Title: Parameterizing dark sector perturbations

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Abstract: When recent observational evidence and the GR+FRW+CDM
model are combined we obtain the result that the Universe is accelerating,
where the acceleration is due to some not-yet-understood "dark
sector". There has been a considerable number of theoretical models
constructed in an attempt to provide an "understanding" of the dark

sector: dark energy and modified gravity theories. The
proliferation of modified gravity and dark energy models has brought to light
the need to construct a "generic" way to parameterize the dark
sector.

We will discuss our new way of approaching this problem,
looking at linearised perturbations. Our approach is inspired by that taken in
particle physics, where the most general modifications to the standard model
are written down for a given field content that is compatible with some assumed
symmetry (which we take to be isotropy of the background spatial sections). Our
emphasis is on constructing a theoretically motivated toolkit which can be used
to extract meaningful information about the dark sector (such as its field
content). We find, for example, that the observational impact of very broad
classes of theories can be encoded by a very small (less than 5) number of
parameters. It is these parameters which we hope to measure with observational
data.

Parameterizing dark sector perturbations

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Outline

➡ Introduction - cosmology & the dark side

➡ Current approaches to parameterizing ignorance

➡ Formalism - our approach

➡ Connection to massive gravity

➡ Equations of state for dark sector perturbations

➡ Outlook

Introduction

Einstein's field equations with FRW metric yield

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \qquad \left(\frac{\dot{a}}{a}\right)^2 \equiv \mathcal{H}^2 = \frac{8\pi G}{3}\rho \qquad \dot{\rho} = -3\mathcal{H}(\rho + P)$$

Also do for fluctuations about FRW (now study structures)

$$g_{\mu\nu} = a^2(\tau)\left[\eta_{\mu\nu} + h_{\mu\nu}(\tau, \mathbf{x})\right]$$

Provides a set of predictions for

Distances to (e.g.) supernovae
Cosmic **M**icrowave **B**ackground fluctuation spectrum
Structure... how many galaxies of each size should there be?

(all quantities which we can & have observed)

Introduction

General Relativity

FRW metric

Standard matter content

Introduction

General Relativity

FRW metric

Standard matter content

$+$

data

$=$

Inconsistent!

(by

Introduction

Re-examine ingredients that went into the cosmological cookbook

Gravity... ?

General Relativity

Geometry... ?

Homogeneous & isotropic

Content... ?

Baryons, photons, neutrinos,...

Maybe GR is not *the* gravitational
theory on large scales?
Applying to cosmology is an
extrapolation of the validity of GR as
the gravitational theory.
Modified gravity

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Inhomogeneous universes

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Maybe there's more to the content of the universe than we realized
Dark matter & dark energy

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Dark matter & dark energy

structure formation & galaxy rotations...

cosmic acceleration

Introduction

Assume FRW is correct

Assume GR is correct

Require *24% dark matter* — Invented!
73% dark energy

$$P = w\rho \quad w_{\mathrm{de}} < -1/3$$

What's the dark energy?

Perhaps "observed" acceleration is actually manifestation of non-GR gravity on large scales...

...lesson from Newton & the Sun...

What's the gravitational theory?

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Introduction

Accelerating universe
Dark energy, inhomogeneous universe or modified gravity?

Same matter content causes
gravity to respond differently

Dark sector
Something causing apparent acceleration

Model zoo
Λ, quintessence, k-essence, galileon, F(R), elastic dark energy, massive gravity, Einstein-aether, ... **Lagrangian engineering**

(See 1106.2476 for a great review)

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The model zoo

$$\mathcal{L} = R - 2\Lambda \qquad \textit{cosmological constant}$$

$$\mathcal{L} = R + F(R)$$

$$\mathcal{L} = R + F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$$

$$\mathcal{L} = R + \partial_\mu\phi\partial^\mu\phi + 2V(\phi) \qquad \textit{quintessence}$$

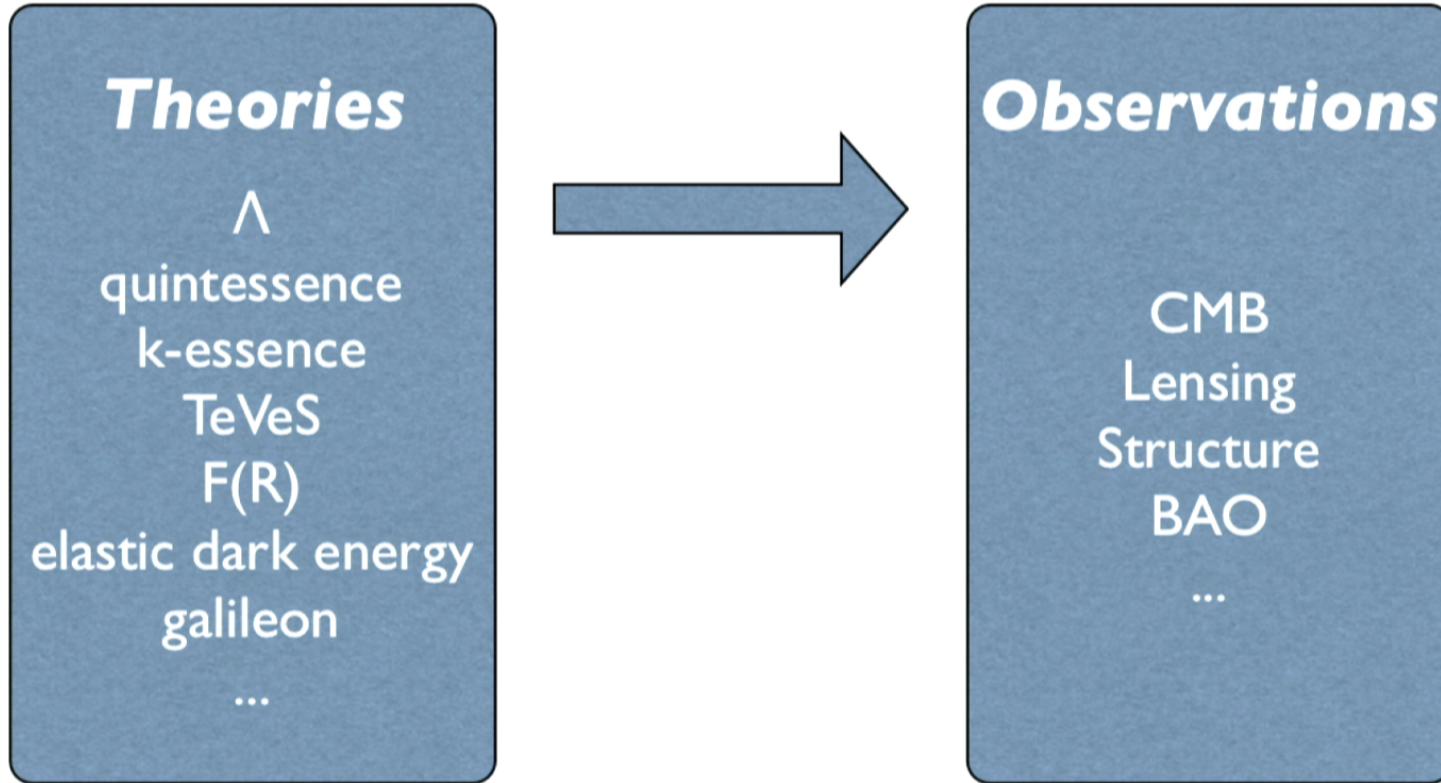$$\mathcal{L} = R + F(R) + \partial_\mu\phi\partial^\mu\phi + 2V(\phi)$$

$$\mathcal{L} = R - 2f\left(-\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi, \phi\right) \qquad \text{K-essence}$$

$$\mathcal{L} = R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \lambda(A^\mu A_\mu - 1) \qquad \textit{Einstein aether}$$

& many more besides

... Lagrangian engineering

The problem

Theories

Λ
quintessence
k-essence
TeVeS
F(R)
elastic dark energy
galileon

...

Observations

CMB
Lensing
Structure
BAO

...

CFHTLenS parameterization: $\quad \mathrm{d}s^2 = (1 + 2\Psi)\mathrm{d}t^2 - a^2(t)(1 - 2\Phi)\mathrm{d}x^2$

$$\Phi(k, a) = \big[1 + \mu(k, a)\big]\Psi_{\mathrm{GR}}(k, a)$$

$$\big[\Phi(k, a) + \Psi(k, a)\big] = \big[1 + \Sigma(k, a)\big]\big[\Psi_{\mathrm{GR}}(k, a) + \Phi_{\mathrm{GR}}(k, a)\big]$$

Pick scale invariant & parameterize out time dependance

$$\Sigma(a) = \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda} \qquad \mu(a) = \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$
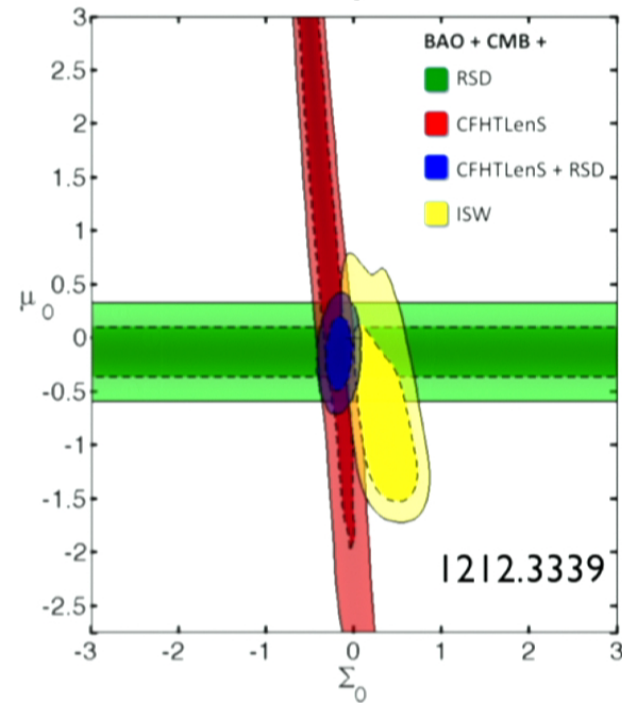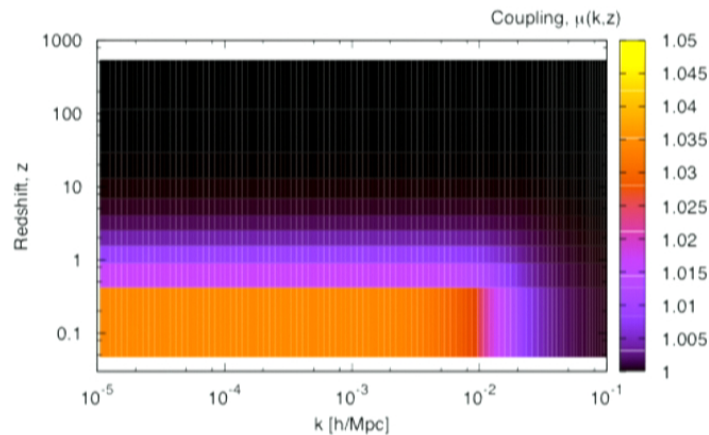
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1212.3339

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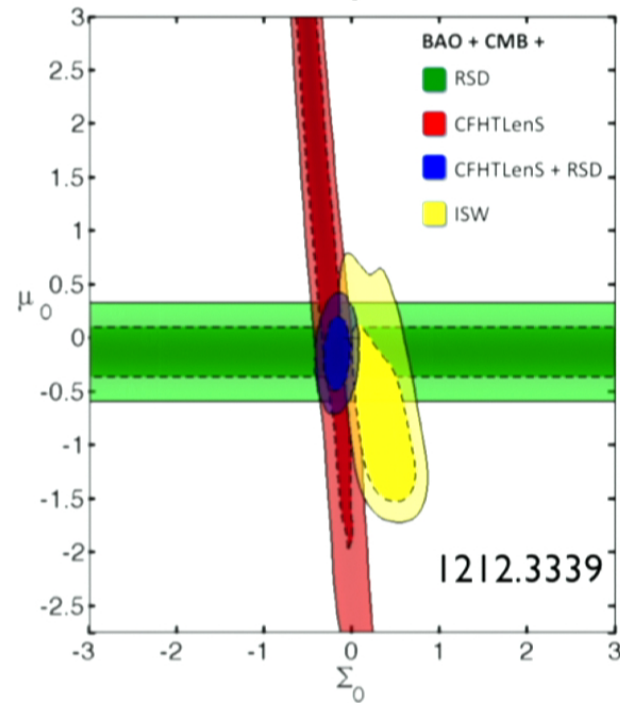
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Pick scale invariant & parameterize out time dependance

$$\Sigma(a) = \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda} \qquad \mu(a) = \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$

Scalar field model: only 5% deviation

1212.3339

Lagrangian for perturbations

Nothing extra: $\mathcal{L} = \mathcal{L}(g_{\mu\nu})$

$$\mathcal{L}_{\{2\}} = \frac{1}{8}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta}$$

$$\delta G_{\mu\nu} = 8\pi G\delta T_{\mu\nu} + \delta U_{\mu\nu}$$

$$\delta U^{\mu\nu} = -\frac{1}{2}\left(\mathcal{W}^{\mu\nu\alpha\beta} + U^{\mu\nu}g^{\alpha\beta}\right)\delta g_{\alpha\beta}$$

has maximum of 5 components in isotropic space

Elastic dark energy, massive gravity, ...

$$\mathcal{W}^{\mu\nu\alpha\beta}_{\mathrm{EDE}} = -\rho u^{\mu}u^{\nu}u^{\alpha}u^{\beta} + P(u^{\mu}u^{\nu}\gamma^{\alpha\beta} + u^{\alpha}u^{\beta}\gamma^{\mu\nu})$$
$$-2P(\gamma^{\alpha(\mu}u^{\nu)}u^{\beta} + \gamma^{\beta(\mu}u^{\nu)}u^{\alpha})$$
$$+(\beta - P - \tfrac{2}{3}\mu)\gamma^{\mu\nu}\gamma^{\alpha\beta} + 2(\mu + P)\gamma^{\mu(\alpha}\gamma^{\beta)\nu}$$

$$\mathcal{W}^{\mu\nu\alpha\beta}_{\mathrm{PF}} = m^{2}\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu(\alpha}g^{\beta)\nu}\right)$$

Lagrangian for perturbations

Scalar fields (one derivative) $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu \phi)$

$$\mathcal{L}_{\{2\}} = \mathcal{A}(\delta\phi)^2 + \mathcal{B}^\mu \delta\phi \nabla_\mu \delta\phi + \frac{1}{2}\mathcal{C}^{\mu\nu}\nabla_\mu\delta\phi\nabla_\nu\delta\phi$$

$$+ \frac{1}{4}\left[\mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi\,\delta g_{\mu\nu} + \mathcal{V}^{\mu\nu}\delta\phi\,\delta g_{\mu\nu} + \frac{1}{2}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta} \right].$$

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}$$

$$\delta U^{\mu\nu} = -\frac{1}{2}\left(\mathcal{V}^{\mu\nu}\delta\phi + \mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi \right) - \frac{1}{2}\left(\mathcal{W}^{\alpha\beta\mu\nu} + g^{\alpha\beta}U^{\mu\nu} \right)\delta g_{\alpha\beta}.$$

2 *3* *5*

(74 before isotropy!)

Quintessence, k-essence, Lorentz-violating theories,...

Lagrangian for perturbations

Formalism can be extended to include...

- Lorentz violating theories ✔
- Massive gravity ✔
- Multi-field dark energy ✔
- High-derivative scalar field theories
- Vector field theories
- Curvature tensor theories
- Bimetric
- Couplings between known & dark sector

& identify "generalized" boundary terms

Lagrangian for perturbations

Scalar fields (one derivative) $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu \phi)$

$$
\begin{aligned}
\mathcal{L}_{\{2\}} =\ & \mathcal{A}(\delta\phi)^2 + \mathcal{B}^\mu \delta\phi \nabla_\mu \delta\phi + \frac{1}{2}\mathcal{C}^{\mu\nu}\nabla_\mu\delta\phi\nabla_\nu\delta\phi \\
& + \frac{1}{4}\left[\mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi\,\delta g_{\mu\nu} + \mathcal{V}^{\mu\nu}\delta\phi\,\delta g_{\mu\nu} + \frac{1}{2}\mathcal{W}^{\mu\nu\alpha\beta}\delta g_{\mu\nu}\delta g_{\alpha\beta} \right].
\end{aligned}
$$

$$
\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}
$$

$$
\delta U^{\mu\nu} = -\frac{1}{2}\left(\underset{2}{\mathcal{V}^{\mu\nu}\delta\phi} + \underset{3}{\mathcal{Y}^{\alpha\mu\nu}\nabla_\alpha\delta\phi} \right) - \frac{1}{2}\left(\underset{5}{\mathcal{W}^{\alpha\beta\mu\nu}} + g^{\alpha\beta}U^{\mu\nu} \right)\delta g_{\alpha\beta}.
$$

(74 before isotropy!)

Quintessence, k-essence, Lorentz-violating theories,...

Connection to massive gravity

$$S_{\{2\}} = \int \mathrm{d}^4 x \sqrt{-g} \left[\Diamond^2 R + 16\pi G \Diamond^2 \mathcal{L}_\mathrm{m} - \tfrac{1}{4} \mathcal{W}^{\mu\nu\alpha\beta} \delta g_{\mu\nu} \delta g_{\alpha\beta} \right].$$

$$\delta g_{\mu\nu} = h_{\mu\nu} + 2\nabla_{(\mu} \xi_{\nu)}$$

$$\mathcal{W}^{\mu\nu\alpha\beta} = A_{\mathcal{W}} u^\mu u^\nu u^\alpha u^\beta + B_{\mathcal{W}} (u^\mu u^\nu \gamma^{\alpha\beta} + u^\alpha u^\beta \gamma^{\mu\nu})$$
$$+ 4 C_{\mathcal{W}} u^{(\alpha} \gamma^{\beta)(\mu} u^{\nu)} + D_{\mathcal{W}} \gamma^{\mu\nu} \gamma^{\alpha\beta} + 2 E_{\mathcal{W}} \gamma^{\mu(\alpha} \gamma^{\beta)\nu}$$

(isotropic space)

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when ξ^0 decouples, components become bulk & shear moduli of elastic medium *5DoFs*

(isotropic space)

$$A_{\mathcal{W}} = -\rho, \quad B_{\mathcal{W}} = -C_{\mathcal{W}} = P, \quad \dot{P} = -\mathcal{H}(P + 3D_{\mathcal{W}} + 2E_{\mathcal{W}})$$

Becomes identical to components of elasticity tensor of relativistic elasticity theory (see Carter...)

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when ξ^i decouples, becomes non-canonical
scalar field theory **3DoFs** $\quad B_{\mathcal{W}} = -C_{\mathcal{W}} = -\rho, \quad D_{\mathcal{W}} = -E_{\mathcal{W}} = -P$

Connection to massive gravity

$$S_{\{2\}} = \int d^4x \sqrt{-g} \left[\lozenge^2 R + 16\pi G \lozenge^2 \mathcal{L}_m - \tfrac{1}{4} \mathcal{W}^{\mu\nu\alpha\beta} \delta g_{\mu\nu} \delta g_{\alpha\beta} \right].$$

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scalar field theory 3DoFs $\quad B_{\mathcal{W}} = -C_{\mathcal{W}} = -\rho. \quad D_{\mathcal{W}} = -E_{\mathcal{W}} = -P$

LV massive gravity with time translation invariance is *elastic dark energy*
... *massive gravity is the manifestation of rigidity of spacetime*

masses determined by bulk & shear moduli of elastic medium

Perturbed dark energy momentum tensor

Everything boils down to

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}$$

$$\delta U^{\mu\nu} = \hat{\mathbb{Y}}^{\mu\nu}\delta\phi + \hat{\mathbb{W}}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta}$$

with

can be traced back to couplings
in the quadratic action

$$\hat{\mathbb{Y}} = \mathbb{A} + \mathbb{B}\nabla + \mathbb{C}\nabla\nabla + \mathbb{D}\nabla\nabla\nabla + \cdots$$

(suppressing indices)

$$\hat{\mathbb{W}} = \mathbb{E} + \mathbb{F}\nabla + \mathbb{G}\nabla\nabla + \mathbb{H}\nabla\nabla\nabla + \cdots$$

Lagrangian contains "too much" information for cosmological
perturbations, but it is useful for understanding what the free
functions mean... *symmetries & couplings*

Perturbed fluid variables

$$\mathcal{L}_{\{2\}} \to \delta U^{\mu\nu}$$

$$\left\{ \begin{array}{c} \delta\phi \\ \nabla_\mu \delta\phi \\ \nabla_\mu \nabla_\nu \delta\phi \\ \vdots \\ \delta g_{\mu\nu} \\ \nabla_\alpha \delta g_{\mu\nu} \\ \nabla_\alpha \nabla_\beta \delta g_{\mu\nu} \\ \vdots \end{array} \right\} \longrightarrow \left\{ \begin{array}{c} \delta \\ \theta \\ \delta P \\ \Pi \end{array} \right\}$$

Phenomenologically "active" variables

**"fundamental" field content
of dark sector**

$$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P)v_{(\mu}u_{\nu)} + \delta P \gamma_{\mu\nu} + \Pi_{\mu\nu}$$

Perturbed fluid variables

$$\mathcal{L}_{\{2\}} \rightarrow \delta U^{\mu\nu}$$

$$\left\{ \begin{array}{c} \delta\phi \\ \nabla_\mu \delta\phi \\ \nabla_\mu \nabla_\nu \delta\phi \\ \vdots \\ \delta g_{\mu\nu} \\ \nabla_\alpha \delta g_{\mu\nu} \\ \nabla_\alpha \nabla_\beta \delta g_{\mu\nu} \\ \vdots \end{array} \right\}$$

direct correspondence (see paper) $\longrightarrow$ $\left\{ \begin{array}{c} \delta \\ \theta \\ \delta P \\ \Pi \end{array} \right\}$ $\longrightarrow$ $\left\{ \begin{array}{c} \delta P \\ \Pi \end{array} \right\}$

Free variables

Phenomenologically "active" variables

"fundamental" field content of dark sector

$$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P)v_{(\mu} u_{\nu)} + \delta P \gamma_{\mu\nu} + \Pi_{\mu\nu}$$

Equations of state for dark sector perturbations

$$\delta U_{\mu\nu} = \delta\rho u_\mu u_\nu + 2(\rho + P)v_{(\mu}u_{\nu)} + \delta P\gamma_{\mu\nu} + \Pi_{\mu\nu}$$

$$\delta g_{\mu\nu} = 2\phi u_\mu u_\nu + 2n_{(\mu}u_{\nu)} + \tfrac{1}{3}h\gamma_{\mu\nu} + h^{(\Pi)}_{\mu\nu}$$

evolve via gravitational field equations

Perturbed conservation equation $\quad \delta(\nabla_\mu U^\mu{}_\nu) = 0$

$$\dot{\delta} = -(1+w)\left(\nabla_\mu v^\mu + \tfrac{1}{2}\dot{h}\right) - 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - w\right)\delta$$

$$\dot{v}_\alpha = -\mathcal{H}(1 - 3w)v_\alpha + \left(\bar{\nabla}_\alpha\phi - \mathcal{H}n_\alpha\right) - \frac{1}{\rho(1+w)}\bar{\nabla}_\alpha\delta P - \frac{w}{1+w}\nabla_\mu\Pi^\mu{}_\alpha$$

$$\delta \equiv \frac{\delta\rho}{\rho} \quad u^\mu u_\mu = -1 \quad u^\mu v_\mu = u^\mu n_\mu = u^\mu\gamma_{\mu\nu} = 0 \quad \Pi^\mu{}_\mu = u^\mu\Pi_{\mu\nu} = 0$$

Equations of state for dark sector perturbations

$$\delta U_{\mu\nu} = \delta\rho\, u_\mu u_\nu + 2(\rho + P)v_{(\mu}u_{\nu)} + \delta P\gamma_{\mu\nu} + \Pi_{\mu\nu}$$

Would like expressions of the form

$$\delta P = \delta P(\delta, \theta, \dot{\delta}, \dot{\theta}, h, \eta, \ldots) \qquad \Pi = \Pi(\delta, \theta, \dot{\delta}, \dot{\theta}, h, \eta, \ldots)$$

... these would close the set of perturbation equations.

... they are *equations of state*

Equations of state for dark sector perturbations

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QUESTIONS:

▸ Are there generic forms of these expressions for wide classes of theories?

▸ Is there a specific form for given field content ...?

Equations of state for dark sector perturbations

Example

$$\delta U^{\mu\nu} = \hat{\mathbb{Y}}^{\mu\nu}\delta\phi + \hat{\mathbb{W}}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta}$$

$$\hat{\mathbb{Y}}^{\mu\nu} = \mathbb{A}^{\mu\nu} + \mathbb{B}^{\alpha\mu\nu}\nabla_\alpha + \mathbb{C}^{\alpha\beta\mu\nu}\nabla_\alpha\nabla_\beta + \mathbb{D}^{\rho\alpha\beta\mu\nu}\nabla_\rho\nabla_\alpha\nabla_\beta,$$

$$\hat{\mathbb{W}}^{\mu\nu\alpha\beta} = \mathbb{E}^{\mu\nu\alpha\beta} + \mathbb{F}^{\rho\mu\nu\alpha\beta}\nabla_\rho$$

Contains: massive gravity, (1 derivative) scalar field theories, KGB

imposed with (a) second order field equations
(b) reparameterization invariance

Equations of state for dark sector perturbations

Example

$$\delta U^{\mu\nu} = \hat{Y}^{\mu\nu}\delta\phi + \hat{W}^{\mu\nu\alpha\beta}\delta g_{\alpha\beta}$$

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Contains: massive gravity, (1 derivative) scalar field theories, KGB

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Perturbed fluid variables:

Activation matrix

$$\begin{pmatrix} \delta \\ \theta^s \\ \delta P \\ w\Pi^s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 & A_{14} \\ A_{21} & A_{22} & 0 & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta\phi \\ \dot{\delta\phi} \\ \ddot{\delta\phi} \\ h \end{pmatrix}$$

scale independant

Rearrange

$$w\Gamma = \omega_1\delta + \omega_2\theta^s + \omega_3\dot{h} + \omega_4\ddot{h},$$

$$w\ldots A_{41}^2(\beta_1\delta + \beta_1 A_1\dot{h} + \beta_2\theta^s)$$

$$w\Gamma = \left(\frac{\delta P}{\delta\rho} - w\right)\delta$$

entropy perturbation

Equations of state for dark sector perturbations

$$\mathcal{X} = -\frac{1}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi$$

$$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X}, \Box\phi) \xrightarrow[\text{field equations}]{\text{second order}} \mathcal{L} = \mathcal{A}(\phi, \mathcal{X})\Box\phi + \mathcal{B}(\phi, \mathcal{X})$$

"Kinetic Gravity Braiding"

Equations of state for dark sector perturbations

$$\mathcal{X} = -\frac{1}{2}\nabla^\mu \phi \nabla_\mu \phi$$

$$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X}, \Box\phi) \xrightarrow[\text{field equations}]{\text{second order}} \mathcal{L} = \mathcal{A}(\phi, \mathcal{X})\Box\phi + \mathcal{B}(\phi, \mathcal{X})$$

"Kinetic Gravity Braiding"

Signatures in activation matrix $A_{22} = A_{34} = A_{41} = A_{42} = 0$

scale dependance parameterized for

$$w\Gamma = (\lambda_1 - w)\delta + (\lambda_2 + \sigma\lambda_5 k^2)\theta^{\mathrm{S}} + \sigma(\lambda_1\lambda_6 - \lambda_3)\dot{h}$$
$$+ \sigma(\lambda_3 \dot{\delta\rho} - \lambda_3\lambda_6\sigma\ddot{h} + \lambda_4\dot{\theta}^{\mathrm{S}})$$

$$\sigma \propto \mathcal{A}_{,\mathcal{X}}$$

All freedom parameterized by 7 time dependent functions
Finding σ = 0 would tell us something important

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$$w\Gamma = \left(\frac{\delta P}{\delta \rho} - w\right)\delta$$

Different "classes" of theories & field contents of the dark sector

Write perturbed pressure as entropy perturbation

$$\delta U^0{}_i \sim v_i = \partial_i \theta$$

$\mathcal{L} = \mathcal{L}(g_{\mu\nu})$

$$w\Gamma = (\kappa - \lambda)\delta \qquad w\Pi = (w - \lambda)\left[\delta - 3(1+w)\varepsilon\eta\right]$$

scale independant

$\mathcal{L} = \mathcal{L}(\phi, \mathcal{X})$

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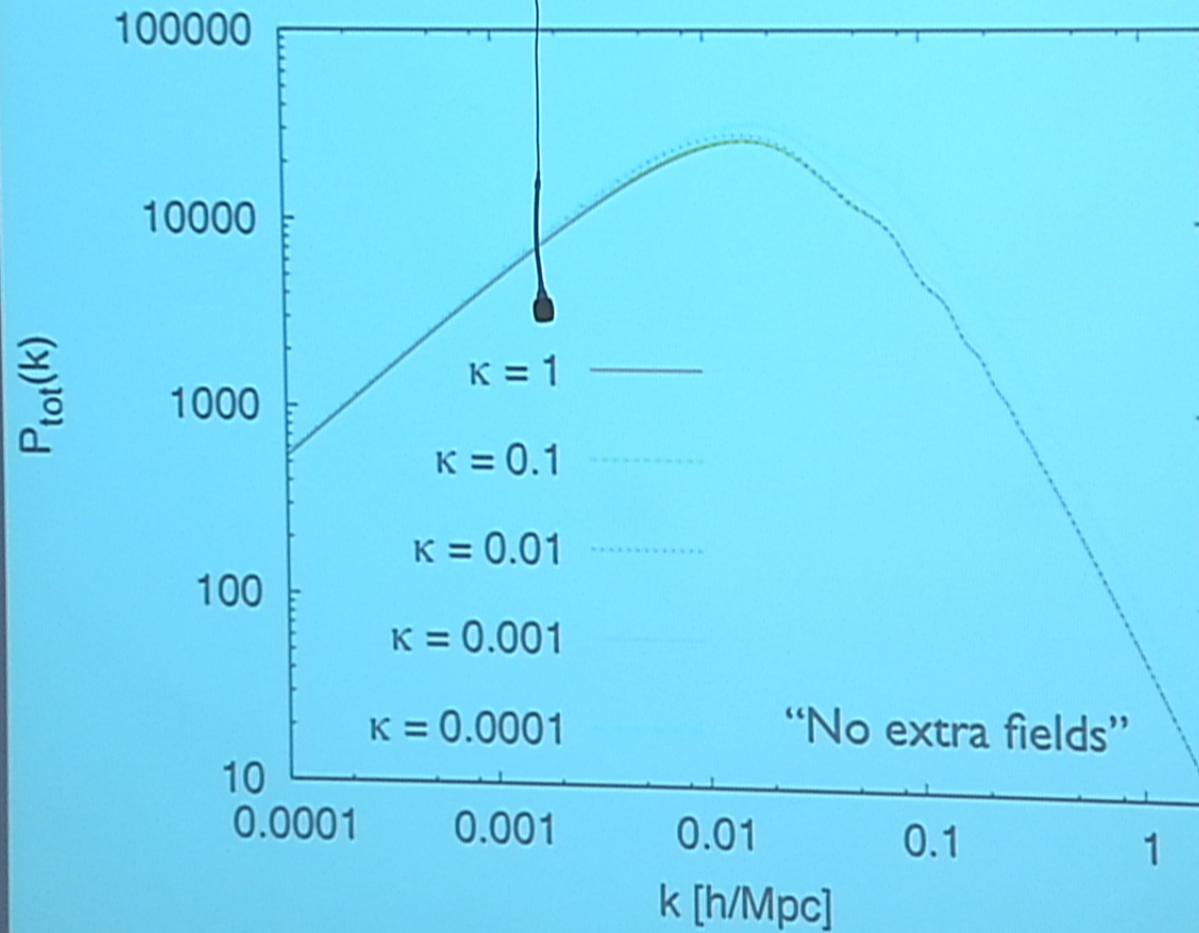# Equations of state for dark sector perturbations

Explicit mappings...

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu})$$

$$\kappa = \frac{D_W + \frac{2}{3}E_W + \frac{1}{3}P}{\rho + B_W} \qquad \lambda = w + \frac{2P - E_W}{\rho + B_W} \qquad \varepsilon = \frac{\rho + B_W}{w(1+w)}$$

$$A_W = m_0^2 = -\rho, \qquad B_W = -2m_1^2 = P, \qquad C_W = 4m_1^2 = -P,$$

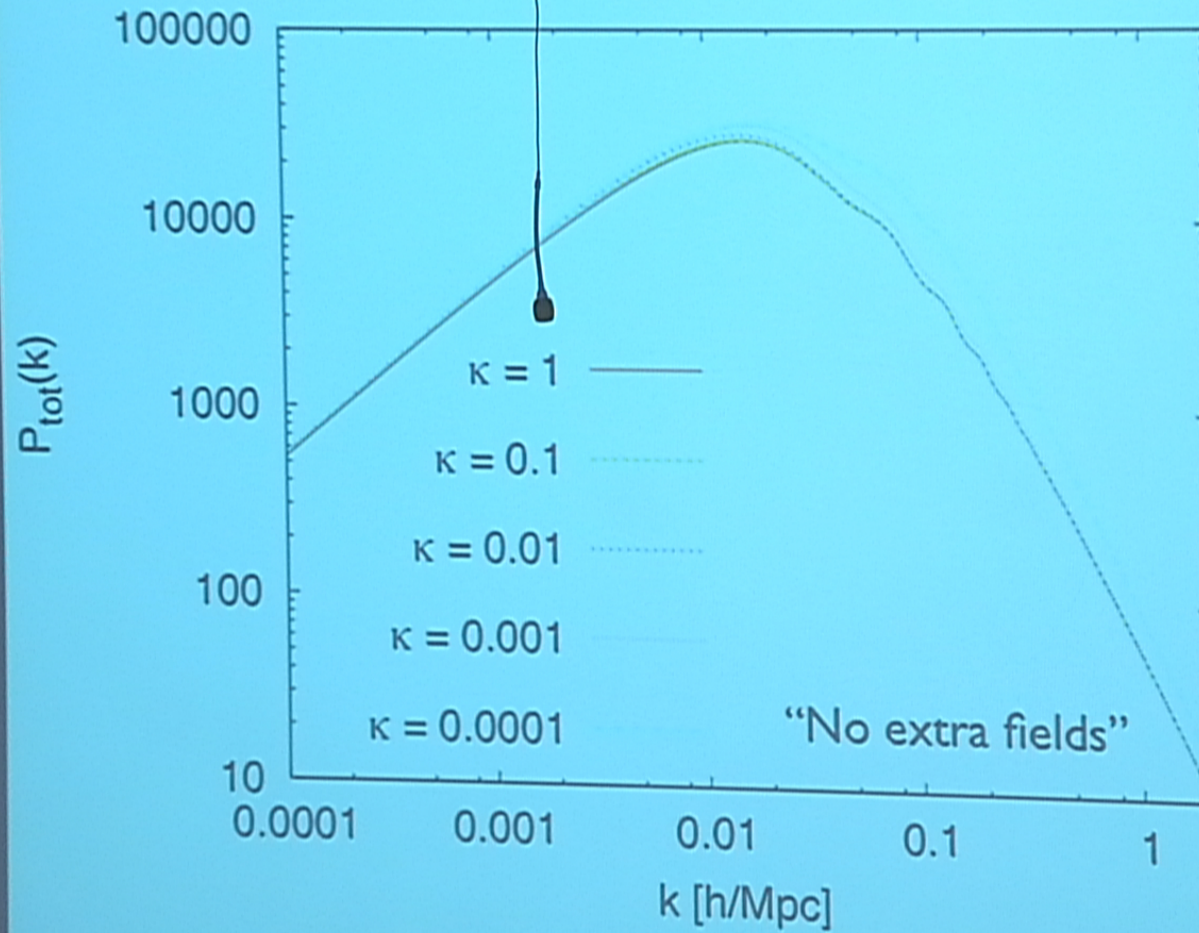$$D_W = 4m_3^2 = \beta - P - \frac{2}{3}\mu, \qquad E_W = -2m_2^2 = \mu + P.$$

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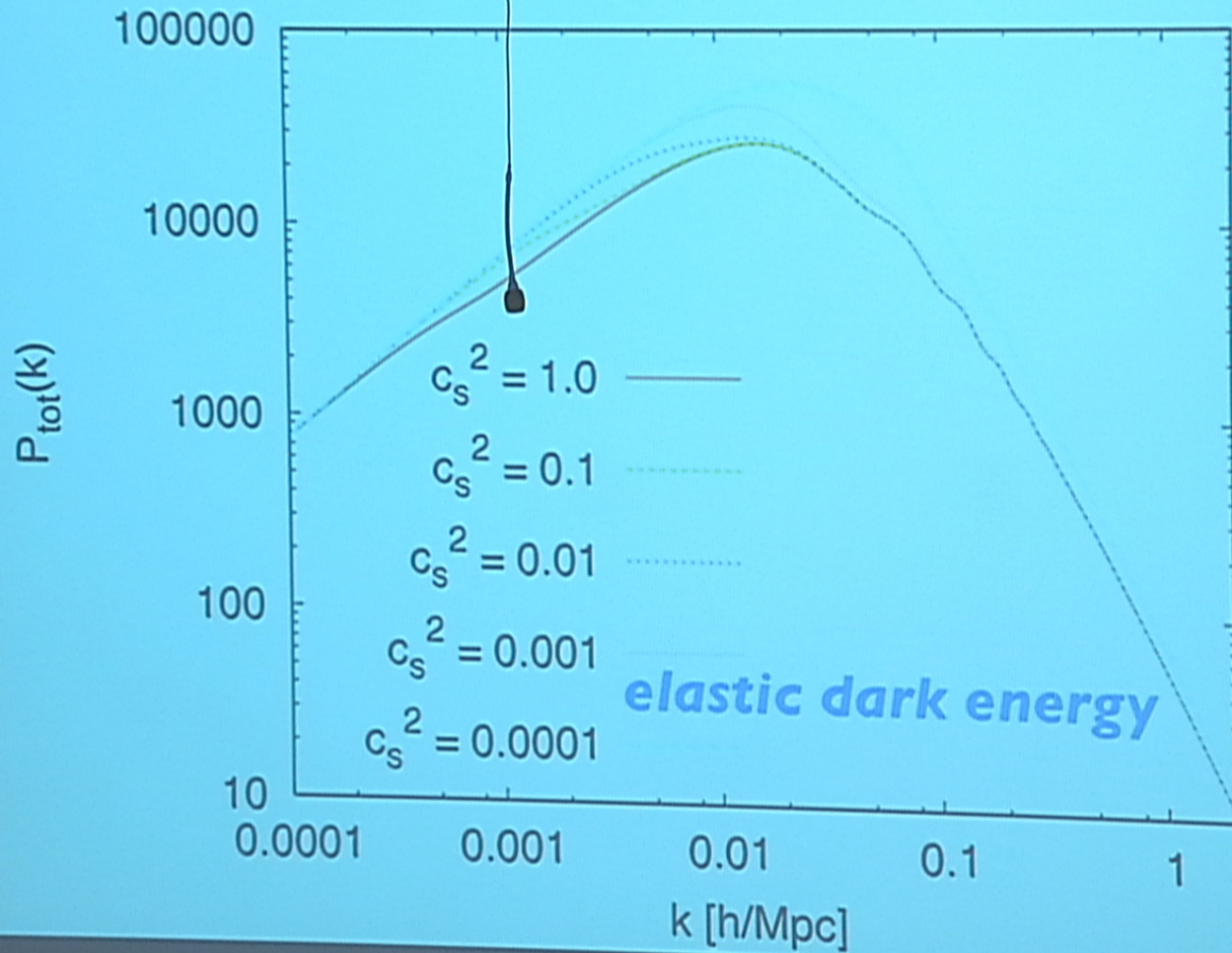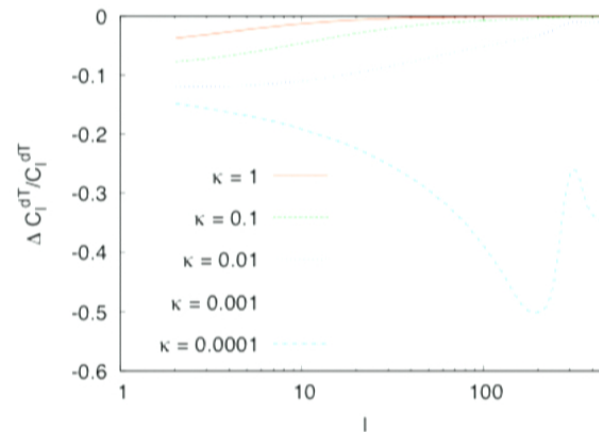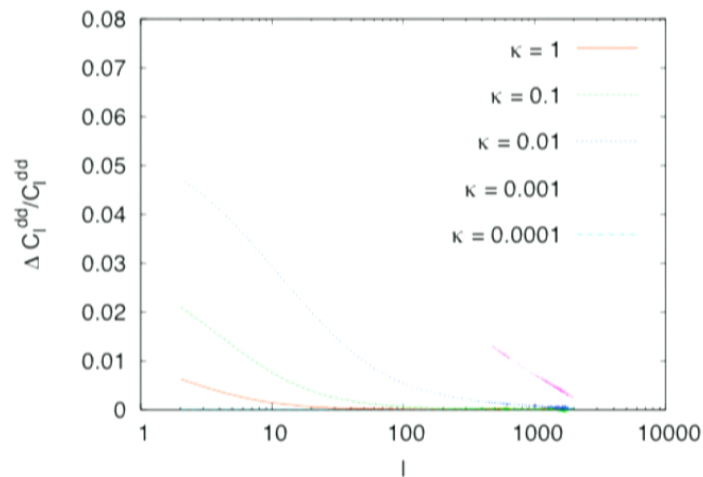$$\mathcal{X} = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$$

Observational signatures

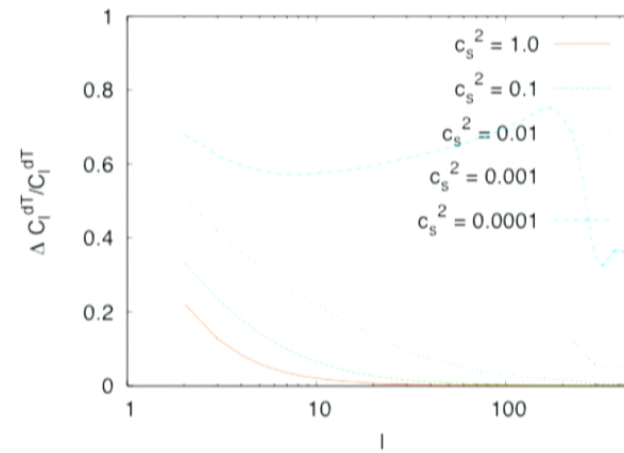$\kappa = 1$

$\kappa = 0.1$

$\kappa = 0.01$

$\kappa = 0.001$

$\kappa = 0.0001$

"No extra fields"

$P_{tot}(k)$

k [h/Mpc]

Observational signatures

Observational signatures

Plot of $P_{tot}(k)$ versus k [h/Mpc], showing curves for:
- $c_s^2 = 1.0$
- $c_s^2 = 0.1$
- $c_s^2 = 0.01$
- $c_s^2 = 0.001$
- $c_s^2 = 0.0001$

elastic dark energy

Observational signatures
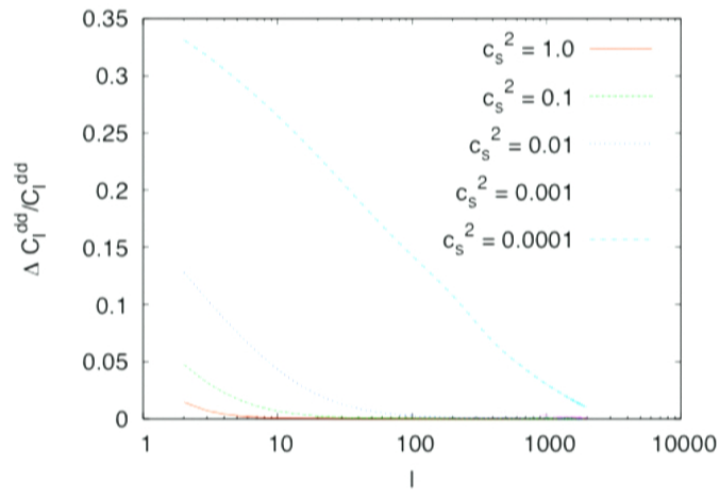
These metric-only theories have distinctive & "large" signatures!

Final remarks

▶ Perturbations in the dark sector need parameterizing

▶ Current methods are not particularly "physical"

▶ We construct Lagrangian for perturbations after picking a field content

▶ Derive general perturbed dark energy momentum tensor

▶ Identify maximal amount of freedom

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