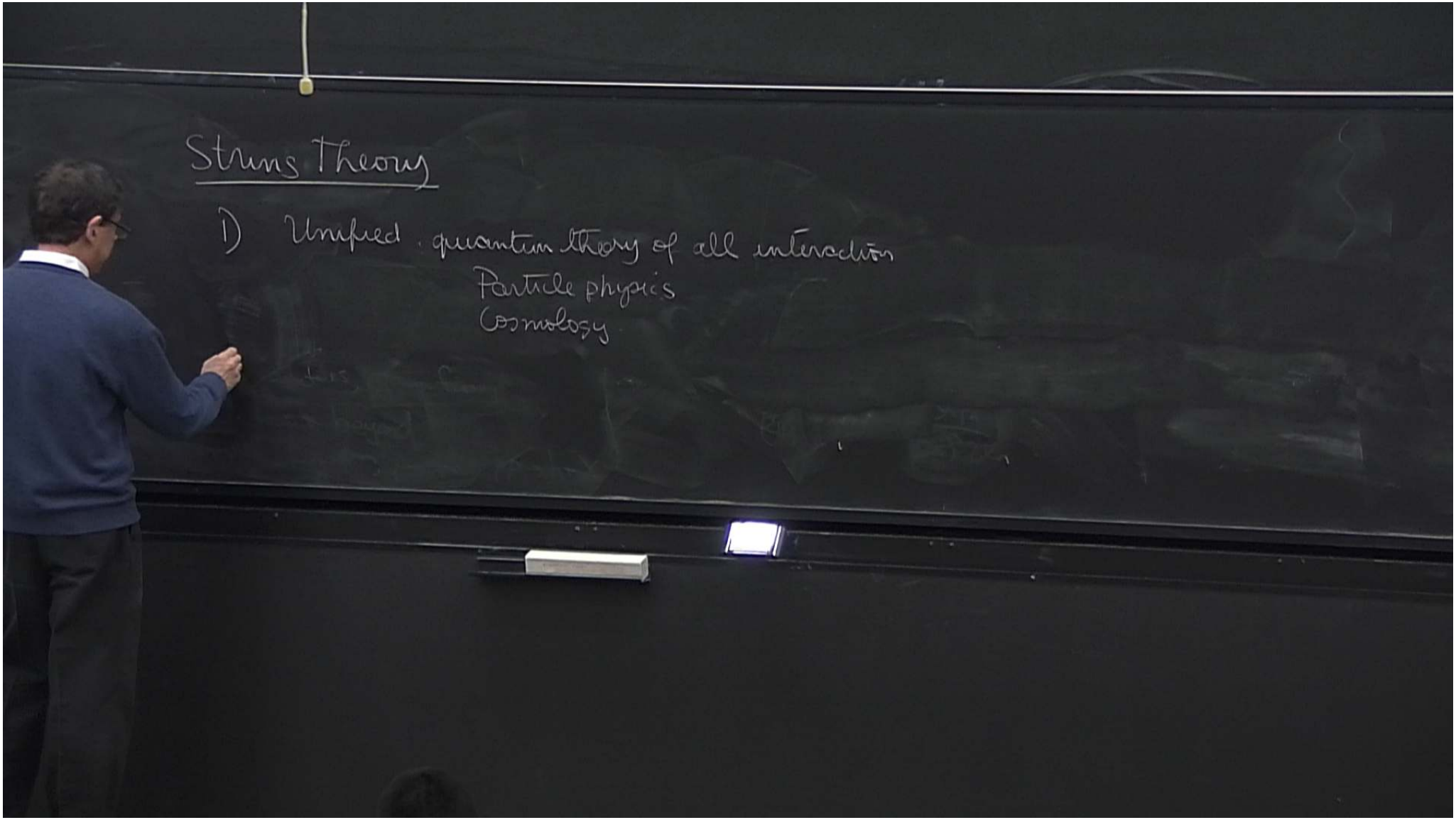
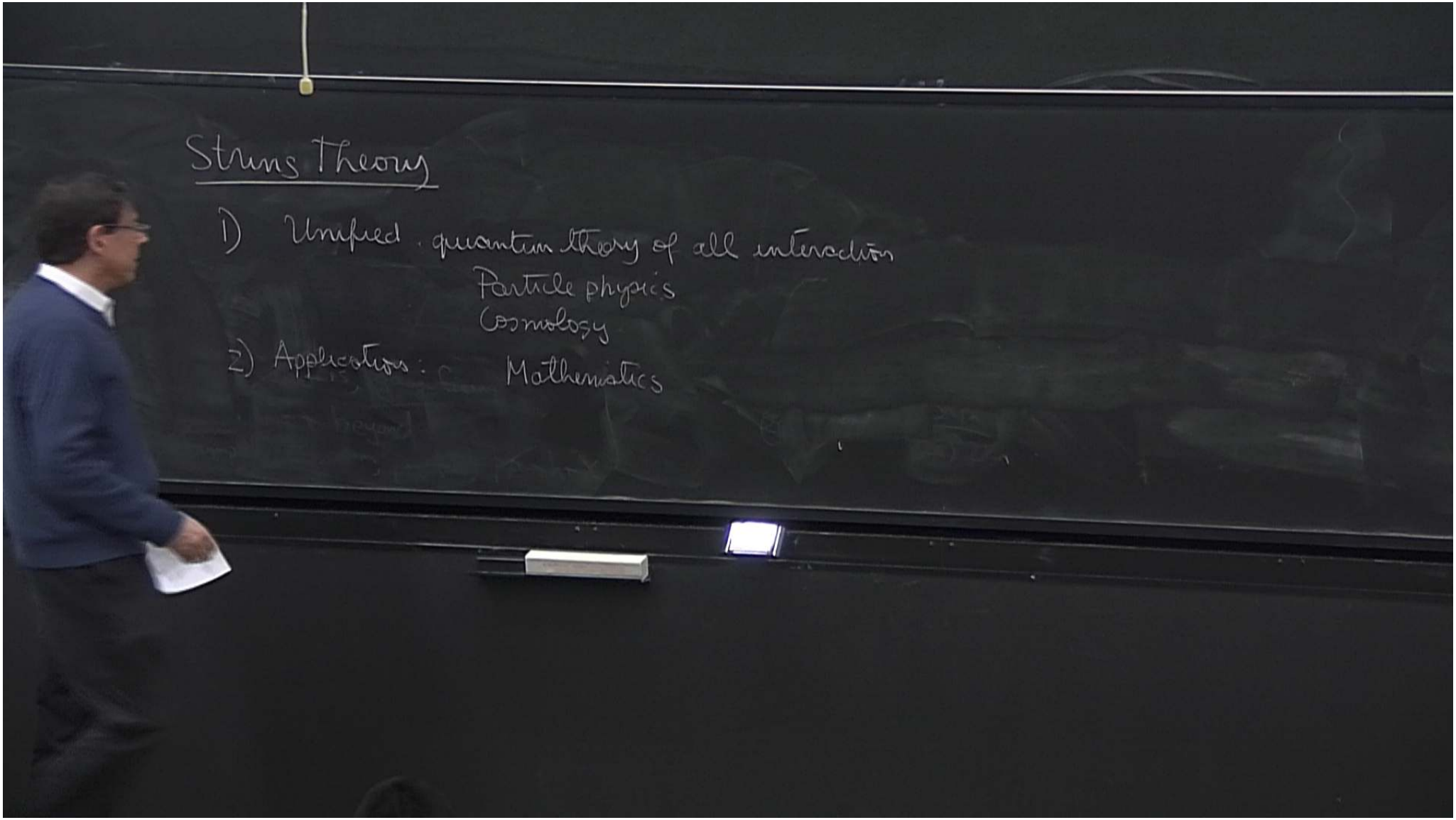
Title: 12/13 PSI - String Theory Review Lecture 1

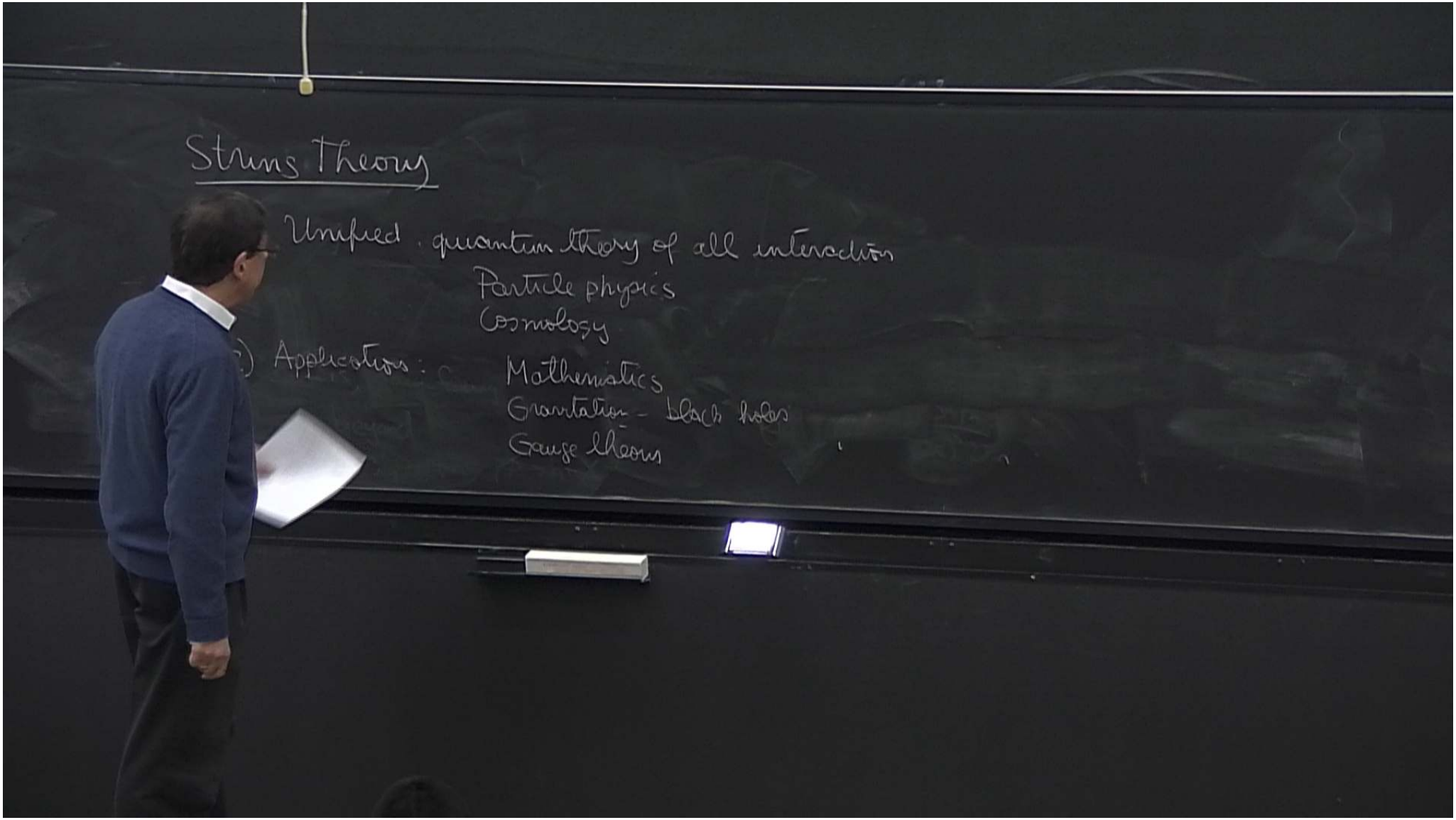
Date: Jan 07, 2013 10:15 AM

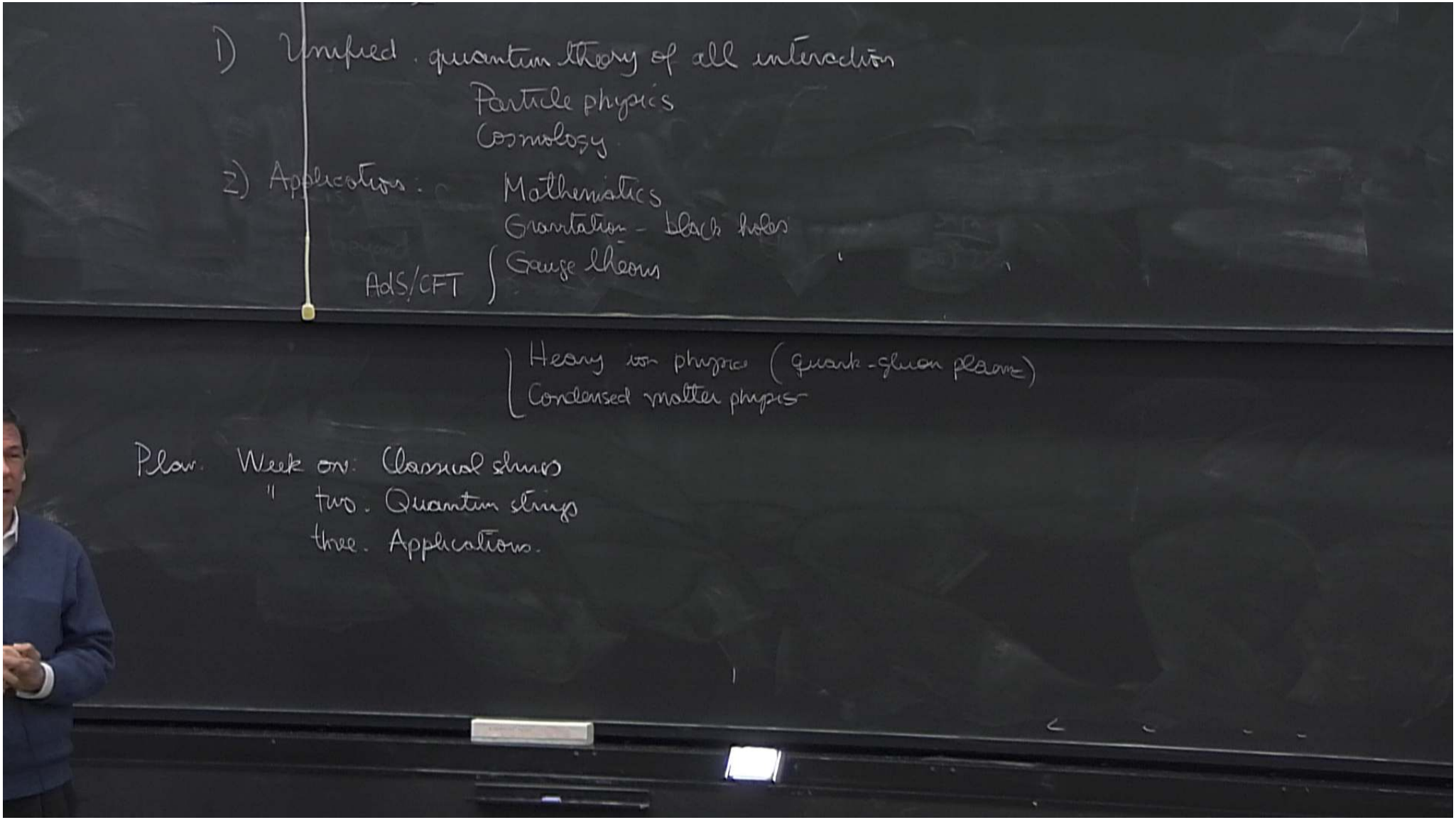
URL: http://pirsa.org/13010043

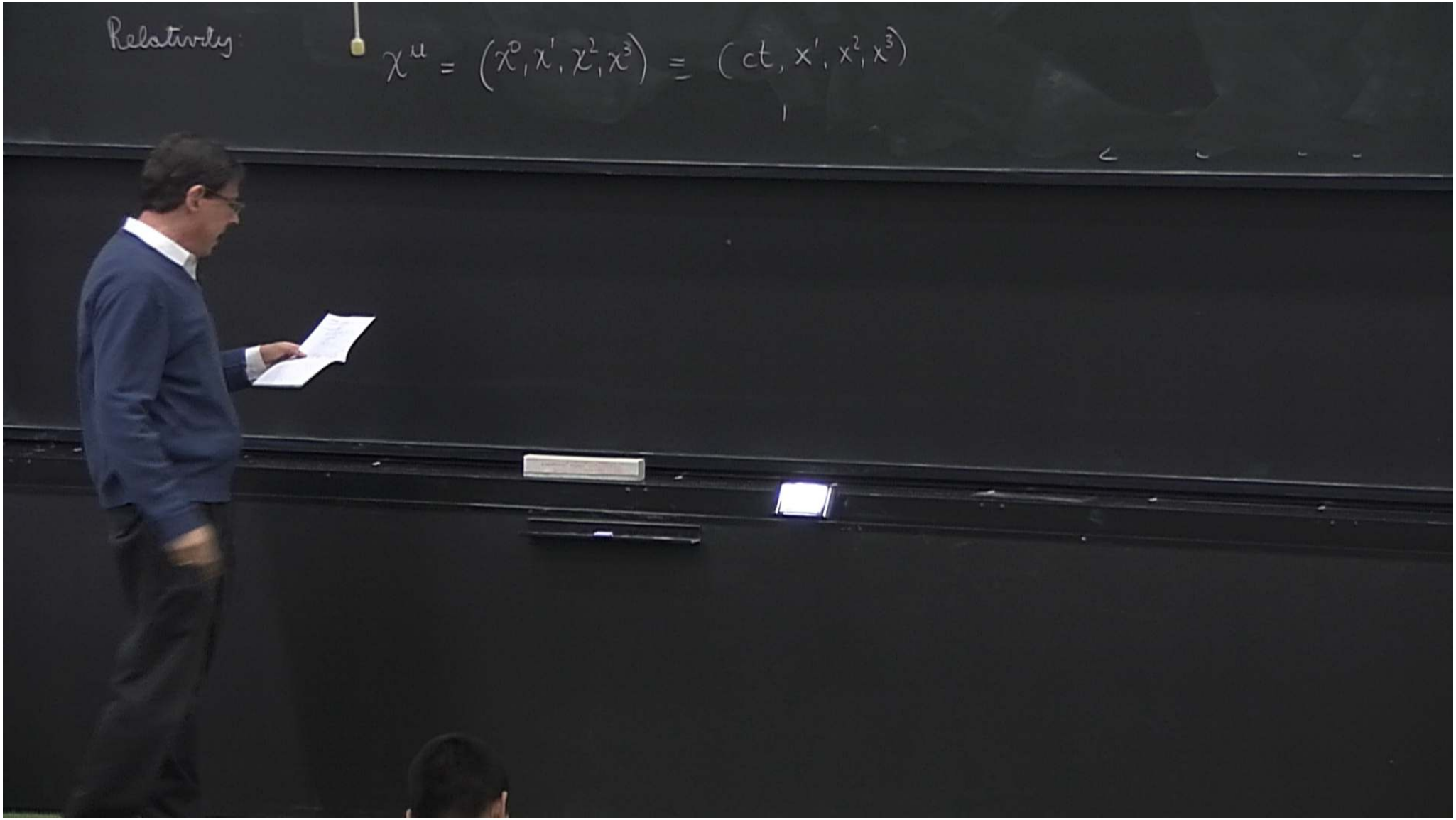
Abstract:

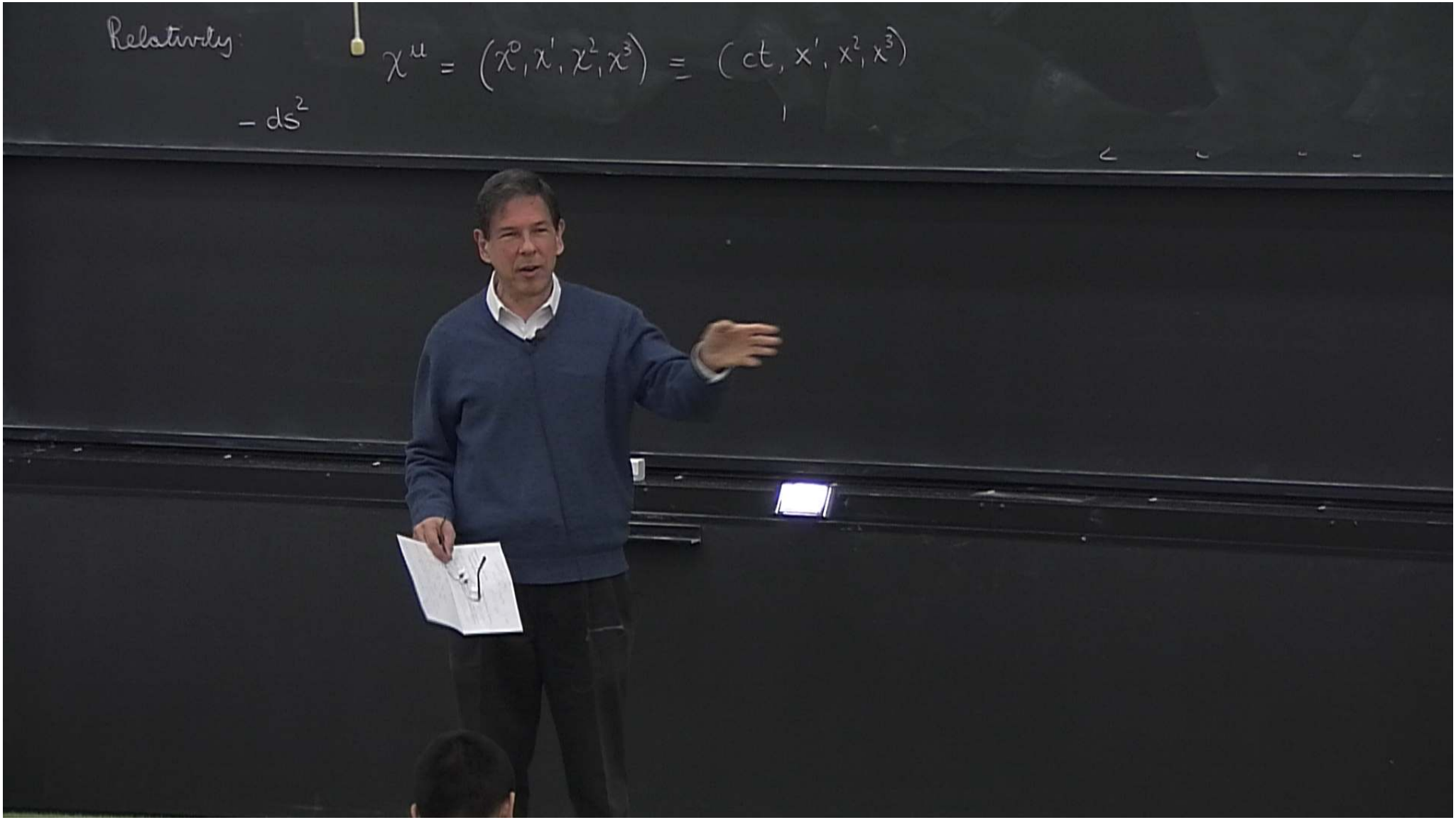

Strins Theory
1) Unified quantum theory of all interaction
Particle physics
Cosmology

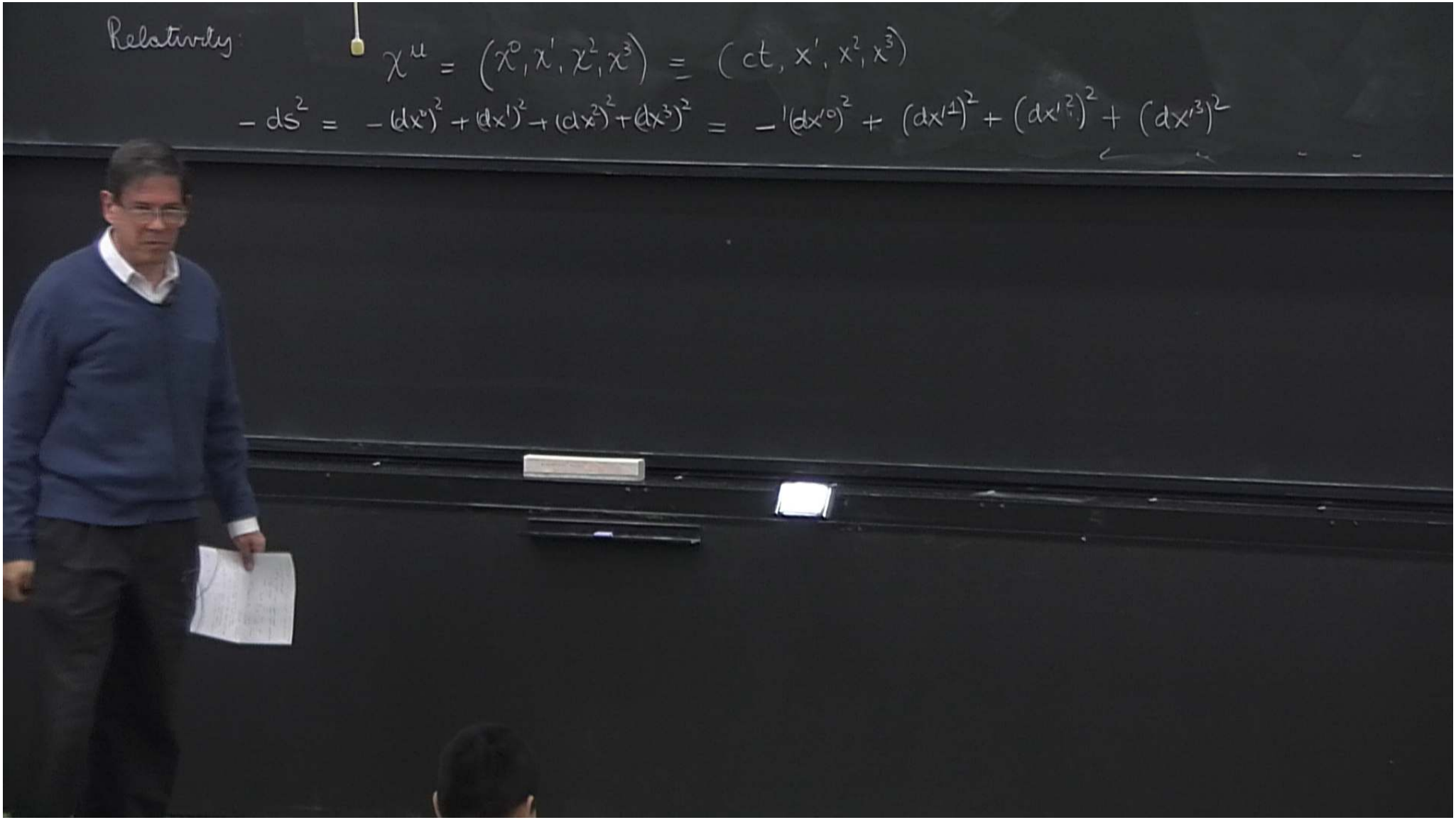

Strings Theory
1) Unified quantum theory of all interaction
Particle physics
Cosmology
2) Applications: Mathematics

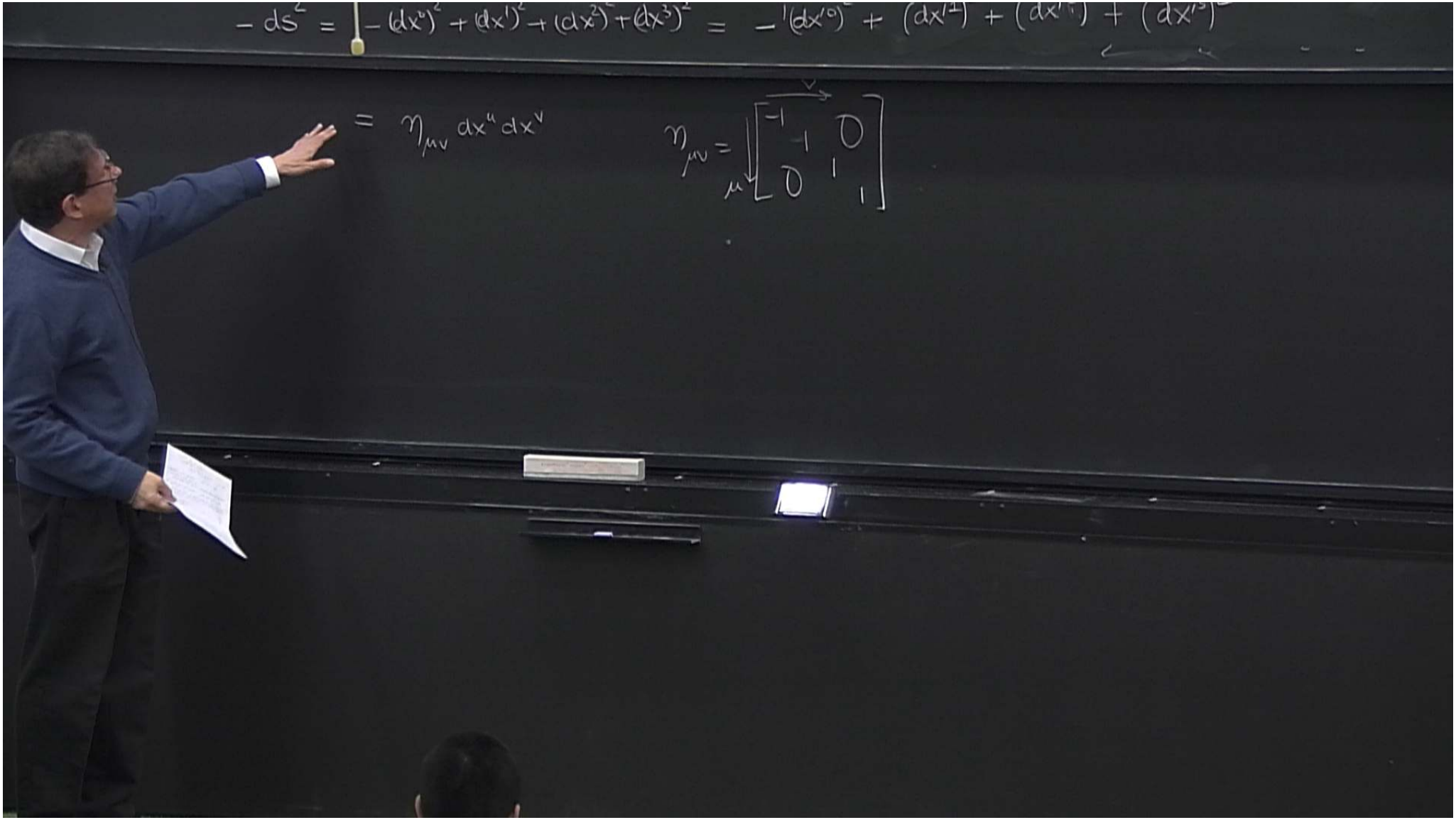

Strins Theory
Unified quantum theory of all interaction
Particle physics
Cosmology
Applications:
Mathematics
Gravitation - black holes
Gauge theors

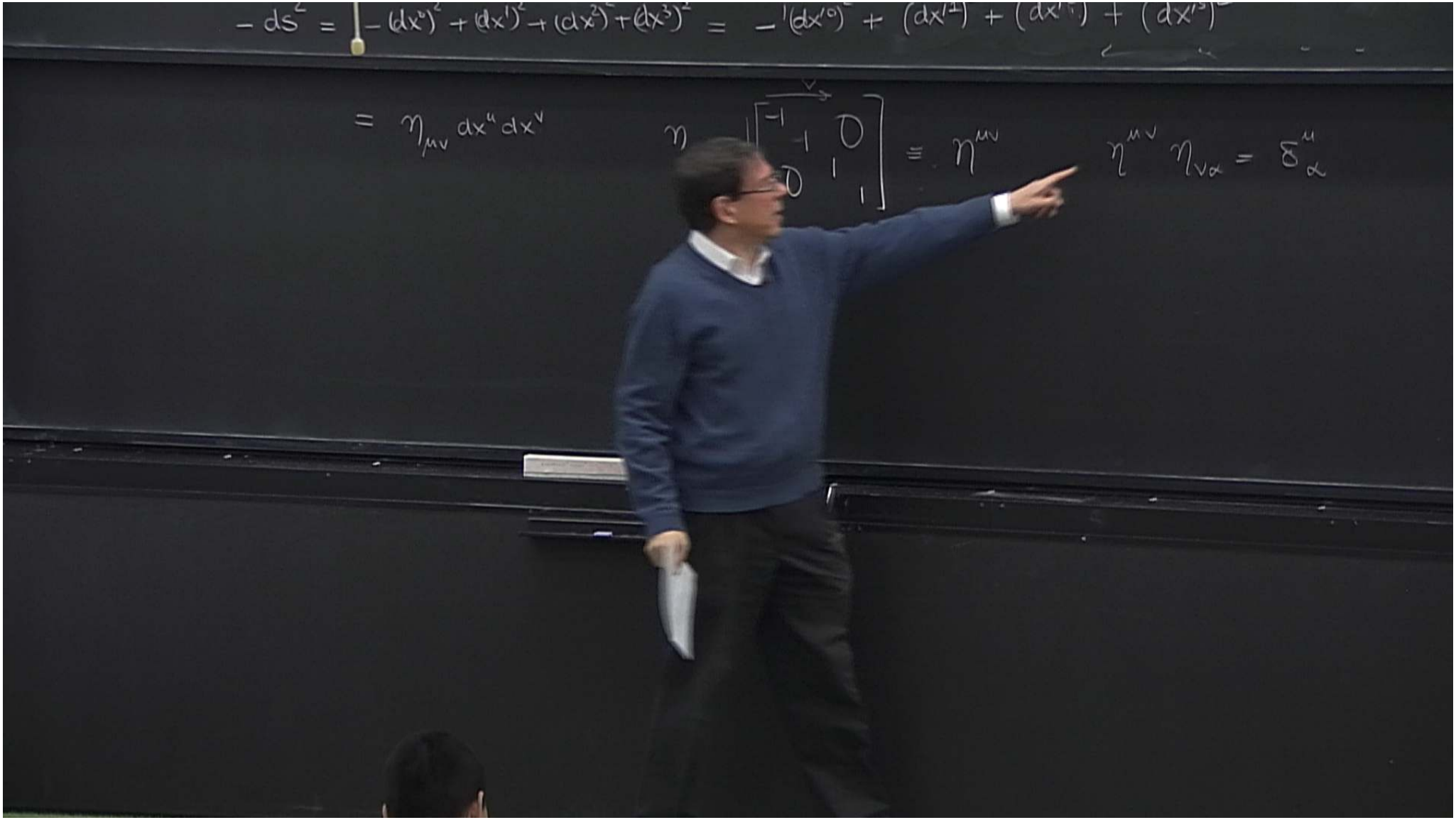

1) Unified quantum theory of all interaction
Particle physics
Cosmology
2) Applications:
Mathematics
Gravitation - black holes
AdS/CFT
Gauge theories
Heavy ion physics (quark-gluon plasma)
Condensed matter physics
Plan. Week one: Classical strings
" two. Quantum strings
three. Applications.


Relativity
$x^\mu = (x^0, x^1, x^2, x^3) \equiv (ct, x^1, x^2, x^3)$


Relativity
$x^\mu = (x^0, x^1, x^2, x^3) \equiv (ct, x^1, x^2, x^3)$
$-ds^2$


Relativity
$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x^1, x^2, x^3)$
$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$


$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$
$= \eta_{\mu\nu}\, dx^\mu dx^\nu$
$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
μ
ν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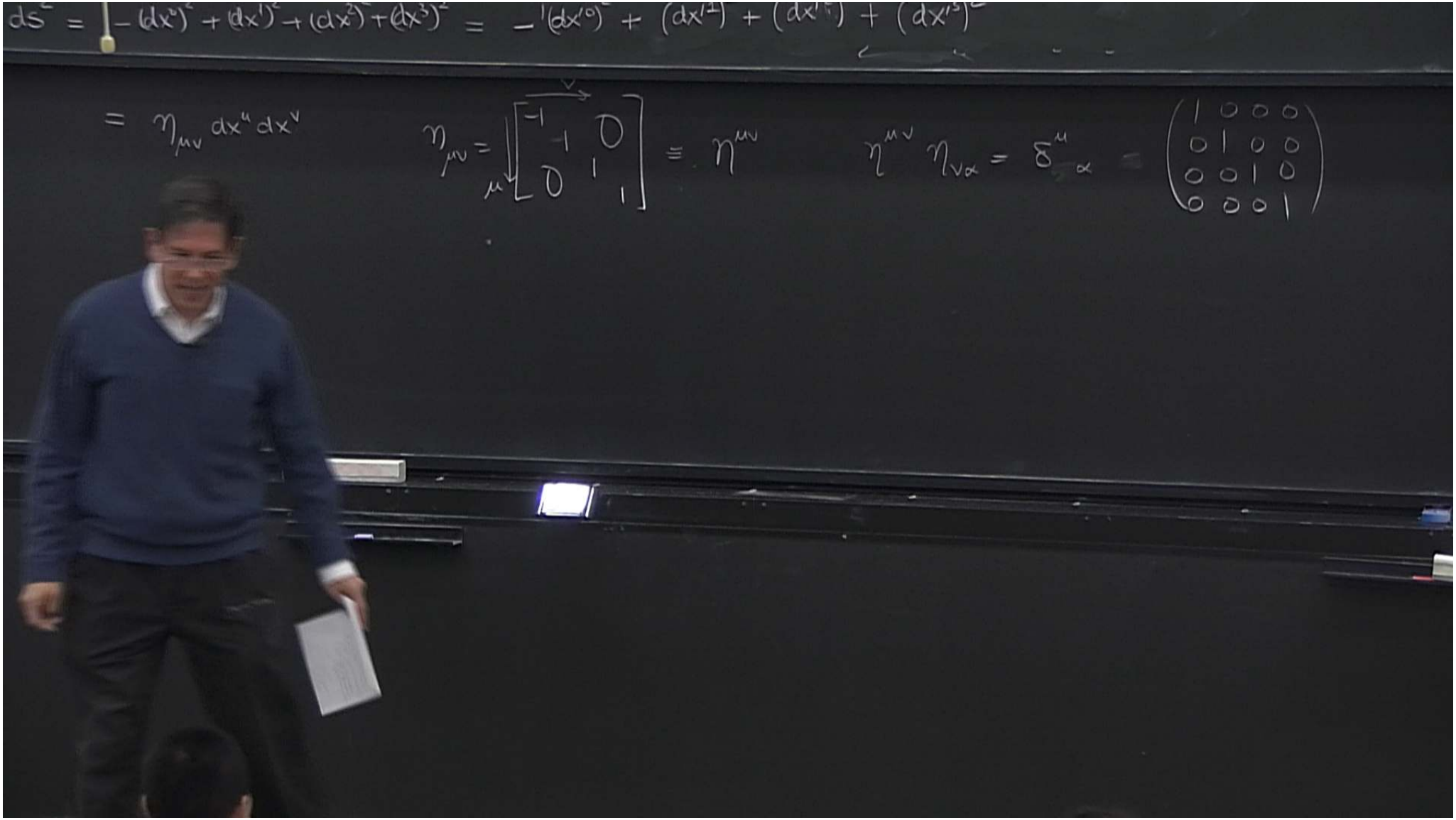
$\eta \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv \eta^{\mu\nu}$
$\eta^{\mu\nu}\,\eta_{\nu\alpha} = \delta^\mu_\alpha$


$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$
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$\eta^{\mu\nu}\,\eta_{\nu\alpha} = \delta^\mu{}_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$

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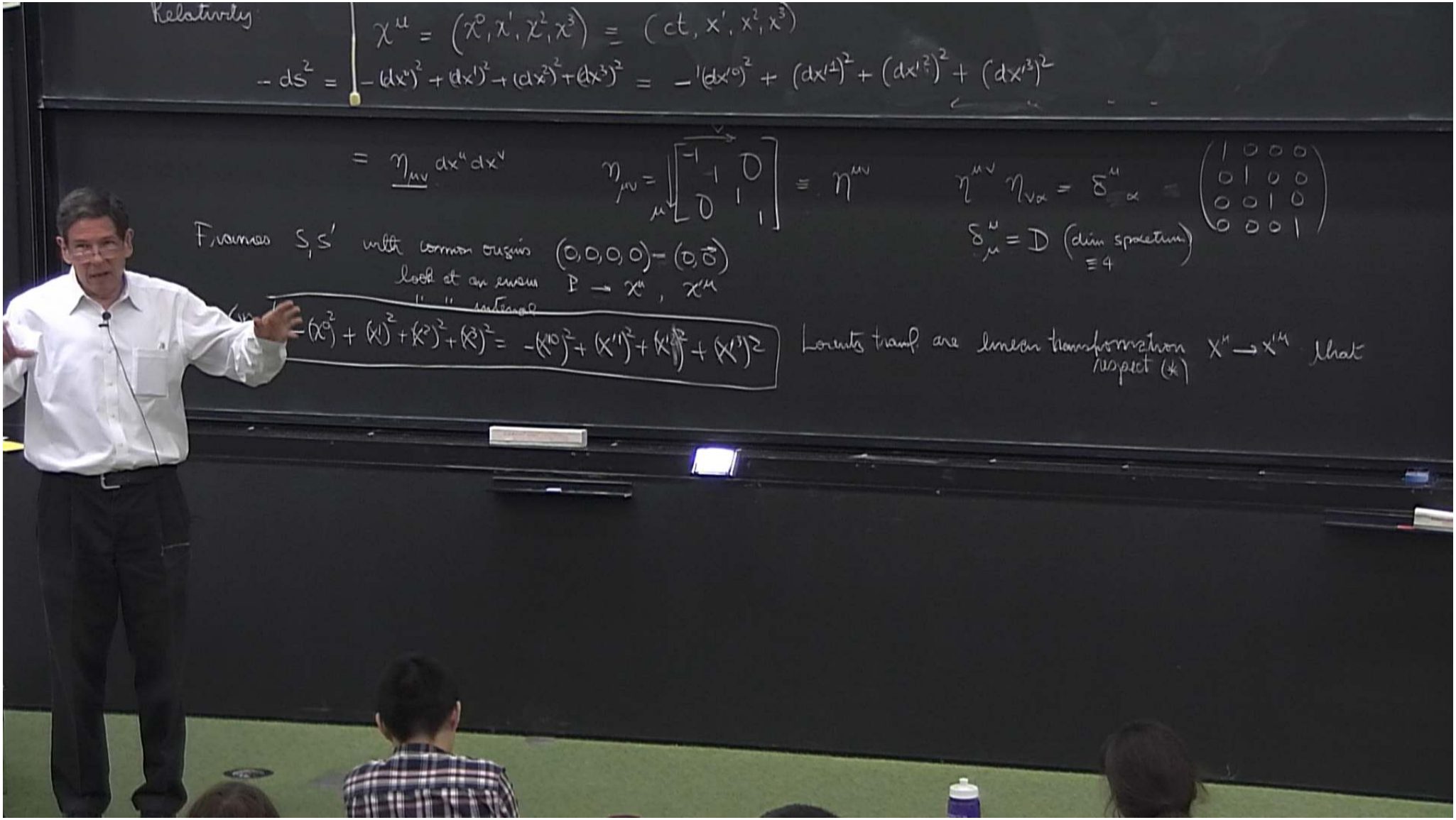
$\eta^{\mu\nu}\eta_{\nu\alpha} = \delta^\mu{}_\alpha$

$\delta^\mu_\mu = D$ (dim spacetime) $\equiv 4$


Relativity
$x^\mu = (x^0, x^1, x^2, x^3) \equiv (ct, x, y, z)$
$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$
$= \eta_{\mu\nu}\, dx^\mu dx^\nu$
$\eta_{\mu\nu} = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \equiv \eta^{\mu\nu}$
$\eta^{\mu\nu}\eta_{\nu\alpha} = \delta^\mu{}_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
$\delta^\mu_\mu = D$ (dim spacetime) $= 4$
Frames S, S' with common origins $(0,0,0,0) = (0,\vec{0})$
look at an event $P \to x^\mu, x'^\mu$
$-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 = -(x'^0)^2 + (x'^1)^2 + (x'^2)^2 + (x'^3)^2$
Lorentz transf. are linear transformation $x^\mu \to x'^\mu$ that
respect (*)

Relativity

$$x^\mu = (x^0, x^1, x^2, x^3) \equiv (ct, x^1, x^2, x^3)$$

$$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$$

$$= \eta_{\mu\nu}\, dx^\mu dx^\nu \qquad \eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv \eta^{\mu\nu} \qquad \eta^{\mu\nu}\eta_{\nu\alpha} = \delta^\mu{}_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

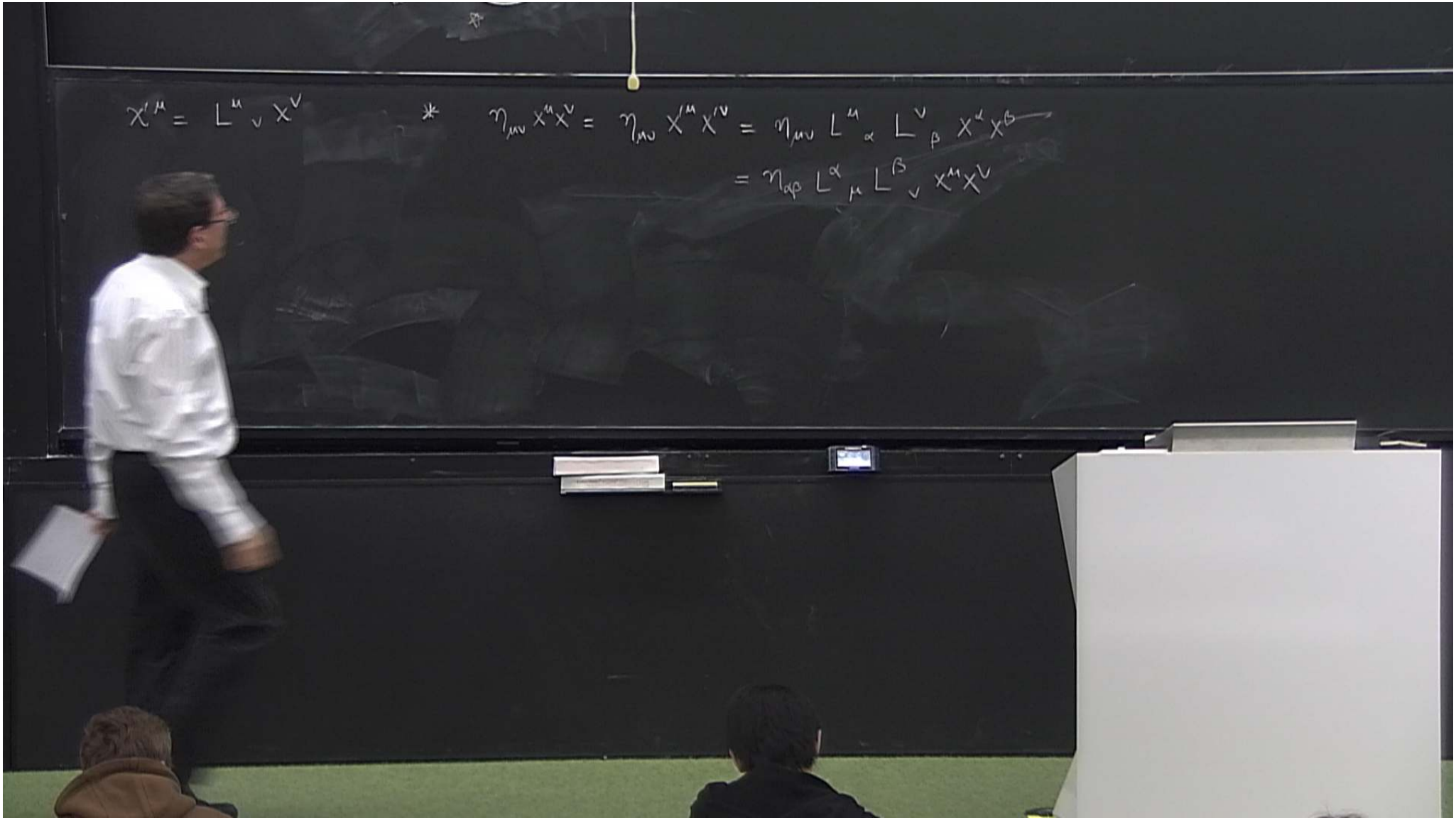
$\delta^\mu_\mu = D$ (dim spacetime) $\equiv 4$

Frames S, S' with common origins $(0,0,0,0) = (0,\vec{0})$

look at an event $P \to x^\mu, x'^\mu$

(*) $$-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 = -(x'^0)^2 + (x'^1)^2 + (x'^2)^2 + (x'^3)^2$$

Lorentz transf. are linear transformation $x^\mu \to x'^\mu$ that respect (*)


$x'^{\mu} = L^{\mu}{}_{\nu} x^{\nu}$
$\eta_{\mu\nu} x^{\mu} x^{\nu} = \eta_{\mu\nu} x'^{\mu} x'^{\nu} = \eta_{\mu\nu} L^{\mu}{}_{\alpha} L^{\nu}{}_{\beta} x^{\alpha} x^{\beta}$
$= \eta_{\alpha\beta} L^{\alpha}{}_{\mu} L^{\beta}{}_{\nu} x^{\mu} x^{\nu}$

$x'^\mu = L^\mu{}_\nu x^\nu$ * $\eta_{\mu\nu} x^\mu x^\nu = \eta_{\mu\nu} x'^\mu x'^\nu = \eta_{\mu\nu} L^\mu{}_\alpha L^\nu{}_\beta x^\alpha x^\beta$

$= \eta_{\alpha\beta} L^\alpha{}_\mu L^\beta{}_\nu x^\mu x^\nu$

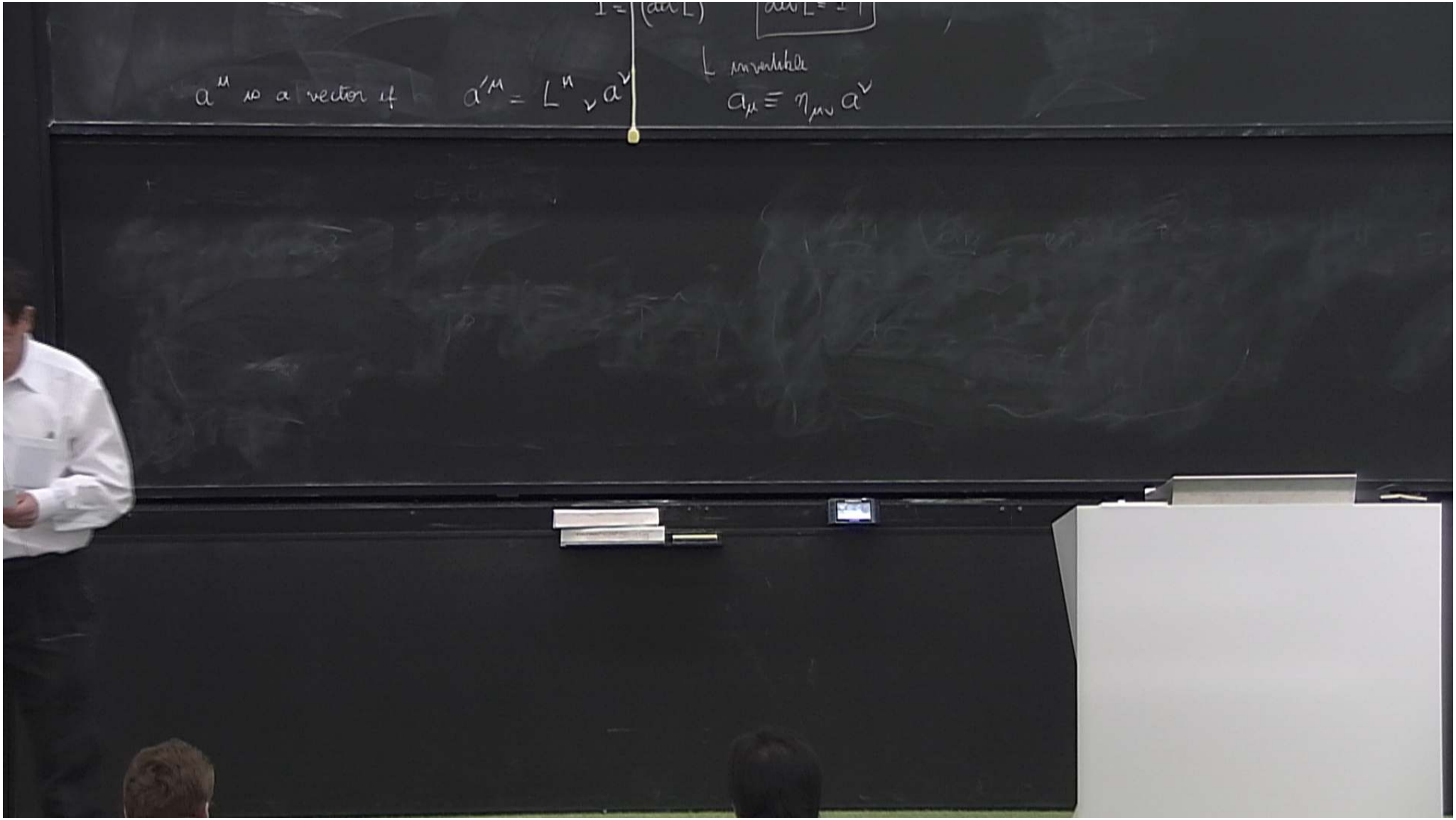
$\eta_{\mu\nu} = L^\alpha{}_\mu \eta_{\alpha\beta} L^\beta{}_\nu$

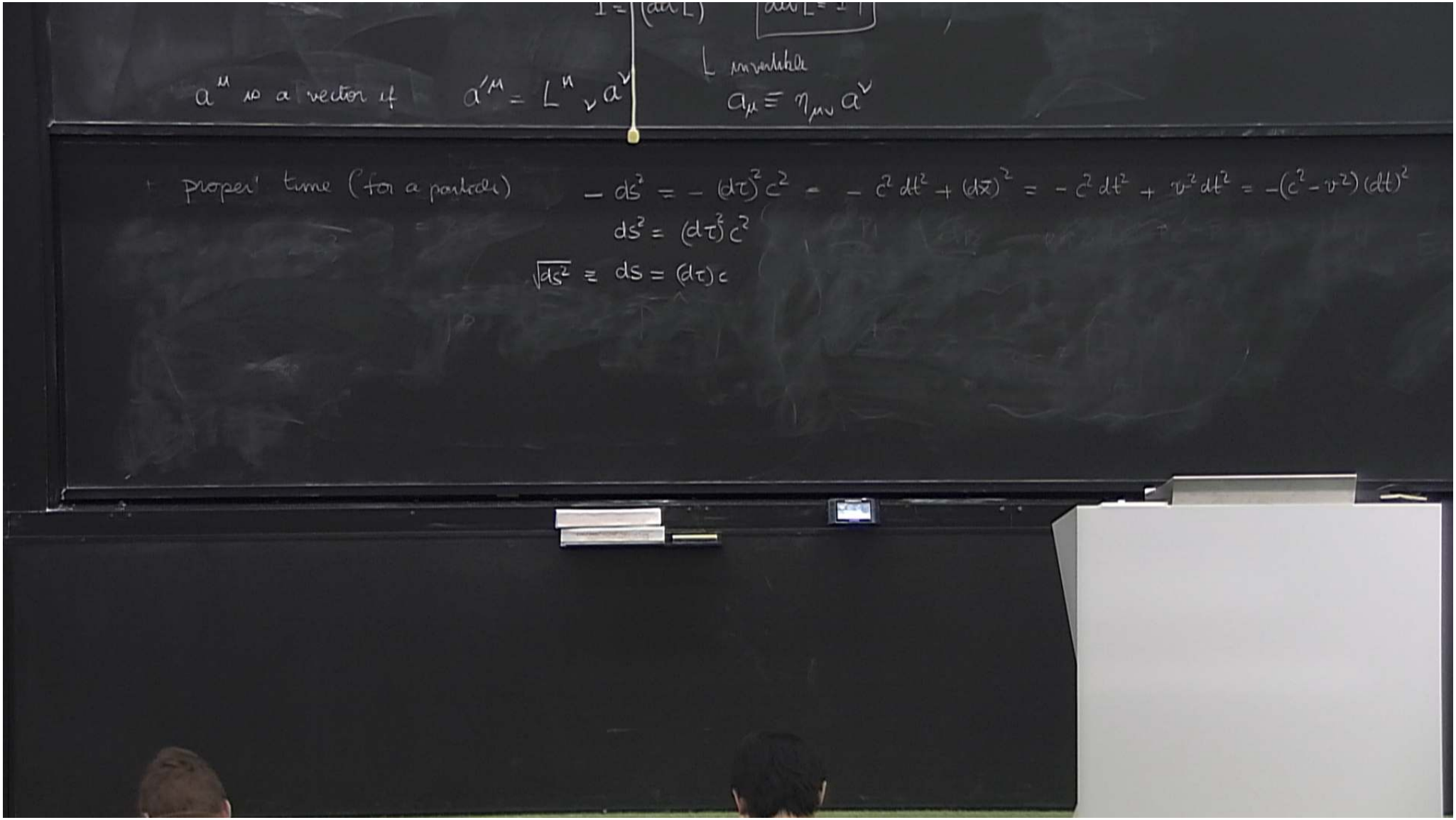
L is a LT iff: $\boxed{\eta = L^T \eta L}$

$\det \eta = (\det L^T)(\det \eta)(\det L)$

$1 = (\det L)^2$ $\boxed{\det L = \pm 1}$

L invertible


a^μ is a vector if $a'^\mu = L^\mu{}_\nu a^\nu$
L invertible
$a_\mu \equiv \eta_{\mu\nu} a^\nu$


a^μ is a vector if $a'^\mu = L^\mu{}_\nu a^\nu$
L invertible
$a_\mu \equiv \eta_{\mu\nu} a^\nu$
proper time (for a particle)
$-ds^2 = -(d\tau)^2 c^2 = -c^2 dt^2 + (d\vec{x})^2 = -c^2 dt^2 + v^2 dt^2 = -(c^2 - v^2)(dt)^2$
$ds^2 = (d\tau)^2 c^2$
$\sqrt{ds^2} \equiv ds = (d\tau)c$

a^μ is a vector if $a'^\mu = L^\mu{}_\nu a^\nu$

L invertible

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$$ds^2 = (d\tau)^2 c^2$$

$$\sqrt{ds^2} \equiv ds = (d\tau)c$$

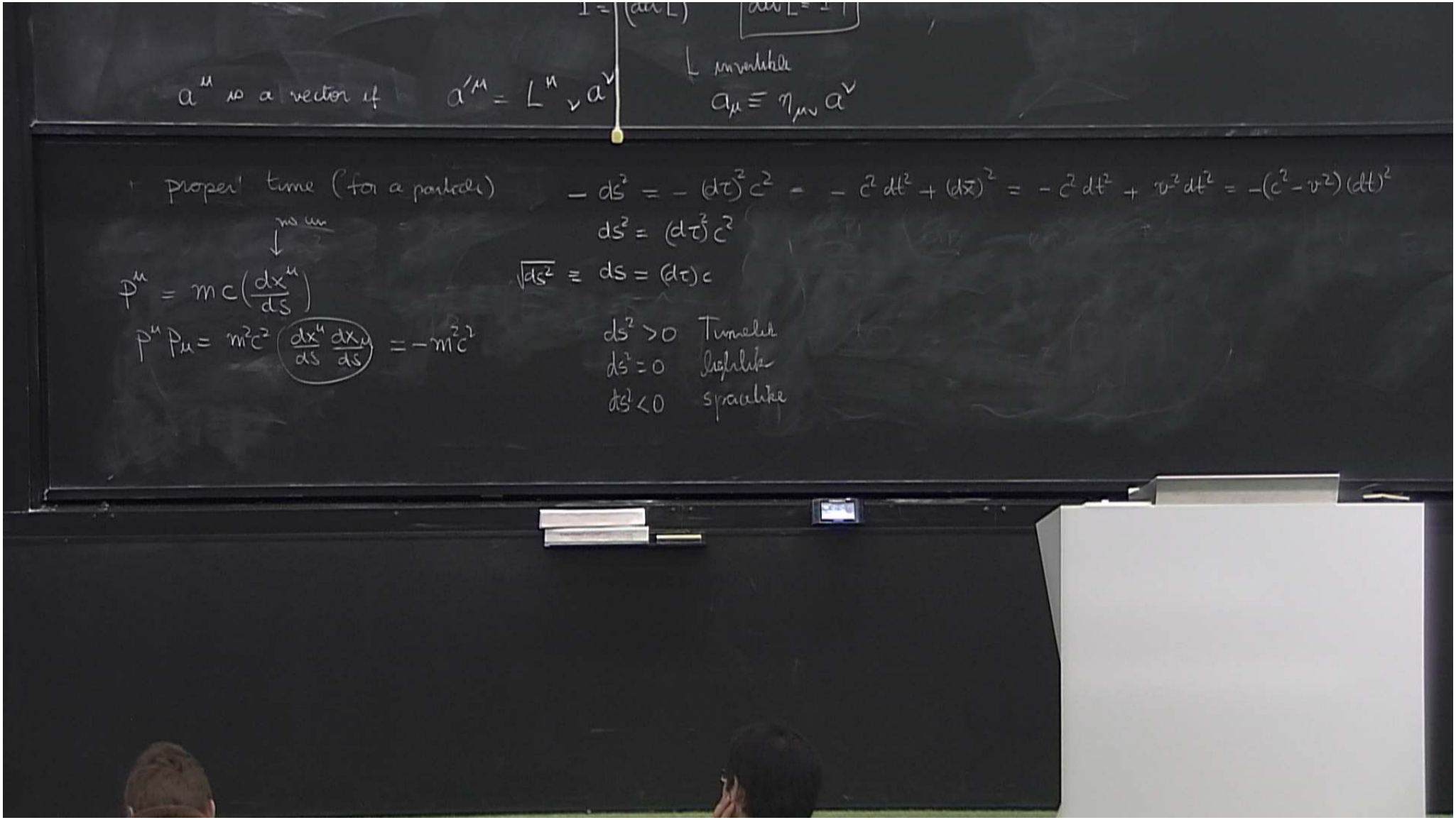
$$p^\mu = mc\left(\frac{dx^\mu}{ds}\right) \quad \text{no } m$$

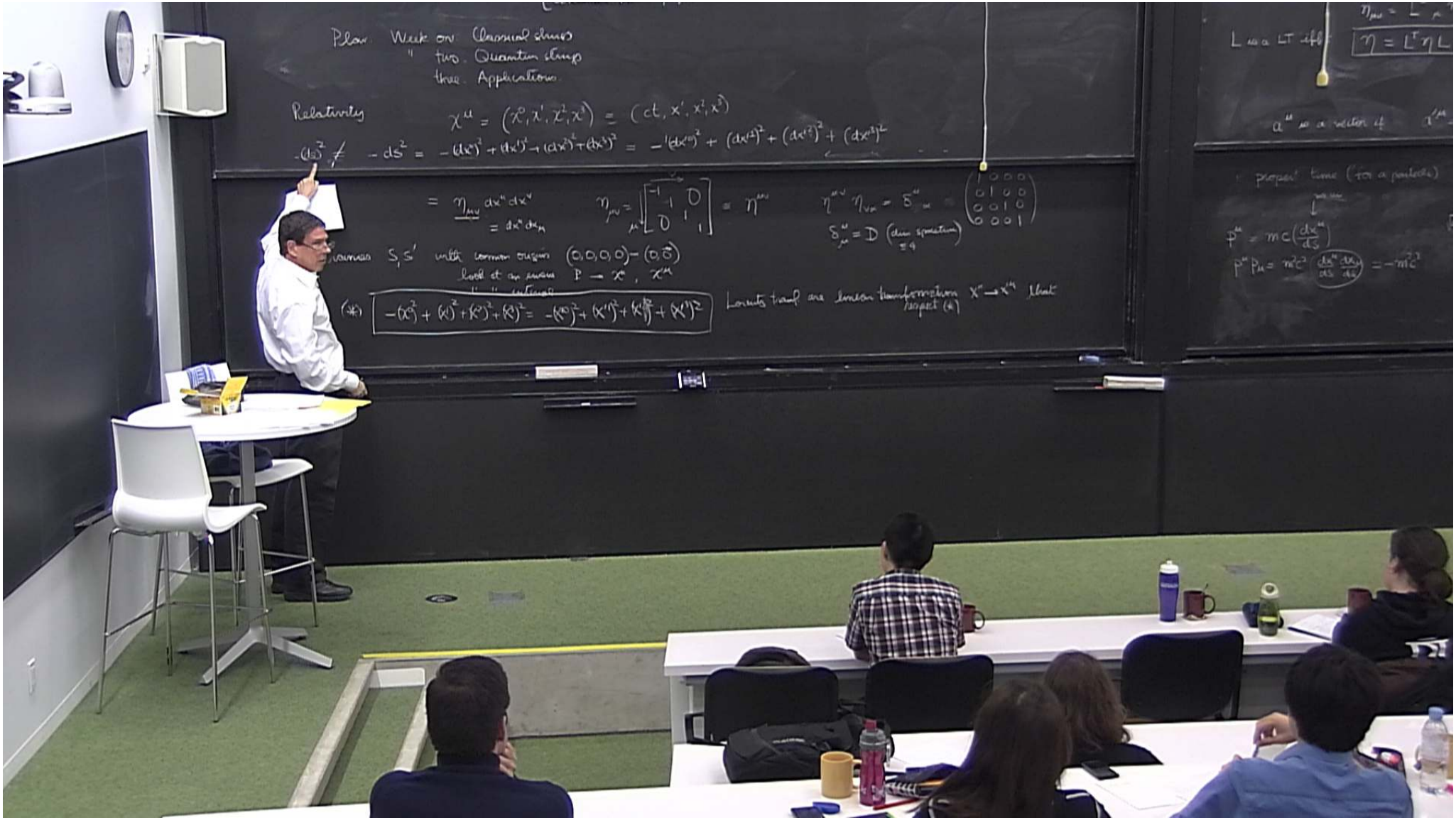
$$p^\mu p_\mu = m^2 c^2 \left(\frac{dx^\mu}{ds}\frac{dx_\mu}{ds}\right) = -m^2 c^2$$

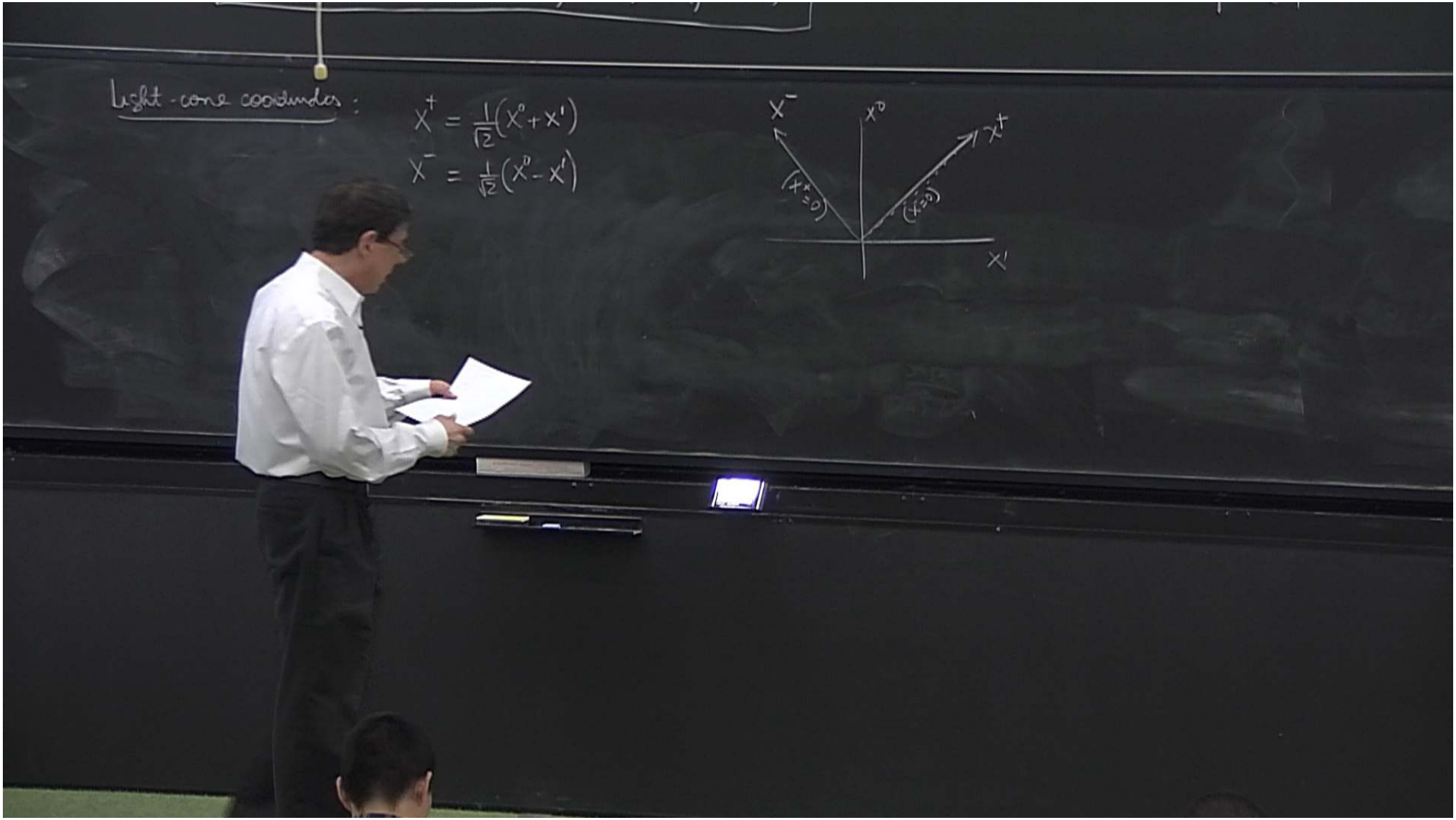
$ds^2 > 0$ Timelike

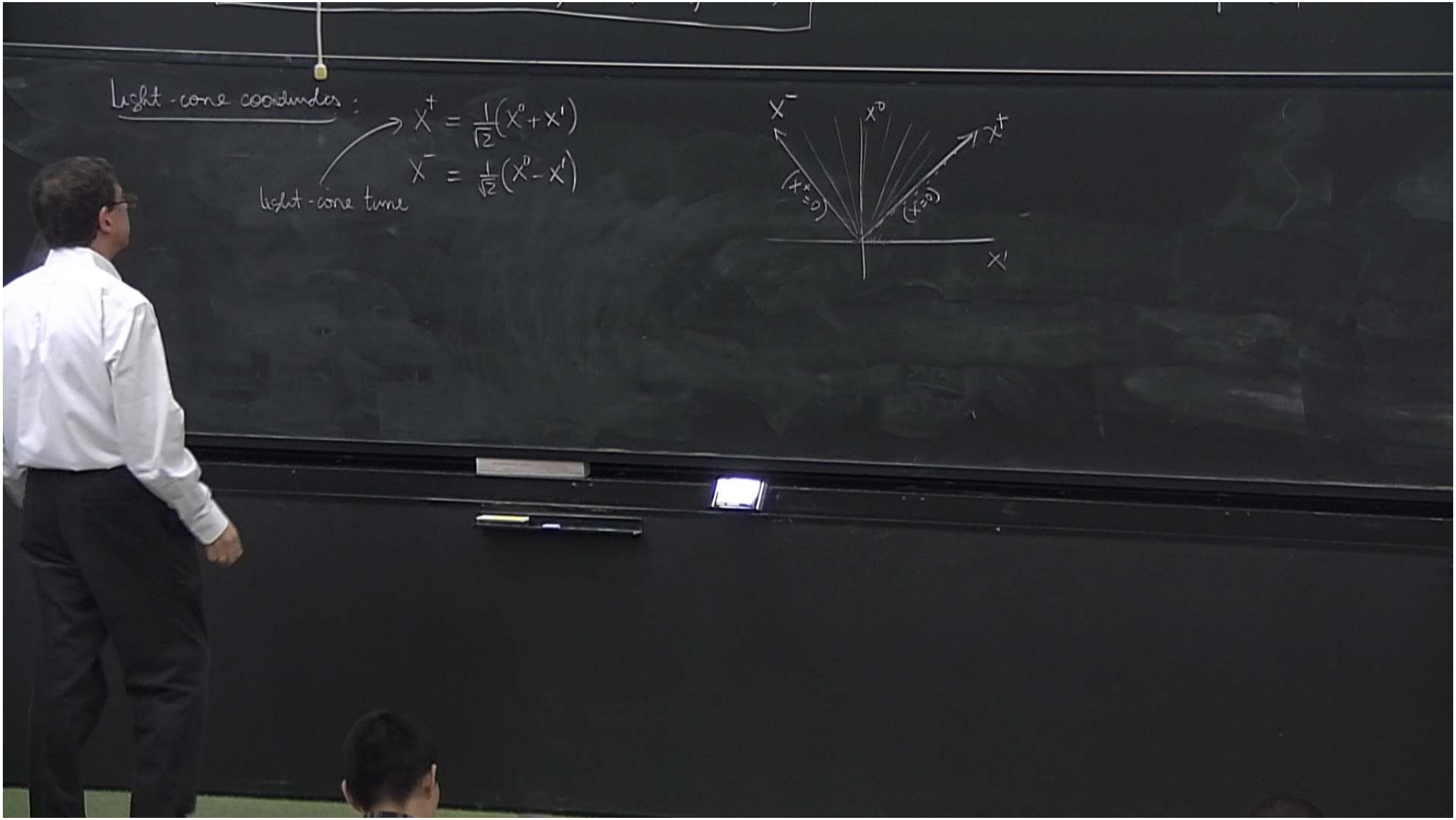
$ds^2 = 0$ lightlike

$ds^2 < 0$ spacelike




Plan Week on Classical stuff
" two Quantum stuff
three Applications
Relativity
Lorentz transf are linear transformation
L is a LT iff
proper time (for a particle)


Light-cone coordinates:
$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$
$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$
X^-
X^0
X^+
$(X^+ = 0)$
$(X^- = 0)$
X^1

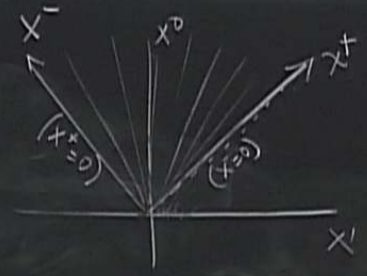

Light-cone coordinates:
$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$
$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$
light-cone time
X^-
X^0
X^+
$(X^+ = 0)$
$(X^- = 0)$
X^1

Light-cone coordinates:

$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$$

$$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$$

light-cone time



$$2\,dx^+\,dx^- = (dx^0)^2 - (dx^1)^2$$

$$-ds^2 = -2\,dx^+\,dx^- + (dx^2)^2 + (dx^3)^2 = \hat{\eta}_{\mu\nu}\,dx^\mu\,dx^\nu$$

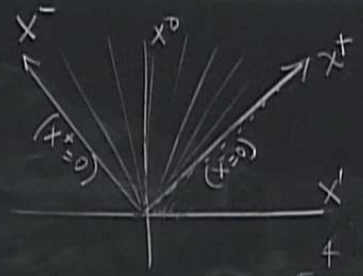
$\mu = +, -, 2, 3$

Light-cone coordinates:

$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$$

$$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$$

light-cone time



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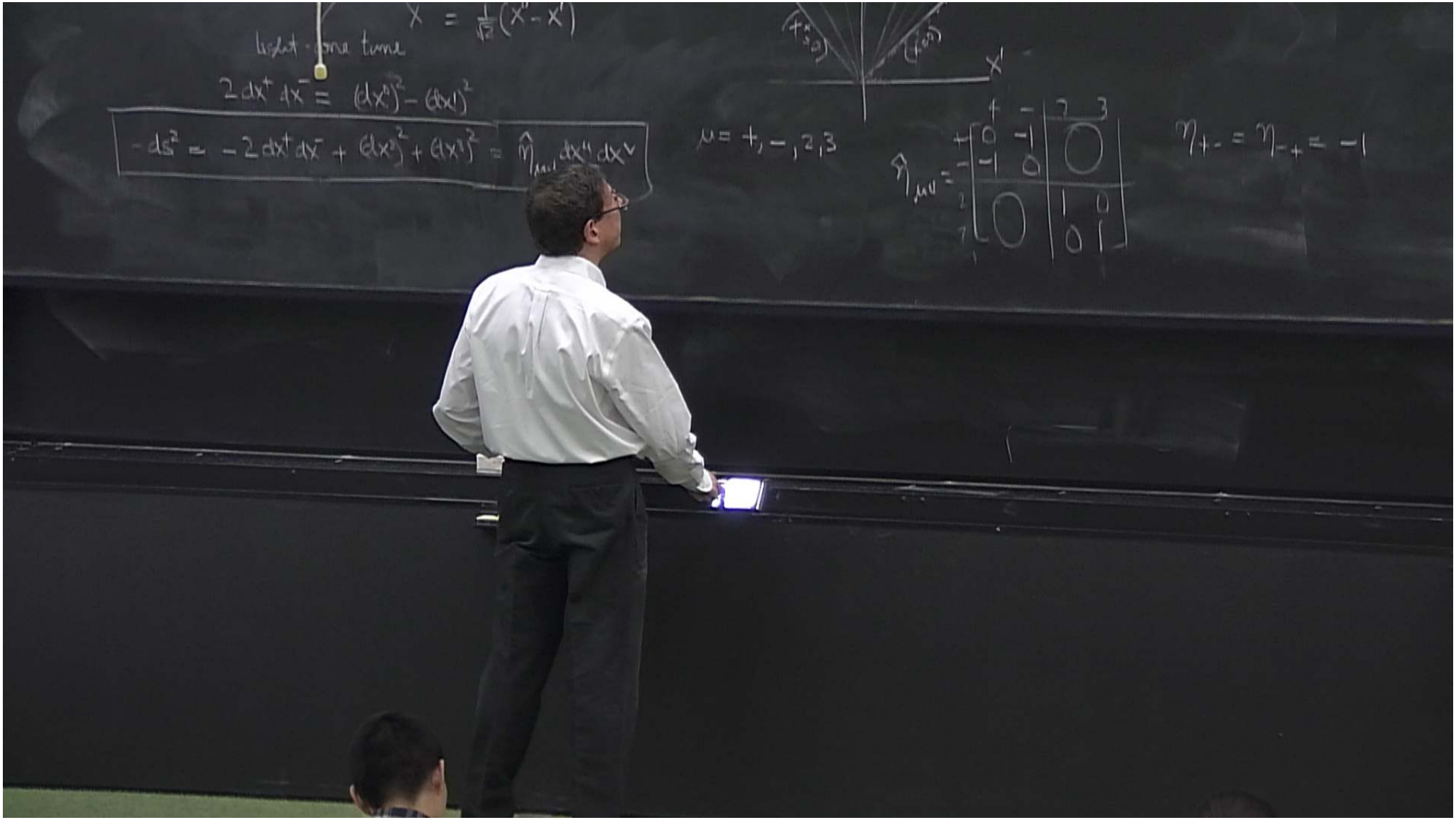
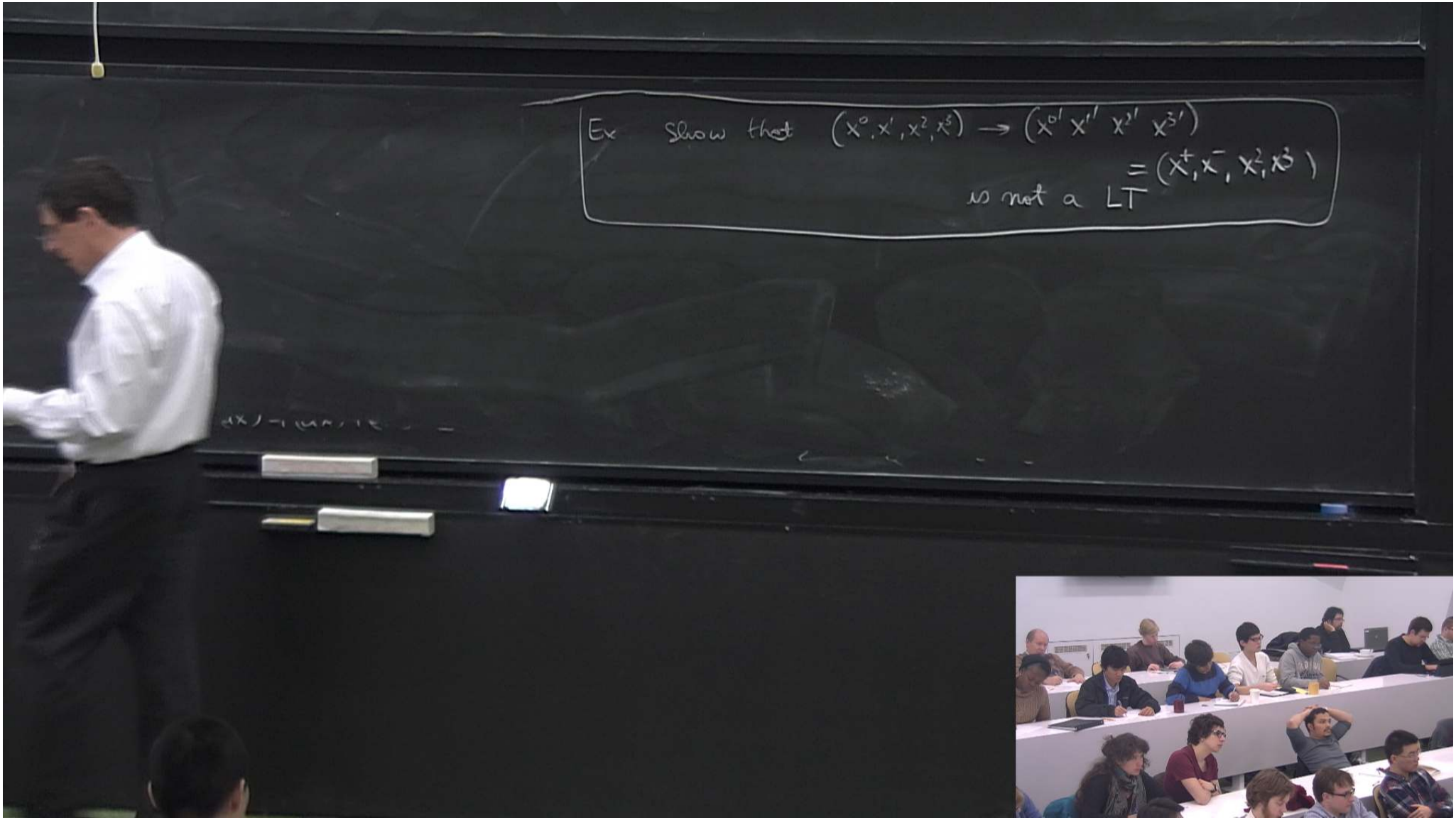
$$-ds^2 = -2\,dx^+\,dx^- + (dx^2)^2 + (dx^3)^2 = \hat{\eta}_{\mu\nu}\,dx^\mu\,dx^\nu$$

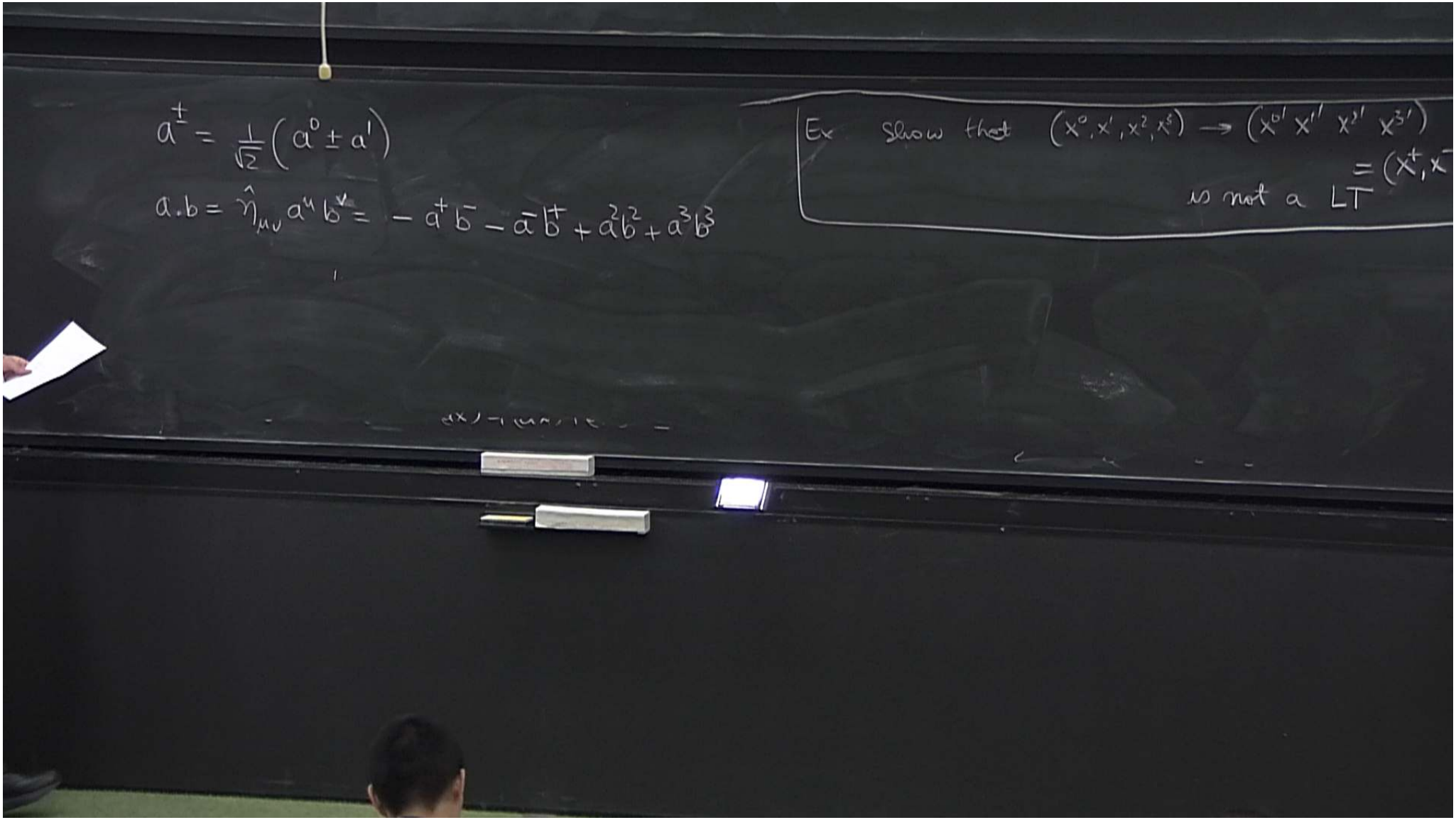
$\mu = +, -, 2, 3$

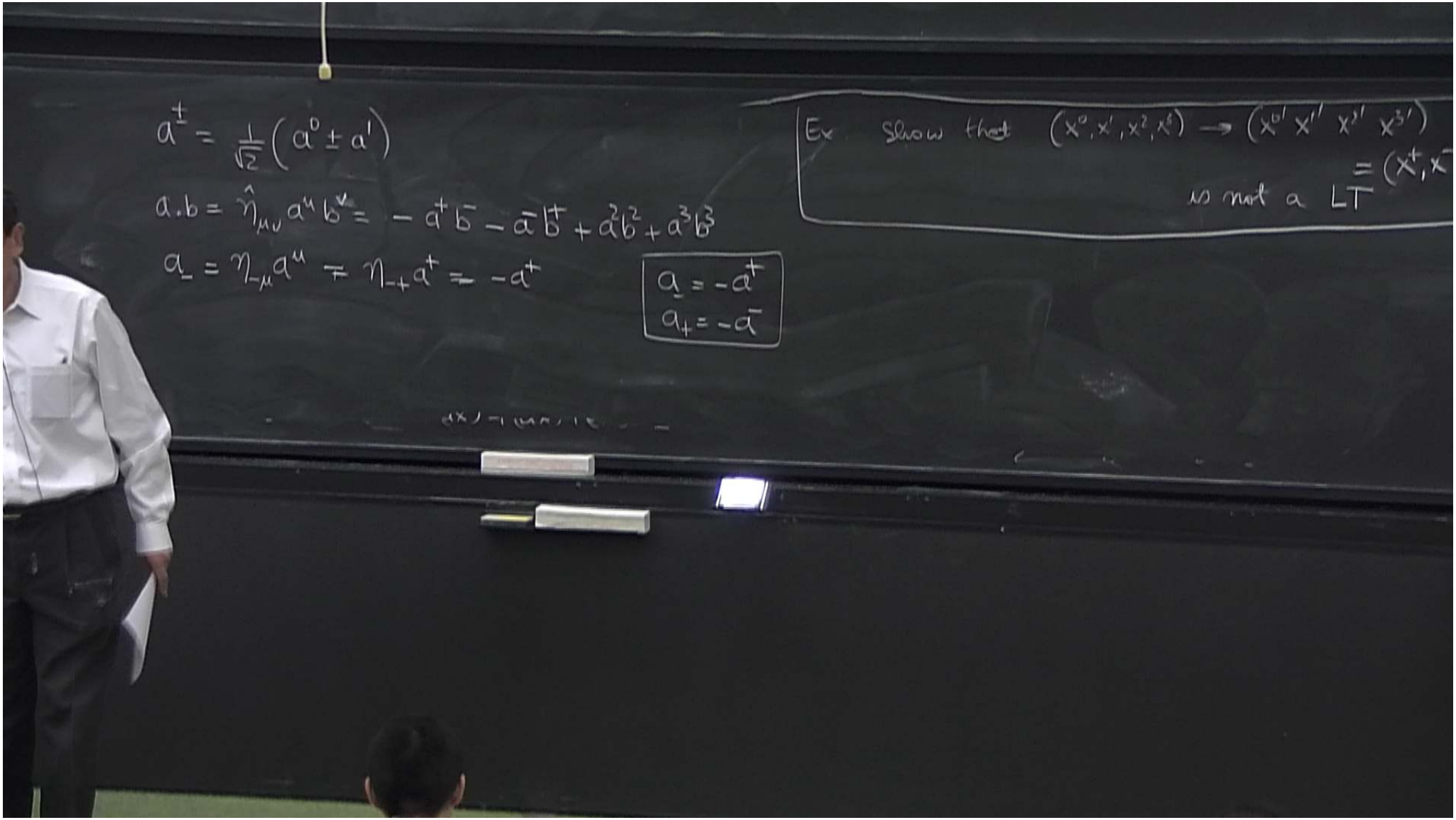
$\hat{\eta}_{\mu\nu}$ =

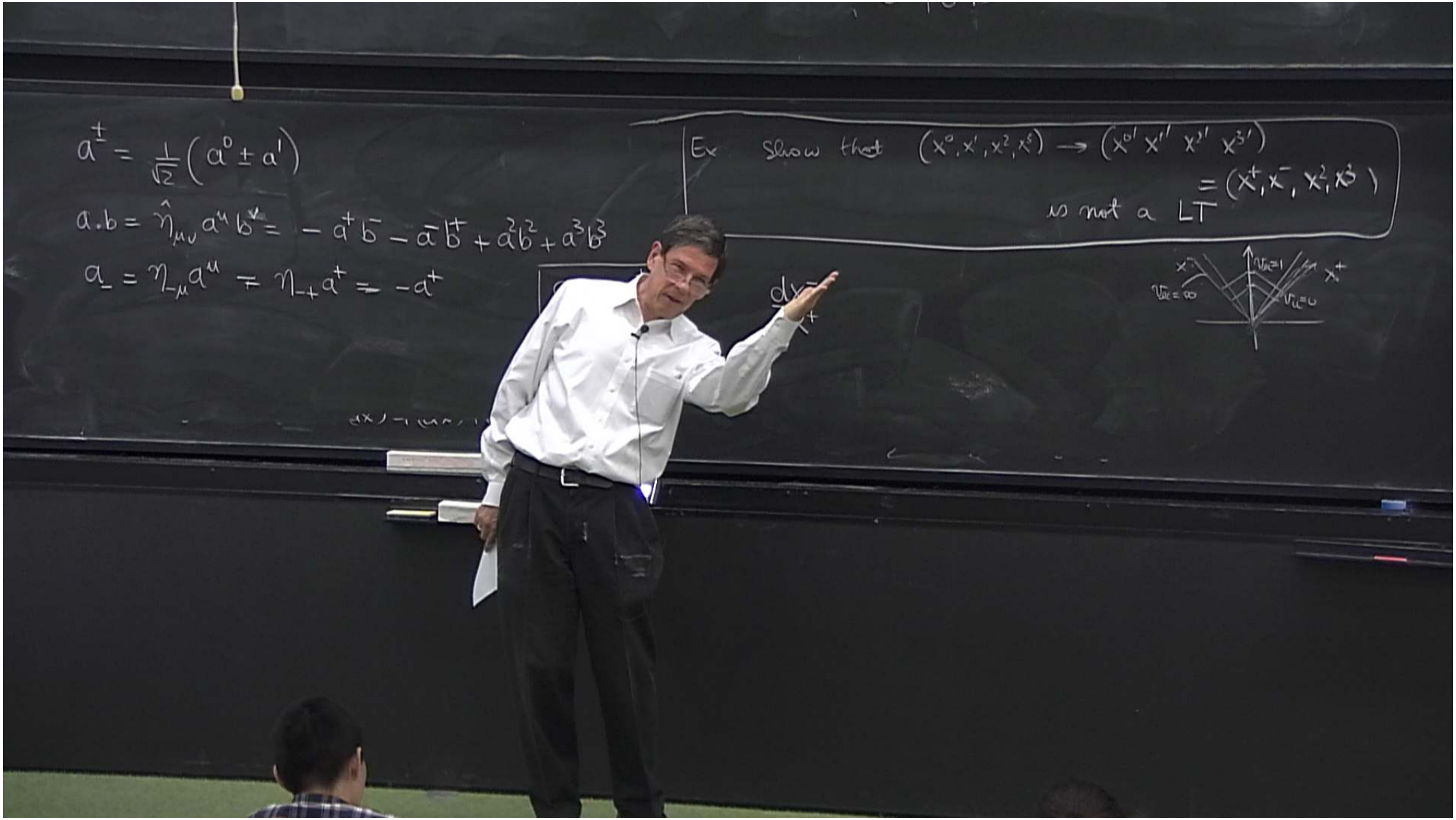
	+	−	2	3
+			0	
−				
2	0		1	0
3			0	1

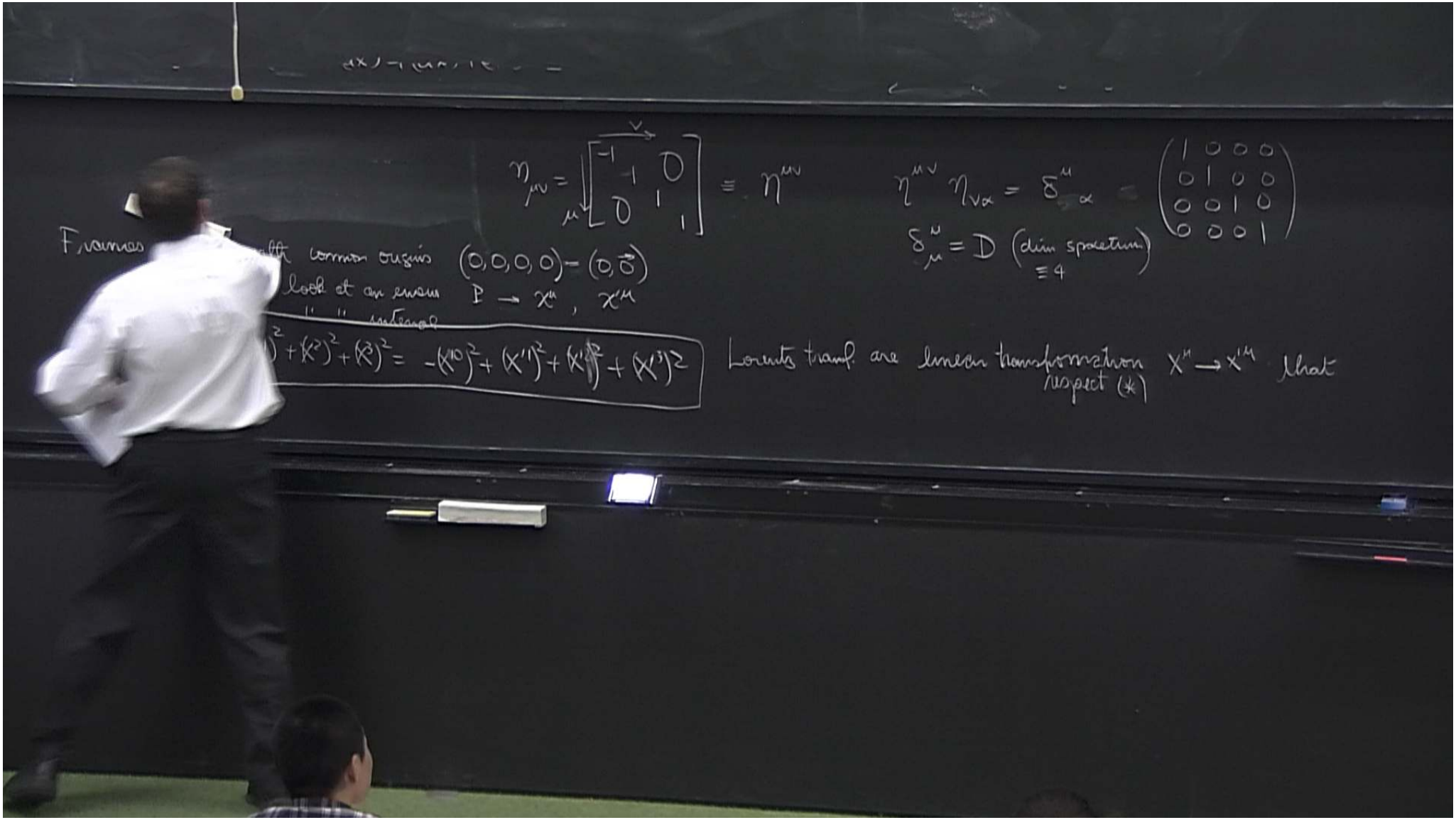

$X^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$
light-cone time
$2\,dx^+\,dx^- = (dx^0)^2 - (dx^1)^2$
$-ds^2 = -2\,dx^+\,dx^- + (dx^2)^2 + (dx^3)^2 = \hat{\eta}_{\mu\nu}\,dx^\mu\,dx^\nu$
$\mu = +, -, 2, 3$
$\eta_{+-} = \eta_{-+} = -1$
x^1

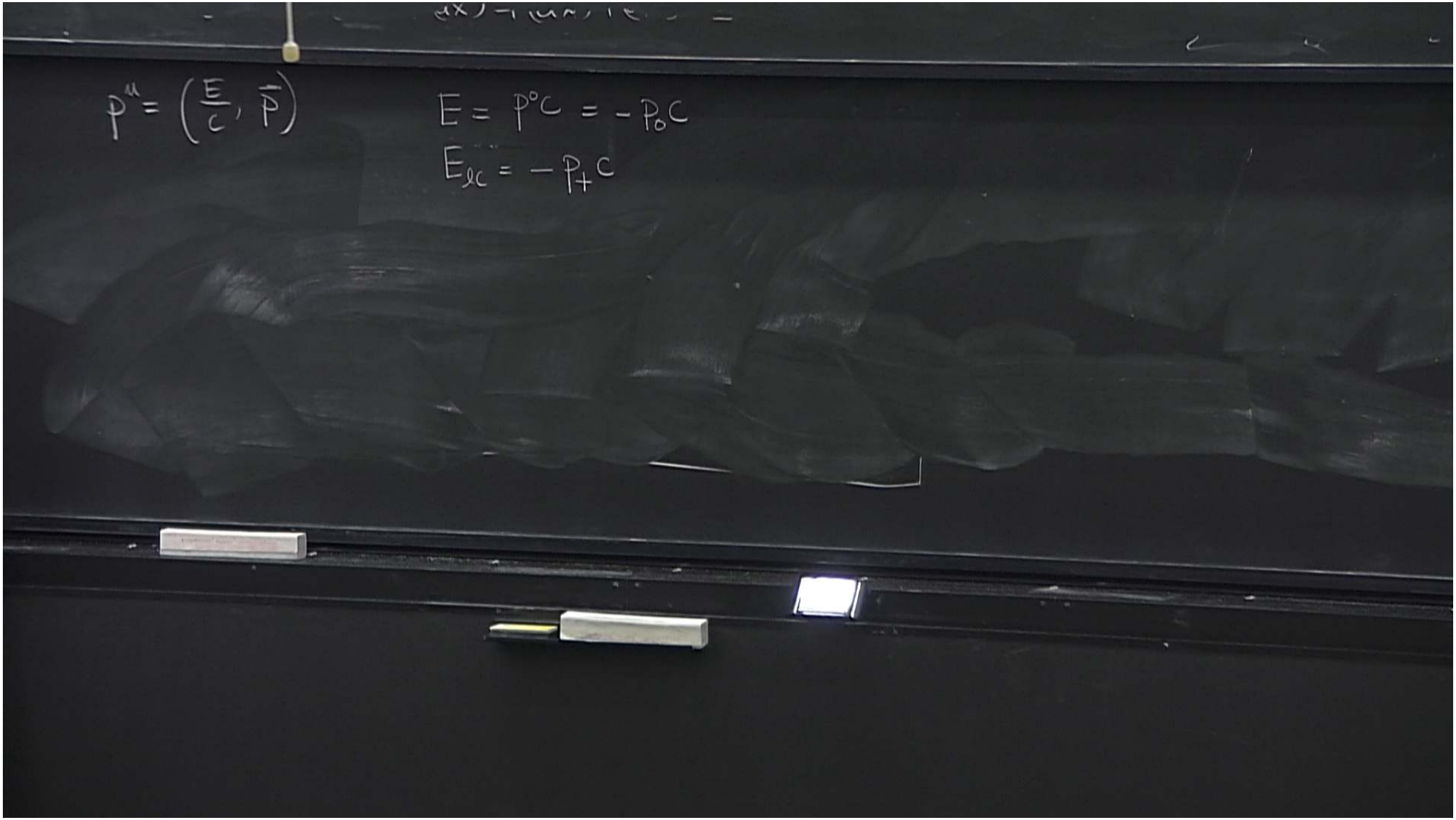

Ex Show that $(x^0, x^1, x^2, x^3) \to (x^{0\prime}\ x^{1\prime}\ x^{2\prime}\ x^{3\prime})$
$= (x^+, x^-, x^2, x^3)$
is not a LT

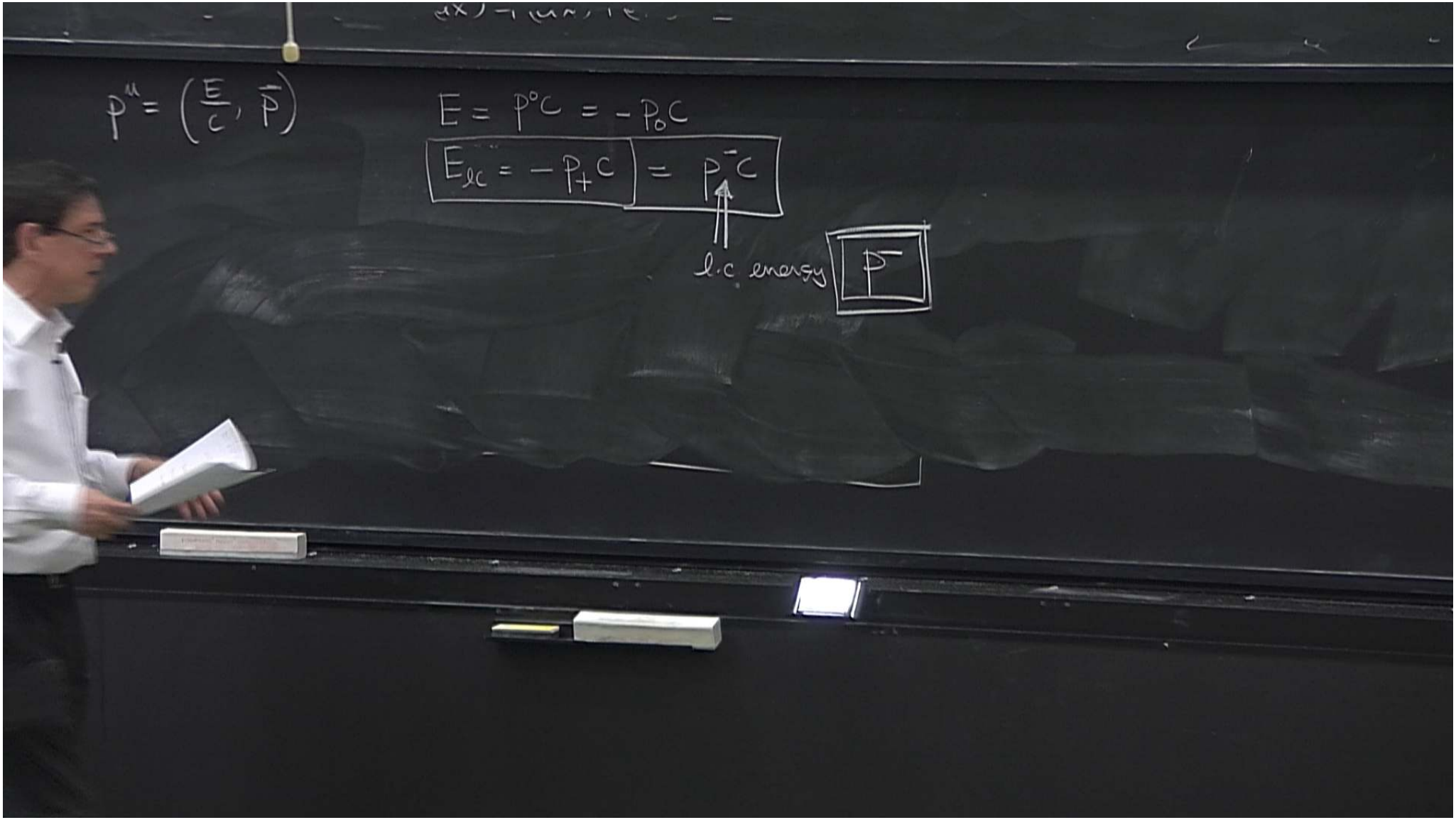

$a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^1)$
$a \cdot b = \hat{\eta}_{\mu\nu} a^\mu b^\nu = -a^+ b^- - a^- b^+ + a^2 b^2 + a^3 b^3$
Ex Show that $(x^0, x^1, x^2, x^3) \to (x^{0\prime}\ x^{1\prime}\ x^{2\prime}\ x^{3\prime})$
$= (x^+, x^-$
is not a LT

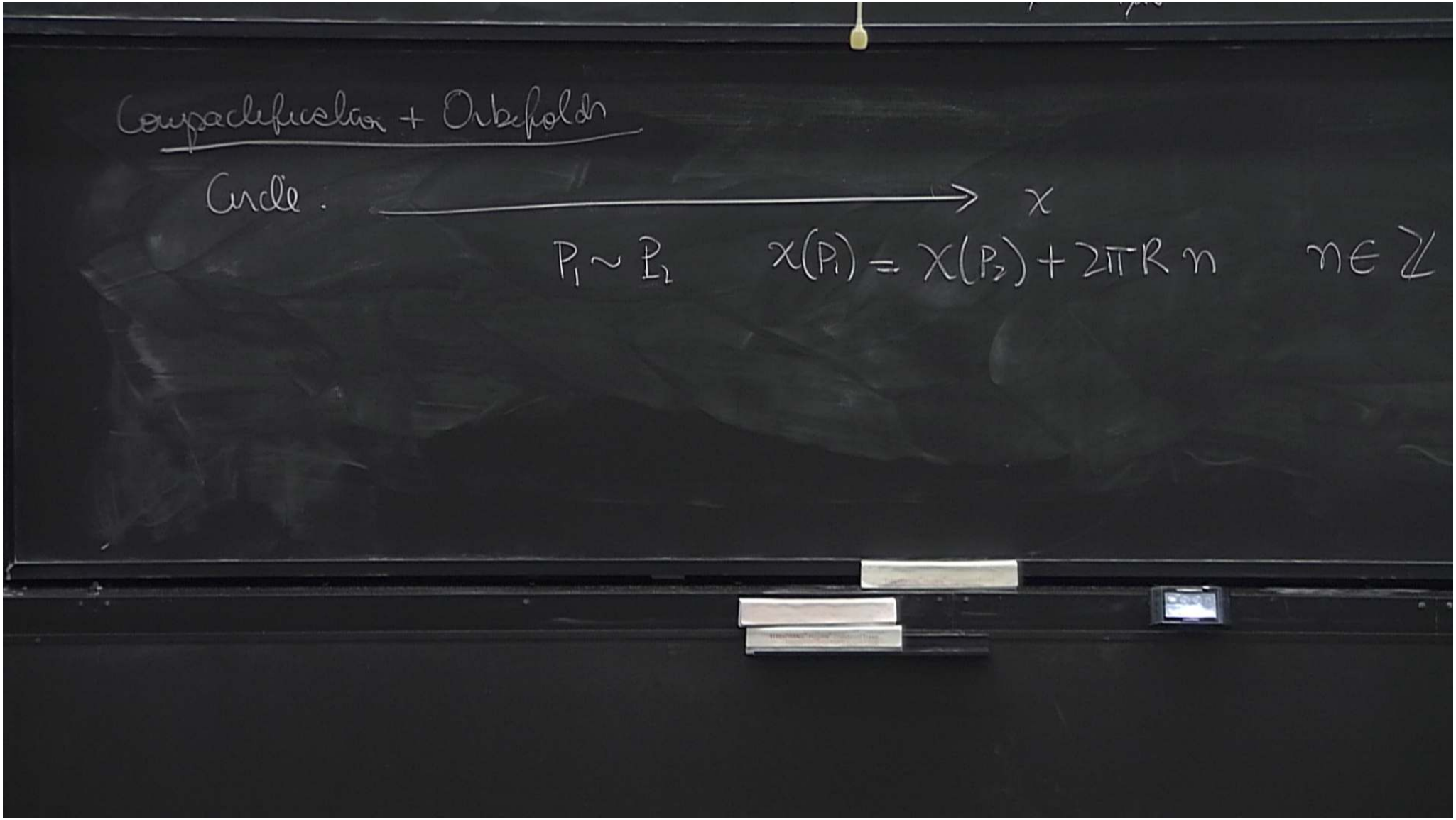

$a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^1)$
$a.b = \hat{\eta}_{\mu\nu} a^\mu b^\nu = -a^+ b^- - a^- b^+ + a^2 b^2 + a^3 b^3$
$a_- = \eta_{-\mu} a^\mu = \eta_{-+} a^+ = -a^+$
$a_- = -a^+$
$a_+ = -a^-$
Ex Show that $(x^0, x^1, x^2, x^3) \to (x^{0\prime} x^{1\prime} x^{2\prime} x^{3\prime})$
$= (x^+, x^-$
is not a LT


$a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^1)$
$a \cdot b = \hat{\eta}_{\mu\nu} a^\mu b^\nu = -a^+ b^- - a^- b^+ + a^2 b^2 + a^3 b^3$
$a_- = \eta_{-\mu} a^\mu = \eta_{-+} a^+ = -a^+$
Ex Show that $(x^0, x^1, x^2, x^3) \to (x^{0\prime} x^{1\prime} x^{2\prime} x^{3\prime})$
$= (x^+, x^-, x^2, x^3)$
is not a LT
x^-
$v_{x^-}=1$
x^+
$v_{x^-}=\infty$
$v_{x^-}=0$


$\eta_{\mu\nu} = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \equiv \eta^{\mu\nu}$
$\eta^{\mu\nu}\eta_{\nu\alpha} = \delta^{\mu}_{\alpha}$
$\delta^{\mu}_{\mu} = D$ (dim spacetime) $\equiv 4$
$(0,0,0,0) - (0,\vec{0})$
$P \to x^{\mu}, x'^{\mu}$
$(x^0)^2 + (x^2)^2 + (x^3)^2 = -(x'^0)^2 + (x'^1)^2 + (x'^2)^2 + (x'^3)^2$
Lorentz transf. are linear transformation $x^{\mu} \to x'^{\mu}$ that
respect (*)


$p^\mu = \left(\frac{E}{c}, \vec{p}\right)$
$E = p^0 c = -p_0 c$
$E_{\gamma c} = -p_+ c$


$p^\mu = \left(\frac{E}{c}, \vec{p}\right)$
$E = p^0 c = -p_0 c$
$E_{lc} = -p_+ c = p^- c$
l.c energy
p^-

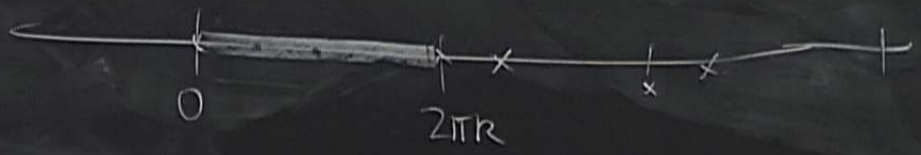

Compactification + Orbifolds
Circle.
x
$P_1 \sim P_2$
$x(P_1) = x(P_2) + 2\pi R\, n$
$n \in \mathbb{Z}$

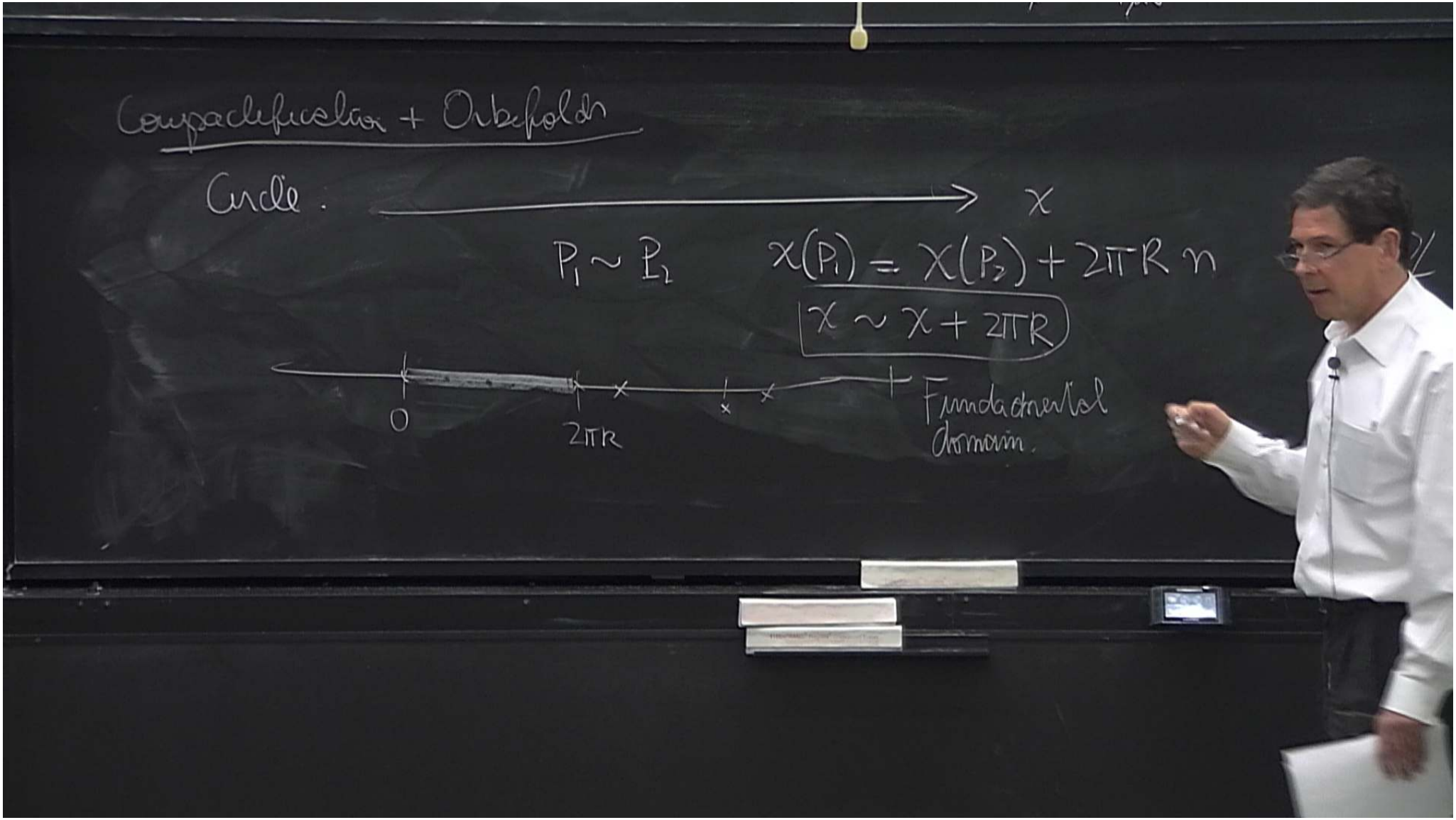
Compactification + Orbifolds

Circle: $\longrightarrow x$

$P_1 \sim P_2 \qquad x(P_1) = x(P_2) + 2\pi R\, n \qquad n \in \mathbb{Z}$

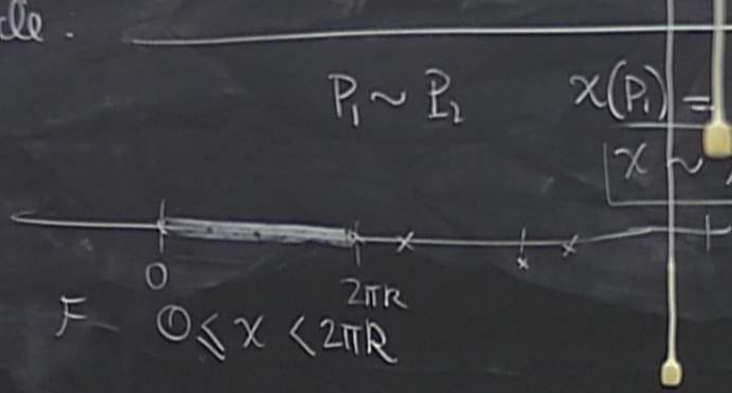
$x \sim x + 2\pi R$




Compactification + Orbifolds
Circle.
x
$P_1 \sim P_2$
$x(P_1) = x(P_2) + 2\pi R\, n$
$x \sim x + 2\pi R$
0
$2\pi R$
Fundamental domain.

Compactification + Orbifold

Circle.



$P_1 \sim P_2 \qquad x(P_1) = x(P_2) + 2\pi R\, n \qquad n \in \mathbb{Z}$

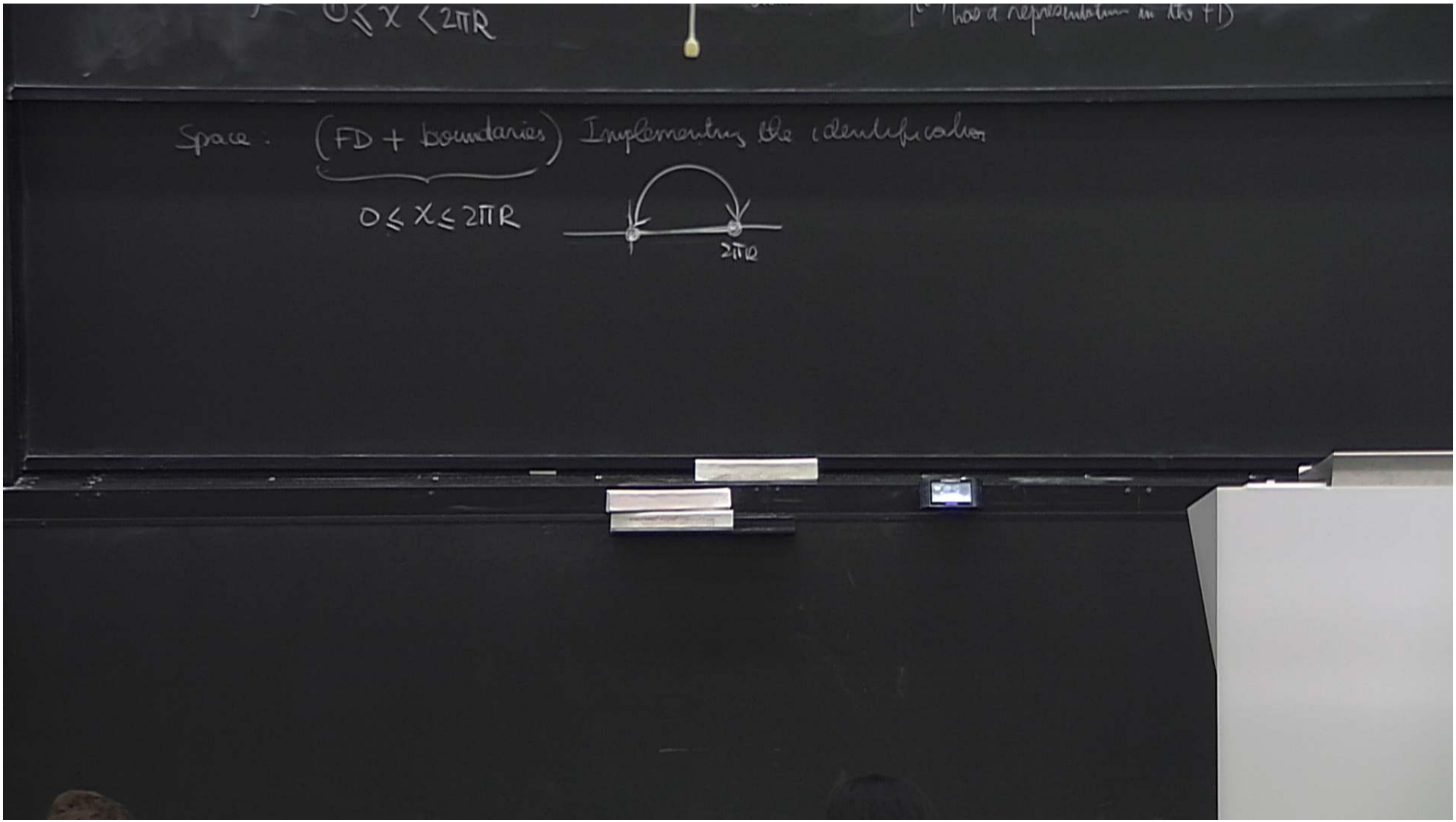
$x \sim x + 2\pi R$

F $0 \leq x < 2\pi R$

Fundamental domain.

(I) no two points are identified

(II) any point in the entire space has a representation in the FD


0 ≤ x < 2πR
has a representation in the FD
Space: (FD + boundaries) Implementing the identification
0 ≤ x ≤ 2πR
2πR

$0 \leq x < 2\pi R$

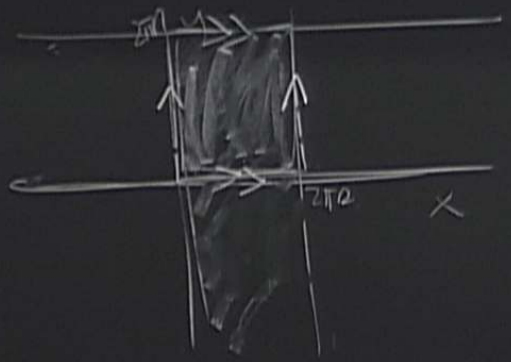
has a representation in this FD)

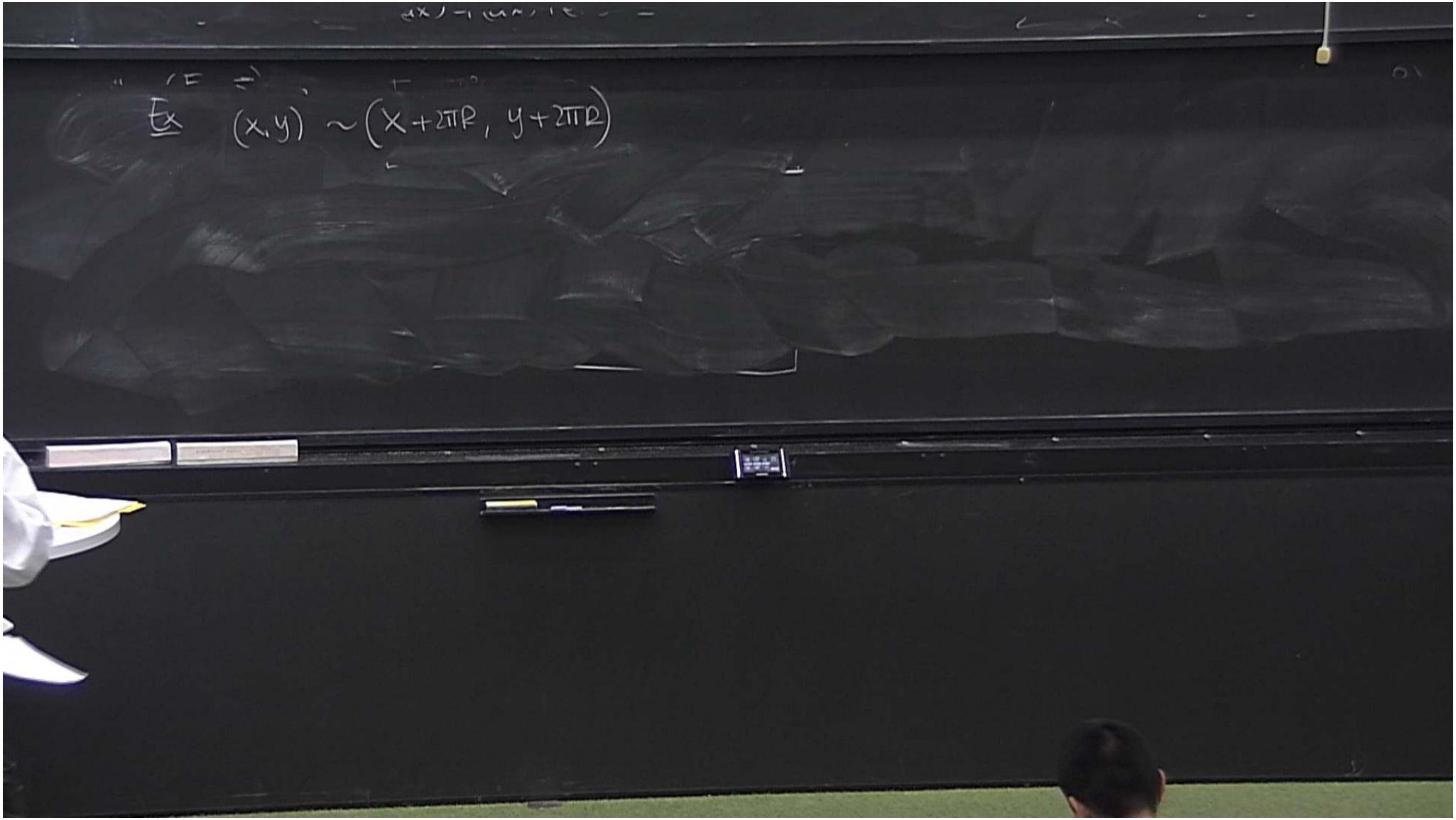
Space: (FD + boundaries) Implementing the identifications

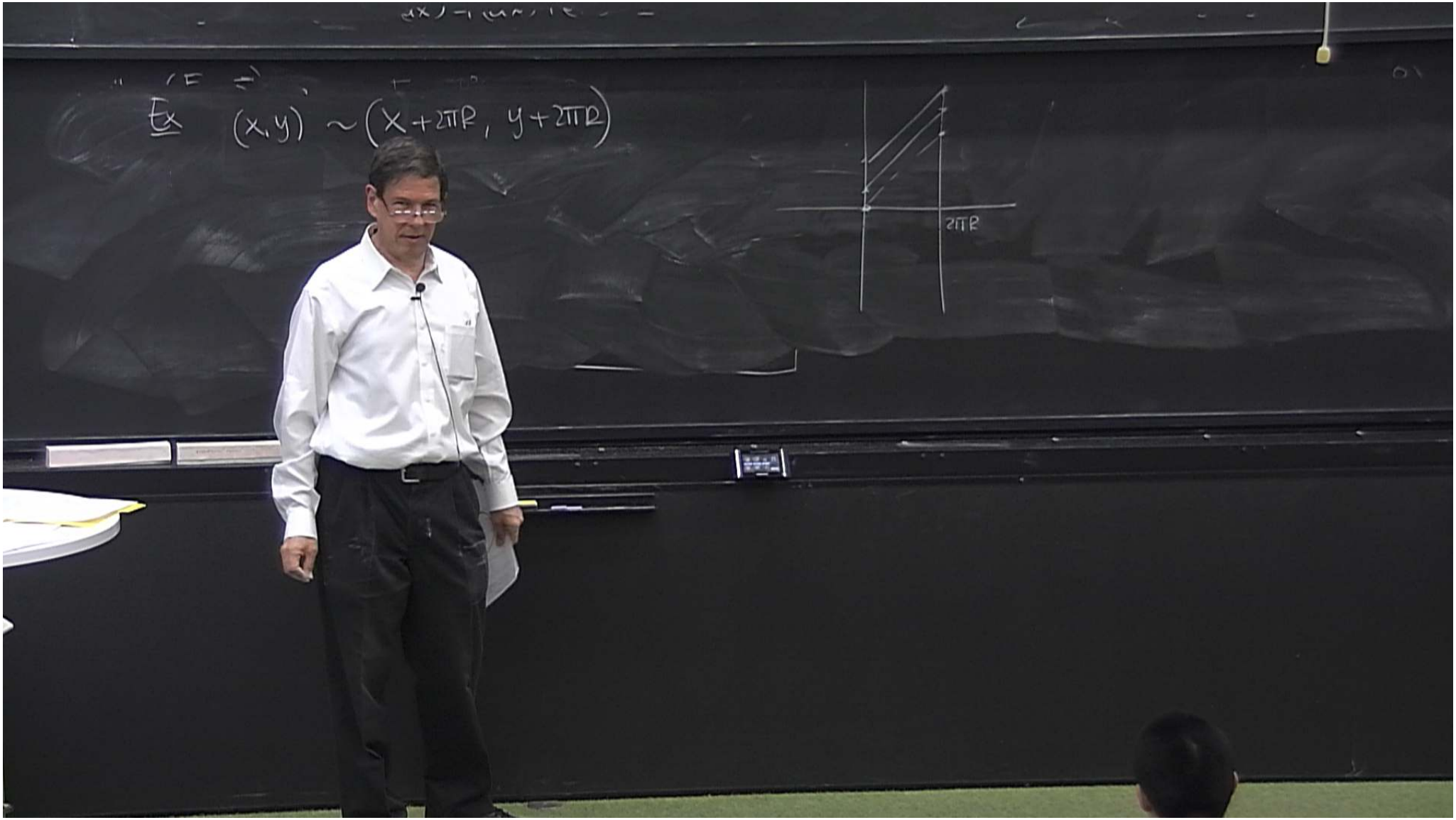
$0 \leq x \leq 2\pi R$

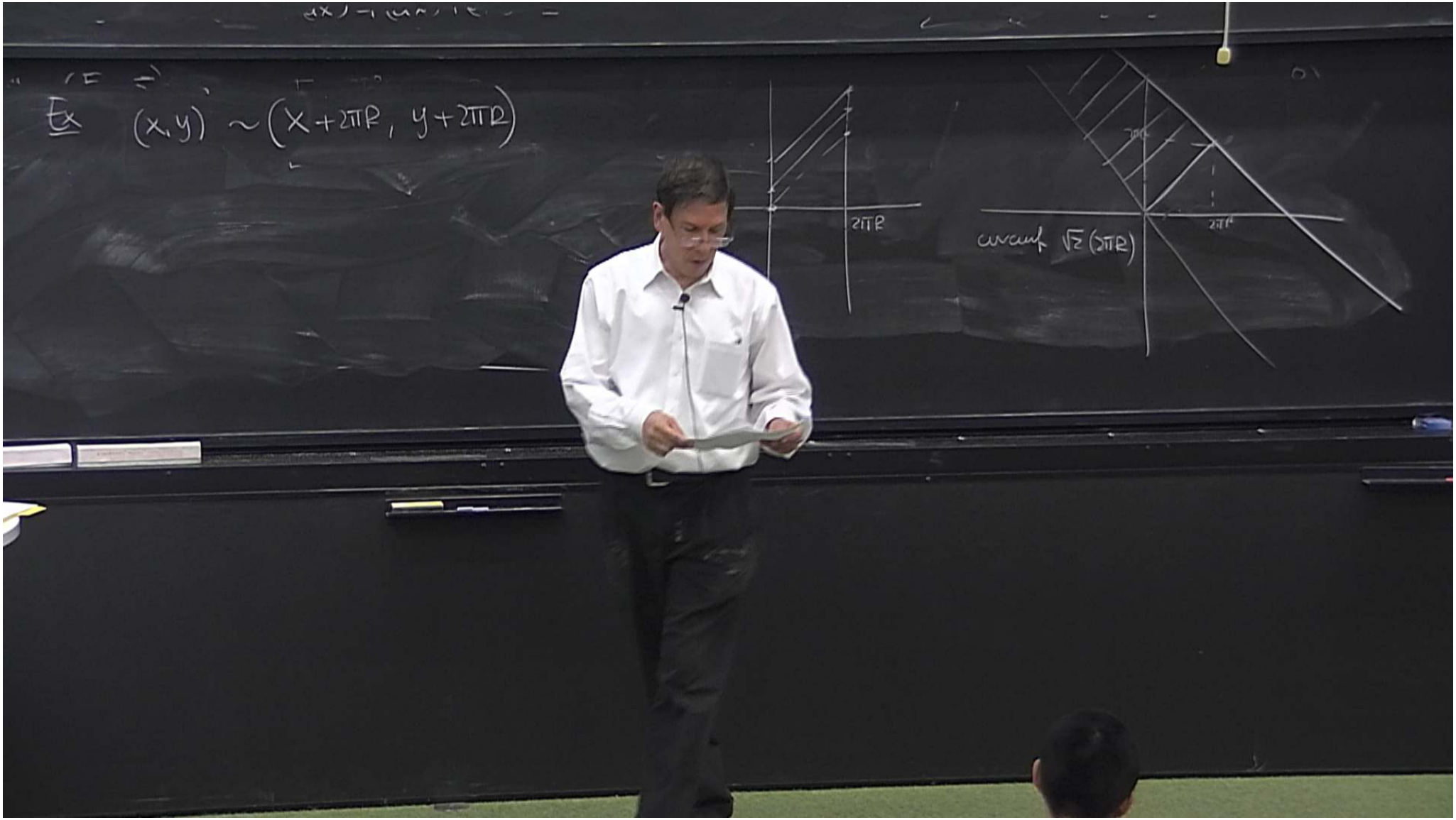
Torus: $(x,y) \sim (x+2\pi R, y)$

$(x,y) \sim (x, y+2\pi R)$




Ex $(x,y) \sim (x+2\pi R, y+2\pi R)$


Ex $(x,y) \sim (x+2\pi R, y+2\pi R)$
$2\pi R$


Ex $(x,y) \sim (x+2\pi R, y+2\pi R)$
$2\pi R$
circumf $\sqrt{2}(2\pi R)$
$2\pi R$