Title: How to Fall into a Black Hole

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Abstract: In this talk I investigate the
"firewall argument", that claims that black hole horizons can
not be smooth.

Using
a holographic model of the black hole horizon as a quantum mechanical
membrane, I show how

to
recover the black hole interior as an emergent
smooth region of space-time. The reconstruction makes

use
of the formalism of quantum error correcting codes. I explain
why the horizon of very old black holes

appears
to be singular, and formulate a complementarity principle that resolves
this firewall paradox.

How to Fall into a Black Hole

Herman Verlinde

Princeton University

PI January 28, 2013

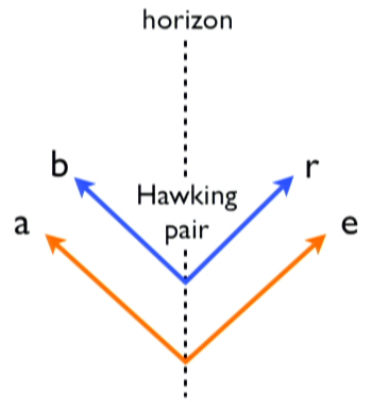


Figure 1: The firewall argument in a nut shell: quanta r and e can escape the black hole thanks to entanglement with Hawking partners b and a. Monogamy prevents the partners to get entangled with other interior quanta. So after the Page time, the black hole is in a maximally mixed state, and a and b can not escape via the normal Hawking process.

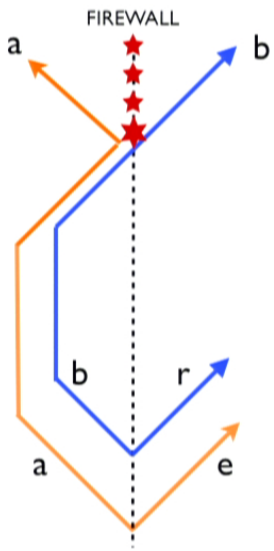
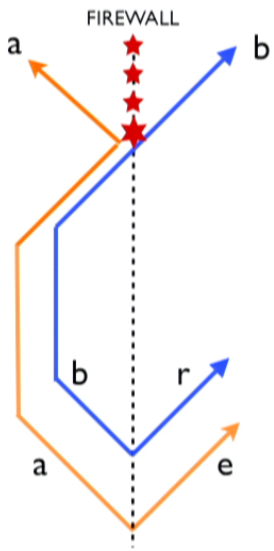


Figure 2: When the Hawking partner b leaves the black hole, it can not be entangled with any mode a right behind the horizon: the horizon turns into a firewall. To avoid this conclusion, one needs a mechanism for allowing entanglement to re-establish inside the black hole.



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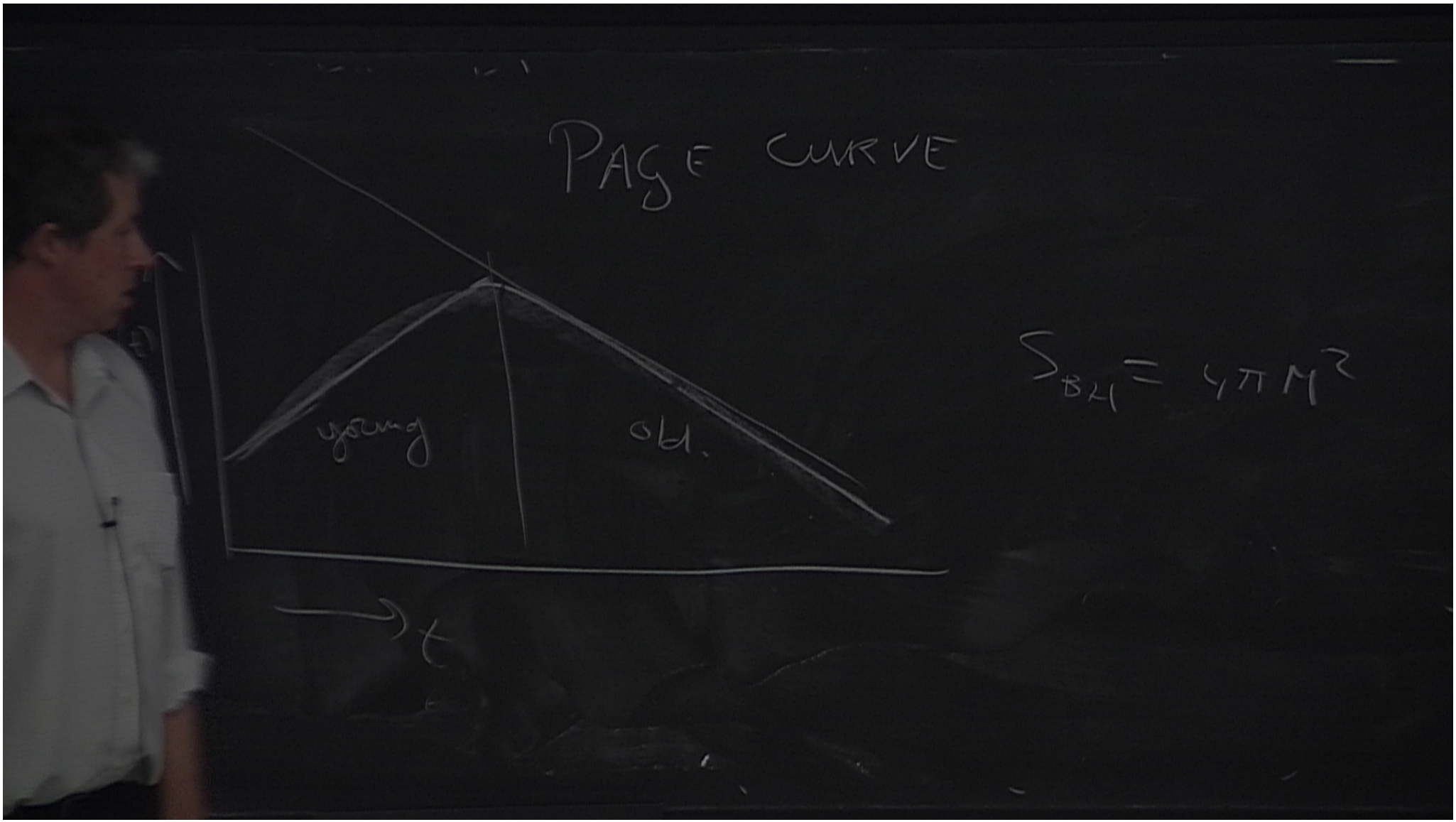
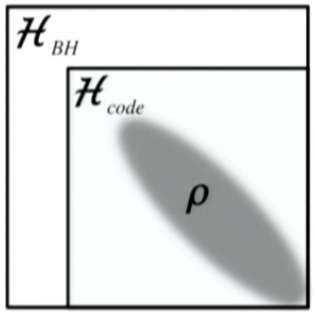

PAGE CURVE
young
old.
$\rightarrow t$
$S_{BH} = 4\pi M^2$

$$|\, i \,\rangle \in \mathcal{H}_M \qquad N = \dim \mathcal{H}_M = e^{S_{BH}}, \qquad S_{BH} = 4\pi M^2.$$

The black hole is in a mixed state:



$$|\Psi\rangle = \sum_{\bar{i} \in \mathcal{H}_{\rm code}} a_{\bar{i}} \,|\, \bar{i} \,\rangle |\Phi_{\bar{i}}\rangle \qquad \leftrightarrow \qquad \rho_{BH} = \sum_{\bar{i} \in \mathcal{H}_{\rm code}} p_{\bar{i}} |\, \bar{i} \,\rangle\langle\, \bar{i} \,|$$

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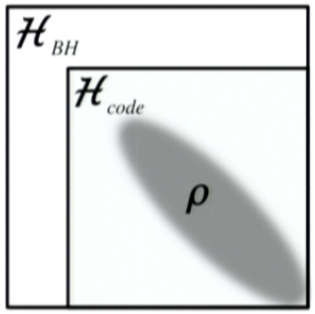

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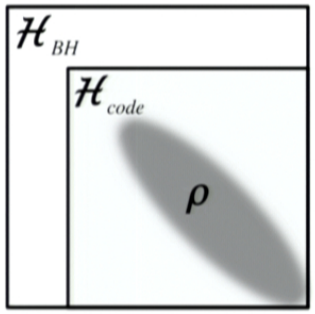

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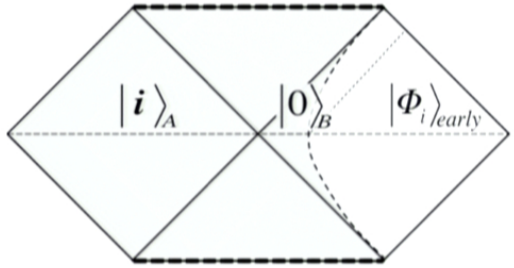


Hilbert space splits up into three sectors:

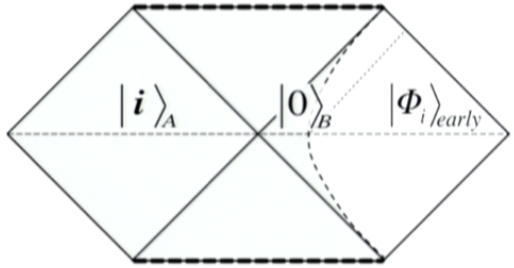
$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_R^{\text{early}}$$

with

$$\mathcal{H}_A = \mathcal{H}_{BH}, \qquad \mathcal{H}_B = \mathcal{H}_R^{\text{late}}.$$

$$\boxed{H = H_{BH} + H_R + H_{int},}$$

Hamiltonian evolution in Interaction Picture



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$$\rho_{AB}(0) = \sum_{\bar{i}} p_{\bar{i}} \, |\, \bar{i} \,\rangle\langle\, \bar{i} \,| \otimes |0\rangle_b\langle 0|,$$

evolves into

$$\rho_{AB}(\tau) = \boldsymbol{U}\, \rho(0)\, \boldsymbol{U}^{\dagger} = \sum_{\boldsymbol{n},\boldsymbol{m}} \rho_{\mathbf{nm}} \, |\boldsymbol{n}\rangle_b\langle \boldsymbol{m}|$$

$$\rho_B = \sum_{\boldsymbol{n}} w_{\boldsymbol{n}} \,|\, \boldsymbol{n} \,\rangle_b\langle\, \boldsymbol{n} \,| \qquad ; \qquad w_{\boldsymbol{n}} = \frac{e^{-\beta E_n}}{Z} \qquad S_B = \log Z + \beta \bar{E}$$

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Time evolution in terms of Kraus Operators C

$$\boldsymbol{U}(\tau)\,|\,i\,\rangle|\,0\,\rangle_b = \sum_{\boldsymbol{n}} \boldsymbol{C_n}\,|\,i\,\rangle\,|\boldsymbol{n}\,\rangle.$$

cf. loss of coherence
of a quantum computer

$$\sum_{\boldsymbol{n}} \boldsymbol{C_n^\dagger C_n} = \mathbb{1}_M.$$

$$\overline{\boldsymbol{C}_n\,\boldsymbol{C}_m^\dagger} = \frac{1}{Z}\,\delta_{nm}\,\mathbb{1}_{M-E_n}$$

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C's are ergodic `random' matrices
with known statistical properties!

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We can compute the Page curve of our model!

$$\rho(E)^2 = \left(\frac{w_E}{N_c} + 2\frac{d_B(E)}{NZ}\right)\rho(E) - \left(\frac{d_B(E)}{NZ}\right)^2 \mathbb{1}_{M-E}$$

$$S(E) = w_E d_B(E)\left[\log\left(\frac{NZ}{d_B(E)}\right) - \sqrt{\frac{y}{2+y}}\log\left(1+y+\sqrt{2y+y^2}\right)\right]$$

$$y = \frac{N}{N_c}\frac{e^{-\beta E}}{2d_B(E)},$$

Two limiting cases:

young BH : $S_{AB} = \log N_c$ $\quad S_A = \log N_c + \log Z + \beta\bar{E}$

old BH : $S_{AB} = \log N,$ $\quad S_A = \log N - \beta\bar{E}$

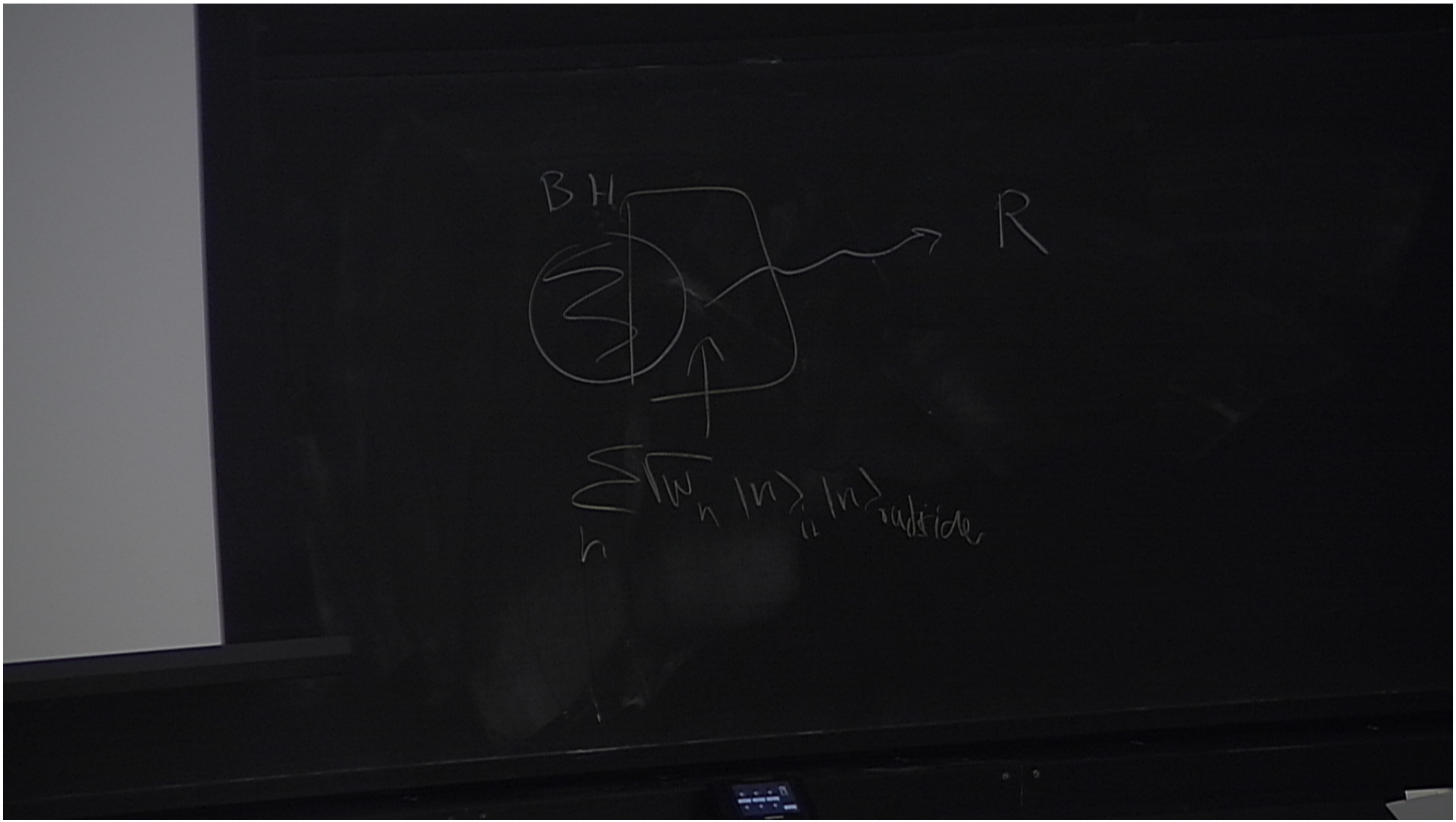
Key Tool: Recovery Super Operator R

$$R_{\boldsymbol{n}} = \frac{1}{\sqrt{w_{\boldsymbol{n}}}} \sum_{\bar{i}} |\,\bar{i}\,\rangle\langle\,\bar{i}\,|\,\boldsymbol{C}_{\boldsymbol{n}}^{\dagger}, \qquad \boldsymbol{R}\,|\,j\,\rangle|0\rangle_a = \sum_{\boldsymbol{n}} R_{\boldsymbol{n}}\,|\,j\,\rangle|\,\boldsymbol{n}\,\rangle_a$$

$$\boldsymbol{R}\,\boldsymbol{U}\,|\,i\,\rangle|0\rangle_a|0\rangle_b = \begin{cases} |\,i\,\rangle\,|\,0_U\rangle & \text{if } |\,i\,\rangle \in \mathcal{H}_{\text{code}} \\ 0 & \text{if } |\,i\,\rangle \notin \mathcal{H}_{\text{code}} \end{cases}$$

$$|\,0_U\rangle = \sum_{\boldsymbol{n}} \sqrt{w_{\boldsymbol{n}}}\,|\,\boldsymbol{n}\rangle_a|\,\boldsymbol{n}\,\rangle_b.$$

Produces the Unruh Vacuum!


BH
R
$\sum_n \sqrt{w_n}\, |n\rangle_{H} |n\rangle_{outside}$

Reconstruction of Interior Operators:

$$\Pi = {}_a\langle 0 | \boldsymbol{R}^\dagger \boldsymbol{R} | 0 \rangle_a, \qquad \Pi^2 = \Pi.$$

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$$\boldsymbol{A} = {}_a\langle 0 | \boldsymbol{R}^\dagger A \boldsymbol{R} | 0 \rangle_a \qquad \boldsymbol{\phi(y)} = {}_a\langle 0 | \boldsymbol{R}^\dagger \phi(y) \boldsymbol{R} | 0 \rangle_a.$$

A acts on black hole Hilbert space
satisfies QFT operator algebra

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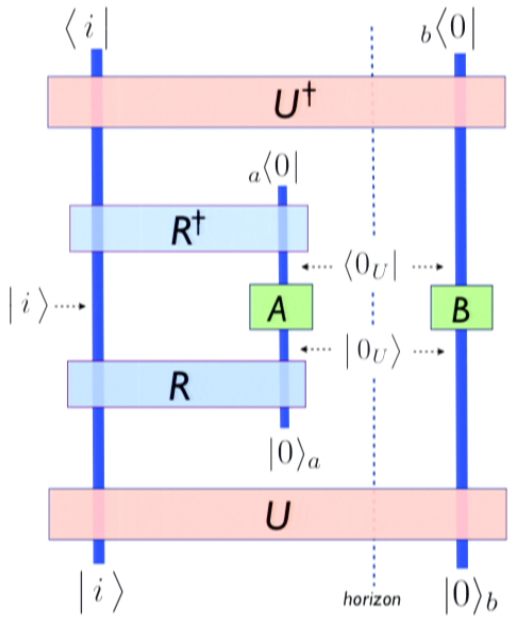


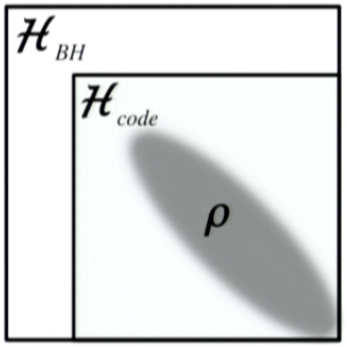
$$\mathrm{tr}\big(\rho\, \boldsymbol{\phi_a}(y)\phi_b(x)\big) \;=\; \big\langle 0_U\big|\phi_a(y)\phi_b(x)\big|\,0_U\big\rangle,$$

Error of the Recovery Operation:

$$\Pi\, \boldsymbol{U} |\, \bar{i}\, \rangle |0\rangle_b = \boldsymbol{U} |\, \bar{i}\, \rangle |0\rangle_b \; + \; \text{error}.$$

$$\text{error} \; = \; \boldsymbol{E}\boldsymbol{U} |\, \bar{i}\, \rangle |0\rangle_b, \quad \text{with} \qquad \boldsymbol{E} = \sum_{\boldsymbol{n}} \frac{N_{\text{code}}}{N_{\boldsymbol{n}}} \, \mathbb{1}_{M-E_{\boldsymbol{n}}}.$$

Reconstruction breaks down for maximally mixed states!



Firewall Paradox

AMPS

Solving the problem:

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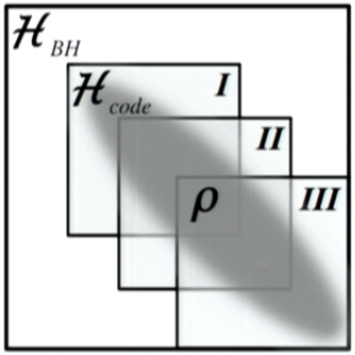
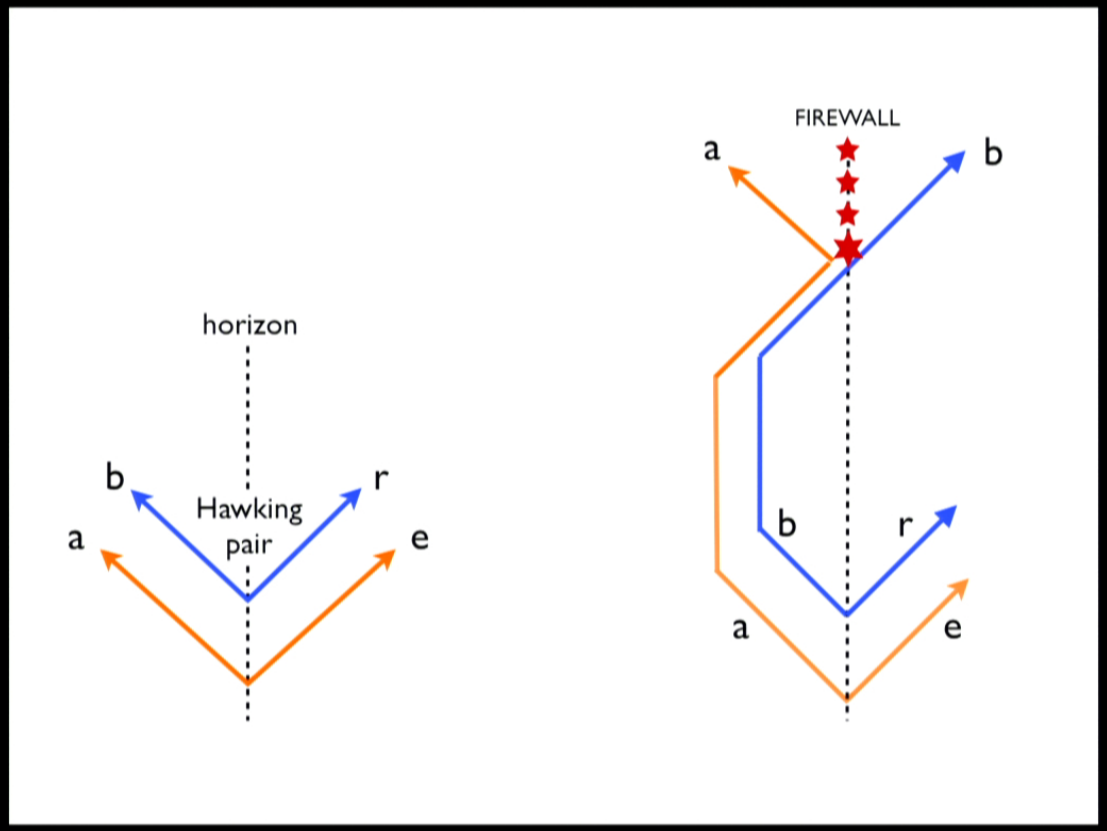
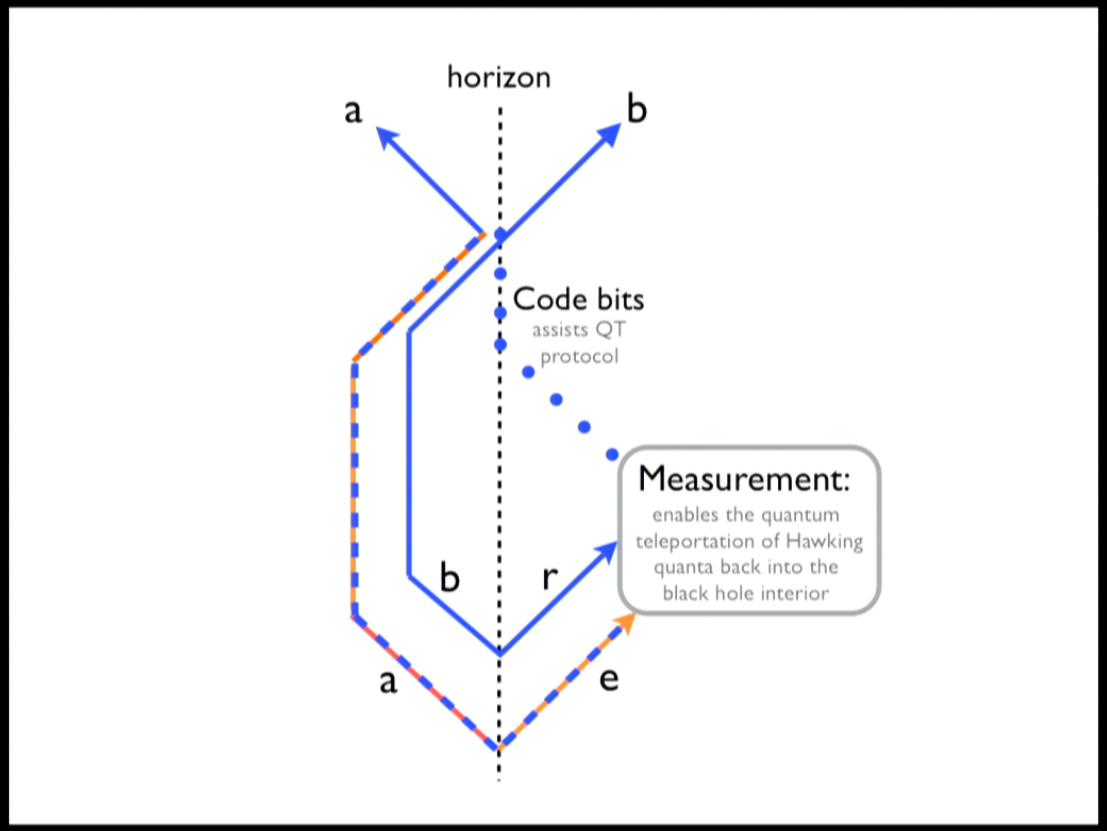


Does this preserve linearity and locality?

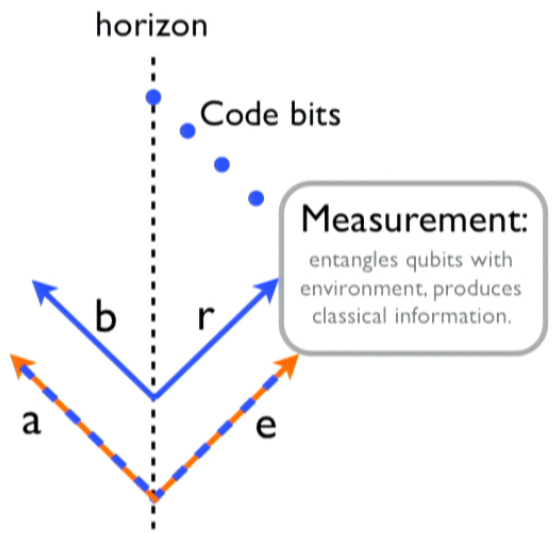

horizon
b
r
Hawking
pair
a
e
FIREWALL
a
b
b
r
a
e

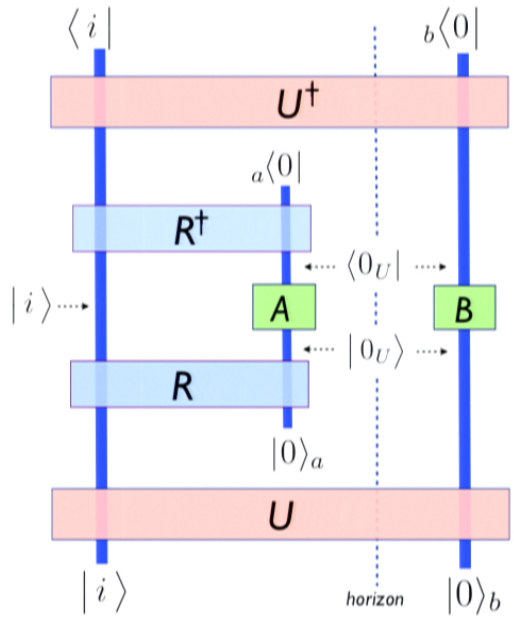

horizon
a
b
Code bits
assists QT
protocol
Measurement:
enables the quantum
teleportation of Hawking
quanta back into the
black hole interior
b
r
a
e

Space-Time Complementarity:

Space-time complementarity: *A variable cut-off scale $\epsilon(x)$ on a Cauchy surface Σ provides a permissible semi-classical description of the second quantized Hilbert space, only when the quantum fluctuations of the local background geometry induced by the corresponding stress-energy fluctuations do not exceed the cut-off scale itself. All critical cut-off scales that saturate this requirement provide complete, complementary descriptions of the Hilbert space.*

Kiem, EV & HV, 1995


horizon
Code bits
Measurement:
entangles qubits with environment, produces classical information.
b
r
a
e



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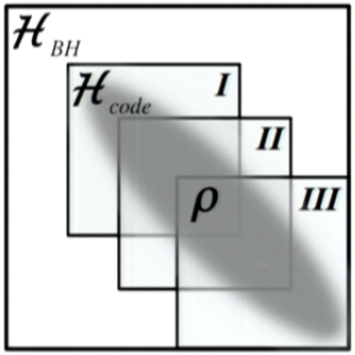

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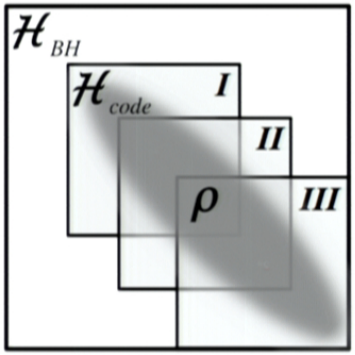

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