Title: Worldsheet form factors in AdS/CFT

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Abstract: The study of the worldsheet S-matrix for AdS_5Ã—S^5 strings was a key step in

the complete determination of the non-pertubrative planar spectrum of anomalous

dimensions for N=4 super-Yang-Mills. To go beyond the spectral problem it is

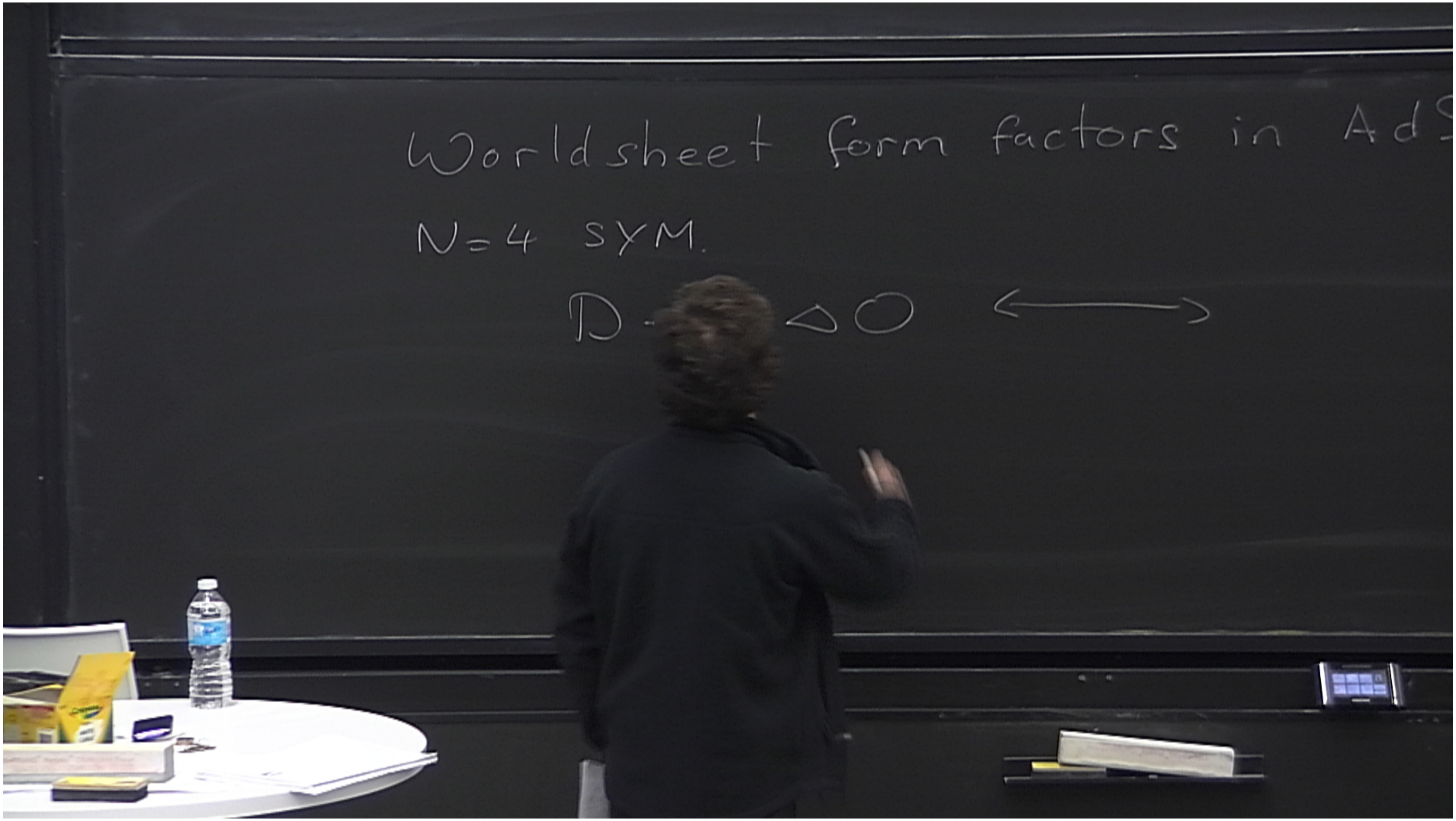
important to consider higher-point worldsheet correlation functions and, as is

standard in many integrable models, one approach is the study of form factors.

We will discuss a set of consistency conditions appropriate to form factors in

the light-cone gauge fixed AdS_5Ã—S^5 string theory. We further discuss the

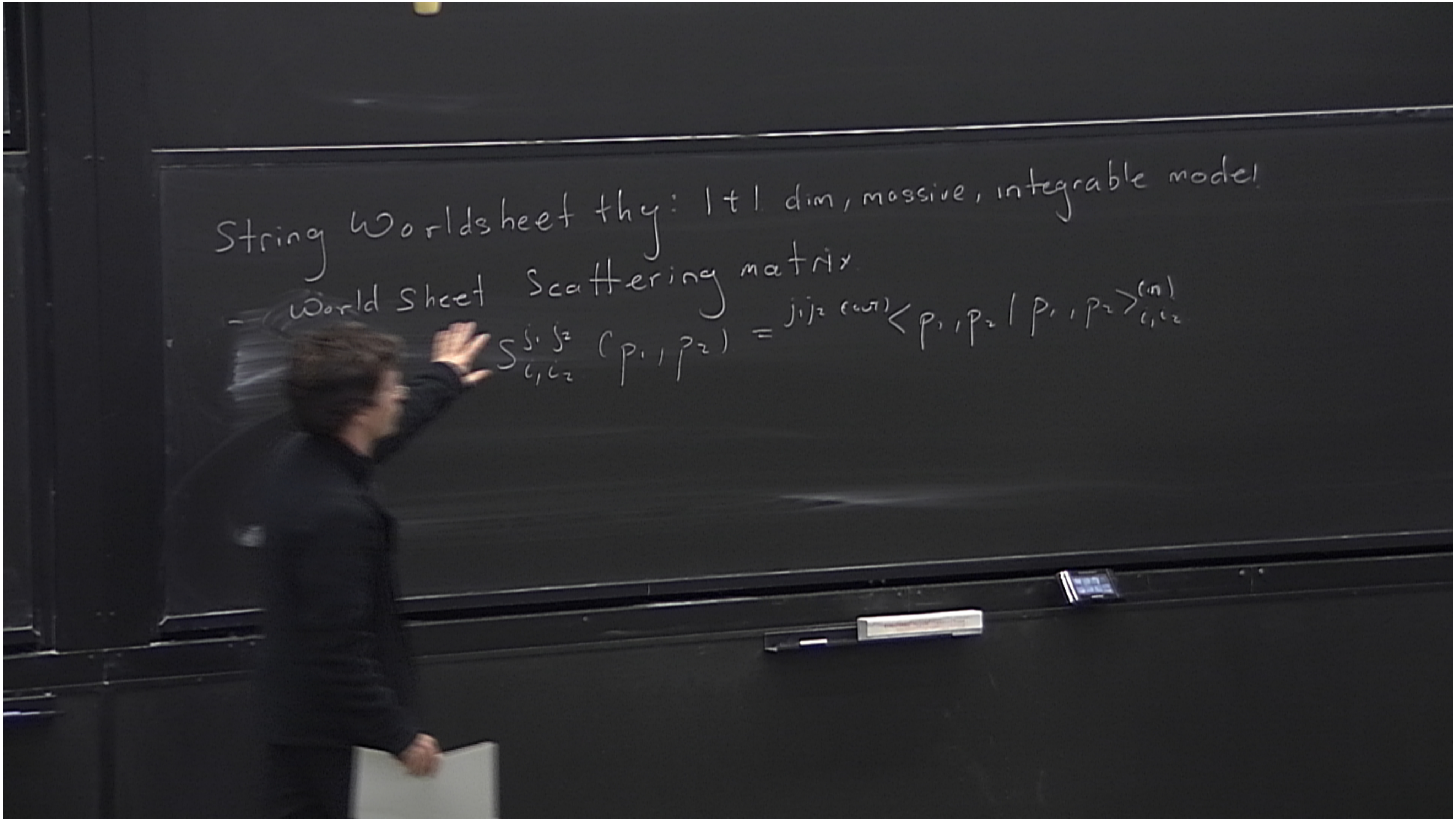
form factors in the weakly coupled dual description, verifying that the relevant conditions naturally hold for the one-loop Heisenberg spin-chain.


Worldsheet form factors in AdS
N=4 SYM.

String Worldsheet thy: 1+1 dim, massive, integrable model

- World Sheet Scattering matrix

$$S^{j_1 j_2}_{i_1 i_2}(p_1, p_2) = {}^{j_1 j_2\,(\text{out})}\langle p_1, p_2 | p_1, p_2 \rangle^{(\text{in})}_{i_1 i_2}$$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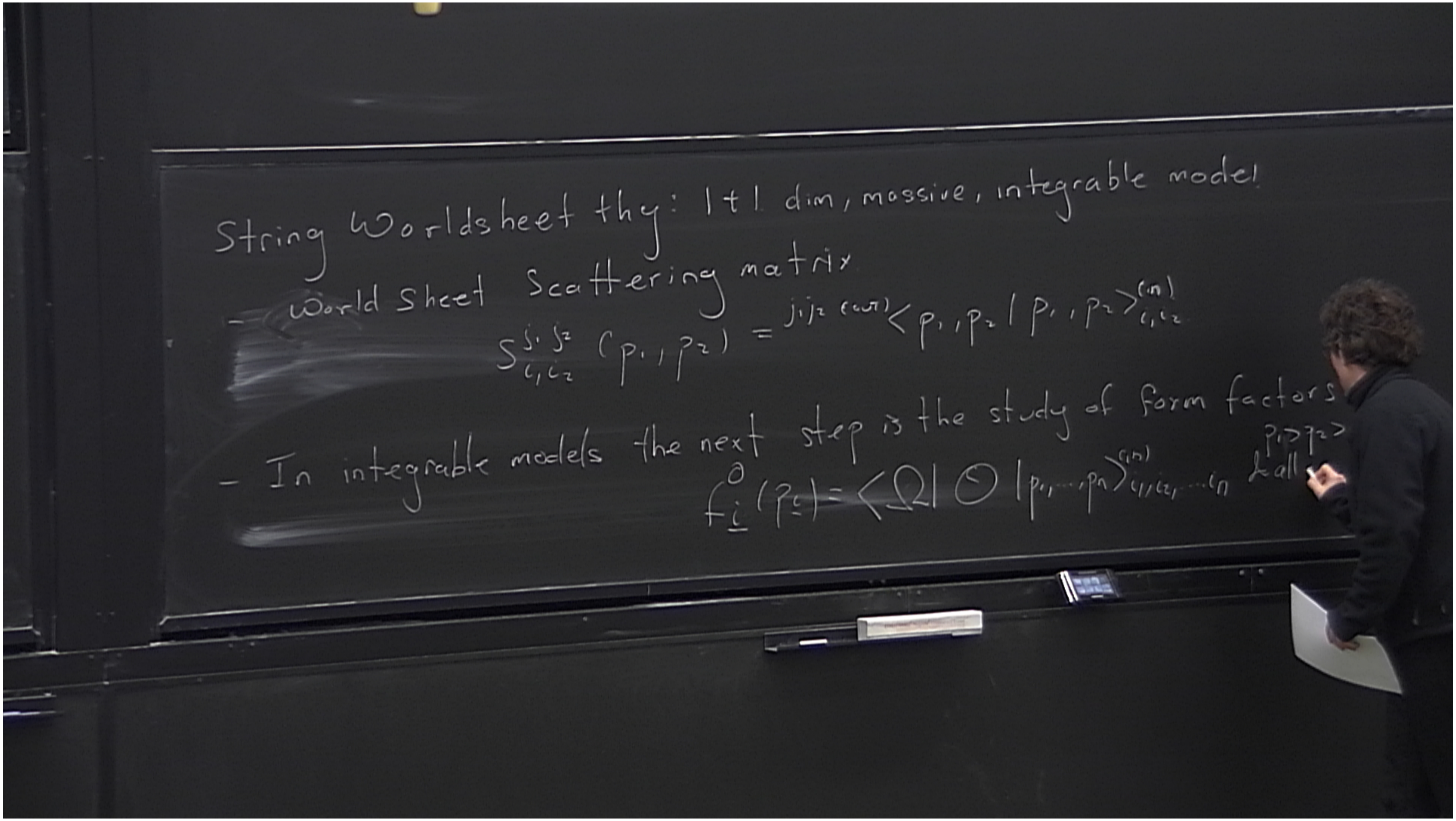
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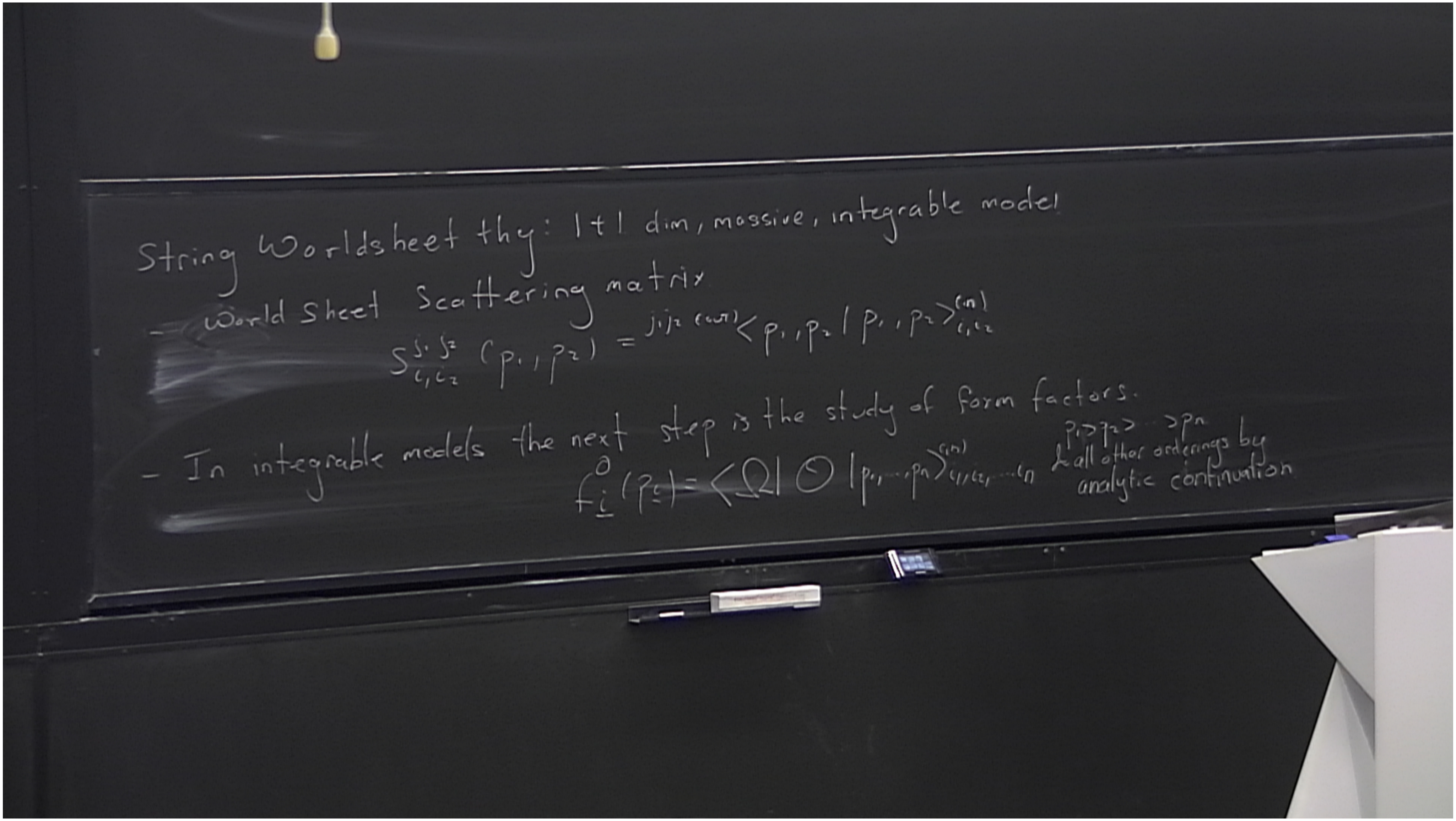
- In integrable models the next step is the study of form factors
$f_{\underline{i}}^{\mathcal{O}}(p_i) = \langle \Omega | \mathcal{O} | p_1, \dots, p_n \rangle^{(in)}_{i_1, i_2, \dots, i_n}$ & all $p_1 > p_2 >$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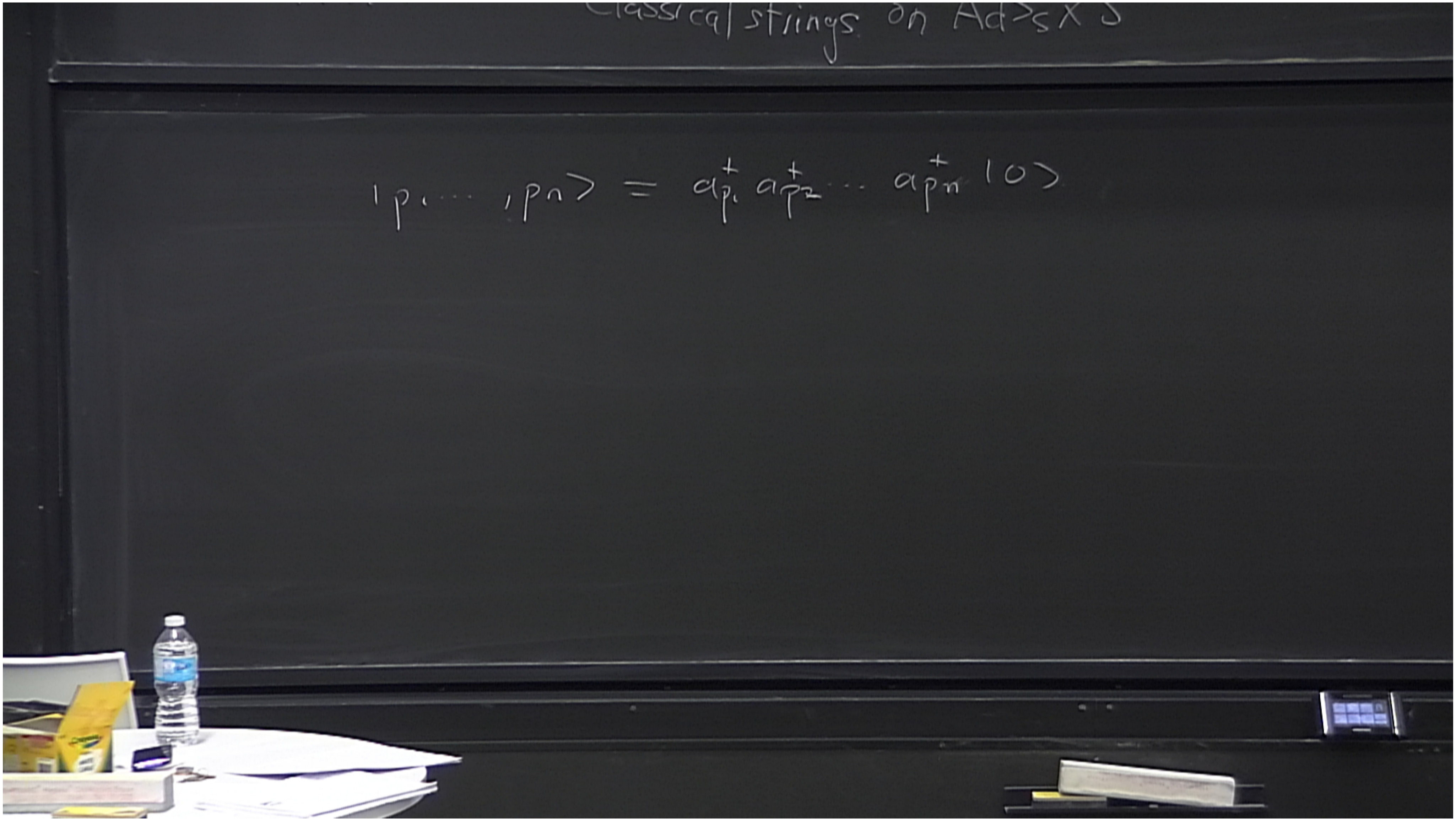
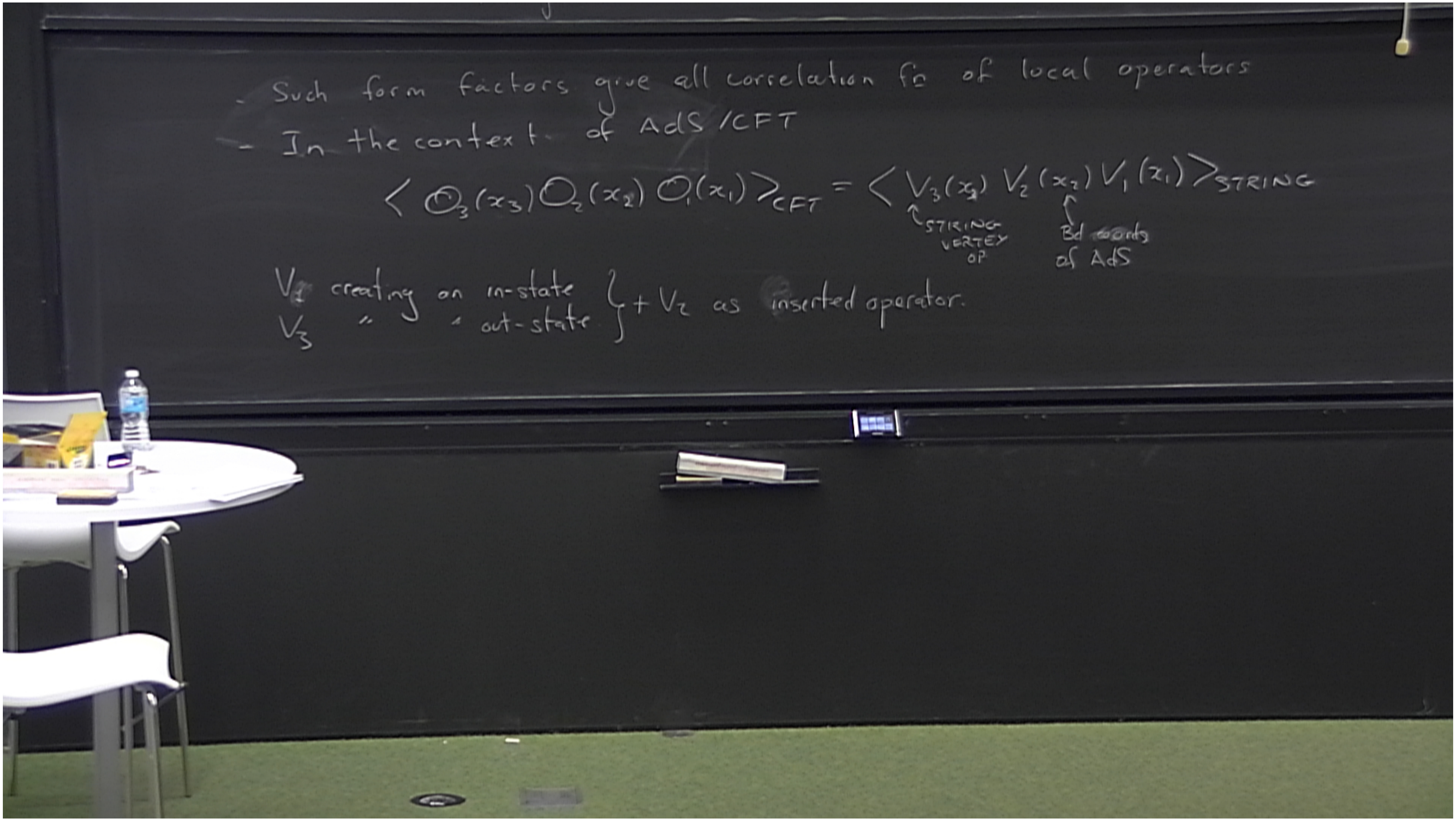
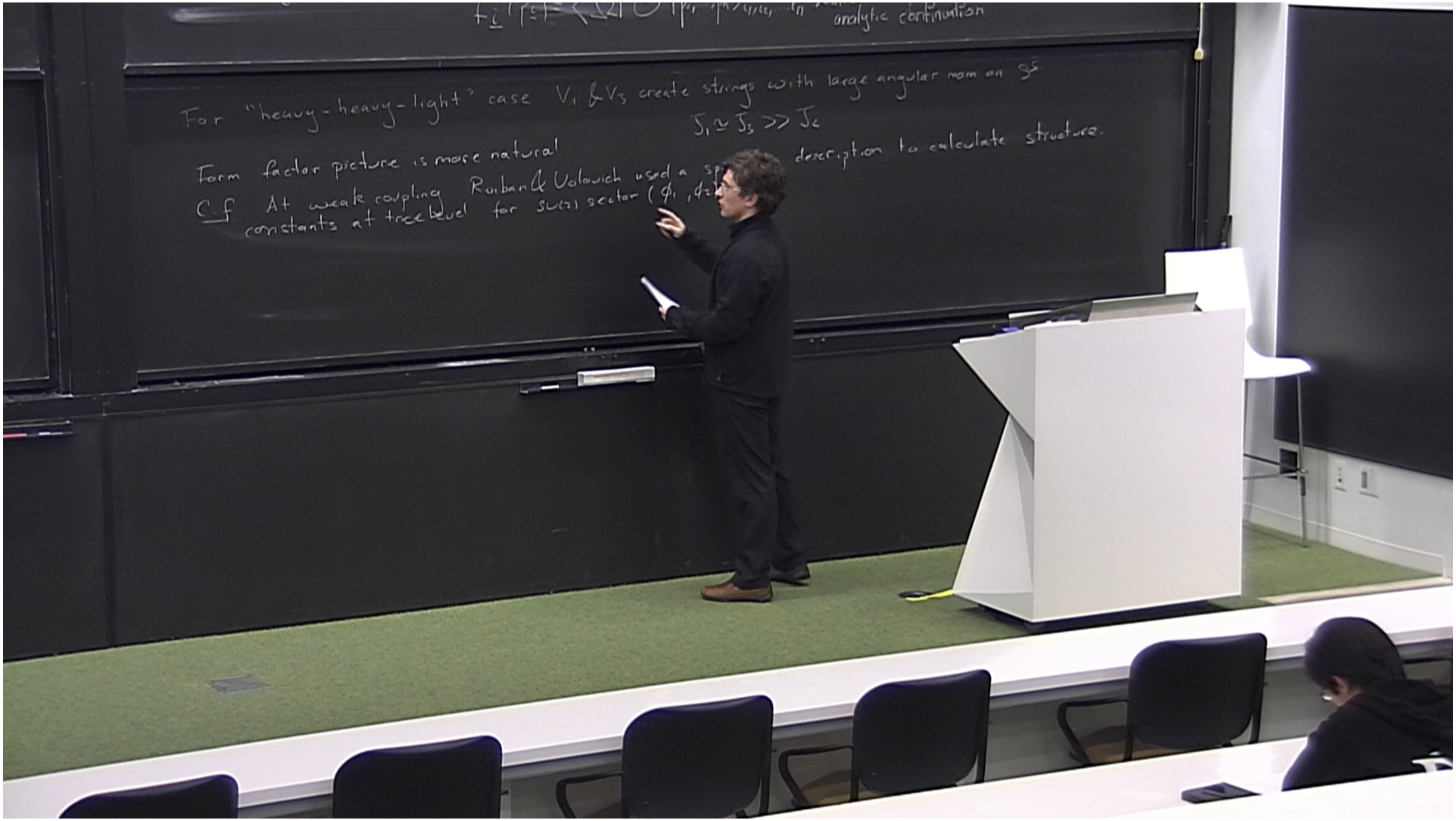
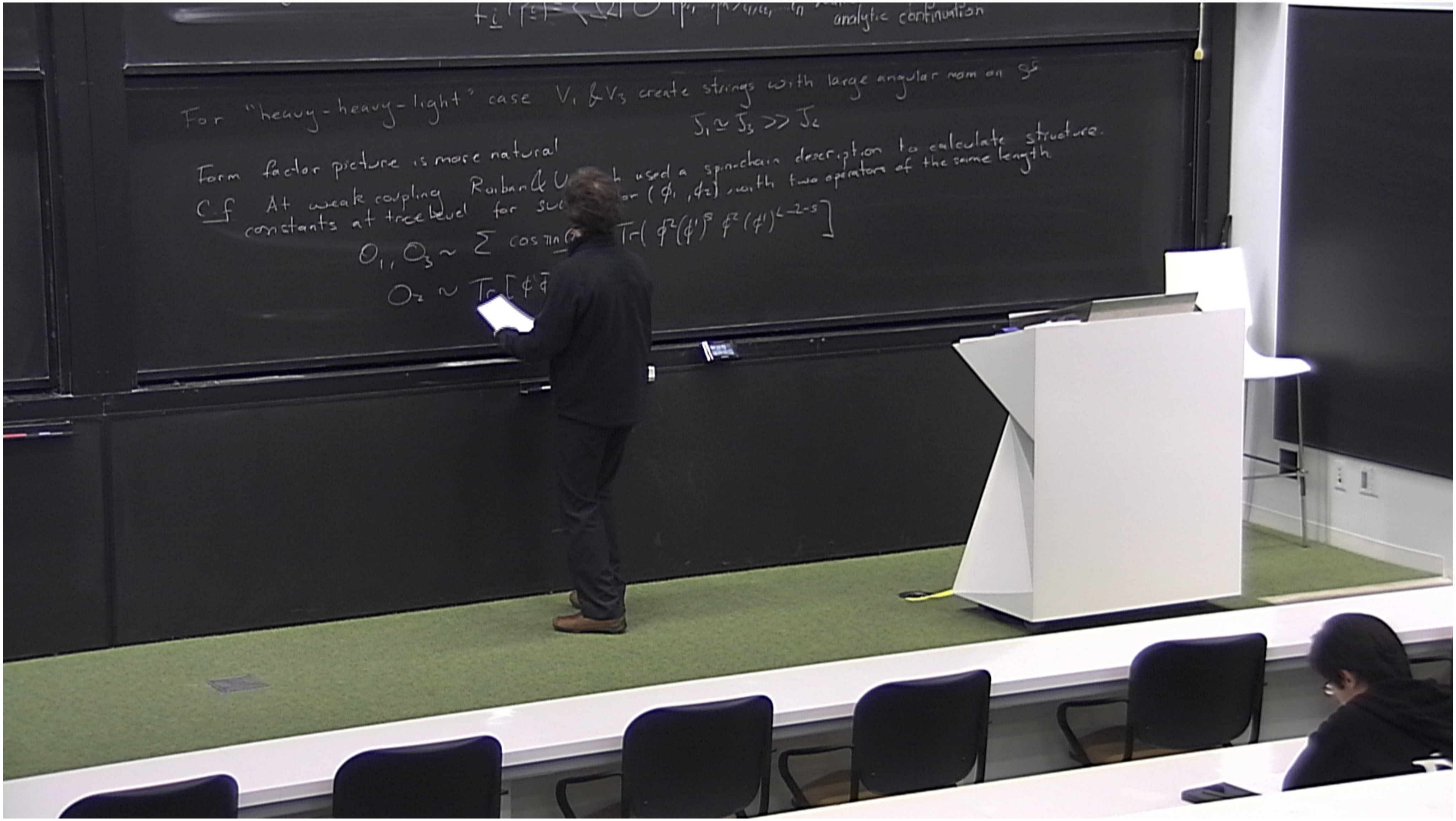
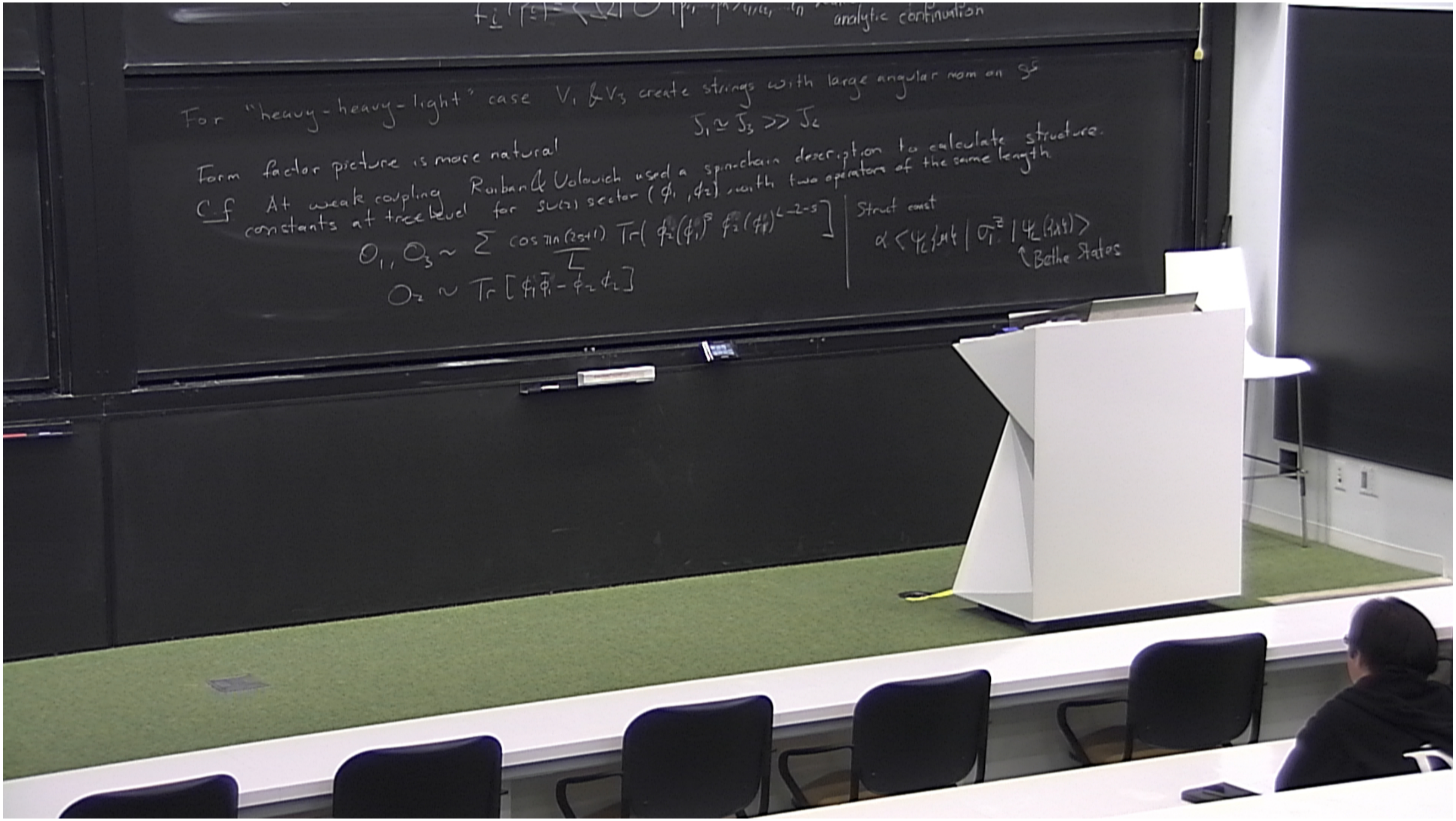
$f^{\mathcal{O}}_{\underline{i}}(p_{\underline{i}}) = \langle \Omega | \mathcal{O} | p_1, \ldots, p_n \rangle^{(in)}_{i_1, i_2, \ldots, i_n}$
$p_1 > p_2 > \ldots > p_n$
& all other orderings by analytic continuation


Classical strings on AdS5 x S5
$|p_1, \cdots, p_n\rangle = a^\dagger_{p_1} a^\dagger_{p_2} \cdots a^\dagger_{p_n} |0\rangle$


- Such form factors give all correlation fn of local operators
- In the context of AdS/CFT
$\langle O_3(x_3) O_2(x_2) O_1(x_1) \rangle_{CFT} = \langle V_3(x_3)\, V_2(x_2)\, V_1(x_1) \rangle_{STRING}$
STRING VERTEX OP
Bd coords of AdS
V_1 creating an in-state
V_3 " " out-state
+ V_2 as inserted operator.


analytic continuation
For "heavy-heavy-light" case V1 & V3 create strings with large angular mom on S5
J1 ≃ J3 >> J2
Form factor picture is more natural
C.f At weak coupling Roiban & Volovich used a sp description to calculate structure.
constants at tree level for SU(2) sector (φ1, φ2)


analytic continuation
For "heavy-heavy-light" case V_1 & V_3 create strings with large angular mom on S^5
$J_1 \simeq J_3 >> J_2$
Form factor picture is more natural
Cf At weak coupling
constants at tree level
used a spin-chain description to calculate structure
with two operators of the same length



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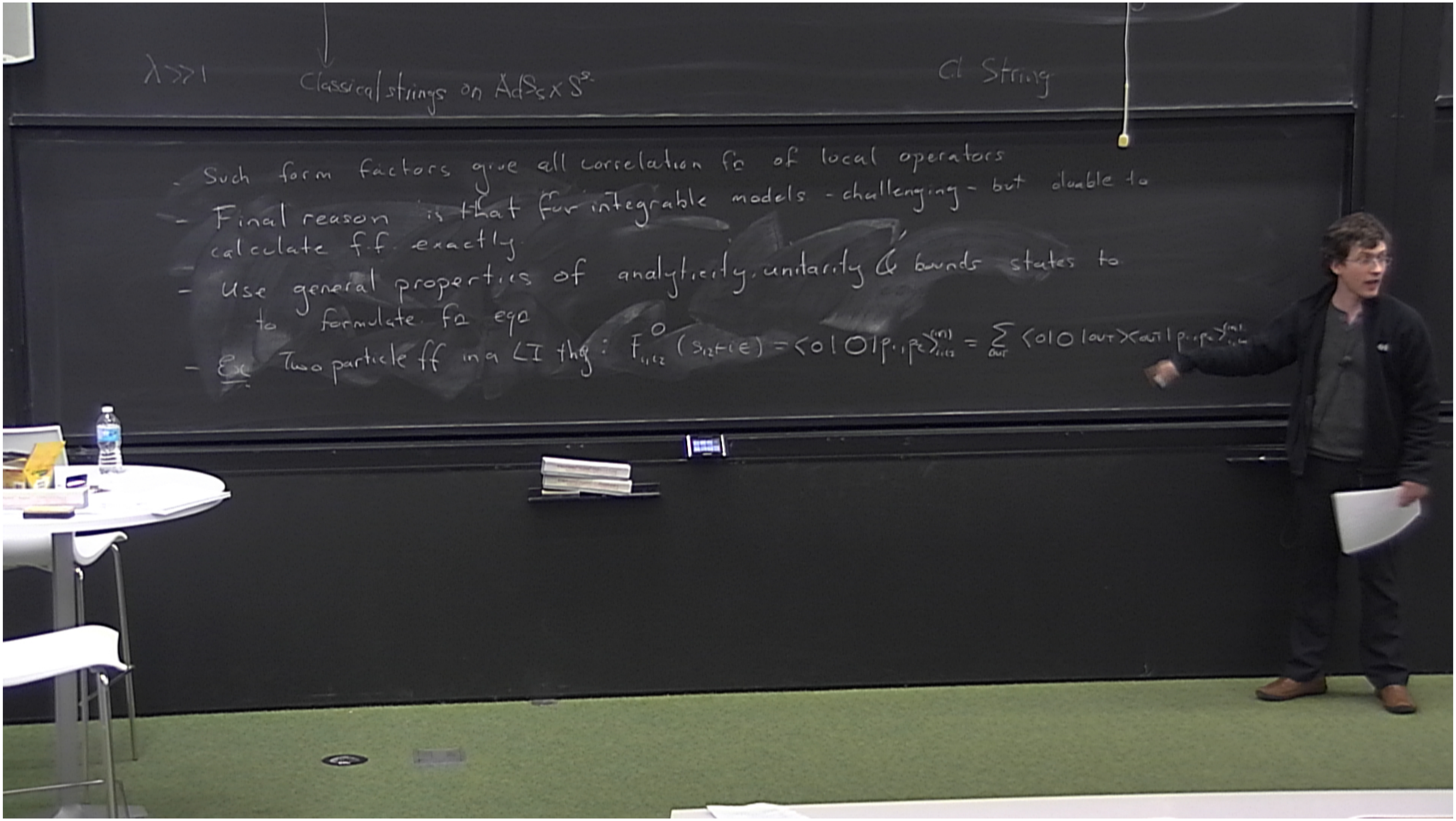
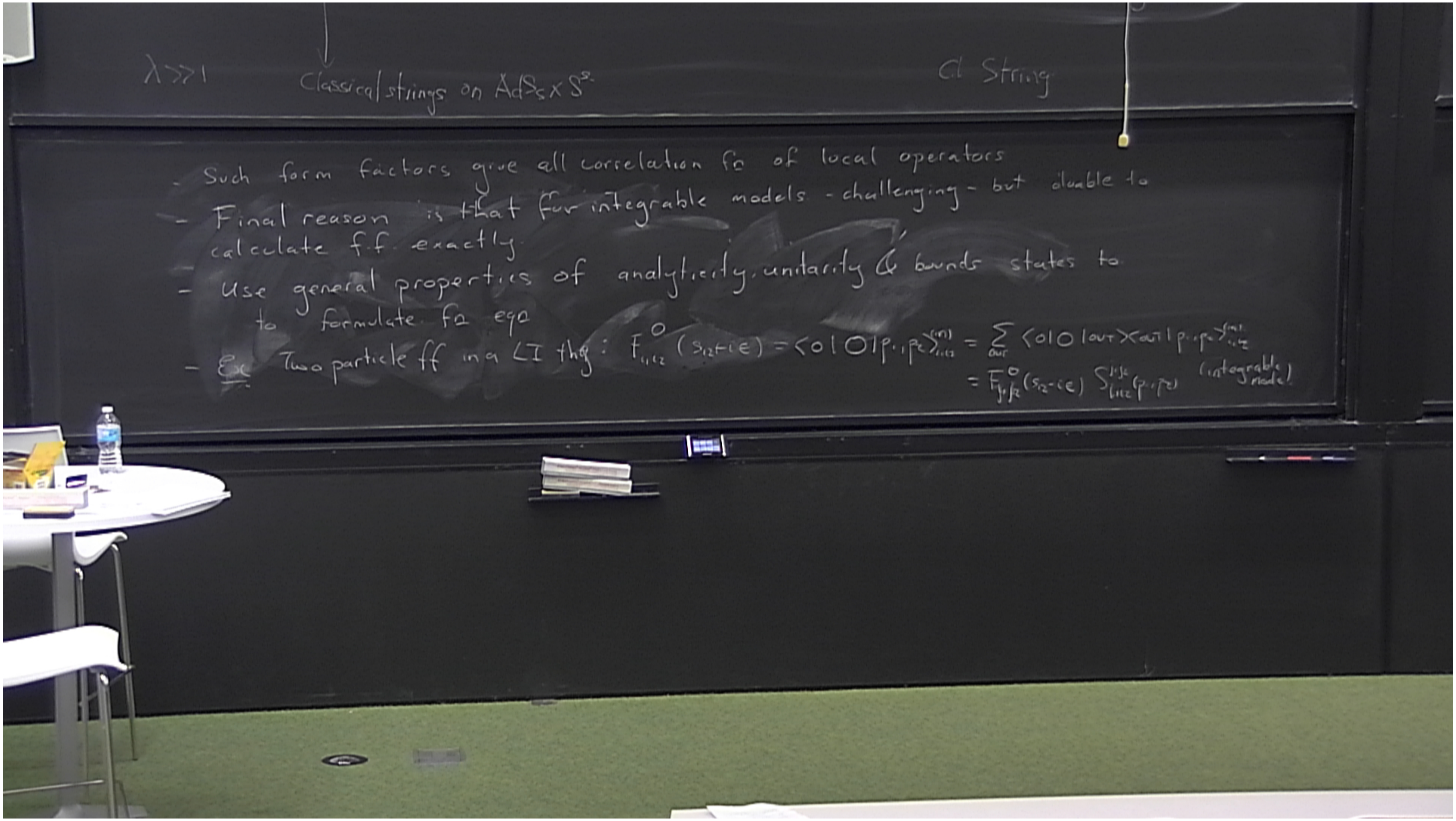
For "heavy-heavy-light" case V1 & V3 create strings with large angular mom on S5
J1 ≃ J3 >> J2
Form factor picture is more natural
C.f At weak coupling Ruiban & Volovich used a spin-chain description to calculate structure constants at tree level for SU(2) sector (φ1, φ2), with two operators of the same length
O1, O3 ~ Σ cos πn(2s+1)/L Tr(φ2 (φ1)^s φ2 (φ1)^(L-2-s)]
O2 ~ Tr [φ1 φ̄1 - φ2 φ̄2]
Struct const
∝ < ψL{u} | σ1^z | ψL{λ} >
Bethe States


λ>>1
Classical strings on AdS5 x S5
Cl String
- Such form factors give all correlation fn of local operators
- Final reason is that for integrable models - challenging - but doable to calculate f.f. exactly
- Use general properties of analyticity, unitarity & bounds states to to formulate fn eqn
- Ex Two particle ff in a LI thy :



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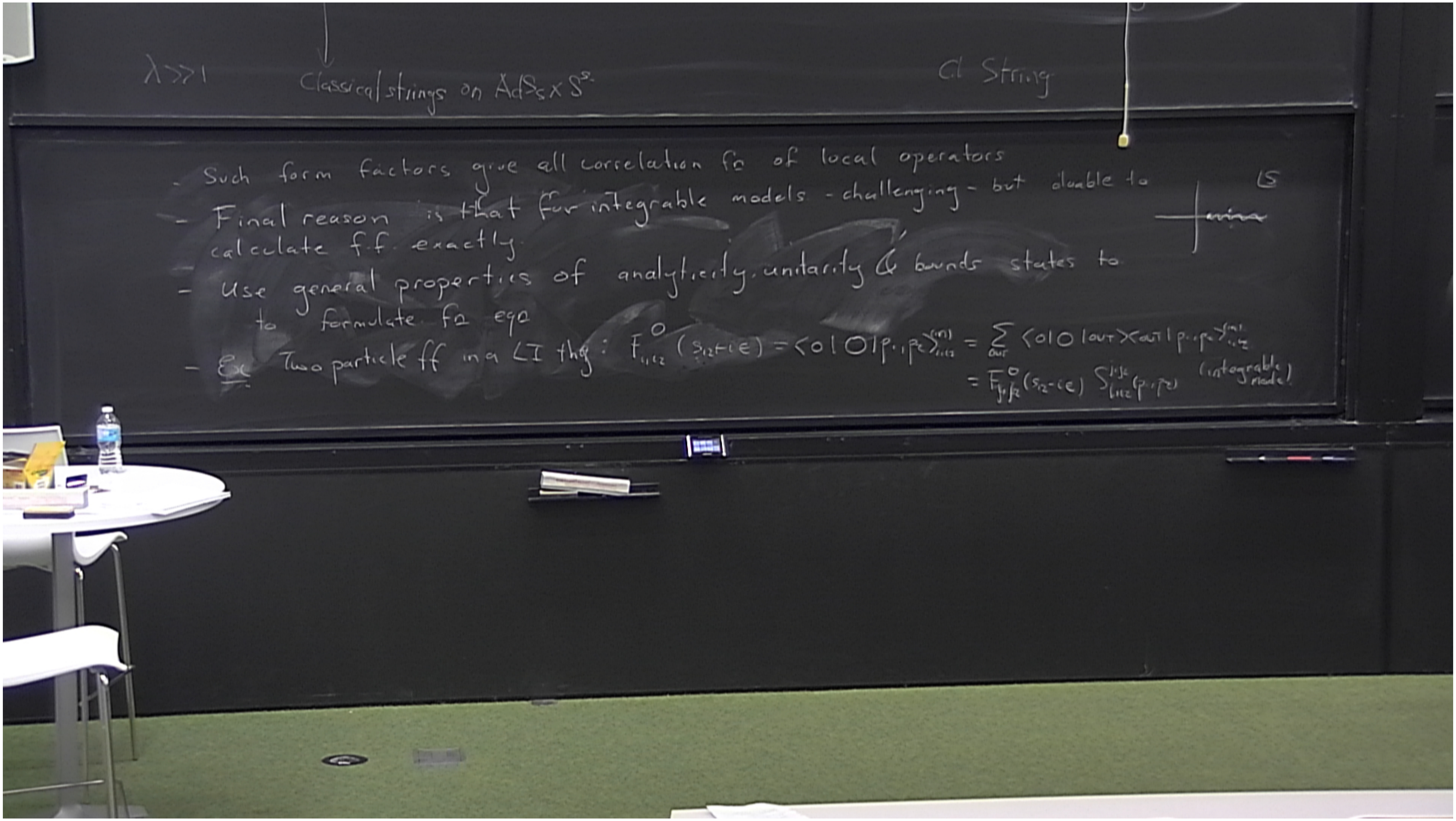
to formulate fn eqn
- Ex Two particle ff in a LI thy : $F^{O}_{i_1 i_2}(s_{12}+i\epsilon) = \langle 0|O|p_1,p_2\rangle^{(in)}_{i_1 i_2} = \sum_{out} \langle 0|O|out\rangle\langle out|p_1,p_2\rangle^{(in)}_{i_1 i_2}$
$= F^{O}_{j_1 j_2}(s_{12}-i\epsilon)\, S^{j_1 j_2}_{i_1 i_2}(p_1,p_2)$ (integrable model)



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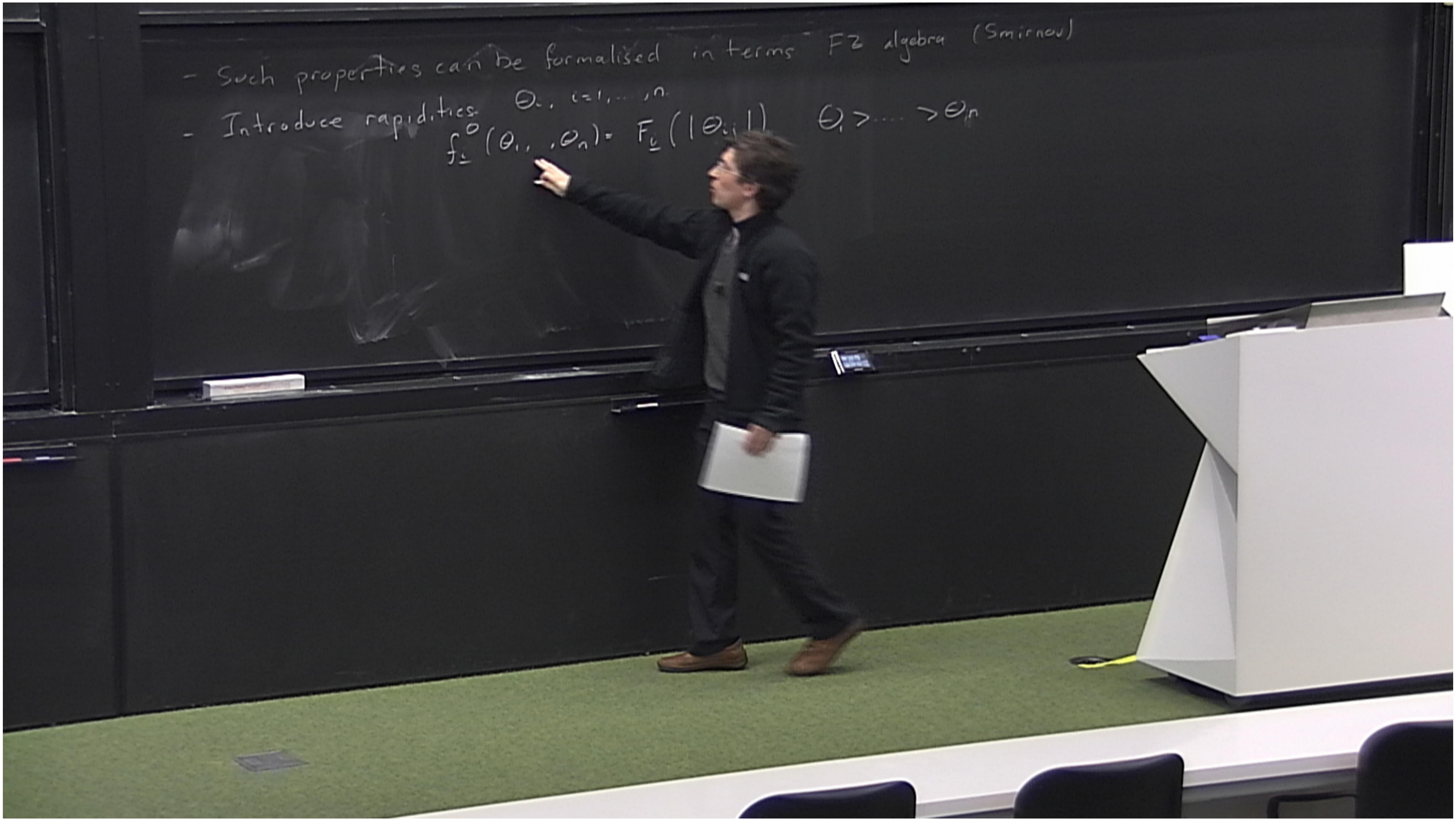
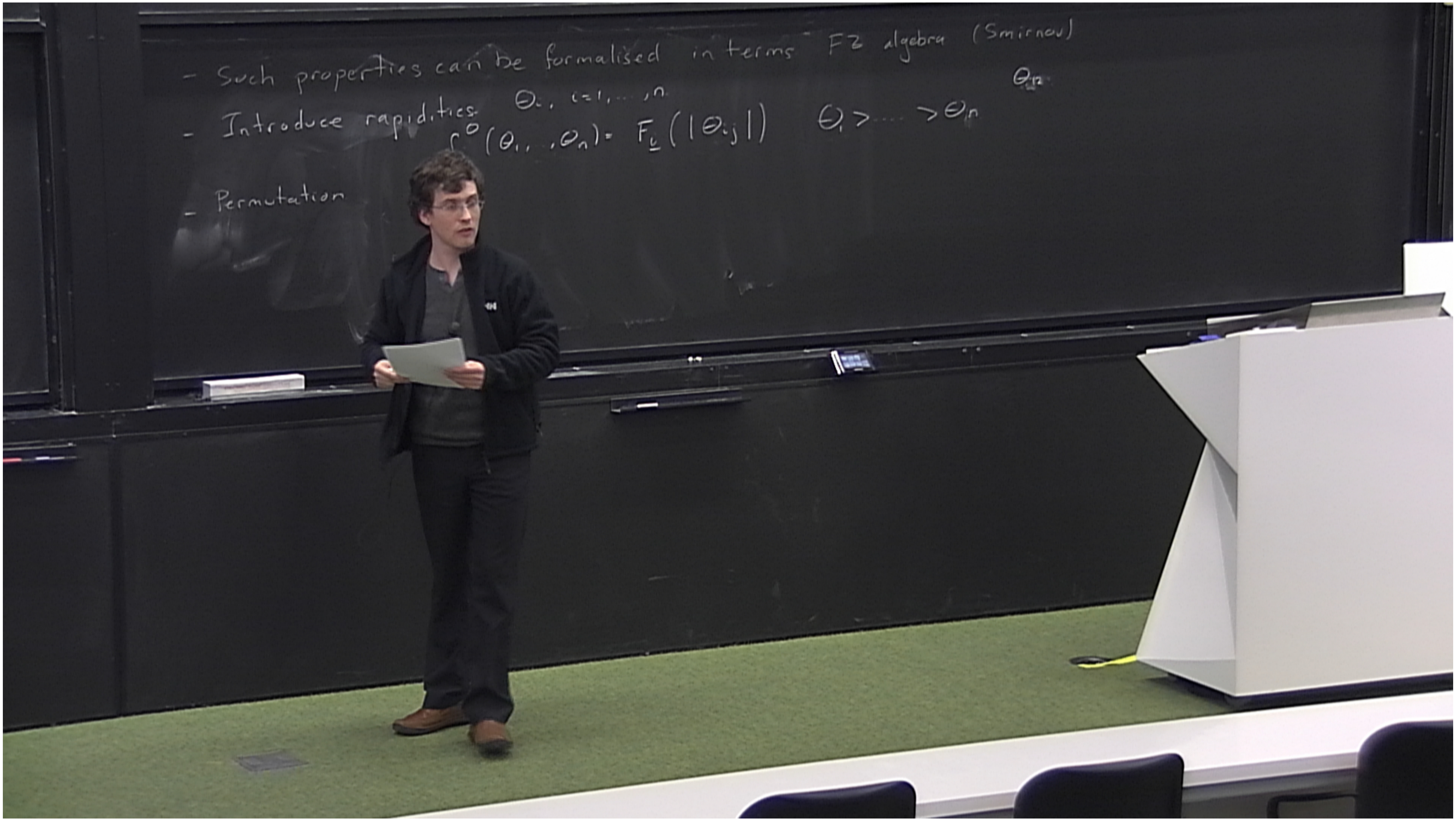
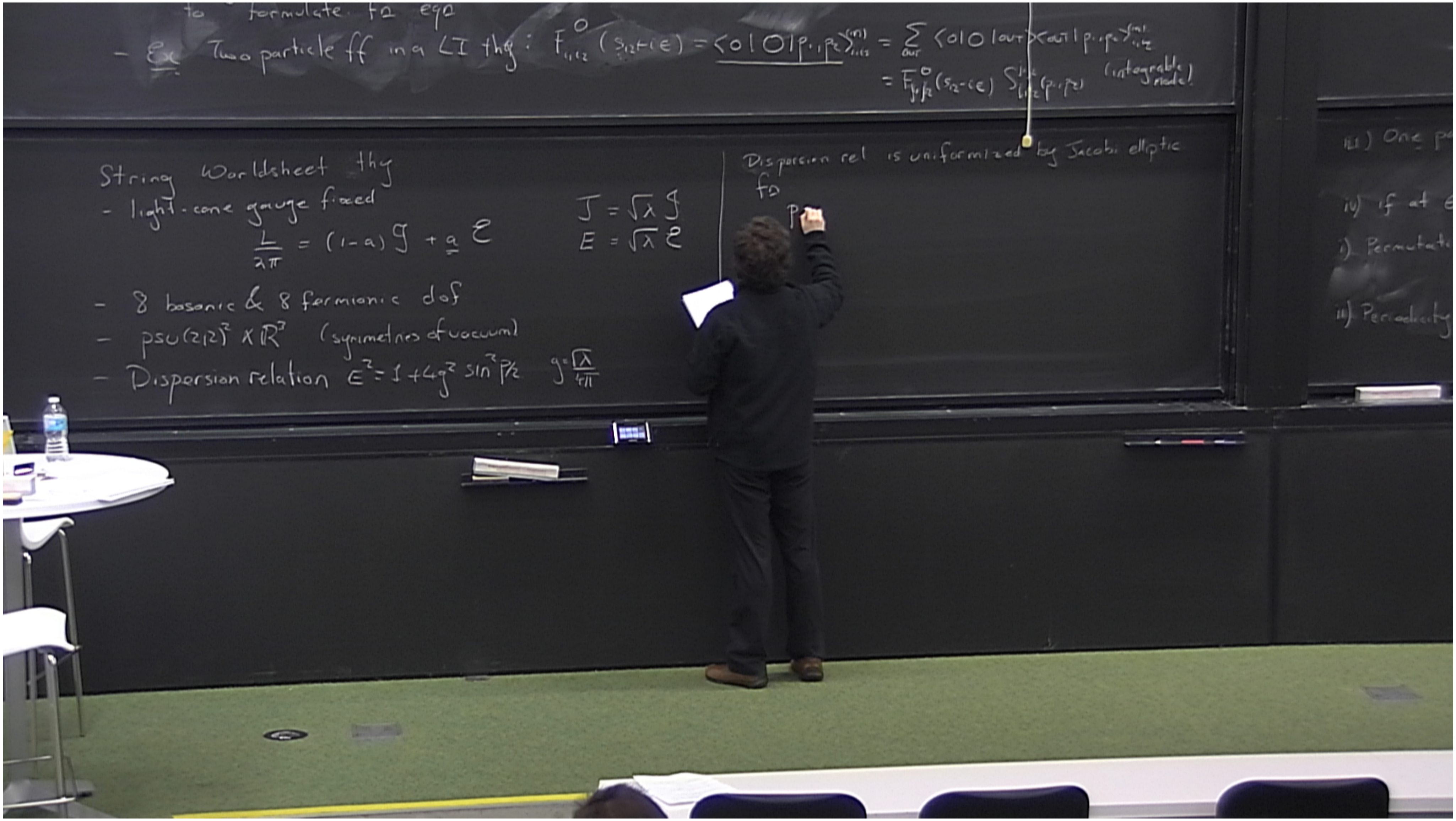

- Such properties can be formalised in terms FZ algebra (Smirnov)
- Introduce rapidities θᵢ, i=1,...,n.
f(θ₁, ,θₙ) = F(|θᵢ|) θ₁ > ... > θₙ


- Such properties can be formalised in terms FZ algebra (Smirnov)
- Introduce rapidities θ_i, i=1, ... , n
f^O(θ_1, .. ,θ_n) = F_L(|θ_ij|) θ_1 > ... > θ_n
- Permutation


to formulate ff eqn
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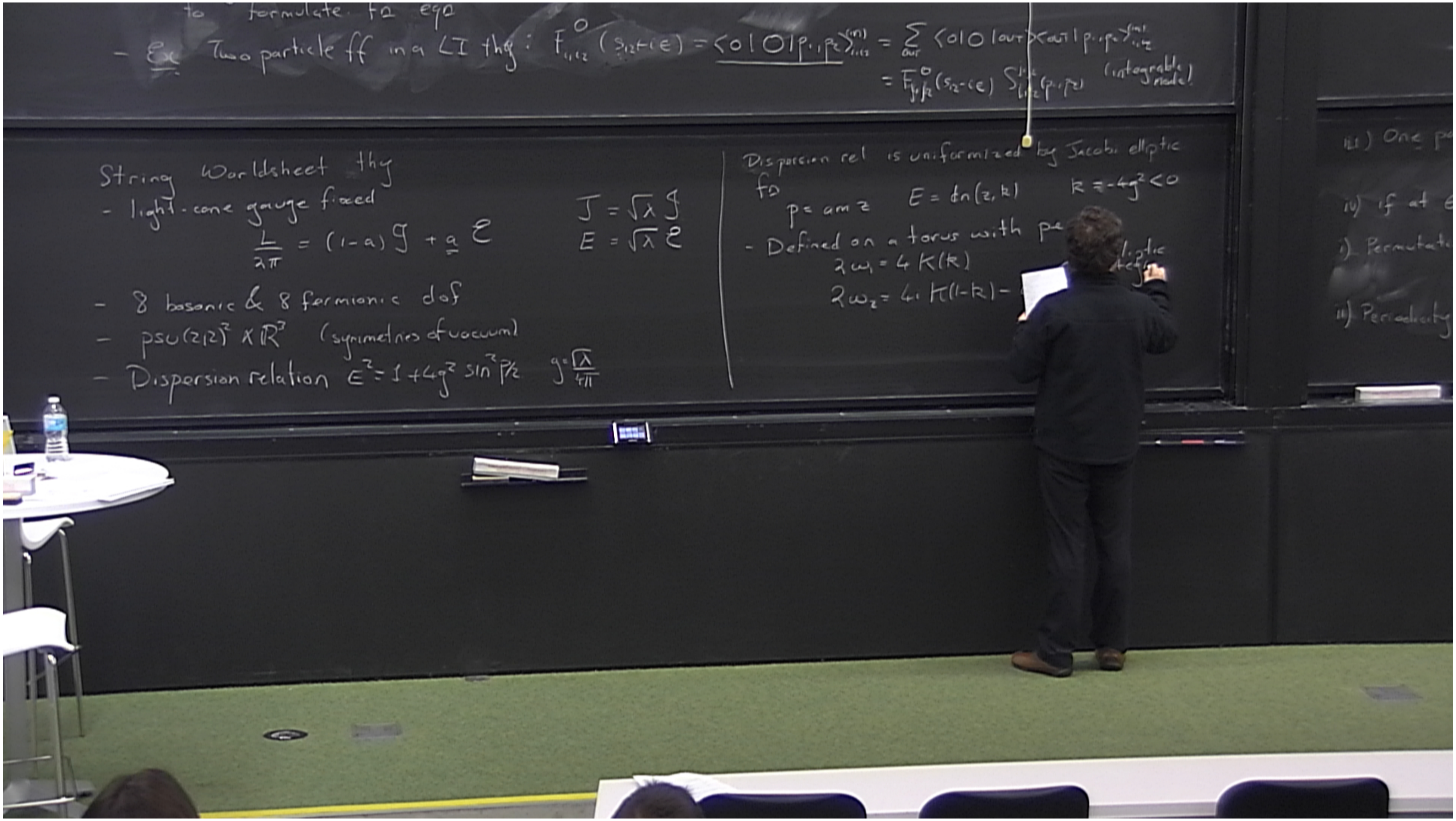
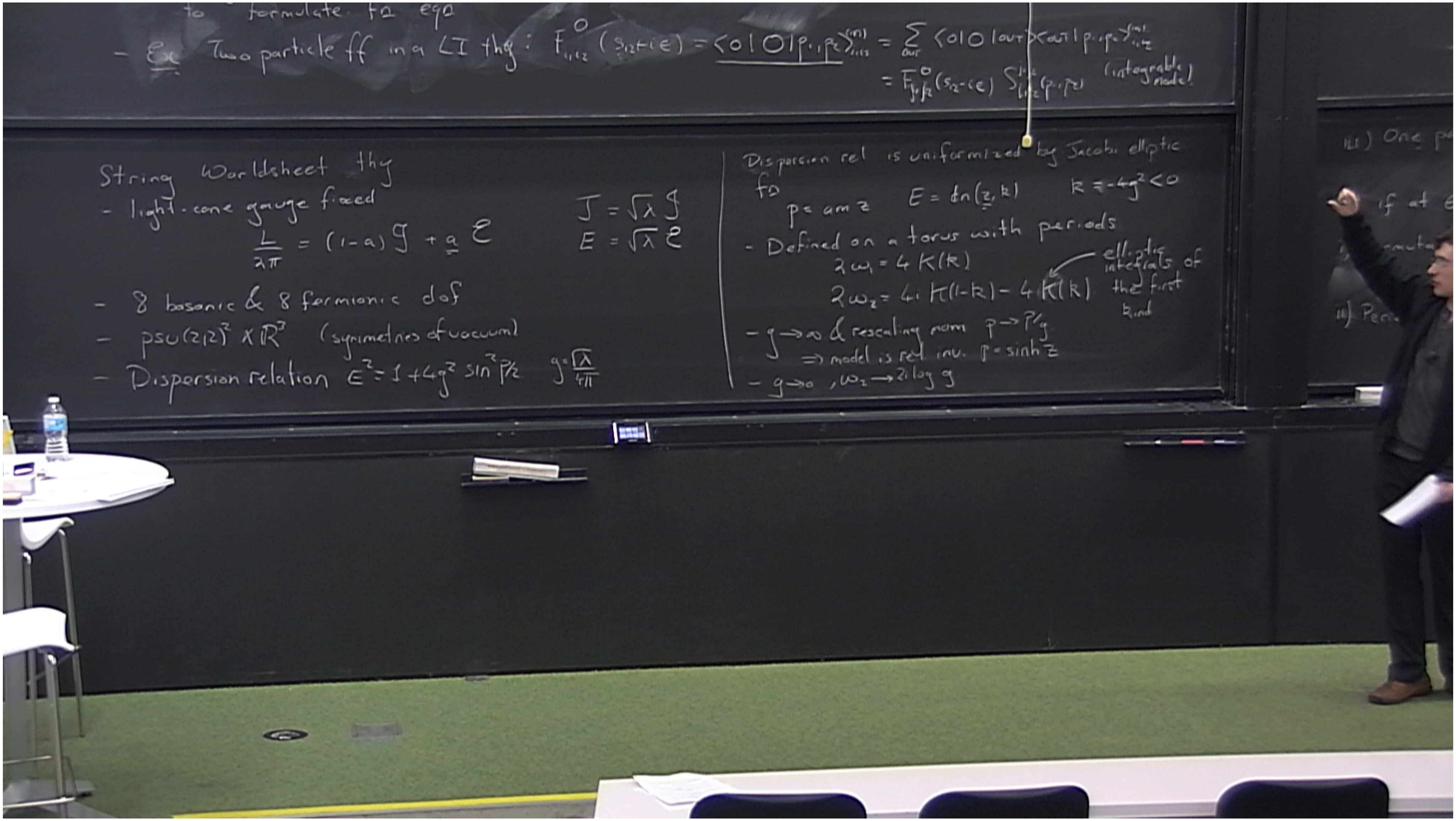
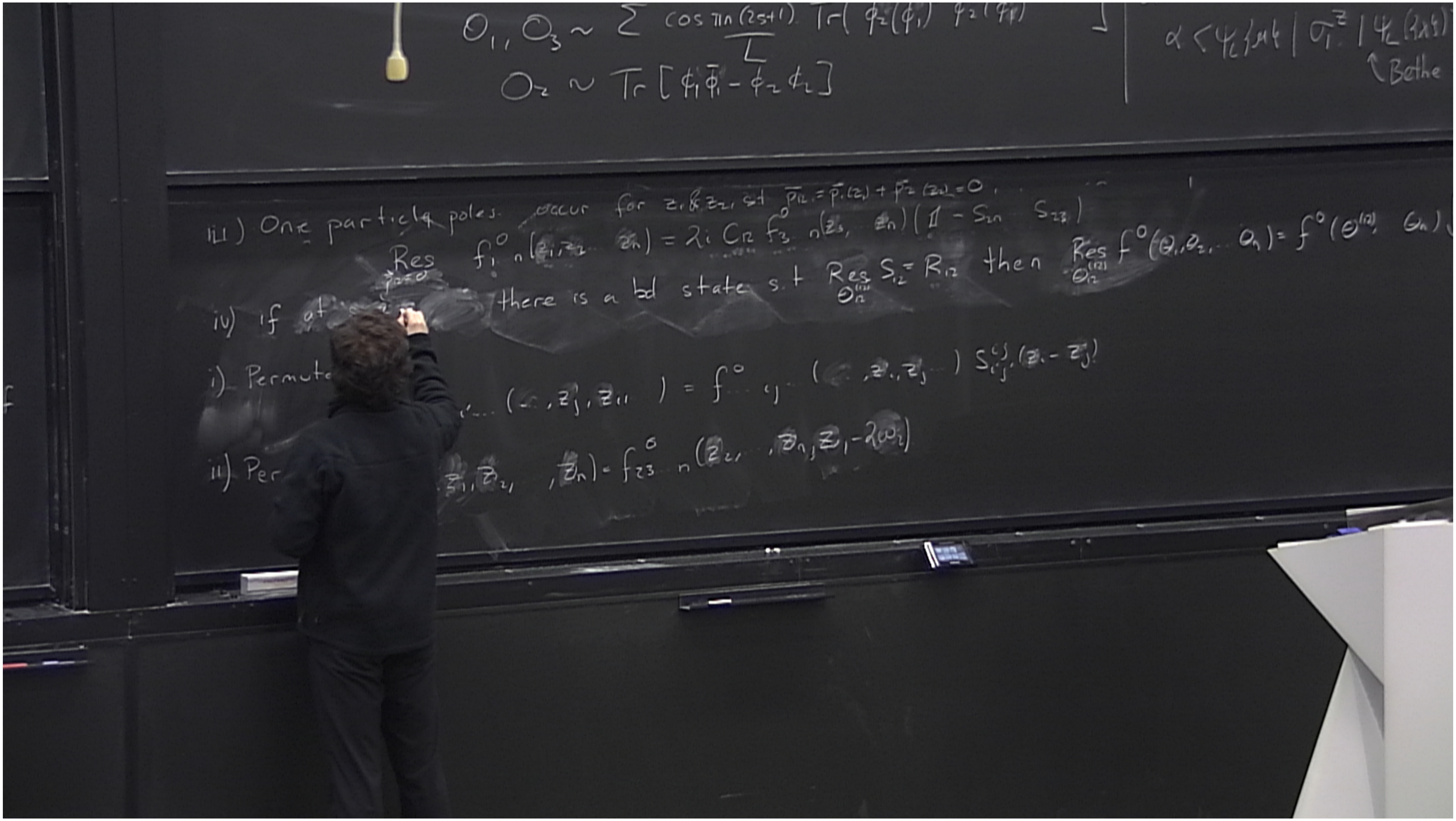
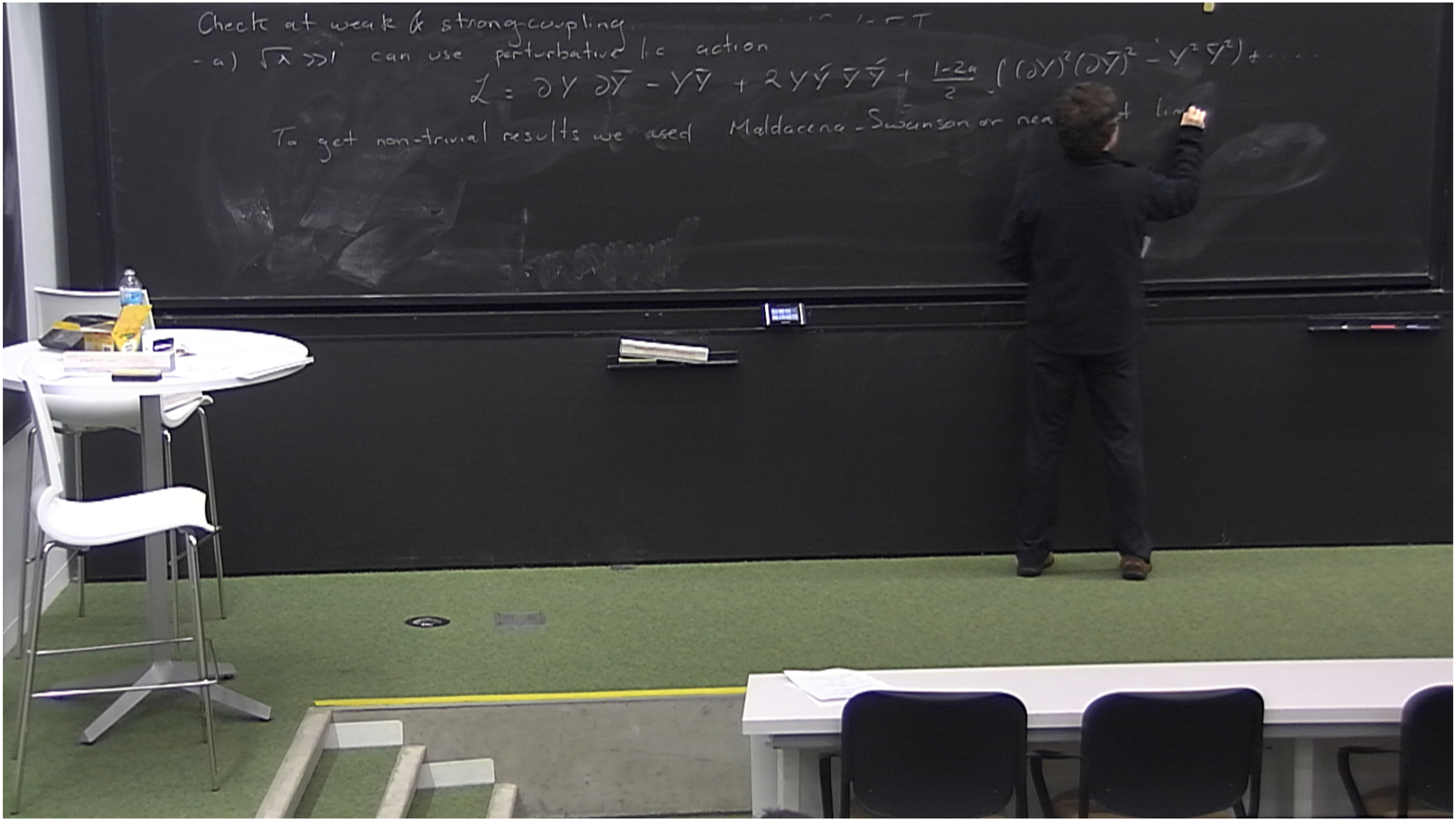
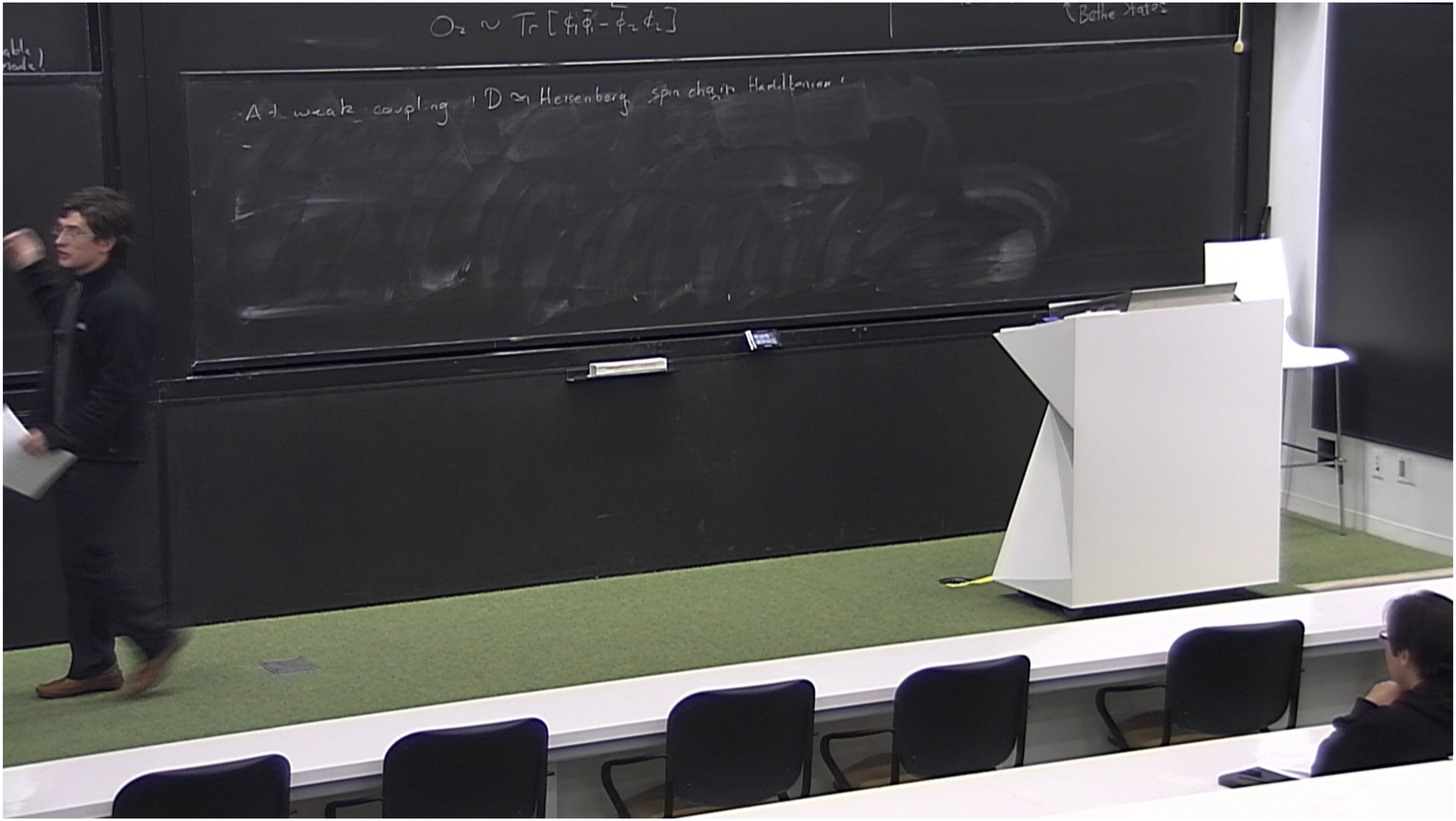
$= F^{O}_{j_1 j_2}(s_{12}-i\epsilon)\, S^{j_1 j_2}_{i_1 i_2}(p_1,p_2)$ (integrable model)
String Worldsheet thy
- light-cone gauge fixed
$\frac{L}{2\pi} = (1-a)\mathcal{J} + a\,\mathcal{E}$
$J = \sqrt{\lambda}\,\mathcal{J}$
$E = \sqrt{\lambda}\,\mathcal{E}$
- 8 bosonic & 8 fermionic dof
- $psu(2|2)^2 \times \mathbb{R}^3$ (symmetries of vacuum)
- Dispersion relation $\epsilon^2 = 1 + 4g^2 \sin^2 p/2$ $g = \frac{\sqrt{\lambda}}{4\pi}$
Dispersion rel is uniformized by Jacobi elliptic fn
p


to formulate FΩ eqn
- Ex Two particle ff in a LI thy
(integrable model)
String Worldsheet thy
- light-cone gauge fixed
L/2π = (1-a) J + a E
J = √λ J
E = √λ E
- 8 bosonic & 8 fermionic dof
- psu(2|2)² x R³ (symmetries of vacuum)
- Dispersion relation E² = 1 + 4g² sin² p/2
g = √λ/4π
Dispersion rel is uniformized by Jacobi elliptic fn
p = am z
E = dn(z, k)
k = -4g² < 0
- Defined on a torus with
2ω₁ = 4 K(k)

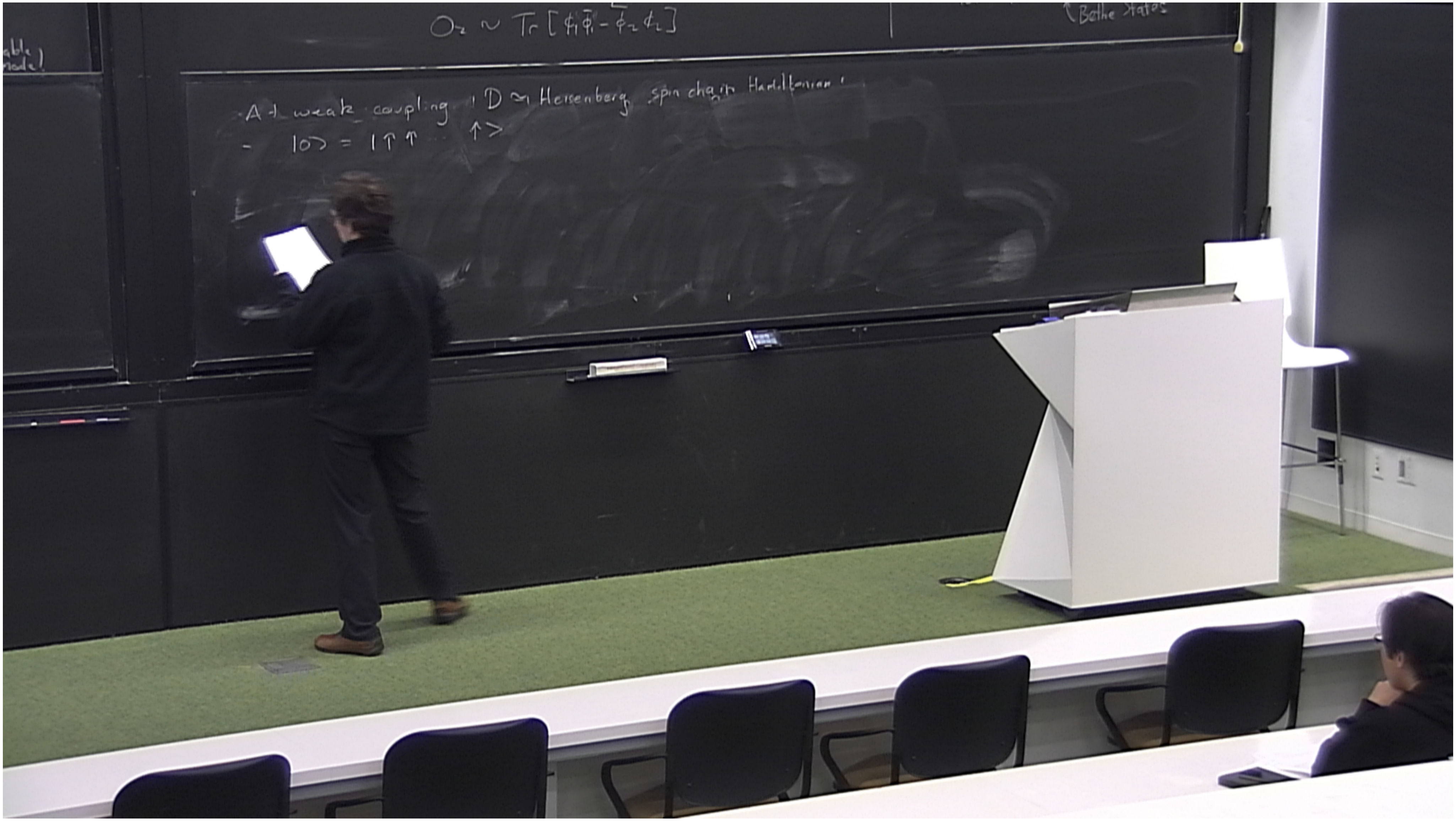

to formulate fa eqn
- Ex Two particle ff in a LI thy
(integrable model)
String Worldsheet thy
- light-cone gauge fixed
- 8 bosonic & 8 fermionic dof
(symmetries of vacuum)
- Dispersion relation
Dispersion rel is uniformized by Jacobi elliptic fn
- Defined on a torus with periods
elliptic integrals of the first kind
& rescaling mom
=> model is rel inv.


O1, O3 ~ Σ cos πn(2s+1)/L Tr(
O2 ~ Tr[
Bethe
iii) One particle poles occur for
Res
there is a bd state s.t
then
iv) if at
i) Permute
ii) Per


Check at weak & strong coupling
- a) √λ >> 1 can use perturbative l.c. action
L = ∂Y ∂Ȳ − YȲ + 2YY' ȲȲ' + (1−2a)/2 ((∂Y)²(∂Ȳ)² − Y²Ȳ²) + ...
To get non-trivial results we used Maldacena - Swanson or nea



Bethe states
At weak coupling : D ≃ Heisenberg spin chain Hamiltonian !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Bethe states
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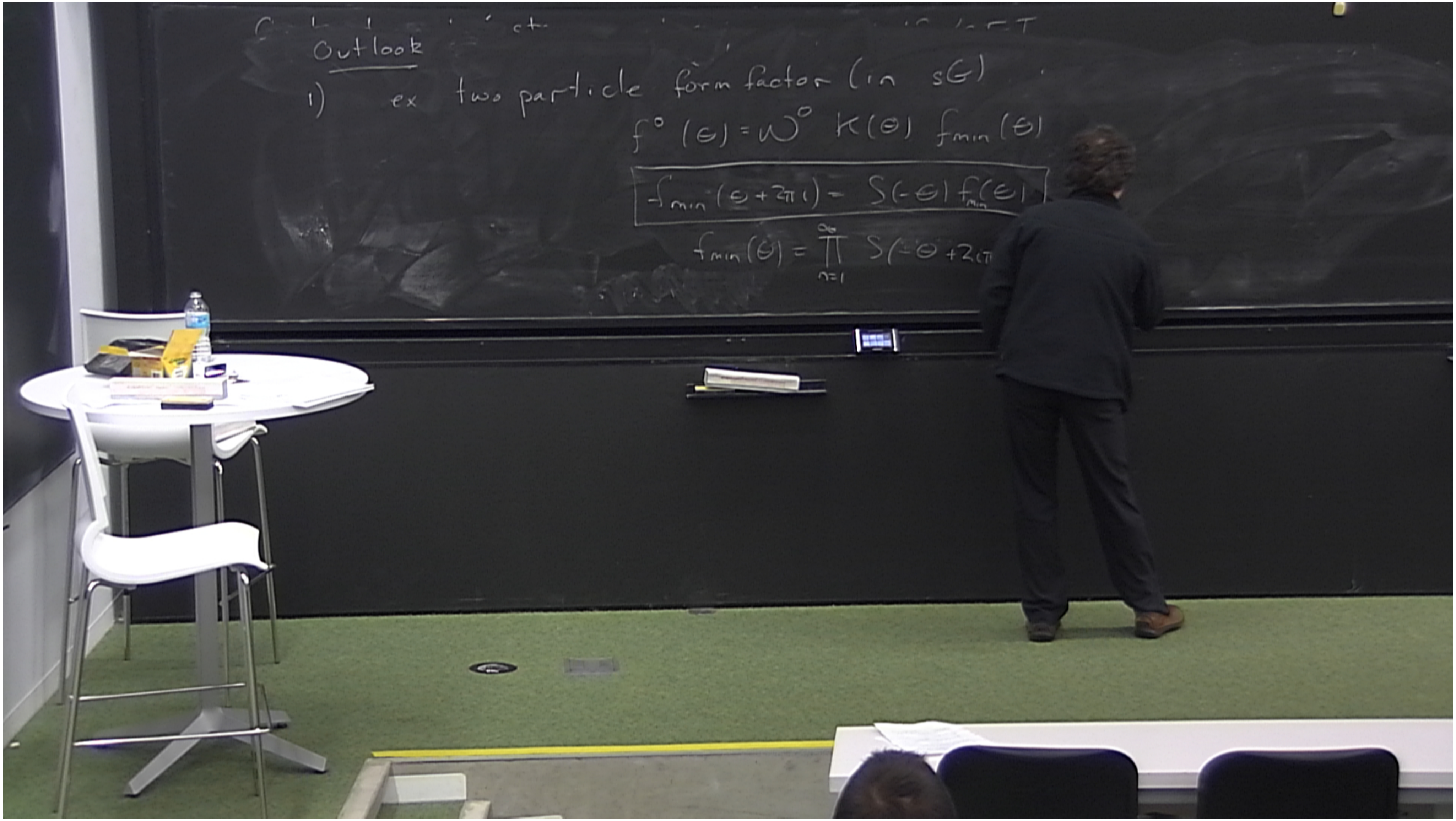
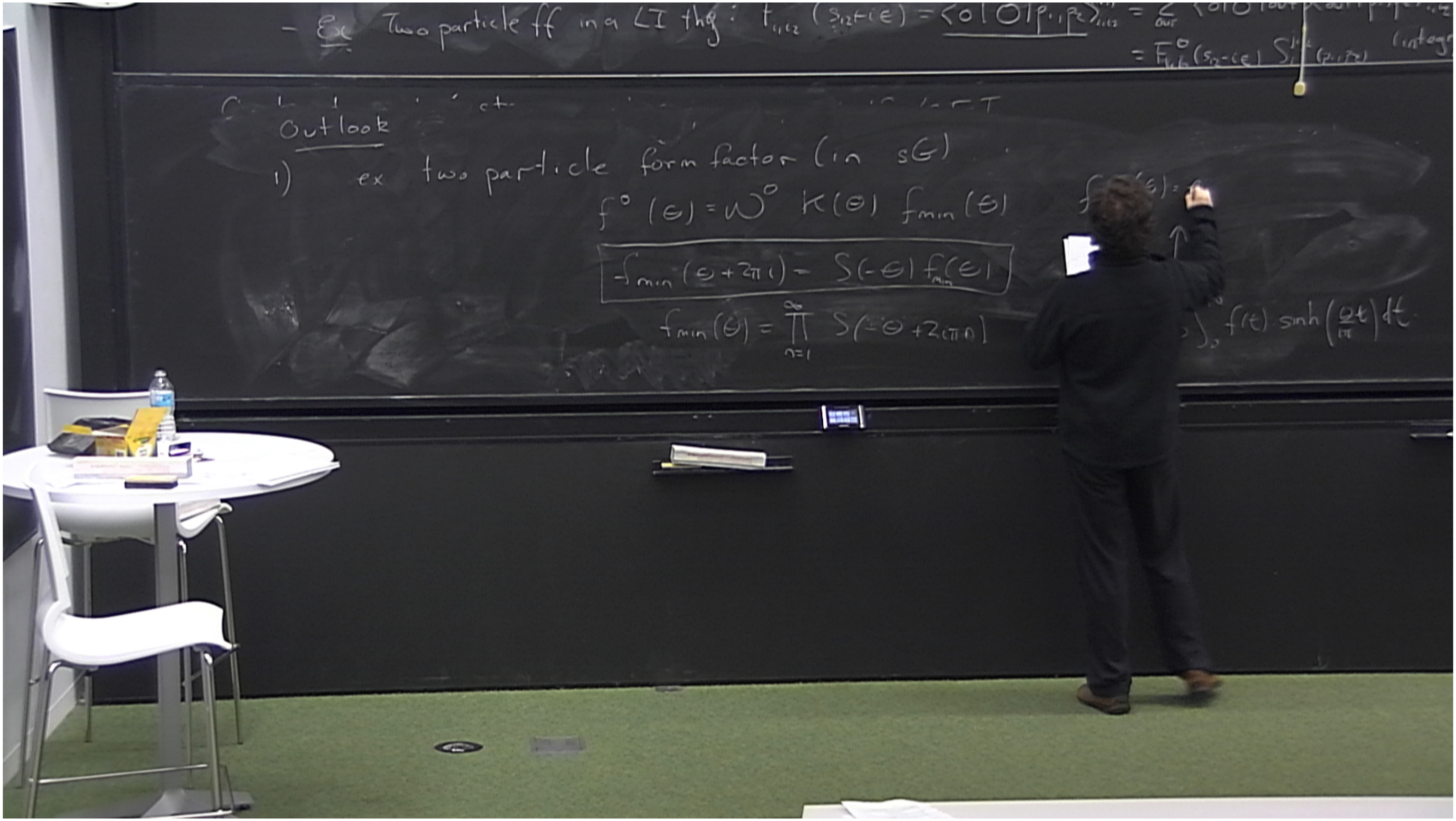
|0> = |↑↑ … ↑>


Outlook
1) ex two particle form factor (in sG)
f°(θ) = N° K(θ) f_min(θ)
f_min(θ + 2πi) = S(-θ) f_min(θ)
f_min(θ) = ∏_{n=1}^{∞} S(-θ + 2iπ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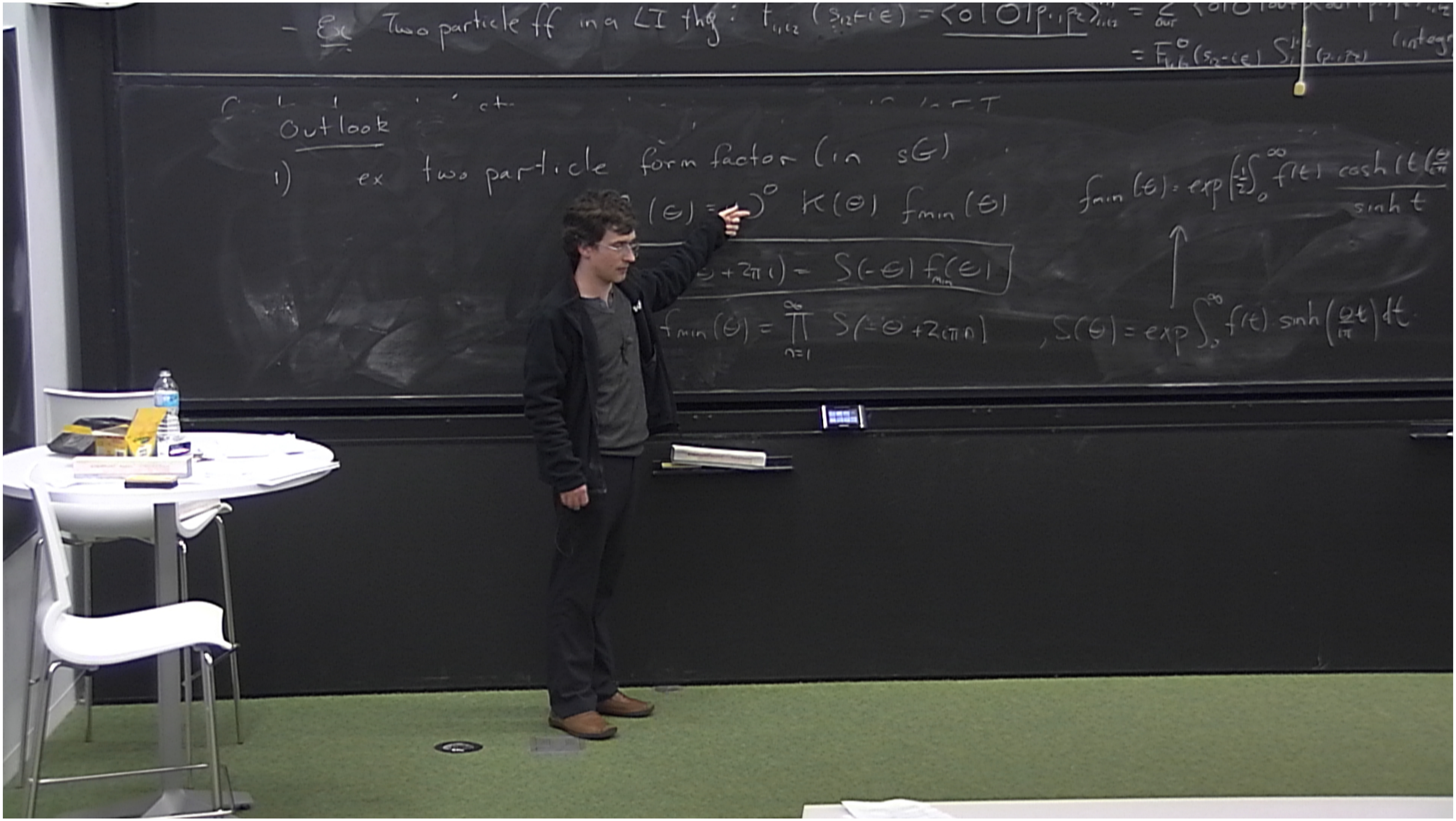
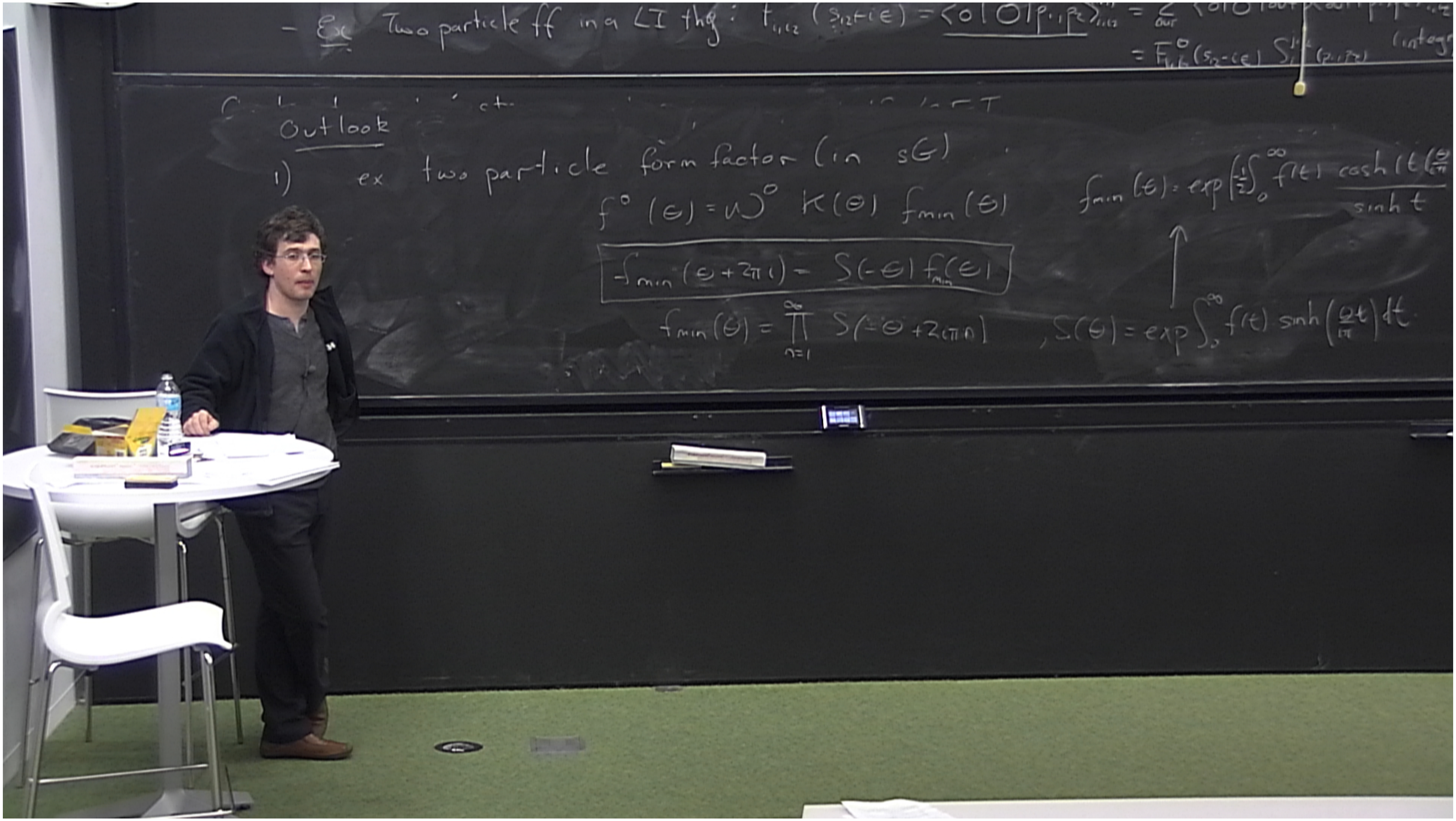
f_min(θ) = ∏ S(-θ+2πin)
n=1


Outlook
1) ex two particle form factor (in sG)
$f_{min}(\theta) = \exp\left(\frac{1}{2}\int_0^\infty f(t)\,\frac{\cosh(t(\ldots))}{\sinh t}\right)$
$f_{min}(\theta) = \prod_{n=1}^{\infty} S(-\theta + 2i\pi n)$
$S(\theta) = \exp\int_0^\infty f(t)\,\sinh\left(\frac{\theta t}{i\pi}\right)dt$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