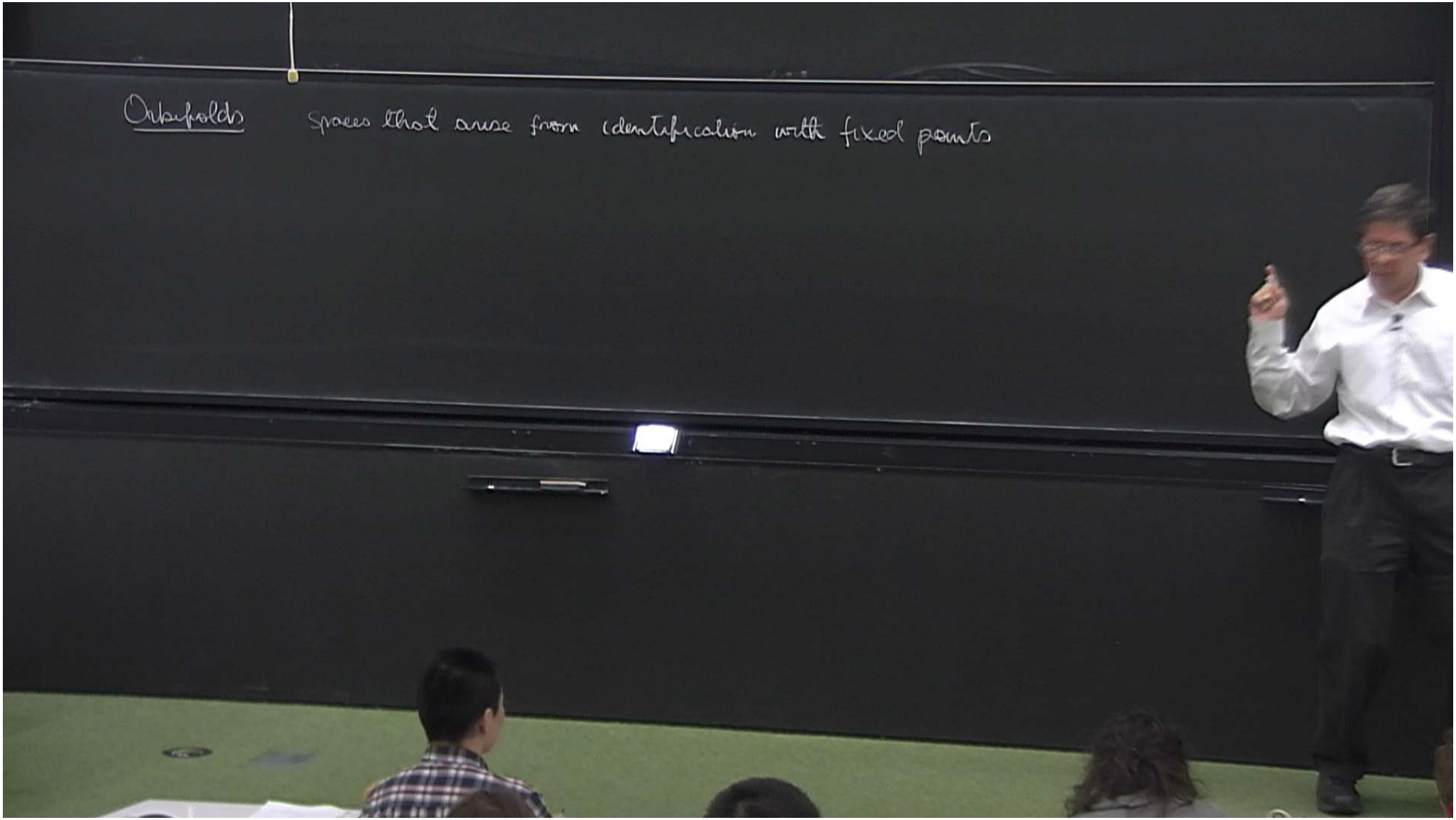
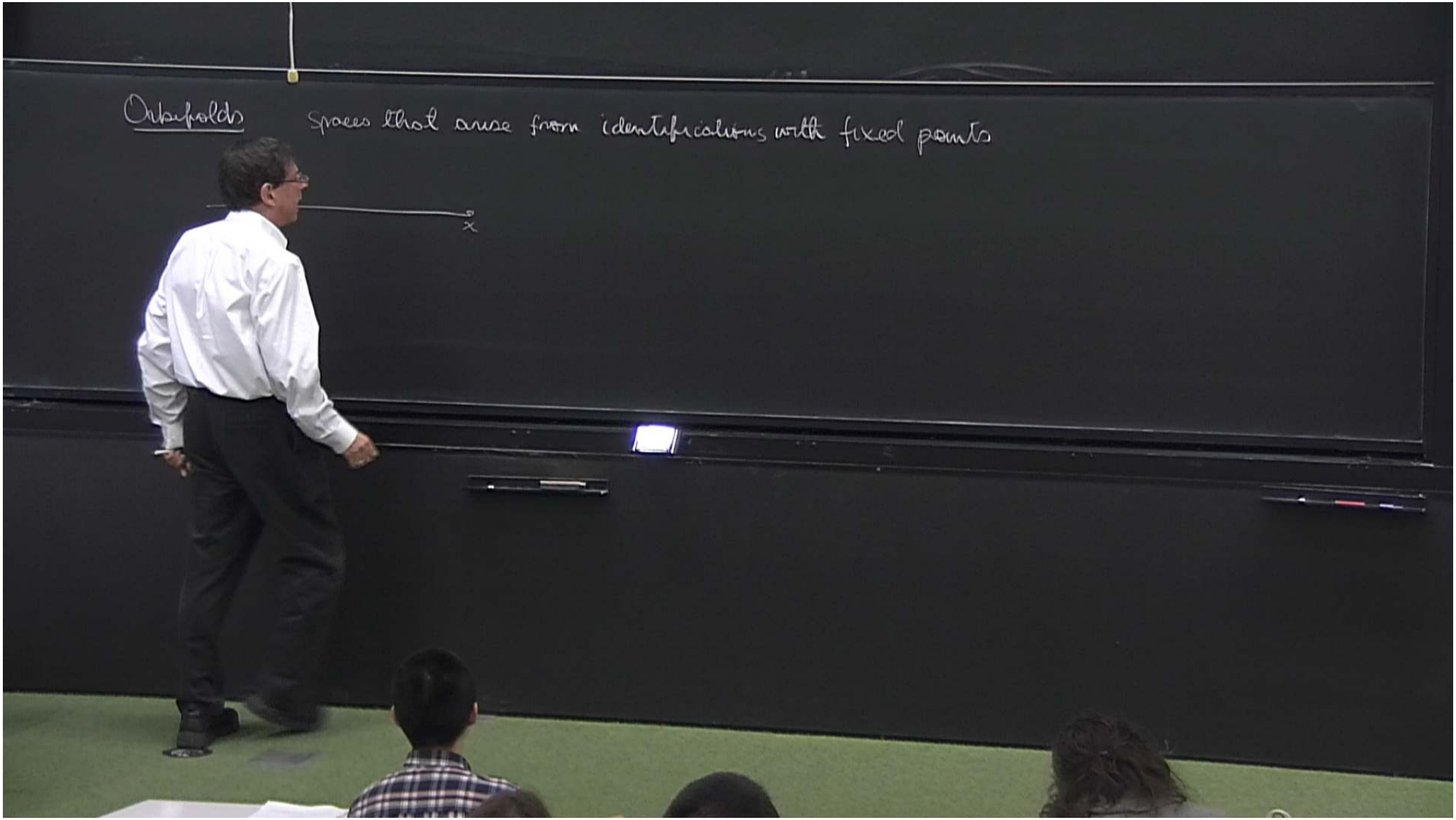
Title: 12/13 PSI - String Theory Review Lecture 2

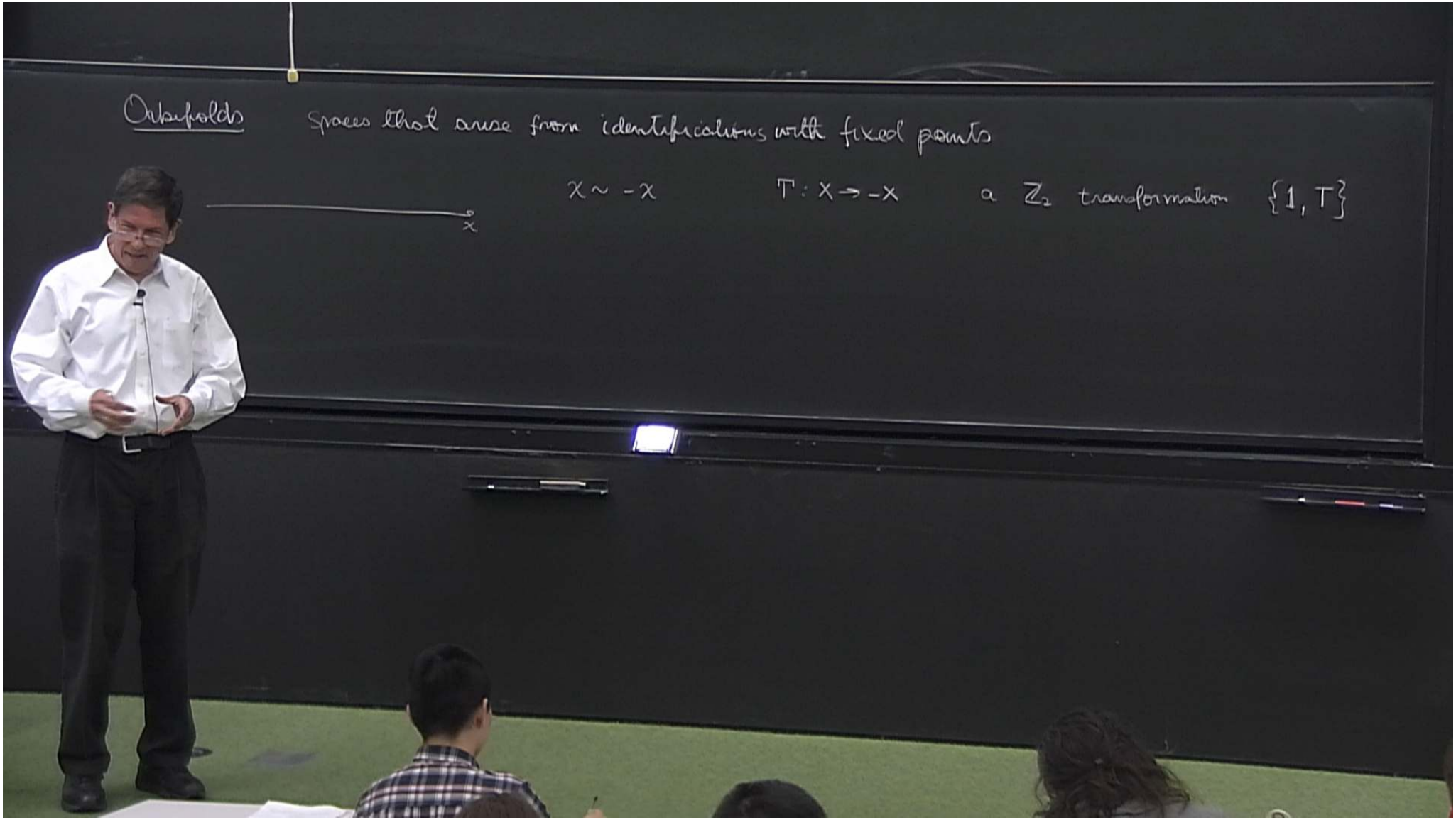
Date: Jan 08, 2013 10:15 AM

URL: http://pirsa.org/13010044

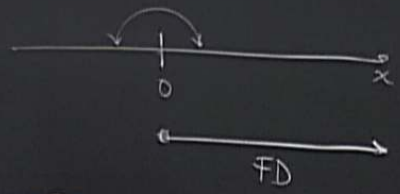
Abstract:


Orbifolds
spaces that arise from identification with fixed points


Orbifolds
Spaces that arise from identifications with fixed points
x


Orbifolds
Spaces that arise from identifications with fixed points
$x \sim -x$
$T: x \to -x$
a $\mathbb{Z}_2$ transformation
$\{1, T\}$
x

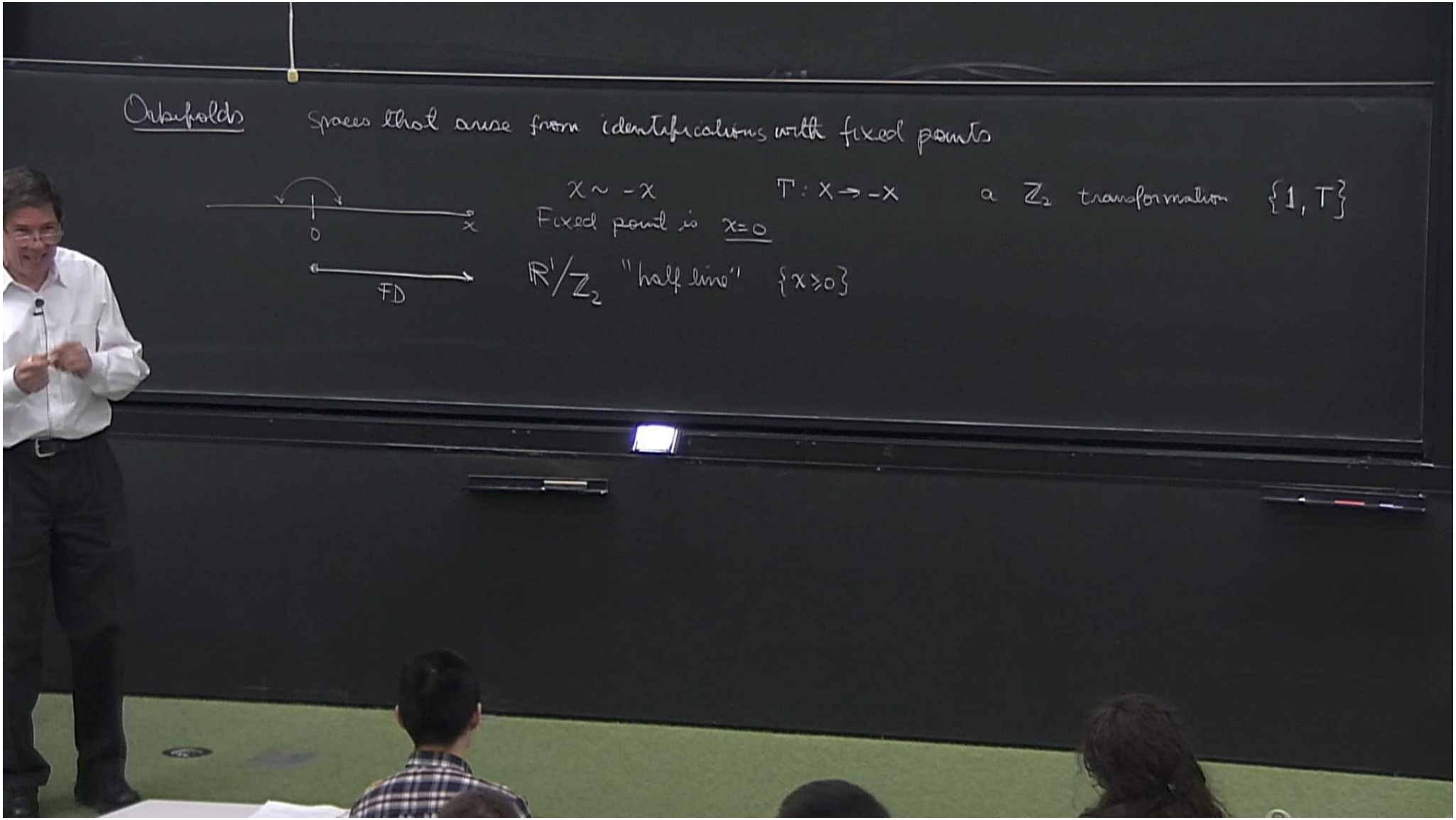
Orbifolds Spaces that arise from identifications with fixed points

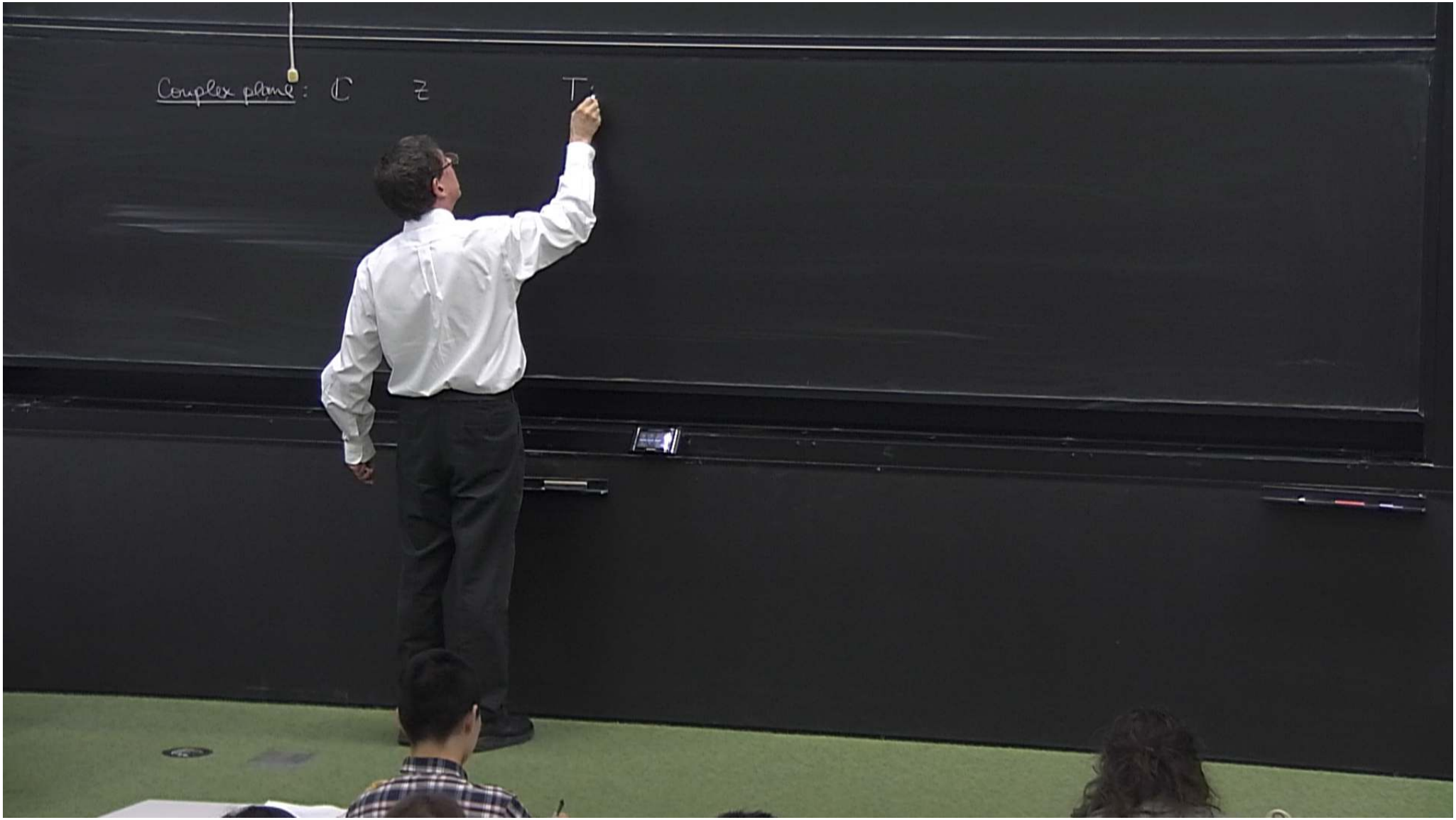


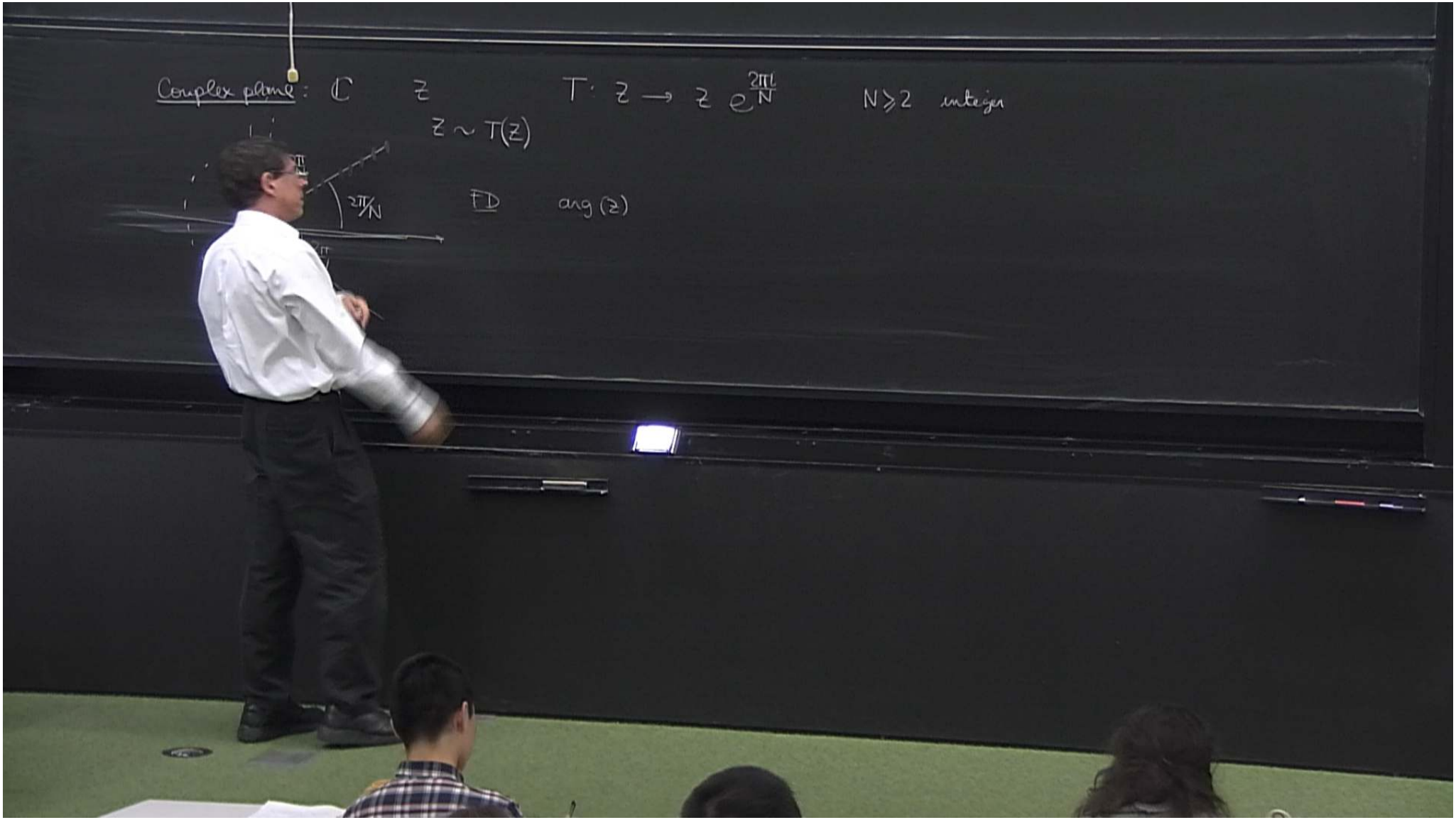
$x \sim -x$ $T: x \to -x$ a $\mathbb{Z}_2$ transformation $\{1, T\}$

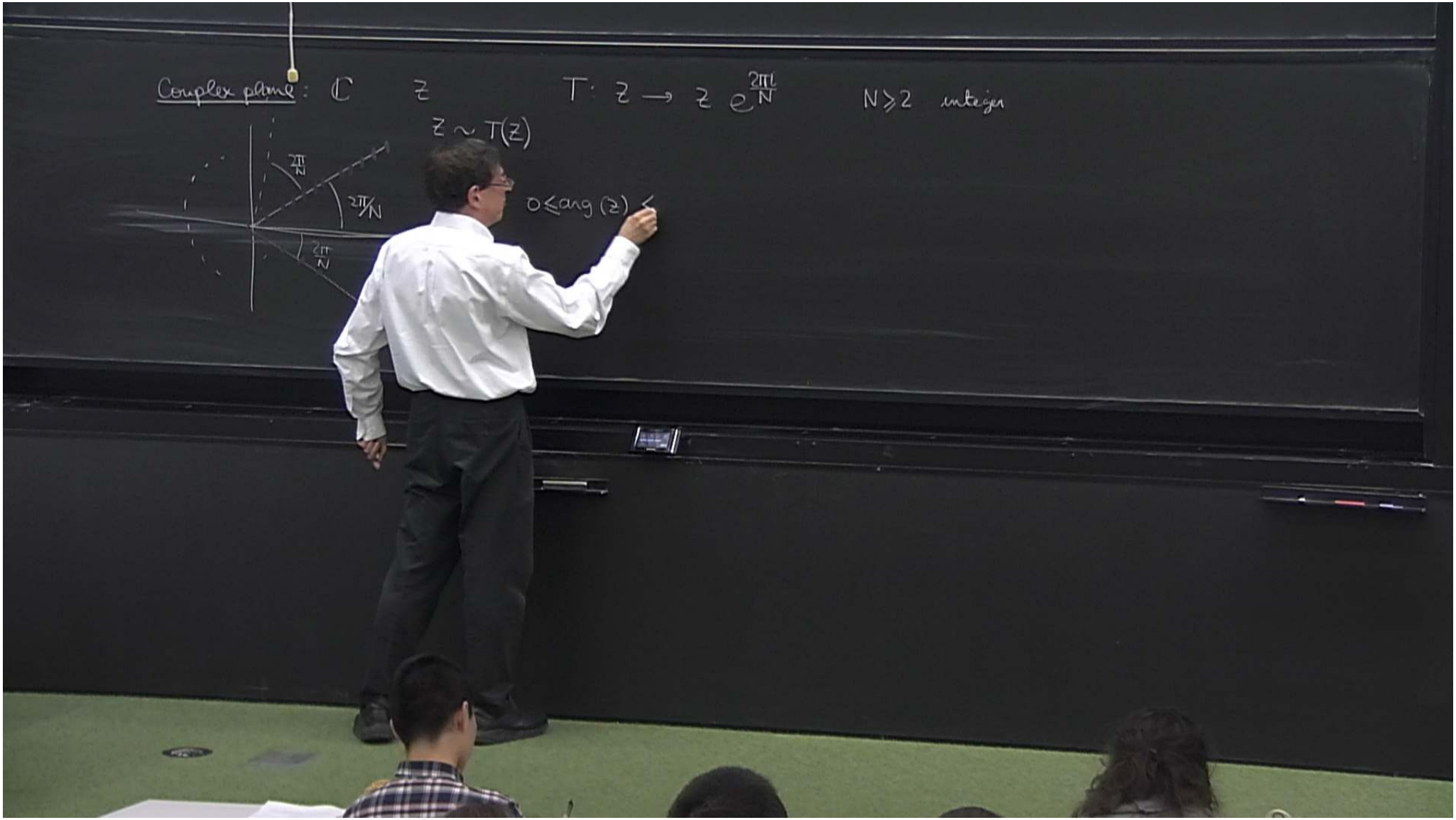
Fixed point is $x=0$

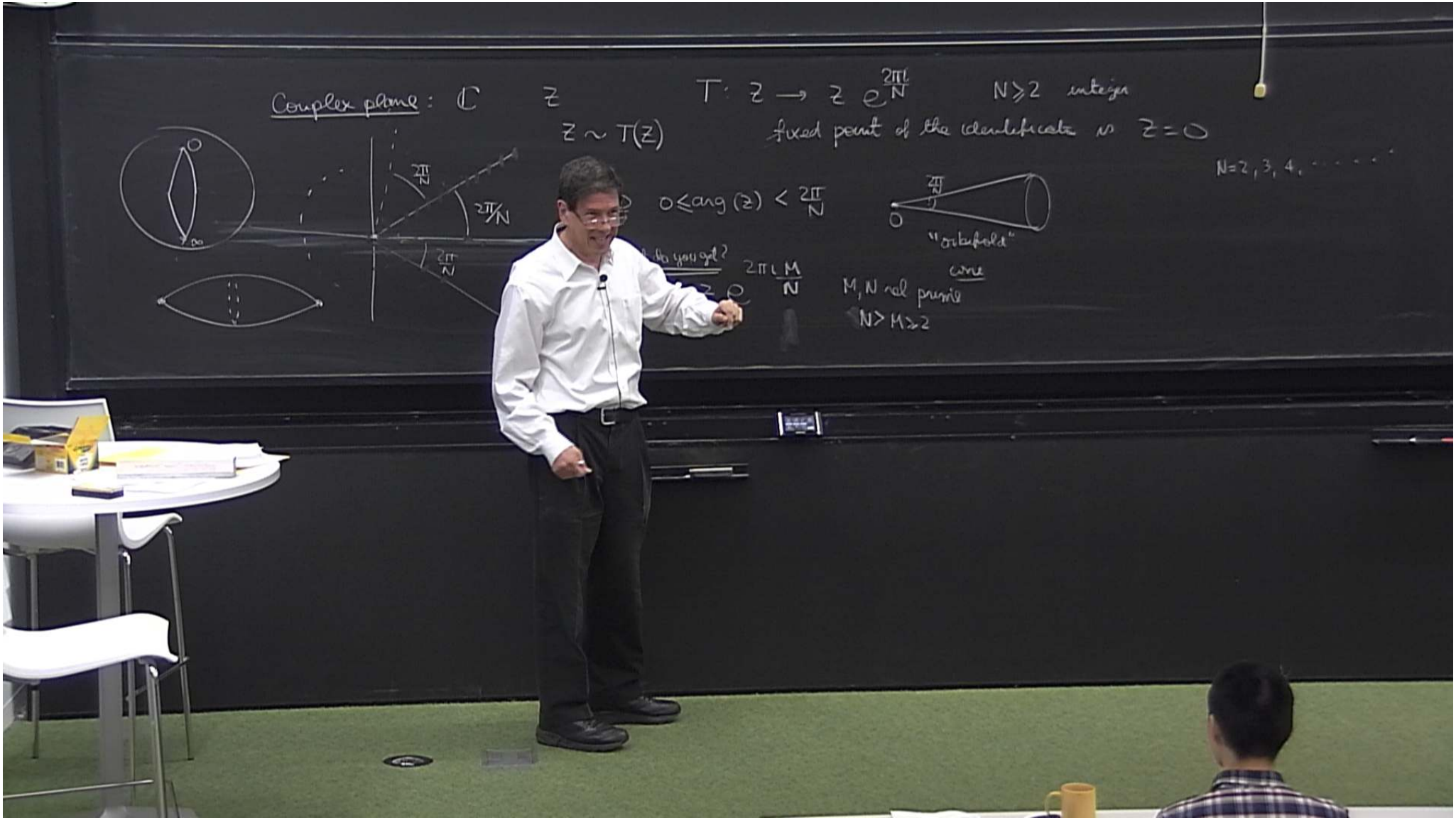
$\mathbb{R}^1/\mathbb{Z}_2$ "half line" $\{x \geq 0\}$

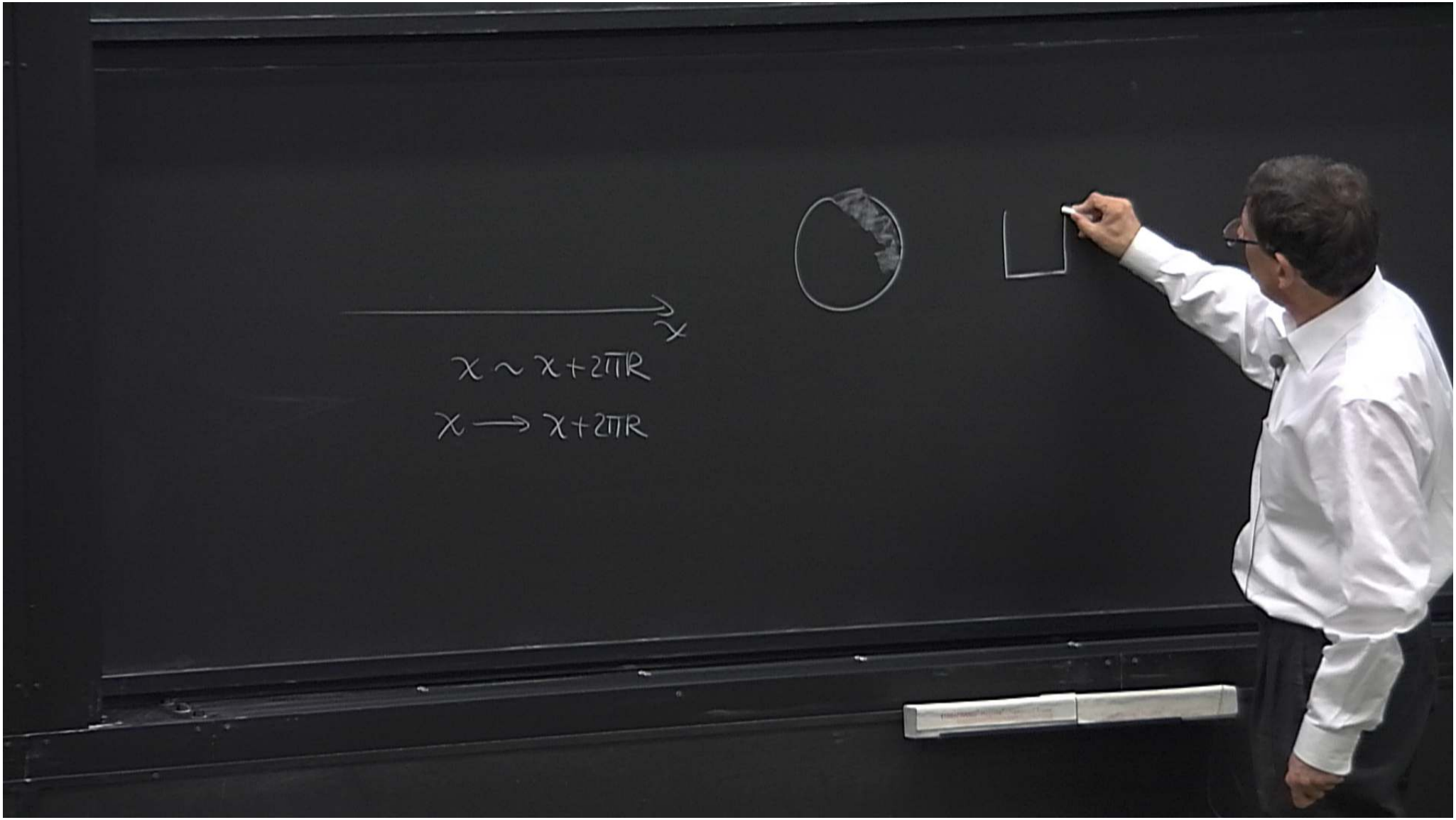

Orbifolds
Spaces that arise from identifications with fixed points
$x \sim -x$
$T: x \to -x$ a $\mathbb{Z}_2$ transformation $\{1, T\}$
0
x
Fixed point is $x=0$
FD
$\mathbb{R}^1/\mathbb{Z}_2$ "half line" $\{x \geq 0\}$

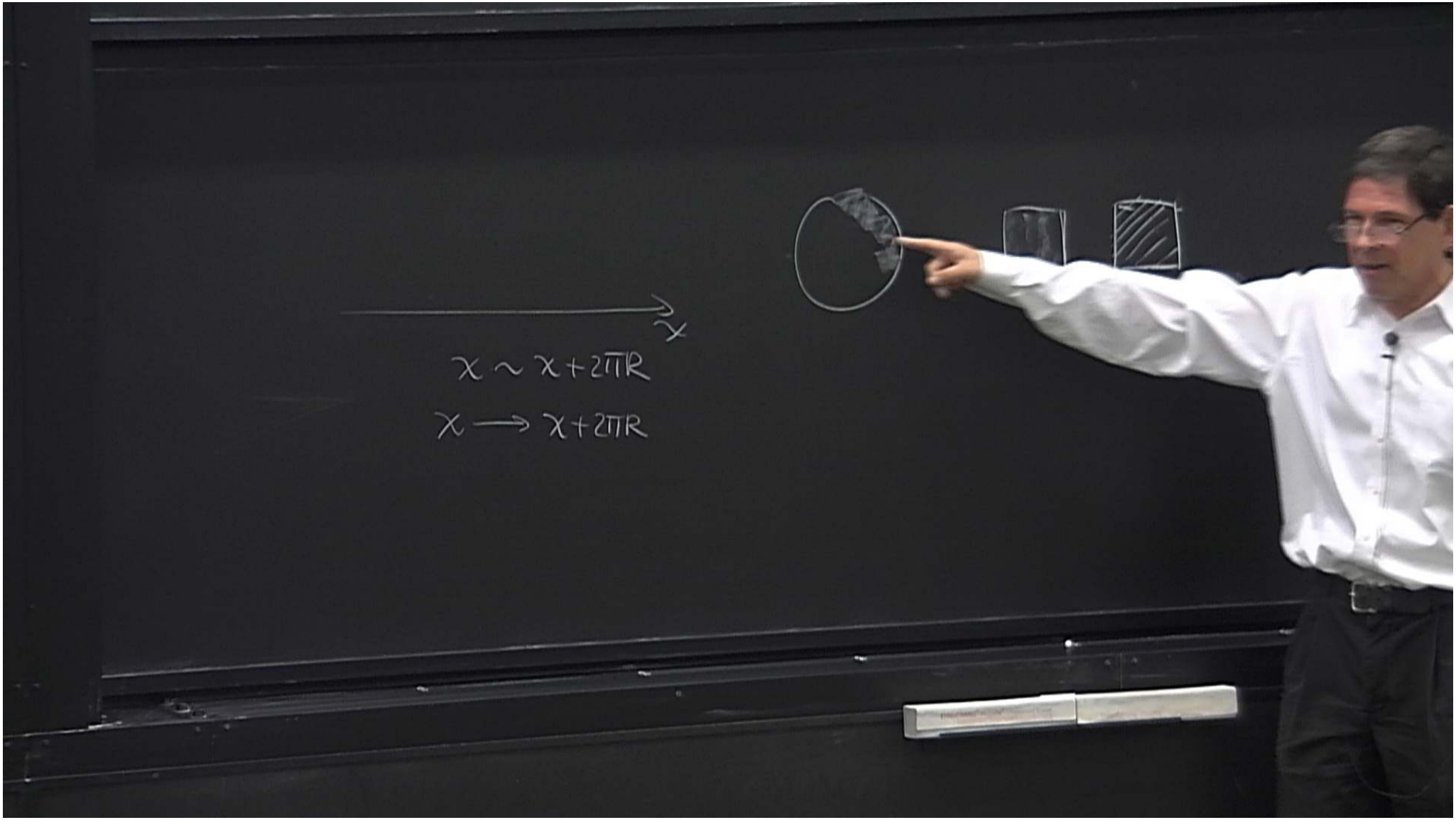

Complex plane : ℂ z

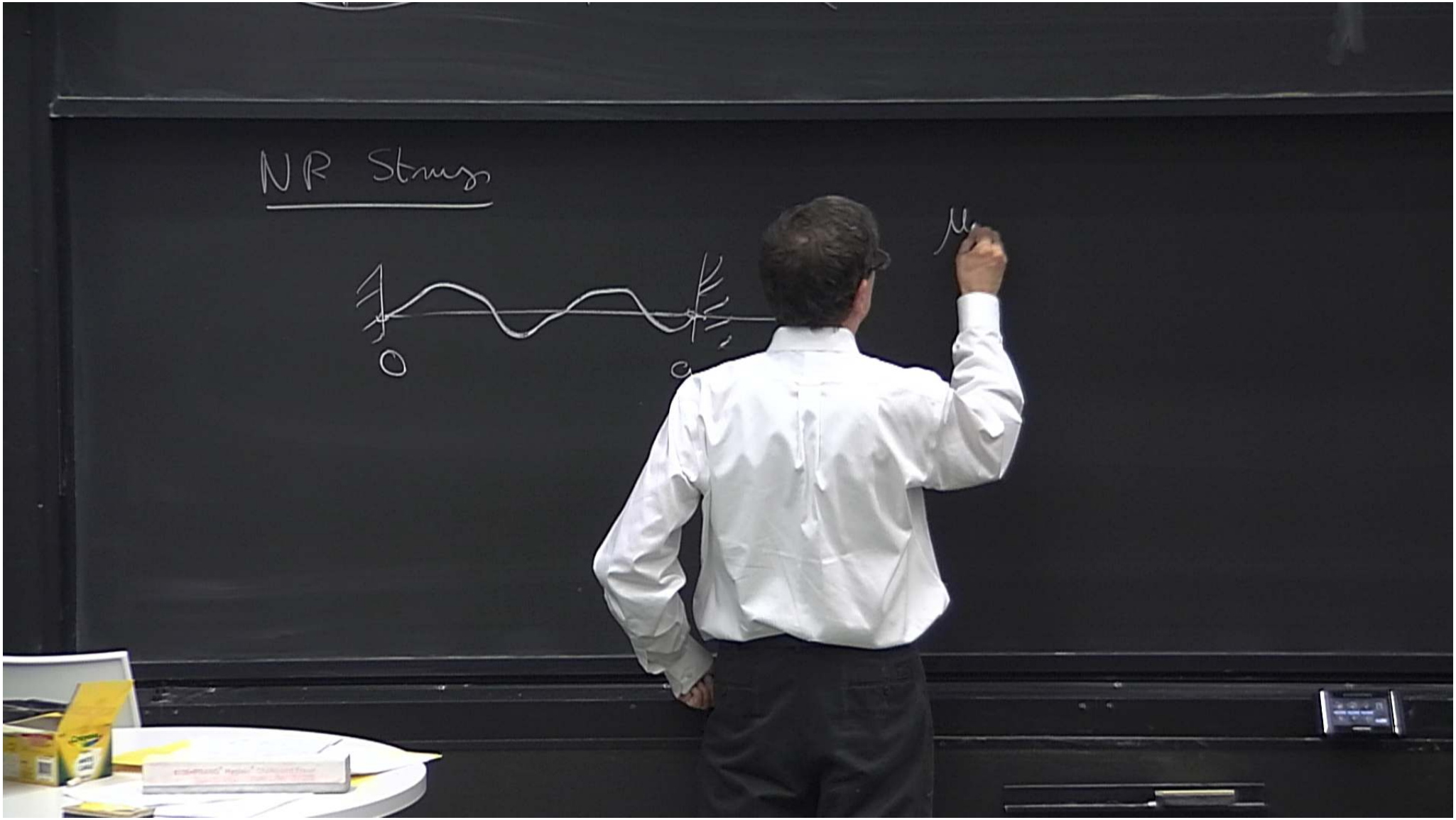

Complex plane: ℂ z
T: z → z e^{2πi/N}
N ≥ 2 integer
z ~ T(z)
2π/N
FD
arg (z)


Complex plane : ℂ z
T : z → z e^{2πi/N}
N ⩾ 2 integer
z ~ T(z)
2π/N
2π/N
2π/N
0 ⩽ arg (z) <

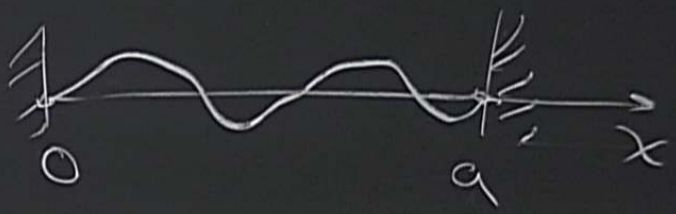

Complex plane : ℂ z
T : z → z e^{2πi/N}
N ≥ 2 integer
z ~ T(z)
fixed point of the identification is z = 0
N = 2, 3, 4, ...
0 ≤ arg (z) < 2π/N
2π/N
2π/N
2π/N
"orbifold"
cone
2πi M/N
M, N rel prime
N > M ≥ 2


x
x ~ x + 2πR
x ⟶ x + 2πR


x
x ~ x + 2πR
x ⟶ x + 2πR

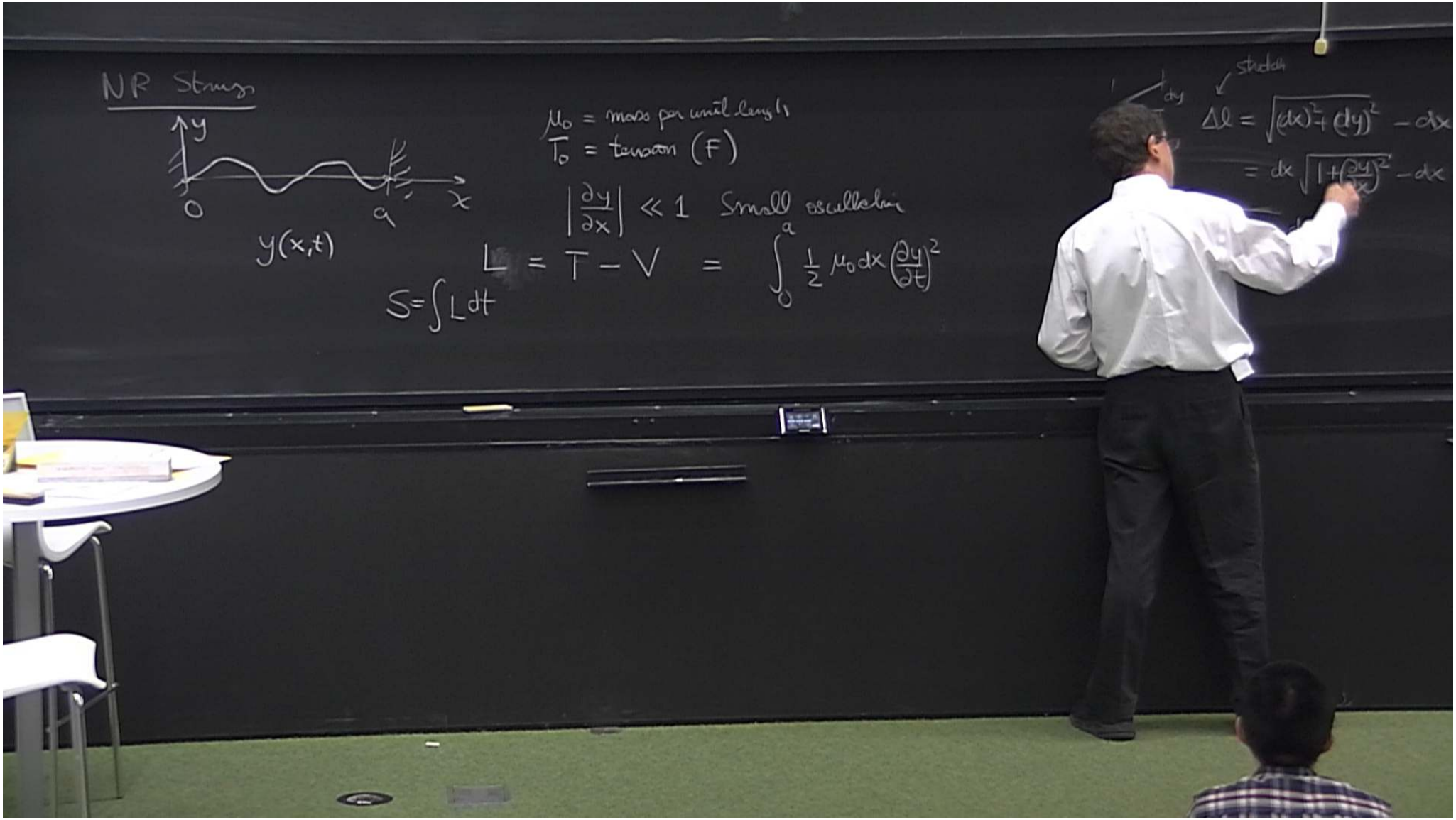

NR Strings

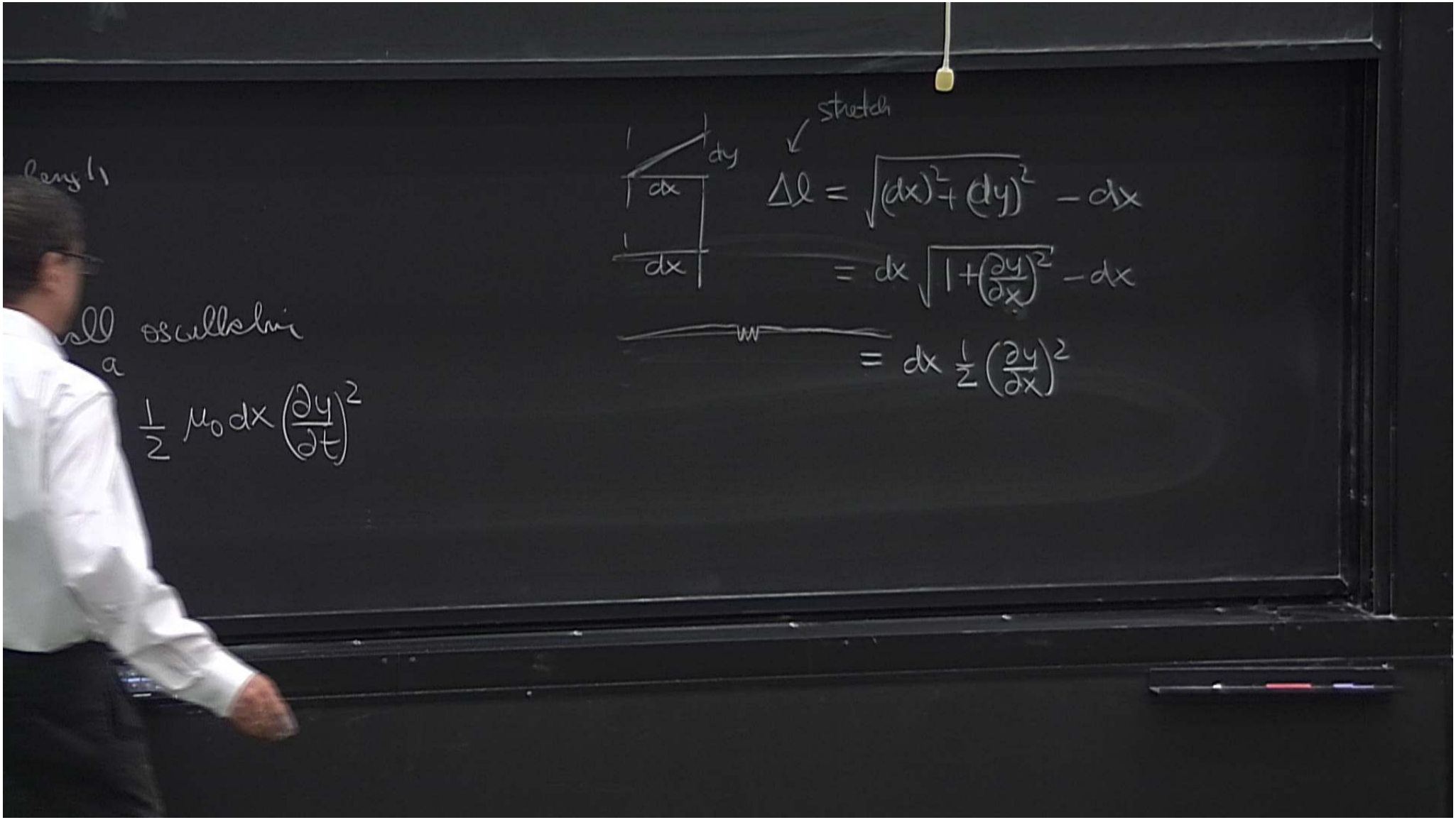
NR Strings

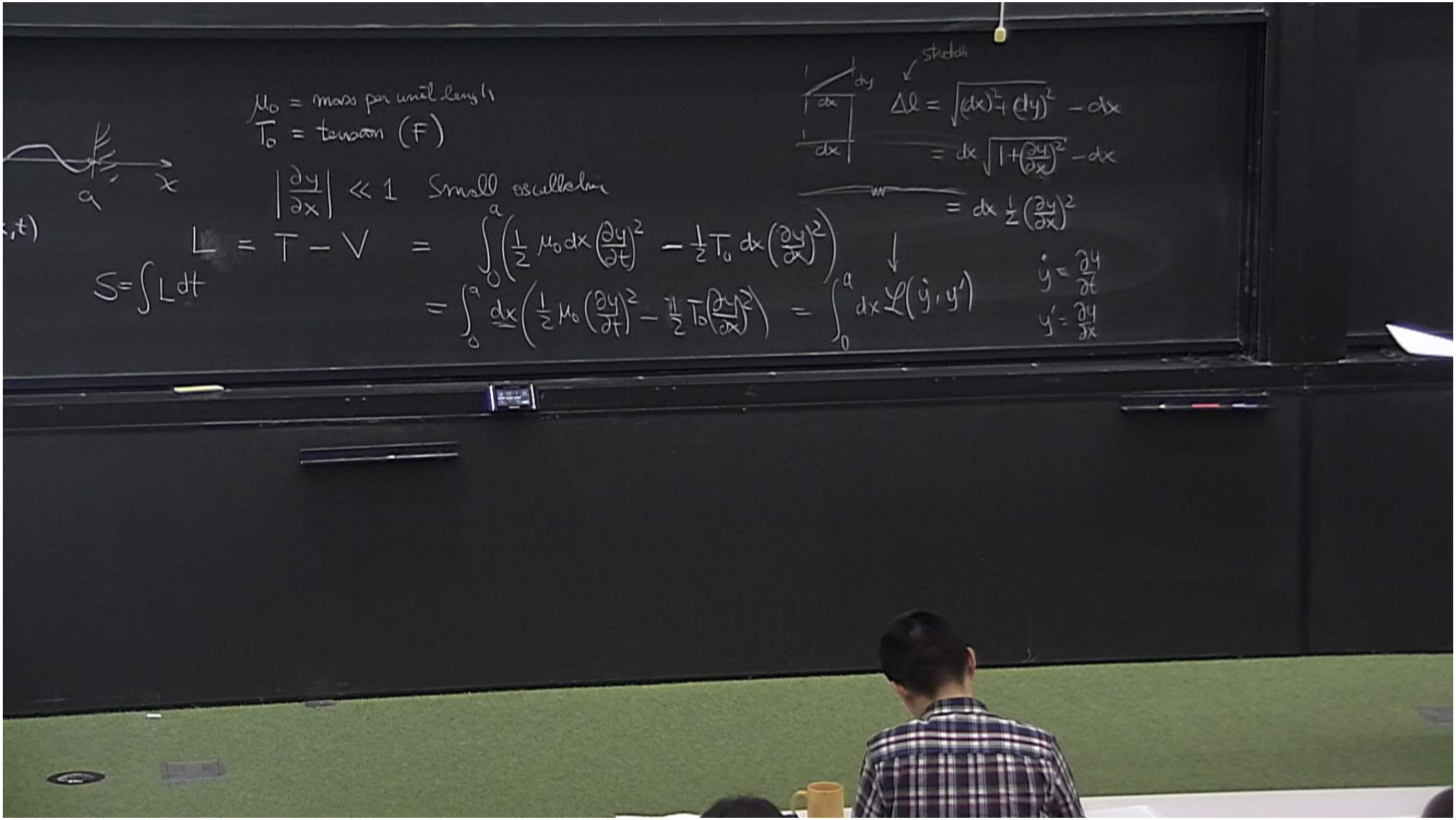


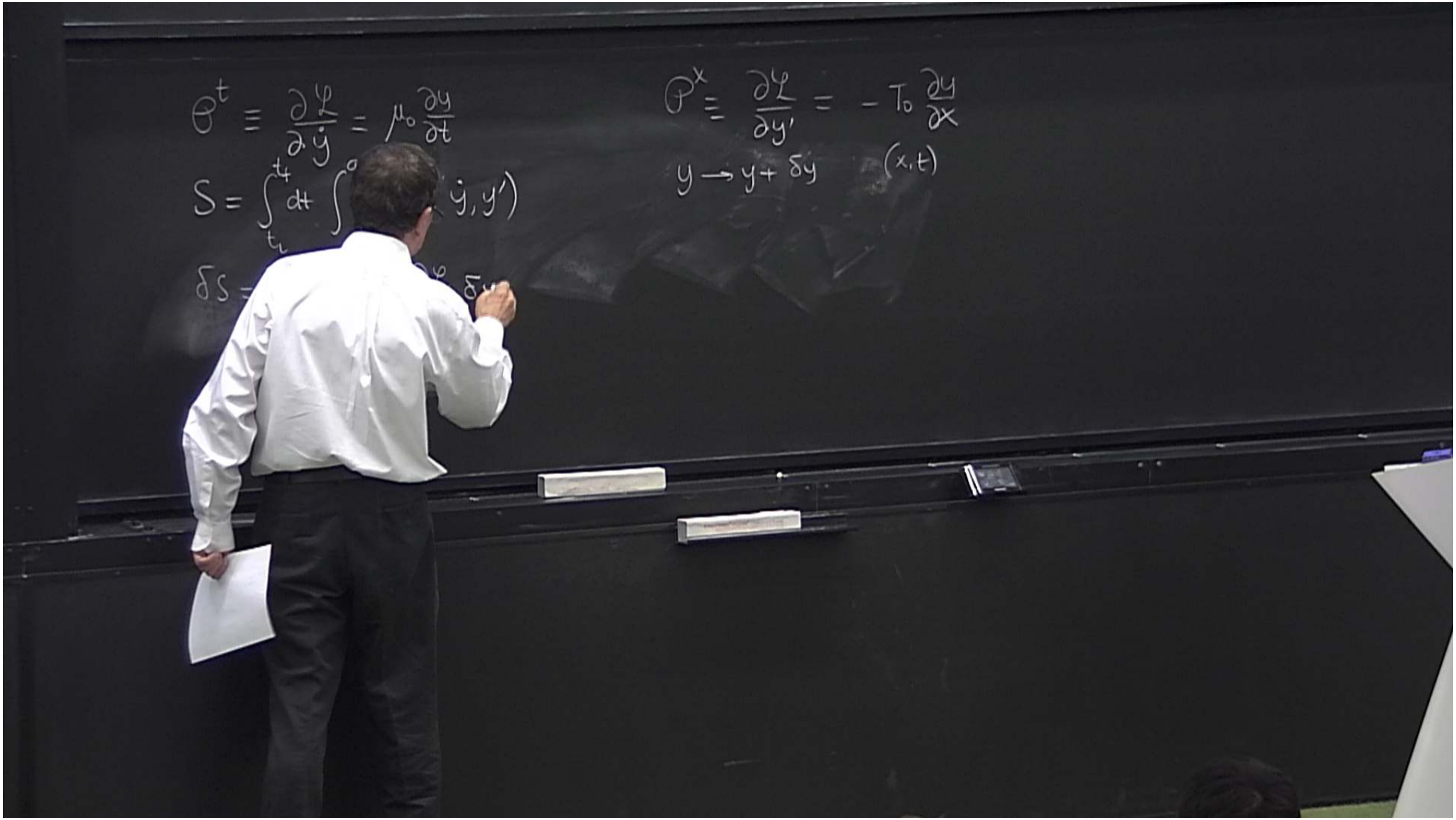
μ_0 = mass per unit length

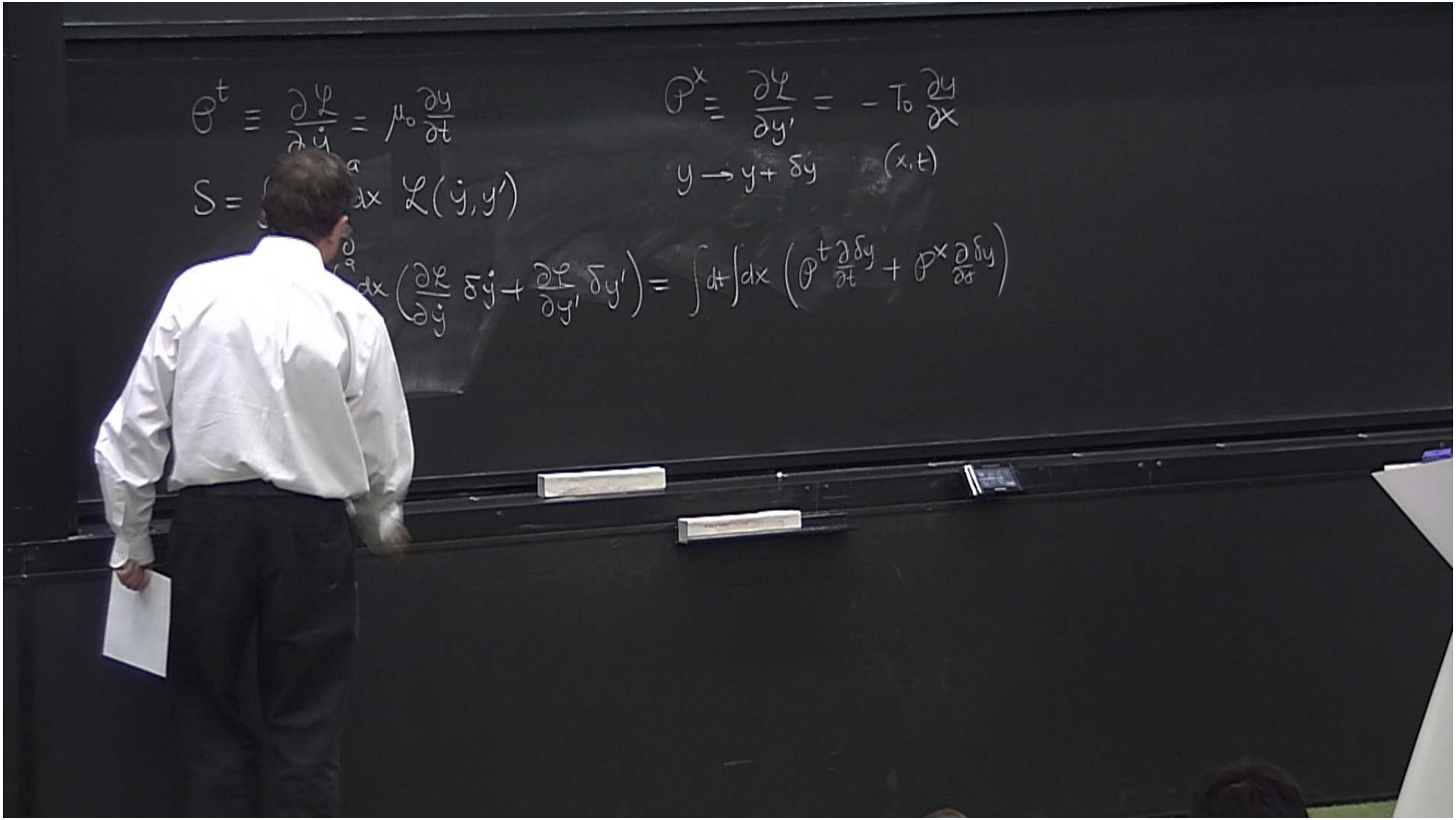
T_0 = tension (F)

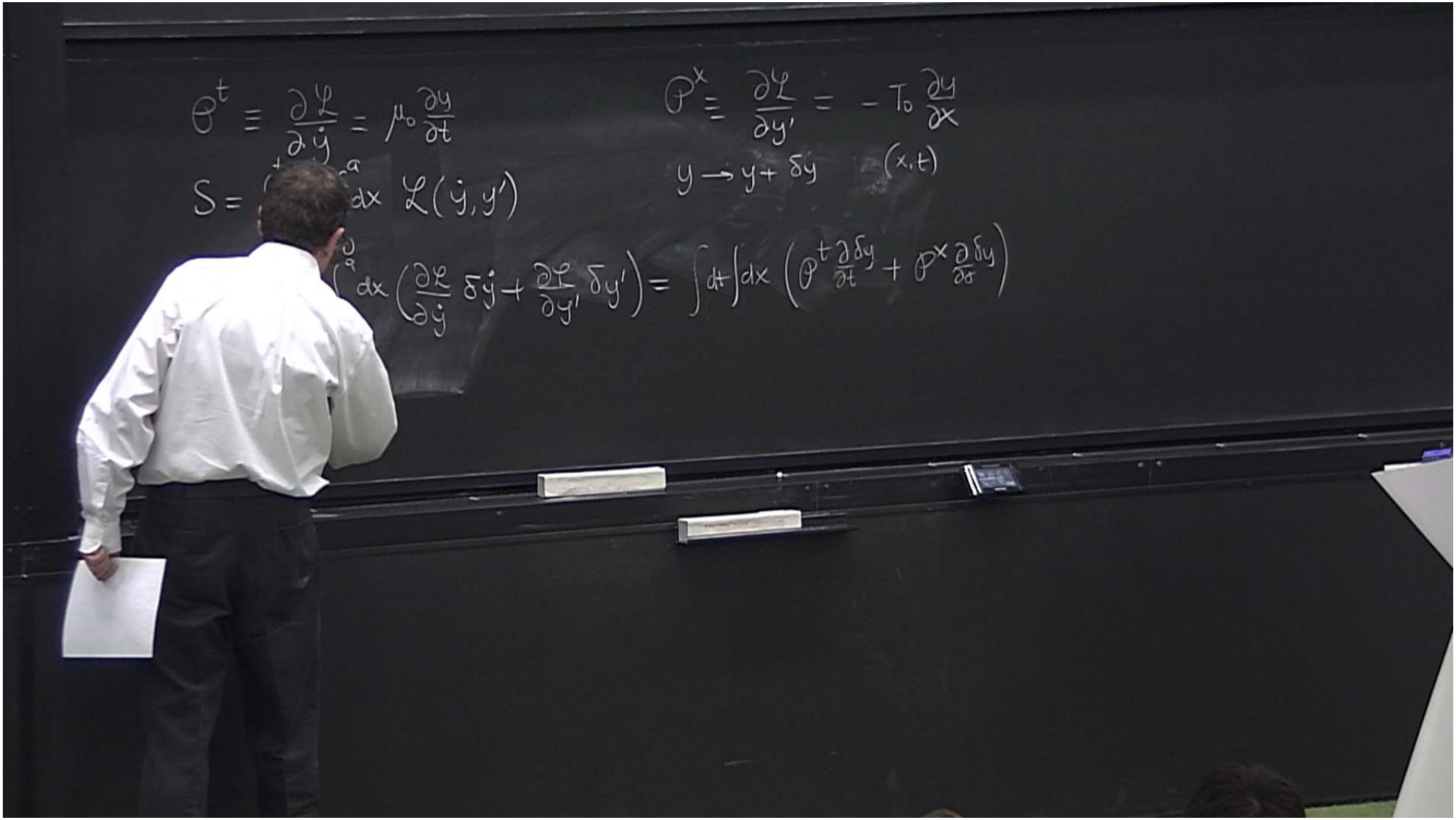

NR Strings
y
0
q
x
y(x,t)
μ_0 = mass per unit length
T_0 = tension (F)
$\left|\frac{\partial y}{\partial x}\right| \ll 1$ Small oscillations
$L = T - V = \int_0^q \frac{1}{2}\mu_0 dx \left(\frac{\partial y}{\partial t}\right)^2$
$S = \int L dt$
stretch
dy
$\Delta l = \sqrt{(dx)^2 + (dy)^2} - dx$
$= dx\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2} - dx$


stretch
dy
dx
dx
$\Delta l = \sqrt{(dx)^2 + (dy)^2} - dx$
$= dx\sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2} - dx$
$= dx\,\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2$
$\frac{1}{2}\mu_0\,dx\left(\frac{\partial y}{\partial t}\right)^2$


μ_0 = mass per unit length
T_0 = tension (F)
$\left|\frac{\partial y}{\partial x}\right| \ll 1$ Small oscillations
$L = T - V = \int_0^a \left(\frac{1}{2}\mu_0 dx \left(\frac{\partial y}{\partial t}\right)^2 - \frac{1}{2} T_0 dx \left(\frac{\partial y}{\partial x}\right)^2\right)$
$= \int_0^a dx \left(\frac{1}{2}\mu_0 \left(\frac{\partial y}{\partial t}\right)^2 - \frac{1}{2} T_0 \left(\frac{\partial y}{\partial x}\right)^2\right) = \int_0^a dx \mathcal{L}(\dot{y}, y')$
$S = \int L dt$
$\dot{y} = \frac{\partial y}{\partial t}$
$y' = \frac{\partial y}{\partial x}$
stretch
$\Delta l = \sqrt{(dx)^2 + (dy)^2} - dx$
$= dx\sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2} - dx$
$= dx \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2$
dy
dx
dx
a
x


$\Theta^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t}$
$\mathcal{P}^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$
$y \to y + \delta y$
(x,t)
$S = \int_{t_i}^{t_f} dt \int$
$\dot{y}, y')$
$\delta S =$
δ


$\mathcal{P}^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t}$
$\mathcal{P}^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$
$y \to y + \delta y \qquad (x,t)$
$S = \int_a^b dx\, \mathcal{L}(\dot{y}, y')$
$\int_a^b dx \left(\frac{\partial \mathcal{L}}{\partial \dot{y}} \delta\dot{y} + \frac{\partial \mathcal{L}}{\partial y'} \delta y' \right) = \int dt \int dx \left(\mathcal{P}^t \frac{\partial \delta y}{\partial t} + \mathcal{P}^x \frac{\partial \delta y}{\partial x} \right)$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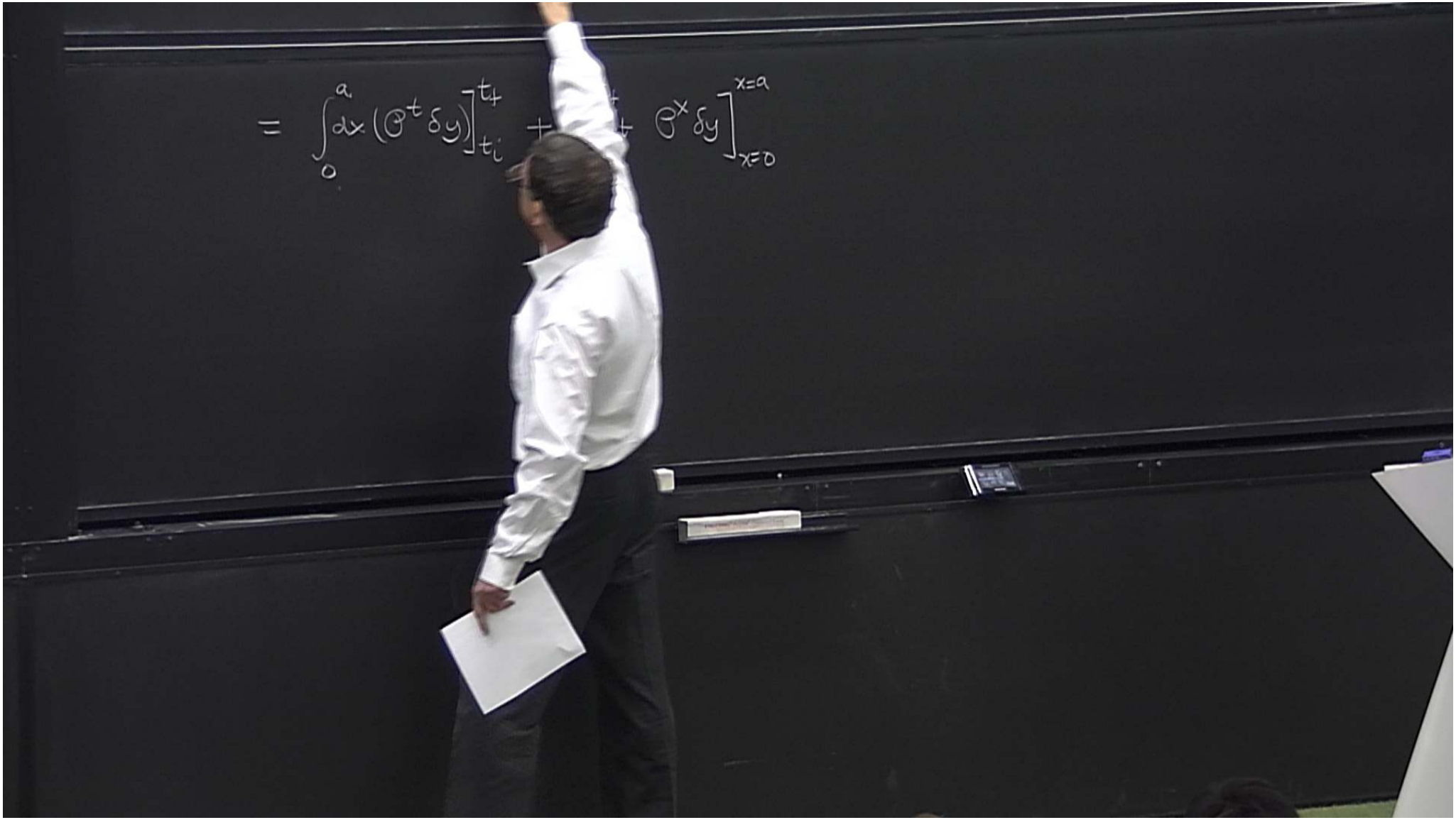
$y \to y + \delta y \qquad (x,t)$
$S = \int dx\, \mathcal{L}(\dot{y}, y')$
$\int dx \left(\frac{\partial \mathcal{L}}{\partial \dot{y}} \delta\dot{y} + \frac{\partial \mathcal{L}}{\partial y'} \delta y' \right) = \int dt \int dx \left(\mathcal{P}^t \frac{\partial \delta y}{\partial t} + \mathcal{P}^x \frac{\partial \delta y}{\partial x} \right)$

$$\mathcal{P}^t \equiv \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu_0 \frac{\partial y}{\partial t} \qquad \mathcal{P}^x \equiv \frac{\partial \mathcal{L}}{\partial y'} = -T_0 \frac{\partial y}{\partial x}$$

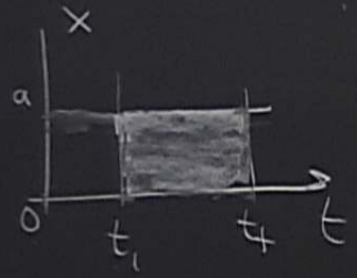
$$S = \int_{t_1}^{t_2} dt \int_0^a dx \; \mathcal{L}(\dot{y}, y') \qquad y \to y + \delta y \qquad (x,t)$$

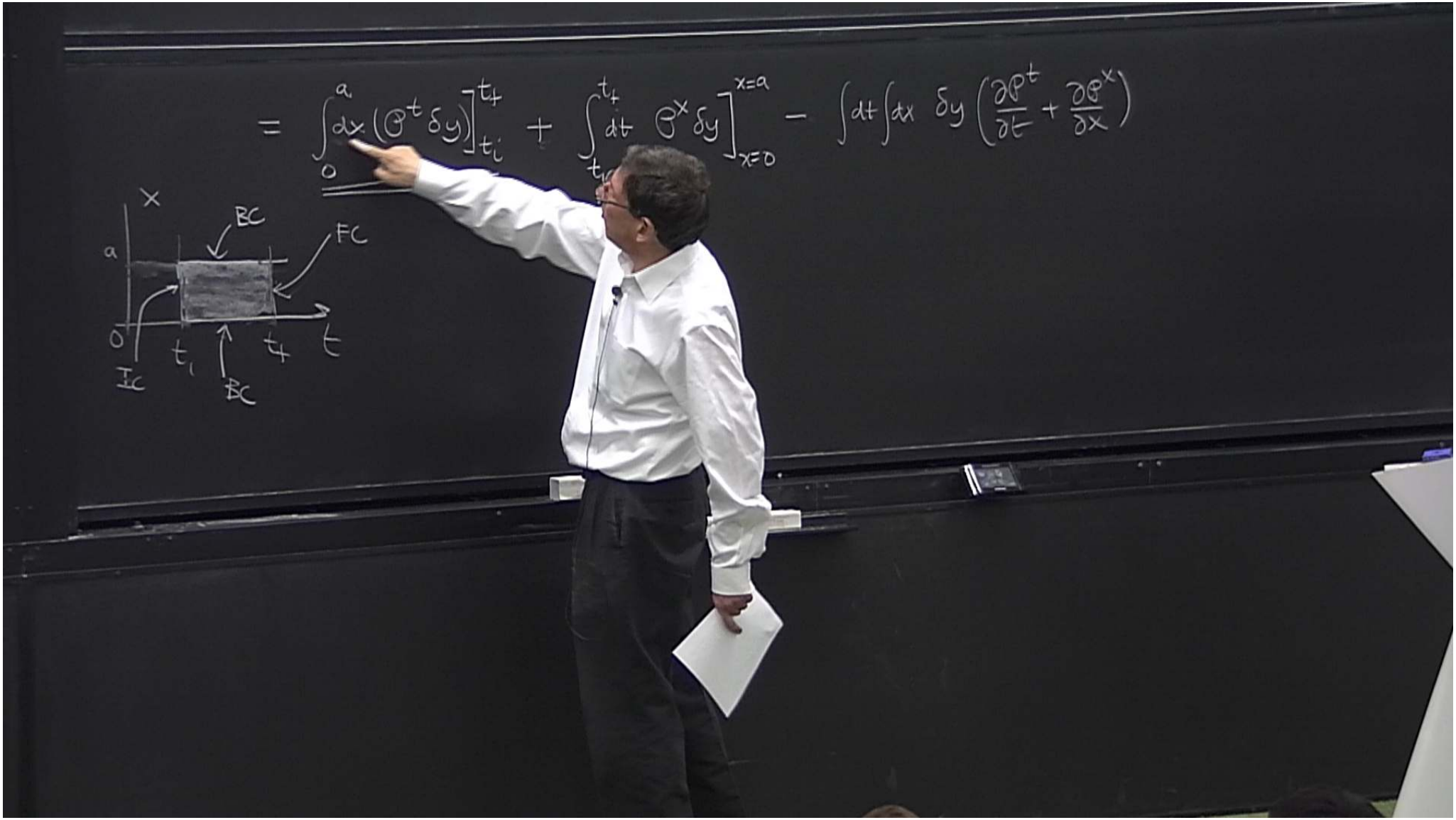
$$\delta S = \int_{t_1}^{t_2} dt \int_0^a dx \left(\frac{\partial \mathcal{L}}{\partial \dot{y}} \delta \dot{y} + \frac{\partial \mathcal{L}}{\partial y'} \delta y' \right) = \int dt \int dx \left(\mathcal{P}^t \frac{\partial \delta y}{\partial t} + \mathcal{P}^x \frac{\partial \delta y}{\partial x} \right)$$

$$= \int dt\, dx \left[\frac{\partial}{\partial t}(\mathcal{P}^t \delta y) + \frac{\partial}{\partial x}(\mathcal{P}^x \delta y) - \delta y \left(\frac{\partial \mathcal{P}^t}{\partial t} + \frac{\partial \mathcal{P}^x}{\partial x} \right) \right]$$


$= \int_0^a dx \, (\Theta^t \delta y)\Big]_{t_i}^{t_f}$
$\Theta^x \delta y \Big]_{x=0}^{x=a}$

$$= \int_0^a dx \left(\mathcal{P}^t \delta y\right)\Big]_{t_i}^{t_f} + \int_{t_i}^{t_f} dt \; \mathcal{P}^x \delta y\Big]_{x=0}^{x=a} - \int dt \int dx \; \delta y \left(\frac{\partial \mathcal{P}^t}{\partial t} + \frac{\partial \mathcal{P}^x}{\partial x}\right)$$

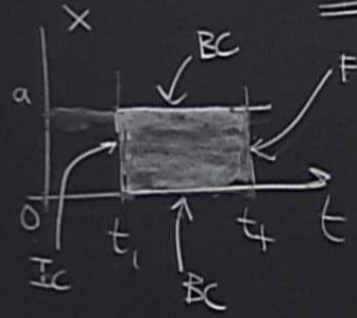


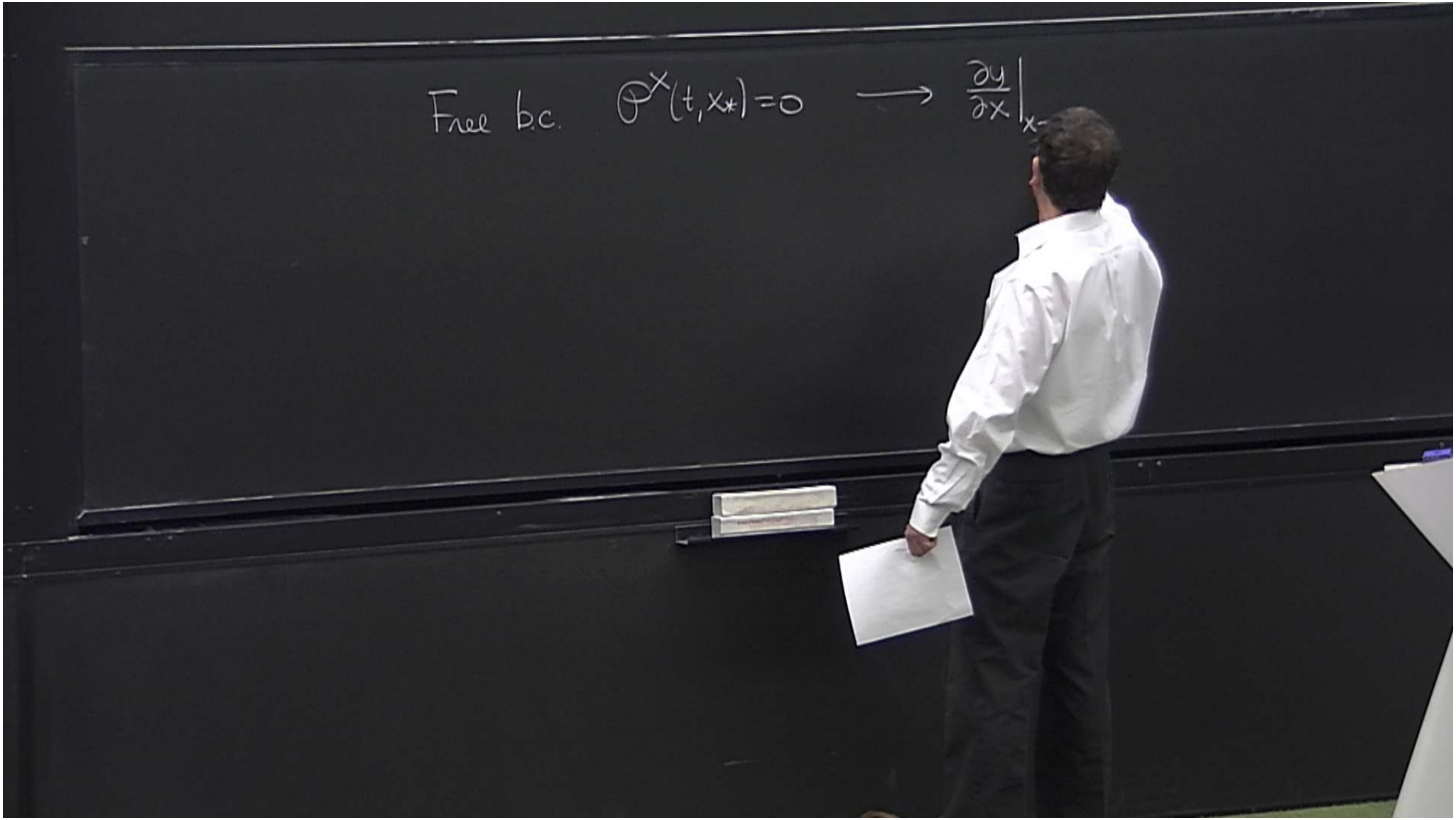

$= \int_0^a dx \left(\Theta^t \delta y\right)\Big]_{t_i}^{t_f} + \int_{t_i}^{t_f} dt \; \Theta^x \delta y\Big]_{x=0}^{x=a} - \int dt \int dx \; \delta y \left(\frac{\partial \rho^t}{\partial t} + \frac{\partial \rho^x}{\partial x}\right)$
X
a
BC
FC
0
t_i
t_f
t
IC
BC

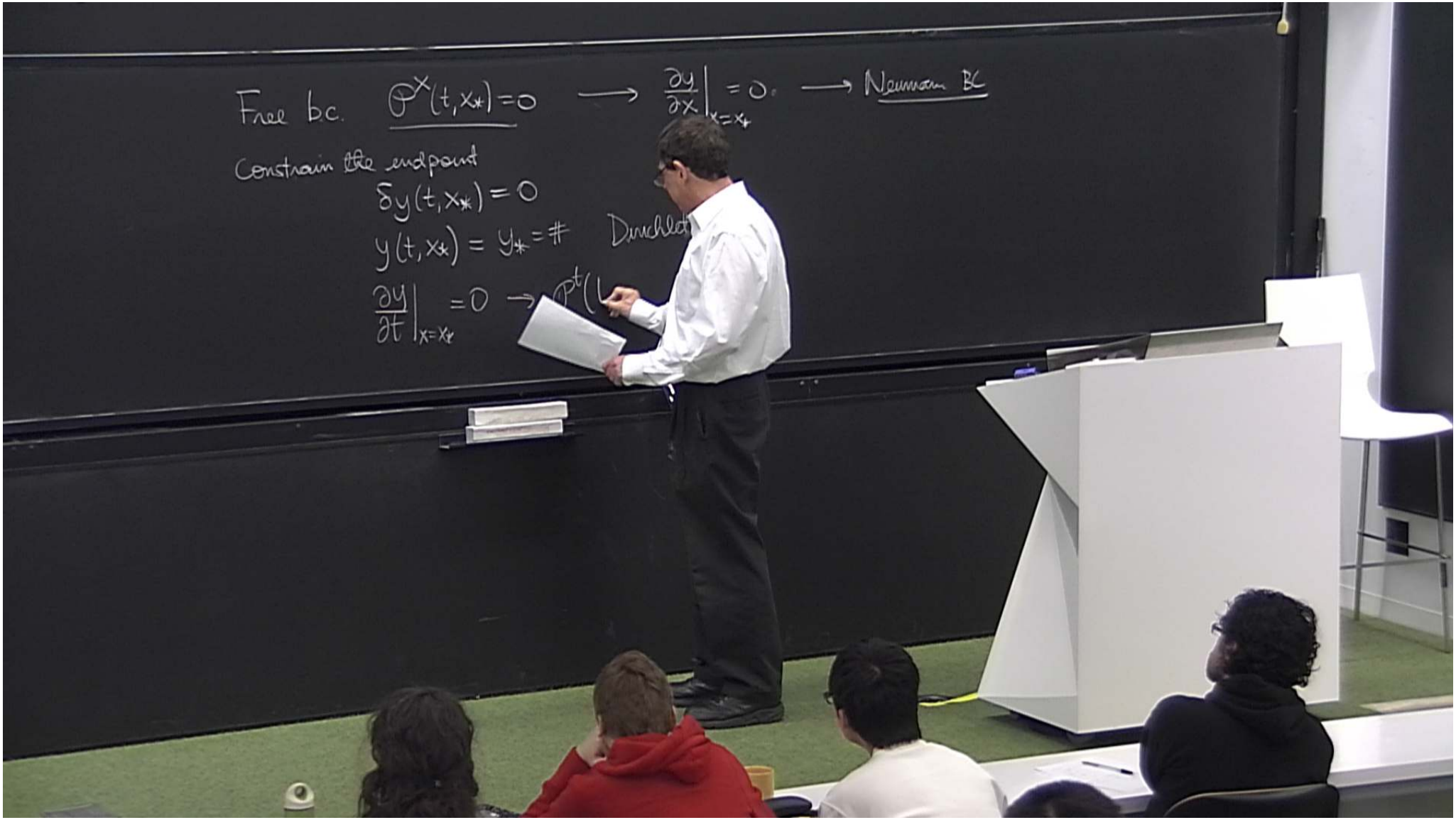
$$= \int_0^a dx \left(\Theta^t \delta y\right)\Big]_{t_i}^{t_f} + \int_{t_i}^{t_f} dt \; \Theta^x \delta y \Big]_{x=0}^{x=a} - \int dt \int dx \; \delta y \left(\frac{\partial \rho^t}{\partial t} + \frac{\partial \rho^x}{\partial x}\right)$$

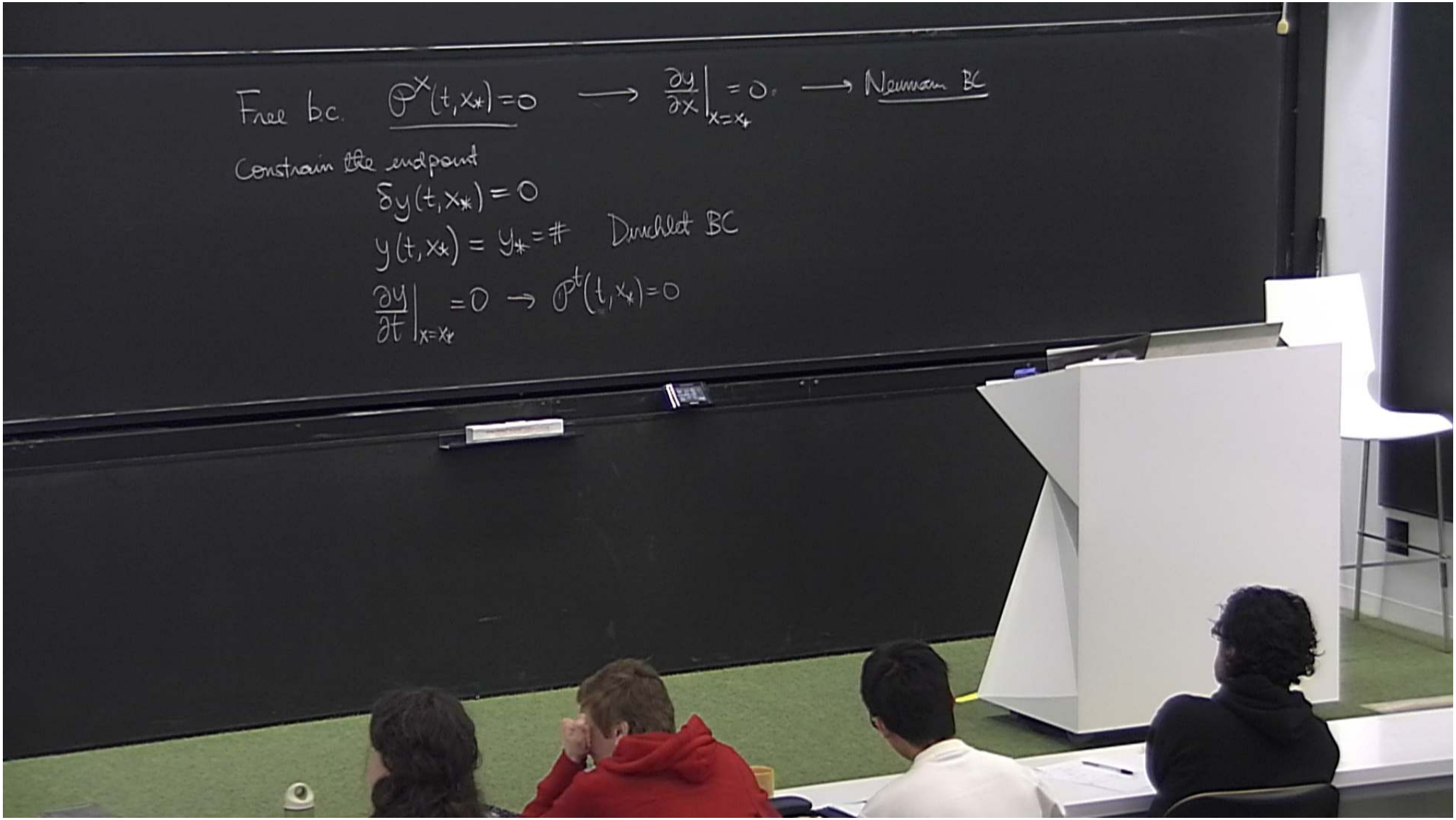
forget it

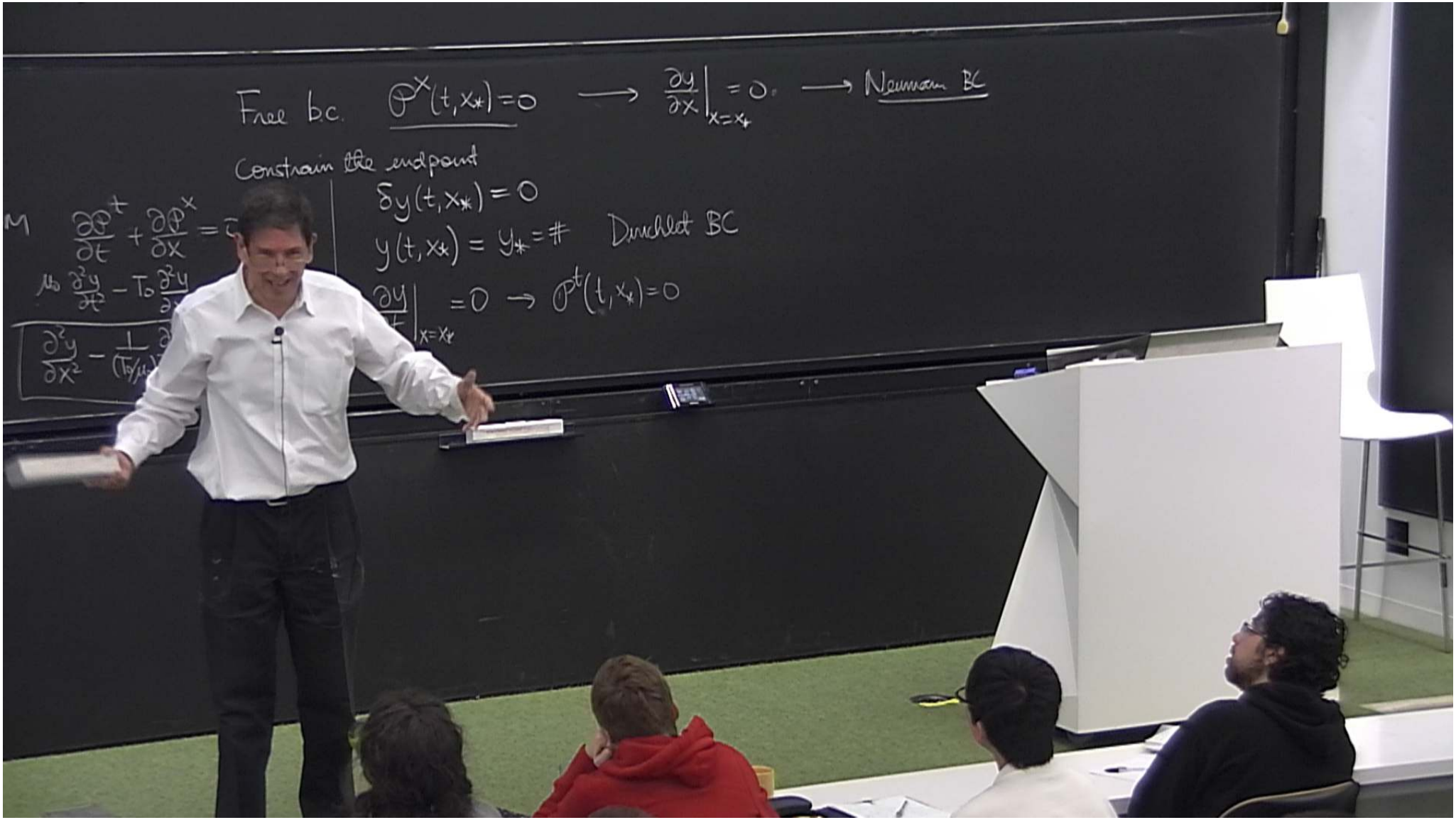
$$\int_{t_i}^{t_f} dt \left(\Theta^x(t, x=a)\, \delta y(t, x=a) - \rho^x(t, x=0)\, \delta y(t, x=0)\right)$$




Free b.c. $\rho^x(t, x_*) = 0 \longrightarrow \frac{\partial y}{\partial x}\Big|_{x-}$


Free b.c. $\mathcal{P}^x(t,x_*)=0$ $\longrightarrow$ $\left.\frac{\partial y}{\partial x}\right|_{x=x_*}=0$ $\longrightarrow$ Neumann BC
Constrain the endpoint
$\delta y(t,x_*)=0$
$y(t,x_*)=y_*=\#$ Dirichlet
$\left.\frac{\partial y}{\partial t}\right|_{x=x_*}=0$ $\rightarrow$ $\mathcal{P}^t($


Free b.c. $\mathcal{P}^x(t,x_*)=0 \longrightarrow \left.\frac{\partial y}{\partial x}\right|_{x=x_*}=0 \longrightarrow$ Neumann BC
Constrain the endpoint
$\delta y(t,x_*)=0$
$y(t,x_*)=y_*=\#$ Dirichlet BC
$\left.\frac{\partial y}{\partial t}\right|_{x=x_*}=0 \rightarrow \mathcal{P}^t(t,x_*)=0$


Free b.c. $\mathcal{P}^x(t,x_*)=0 \longrightarrow \left.\frac{\partial y}{\partial x}\right|_{x=x_*}=0 \longrightarrow$ Neumann BC
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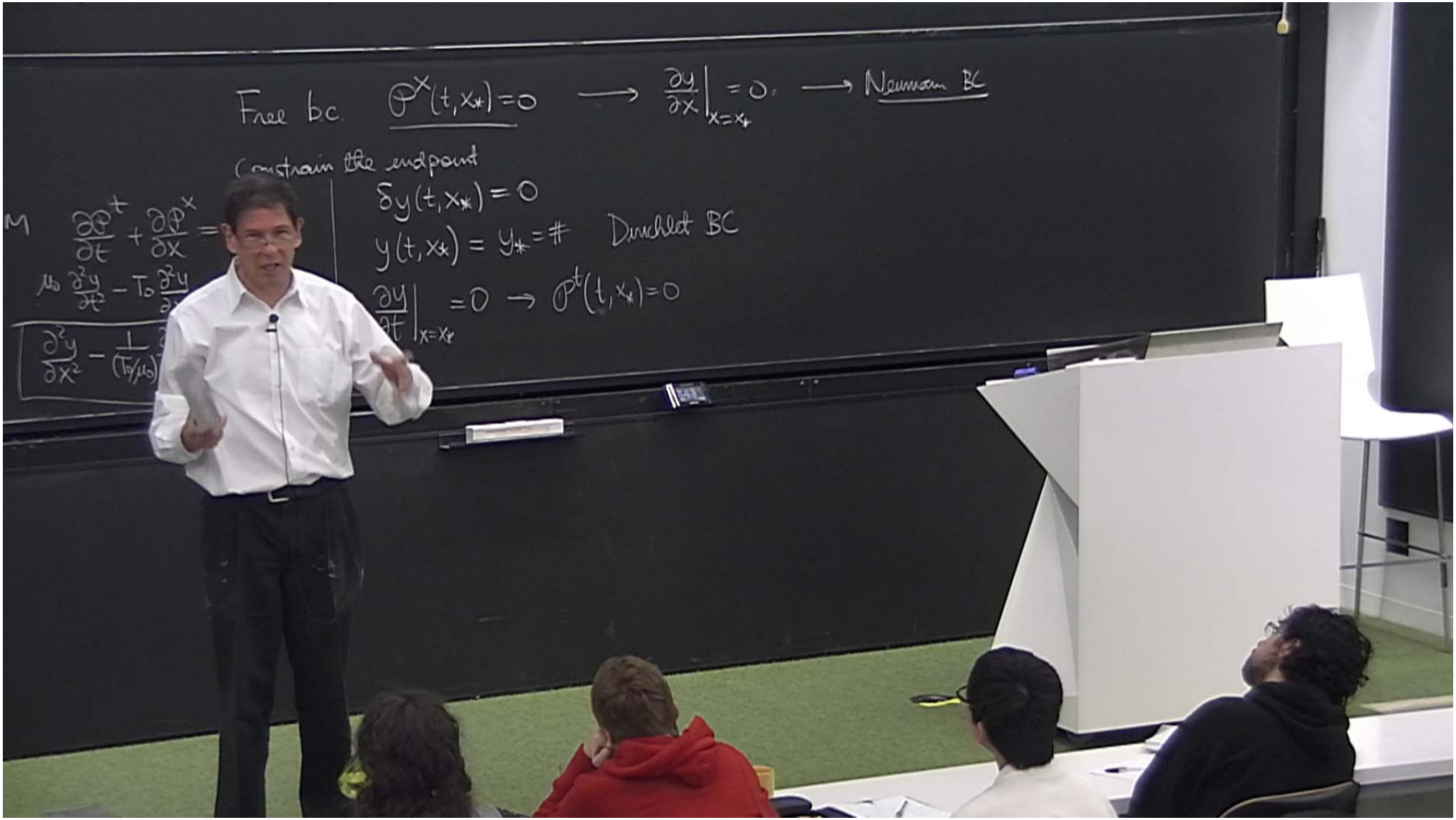
Free b.c. $\mathcal{P}^x(t,x_*)=0 \longrightarrow \left.\frac{\partial y}{\partial x}\right|_{x=x_*}=0 \longrightarrow$ Neumann BC
Constrain the endpoint
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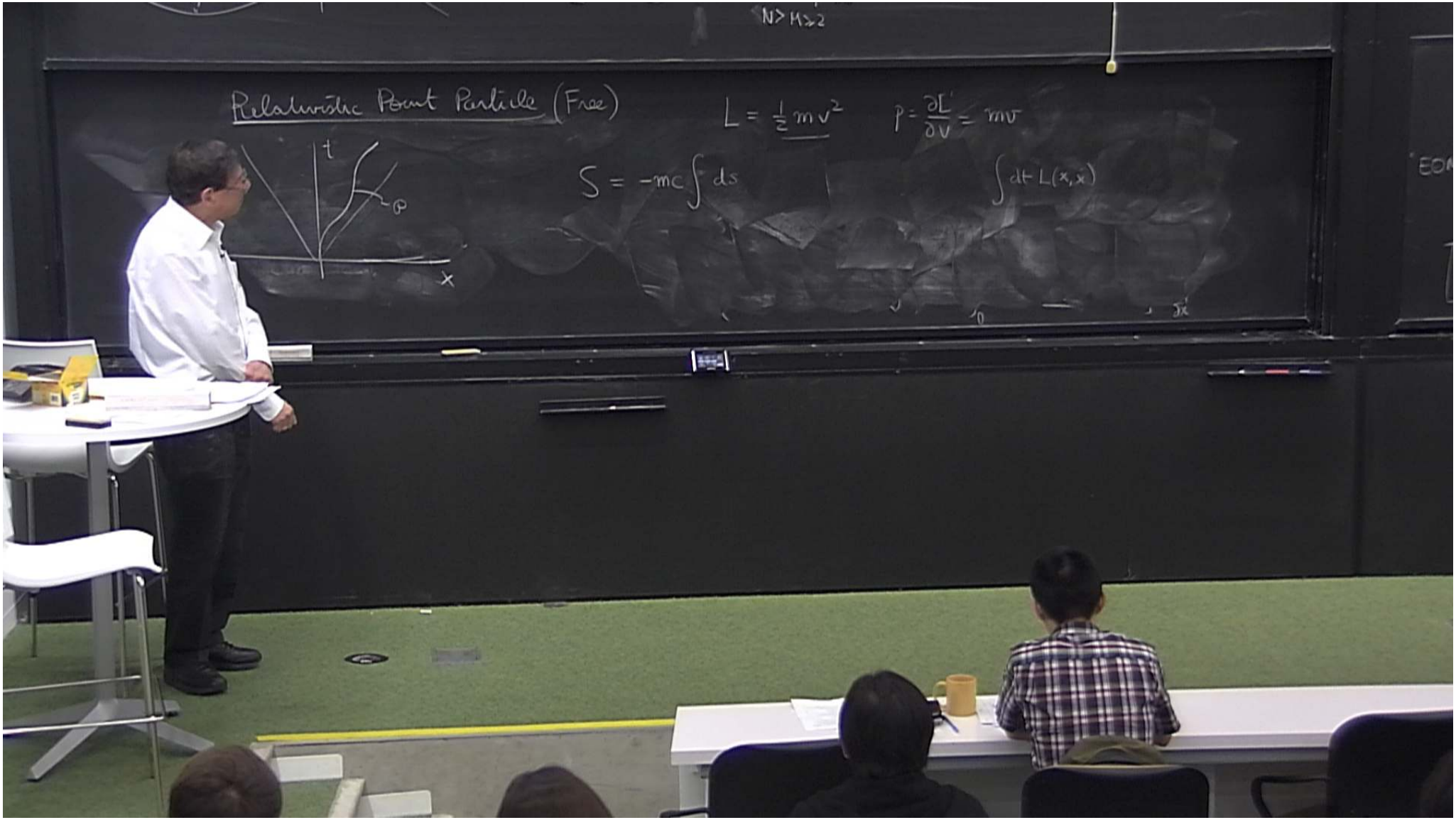
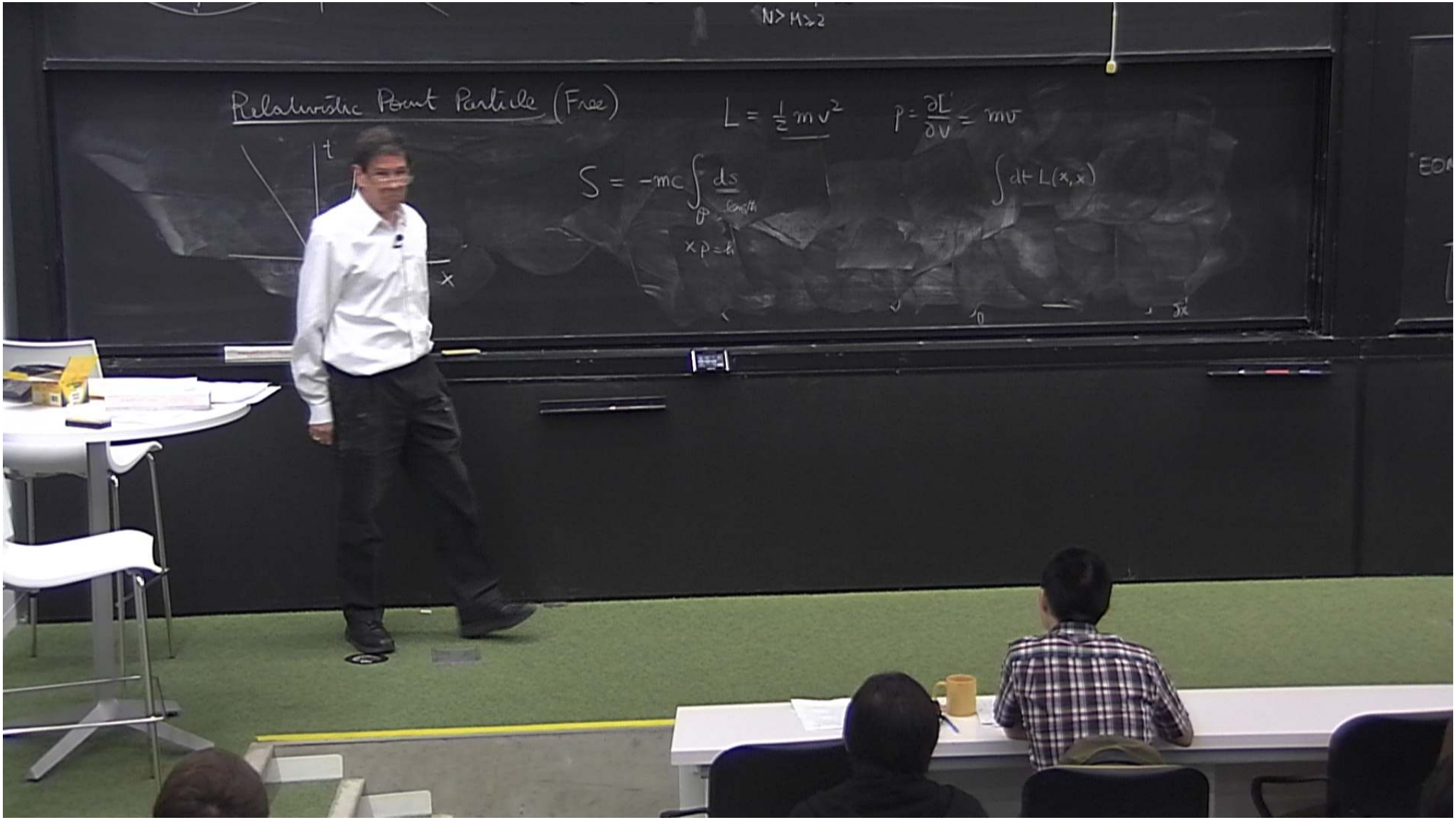
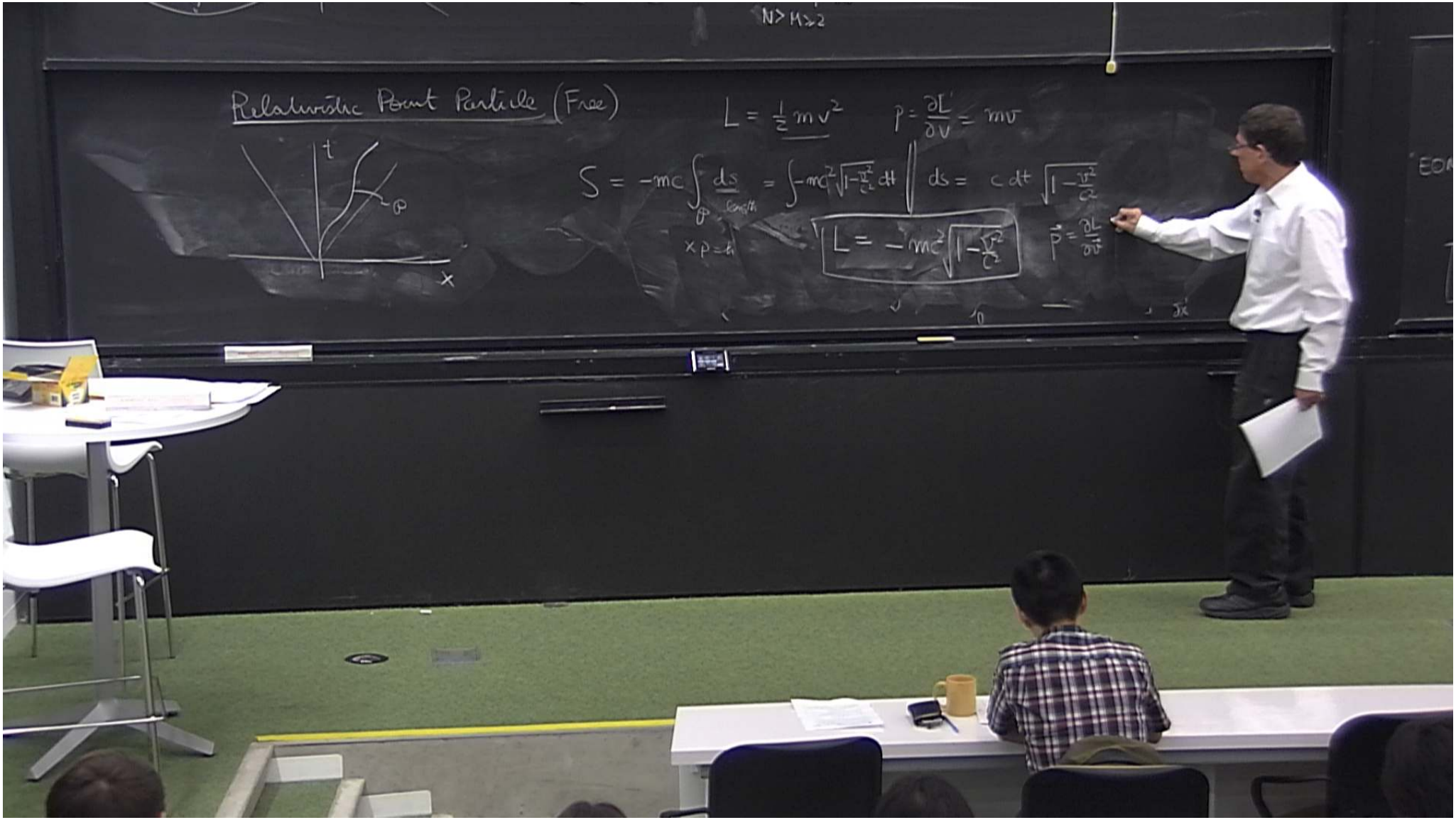
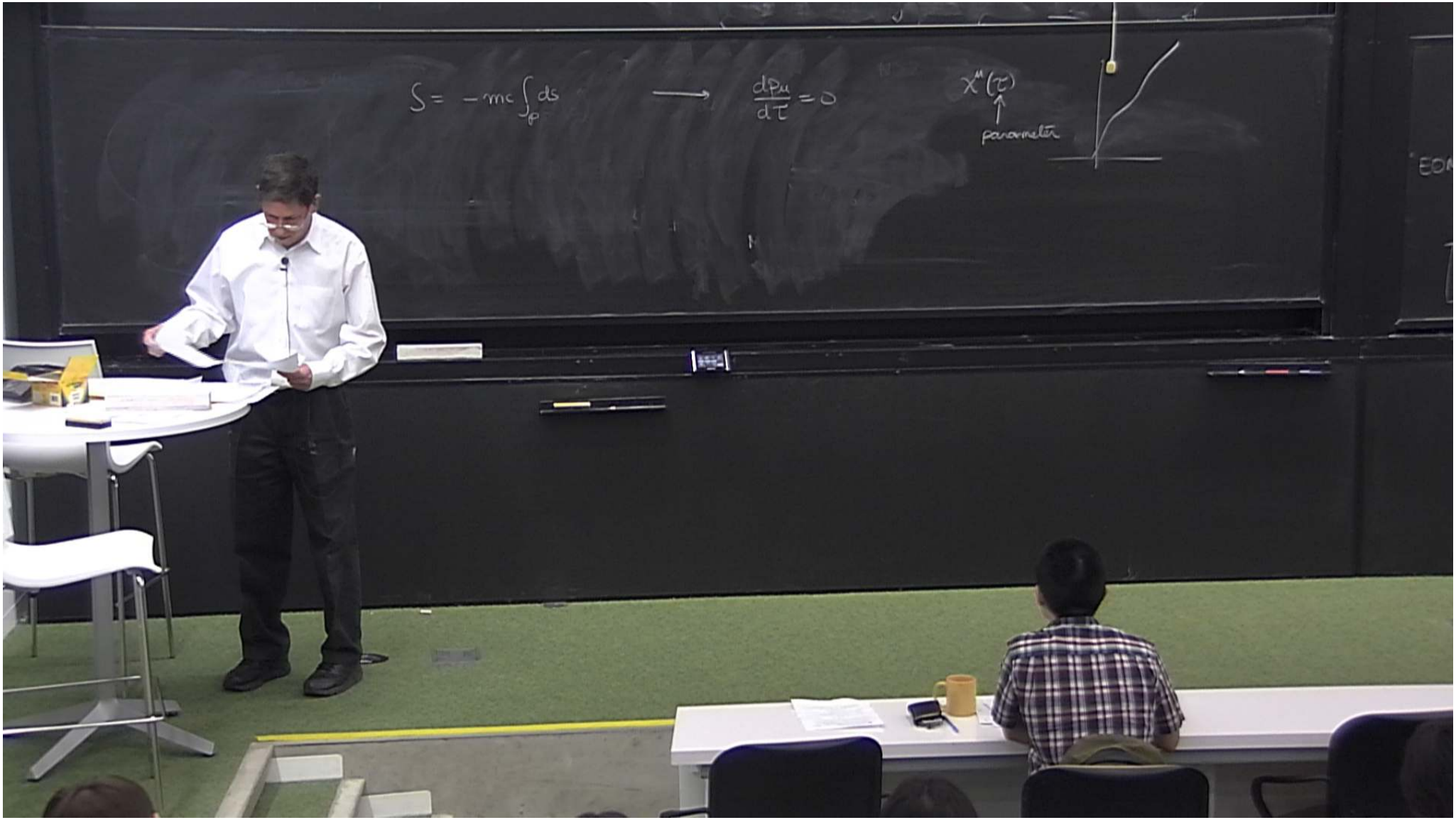
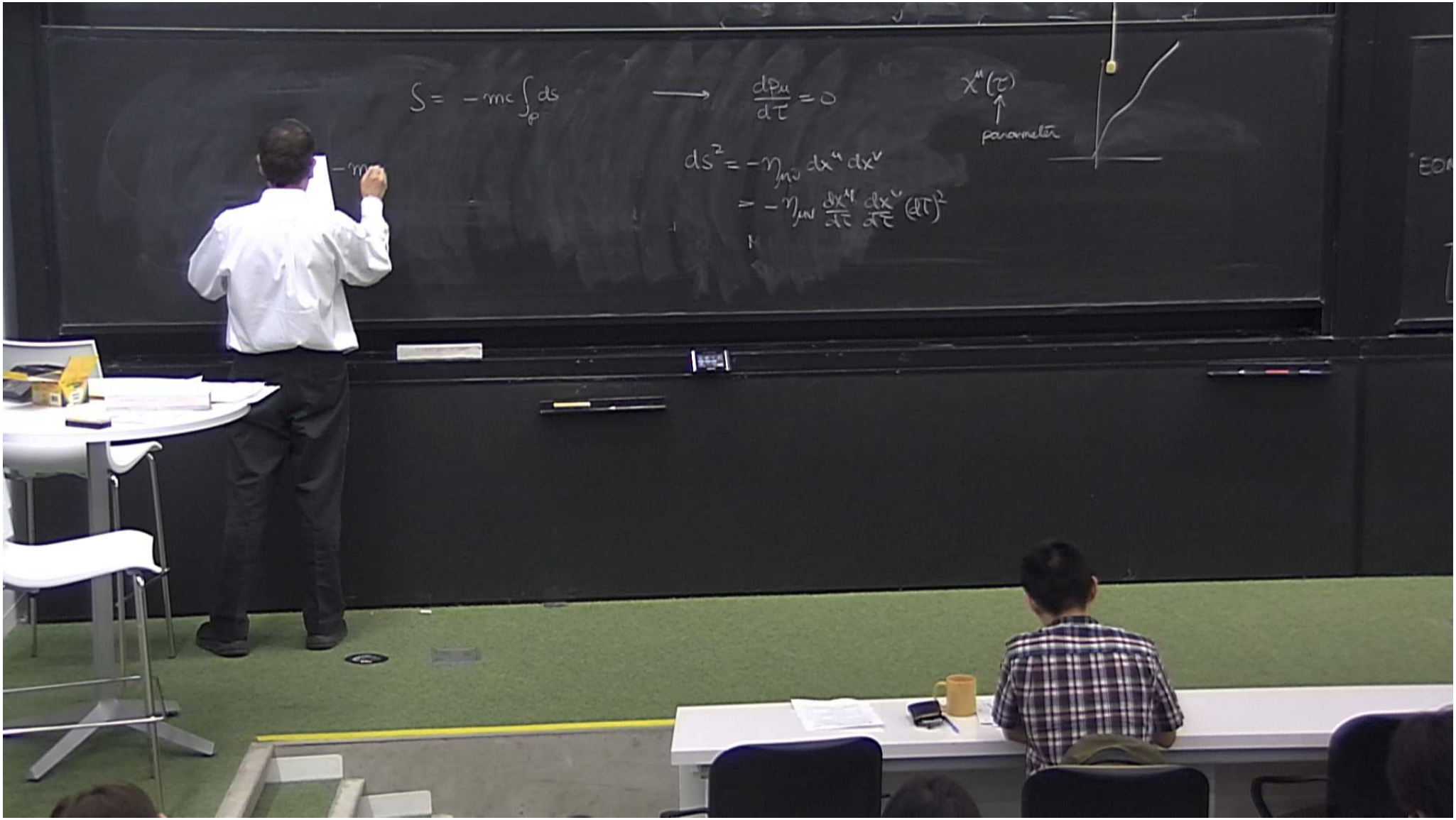
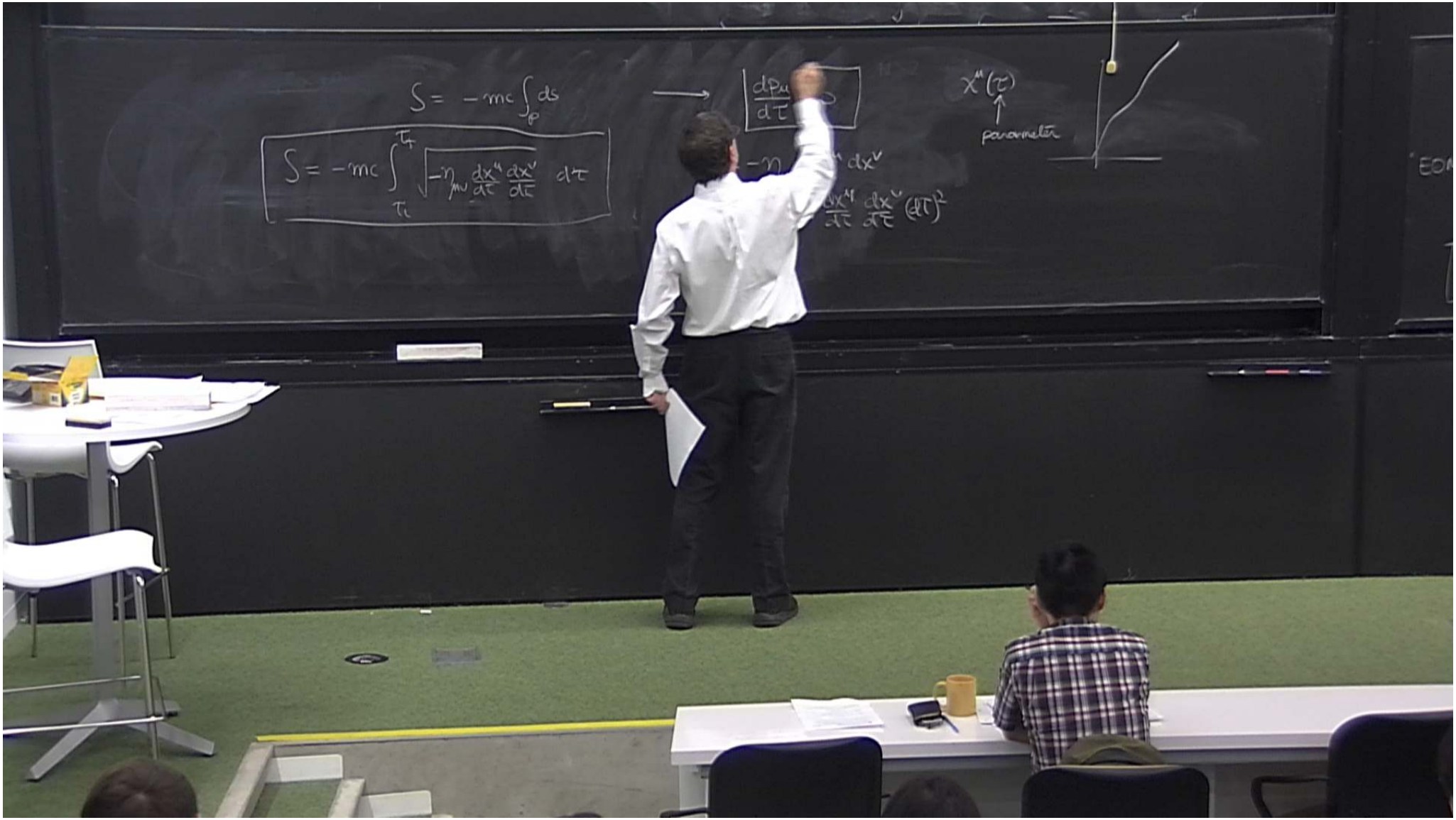

N> M>2
Relativistic Point Particle (Free)
L = ½ mv²
p = ∂L/∂v = mv
t
x
p
S = -mc ∫ ds
∫ dt L(x,ẋ)

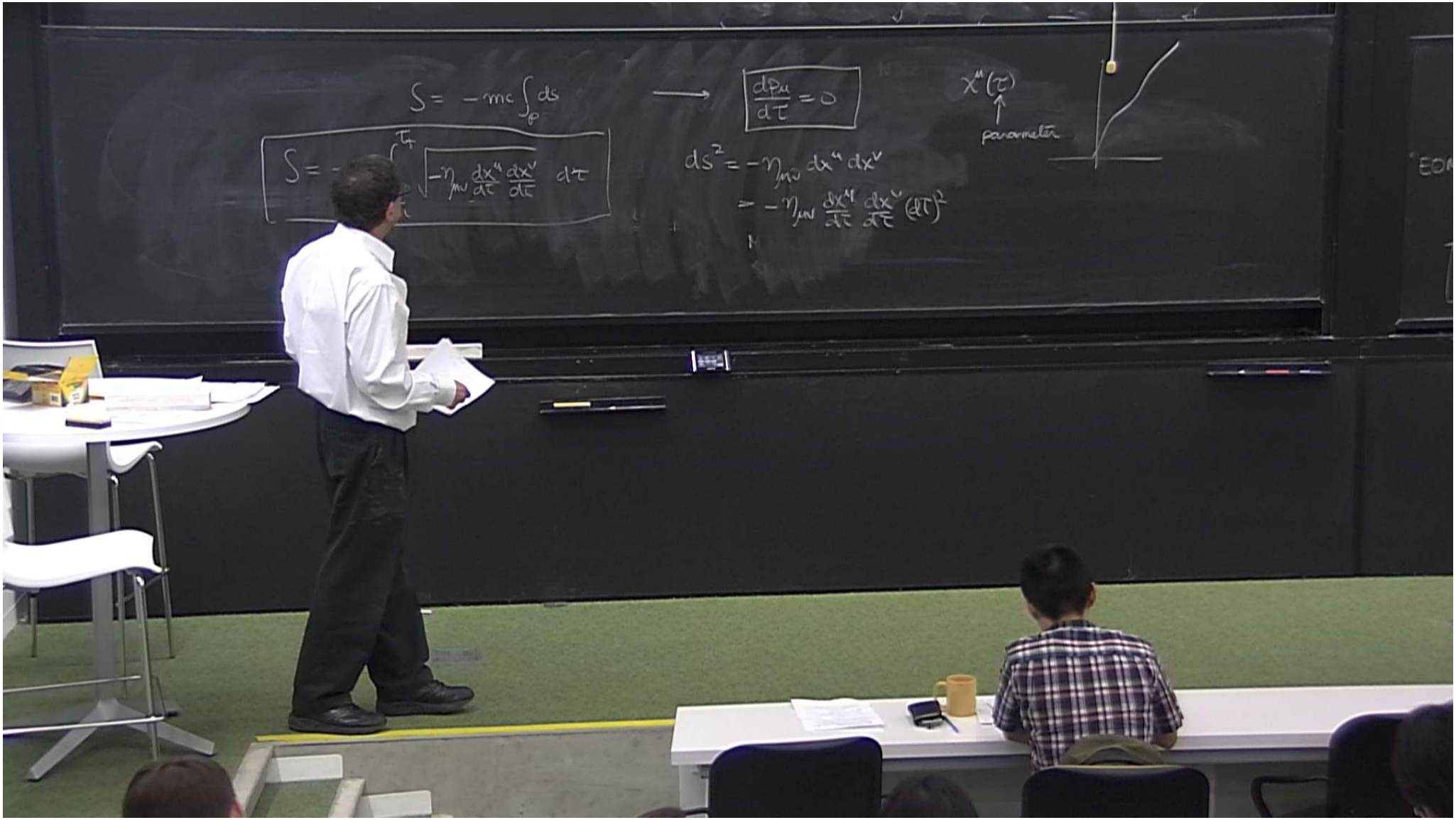

N > M ≥ 2
Relativistic Point Particle (Free)
$L = \frac{1}{2}mv^2$
$p = \frac{\partial L}{\partial v} = mv$
$S = -mc\int \frac{ds}{\text{const}}$
$\int dt\, L(x, \dot{x})$
t
x
x_0
x_1
δx

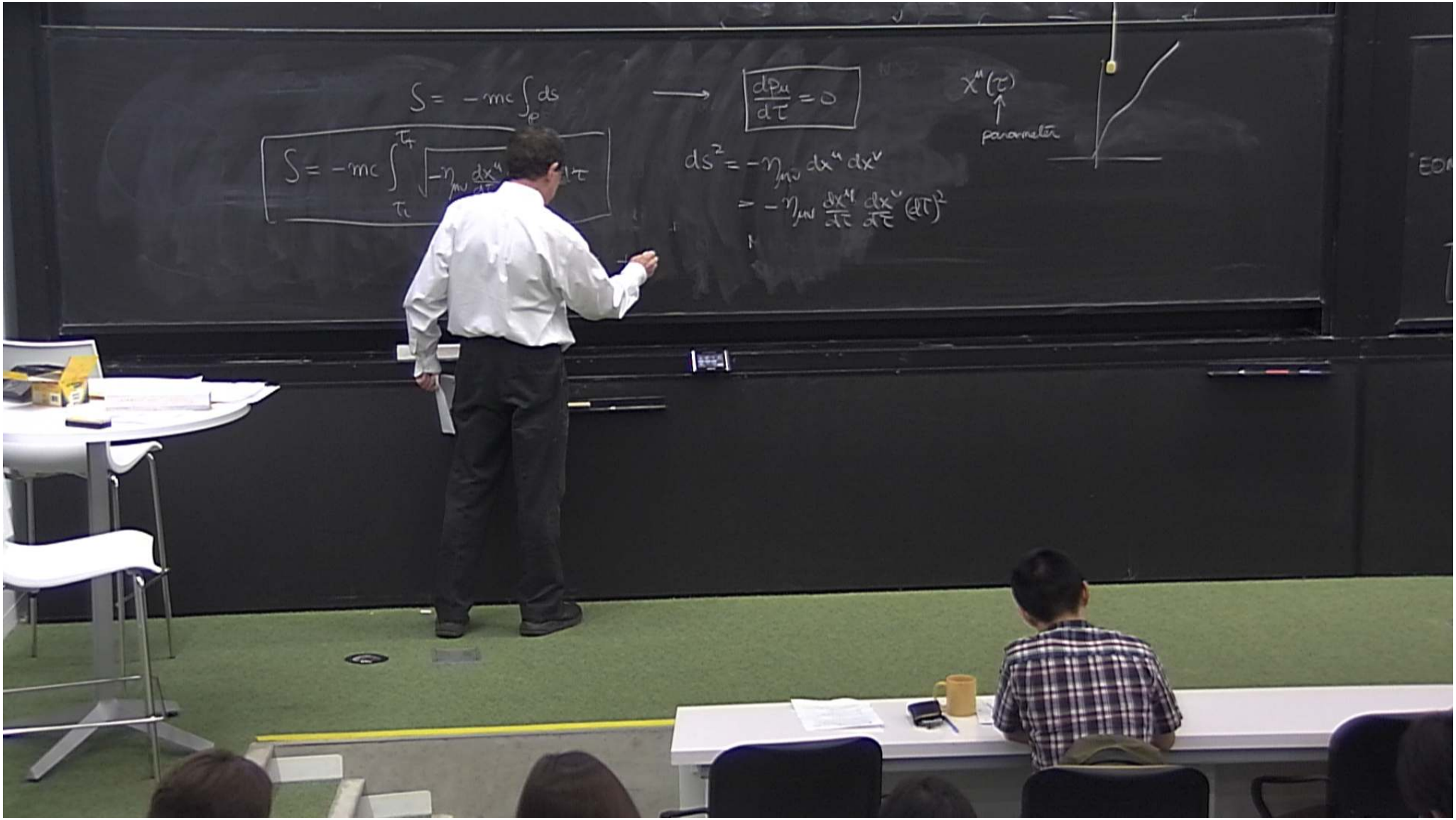

N > M ≥ 2
Relativistic Point Particle (Free)
$L = \frac{1}{2}mv^2$
$p = \frac{\partial L}{\partial v} = mv$
$S = -mc\int \frac{ds}{\text{const}} = \int -mc^2\sqrt{1-\frac{v^2}{c^2}}\,dt$
$ds = c\,dt\sqrt{1-\frac{v^2}{c^2}}$
$L = -mc^2\sqrt{1-\frac{v^2}{c^2}}$
$\vec{p} = \frac{\partial L}{\partial \vec{v}}$
t
x
p


$S = -mc\int_p ds$
$\frac{dP_\mu}{d\tau} = 0$
$x^\mu(\tau)$
parameter


$S = -mc\int_p ds$
$\longrightarrow \frac{dP_\mu}{d\tau} = 0$
$x^\mu(\tau)$
parameter
$ds^2 = -\eta_{\mu\nu}\, dx^\mu dx^\nu$
$= -\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}(d\tau)^2$


$S = -mc\int_P ds$
$S = -mc\int_{\tau_i}^{\tau_f}\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\,d\tau$
$\frac{dp_\mu}{d\tau}$
$x^\mu(\tau)$
parameter


$S = -mc\int_P ds$
$\frac{dp_\mu}{d\tau} = 0$
$S = -\int_{\tau_i}^{\tau_f} \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\, d\tau$
$ds^2 = -\eta_{\mu\nu}\, dx^\mu dx^\nu$
$= -\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}(d\tau)^2$
$x^\mu(\tau)$
parameter


$S = -mc\int_P ds$
$\frac{dP_\mu}{d\tau} = 0$
$S = -mc\int_{\tau_i}^{\tau_f} \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\tau}} \, d\tau$
$ds^2 = -\eta_{\mu\nu}\, dx^\mu dx^\nu$
$= -\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}(d\tau)^2$
$x^\mu(\tau)$
parameter