

Title: 12/13 PSI - Standard Model Review Lecture 7

Date: Jan 15, 2013 09:00 AM

URL: <http://pirsa.org/13010032>

Abstract:

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + i \bar{\psi}^{(0)} \gamma^\mu (\partial_\mu - i e^{(0)} A_\mu^{(0)}) \psi^{(0)} - m_e^{(0)} \bar{\psi}^{(0)} \psi^{(0)}$$

$n = 4 - \epsilon$
 space-time dimensions

$$A_\mu = \frac{1}{\sqrt{Z_A}} A_\mu^{(0)}, \quad m_e^{(0)} = \frac{1}{Z_m} m_e, \quad e = \frac{1}{Z_e} \mu^{-\epsilon/2} e^{(0)}$$

subtraction pt

$$Z_j = 1 + \frac{g^2}{\epsilon} + \text{higher order}$$

vacuum expectation values of
 determines the a_j , gives

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + i \bar{\psi} \gamma^\mu (\partial_\mu - i e \mu^{\epsilon/2} A_\mu) \psi - m \bar{\psi} \psi + \mathcal{L}_{ct}$$

$$\mathcal{L}_{ct} = \frac{1}{4} (Z_4 - 1) F_{\mu\nu}^2 + (Z_4 Z_e \sqrt{Z_A}) \mu^{\epsilon/2} e \bar{\psi} \gamma^\mu A_\mu \psi - m_e (Z_m Z_4 - 1) \bar{\psi} \psi$$

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu (\partial_\mu - i e A_\mu) \psi - m_e \bar{\psi} \psi$$

$$A_\mu = \frac{1}{\sqrt{Z_A}} A_\mu^{(0)}, \quad \psi^{(0)} = \frac{1}{\sqrt{Z_\psi}} \psi, \quad m_e^{(0)} = \frac{1}{Z_m} m_e, \quad e = \frac{1}{Z_e} e^{(0)} \xrightarrow{\text{subtraction}} \frac{e^{(0)}}{Z_e}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu (\partial_\mu - i e A_\mu) \psi - m \bar{\psi} \psi + \mathcal{L}_{\text{ct}}$$

$$\mathcal{L}_{\text{ct}} = -\frac{1}{4} (Z_A - 1) F_{\mu\nu} F^{\mu\nu} + (Z_\psi - 1) \bar{\psi} \gamma^\mu \partial_\mu \psi + (Z_m - 1) m \bar{\psi} \psi + (Z_e - 1) e \bar{\psi} \gamma^\mu A_\mu \psi$$

$n = 4 - \epsilon$
space time dim

$$Z_j = 1 + \frac{g^2}{\epsilon}$$

vacuum e
determ

$$\int \frac{d^4 q}{(2\pi)^4} \frac{(q^2)^x}{(q^2 - M^2)^B} = \frac{1}{2^{n/2}} \Gamma(1+B) (M^2)^{-B+n/2} \frac{\Gamma(\alpha+n/2) \Gamma(B-\alpha-n/2)}{\Gamma(n/2) \Gamma(B)}$$

on dices
beyond 1-loop

products of fields finite as $\epsilon \rightarrow 0$
finite S-matrix as $\epsilon \rightarrow 0$

$$\mathcal{L}_{ED} = -\frac{1}{4} F_{\mu\nu}^{(0)} F^{(0)\mu\nu} + i \bar{\psi}^{(0)} \gamma^\mu (\partial_\mu - i e^{(0)} A_\mu^{(0)}) \psi^{(0)} - m_e^{(0)} \bar{\psi}^{(0)} \psi^{(0)}$$

$$A_\mu = \frac{1}{\sqrt{Z_A}} A_\mu^{(0)}, \quad \psi^{(0)} = \frac{1}{\sqrt{Z_\psi}} \psi$$

$n = 4 - \epsilon$
 ↗ space-time dimensions

$$\int \frac{d^n q}{(2\pi)^n} \frac{(q^2)^\alpha}{(q^2 - M^2)^\beta}$$

$$Z_j = 1 + \frac{a_j e^2}{\epsilon} + \text{higher orders in } e^2 \text{ be}$$

vacuum expectation values of T product
 determines the a_j , gives finite S-m

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu (\partial_\mu - i e^{(0)} A_\mu^{(0)}) \psi - m \bar{\psi} \psi + \mathcal{L}_{ct}$$

$$\mathcal{L}_{ct} = -\frac{1}{4} (Z_A - 1) F_{\mu\nu} F^{\mu\nu} + (Z_\psi - 1) \bar{\psi} \gamma^\mu \partial_\mu \psi + (Z_4 Z_e \sqrt{Z_A}) a^{e/2} e \bar{\psi} \gamma^\mu A_\mu \psi - m_e (Z_m Z_\psi - 1) \bar{\psi} \psi$$

$$-ie\bar{\psi}(x)A_\mu(x)\psi(x) - m_e\bar{\psi}(x)\psi(x)$$

$$m_e^{(0)} = \frac{1}{Z_m} m_e \quad e = \frac{e^{(0)}}{Z_3} \rightarrow \text{subtraction pt}$$

$$-ie\bar{\psi}(x)\gamma^\mu A_\mu(x)\psi(x) + \mathcal{L}_{ct}$$

$$(Z_4 - 1)\bar{\psi}\gamma^\mu\psi + (Z_2\sqrt{Z_A})m_e^{1/2}e\bar{\psi}\gamma^\mu A_\mu\psi - m_e(Z_m Z_4 - 1)\bar{\psi}\psi$$

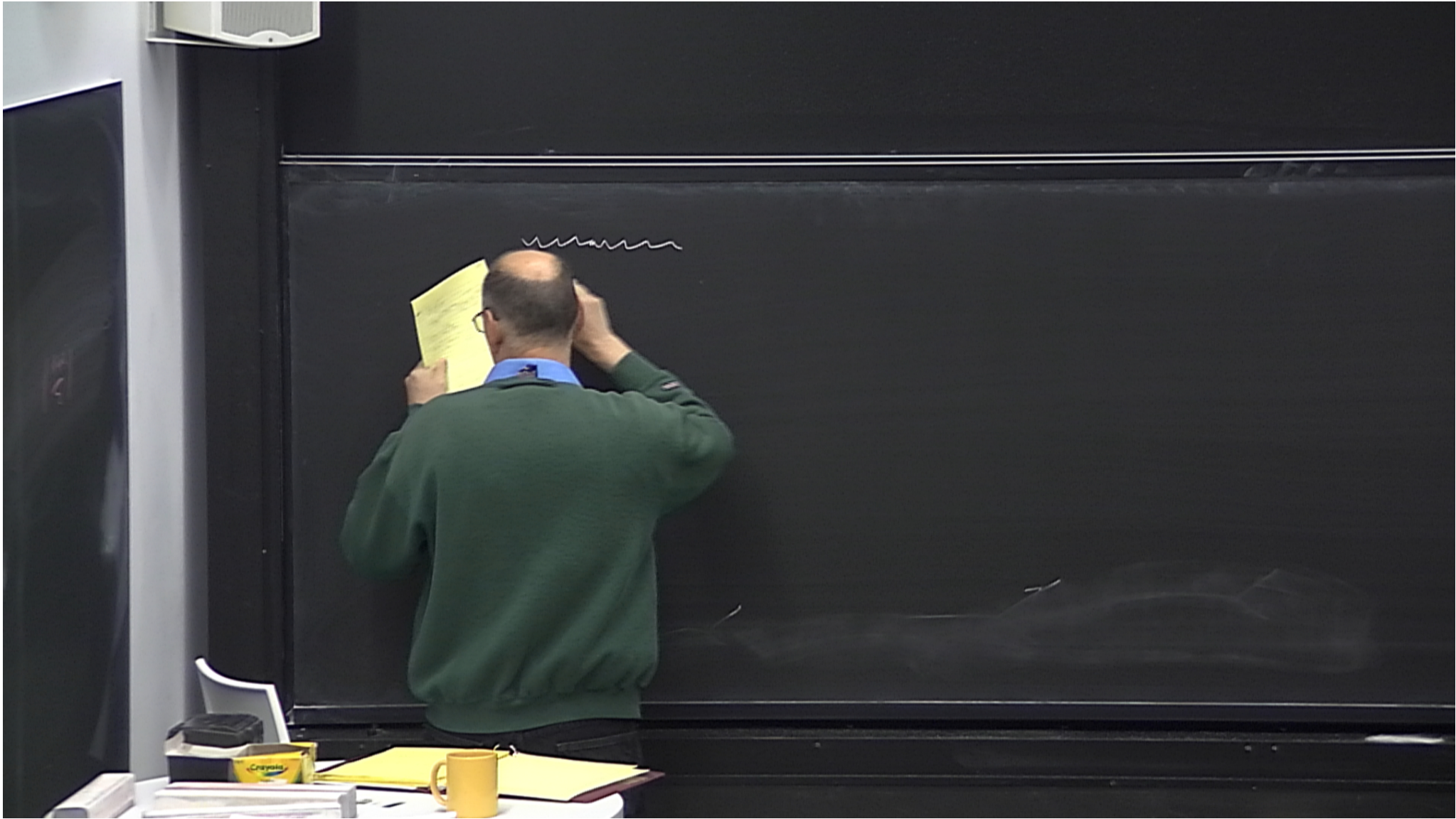
$$n = 4 - \epsilon$$

space time dimensions

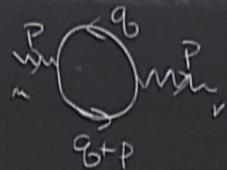
$$\int \frac{d^n q}{(2\pi)^n} \frac{(q^2)^\alpha}{(q^2 - M^2)^B} = \frac{1}{Z^{n/2}} \Gamma(1+B) (M^2)^{\alpha-B+\frac{n}{2}} \frac{\Gamma(\alpha+\frac{n}{2})\Gamma(B-\alpha-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(B)}$$

$$Z_j = 1 + \frac{ae^2}{\epsilon} + \text{higher orders in } e^2 \text{ beyond 1-loop}$$

vacuum expectation values of T products of fields finite as $\epsilon \rightarrow 0$
determines the a_j , gives finite S-matrix as $\epsilon \rightarrow 0$

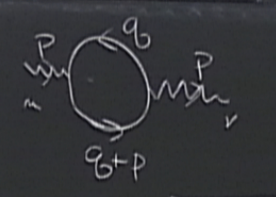


$$P_{\mu\nu} = i(\not{p}_A - 1)(p_\mu p_\nu - p^2 g_{\mu\nu})$$



$$(ie)^2 \mu^\epsilon \int \frac{d^4 q}{(2\pi)^4} \text{Tr}[\gamma_\nu (\not{q} + \not{p}) \gamma_\mu (\not{q})]$$

$$i(Z_A - 1)(P_\mu P_\nu - P^2 \eta_{\mu\nu})$$



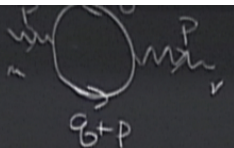
$$(-)(ie)^2 \mu^\epsilon \int \frac{d^n q}{(2\pi)^n} \frac{\text{Tr}[\gamma_\nu (q + \cancel{P} + m_c) \gamma_\mu (q + m_c)]}{[q^2 - m_c^2 + i0^+][q + P]^2}$$

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2}$$

$$\mu^\epsilon = 1 + \epsilon \ln \mu + \dots \approx 1$$

$$-4e^2 \{ \cancel{P}_\mu \cancel{q}_\nu + (\cancel{q} + \cancel{P})_\nu \cancel{q}_\mu + (\cancel{q} + \cancel{P})_\mu \cancel{q}_\nu + m_c^2 \eta_{\mu\nu} \}$$

$(Z_A - 1) (P_\mu P_\nu - \delta_{\mu\nu})$
 $\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2}$


 $\mu^E = 1 + \epsilon \ln \mu + \dots \approx 1$

$(-)(ie)^2 \mu^E \int \frac{d^n q}{(2\pi)^n} \frac{\text{Tr} [\gamma_\nu (\not{q} + \not{P} + m_e) \gamma_\mu (\not{q} + m_e)]}{[(q+p)^2 - m_e^2 + i0] [q^2 - m_e^2 + i0]}$
 $-4e^2 \int_0^1 dx \int \frac{d^n q}{(2\pi)^n} \left\{ (q+p)_\mu q_\nu + (q+p)_\nu q_\mu - (q+p) \cdot q \eta_{\mu\nu} + m_e^2 \eta_{\mu\nu} \right\}$
 $\frac{1}{[(q+xp)^2 + p^2 x(1-x) - m_e^2 + i0]^2}$



$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax+b(1-x)]^2}$$

$$M^E = 1 + \epsilon \ln \mu + \dots \approx 1$$

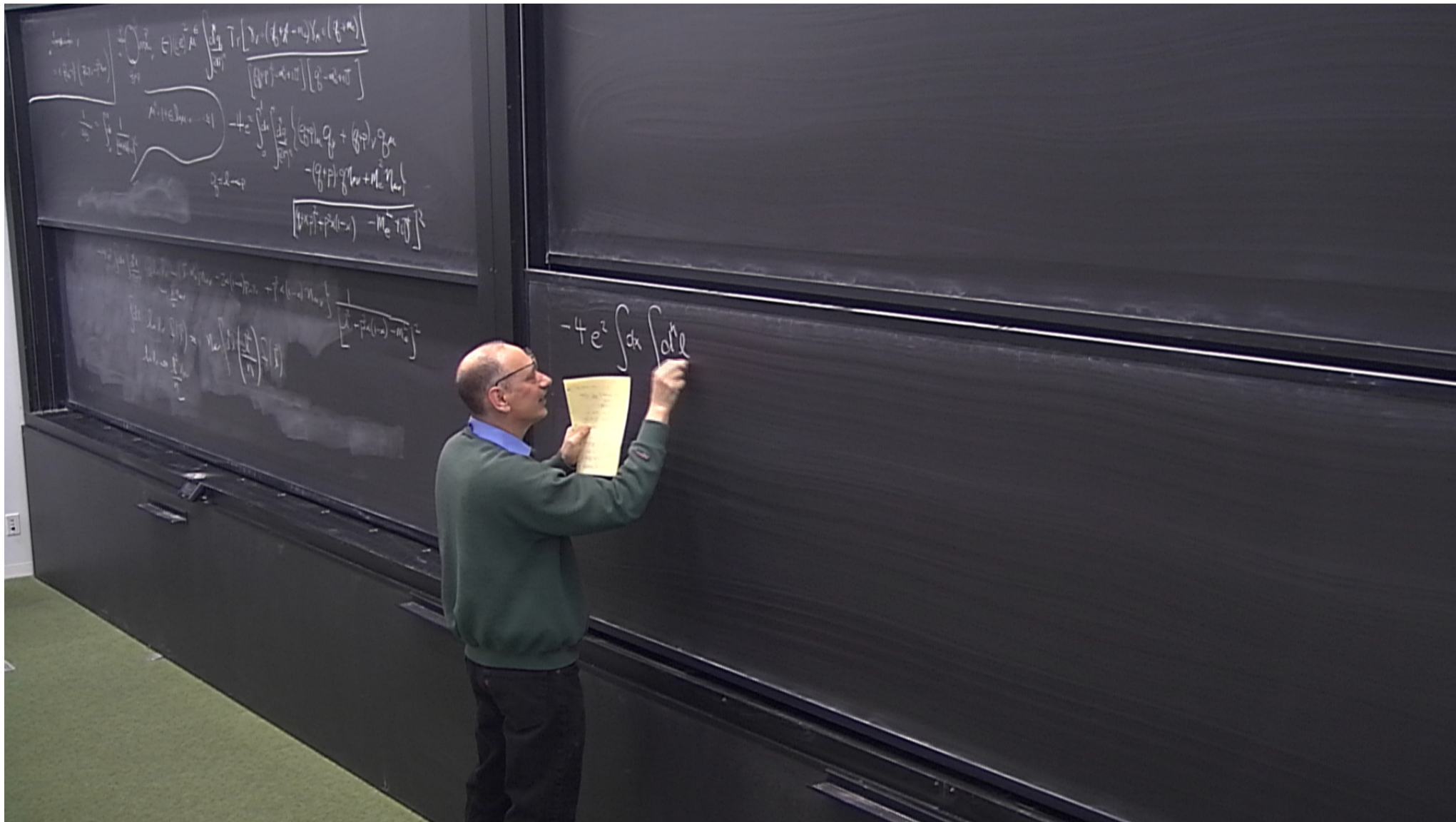
$$q = l - xp$$

$$-4e^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \left\{ (q+p)_\mu q_\nu + (q+p)_\nu q_\mu - (q+p)_\mu q_\nu + m_e^2 \eta_{\mu\nu} \right\} \frac{1}{[q^2 + p^2 x(1-x) - m_e^2 + i0]^2}$$

$$-4e^2 \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \left\{ 2l_\mu l_\nu - \left(l^2 - m_e^2 \right) \eta_{\mu\nu} - 2x(1-x) p_\mu p_\nu + p^2 x(1-x) \eta_{\mu\nu} \right\} \frac{1}{[l^2 + p^2 x(1-x) - m_e^2]^2}$$

$$\int d^4 l \, l_\mu l_\nu f(l^2) = \eta_{\mu\nu} \int d^4 l \left(\frac{l^2}{n} \right) f(l^2)$$

$$l_\mu l_\nu \rightarrow \frac{l^2}{n} \eta_{\mu\nu}$$



$$-4e^2 \int dx \int \frac{d^n l}{(2\pi)^n} \left\{ -\frac{1}{2} n_{\mu\nu} l^2 + [-2x(1-x)p_\mu p_\nu + (\vec{p}^2 x(1-x) + m_e^2) n_{\mu\nu}] \right.$$

$$-4e^2 \int dx \int \frac{d^n l}{(2\pi)^n} \left\{ -\frac{1}{2} \eta_{\mu\nu} l^2 + [-2x(1-x) p_\mu p_\nu + (p^2 x(1-x) + m_e^2) \eta_{\mu\nu}] \frac{1}{[l^2 + p^2 x(1-x) - m_e^2]^2} \right\}$$

$$-4e^2 \int dx \int \frac{d^n l}{(2\pi)^n} \left\{ -\frac{1}{2} \eta_{\mu\nu} l^2 + [-2x(1-x)p_\mu p_\nu + (p^2 x(1-x) + m_e^2) \eta_{\mu\nu}] \frac{1}{[l^2 + p^2 x(1-x) + m_e^2]^2} \right\}$$

Do integral:

$$\int \frac{d^n l}{(2\pi)^n} \frac{1}{[l^2 + p^2 x(1-x) + m_e^2]^2}$$

$$-4e^2 \int dx \int \frac{d^n l}{(2\pi)^n} \left\{ -\frac{1}{2} n_{\mu\nu} l^2 + [-2x(1-x)p_\mu p_\nu + (p^2 x(1-x) + m_e^2) n_{\mu\nu}] \frac{1}{[l^2 + p^2 x(1-x) - m_e^2]^2} \right\}$$

Do integral:

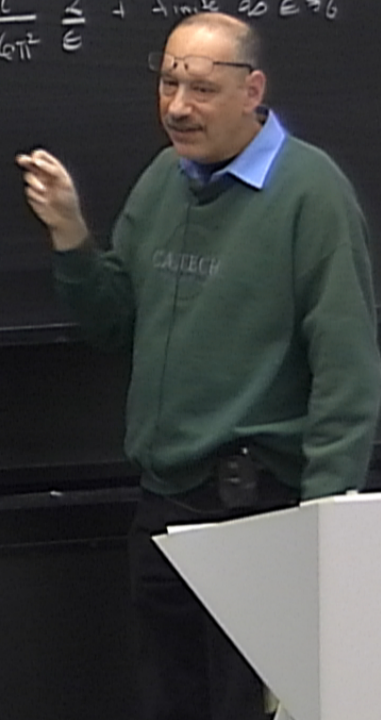
$$\int \frac{d^n l}{(2\pi)^2} \frac{l^2}{[l^2 + p^2 x(1-x) - m_e^2]^2} = \frac{i(-1)}{2^n \pi^{n/2}} (m_e^2 - p^2 x(1-x))^{1-2+n/2} \frac{\Gamma(1+n/2) \Gamma(1-n/2)}{\Gamma(n/2) \Gamma(2)}$$

$$\Gamma(n)$$

$$\Gamma(1+\epsilon/2) = \frac{\epsilon}{2} \Gamma\left(\frac{\epsilon}{2}\right)$$

$$\Gamma\left(\frac{\epsilon}{2}\right) = \frac{2}{\epsilon} \Gamma(1+\epsilon/2) \quad \left. \begin{array}{l} -\gamma_E \\ \end{array} \right\} \\ = \frac{2}{\epsilon} \left[1 + \frac{\epsilon}{2} \Gamma'(1) + \dots \right]$$

$$\int \frac{d^N l}{(2\pi)^N} \frac{1}{[l^2 + p^2 x(1-x)]^2} = \frac{i}{16\pi^2} \frac{2}{\epsilon} + \text{finite as } \epsilon \rightarrow 0$$



$$\Gamma(1+\epsilon/2) = \frac{\epsilon}{2} \Gamma\left(\frac{\epsilon}{2}\right)$$

$$\Gamma(\epsilon/2) = \frac{2}{\epsilon} \Gamma(1+\epsilon/2) \quad \left. \begin{array}{l} -\gamma_E \\ \end{array} \right\}$$

$$= \frac{2}{\epsilon} \left[1 + \frac{\epsilon}{2} \Gamma'(1) + \dots \right]$$

Put together the pieces and get

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{[l^2 + p^2 x(1-x)]^2} = \frac{i}{16\pi^2} \frac{2}{\epsilon} + \text{finite as } \epsilon \rightarrow 0$$

$$\left(-\frac{e^2}{16\pi^2} \left(\frac{\gamma}{3} \right) (p^2 \eta_{\mu\nu} - p_\mu p_\nu) + \text{finite as } \epsilon \rightarrow 0 \right)$$

