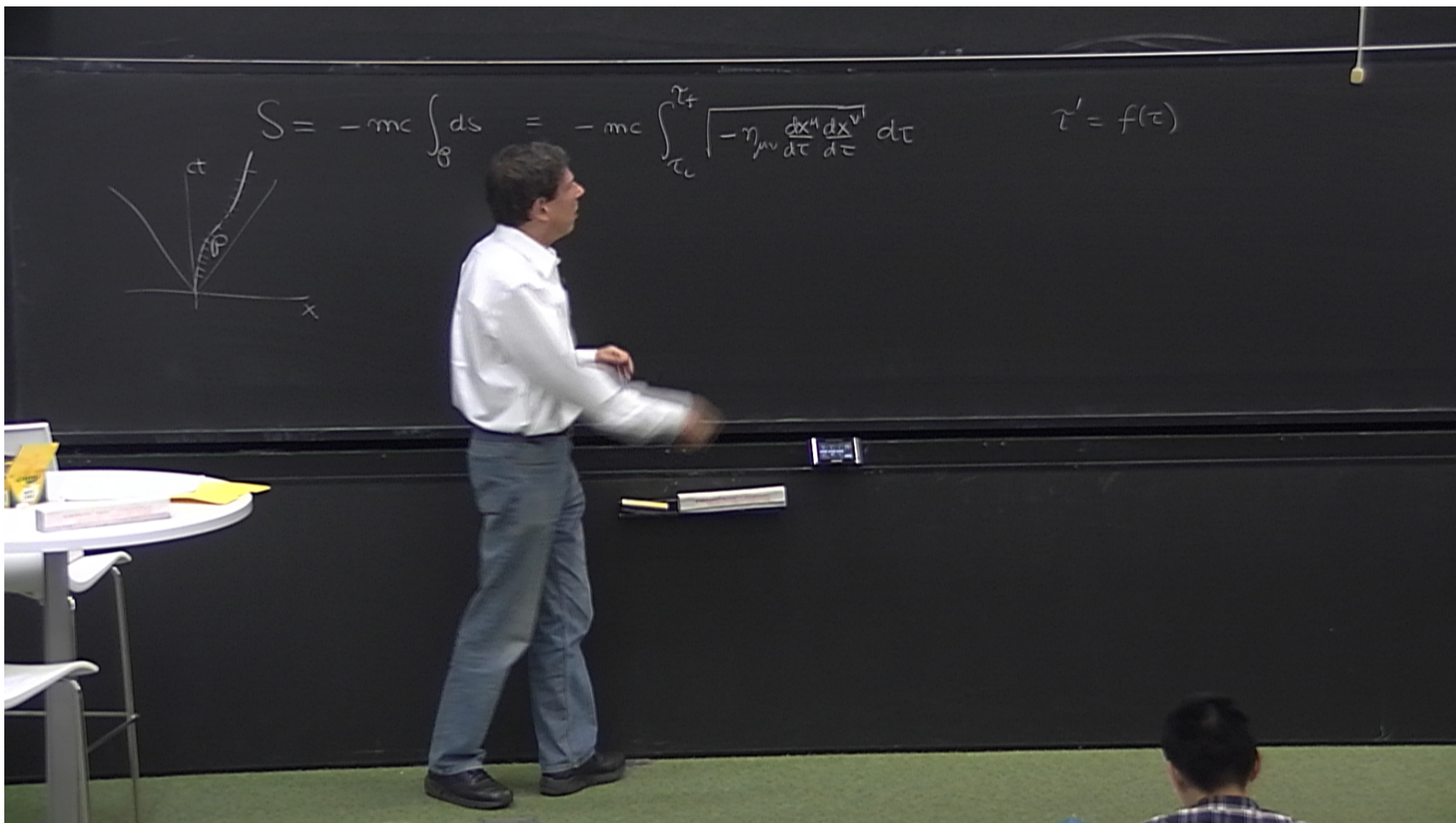
Date: Jan 09, 2013 10:15 AM

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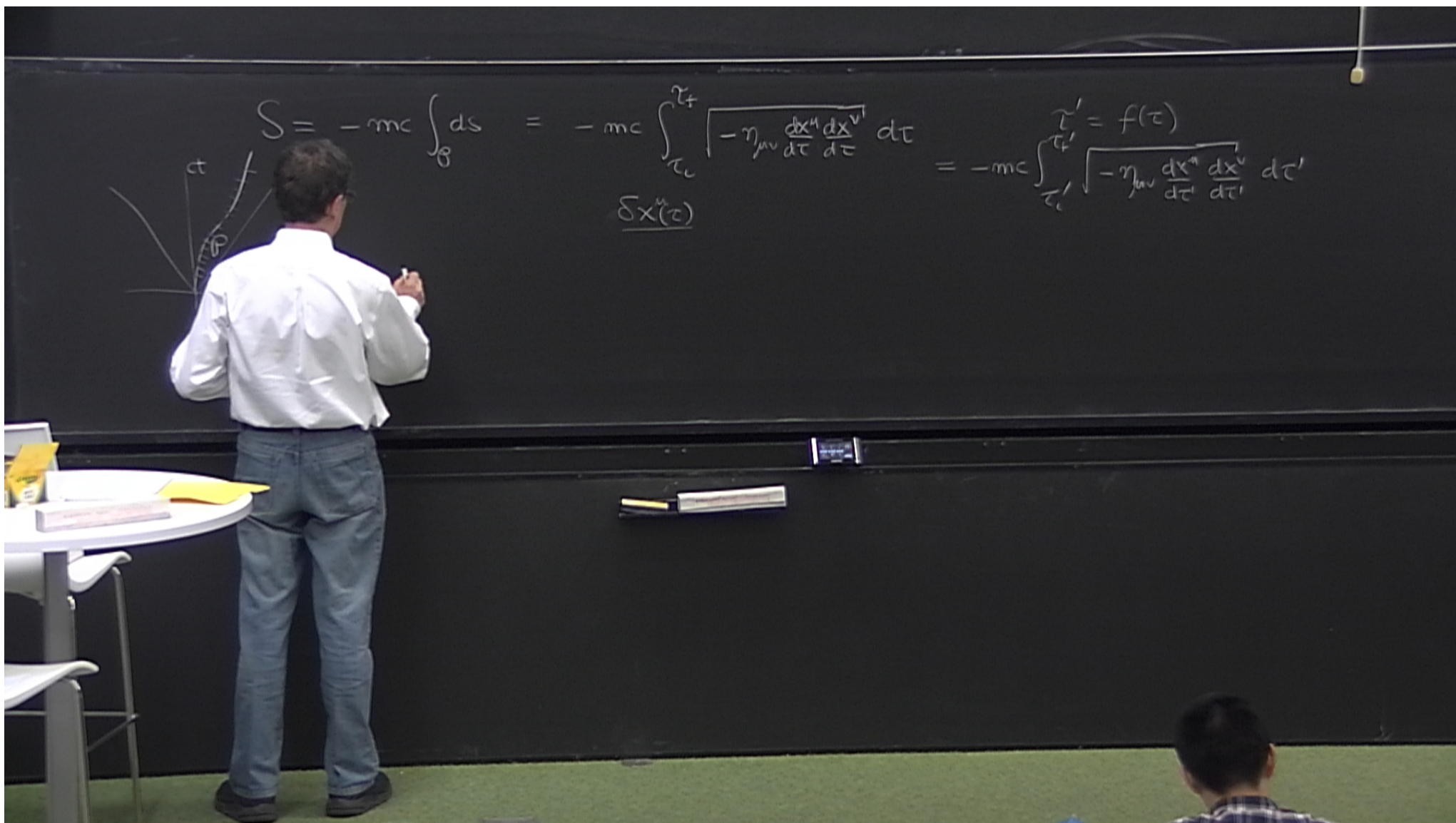
Abstract:

$$S = -mc \int_{\theta} ds = -mc \int_{\tau_i}^{\tau_f} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau$$





$$S = -mc \int_{\theta} ds = -mc \int_{\tau_c}^{\tau_f} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = -mc \int_{\tau'_c}^{\tau'_f} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau'} \frac{dx^\nu}{d\tau'}} d\tau' \quad \tau' = f(\tau)$$



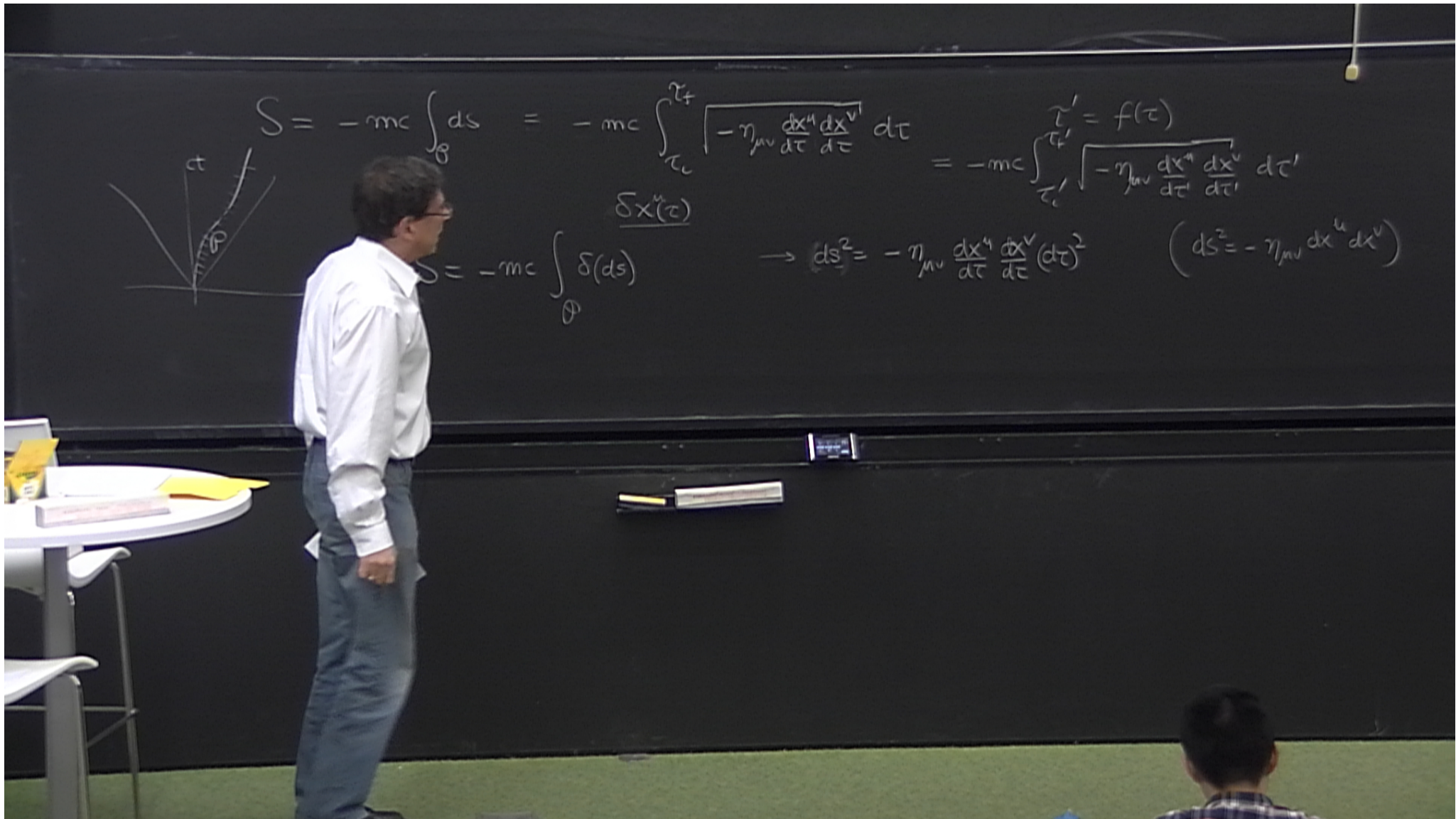
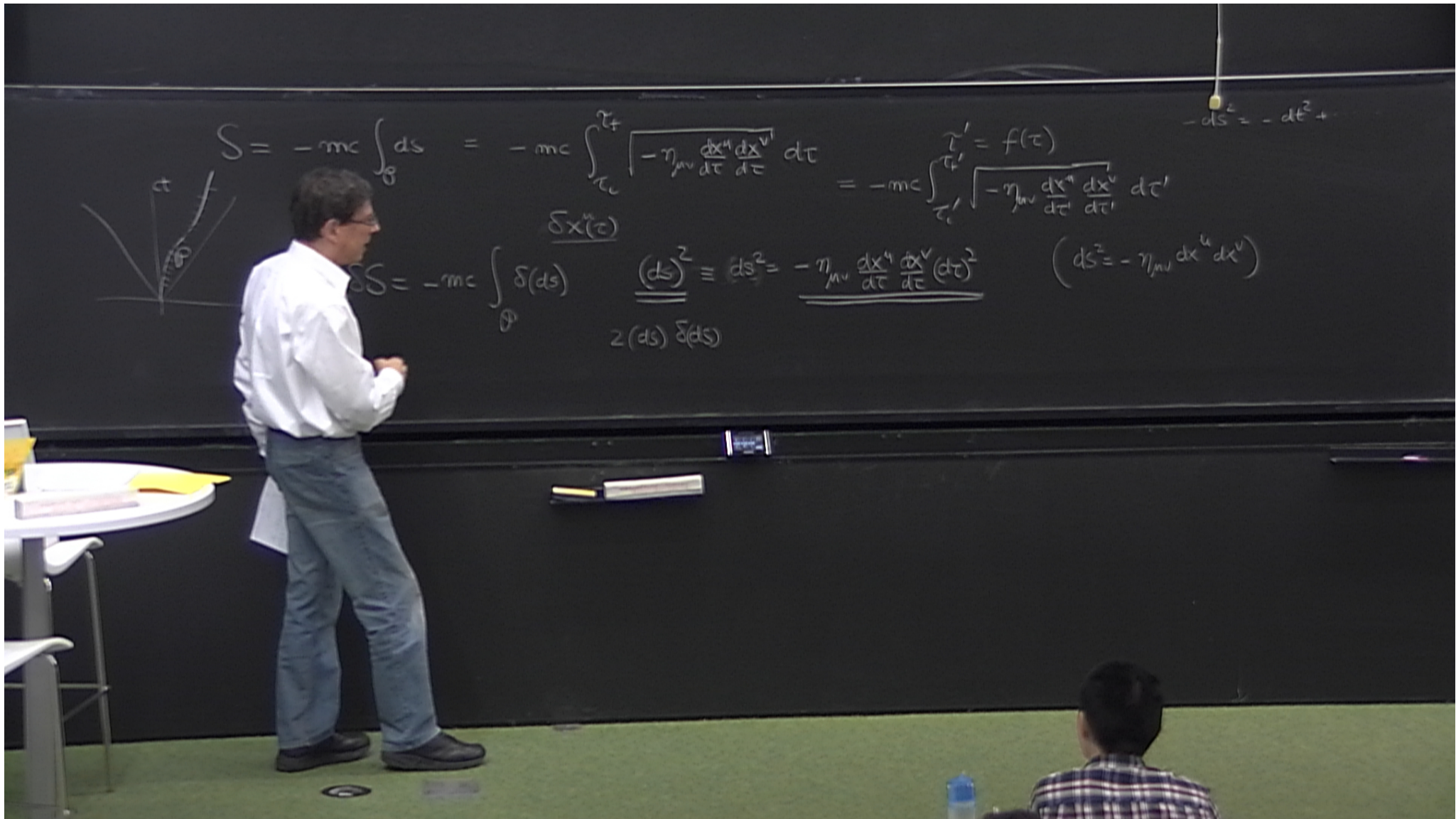


Diagram on the left: A coordinate system with a vertical axis labeled ct and a horizontal axis. A curve starts at the origin and goes upwards and to the right. A dashed line segment connects the origin to a point on the curve, and the angle between the vertical axis and this segment is labeled θ .

$$S = -mc \int_{\emptyset} ds = -mc \int_{\tau_c}^{\tau_+} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = -mc \int_{\tau'_c}^{\tau'_+} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau'} \frac{dx^\nu}{d\tau'}} d\tau' \quad \tau' = f(\tau)$$

$\underline{\delta x^u(\tau)}$

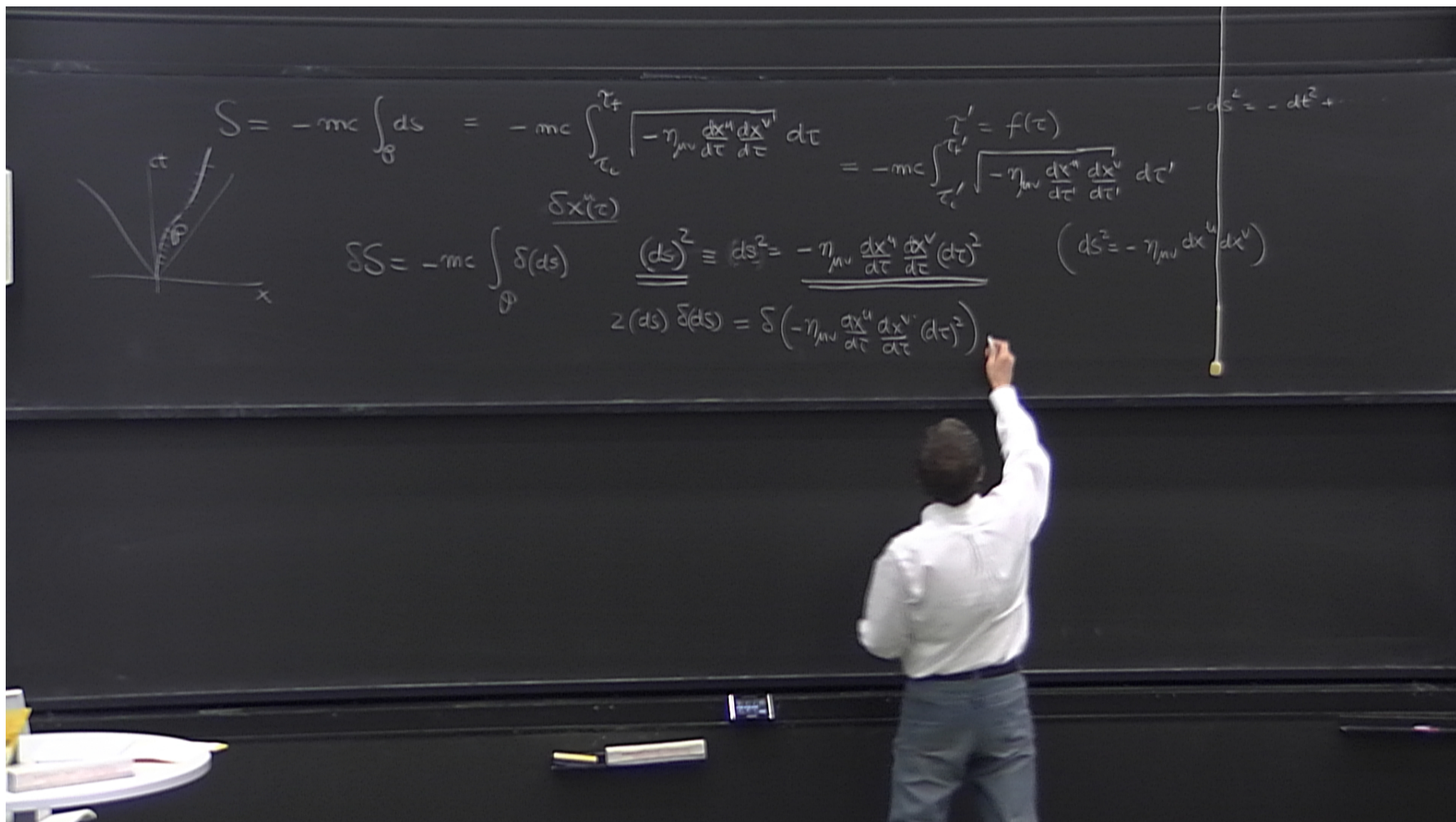
$$S = -mc \int_{\emptyset} \delta(ds) \quad \rightarrow \quad ds^2 = -\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} (d\tau)^2 \quad (ds^2 = -\eta_{\mu\nu} dx^\mu dx^\nu)$$

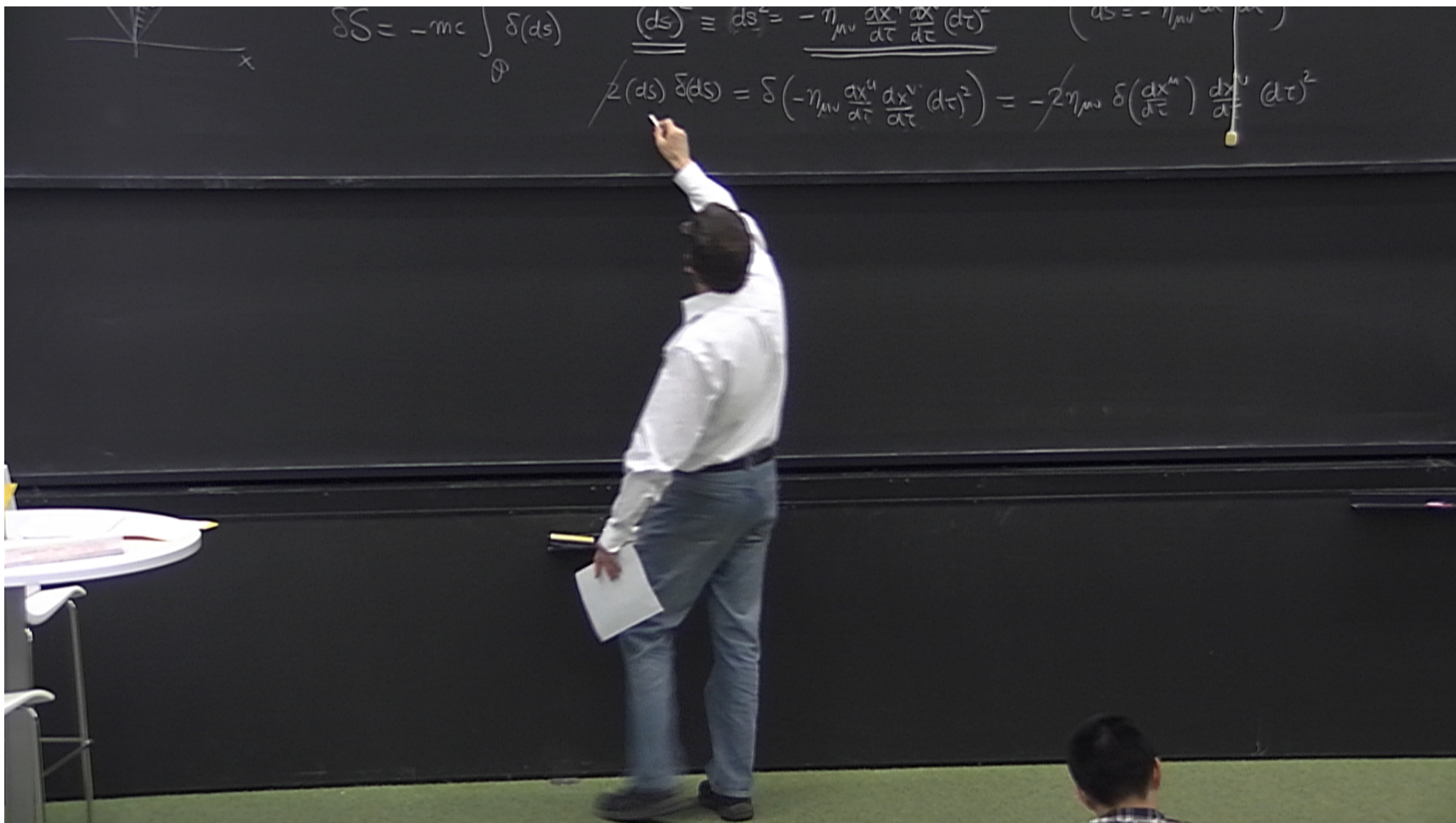


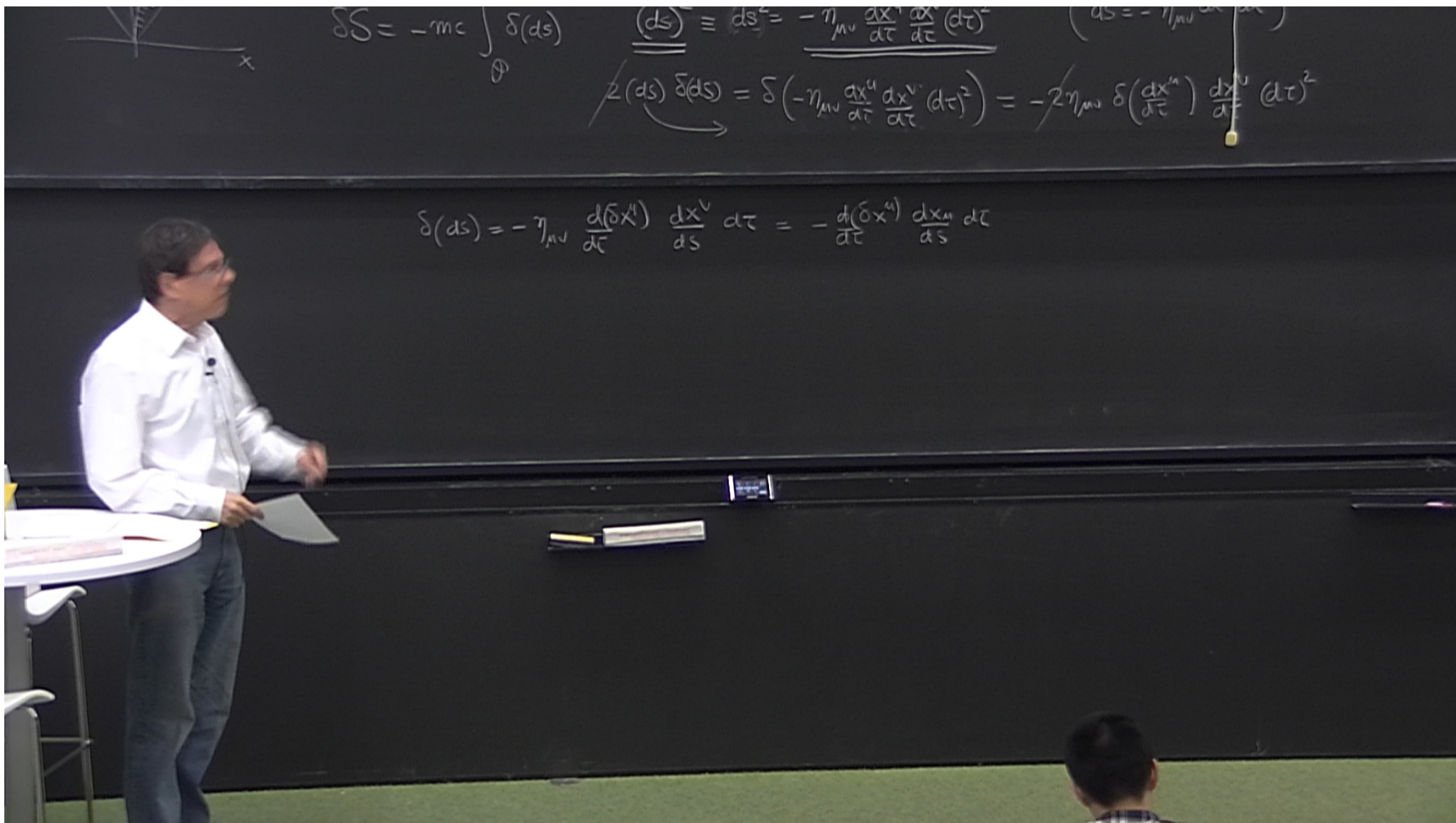
$$S = -mc \int_{\mathcal{P}} ds = -mc \int_{\tau_c}^{\tau_f} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = -mc \int_{\tau'_c}^{\tau'_f} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau'} \frac{dx^\nu}{d\tau'}} d\tau' \quad \tau' = f(\tau)$$

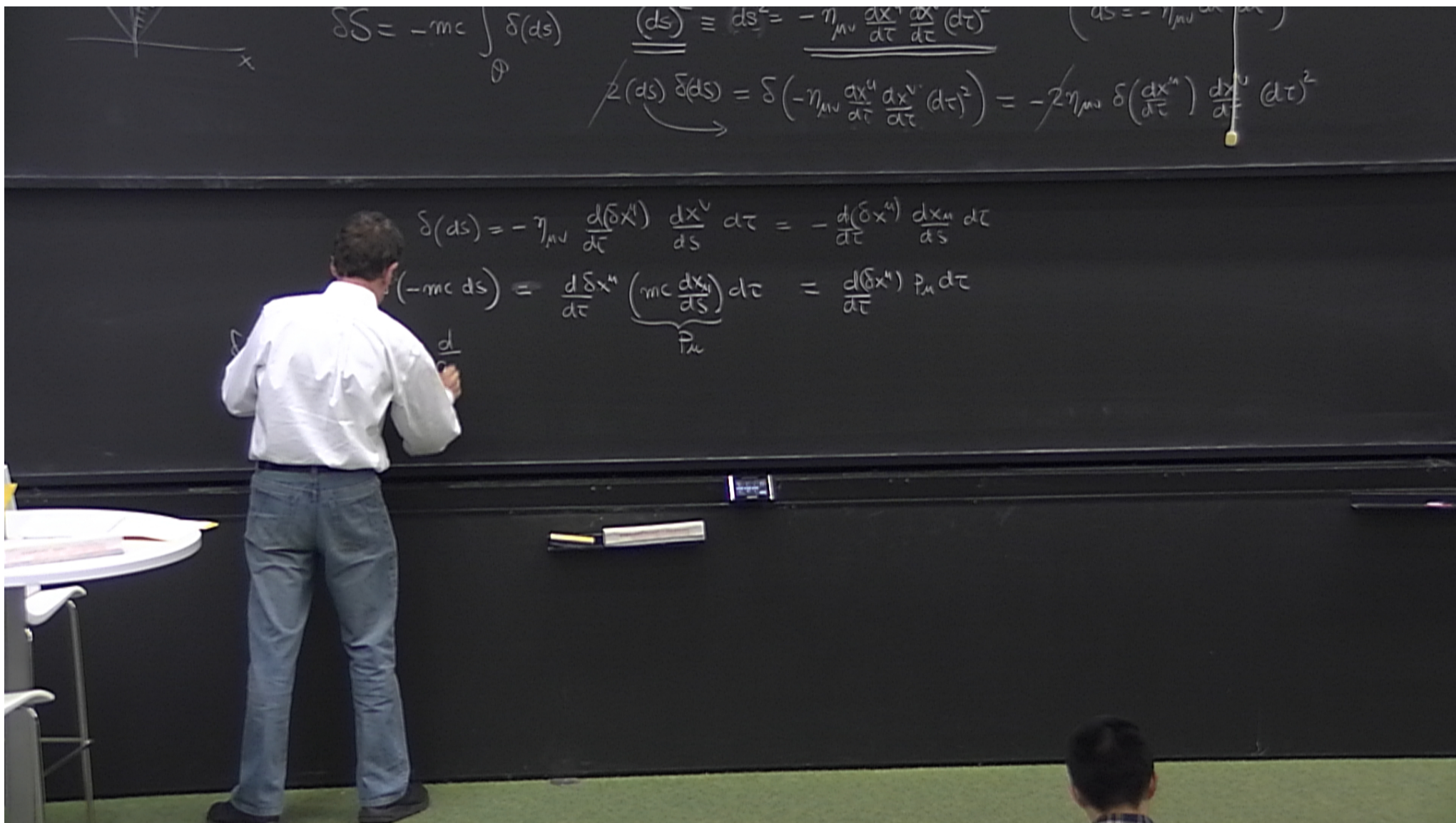
$$\delta S = -mc \int_{\mathcal{P}} \delta(ds) \quad \frac{(ds)^2}{2(ds)\delta(ds)} \equiv ds^2 = -\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} (d\tau)^2 \quad (ds^2 = -\eta_{\mu\nu} dx^\mu dx^\nu)$$

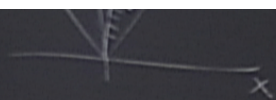
$$-ds^2 = -dt^2 + \dots$$











$$\delta S = -mc \int \delta(ds)$$

$$(ds)^2 \equiv ds^2 = -\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} (d\tau)^2$$

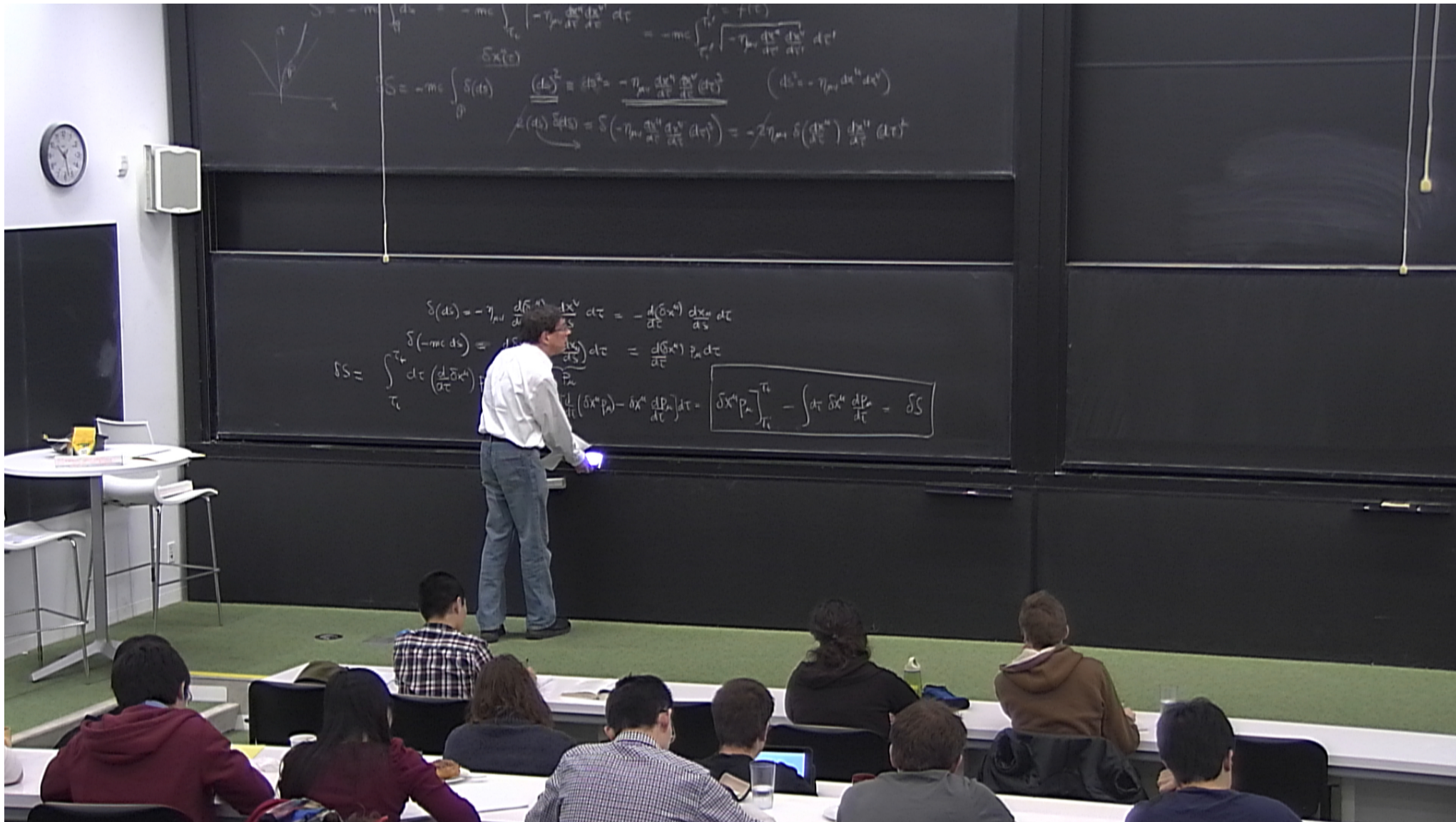
$$(ds = -\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} d\tau)$$

$$\delta(ds) = \delta\left(-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} (d\tau)^2\right) = -2\eta_{\mu\nu} \delta\left(\frac{dx^\mu}{d\tau}\right) \frac{dx^\nu}{d\tau} (d\tau)^2$$

$$\delta(ds) = -\eta_{\mu\nu} \frac{d(\delta x^\mu)}{d\tau} \frac{dx^\nu}{ds} d\tau = -\frac{d(\delta x^\mu)}{d\tau} \frac{dx_\mu}{ds} d\tau$$

$$\delta(-mc ds) = \frac{d\delta x^\mu}{d\tau} \underbrace{\left(mc \frac{dx_\mu}{ds}\right)}_{p_\mu} d\tau = \frac{d(\delta x^\mu)}{d\tau} p_\mu d\tau$$

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d\delta x^\mu}{d\tau}\right) p_\mu = \int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau}(\delta x^\mu p_\mu) - \delta x^\mu \frac{dp_\mu}{d\tau}\right] d\tau = \left[\delta x^\mu p_\mu\right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} \delta x^\mu \frac{dp_\mu}{d\tau} d\tau$$



$$\delta(ds) = -\eta_{\mu\nu} \frac{d(\delta x^\mu)}{d\tau} \frac{dx^\nu}{ds} d\tau = -\frac{d(\delta x^\mu)}{d\tau} \frac{dx_\mu}{ds} d\tau$$

$$\delta(-mc ds) = \frac{d\delta x^\mu}{d\tau} \left(mc \frac{dx_\mu}{d\tau} \right) d\tau = \frac{d(\delta x^\mu)}{d\tau} p_\mu d\tau$$

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d\delta x^\mu}{d\tau} \right) p_\mu = \left[\delta x^\mu p_\mu \right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau \delta x^\mu \frac{dp_\mu}{d\tau} = \delta S$$

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) p_\mu = \int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau} (\delta x^\mu p_\mu) - \delta x^\mu \frac{dp_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu p_\mu \right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau \delta x^\mu \frac{dp_\mu}{d\tau} = \delta S$$

$$\frac{dp_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{d\tau} = 0$$



$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) P_\mu = \int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau} (\delta x^\mu P_\mu) - \delta x^\mu \frac{dP_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu P_\mu \right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau \delta x^\mu \frac{dP_\mu}{d\tau} = \delta S$$

$$\frac{dP_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{ds} = 0 \xrightarrow{\tau=s} \frac{d^2 x^\mu}{ds^2} = 0$$

$$\frac{d}{d\tau} \frac{dx^\mu}{d\tau} \neq 0$$

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) p_\mu = \int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau} (\delta x^\mu p_\mu) - \delta x^\mu \frac{dp_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu p_\mu \right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau \delta x^\mu \frac{dp_\mu}{d\tau} = \delta S$$

$$\frac{dp_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{ds} = 0 \xrightarrow{\tau=s} \frac{d^2 x^\mu}{ds^2} = 0$$

$$\frac{d}{d\tau} \frac{dx^\mu}{d\tau} \neq 0$$

$$S = -mc \int_{\mathcal{P}} ds + \frac{q}{c} \int_{\mathcal{P}} A_\mu(x) dx^\mu$$

$$\rightarrow \delta S = 0 \quad \frac{dp_\mu}{ds} = \frac{q}{c} \bar{F}_{\mu\nu} \frac{dx^\nu}{ds}$$

Looney force law

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) p_\mu = \int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau} (\delta x^\mu p_\mu) - \delta x^\mu \frac{dp_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu p_\mu \right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau \delta x^\mu \frac{dp_\mu}{d\tau} = \delta S$$

$$\frac{dp_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{ds} = 0 \xrightarrow{\tau=s} \frac{d^2 x^\mu}{ds^2} = 0$$

$$\frac{d}{d\tau} \frac{dx^\mu}{d\tau} \neq 0$$

$$S = -mc \int_{\mathcal{P}} ds + \frac{q}{c} \int_{\mathcal{P}} A_\mu(x) dx^\mu \quad \checkmark$$

$$\rightarrow \delta S = 0 \quad \frac{dp_\mu}{ds} = \frac{q}{c} \bar{F}_{\mu\nu} \frac{dx^\nu}{ds} \quad \checkmark$$

Looney force law

$$\delta S = \int_{\tau_1}^{\tau_2} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) P_\mu = \int_{\tau_1}^{\tau_2} \left[\frac{d}{d\tau} (\delta x^\mu P_\mu) - \delta x^\mu \frac{dP_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu P_\mu \right]_{\tau_1}^{\tau_2} - \int_{\tau_1}^{\tau_2} d\tau \delta x^\mu \frac{dP_\mu}{d\tau} = \delta S$$

$$\frac{dP_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{ds} = 0 \xrightarrow{\tau=s} \frac{d^2 x^\mu}{ds^2} = 0$$

$$S = -mc \int_{\mathcal{P}} ds + \frac{q}{c} \int_{\mathcal{P}} A_\mu(x) dx^\mu \quad \checkmark$$

$$\rightarrow \delta S = 0 \quad \frac{dP_\mu}{ds} = \frac{q}{c} \overline{F}_{\mu\nu} \frac{dx^\nu}{ds} \quad \checkmark$$

Looney force law

$$q_\nu(x) dx^\mu dx^\nu$$

$$\delta S = \int_{T_i}^{T_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) P_\mu = \int_{T_i}^{T_f} \left[\frac{d}{d\tau} (\delta x^\mu P_\mu) - \delta x^\mu \frac{dP_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu P_\mu \right]_{T_i}^{T_f} - \int_{T_i}^{T_f} d\tau \delta x^\mu \frac{dP_\mu}{d\tau} = \delta S$$

$$\frac{dP_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{ds} = 0 \xrightarrow{\tau=s} \frac{d^2 x^\mu}{ds^2} = 0$$

$$S = -mc \int_{\mathcal{P}} ds + \frac{q}{c} \int_{\mathcal{P}} A_\mu(x) dx^\mu \quad \checkmark$$

$$\rightarrow \delta S = 0 \quad \frac{dP_\mu}{ds} = \frac{q}{c} F_{\mu\nu} \frac{dx^\nu}{ds} \quad \checkmark$$

Lorentz force law

$$\frac{d}{d\tau} \frac{dx^\mu}{d\tau} \neq 0$$

$$ds^2 = -g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) P_\mu = \int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau} (\delta x^\mu P_\mu) - \delta x^\mu \frac{dP_\mu}{d\tau} \right] d\tau = \left[\delta x^\mu P_\mu \right]_{\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau \delta x^\mu \frac{dP_\mu}{d\tau} = \delta S$$

$$\frac{dP_\mu}{d\tau} = 0 \Rightarrow \frac{d}{d\tau} \frac{dx^\mu}{ds} = 0 \xrightarrow{\tau=s} \frac{d^2 x^\mu}{ds^2} = 0$$

$$\frac{d}{d\tau} \frac{dx^\mu}{d\tau} \neq 0$$

$$ds^2 = -g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$S = -mc \int_{\mathcal{P}} ds + \frac{q}{c} \int_{\mathcal{P}} A_\mu(x) dx^\mu \quad \checkmark$$

$$\rightarrow \delta S = 0$$

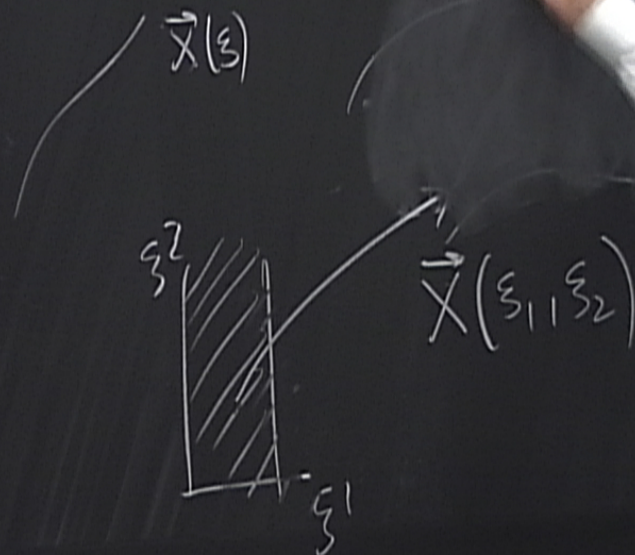
$$\frac{dP_\mu}{ds} = \frac{q}{c} F_{\mu\nu} \frac{dx^\nu}{ds} \quad \checkmark$$

Lowty force law

Strings Use the "area", begin with surfaces in space

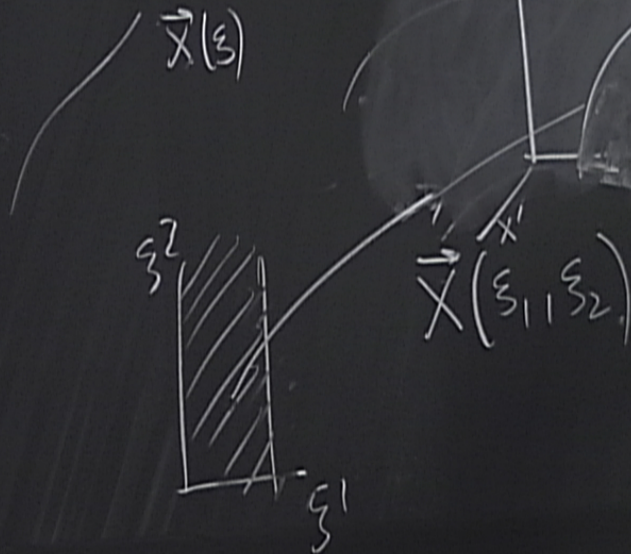


Strategy Use the "area", begin with surfaces in space



Strings

Use the "area", begin with surfaces in space



Strings

Use the "area", begin with surfaces in space

$$\vec{X}(\xi)$$

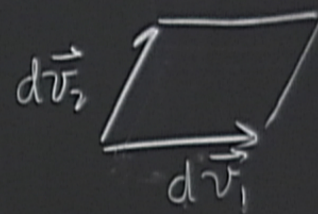
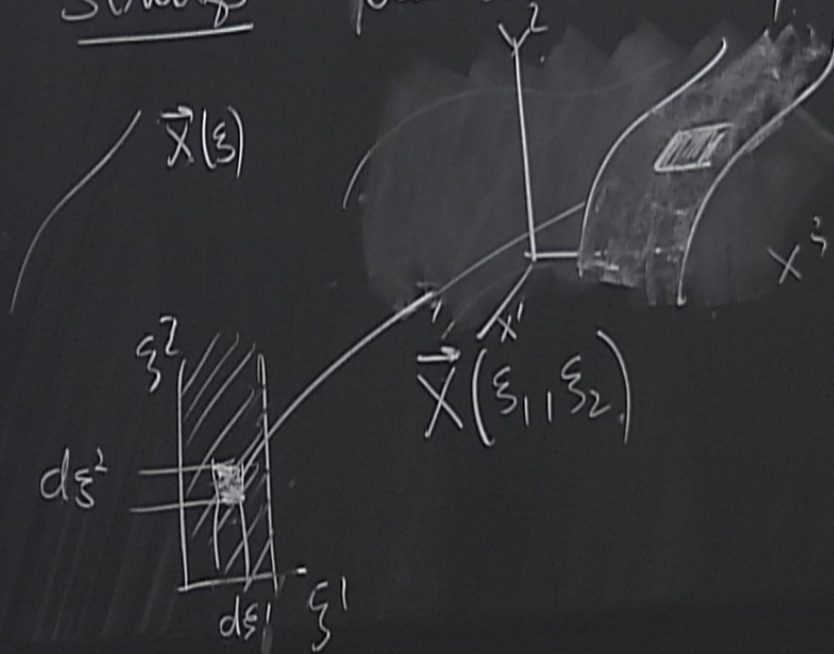


$$\vec{X}(\xi_1, \xi_2)$$



Strings

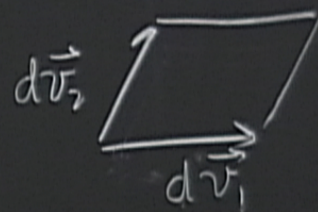
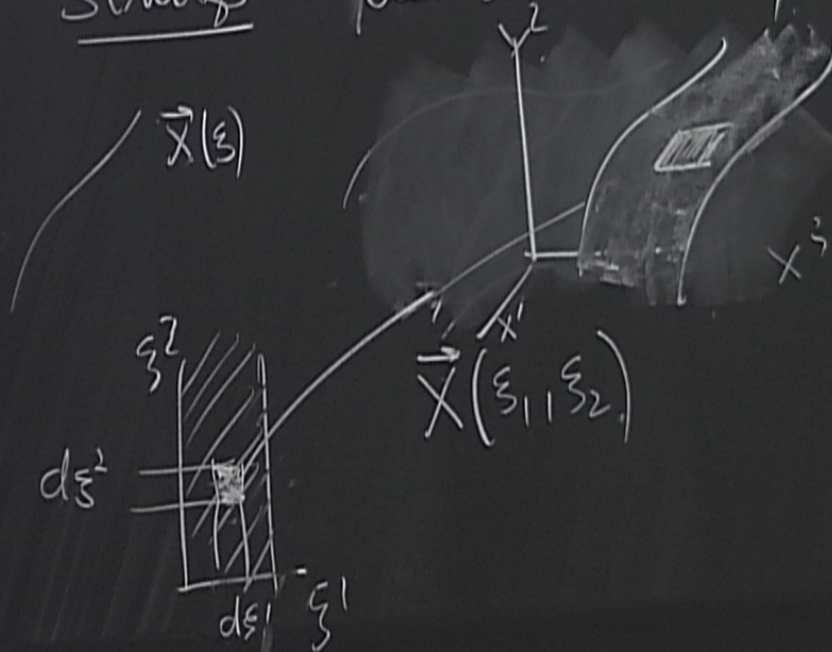
Use the "area", begin with surfaces in space



$d\vec{v}$

Strings

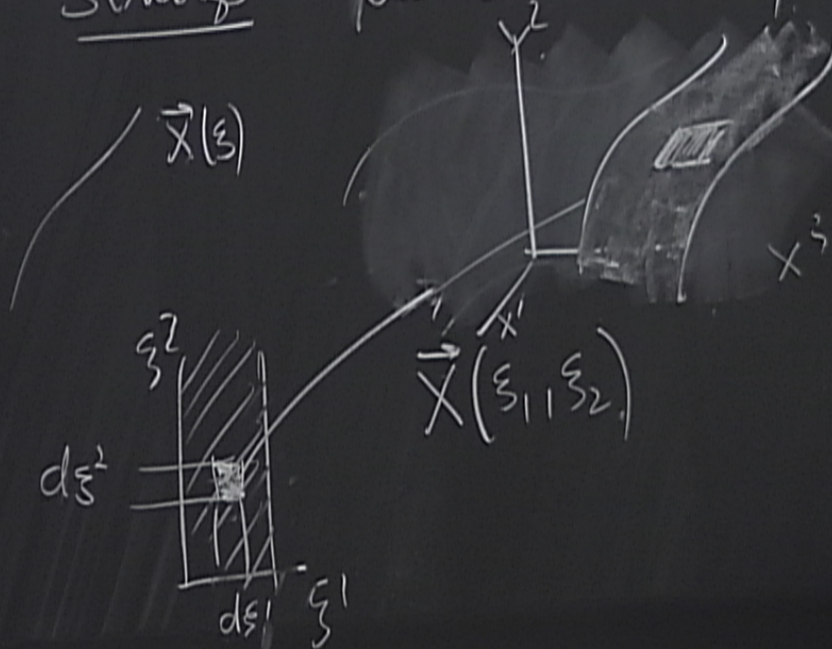
Use the "area", begin with surfaces in space



$$d\vec{v}_i = \frac{\partial \vec{X}}{\partial \xi^i} d\xi^i$$

Strings

Use the "area", begin with surfaces in space



$$d\vec{v}_1$$

$$d\vec{v}_1 = \frac{\partial \vec{X}}{\partial \xi^1} d\xi^1$$

$$d\vec{v}_2 = \frac{\partial \vec{X}}{\partial \xi^2} d\xi^2$$

$$dA = |d\vec{v}_1 \times d\vec{v}_2|$$

dA

$$\begin{aligned}
 d\vec{s} &= d\vec{x} \cdot d\vec{x} = \left(\frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \right) d\xi^i d\xi^j & g_{ij}(\xi) &= \frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \\
 &= g_{ij}(\xi) d\xi^i d\xi^j \\
 d &= \sqrt{g_{11}g_{22} - g_{12}^2} d\xi^1 d\xi^2 = \sqrt{\det g} d\xi^1 d\xi^2 \\
 |dA| &= \sqrt{g} d\xi^1 d\xi^2 \quad g \equiv \det g_{ij}(\xi)
 \end{aligned}$$

$$\begin{aligned}
 d\vec{s} &= g_{ij}(\xi) d\xi^i d\xi^j = \tilde{g}_{ij}(\tilde{\xi}) d\tilde{\xi}^i d\tilde{\xi}^j = \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j} d\xi^i d\xi^j \\
 g_{ij}(\xi) &= \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j}
 \end{aligned}$$

$$\begin{aligned}
 d\vec{s} &= d\vec{x} \cdot d\vec{x} = \left(\frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \right) d\xi^i d\xi^j & g_{ij}(\xi) &= \frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \\
 &= g_{ij}(\xi) d\xi^i d\xi^j \\
 d &= \sqrt{g_{11}g_{22} - g_{12}^2} d\xi^1 d\xi^2 = \sqrt{\det g} d\xi^1 d\xi^2 \\
 |dA| &= \sqrt{g} d\xi^1 d\xi^2 \quad g \equiv \det g_{ij}(\xi)
 \end{aligned}$$

$$\begin{aligned}
 d\vec{s} &= g_{ij}(\xi) d\xi^i d\xi^j = \tilde{g}_{ij}(\tilde{\xi}) d\tilde{\xi}^i d\tilde{\xi}^j = \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j} d\xi^i d\xi^j \\
 g_{ij}(\xi) &= \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j}
 \end{aligned}$$

$$\begin{aligned}
 d\vec{s} &= d\vec{x} \cdot d\vec{x} = \left(\frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \right) d\xi^i d\xi^j & g_{ij}(\xi) &= \frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \\
 &= g_{ij}(\xi) d\xi^i d\xi^j \\
 d &= \sqrt{g_{11}g_{22} - g_{12}^2} d\xi^1 d\xi^2 = \sqrt{\det g} d\xi^1 d\xi^2 \\
 dA &= \sqrt{g} d\xi^1 d\xi^2 \quad g = \det g_{ij}(\xi)
 \end{aligned}$$

$$d\vec{s}^2 = g_{ij}(\xi) d\xi^i d\xi^j = \tilde{g}_{ij}(\tilde{\xi}) d\tilde{\xi}^i d\tilde{\xi}^j = \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j} d\xi^i d\xi^j$$

$$g_{ij}(\xi) = \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j}$$

Ex

$$\int d\xi^1 d\xi^2$$

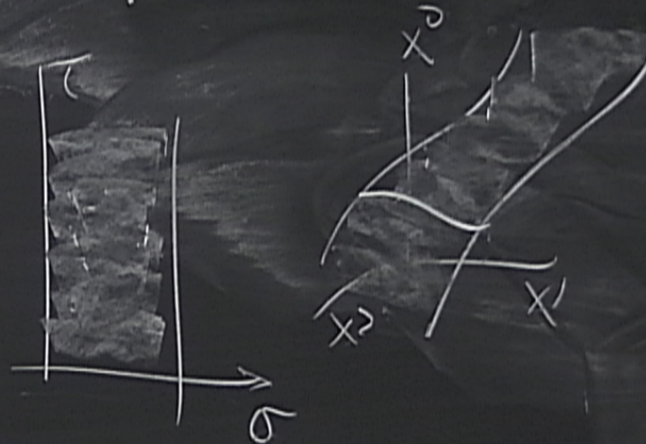
$$\begin{aligned}
 d\vec{s} &= d\vec{x} \cdot d\vec{x} = \left(\frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \right) d\xi^i d\xi^j & g_{ij}(\xi) &= \frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} \\
 &= g_{ij}(\xi) d\xi^i d\xi^j \\
 d &= \sqrt{g_{11}g_{22} - g_{12}^2} d\xi^1 d\xi^2 = \sqrt{\det g} d\xi^1 d\xi^2 \\
 dA &= \sqrt{g} d\xi^1 d\xi^2 \quad g = \det g_{ij}(\xi)
 \end{aligned}$$

$$d\vec{s}^2 = g_{ij}(\xi) d\xi^i d\xi^j = \tilde{g}_{ij}(\tilde{\xi}) d\tilde{\xi}^i d\tilde{\xi}^j = \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j} d\xi^i d\xi^j$$

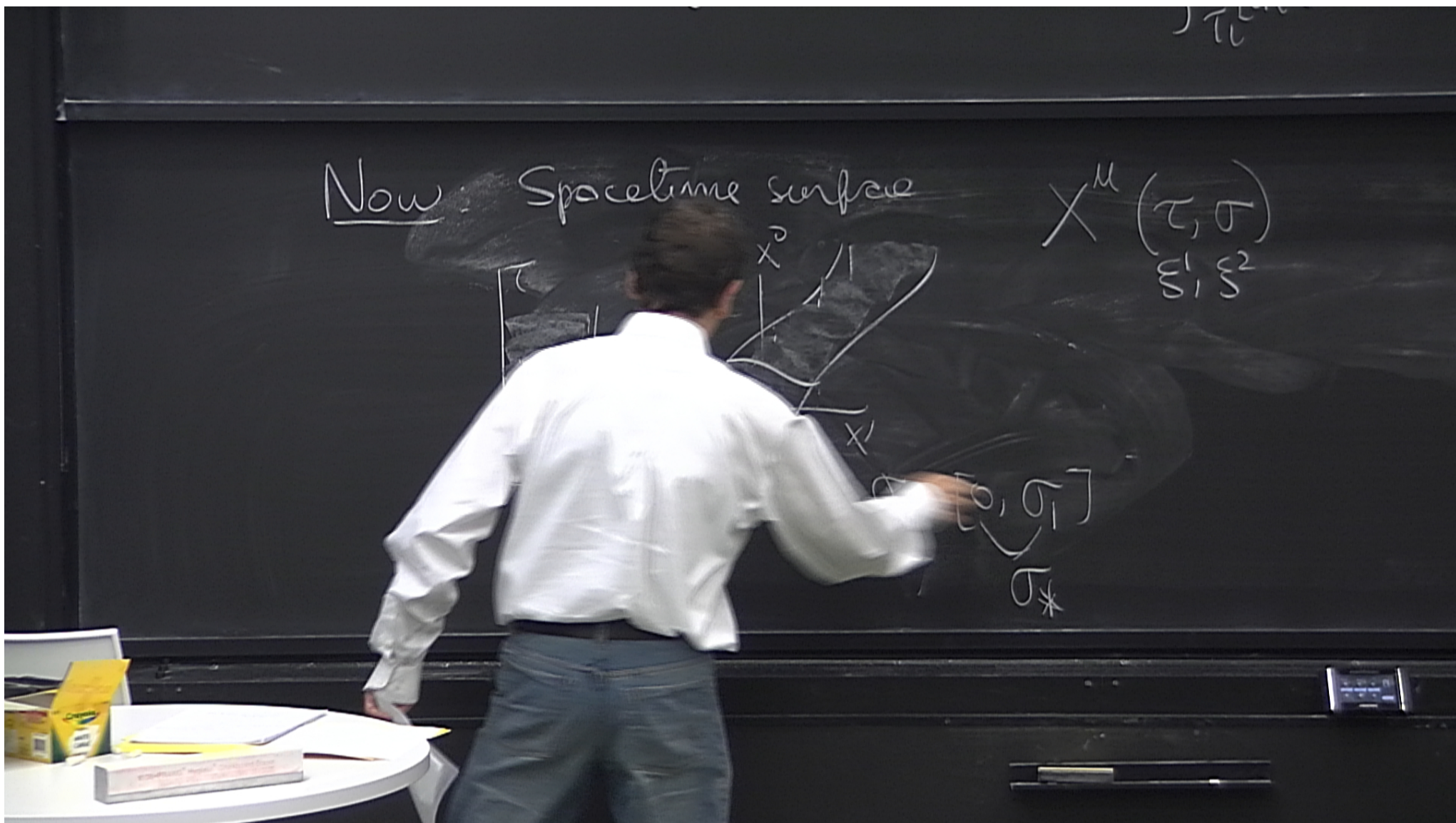
$$g_{ij}(\xi) = \tilde{g}_{pq}(\tilde{\xi}) \frac{\partial \tilde{\xi}^p}{\partial \xi^i} \frac{\partial \tilde{\xi}^q}{\partial \xi^j} \quad \left(dx \sqrt{g} \right)$$

$$\text{Ex} \quad \int d\xi^1 d\xi^2 \sqrt{g} = \int d\tilde{\xi}^1 d\tilde{\xi}^2 \sqrt{\tilde{g}}$$

Now: Spacetime surface

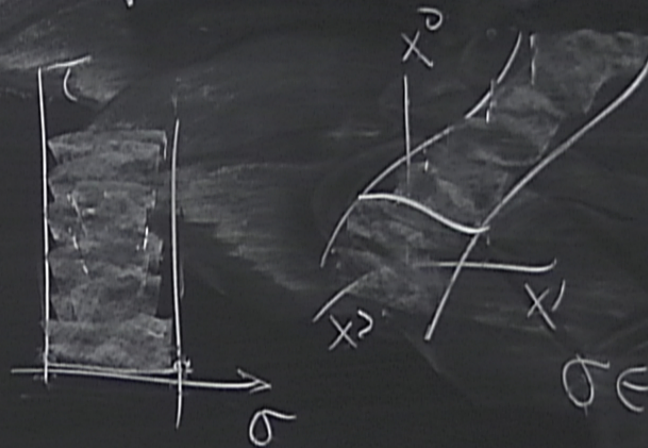


$$X^\mu(\tau, \sigma)$$
$$\xi^1, \xi^2$$



Now: Spacetime surface

$$X^\mu(\tau, \sigma)$$
$$\xi^1, \xi^2$$

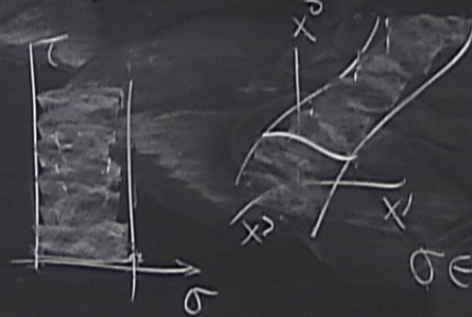


$$\sigma \in [0, \sigma_1]$$
$$\sigma_*$$



$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left(\frac{d}{d\tau} \delta x^\mu \right) P_\mu = \overbrace{\int_{\tau_i}^{\tau_f} \left[\frac{d}{d\tau} (\delta x^\mu P_\mu) - \delta x^\mu \frac{dP_\mu}{d\tau} \right] d\tau}^{P_\mu} = \left[\delta x^\mu P_\mu \right]_{\tau_i}^{\tau_f} - \int d$$

Now: Spacetime surface

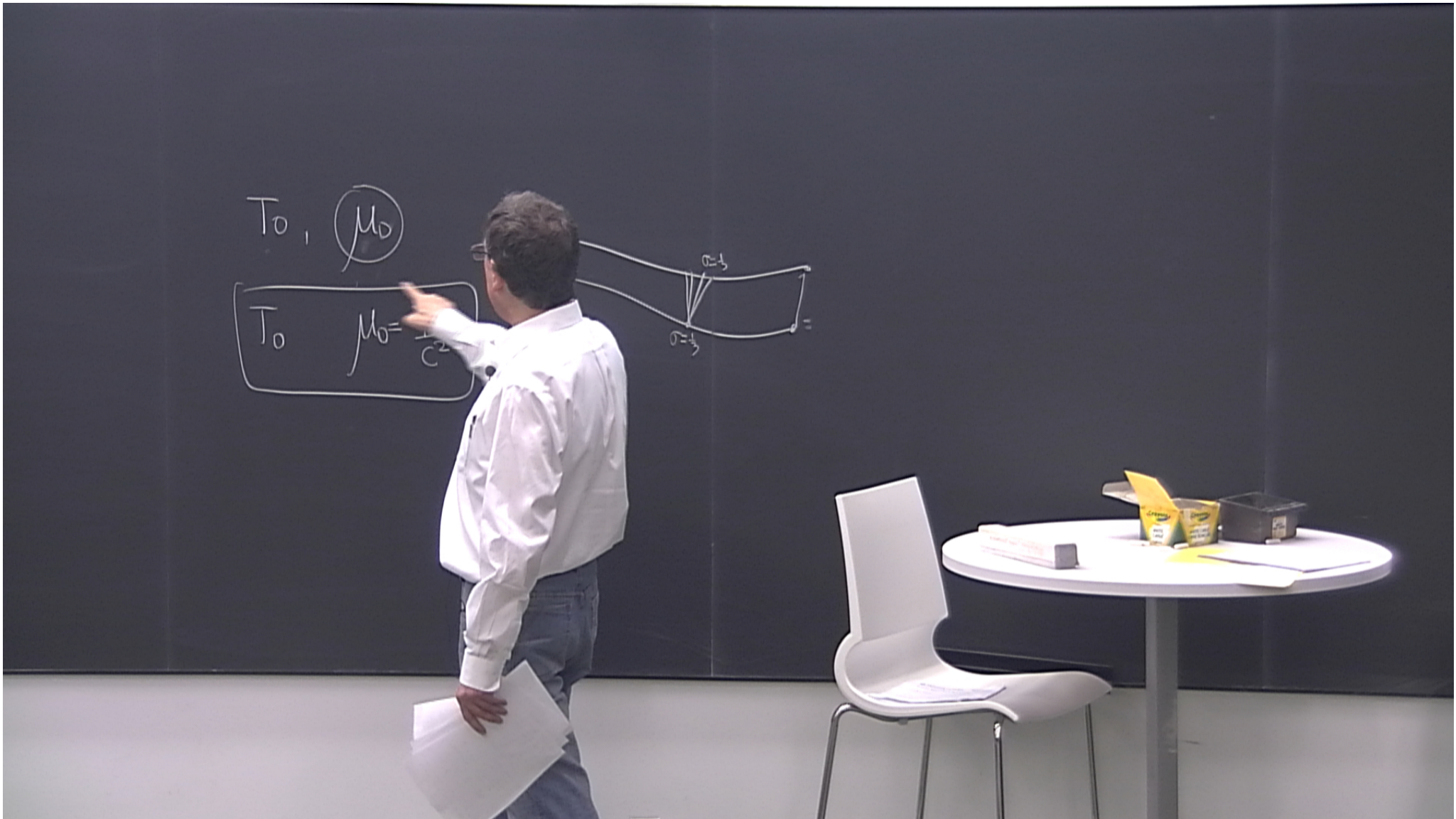


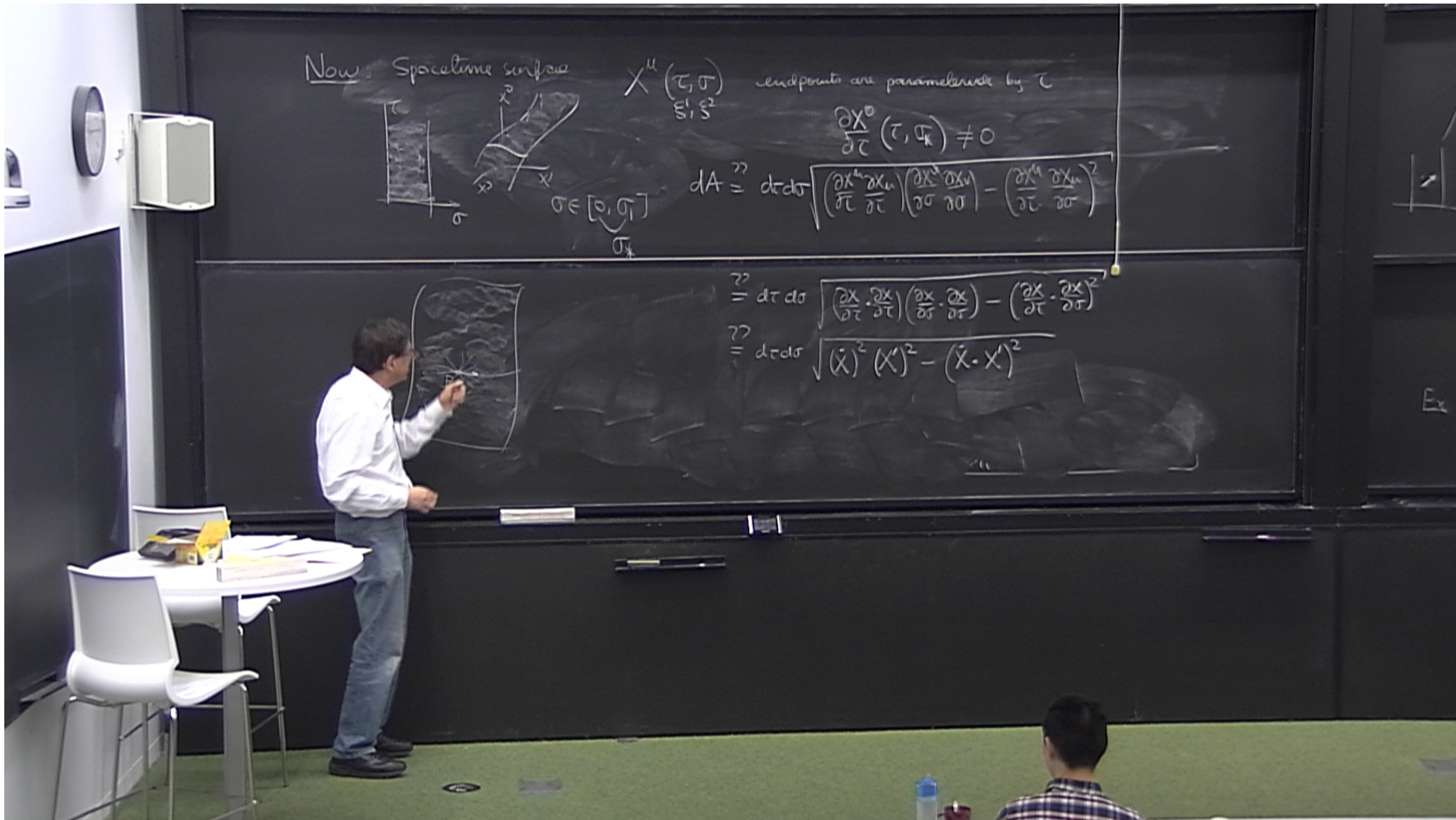
$$\sigma \in [0, \sigma_1]$$

$$X^\mu(\tau, \sigma)$$

endpoints are parameterized by τ

$$\frac{\partial X^0}{\partial \tau}(\tau, \sigma_*) \neq 0$$







T_0, μ_0

$T_0 \quad \mu_0 = \frac{T_0}{c^2}$

