

Title: 12/13 PSI - String Theory Review Lecture 1

Date: Jan 07, 2013 10:15 AM

URL: <http://pirsa.org/13010043>

Abstract:

Strings Theory

- 1) Unified quantum theory of all interaction
Particle physics
Cosmology

Strings Theory

- 1) Unified quantum theory of all interaction
Particle physics
Cosmology
- 2) Applications: Mathematics

Strings Theory

Unified quantum theory of all interaction

Particle physics

Cosmology

Applications:

Mathematics

Gravitation - black holes

Gauge theories

1) Unified quantum theory of all interaction

Particle physics
Cosmology

2) Applications:

Mathematics

Gravitation - black holes

AdS/CFT } Gauge theories

} Heavy ion physics (quark-gluon plasma)
Condensed matter physics

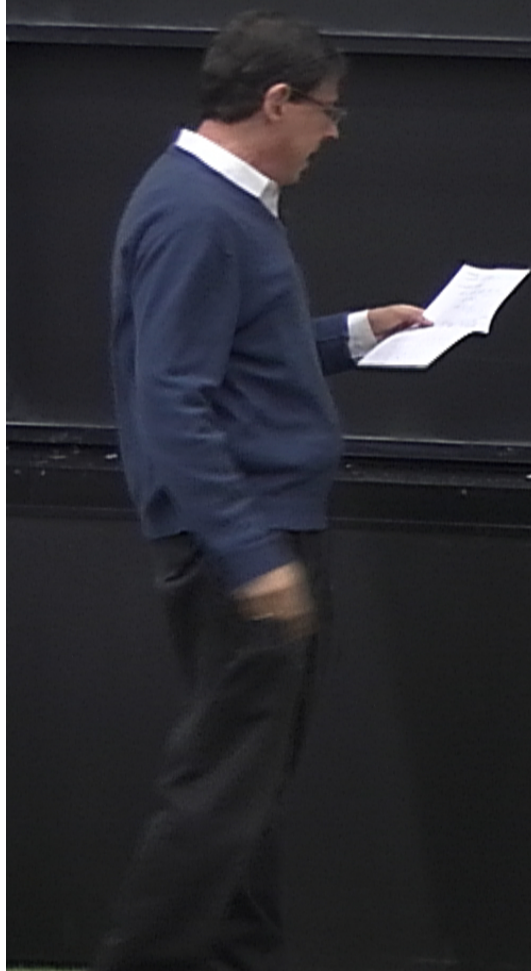
Plan. Week one: Classical strings

" two: Quantum strings

three: Applications.

Relativity:

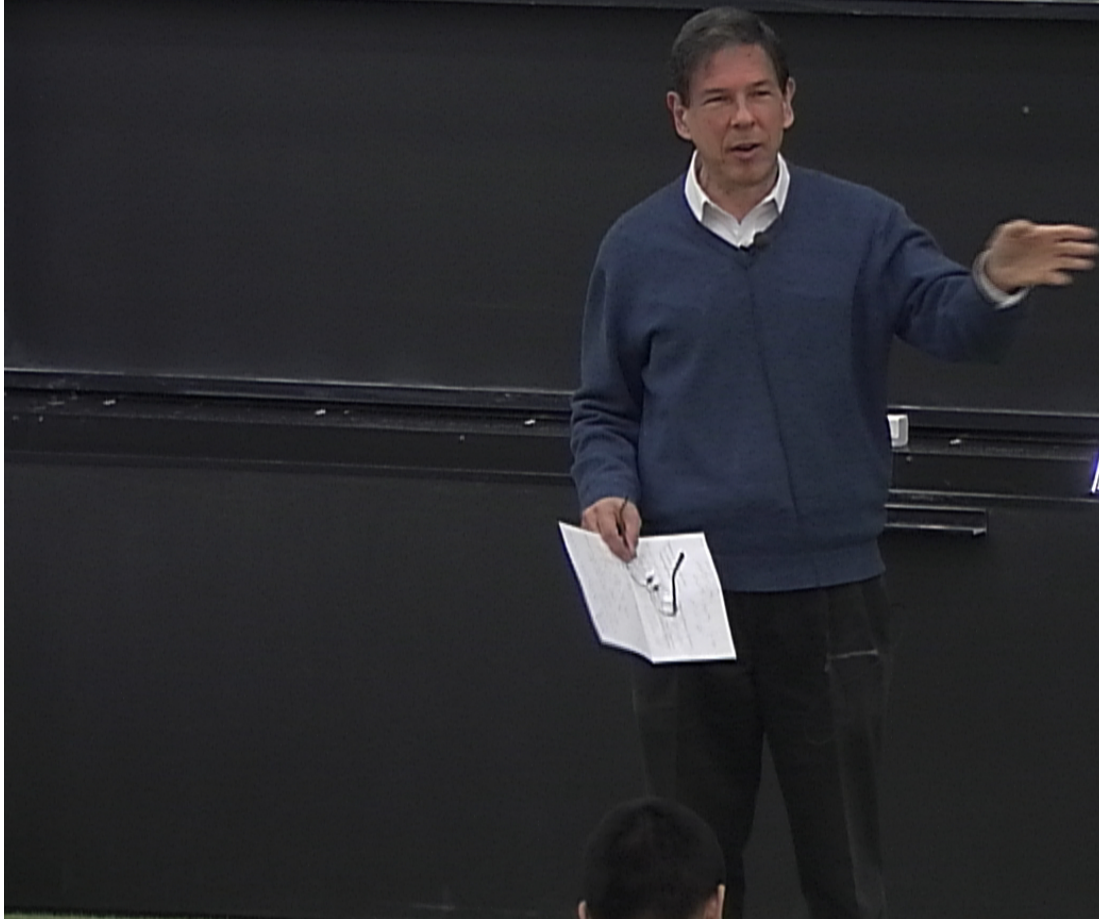
$$\chi^\mu = (\chi^0, \chi^1, \chi^2, \chi^3) = (ct, x^1, x^2, x^3)$$



Relativity:

$$-ds^2$$

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x^1, x^2, x^3)$$



Relativity:

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x', x^2, x^3)$$

$$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$$

$$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -1(dx^{1'0})^2 + (dx^{1'1})^2 + (dx^{1'2})^2 + (dx^{1'3})^2$$

$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = \begin{matrix} & \xrightarrow{\nu} \\ \downarrow \mu & \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$-ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$$

$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \equiv \eta^{\mu\nu}$$

$$\eta^{\mu\nu} \eta_{\nu\alpha} = \delta^\mu_\alpha$$

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 + (dx'^3)^2$$

$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

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$$\eta^{\mu\nu} \eta_{\nu\alpha} = \delta^\mu_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = -1(dx^{0'})^2 + (dx^{1'})^2 + (dx^{2'})^2 + (dx^{3'})^2$$

$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = \begin{matrix} & \begin{matrix} \mu \downarrow \end{matrix} \\ \begin{matrix} \nu \downarrow \end{matrix} & \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix} \equiv \eta^{\mu\nu}$$

$$\eta^{\mu\nu} \eta_{\nu\alpha} = \delta^\mu_\alpha \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\delta^\mu_\mu = D \quad \begin{matrix} (\text{dim spacetime}) \\ \equiv 4 \end{matrix}$$

Relativity: $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x^1, x^2, x^3)$

$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \equiv \eta^{\mu\nu}$$

$$\eta^{\mu\nu} \eta_{\nu\alpha} = \delta^\mu_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\delta^\mu_\mu = D \text{ (dim spacetime)}$$

$$\equiv 4$$

F , values S_i, S'_i with common origin
look at an ensemble $P \rightarrow X^M, X'^M$

$$(*) \quad -(\overset{1}{x})^2 + (\overset{1}{x})^2 + (\overset{2}{x})^2 + (\overset{2}{x})^2 = -(\overset{10}{x})^2 + (\overset{11}{x})^2 + (\overset{11}{x})^2 + (\overset{11}{x})^2$$

Lorentz transf are linear transformation $X^\mu \rightarrow X'^\mu$ that respect (*)

$$x'^{\mu} = L^{\mu}_{\nu} x^{\nu}$$

$$\begin{aligned} * \quad \eta_{\mu\nu} x^{\mu} x^{\nu} &= \eta_{\mu\nu} x'^{\mu} x'^{\nu} = \eta_{\mu\nu} L^{\mu}_{\alpha} L^{\nu}_{\beta} x^{\alpha} x^{\beta} \\ &= \eta_{\alpha\beta} L^{\alpha}_{\mu} L^{\beta}_{\nu} x^{\mu} x^{\nu} \end{aligned}$$

$$x'^{\mu} = L^{\mu}_{\nu} x^{\nu}$$

$$\begin{aligned} * \quad \eta_{\mu\nu} x'^{\mu} x'^{\nu} &= \eta_{\mu\nu} x^{\mu} x^{\nu} = \eta_{\mu\nu} L^{\mu}_{\alpha} L^{\nu}_{\beta} x^{\alpha} x^{\beta} \\ &= \eta_{\alpha\beta} L^{\alpha}_{\mu} L^{\beta}_{\nu} x^{\mu} x^{\nu} \end{aligned}$$

$$\eta_{\mu\nu} = L^{\alpha}_{\mu} \eta_{\alpha\beta} L^{\beta}_{\nu}$$

L is a LT iff:

$$\boxed{\eta = L^T \eta L}$$

$$\det \eta = (\det L^T) (\det \eta) (\det L)$$

$$1 = (\det L)^2 \quad \boxed{\det L = \pm 1}$$

L invertible

a^μ is a vector if

$$a'^\mu = L^\mu_\nu a^\nu$$

$$I = (\det L)$$

$$\det L = 1$$

L invertible

$$a_\mu \equiv \eta_{\mu\nu} a^\nu$$

a^μ is a vector if

$$a'^\mu = L^\mu_\nu a^\nu$$

$$I = (a^\mu L_\mu)$$

$$a^\mu L_\mu = I$$

L invertible

$$a_\mu \equiv \eta_{\mu\nu} a^\nu$$

proper time (for a particle)

$$-ds^2 = -(d\tau)^2 c^2 = -c^2 dt^2 + (d\vec{x})^2 = -c^2 dt^2 + v^2 dt^2 = -(c^2 - v^2)(dt)^2$$

$$ds^2 = (d\tau)^2 c^2$$

$$\sqrt{ds^2} \equiv ds = (d\tau) c$$

a^μ is a vector if

$$a'^\mu = L^\mu_\nu a^\nu$$

L invertible

$$a_\mu \equiv \eta_{\mu\nu} a^\nu$$

proper time (for a particle)

no sum

$$p^\mu = m c \left(\frac{dx^\mu}{ds} \right)$$

$$p^\mu p_\mu = m^2 c^2 \left(\frac{dx^\mu}{ds} \frac{dx_\mu}{ds} \right) = -m^2 c^2$$

$$-ds^2 = -(d\tau)^2 c^2 = -c^2 dt^2 + (d\vec{x})^2 = -c^2 dt^2 + v^2 dt^2 = -(c^2 - v^2) (dt)^2$$

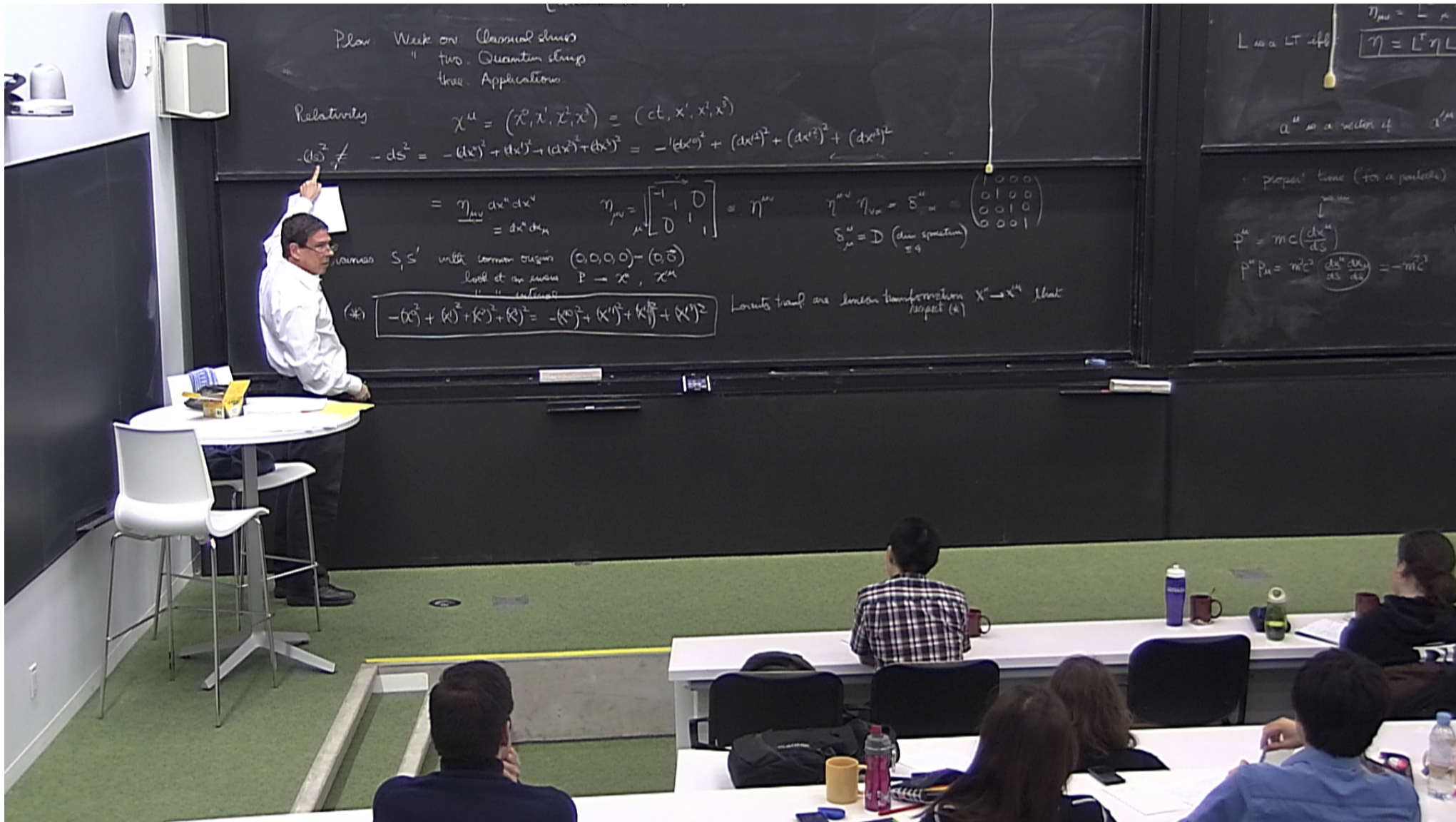
$$ds^2 = (d\tau)^2 c^2$$

$$\sqrt{ds^2} \equiv ds = (d\tau) c$$

$$ds^2 > 0 \quad \text{Timelike}$$

$$ds^2 = 0 \quad \text{lightlike}$$

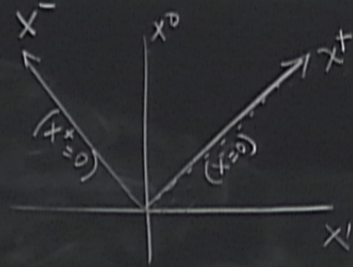
$$ds^2 < 0 \quad \text{spacelike}$$



light-cone coordinates:

$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$$

$$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$$

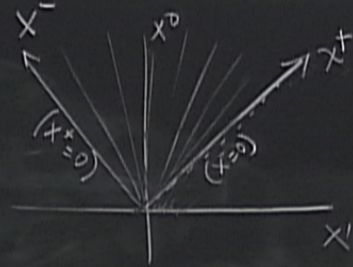


light-cone coordinates:

$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$$

$$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$$

light-cone time



light-cone coordinates:

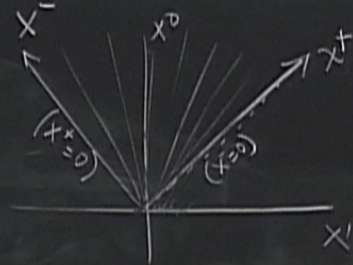
$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$$

$$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$$

light-cone time

$$2dx^+dx^- = (dx^0)^2 - (dx^1)^2$$

$$-ds^2 = -2dx^+dx^- + (dx^2)^2 + (dx^3)^2 = \hat{\eta}_{\mu\nu}dx^\mu dx^\nu$$



$$\mu = +, -, 2, 3$$

light-cone coordinates:

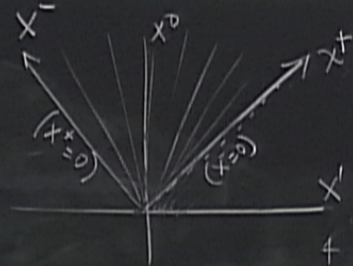
$$X^+ = \frac{1}{\sqrt{2}}(X^0 + X^1)$$

$$X^- = \frac{1}{\sqrt{2}}(X^0 - X^1)$$

light-cone time

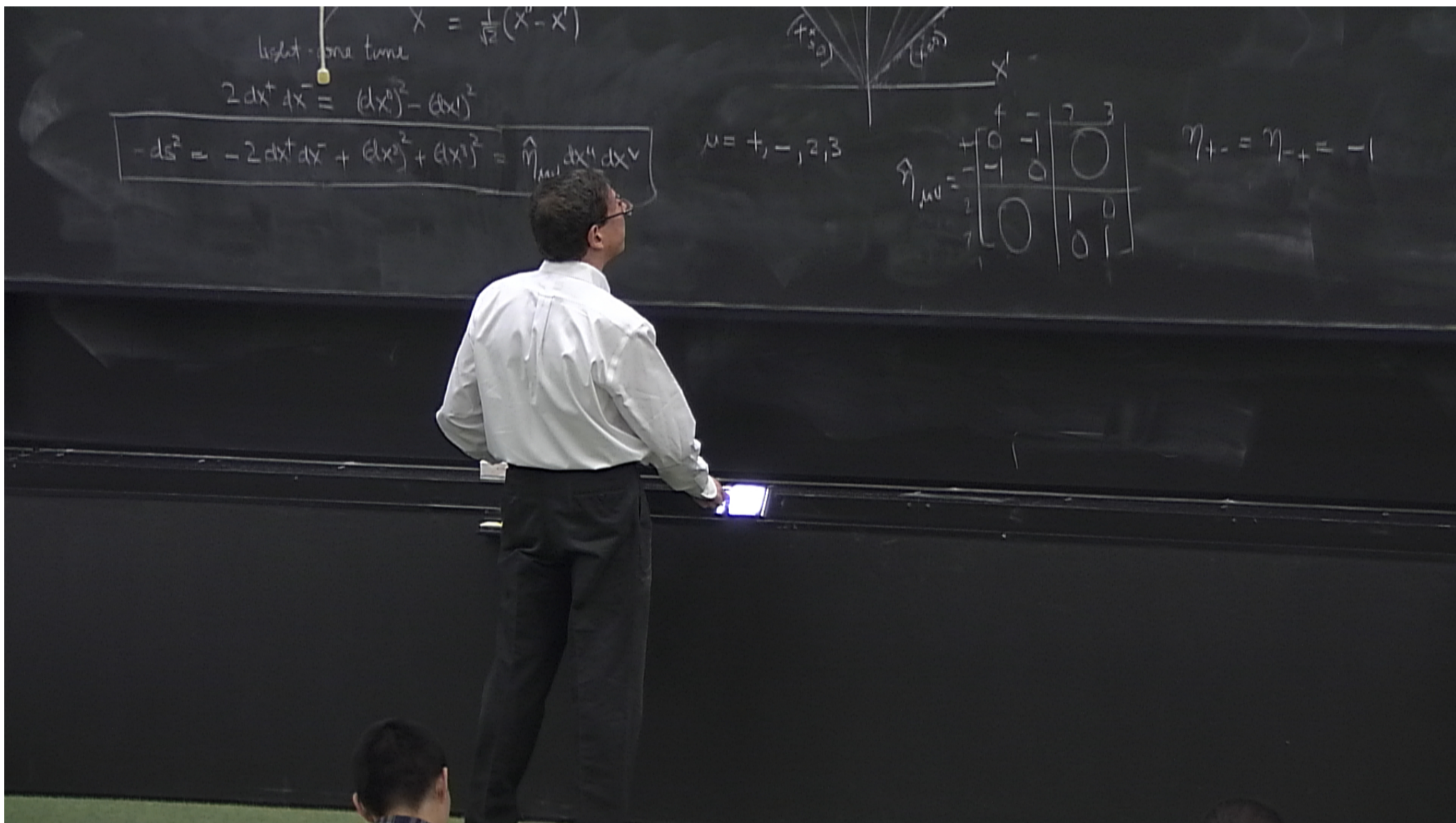
$$2 dx^+ dx^- = (dx^0)^2 - (dx^1)^2$$

$$-ds^2 = -2 dx^+ dx^- + (dx^2)^2 + (dx^3)^2 = \hat{\eta}_{\mu\nu} dx^\mu dx^\nu$$



$\mu = +, -, 2, 3$

$$\hat{\eta}_{\mu\nu} = \begin{array}{c|cc} & + & - & 2 & 3 \\ \hline + & & & & \\ - & & & & \\ \hline 2 & & & 1 & 0 \\ 3 & & & 0 & 1 \end{array}$$



Ex Show that $(x^0, x^1, x^2, x^3) \rightarrow (x^{0'}, x^{1'}, x^{2'}, x^{3'})$
 $= (x^+, x^-, x^2, x^3)$
is not a LT



$$a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^1)$$

$$a \cdot b = \hat{\eta}_{\mu\nu} a^{\mu} b^{\nu} = -a^+ b^- - a^- b^+ + a^2 b^2 + a^3 b^3$$

Ex: Show that $(x^0, x^1, x^2, x^3) \rightarrow (x^{0'}, x^{1'}, x^{2'}, x^{3'})$
 $= (x^+, x^-)$
 is not a LT

$$a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^1)$$

$$a \cdot b = \hat{\eta}_{\mu\nu} a^{\mu} b^{\nu} = -a^+ b^- - a^- b^+ + a^2 b^2 + a^3 b^3$$

$$a_- = \eta_{-\mu} a^{\mu} = \eta_{-+} a^+ = -a^+$$

$$\boxed{\begin{aligned} a_- &= -a^+ \\ a_+ &= -a^- \end{aligned}}$$

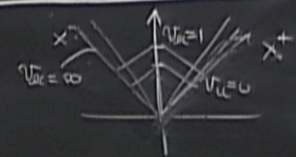
Ex. Show that $(x^0, x^1, x^2, x^3) \rightarrow (x^{0'}, x^{1'}, x^{2'}, x^{3'})$
 $= (x^+, x^-)$
 is not a LT

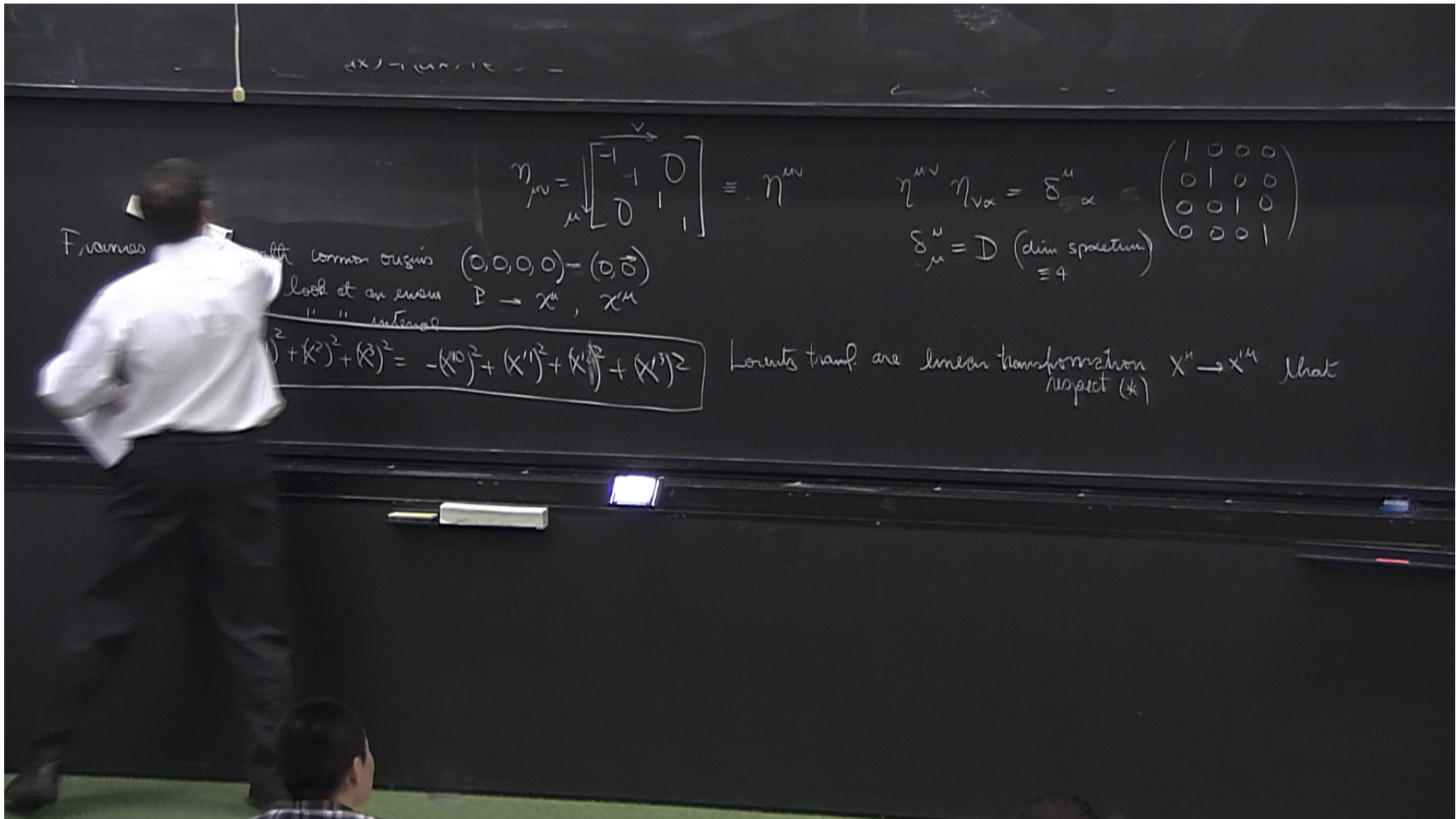
$$a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^1)$$

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$$a_- = \eta_{-\mu} a^{\mu} = \eta_{-+} a^+ = -a^+$$

Ex Show that $(x^0, x^1, x^2, x^3) \rightarrow (x^{0'}, x^{1'}, x^{2'}, x^{3'})$
 $= (x^+, x^-, x^2, x^3)$
 is not a LT





$x(t) = (x, y, z, t)$

$$\eta_{\mu\nu} = \begin{matrix} & \xrightarrow{v} \\ \mu \downarrow & \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix} \equiv \eta^{\mu\nu}$$

$$\eta^{\mu\nu} \eta_{\nu\alpha} = \delta^{\mu}_{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\delta^{\mu}_{\mu} = D \text{ (dim spacetime)} = 4$$

Frames

all have common origins $(0,0,0,0) = (0,0)$
look at an event $P \rightarrow x^{\mu}, x'^{\mu}$
" " " " " "

$$(x^0)^2 + (x^1)^2 + (x^2)^2 = -(x'^0)^2 + (x'^1)^2 + (x'^2)^2 + (x'^3)^2$$

Lorentz transf. are linear transformation $x^{\mu} \rightarrow x'^{\mu}$ that respect (*)

$$p^\mu = \left(\frac{E}{c}, \vec{p} \right)$$

$$E = p^0 c = -p_0 c$$

$$E_{lc} = -p_+ c$$

$$p^\mu = \left(\frac{E}{c}, \vec{p} \right)$$

$$E = p^0 c = -p_0 c$$

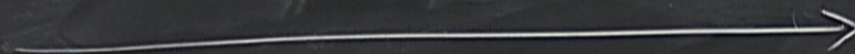
$$E_{lc} = -p_+ c = p_- c$$

l.c. energy

$$p_-$$

Compactification + Orbifolds

Circle:



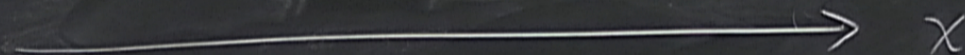
x

$$P_1 \sim P_2$$

$$x(P_1) = x(P_2) + 2\pi R n \quad n \in \mathbb{Z}$$

Compactification + Orbifolds

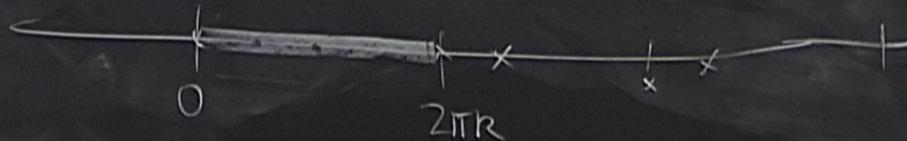
Circle:



$$P_1 \sim P_2$$

$$x(P_1) = x(P_2) + 2\pi R n \quad n \in \mathbb{Z}$$

$$x \sim x + 2\pi R$$



Compactification + Orbifolds

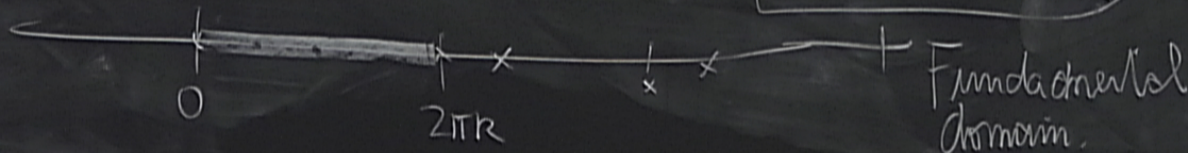
Circle:

$\xrightarrow{\quad\quad\quad} x$

$$P_1 \sim P_2$$

$$x(P_1) = x(P_2) + 2\pi R n$$

$$x \sim x + 2\pi R$$



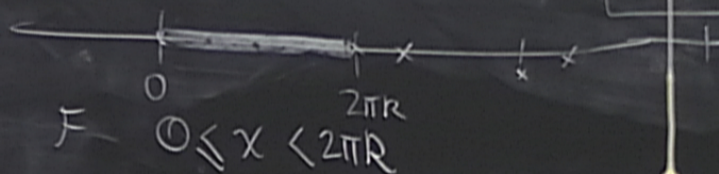
Compactification + Orbifolds

Circle:

$$P_1 \sim P_2$$

$$x(P_1) = x(P_2) + 2\pi R n \quad n \in \mathbb{Z}$$

$$x \sim x + 2\pi R$$



Fundamental domain.

- (i) no two points are identified
- (ii) any point in the entire space has a representation in the FD

$$0 \leq x < 2\pi R$$

ψ has a representation in the FD

Space: (FD + boundaries) Implementing the identification

$$0 \leq x \leq 2\pi R$$

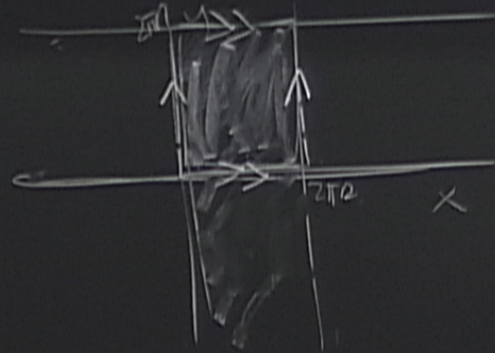


$$0 \leq x < 2\pi R$$

ψ has a representation in the FD

Space: (FD + boundaries) Implementing the identification

$$0 \leq x \leq 2\pi R$$



Torus: $(x, y) \sim (x + 2\pi R, y)$
 $(x, y) \sim (x, y + 2\pi R)$

$$\underline{Ex} \quad (x, y) \sim (X + 2\pi R, y + 2\pi R)$$

$$\underline{E_x} \quad (x, y) \sim (x + 2\pi R, y + 2\pi R)$$

