Title: Parameterizing dark sector perturbations

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Abstract: When recent observational evidence and the GR+FRW+CDM model are combined we obtain the result that the Universe is accelerating, where the acceleration is due to some not-yet-understood "dark sector". There has been a considerable number of theoretical models constructed in an attempt to provide an "understanding" of the dark

sector: dark energy and modified gravity theories. The proliferation of modified gravity and dark energy models has brought to light the need to construct a "generic" way to parameterize the dark sector.

We will discuss our new way of approaching this problem, looking at linearised perturbations. Our approach is inspired by that taken in particle physics, where the most general modifications to the standard model are written down for a given field content that is compatible with some assumed symmetry (which we take to be isotropy of the background spatial sections). Our emphasis is on constructing a theoretically motivated toolkit which can be used to extract meaningful information about the dark sector (such as its field content). We find, for example, that the observational impact of very broad classes of theories can be encoded by a very small (less than 5) number of parameters. It is these parameters which we hope to measure with observational data.

Parameterizing dark sector perturbations

Jonathan A. Pearson
with Richard Battye (Manchester)

**Centre for Particle Theory,
Durham University**

1203.0398
1205.3611
1301.xxxx

jonathan.pearson@durham.ac.uk @jpoffline www.jpoffline.com

Outline

- Introduction - cosmology & the dark side
- Current approaches to parameterizing ignorance
- Formalism - our approach
- Connection to massive gravity
- Equations of state for dark sector perturbations
- Outlook

Introduction

Einstein's field equations with FRW metric yield

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \qquad \left(\frac{\dot{a}}{a}\right)^2 \equiv \mathcal{H}^2 = \frac{8\pi G}{3}\rho \qquad \dot{\rho} = -3\mathcal{H}(\rho + P)$$

Also do for fluctuations about FRW (now study structures)

$$g_{\mu\nu} = a^2(\tau)\left[\eta_{\mu\nu} + h_{\mu\nu}(\tau, \mathbf{x})\right]$$

Provides a set of predictions for

Distances to (e.g.) supernovae
Cosmic **M**icrowave **B**ackground fluctuation spectrum
Structure... how many galaxies of each size should there be?

(all quantities which we can & have observed)

Introduction

General Relativity

FRW metric

Standard matter content

Introduction

General Relativity

FRW metric

Standard matter content

+

data

=

Inconsistent!

(by

Introduction

Re-examine ingredients that went into the cosmological cookbook

Gravity... ?

General Relativity

Geometry... ?

Homogeneous & isotropic

Content... ?

Baryons, photons, neutrinos,...

Maybe GR is not *the* gravitational theory on large scales?
Applying to cosmology is an extrapolation of the validity of GR as the gravitational theory.
Modified gravity

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Inhomogeneous universes

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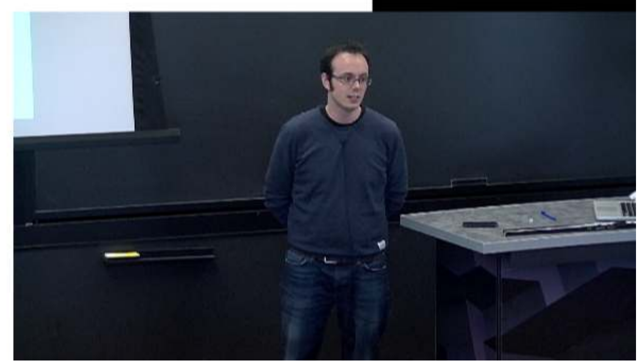

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Dark matter & dark energy

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Dark matter & dark energy

↙ structure formation & galaxy rotations...

↘ cosmic acceleration

Introduction

Assume FRW is correct

Assume GR is correct

Require 24% *dark matter*
73% *dark energy*

Invented!

$P = w\rho \quad w_{\rm de} < -1/3$

What's the dark energy?

Perhaps "observed" acceleration is actually manifestation of non-GR gravity on large scales...

... lesson from Newton & the Sun...

What's the gravitational theory?

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Accelerating universe

Dark energy, inhomogeneous universe or modified gravity?

Same matter content causes
gravity to respond differently

Dark sector

Something causing apparent acceleration

Model zoo

Λ, quintessence, k-essence, galileon, F(R), elastic dark energy, massive gravity, Einstein-aether, ... **Lagrangian engineering**

(See 1106.2476 for a great review)

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The model zoo

$\mathcal{L} = R - 2\Lambda$ *cosmological constant*

$\mathcal{L} = R + F(R)$

$\mathcal{L} = R + F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$

$\mathcal{L} = R + \partial_\mu\phi\partial^\mu\phi + 2V(\phi)$ *quintessence*

$\mathcal{L} = R + F(R) + \partial_\mu\phi\partial^\mu\phi + 2V(\phi)$

$\mathcal{L} = R - 2f(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi, \phi)$ K-*essence*

$\mathcal{L} = R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \lambda(A^\mu A_\mu - 1)$ *Einstein aether*

& many more besides

... Lagrangian engineering

The problem

Theories

Λ
quintessence
k-essence
TeVeS
F(R)
elastic dark energy
galileon
...

→

Observations

CMB
Lensing
Structure
BAO
...

CFHTLenS parameterization:

$$\mathrm{d}s^2 = (1+2\Psi)\mathrm{d}t^2 - a^2(t)(1-2\Phi)\mathrm{d}x^2$$

$$\Phi(k,a) = \left[1+\mu(k,a)\right]\Psi_{\mathrm{GR}}(k,a)$$

$$\left[\Phi(k,a)+\Psi(k,a)\right] = \left[1+\Sigma(k,a)\right]\left[\Psi_{\mathrm{GR}}(k,a)+\Phi_{\mathrm{GR}}(k,a)\right]$$

Pick scale invariant & parameterize out time dependance

$$\Sigma(a) = \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda} \qquad \mu(a) = \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$

CFHTLenS parameterization:

$$ds^2 = (1+2\Psi)dt^2 - a^2(t)(1-2\Phi)dx^2$$

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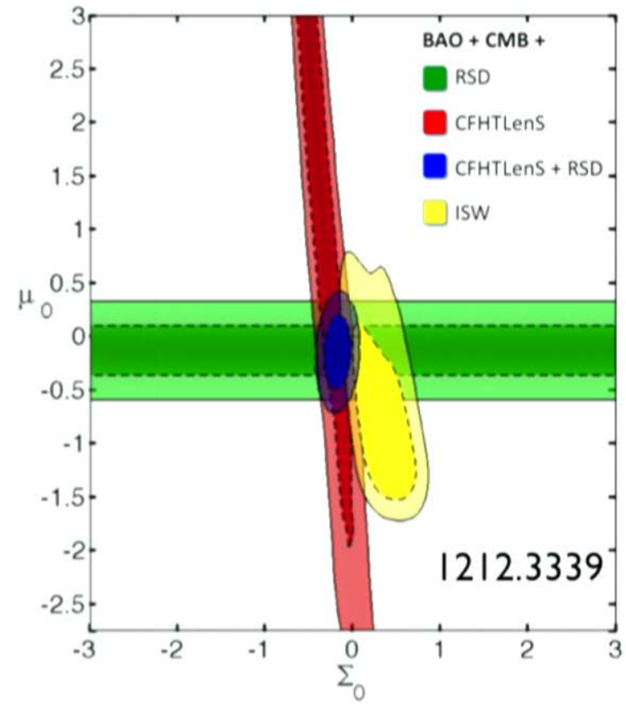

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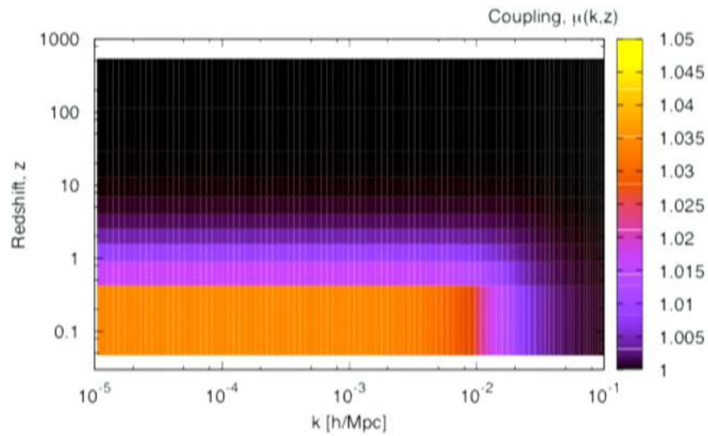
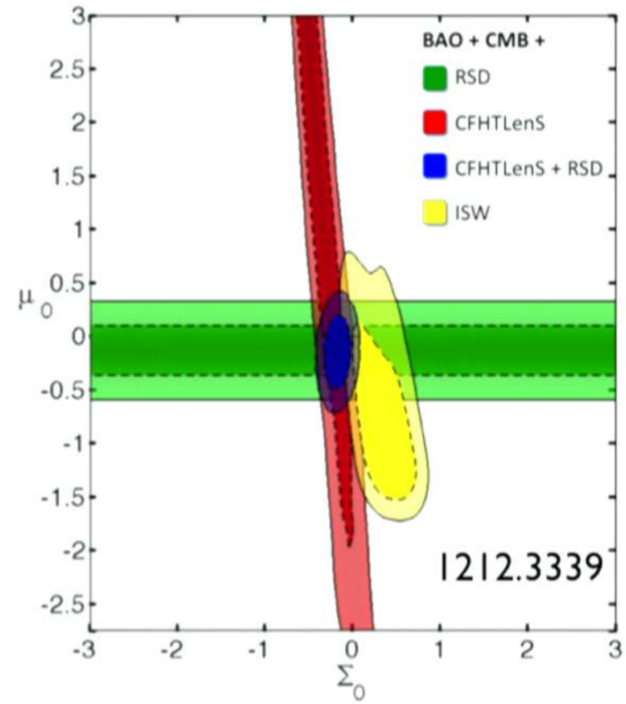


Scalar field model: only 5% deviation



Lagrangian for perturbations

Nothing extra: $\mathcal{L} = \mathcal{L}(g_{\mu\nu})$

$$\mathcal{L}_{\{2\}} = \frac{1}{8}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta}$$

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}$$

$$\delta U^{\mu\nu} = -\frac{1}{2}\left(\mathcal{W}^{\mu\nu\alpha\beta} + U^{\mu\nu}g^{\alpha\beta}\right)\delta g_{\alpha\beta}$$

has maximum of 5 components in isotropic space

Elastic dark energy, massive gravity, ...

$$\begin{aligned} \mathcal{W}^{\mu\nu\alpha\beta}_{\rm EDE} &= -\rho u^\mu u^\nu u^\alpha u^\beta + P\left(u^\mu u^\nu \gamma^{\alpha\beta} + u^\alpha u^\beta \gamma^{\mu\nu}\right) \\ &\quad -2P\left(\gamma^{\alpha(\mu}u^{\nu)}u^\beta + \gamma^{\beta(\mu}u^{\nu)}u^\alpha\right) \\ &\quad +\left(\beta - P - \tfrac{2}{3}\mu\right)\gamma^{\mu\nu}\gamma^{\alpha\beta} + 2(\mu + P)\gamma^{\mu(\alpha}\gamma^{\beta)\nu} \end{aligned}$$

$$\mathcal{W}^{\mu\nu\alpha\beta}_{\rm PF} = m^2\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu(\alpha}g^{\beta)\nu}\right)$$

Lagrangian for perturbations

Scalar fields (one derivative) $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu\phi)$

$$\mathcal{L}_{\{2\}} = \mathcal{A}(\delta\phi)^2 + \mathcal{B}^\mu \delta\phi\nabla_\mu\delta\phi + \frac{1}{2}\mathcal{C}^{\mu\nu}\nabla_\mu\delta\phi\nabla_\nu\delta\phi + \frac{1}{4}\left[\mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi\delta g_{\mu\nu} + \mathcal{V}^{\mu\nu}\delta\phi\delta g_{\mu\nu} + \frac{1}{2}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta}\right].$$

$$\delta G_{\mu\nu} = 8\pi G\delta T_{\mu\nu} + \delta U_{\mu\nu}$$

$$\delta U^{\mu\nu} = -\frac{1}{2}\left(\underset{2}{\mathcal{V}^{\mu\nu}}\delta\phi + \underset{3}{\mathcal{Y}^{\alpha\mu\nu}}\nabla_\alpha\delta\phi\right) - \frac{1}{2}\left(\underset{5}{\mathcal{W}^{\alpha\beta\mu\nu}} + g^{\alpha\beta}U^{\mu\nu}\right)\delta g_{\alpha\beta}.$$

(74 before isotropy!)

Quintessence, k-essence, Lorentz-violating theories,...

Lagrangian for perturbations

Formalism can be extended to include...

- Lorentz violating theories ✔
- Massive gravity ✔
- Multi-field dark energy ✔
- High-derivative scalar field theories
- Vector field theories
- Curvature tensor theories
- Bimetric
- Couplings between known & dark sector

& identify "generalized" boundary terms

Lagrangian for perturbations

Scalar fields (one derivative) $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu \phi)$

$$\begin{aligned}\mathcal{L}_{\{2\}} &= \mathcal{A}(\delta\phi)^2 + \mathcal{B}^\mu \delta\phi \nabla_\mu \delta\phi + \frac{1}{2}\mathcal{C}^{\mu\nu}\nabla_\mu\delta\phi\nabla_\nu\delta\phi \\ &+ \frac{1}{4}\left[\mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi\delta g_{\mu\nu} + \mathcal{V}^{\mu\nu}\delta\phi\delta g_{\mu\nu} + \frac{1}{2}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta}\right].\end{aligned}$$

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(74 before isotropy!)

Quintessence, k-essence, Lorentz-violating theories,...

Connection to massive gravity

$$S_{\{2\}} = \int \mathrm{d}^4x\sqrt{-g}\left[\Diamond^2 R + 16\pi G \Diamond^2 \mathcal{L}_\mathrm{m} - \tfrac{1}{4}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta}\right].$$

$$\delta g_{\mu\nu} = h_{\mu\nu} + 2\nabla_{(\mu}\xi_{\nu)}$$

$$\mathcal{W}^{\mu\nu\alpha\beta} = A_\mathcal{W} u^\mu u^\nu u^\alpha u^\beta + B_\mathcal{W}(u^\mu u^\nu \gamma^{\alpha\beta} + u^\alpha u^\beta \gamma^{\mu\nu}) + 4C_\mathcal{W} u^{(\alpha}\gamma^{\beta)(\mu}u^{\nu)} + D_\mathcal{W}\gamma^{\mu\nu}\gamma^{\alpha\beta} + 2E_\mathcal{W}\gamma^{\mu(\alpha}\gamma^{\beta)\nu}$$

(isotropic space)

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(isotropic space)

when ξ^0 decouples, components become bulk & shear moduli of elastic medium *5DoFs*

$$A_{\mathcal{W}} = -\rho, \quad B_{\mathcal{W}} = -C_{\mathcal{W}} = P, \quad \dot{P} = -\mathcal{H}(P + 3D_{\mathcal{W}} + 2E_{\mathcal{W}})$$

Becomes identical to components of elasticity tensor of relativistic elasticity theory (see Carter...)

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when ξ^i decouples, becomes non-canonical scalar field theory *3DoFs* $B_{\mathcal{W}} = -C_{\mathcal{W}} = -\rho, \quad D_{\mathcal{W}} = -E_{\mathcal{W}} = -P$

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when ξ^i decouples, becomes non-canonical scalar field theory *3DoFs* $B_{\mathcal{W}} = -C_{\mathcal{W}} = -\rho, \quad D_{\mathcal{W}} = -E_{\mathcal{W}} = -P$

LV massive gravity with time translation invariance is elastic dark energy

... *massive gravity is the manifestation of rigidity of spacetime*

masses determined by bulk & shear moduli of elastic medium

Perturbed dark energy momentum tensor

Everything boils down to $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}$

$$\delta U^{\mu\nu} = \hat{\mathbb{Y}}^{\mu\nu}\delta\phi + \hat{\mathbb{W}}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta}$$

with

can be traced back to couplings in the quadratic action

$$\hat{\mathbb{Y}} = \mathbb{A} + \mathbb{B}\nabla + \mathbb{C}\nabla\nabla + \mathbb{D}\nabla\nabla\nabla + \cdots$$

(suppressing indices)

$$\hat{\mathbb{W}} = \mathbb{E} + \mathbb{F}\nabla + \mathbb{G}\nabla\nabla + \mathbb{H}\nabla\nabla\nabla + \cdots$$

Lagrangian contains "too much" information for cosmological perturbations, but it is useful for understanding what the free functions mean... *symmetries* & *couplings*

Perturbed fluid variables

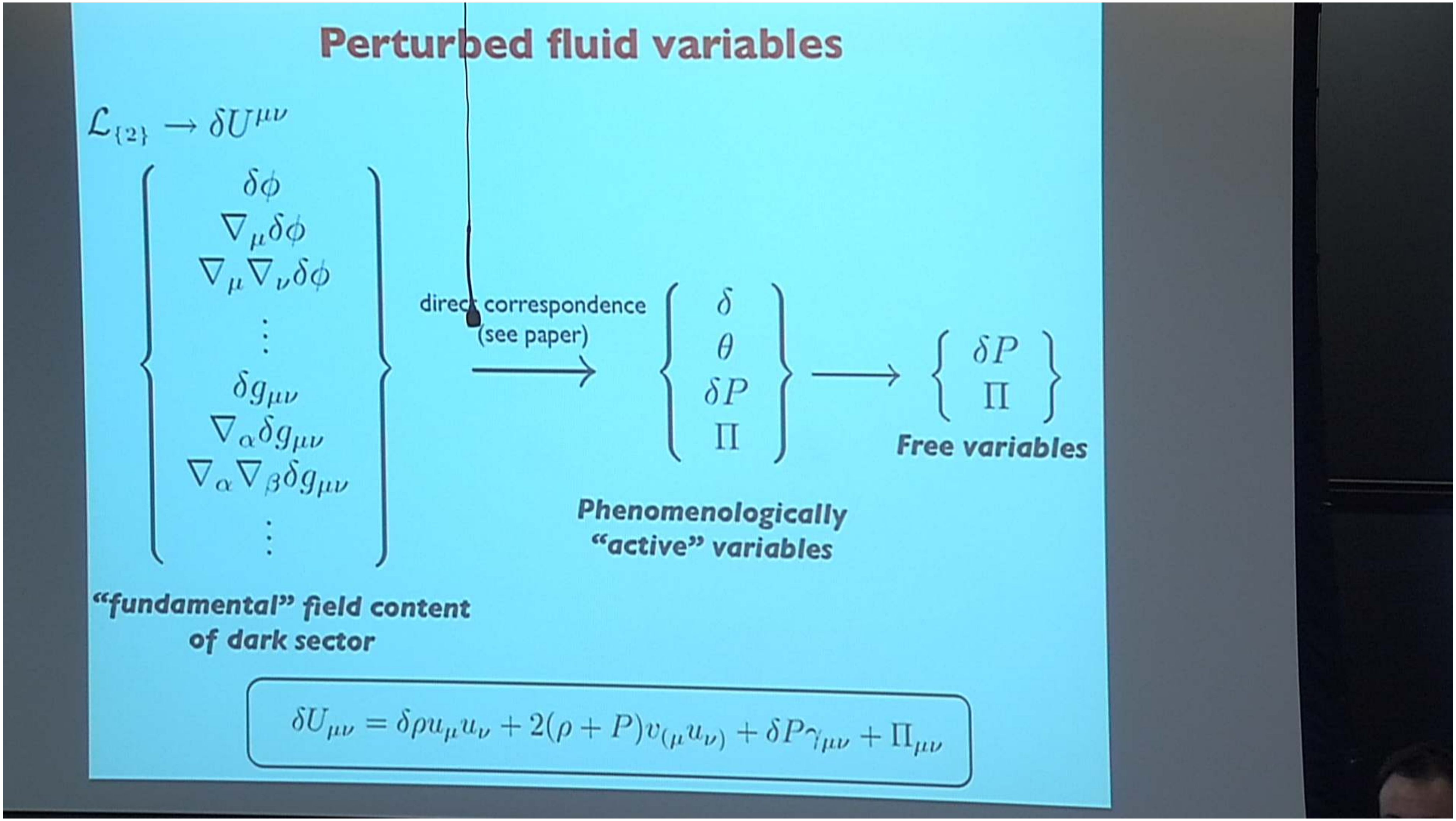
$$\mathcal{L}_{\{2\}} \rightarrow \delta U^{\mu\nu}$$

$$\left\{ \begin{array}{c} \delta\phi \\ \nabla_\mu \delta\phi \\ \nabla_\mu \nabla_\nu \delta\phi \\ \vdots \\ \delta g_{\mu\nu} \\ \nabla_\alpha \delta g_{\mu\nu} \\ \nabla_\alpha \nabla_\beta \delta g_{\mu\nu} \\ \vdots \end{array} \right\} \longrightarrow \left\{ \begin{array}{c} \delta \\ \theta \\ \delta P \\ \Pi \end{array} \right\}$$

"fundamental" field content of dark sector

Phenomenologically "active" variables

$$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P) v_{(\mu} u_{\nu)} + \delta P \gamma_{\mu\nu} + \Pi_{\mu\nu}$$


Perturbed fluid variables
$\mathcal{L}_{\{2\}} \to \delta U^{\mu\nu}$
$\delta\phi$, $\nabla_\mu \delta\phi$, $\nabla_\mu \nabla_\nu \delta\phi$, $\vdots$, $\delta g_{\mu\nu}$, $\nabla_\alpha \delta g_{\mu\nu}$, $\nabla_\alpha \nabla_\beta \delta g_{\mu\nu}$, $\vdots$
"fundamental" field content of dark sector
direct correspondence (see paper)
δ, θ, δP, Π
Phenomenologically "active" variables
δP, Π
Free variables
$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P) v_{(\mu} u_{\nu)} + \delta P \gamma_{\mu\nu} + \Pi_{\mu\nu}$

Equations of state for dark sector perturbations

$$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P)v_{(\mu}u_{\nu)} + \delta P\gamma_{\mu\nu} + \Pi_{\mu\nu}$$

$$\delta g_{\mu\nu} = 2\phi u_\mu u_\nu + 2n_{(\mu}u_{\nu)} + \tfrac{1}{3}h\gamma_{\mu\nu} + h^{(\Pi)}_{\mu\nu}$$

evolve via gravitational field equations

Perturbed conservation equation $\delta(\nabla_\mu U^\mu{}_\nu) = 0$

$$\dot{\delta} = -(1+w)\left(\nabla_\mu v^\mu + \frac{1}{2}\dot{h}\right) - 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - w\right)\delta$$

$$\dot{v}_\alpha = -\mathcal{H}(1-3w)v_\alpha + (\bar{\nabla}_\alpha\phi - \mathcal{H}n_\alpha) - \frac{1}{\rho(1+w)}\bar{\nabla}_\alpha\delta P - \frac{w}{1+w}\nabla_\mu\Pi^\mu{}_\alpha$$

$\delta \equiv \dfrac{\delta\rho}{\rho}$ $\quad u^\mu u_\mu = -1 \quad u^\mu v_\mu = u^\mu n_\mu = u^\mu\gamma_{\mu\nu} = 0 \quad \Pi^\mu{}_\mu = u^\mu\Pi_{\mu\nu} = 0$

Equations of state for dark sector perturbations

$$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P)v_{(\mu}u_{\nu)} + \delta P\gamma_{\mu\nu} + \Pi_{\mu\nu}$$

Would like expressions of the form

$$\delta P = \delta P(\delta, \theta, \dot{\delta}, \dot{\theta}, h, \eta, \ldots) \qquad \Pi = \Pi(\delta, \theta, \dot{\delta}, \dot{\theta}, h, \eta, \ldots)$$

... these would close the set of perturbation equations.

... they are *equations of state*

Equations of state for dark sector perturbations

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QUESTIONS:

- Are there generic forms of these expressions for wide classes of theories?
- Is there a specific form for given field content ...?

Equations of state for dark sector perturbations

Example

$$\delta U^{\mu\nu} = \hat{\mathbb{Y}}^{\mu\nu}\delta\phi + \hat{\mathbb{W}}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta}$$

$$\hat{\mathbb{Y}}^{\mu\nu} = \mathbb{A}^{\mu\nu} + \mathbb{B}^{\alpha\mu\nu}\nabla_\alpha + \mathbb{C}^{\alpha\beta\mu\nu}\nabla_\alpha\nabla_\beta + \mathbb{D}^{\rho\alpha\beta\mu\nu}\nabla_\rho\nabla_\alpha\nabla_\beta,$$

$$\hat{\mathbb{W}}^{\mu\nu\alpha\beta} = \mathbb{E}^{\mu\nu\alpha\beta} + \mathbb{F}^{\rho\mu\nu\alpha\beta}\nabla_\rho$$

Contains: massive gravity, (1derivative) scalar field theories, KGB

imposed with (a) second order field equations
(b) reparameterization invariance

Equations of state for dark sector perturbations

Example

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Perturbed fluid variables:

Activation matrix

$$\begin{pmatrix} \delta \\ \theta^{\rm s} \\ \delta P \\ w\Pi^{\rm s} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 & A_{14} \\ A_{21} & A_{22} & 0 & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta\phi \\ \dot{\delta\phi} \\ \ddot{\delta\phi} \\ \dot{h} \end{pmatrix}$$

scale independant

Rearrange

$$w\Gamma = \omega_1\delta + \omega_2\theta^{\rm s} + \omega_3\dot{h} + \omega_4\ddot{h},$$

$$w \ldots A_{41}^2(\beta_1\delta + \beta_1 A_{14}\dot{h} + \beta_2\theta^{\rm S})$$

$$w\Gamma = \left(\frac{\delta P}{\delta\rho} - w\right)\delta$$

entropy perturbation

Equations of state for dark sector perturbations

$$\mathcal{X} = -\frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi$$

$$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X}, \Box\phi) \xrightarrow[\text{field equations}]{\text{second order}} \mathcal{L} = \mathcal{A}(\phi, \mathcal{X})\Box\phi + \mathcal{B}(\phi, \mathcal{X})$$

"*Kinetic Gravity Braiding*"

Equations of state for dark sector perturbations

$$\mathcal{X} = -\frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi$$

$$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X}, \Box\phi) \xrightarrow[\text{field equations}]{\text{second order}} \mathcal{L} = \mathcal{A}(\phi, \mathcal{X})\Box\phi + \mathcal{B}(\phi, \mathcal{X})$$

"Kinetic Gravity Braiding"

Signatures in activation matrix $A_{22} = A_{34} = A_{41} = A_{42} = 0$

scale dependance parameterized for

$$w\Gamma = (\lambda_1 - w)\delta + (\lambda_2 + \sigma\lambda_5 k^2)\theta^{\mathrm{S}} + \sigma(\lambda_1\lambda_6 - \lambda_3)\dot{h} + \sigma(\lambda_3\dot{\delta\rho} - \lambda_3\lambda_6\sigma\ddot{h} + \lambda_4\dot{\theta}^{\mathrm{S}})$$

$$\sigma \propto \mathcal{A}_{,\mathcal{X}}$$

All freedom parameterized by 7 time dependent functions
Finding $\sigma = 0$ would tell us something important

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$$w\Gamma = \left(\frac{\delta P}{\delta \rho} - w\right)\delta$$

Write perturbed pressure as entropy perturbation

$$\delta U^0{}_i \sim v_i = \partial_i \theta$$

Different "classes" of theories & field contents of the dark sector

$\mathcal{L} = \mathcal{L}(g_{\mu\nu})$

$w\Gamma = (\kappa -$ [illegible]$)\delta \qquad w\Pi = (w - \lambda)\left[\delta - 3(1+w)\varepsilon\eta\right]$

scale independant

$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X})$

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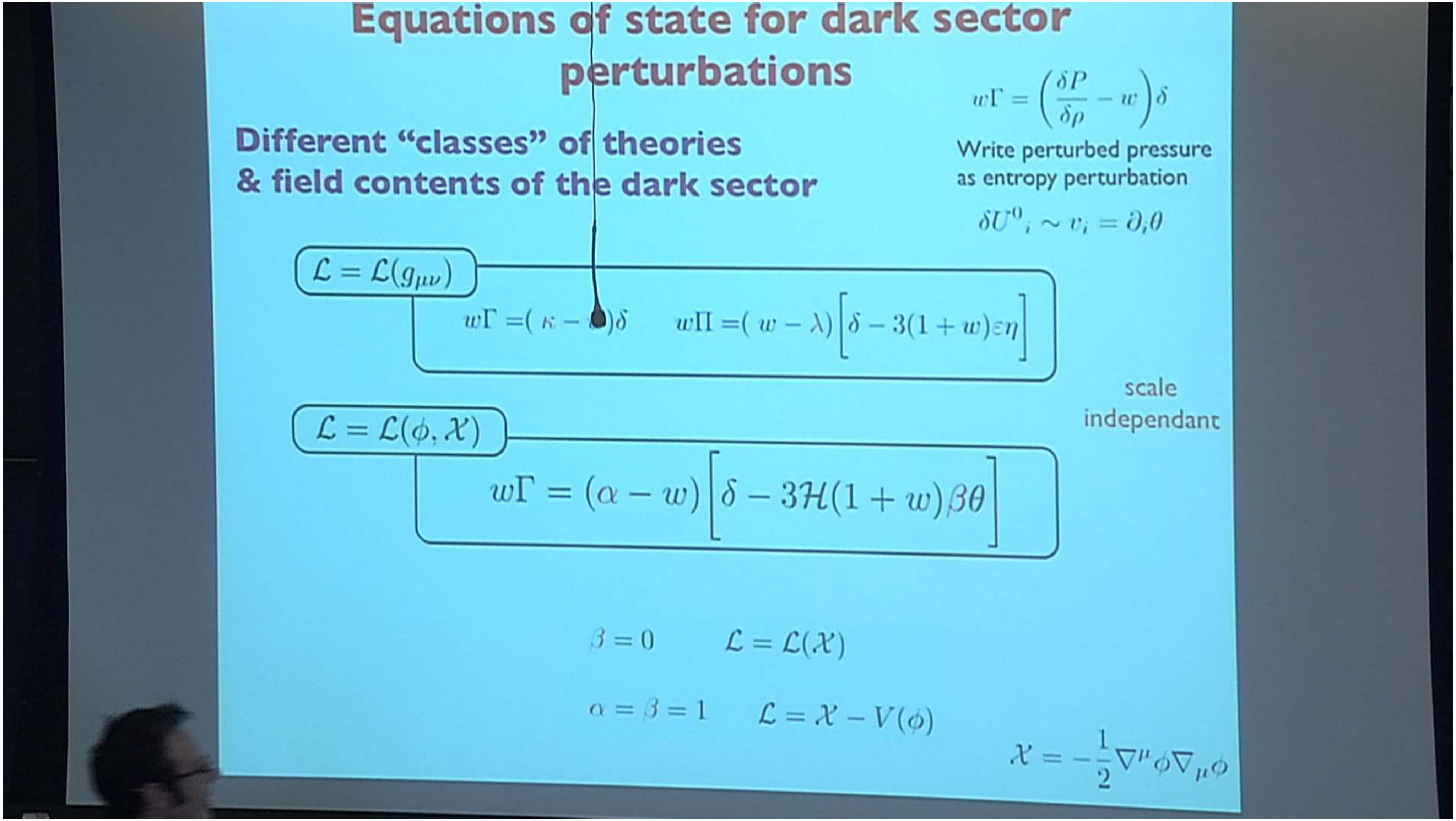
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Equations of state for dark sector perturbations

Explicit mappings...

$\mathcal{L} = \mathcal{L}(g_{\mu\nu})$

$$\kappa = \frac{D_{\mathcal{W}} + \frac{\text{[illegible]}}{3} E_{\mathcal{W}} + \frac{1}{3}P}{\rho + B_{\mathcal{W}}} \qquad \lambda = w + \frac{2P - E_{\mathcal{W}}}{\rho + B_{\mathcal{W}}} \qquad \varepsilon = \frac{\rho + B_{\mathcal{W}}}{w(1+w)}$$

$$A_{\mathcal{W}} = m_0^2 = -\rho, \qquad B_{\mathcal{W}} = -2m_1^2 = P, \qquad C_{\mathcal{W}} = 4m_1^2 = -P,$$

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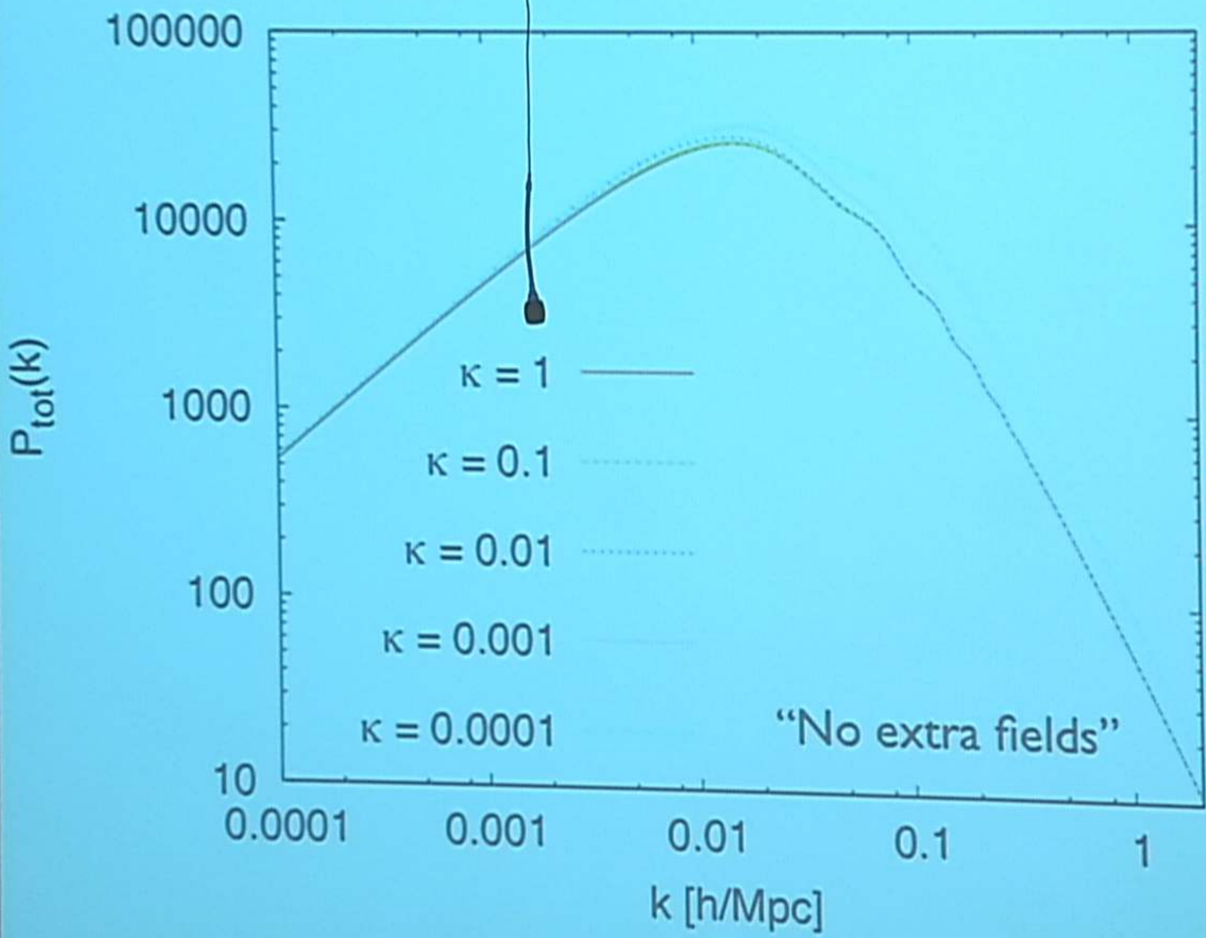
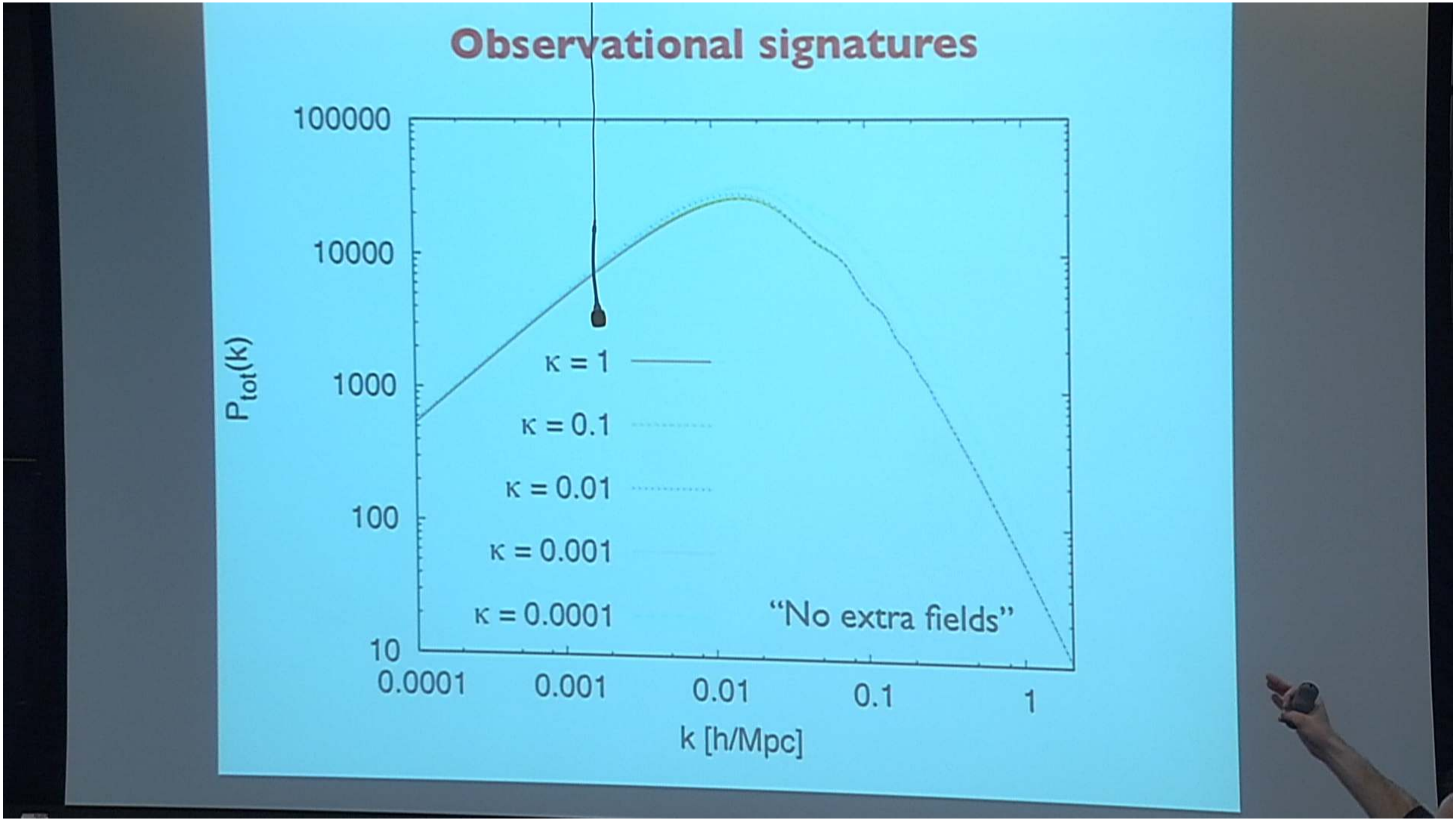
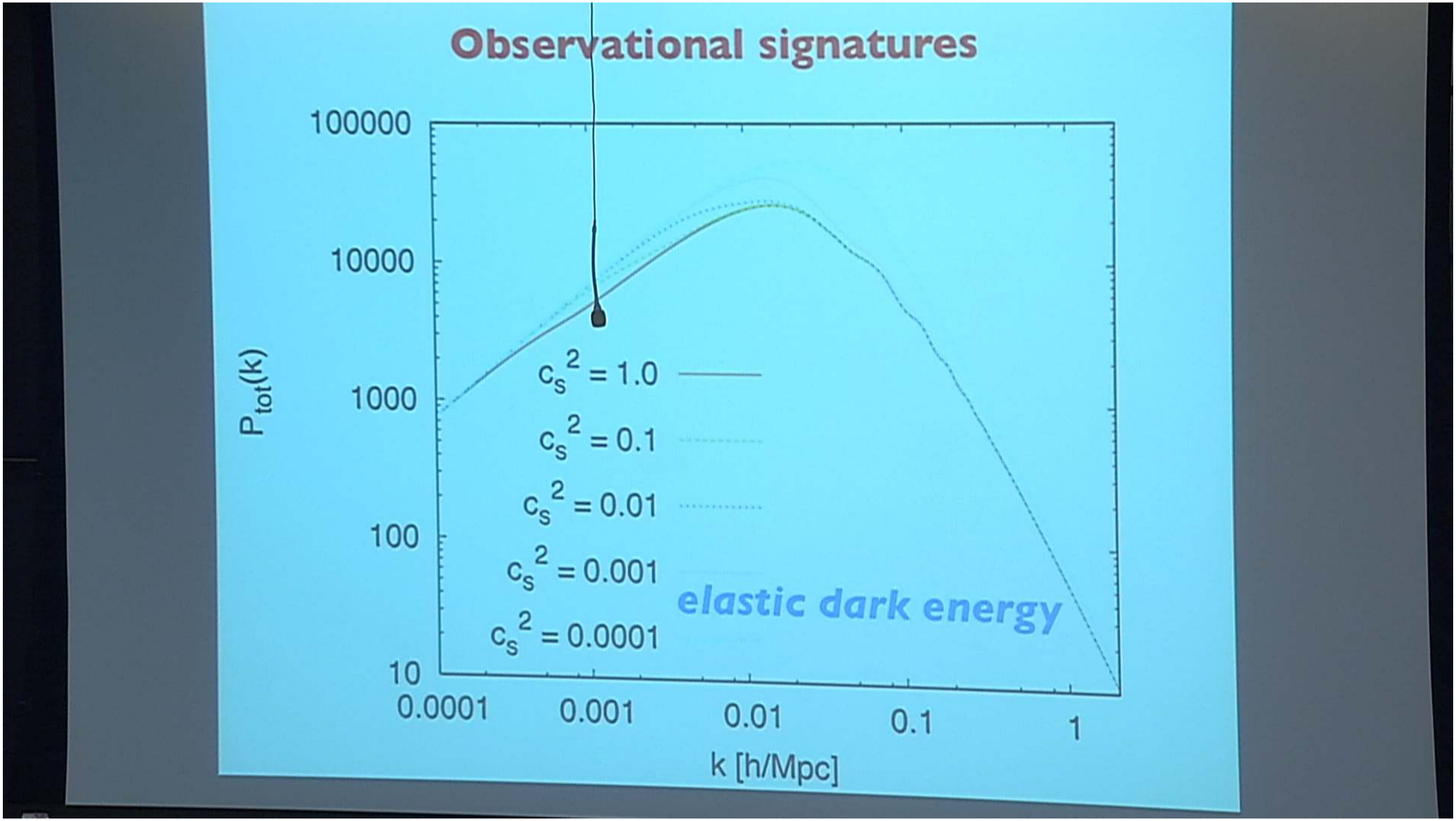
Observational signatures





Observational signatures
100000
10000
1000
100
10
$P_{tot}(k)$
κ = 1
κ = 0.1
κ = 0.01
κ = 0.001
κ = 0.0001
"No extra fields"
0.0001
0.001
0.01
0.1
1
k [h/Mpc]



Observational signatures
100000
10000
1000
100
10
$P_{tot}(k)$
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0.001
0.01
0.1
1
k [h/Mpc]
$c_s^2 = 1.0$
$c_s^2 = 0.1$
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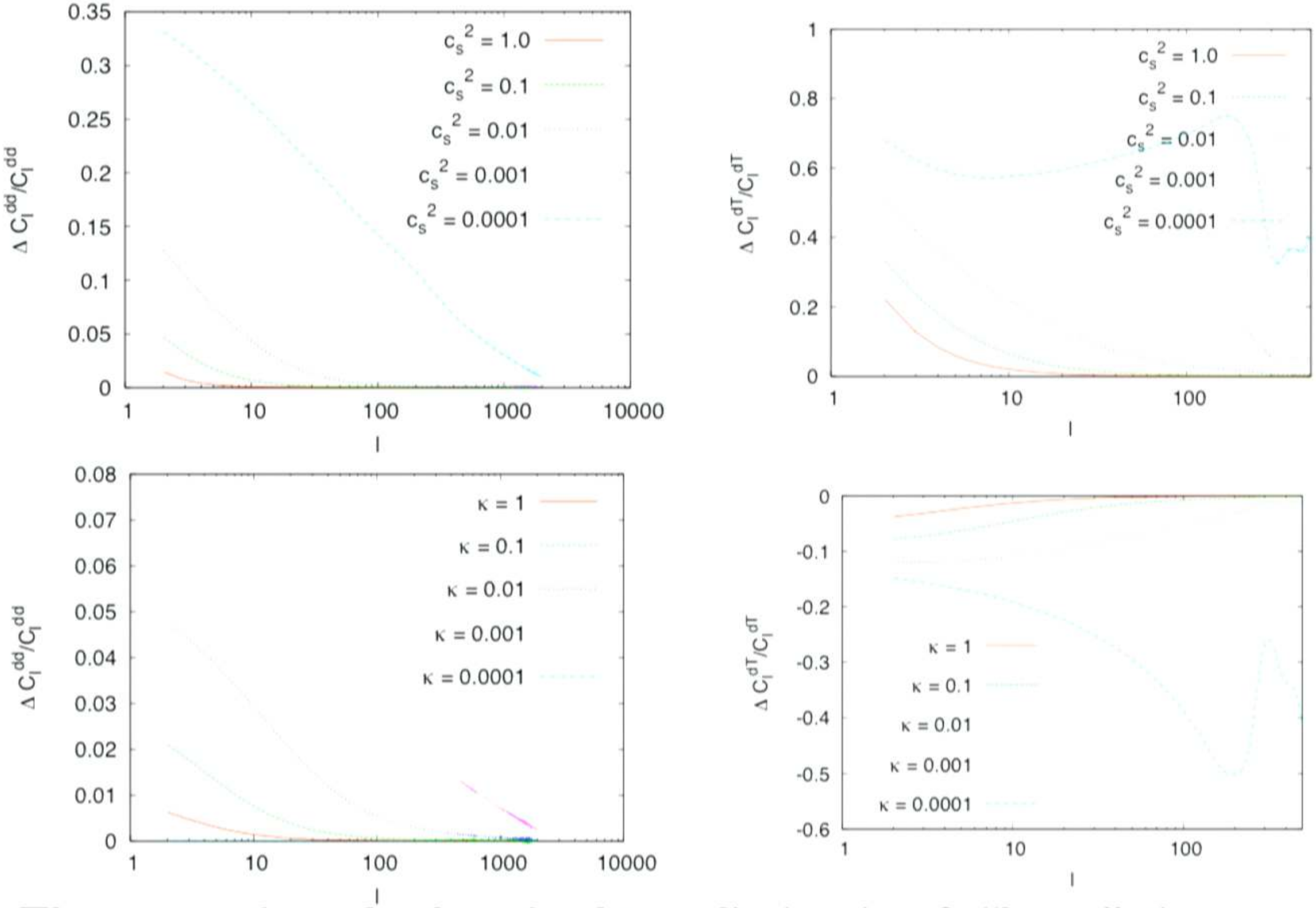
$c_s^2 = 0.001$
$c_s^2 = 0.0001$
elastic dark energy

Observational signatures



These metric-only theories have distinctive & "large" signatures!

Final remarks

- Perturbations in the dark sector need parameterizing
- Current methods are not particularly "physical"
- We construct Lagrangian for perturbations after picking a field content
- Derive general perturbed dark energy momentum tensor
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Still developing

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