

Title: 12/13 PSI - String Theory Review Lecture 2

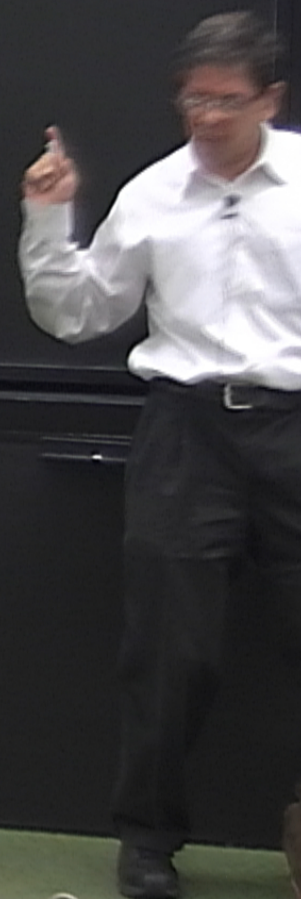
Date: Jan 08, 2013 10:15 AM

URL: <http://pirsa.org/13010044>

Abstract:

Orbifolds

spaces that arise from identification with fixed points



Orbifolds

spaces that arise from identifications with fixed points

$\frac{p}{q}$

x

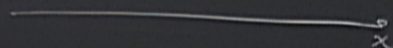
Orbifolds

spaces that arise from identifications with fixed points

$$x \sim -x$$

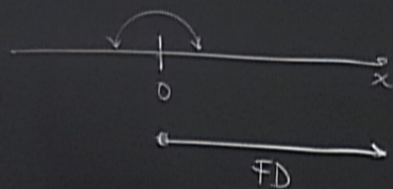
$$T: x \rightarrow -x$$

a $\mathbb{Z}_2$ transformation $\{1, T\}$



Orbifolds

spaces that arise from identifications with fixed points



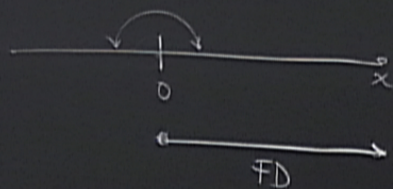
$x \sim -x$
Fixed point is $x=0$

$T: x \rightarrow -x$ a $\mathbb{Z}_2$ transformation $\{1, T\}$

$\mathbb{R}/\mathbb{Z}_2$ "half line" $\{x \geq 0\}$

Orbifolds

spaces that arise from identifications with fixed points



$x \sim -x$
Fixed point is $x=0$

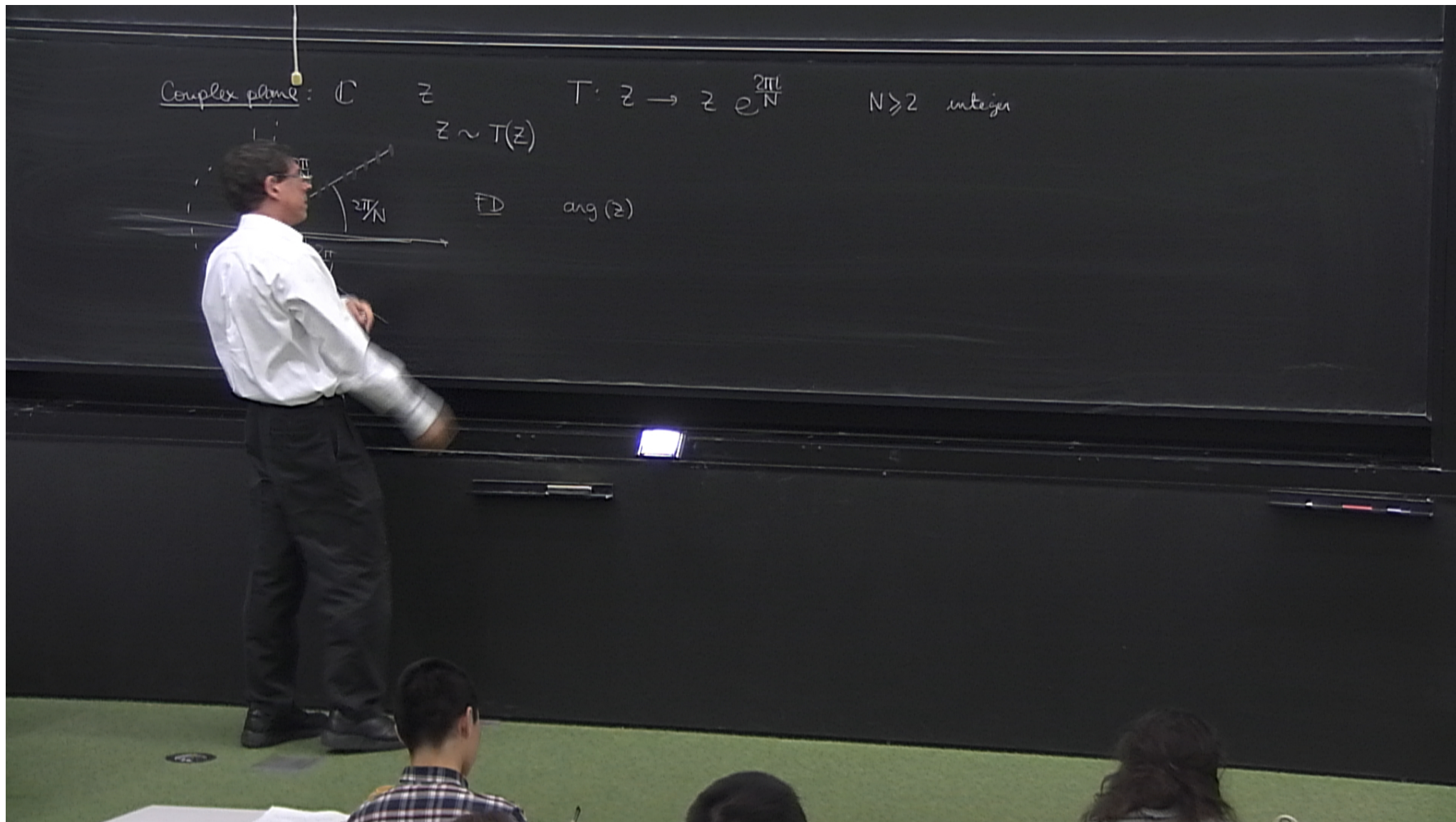
$T: x \rightarrow -x$ a $\mathbb{Z}_2$ transformation $\{1, T\}$

$\mathbb{R}/\mathbb{Z}_2$ "half line" $\{x \geq 0\}$

Complex plane: $\mathbb{C}$ z T

Complex plane: $\mathbb{C}$ z $T: z \rightarrow z e^{\frac{2\pi i}{N}}$ $N \geq 2$ integer
 $z \sim T(z)$

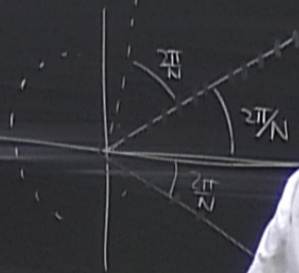
FD $\arg(z)$

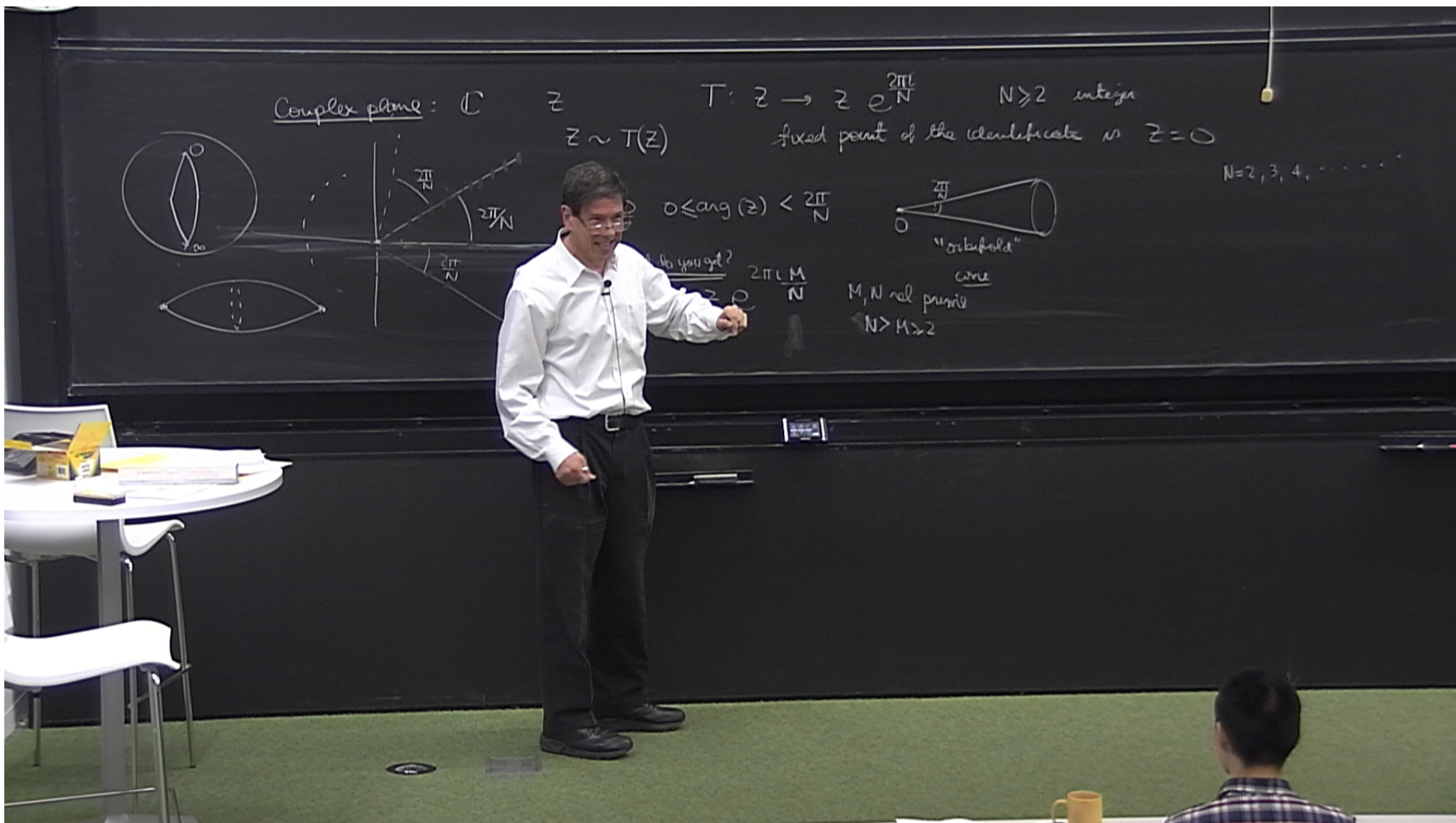


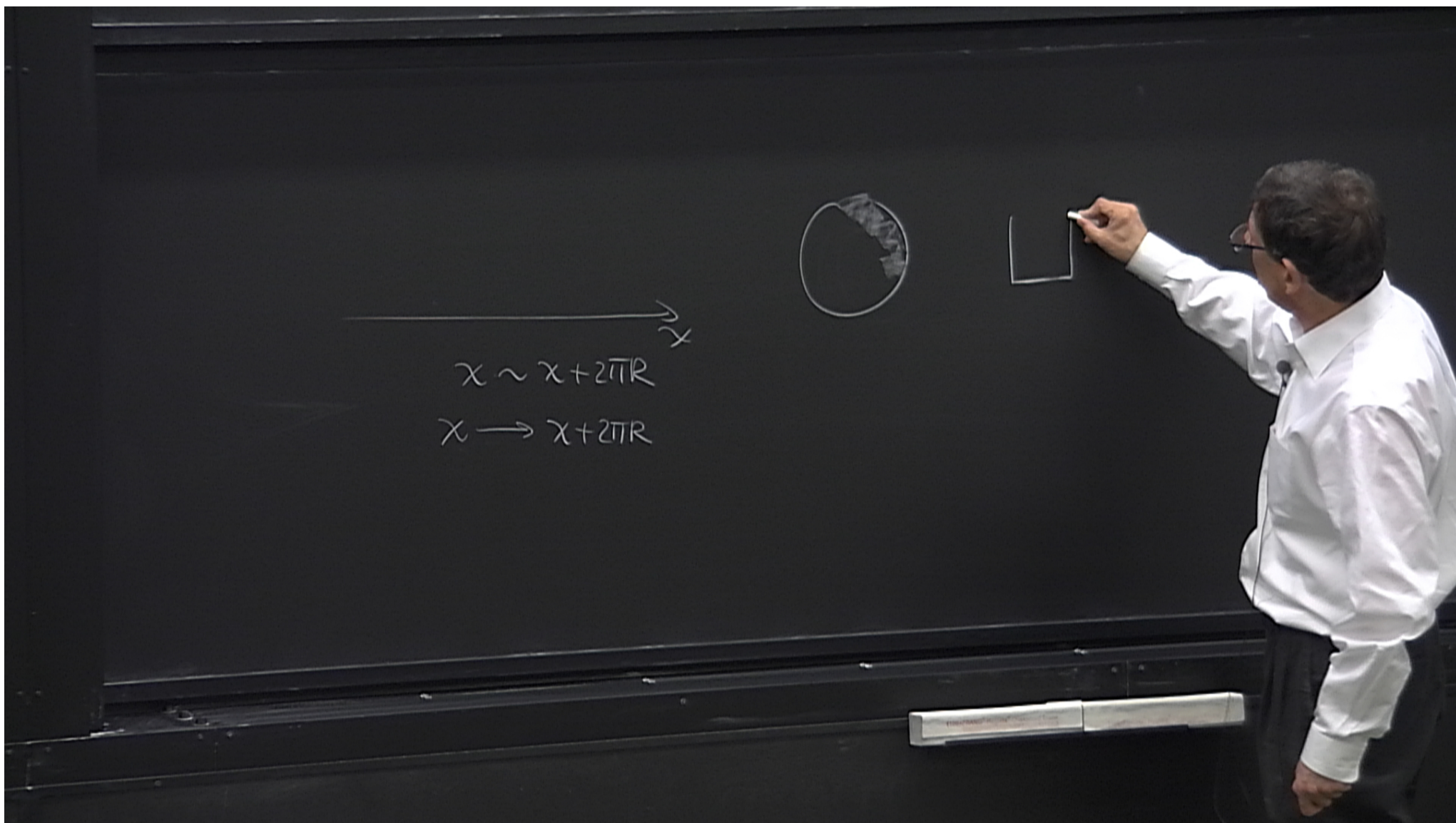
Complex plane: $\mathbb{C}$ z $T: z \rightarrow z e^{\frac{2\pi i}{N}}$ $N \geq 2$ integer

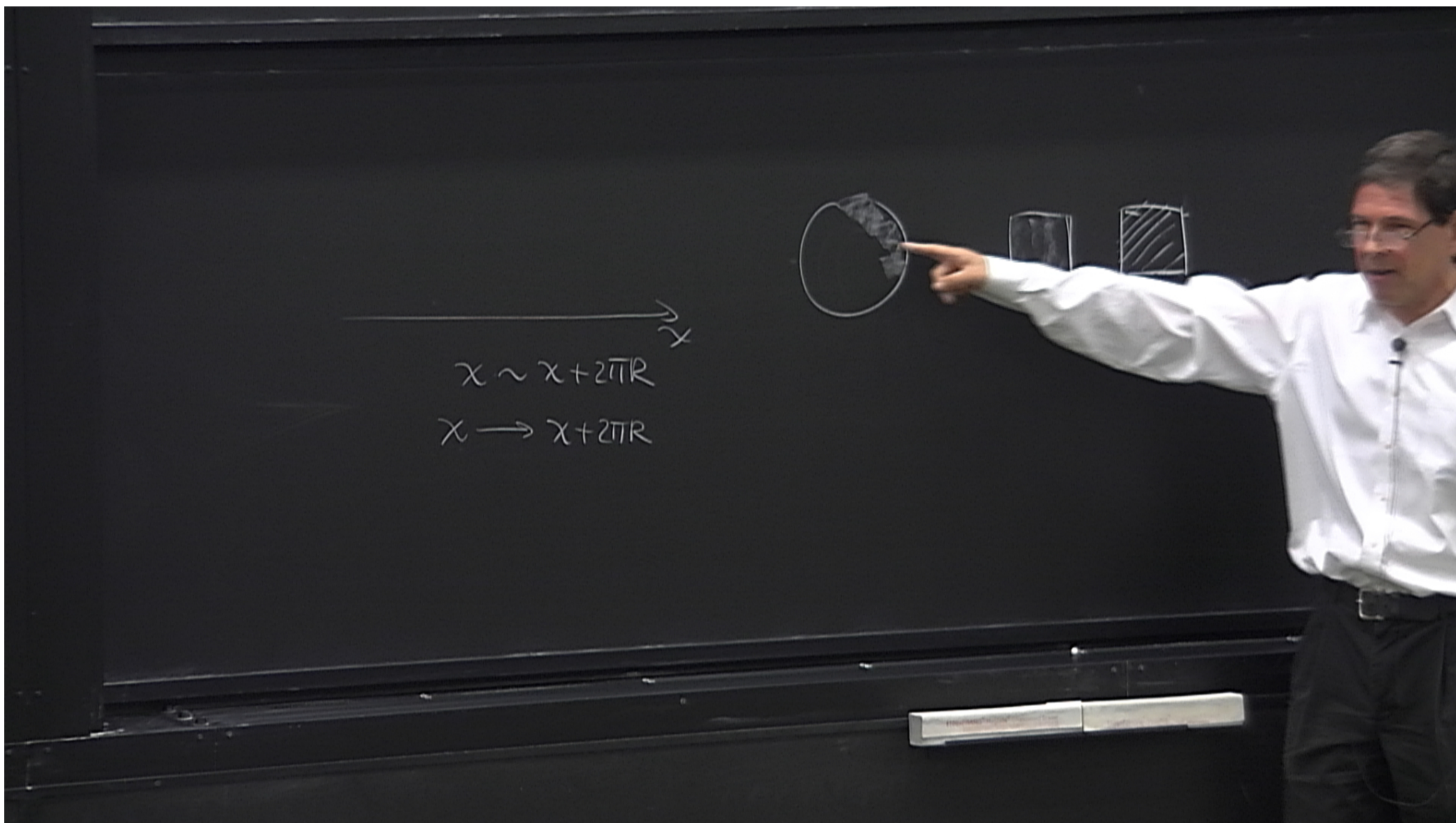
$$z \sim T(z)$$

$$0 \leq \arg(z) <$$

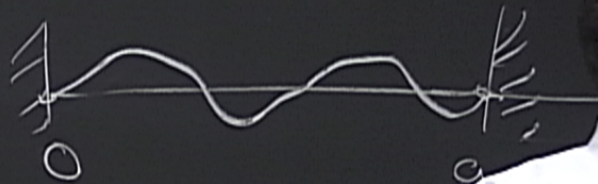






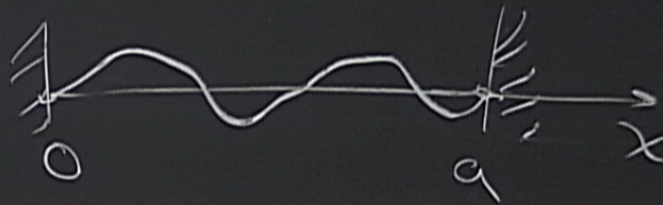


NR Strings



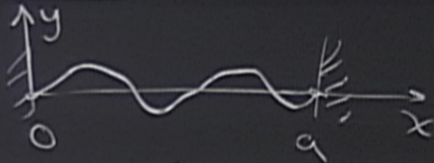
μ

NR Strings



μ_0 = mass per unit length
 T_0 = tension (F)

NR Strings



$y(x,t)$

$\mu_0 = \text{mass per unit length}$

$T_0 = \text{tension (F)}$

$\left| \frac{\partial y}{\partial x} \right| \ll 1$ Small oscillations

$$S = \int L dt \quad L = T - V = \int_0^a \frac{1}{2} \mu_0 dx \left(\frac{\partial y}{\partial t} \right)^2$$

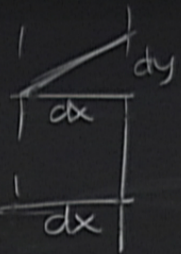
stretch

$$\Delta l = \sqrt{(dx)^2 + (dy)^2} - dx$$

$$= dx \sqrt{1 + \left(\frac{\partial y}{\partial x} \right)^2} - dx$$

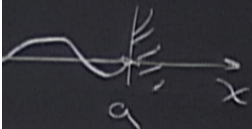
length

stretch


$$\begin{aligned}\Delta l &= \sqrt{(dx)^2 + (dy)^2} - dx \\ &= dx \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2} - dx \\ &\approx dx \frac{1}{2} \left(\frac{\partial y}{\partial x}\right)^2\end{aligned}$$

all oscillation
a

$$\frac{1}{2} \mu_0 dx \left(\frac{\partial y}{\partial t}\right)^2$$



μ_0 = mass per unit length
 T_0 = tension (F)

$\left| \frac{\partial y}{\partial x} \right| \ll 1$ Small oscillation

$y(x, t)$

$$L = T - V = \int_0^a \left(\frac{1}{2} \mu_0 dx \left(\frac{\partial y}{\partial t} \right)^2 - \frac{1}{2} T_0 dx \left(\frac{\partial y}{\partial x} \right)^2 \right)$$

$$S = \int L dt = \int_0^a dx \left(\frac{1}{2} \mu_0 \left(\frac{\partial y}{\partial t} \right)^2 - \frac{1}{2} T_0 \left(\frac{\partial y}{\partial x} \right)^2 \right) = \int_0^a dx \mathcal{L}(\dot{y}, y')$$

stretch

$$\Delta l = \sqrt{(dx)^2 + (dy)^2} - dx$$

$$= dx \sqrt{1 + \left(\frac{\partial y}{\partial x} \right)^2} - dx$$

$$= dx \frac{1}{2} \left(\frac{\partial y}{\partial x} \right)^2$$

$$\dot{y} = \frac{\partial y}{\partial t}$$

$$y' = \frac{\partial y}{\partial x}$$

$$\Theta^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t}$$

$$S = \int_{t_i}^{t_f} dt \int_{x_i}^{x_f} \mathcal{L}(\dot{y}, y')$$

$$\delta S = \int_{t_i}^{t_f} dt \int_{x_i}^{x_f} \delta \mathcal{L}$$

$$\Theta^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$$

$$y \rightarrow y + \delta y \quad (x, t)$$

$$\Theta^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t}$$

$$S = \int dx \mathcal{L}(\dot{y}, y')$$

$$\mathcal{P}^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$$

$$y \rightarrow y + \delta y \quad (x, t)$$

$$\int dx \left(\frac{\partial \mathcal{L}}{\partial \dot{y}} \delta \dot{y} + \frac{\partial \mathcal{L}}{\partial y'} \delta y' \right) = \int dt \int dx \left(\mathcal{P}^t \frac{\partial \delta y}{\partial t} + \mathcal{P}^x \frac{\partial \delta y}{\partial x} \right)$$

$$\Theta^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t}$$

$$S = \int_{t_0}^{t_1} dx \mathcal{L}(\dot{y}, y')$$

$$\mathcal{P}^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$$

$$y \rightarrow y + \delta y \quad (x, t)$$

$$\int_{t_0}^{t_1} dx \left(\frac{\partial \mathcal{L}}{\partial \dot{y}} \delta \dot{y} + \frac{\partial \mathcal{L}}{\partial y'} \delta y' \right) = \int dt \int dx \left(\mathcal{P}^t \frac{\partial \delta y}{\partial t} + \mathcal{P}^x \frac{\partial \delta y}{\partial x} \right)$$

$$\mathcal{P}^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t}$$

$$\mathcal{P}^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$$

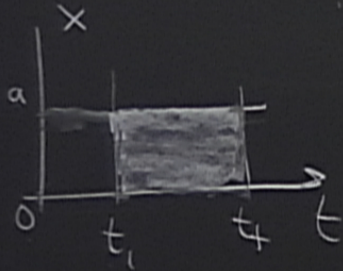
$$S = \int_{t_i}^{t_f} dt \int_a^b dx \mathcal{L}(\dot{y}, y')$$

$$y \rightarrow y + \delta y \quad (x, t)$$

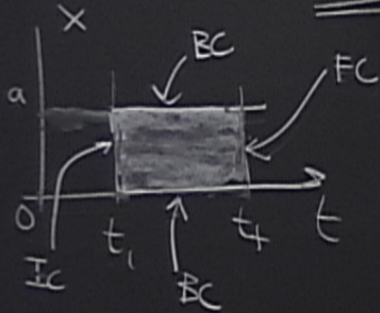
$$\begin{aligned} \delta S &= \int_{t_i}^{t_f} dt \int_a^b dx \left(\frac{\partial \mathcal{L}}{\partial \dot{y}} \delta \dot{y} + \frac{\partial \mathcal{L}}{\partial y'} \delta y' \right) = \int dt \int dx \left(\mathcal{P}^t \frac{\partial \delta y}{\partial t} + \mathcal{P}^x \frac{\partial \delta y}{\partial x} \right) \\ &= \int dt \int dx \left[\frac{\partial}{\partial t} (\mathcal{P}^t \delta y) + \frac{\partial}{\partial x} (\mathcal{P}^x \delta y) - \delta y \left(\frac{\partial \mathcal{P}^t}{\partial t} + \frac{\partial \mathcal{P}^x}{\partial x} \right) \right] \end{aligned}$$

$$= \int_0^a dx (\Theta^t \delta y) \Big|_{t_i}^{t_f} + \int_{x=0}^{x=a} \Theta^x \delta y$$

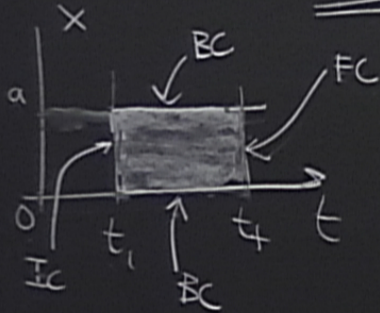
$$= \int_0^a dx (\Theta^t \delta y) \Big|_{t_i}^{t_f} + \int_{t_i}^{t_f} dt \Theta^x \delta y \Big|_{x=0}^{x=a} - \int dt \int dx \delta y \left(\frac{\partial \Theta^t}{\partial t} + \frac{\partial \Theta^x}{\partial x} \right)$$



$$= \int_0^a dx (\Theta^t \delta y) \Big|_{t_i}^{t_f} + \int_{t_i}^{t_f} dt \Theta^x \delta y \Big|_{x=0}^{x=a} - \int dt \int dx \delta y \left(\frac{\partial \mathcal{P}^t}{\partial t} + \frac{\partial \mathcal{P}^x}{\partial x} \right)$$



$$= \int_0^a dx \left(\Theta^t \delta y \right) \Big|_{t_i}^{t_f} + \underbrace{\int_{t_i}^{t_f} dt \left(\Theta^x \delta y \right) \Big|_{x=0}^{x=a}}_{\substack{\text{forget it} \\ \int_{t_i}^{t_f} dt \left(\Theta^x(t, x=a) \delta y(t, x=a) - \Theta^x(t, x=0) \delta y(t, x=0) \right)}}$$



Free b.c. $\Phi^X(t, x_*) = 0 \longrightarrow \frac{\partial y}{\partial x} \Big|_x$

Free bc. $\underline{\phi^x(t, x_*) = 0} \rightarrow \left. \frac{\partial y}{\partial x} \right|_{x=x_*} = 0 \rightarrow \underline{\text{Neumann BC}}$

Constrain the endpoint

$$\delta y(t, x_*) = 0$$

$$y(t, x_*) = y_* = \# \quad \text{Dirichlet}$$

$$\left. \frac{\partial y}{\partial t} \right|_{x=x_*} = 0 \rightarrow \phi^t(L)$$

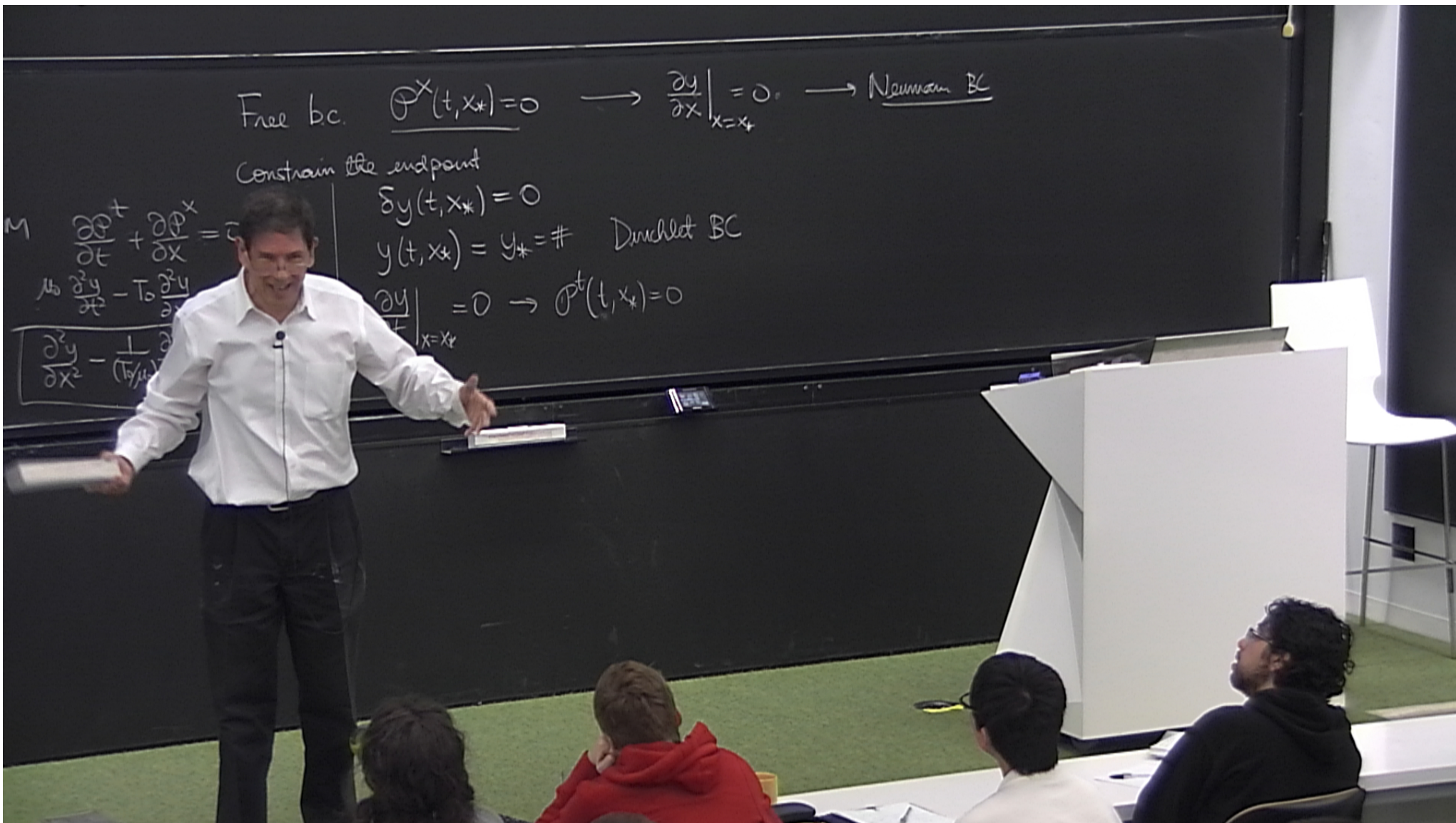
Free b.c. $\phi^x(t, x_*) = 0 \rightarrow \frac{\partial y}{\partial x} \Big|_{x=x_*} = 0 \rightarrow \text{Neumann BC}$

Constrain the endpoint

$$\delta y(t, x_*) = 0$$

$$y(t, x_*) = y_* = \# \quad \text{Dirichlet BC}$$

$$\frac{\partial y}{\partial t} \Big|_{x=x_*} = 0 \rightarrow \phi^t(t, x_*) = 0$$



Free bc. $\phi^x(t, x_*) = 0 \rightarrow \frac{\partial y}{\partial x} \Big|_{x=x_*} = 0 \rightarrow \text{Neumann BC}$

Constrain the endpoint

$$\delta y(t, x_*) = 0$$

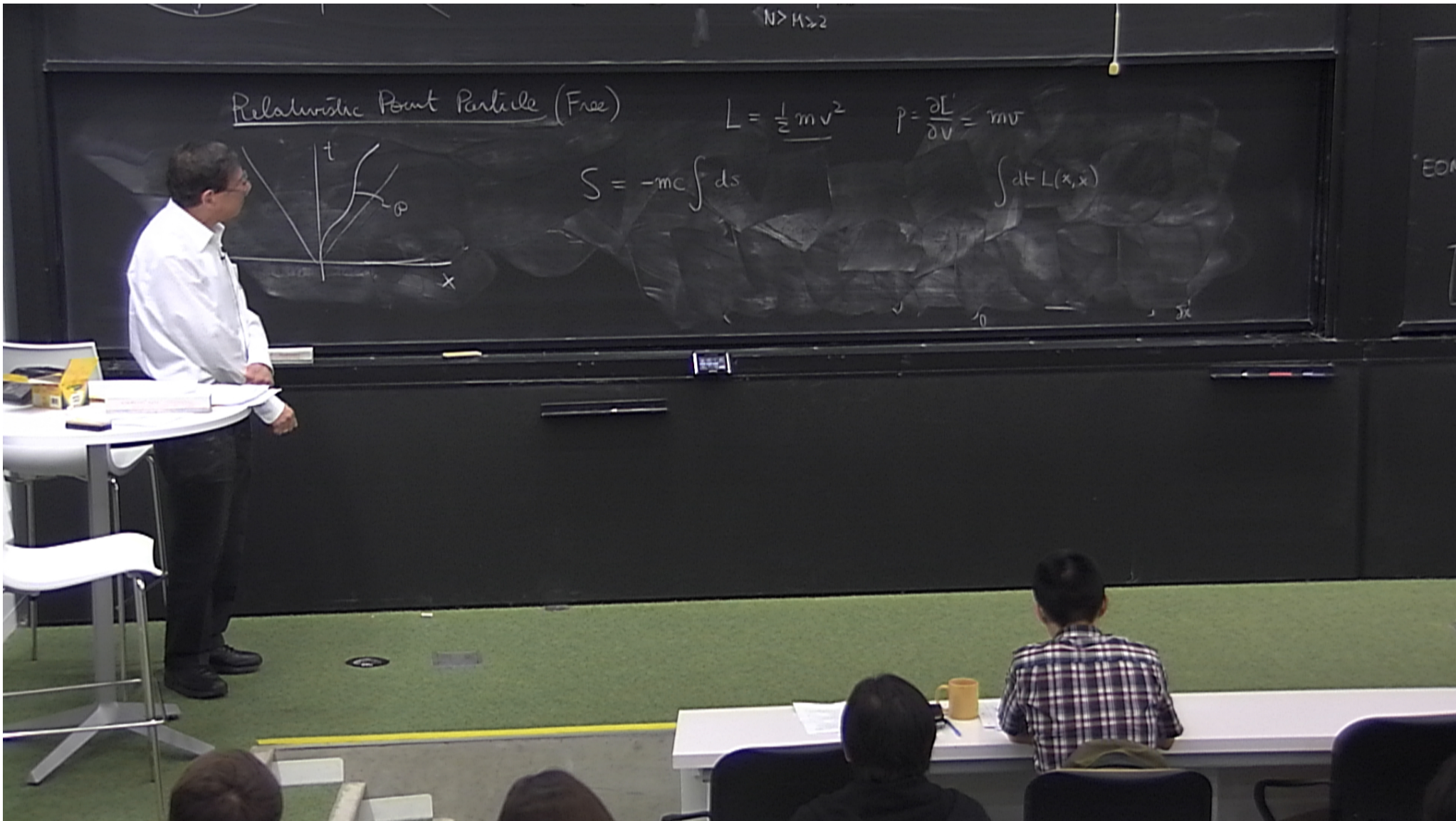
$$y(t, x_*) = y_* = \# \quad \text{Dirichlet BC}$$

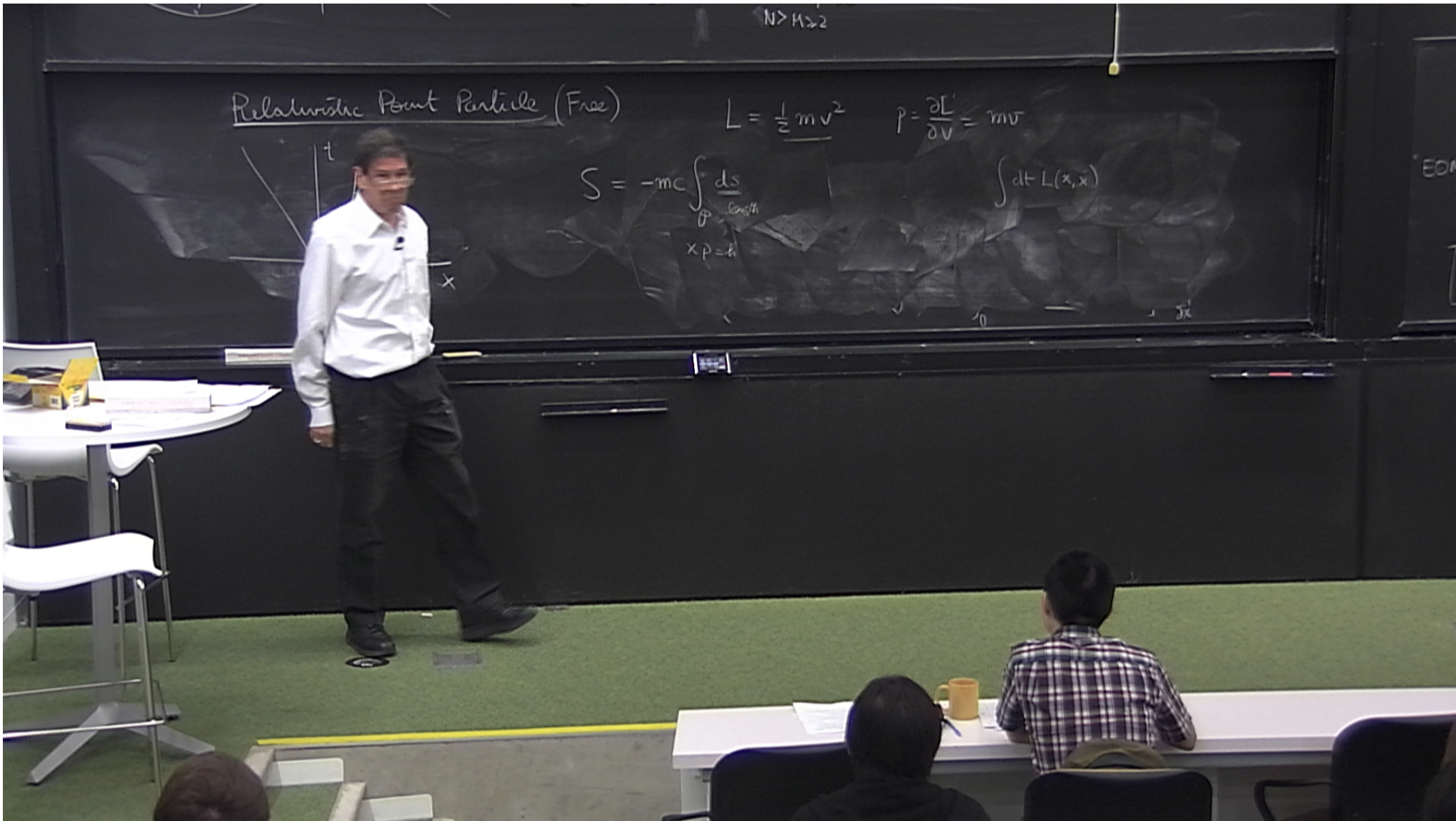
$$\frac{\partial y}{\partial t} \Big|_{x=x_*} = 0 \rightarrow \phi^t(t, x_*) = 0$$

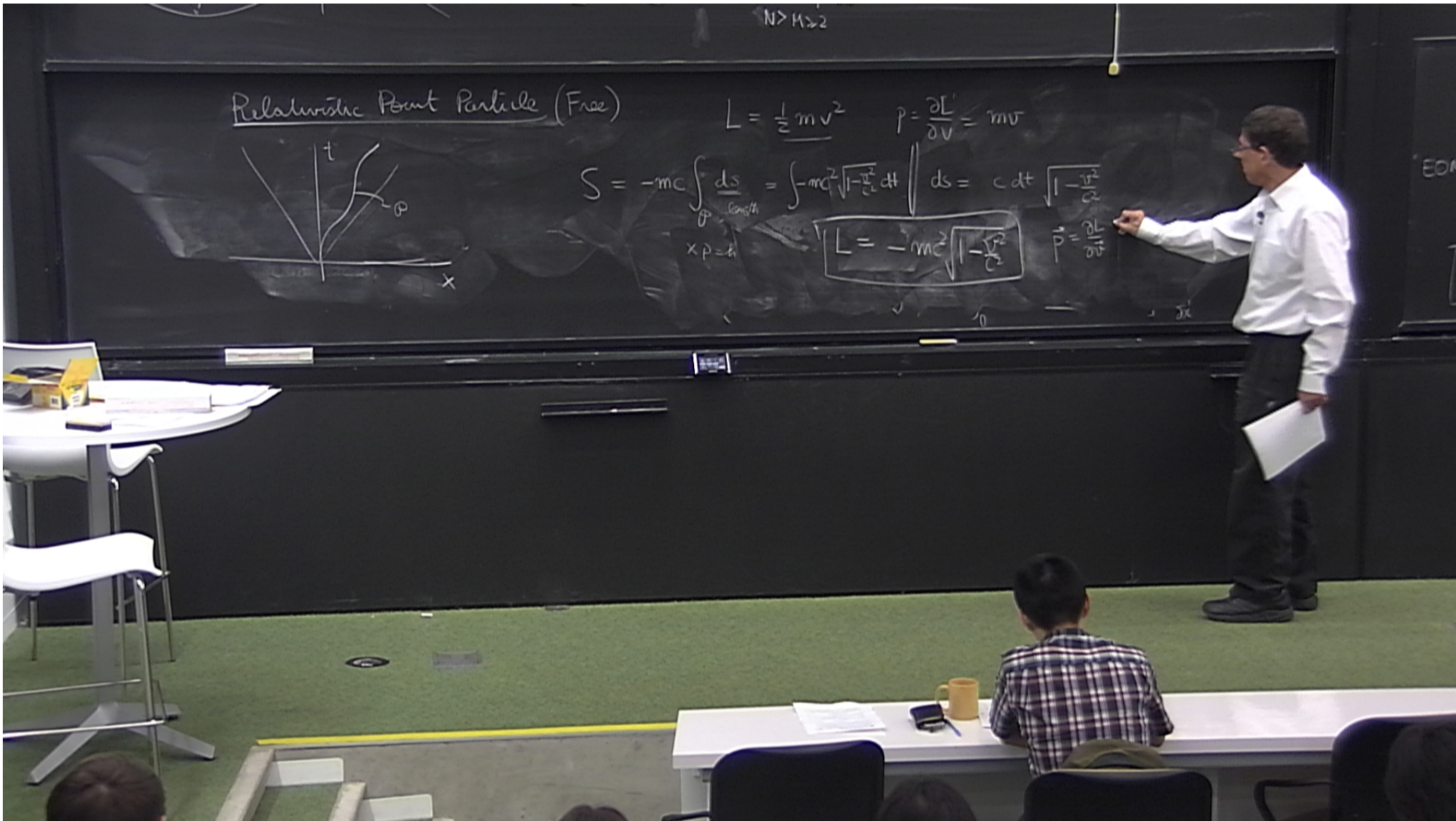
$$M \frac{\partial \phi^t}{\partial t} + \frac{\partial \phi^x}{\partial x} =$$

$$\mu_0 \frac{\partial^2 y}{\partial t^2} - T_0 \frac{\partial^2 y}{\partial x^2}$$

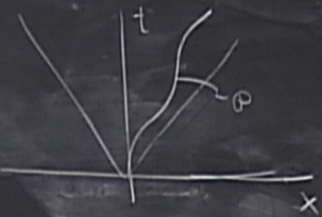
$$\left[\frac{\partial^2 y}{\partial x^2} - \frac{1}{(T_0/\mu_0)} \frac{\partial^2 y}{\partial t^2} \right]$$







Relativistic Point Particle (Free)



$$L = \frac{1}{2} m v^2 \quad p = \frac{\partial L}{\partial v} = m v$$

$$S = -m c \int \frac{ds}{\hbar} = \int -m c^2 \sqrt{1 - \frac{v^2}{c^2}} dt \quad ds = c dt \sqrt{1 - \frac{v^2}{c^2}}$$
$$x p = \hbar$$
$$L = -m c^2 \sqrt{1 - \frac{v^2}{c^2}} \quad \vec{p} = \frac{\partial L}{\partial \vec{v}}$$

