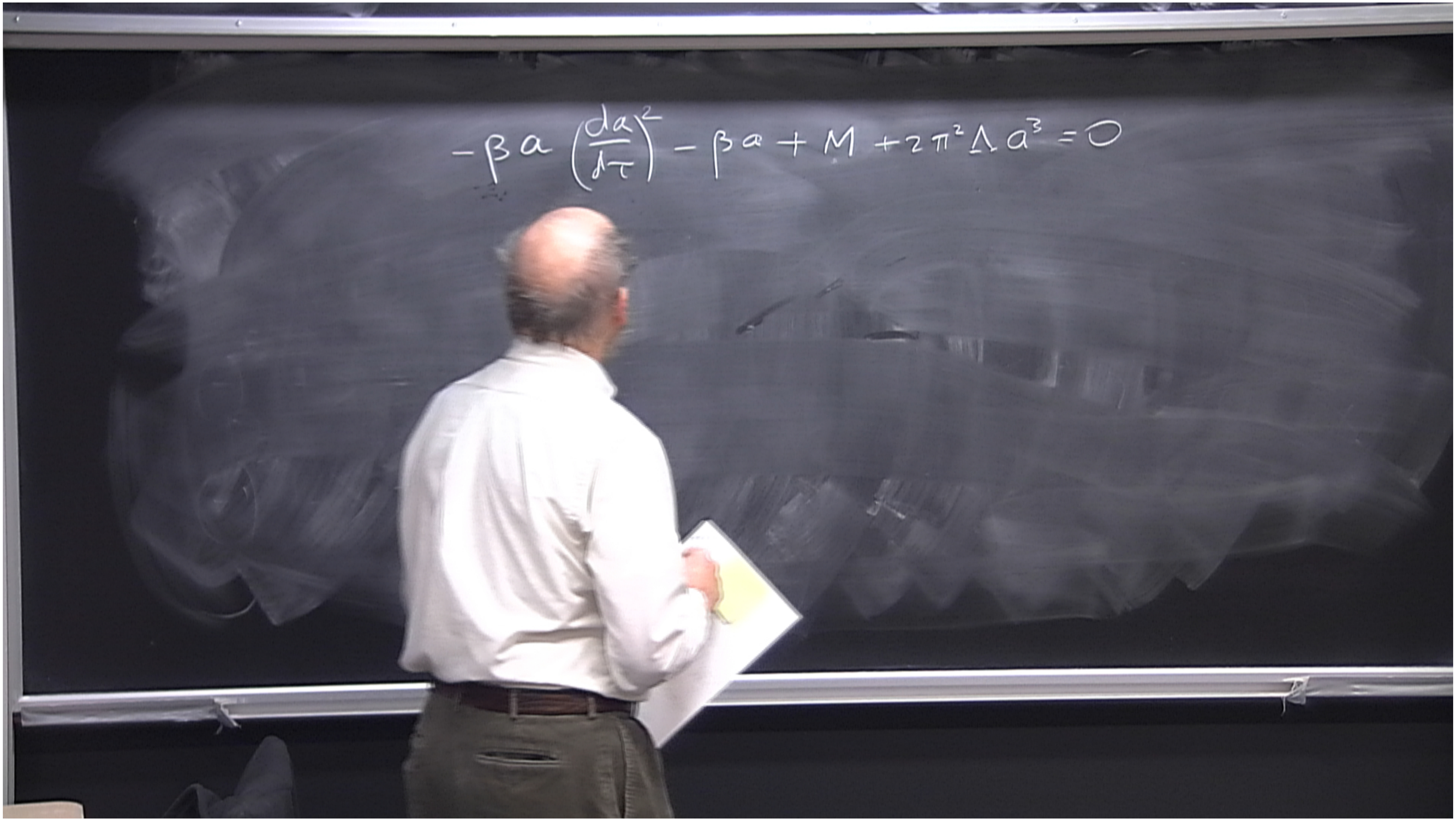
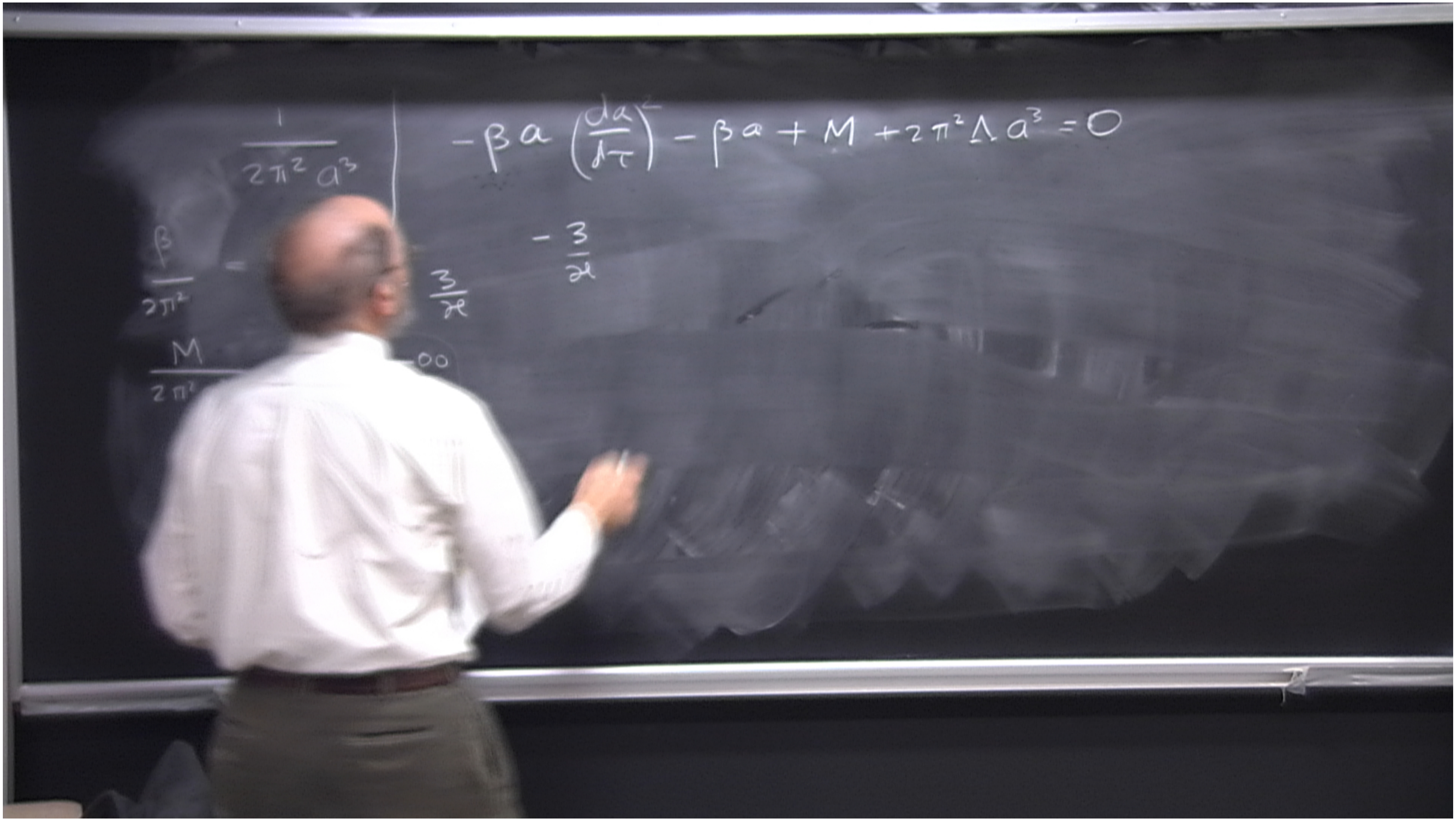
Title: Advanced General Relativity - Lecture 4

Date: Jan 31, 2013 05:00 PM

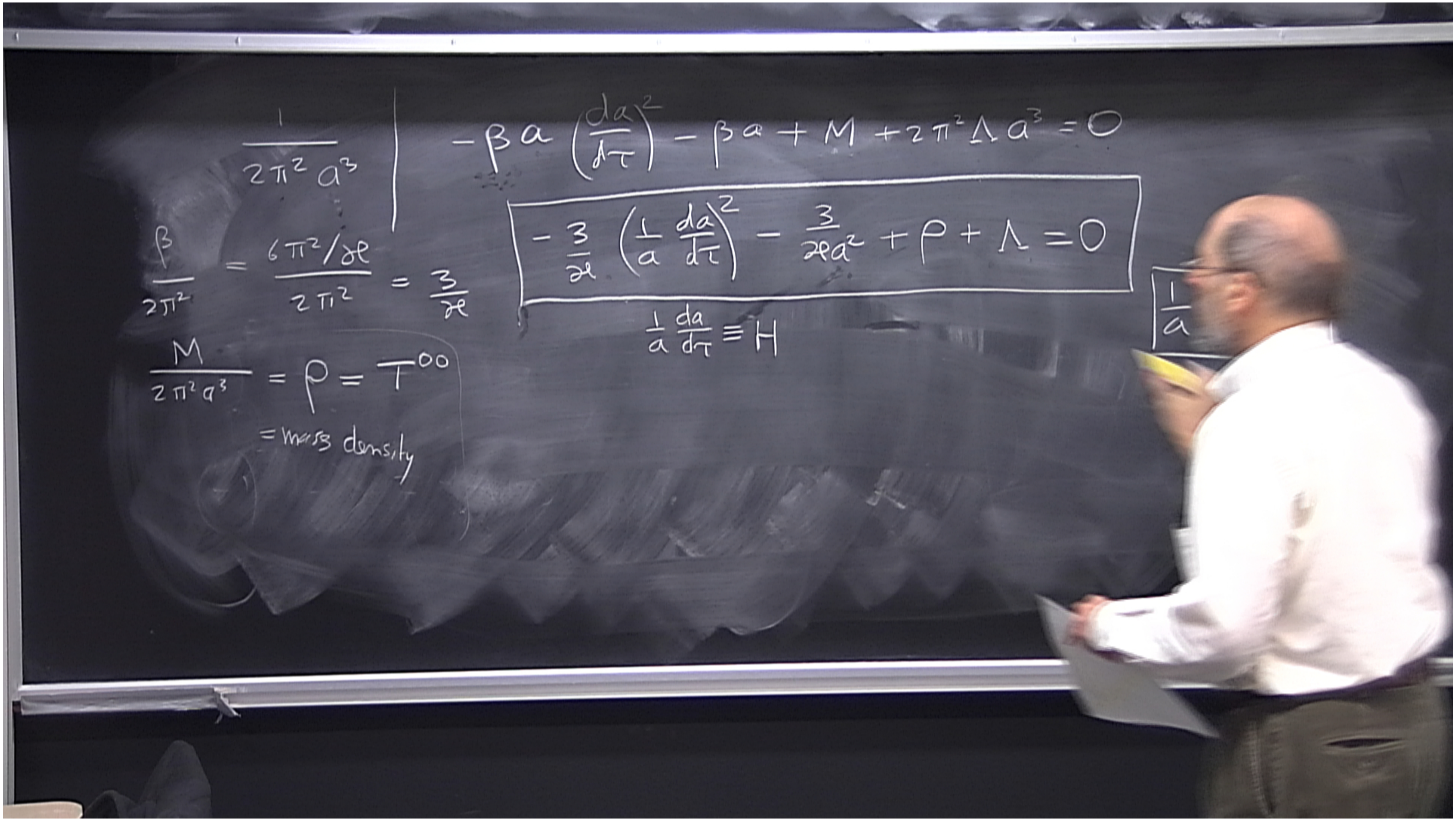
URL: http://pirsa.org/13010015

Abstract:

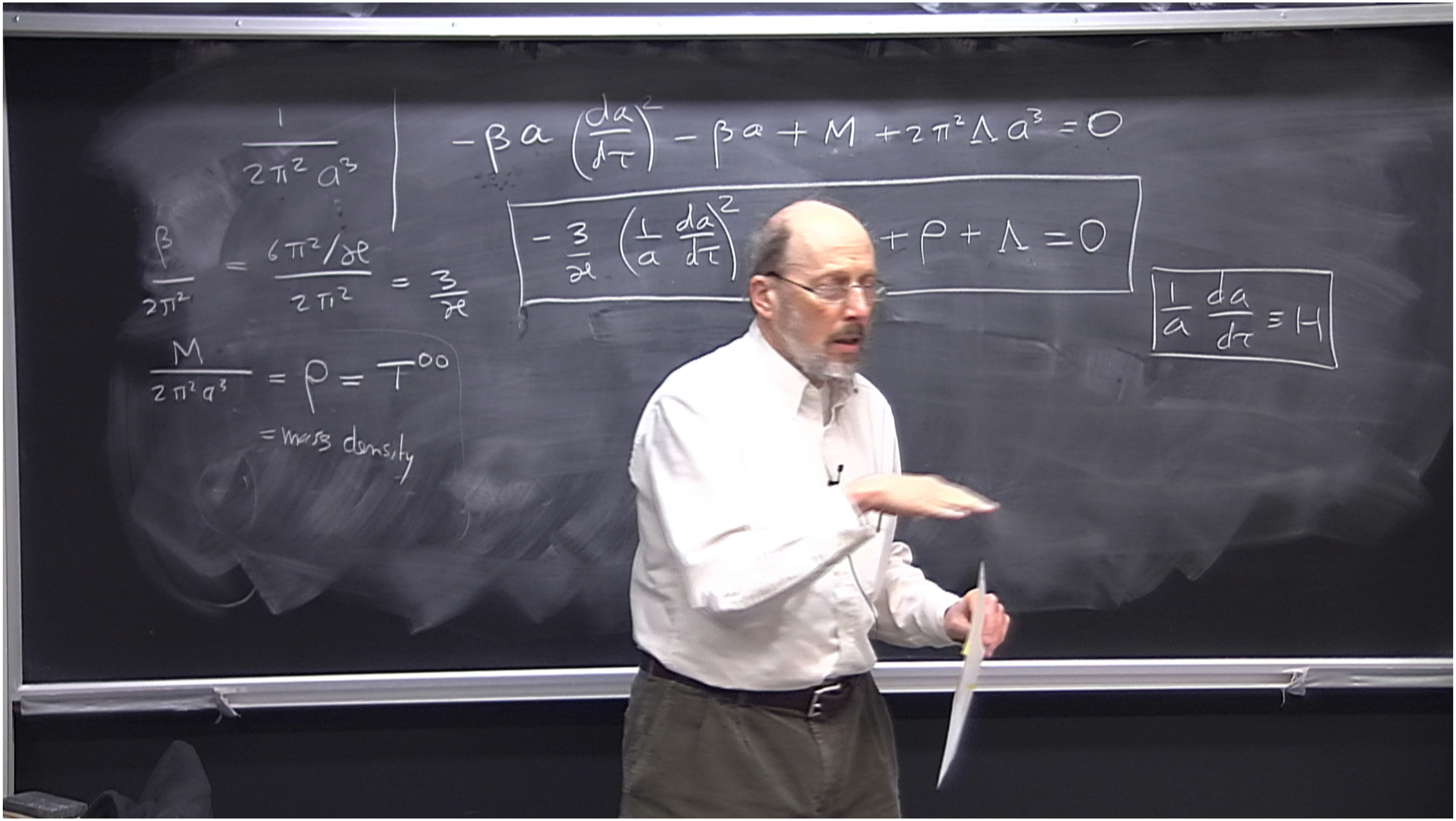

$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$

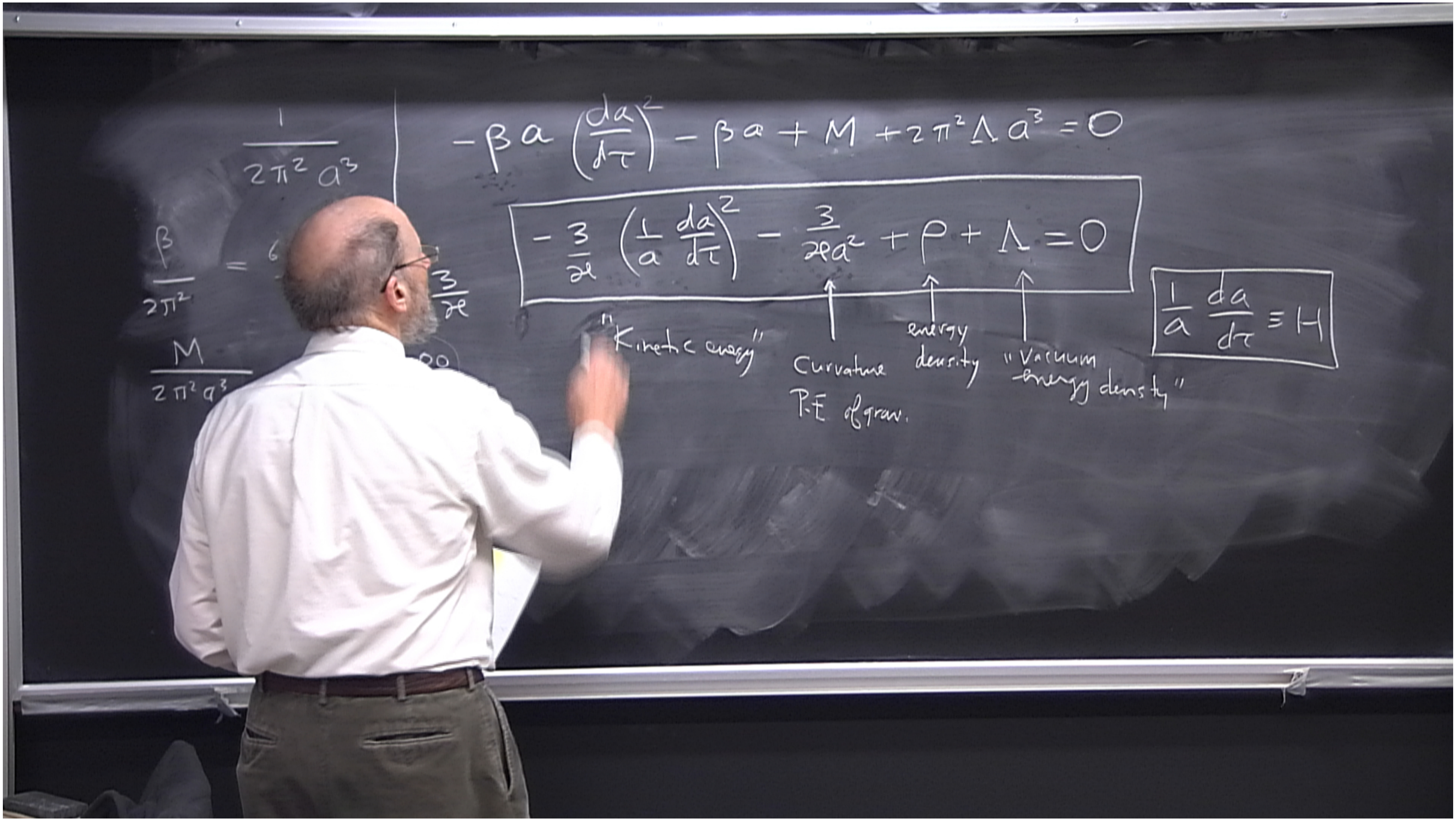


$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$
$\frac{1}{2\pi^2 a^3}$
$\frac{\beta}{2\pi^2}$
$\frac{M}{2\pi^2}$
$\frac{3}{\varkappa}$
$-\frac{3}{\varkappa}$



$\frac{1}{2\pi^2 a^3}$
$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$
$\boxed{-\frac{3}{\varkappa}\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 - \frac{3}{\varkappa a^2} + \rho + \Lambda = 0}$
$\frac{\beta}{2\pi^2} = \frac{6\pi^2/\varkappa}{2\pi^2} = \frac{3}{\varkappa}$
$\frac{1}{a}\frac{da}{d\tau} \equiv H$
$\frac{1}{a}$
$\frac{M}{2\pi^2 a^3} = \rho = T^{00}$
= mass density


1 / 2π² a³
−βa (da/dτ)² − βa + M + 2π² Λ a³ = 0
− 3/ϰ (1/a da/dτ)² + ρ + Λ = 0
1/a da/dτ = H
β / 2π² = (6π²/ϰ) / 2π² = 3/ϰ
M / 2π² a³ = ρ = T⁰⁰
= mass density


$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$
$-\frac{3}{\varkappa}\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 - \frac{3}{\varkappa a^2} + \rho + \Lambda = 0$
"Kinetic energy"
Curvature
P.E of grav.
energy density
"Vacuum energy density"
$\frac{1}{a}\frac{da}{d\tau} = H$
$\frac{1}{2\pi^2 a^3}$
$\frac{\beta}{2\pi^2} =$
$\frac{3}{\varkappa}$
$\frac{M}{2\pi^2 a^3}$

$$\frac{1}{2\pi^2 a^3}$$

$$\frac{\beta}{2\pi^2} = \frac{6\pi^2/\varkappa}{2\pi^2} = \frac{3}{\varkappa}$$

$$\frac{M}{2\pi^2 a^3} = \rho = T^{00}$$

= mass density

$$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$$

$$\boxed{-\frac{3}{\varkappa}\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 - \frac{3}{\varkappa a^2} + \rho + \Lambda = 0}$$

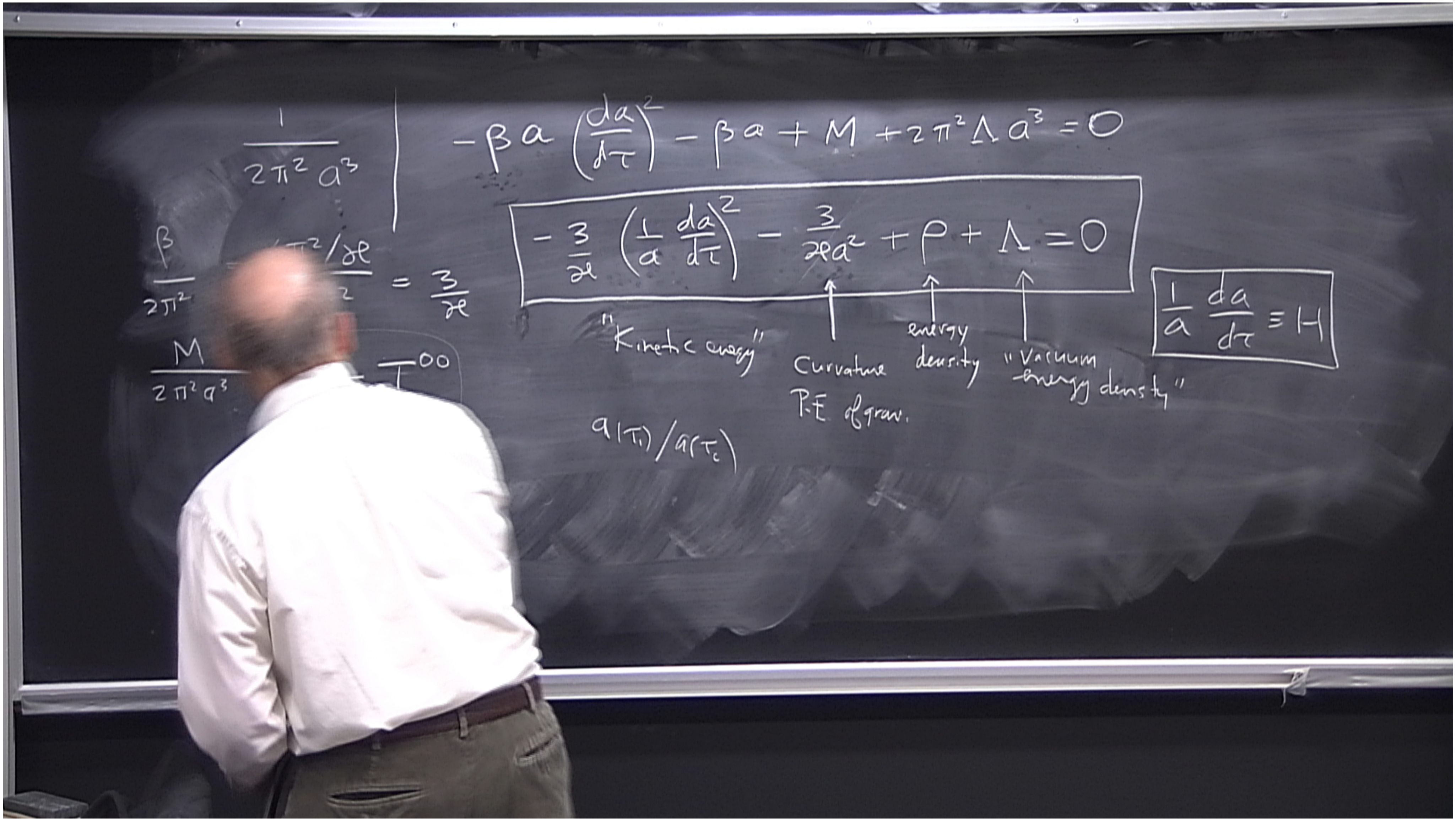
"Kinetic energy"

Curvature P.E of grav.

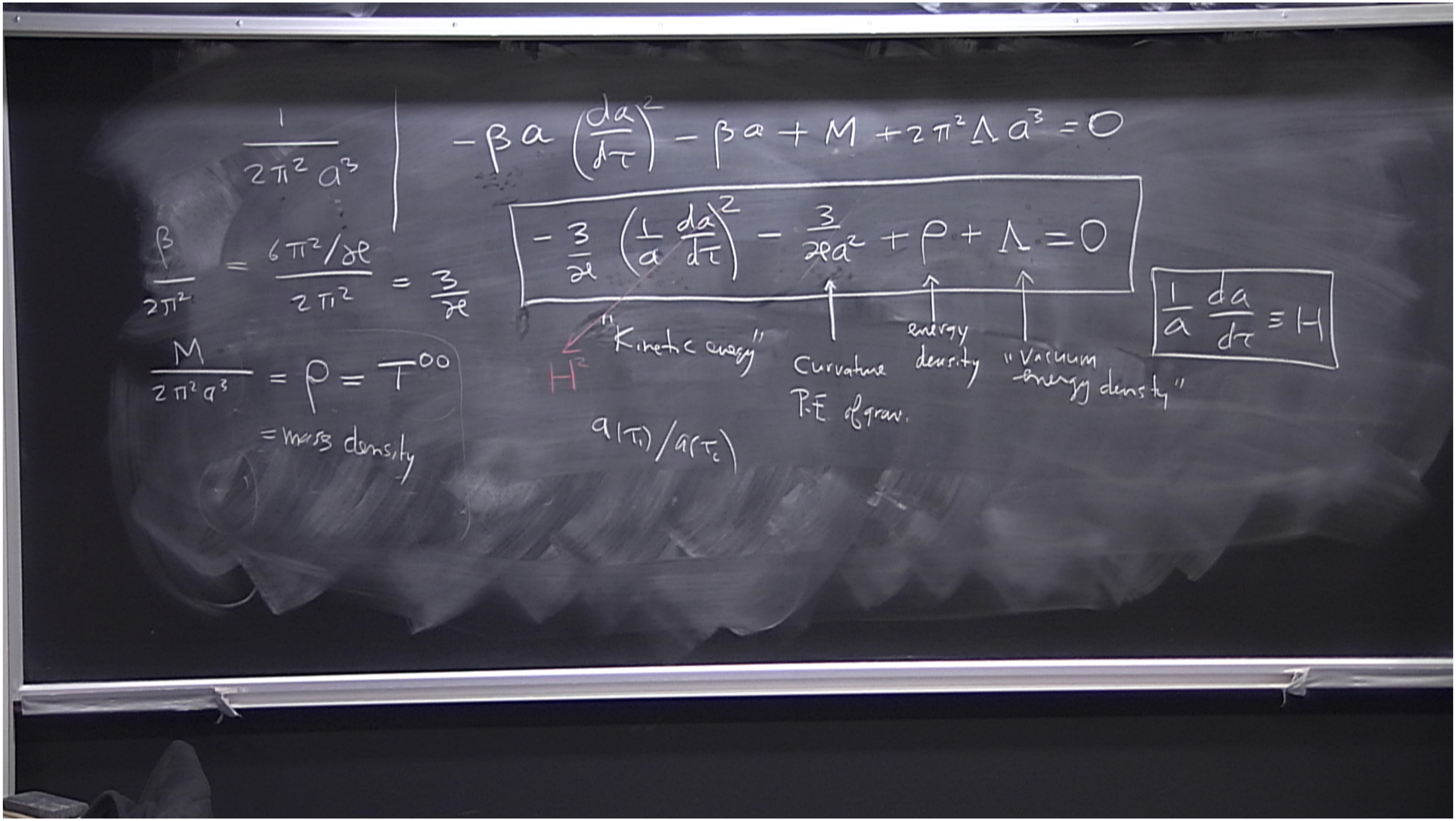
energy density

"Vacuum energy density"

$$\boxed{\frac{1}{a}\frac{da}{d\tau} = H}$$



$\frac{1}{2\pi^2 a^3}$
$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$
$-\frac{3}{\varkappa}\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 - \frac{3}{\varkappa a^2} + \rho + \Lambda = 0$
$= \frac{3}{\varkappa}$
$\frac{1}{a}\frac{da}{d\tau} = H$
"Kinetic energy"
Curvature
P.E. of grav.
energy density
"Vacuum energy density"
$\frac{M}{2\pi^2 a^3}$
$a(\tau_1)/a(\tau_2)$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$\frac{M}{2\pi^2 a^3} = \rho = T^{00}$
= mass density
H^2
"Kinetic energy"
Curvature
P.E. of grav.
energy
density
"Vacuum
energy density"
$a(\tau_1)/a(\tau_2)$

$$\frac{1}{2\pi^2 a^3}$$

$$\frac{\beta}{2\pi^2} = \frac{6\pi^2/\varkappa}{2\pi^2} = \frac{3}{\varkappa}$$

$$\frac{M}{2\pi^2 a^3} = \rho = T^{00}$$

= mass density

$$-\beta a \left(\frac{da}{d\tau}\right)^2 - \beta a + M + 2\pi^2 \Lambda a^3 = 0$$

$$\boxed{-\frac{3}{\varkappa}\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 - \frac{3}{\varkappa a^2} + \rho + \Lambda = 0}$$

H^2 "Kinetic energy"

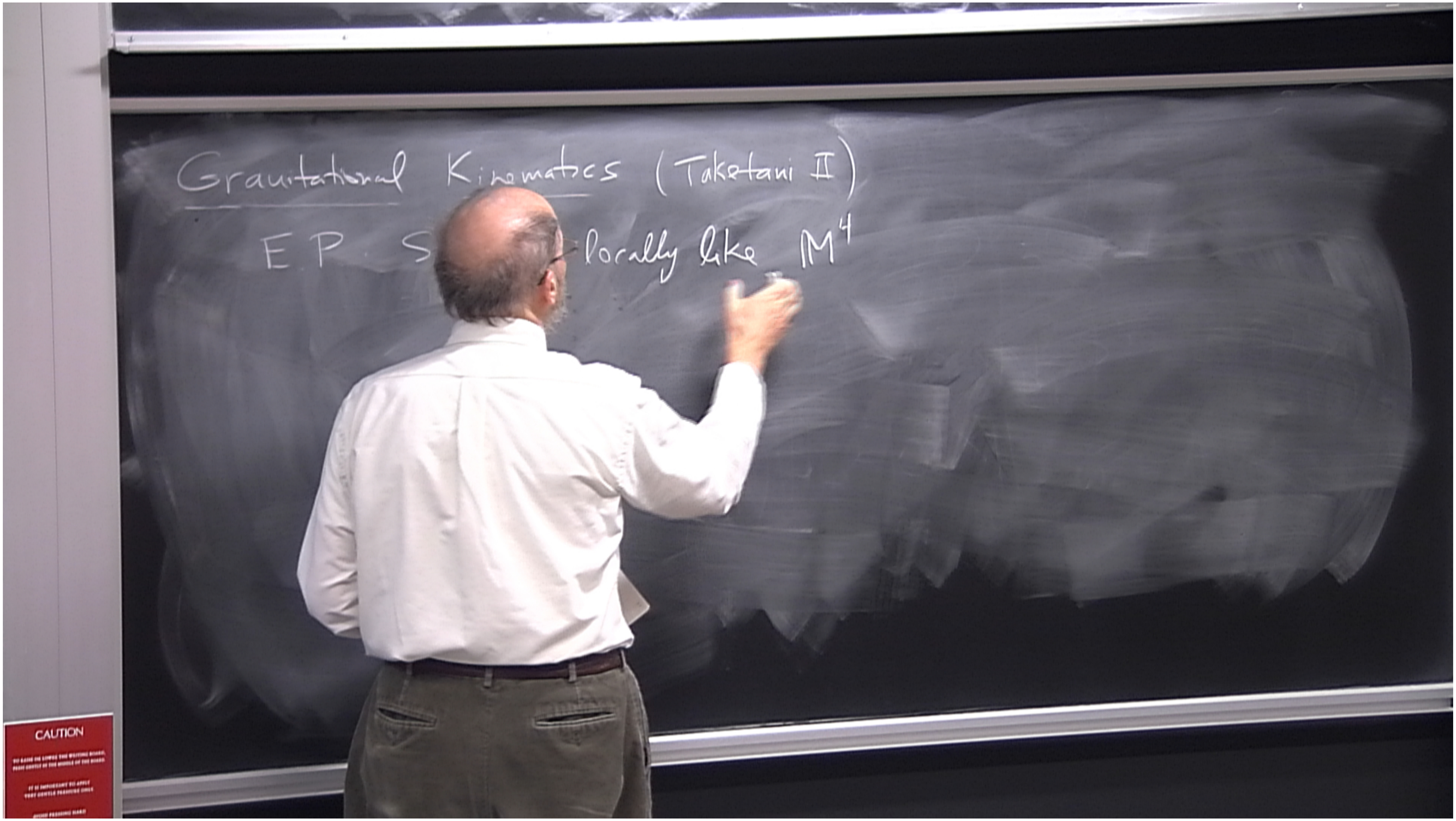
Curvature P.E. of grav.

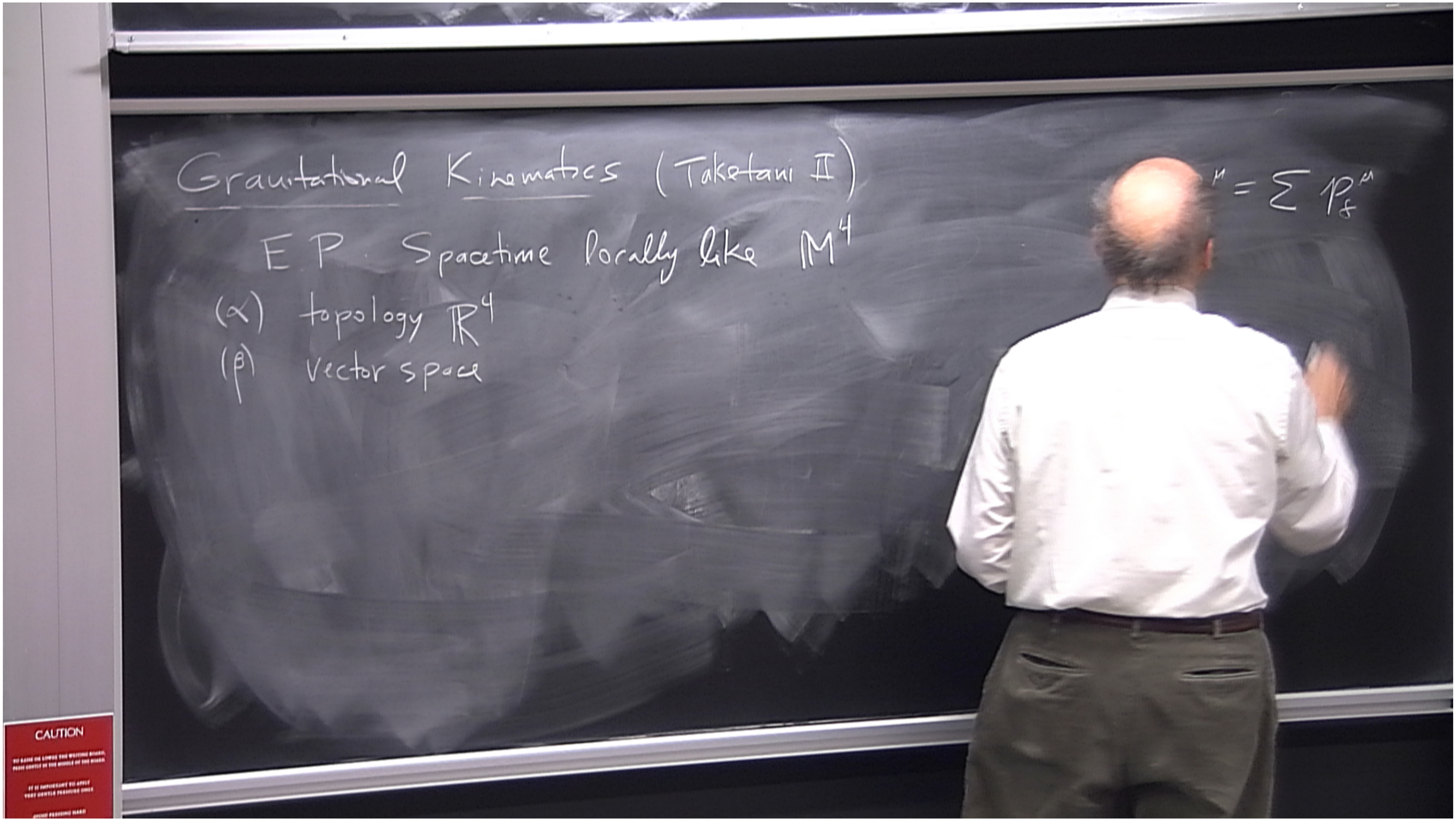
energy density

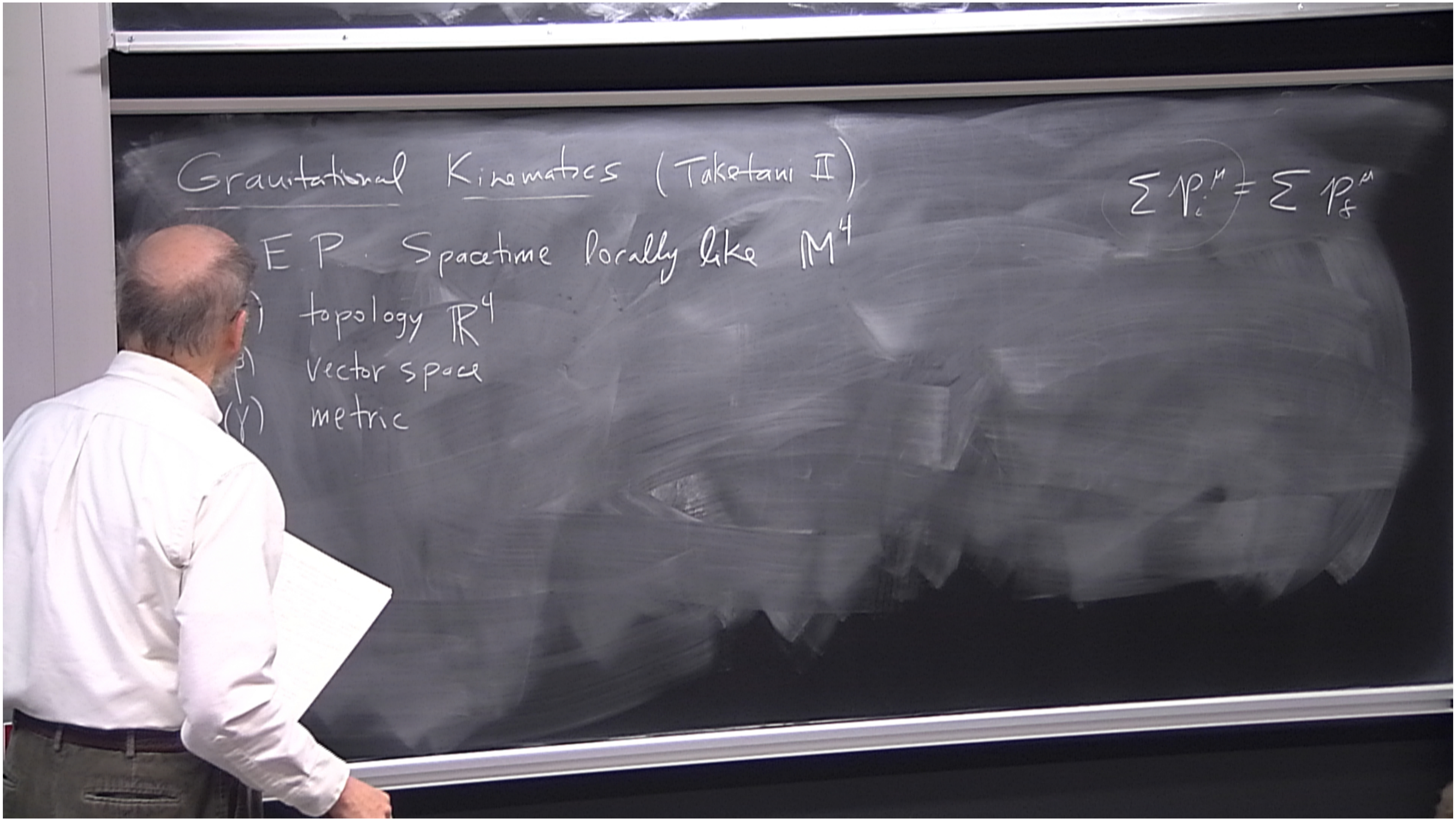
"Vacuum energy density"

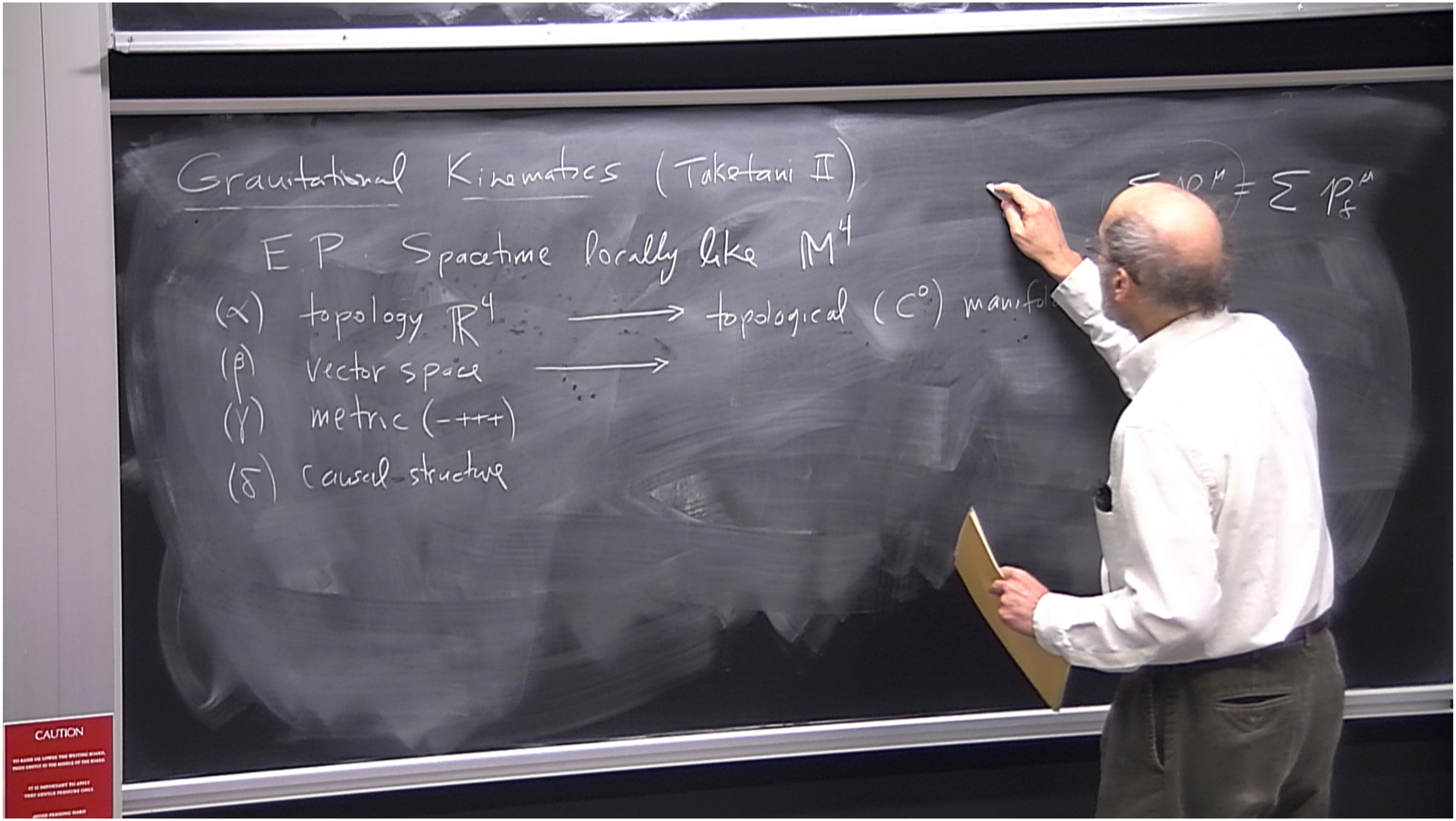
$$\boxed{\frac{1}{a}\frac{da}{d\tau} \equiv H}$$

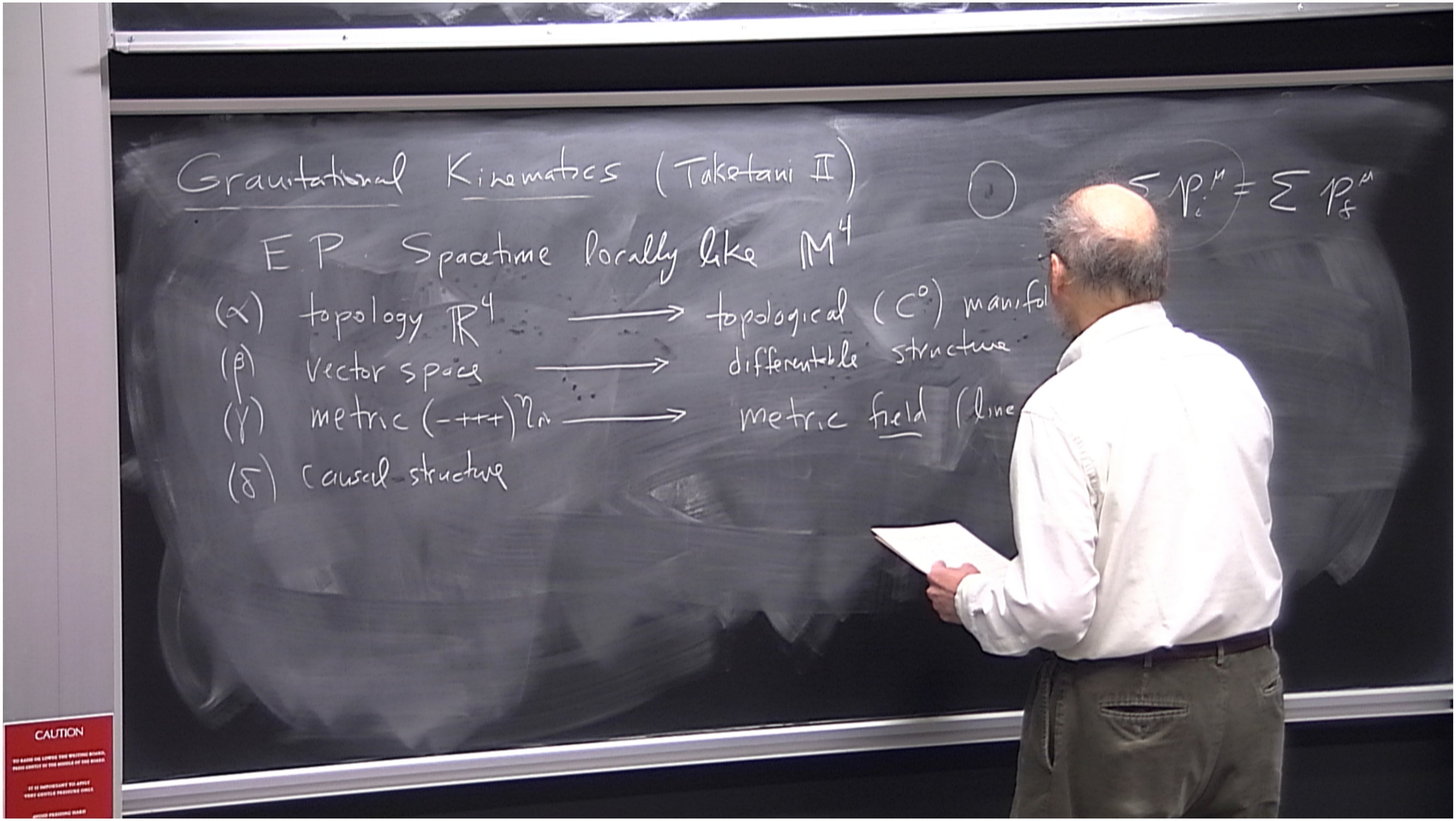
$$a(\tau_1)/a(\tau_2)$$

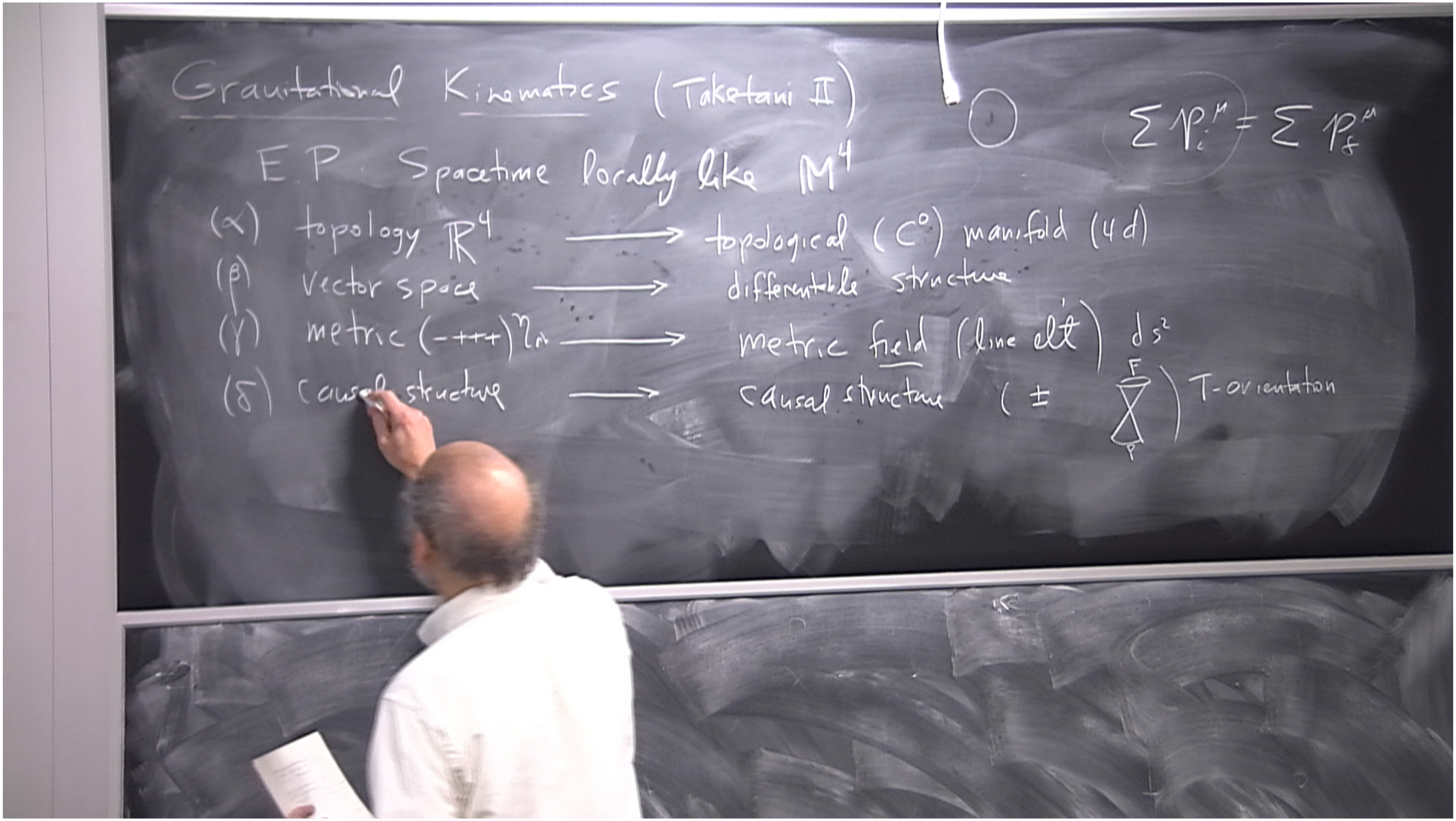

Gravitational Kinematics (Taketani II)
E.P.
locally like M^4


Gravitational Kinematics (Taketani II)
E.P. Spacetime locally like M^4
(α) topology R^4
(β) vector space
CAUTION

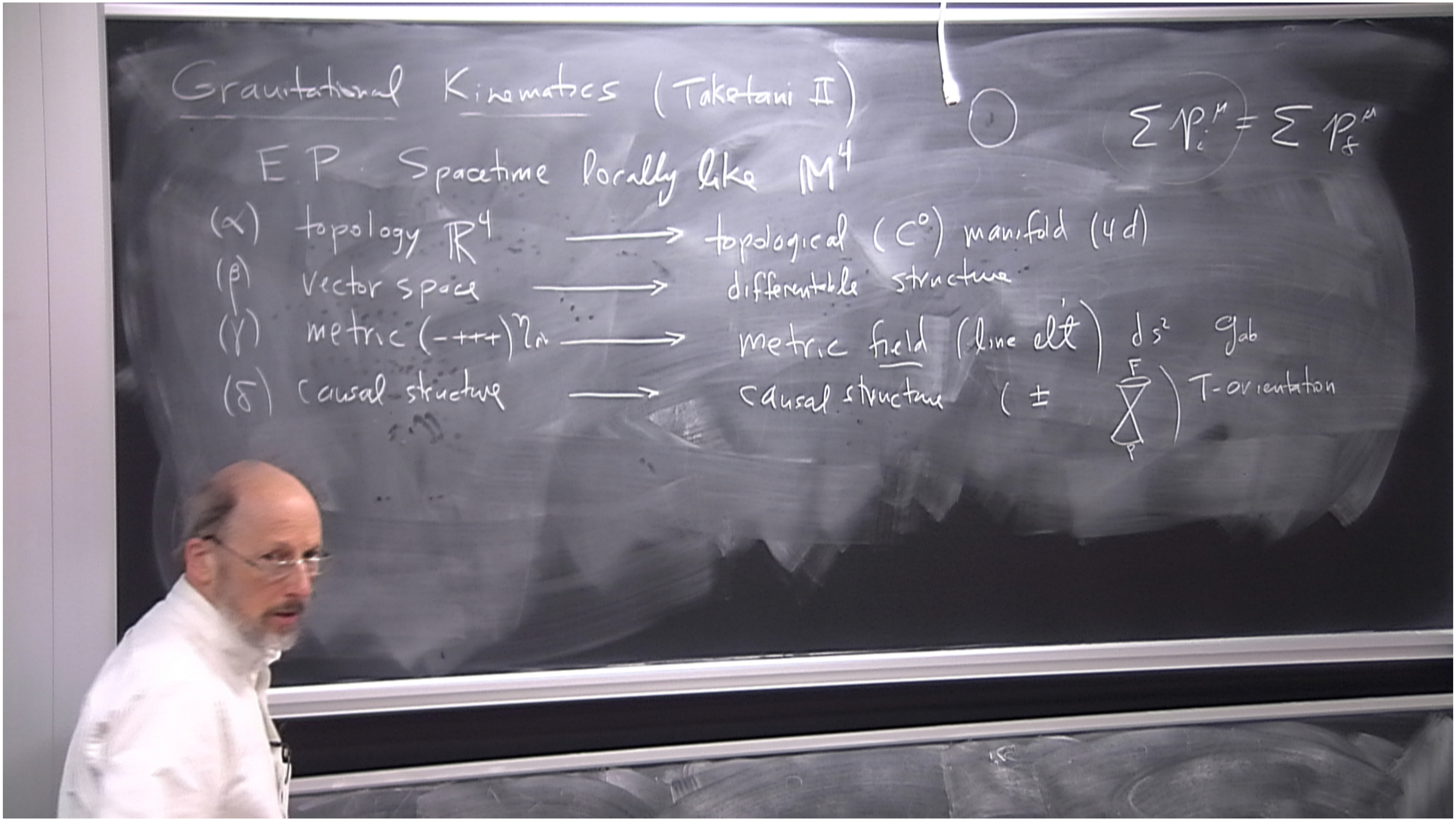

Gravitational Kinematics (Taketani II)
$\sum p_i^\mu = \sum p_f^\mu$
EP: Spacetime locally like $\mathbb{M}^4$
topology $\mathbb{R}^4$
vector space
(γ) metric


Gravitational Kinematics (Taketani II)
E.P. Spacetime locally like $\mathbb{M}^4$
(α) topology $\mathbb{R}^4$ → topological (C^0) manifo
(β) vector space →
(γ) metric (−+++)
(δ) causal structure
$= \Sigma P_f^\mu$
CAUTION


Gravitational Kinematics (Taketani II)
EP: Spacetime locally like M^4
(α) topology $\mathbb{R}^4$ → topological (C^0) manifol
(β) vector space → differentiable structure
(γ) metric (−+++) $\eta_{\mu\nu}$ → metric field (line
(δ) causal structure
$\Sigma p_i^\mu = \Sigma p_f^\mu$
CAUTION



Gravitational Kinematics (Taketani II)
$\Sigma p_i^\mu = \Sigma p_f^\mu$
E.P. Spacetime locally like $\mathbb{M}^4$
(α) topology $\mathbb{R}^4$ → topological (C^0) manifold (4d)
(β) vector space → differentiable structure
(γ) metric (-+++) $\eta_{\mu\nu}$ → metric field (line elt) ds^2
(δ) causal structure → causal structure ($\pm$) T-orientation
F
P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$\sum p_i^\mu = \sum p_f^\mu$
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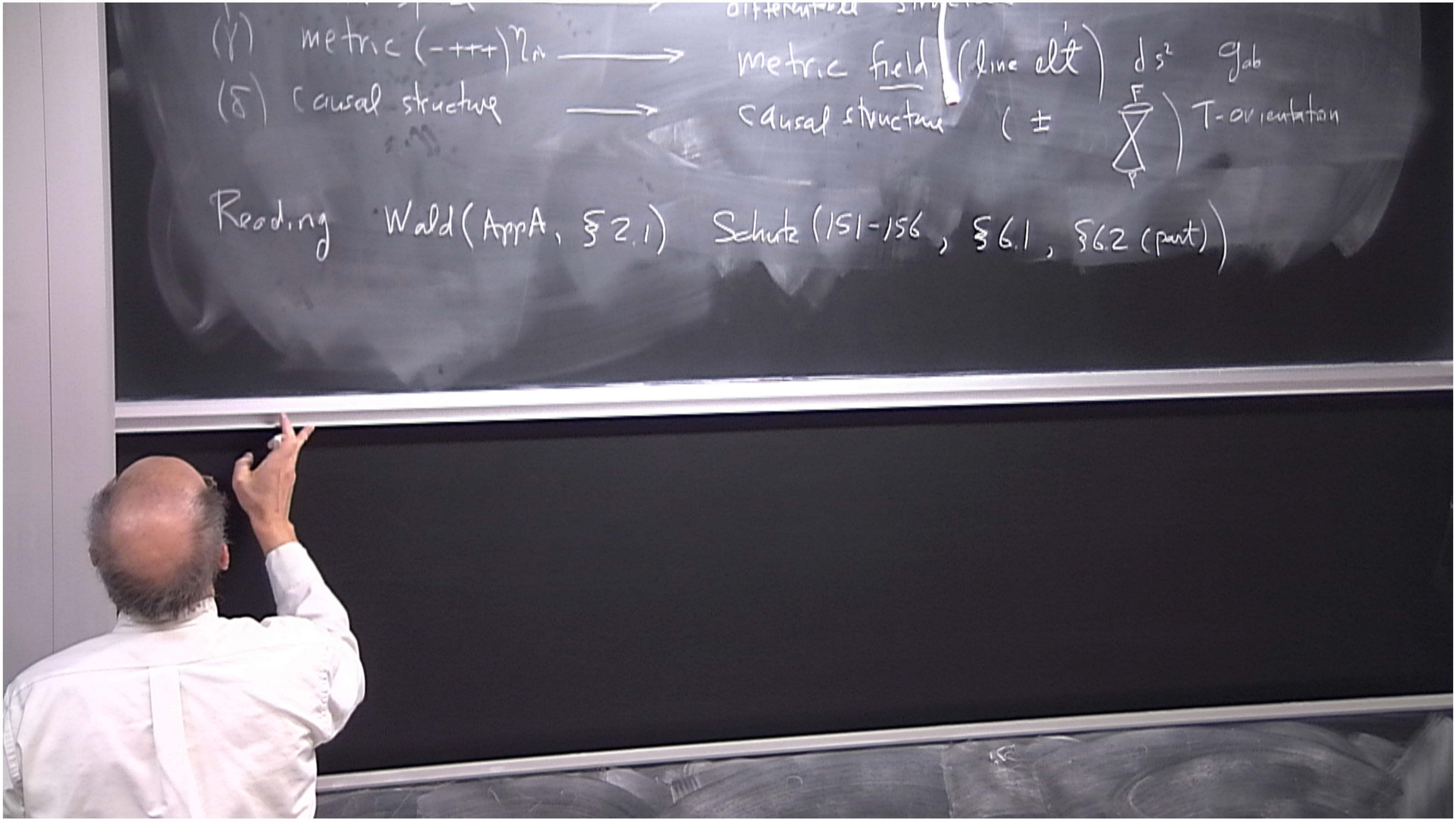
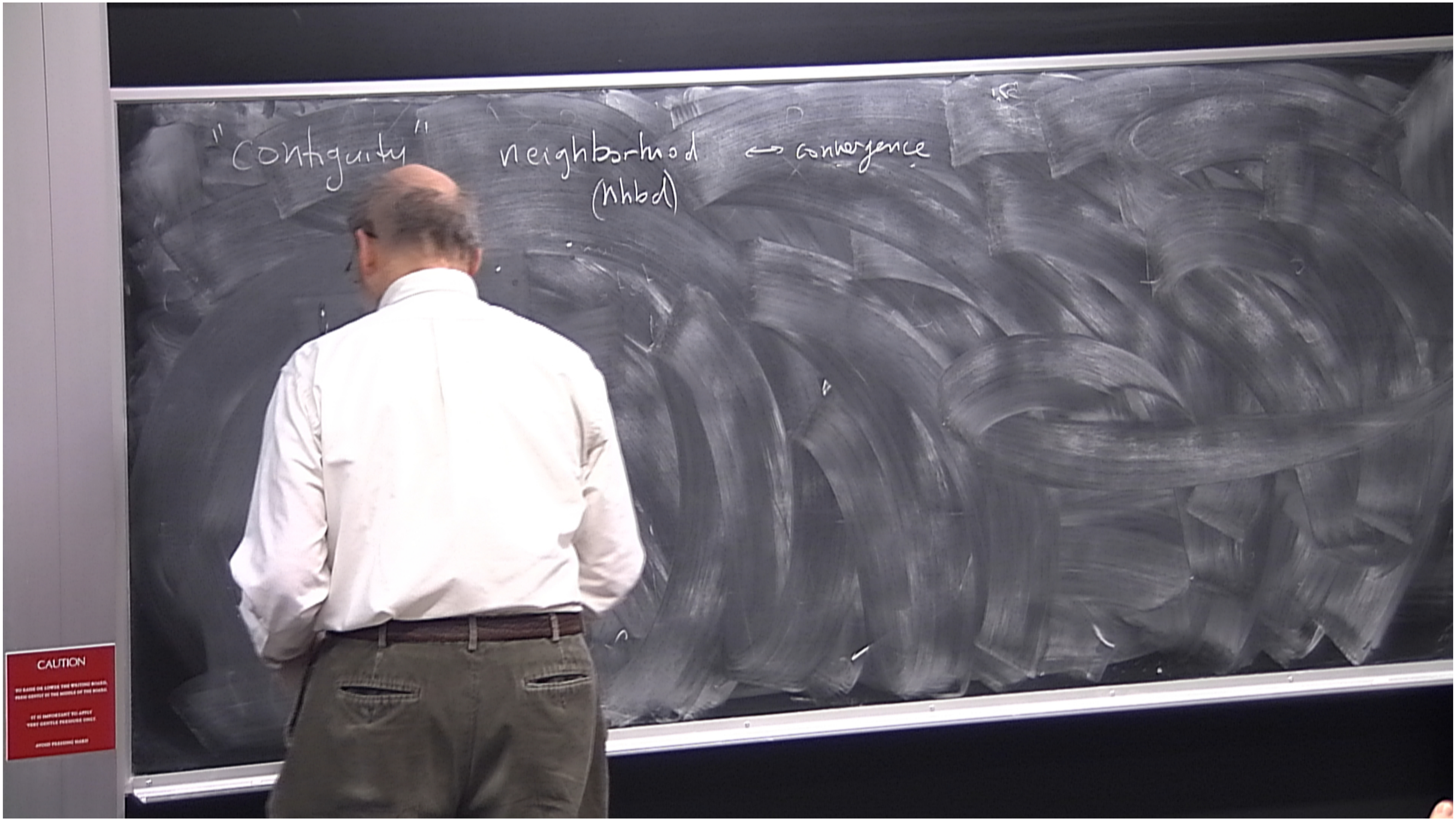
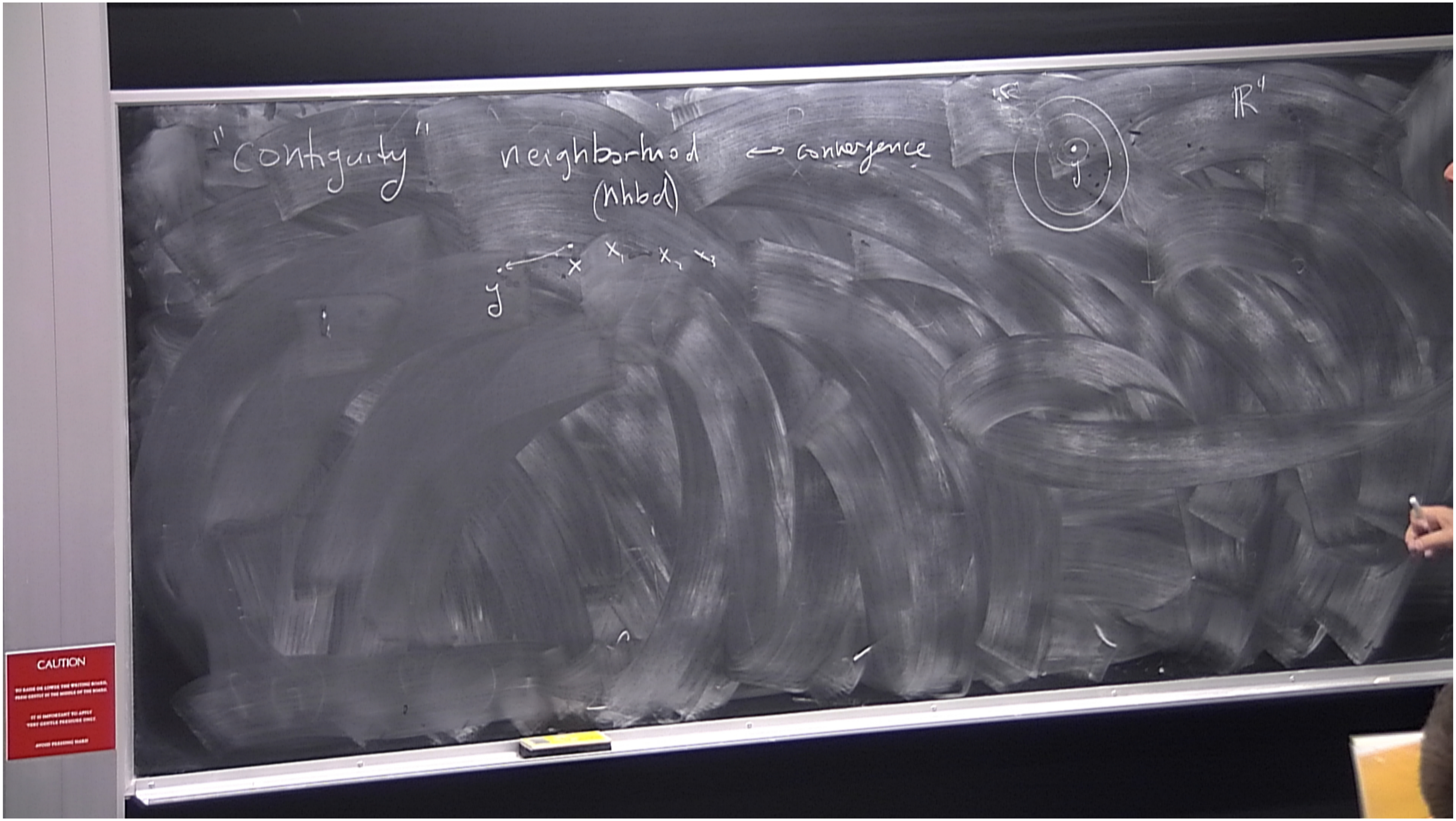
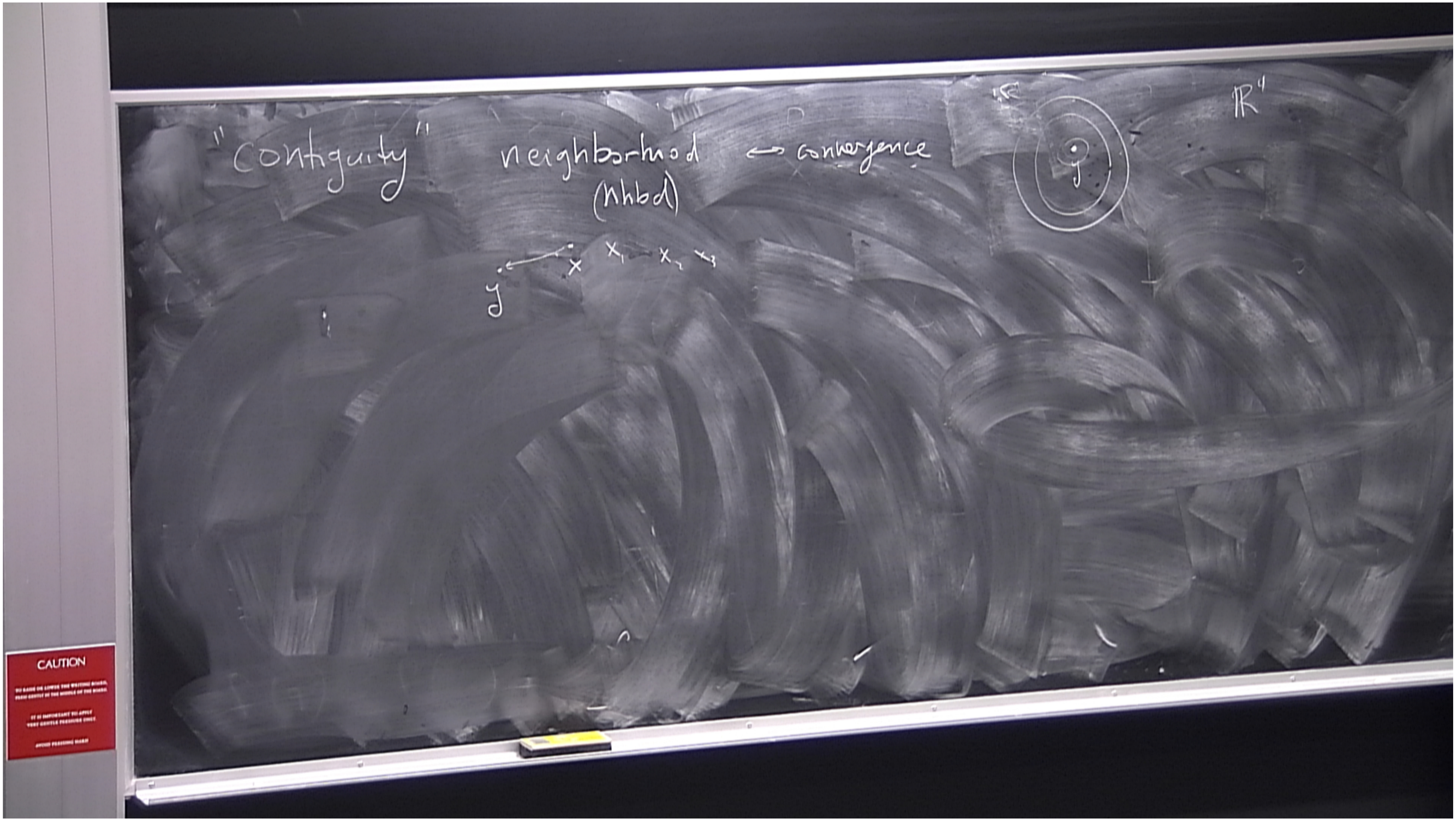
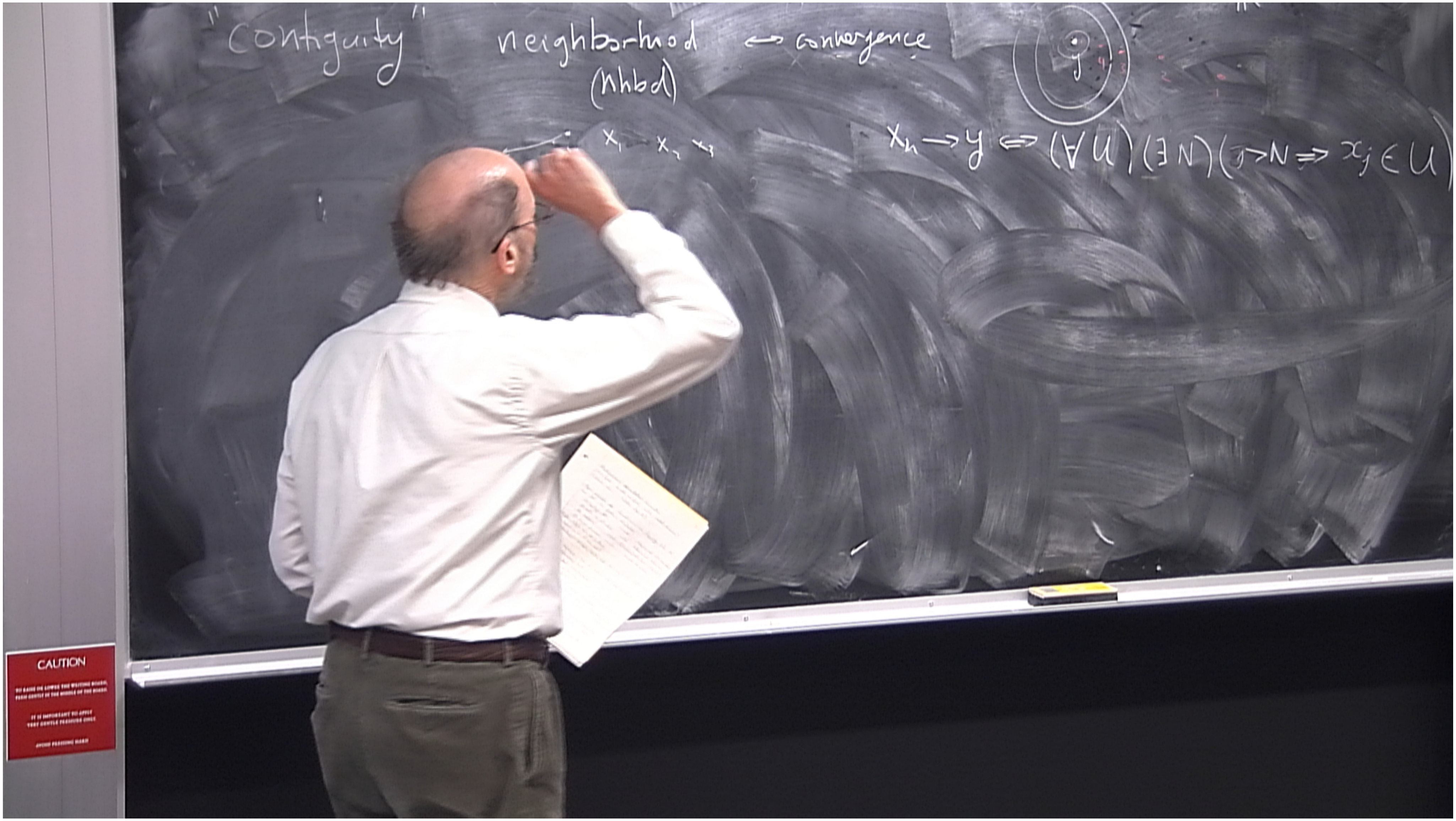
(γ) metric $(-+++)\,\eta_{\mu\nu}$ ⟶ metric field (line elt) ds^2 g_{ab}
(δ) causal structure ⟶ causal structure (± F P) T-orientation

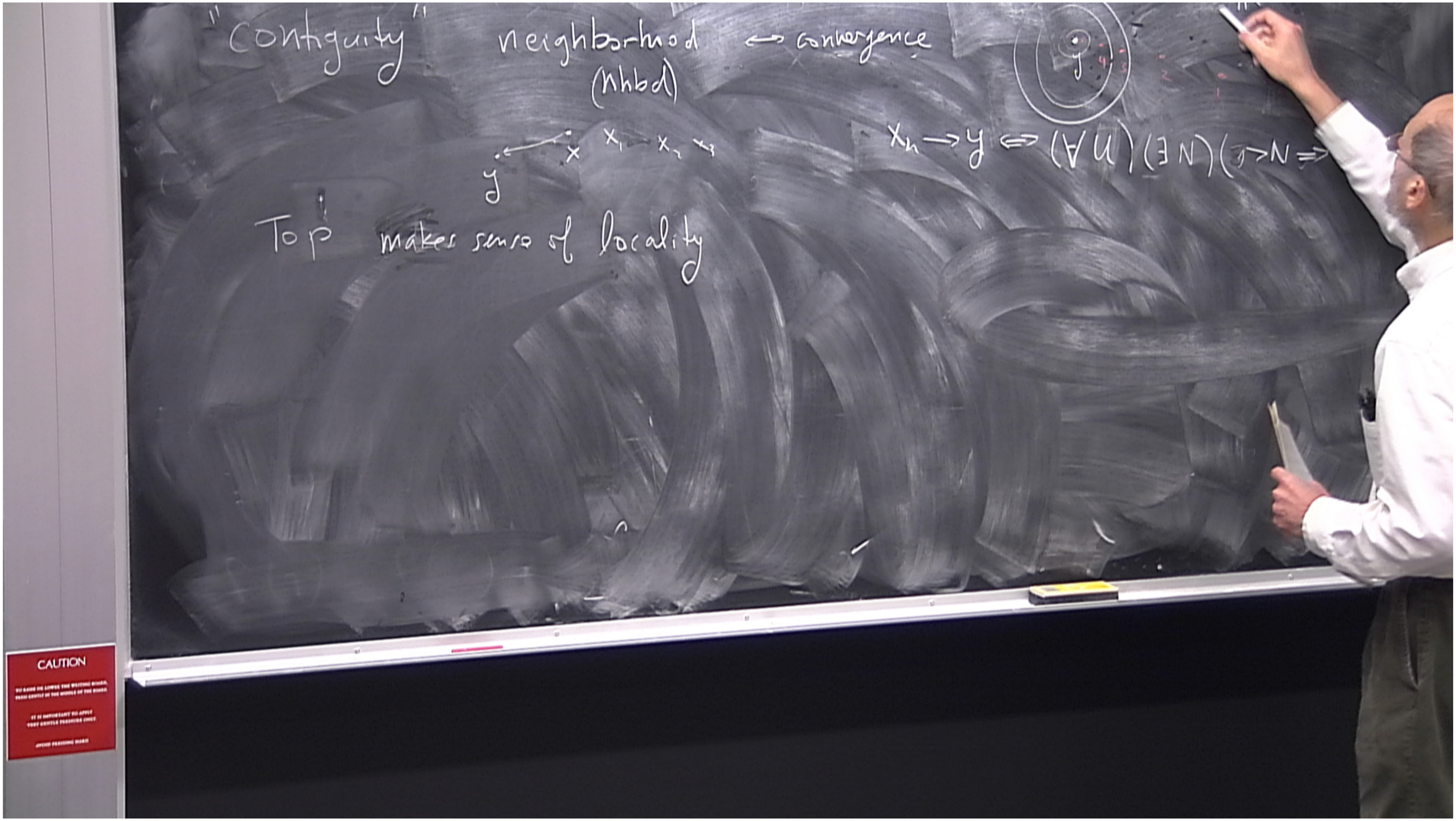

(γ) metric (-+++) η_ab → metric field (line elt) ds² g_ab
(δ) causal structure → causal structure (± F P) T-orientation
Reading Wald (App A, §2.1) Schutz (151-156, §6.1, §6.2 (part))

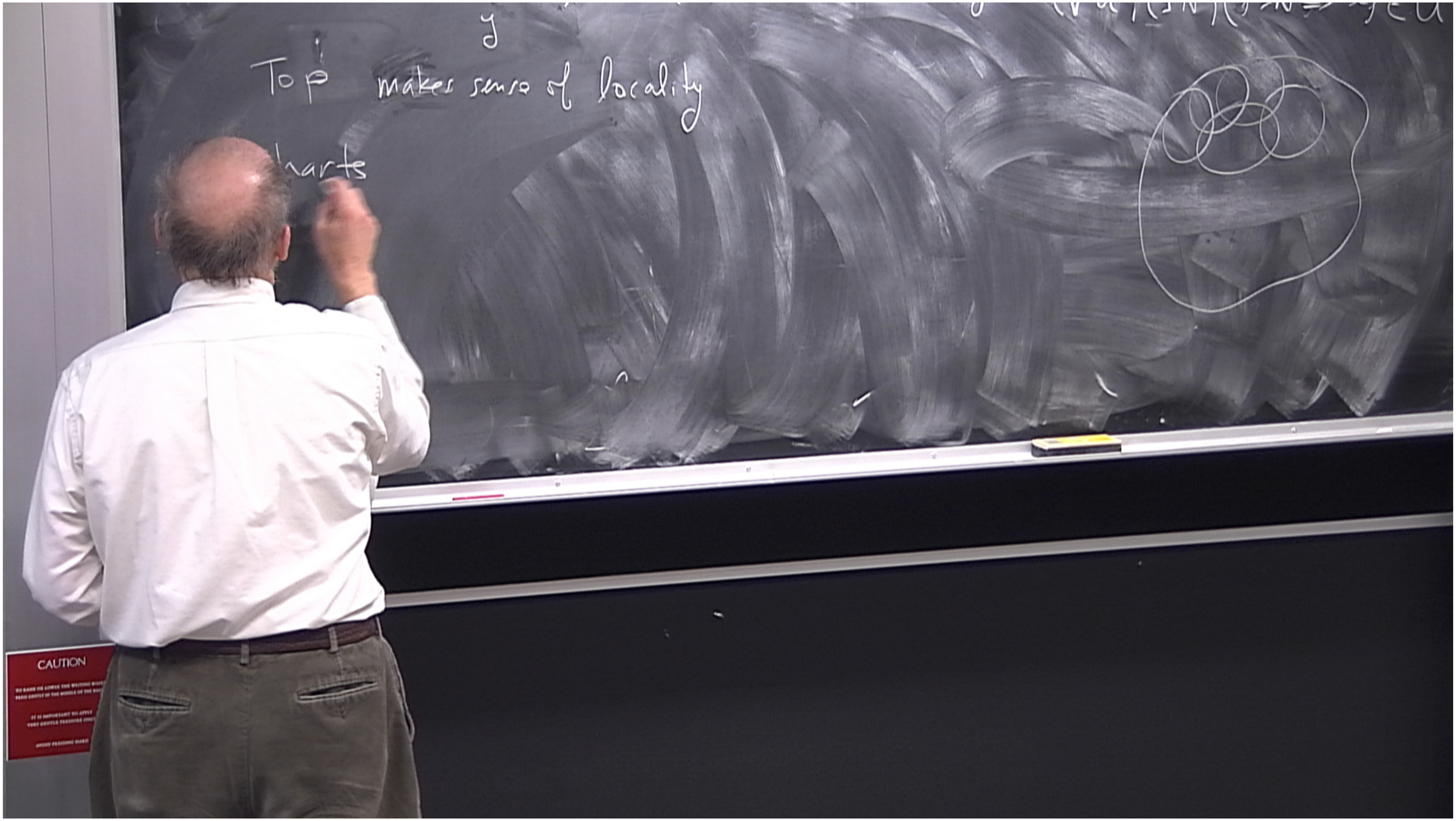

"contiguity"
neighborhood ⟷ convergence
(nhbd)
CAUTION

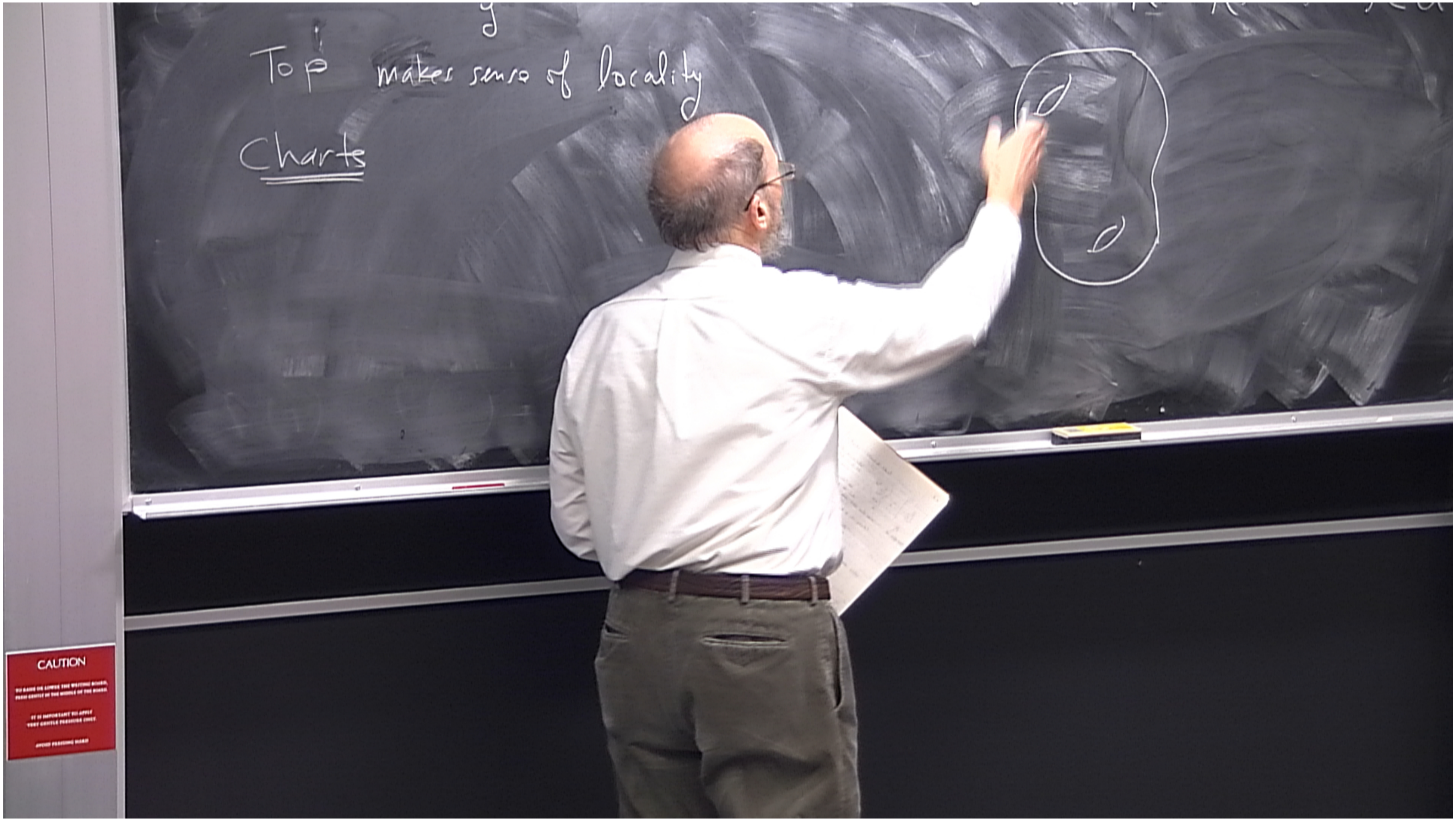

"contiguity"
neighborhood
(nhbd)
convergence
CAUTION

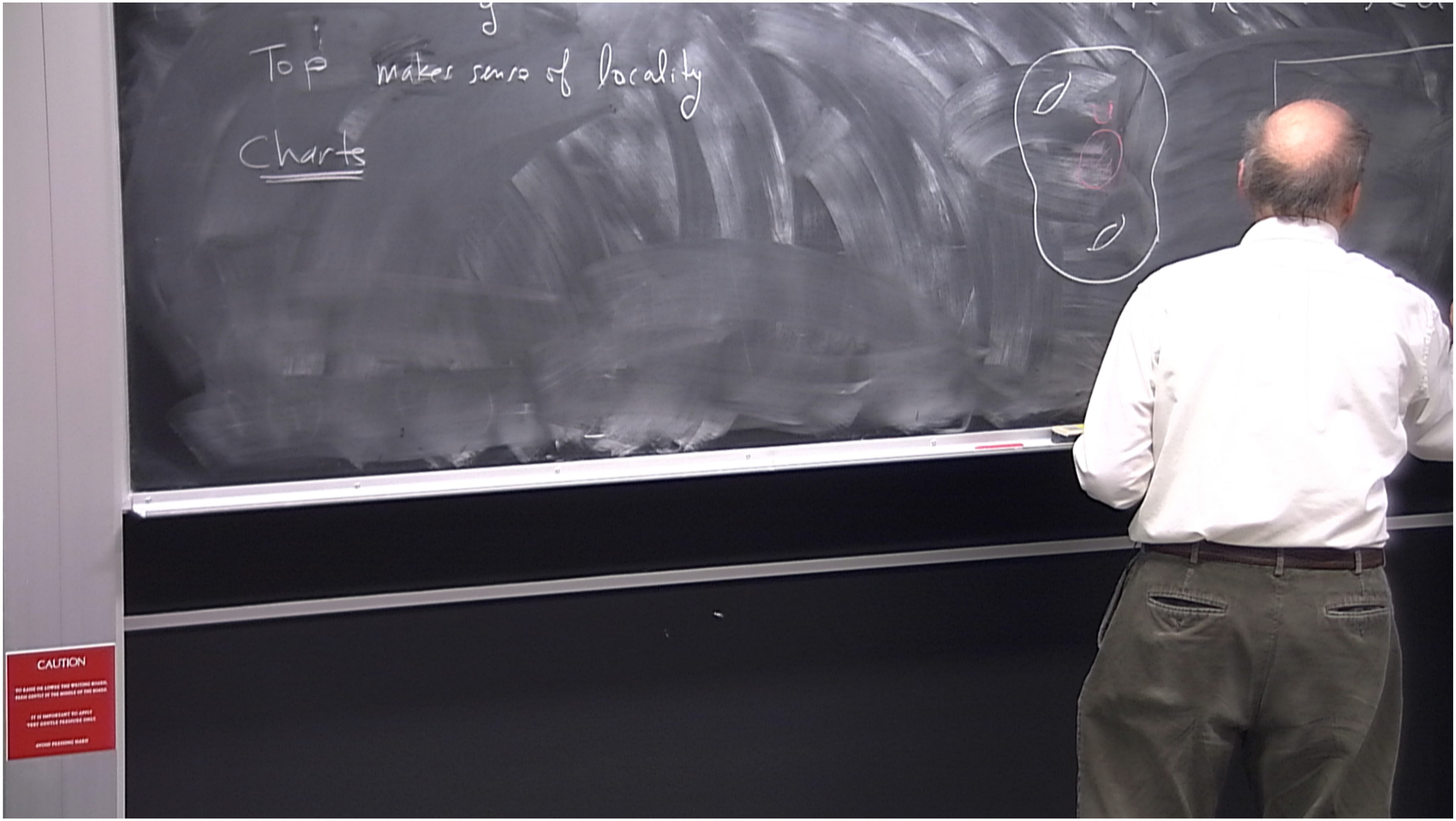

"contiguity"
neighborhood
(nbhd)
⟷ convergence
y
x
x_1
x_2
x_3
$\mathbb{R}^4$
CAUTION

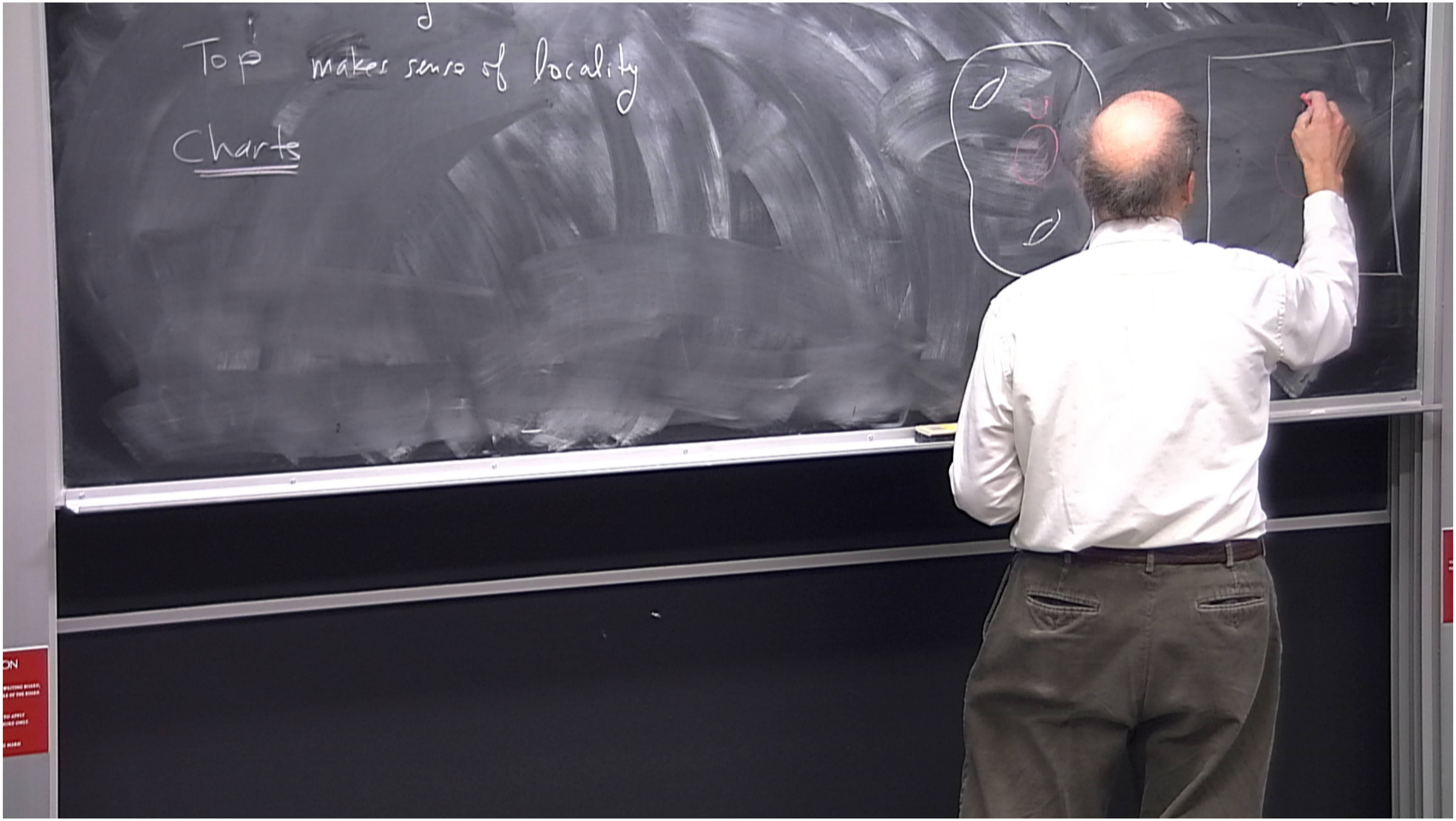

"Contiguity"
neighborhood (nhbd) ⟺ convergence
x1 x2 x3
xn → y ⟺ (∀U)(∃N)(j>N ⇒ xj ∈ U)
CAUTION

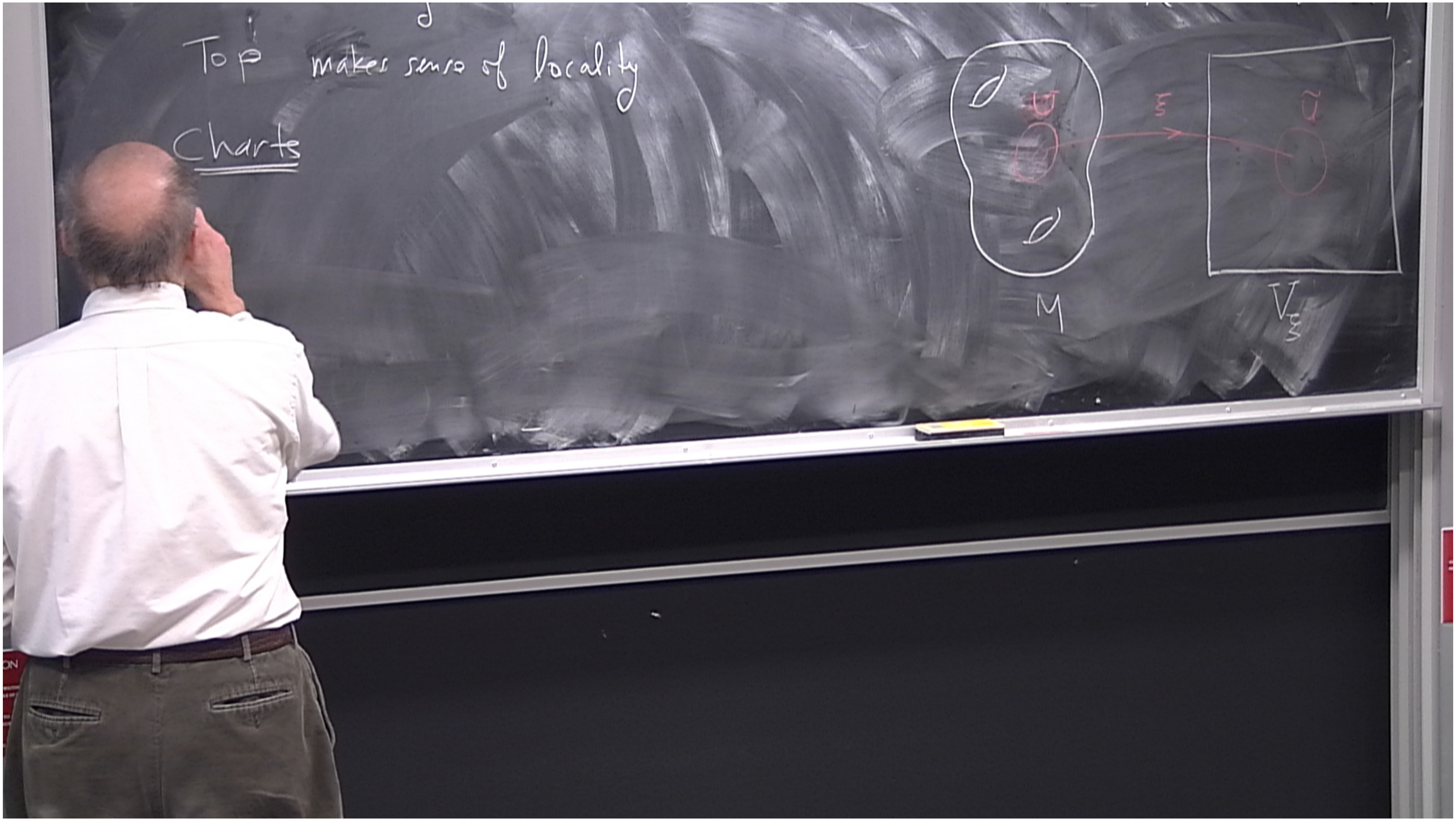

"Contiguity"
neighborhood ⟺ convergence
(nhbd)
Top makes sense of locality
xn → y ⟺ (∀U)(∃N)(n>N ⇒
CAUTION


Top makes sense of locality
CAUTION


Top makes sense of locality
Charts
CAUTION


Top makes sense of locality
Charts
CAUTION

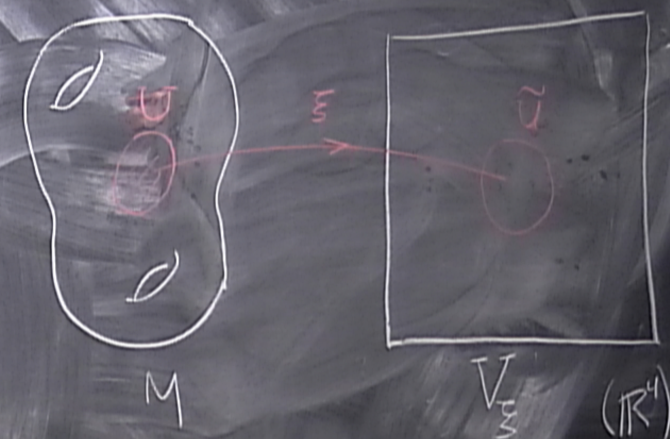

Top makes sense of locality
Charts

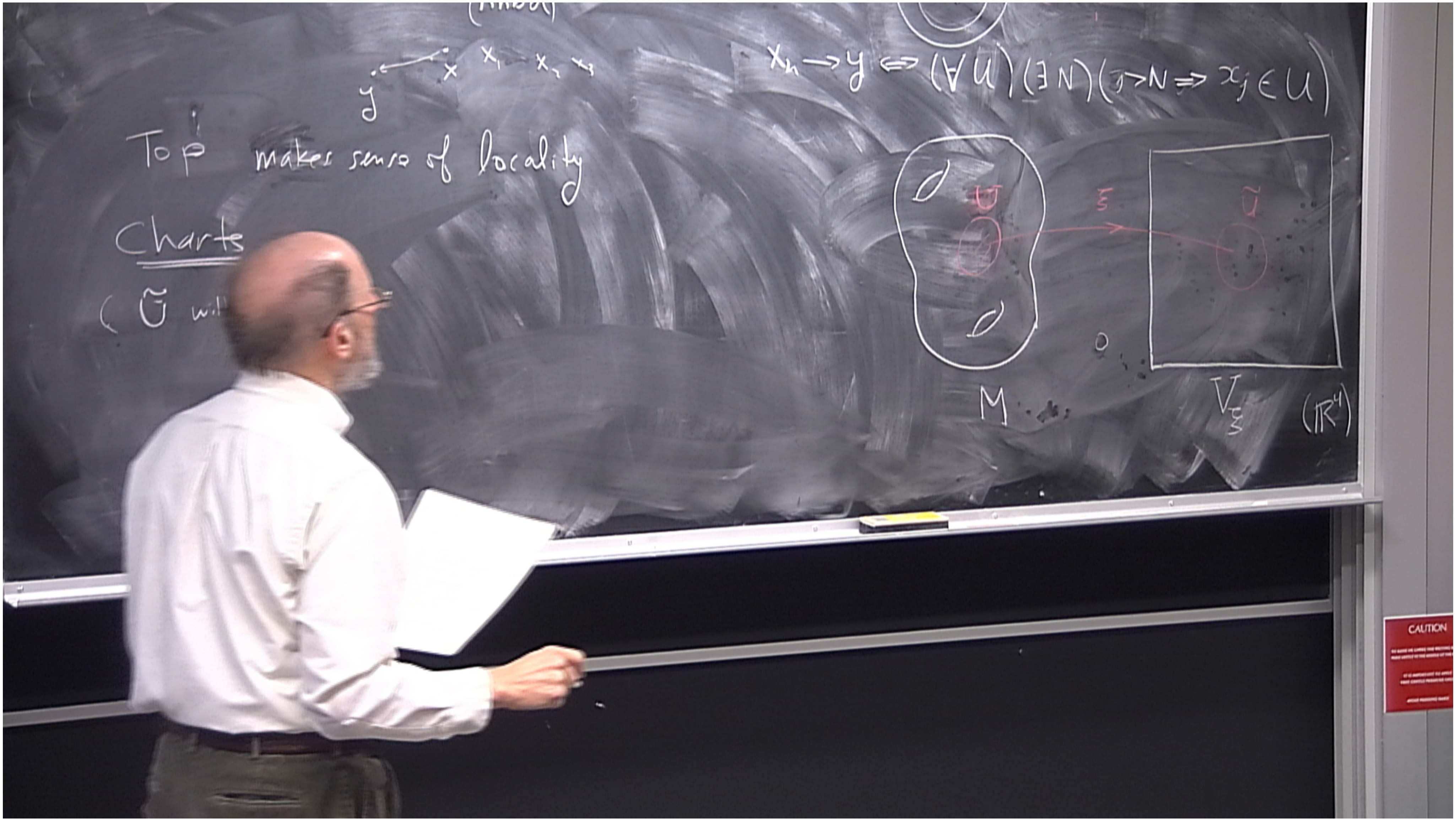

Top makes sense of locality
Charts
U
ξ
Ũ
M
V_ξ

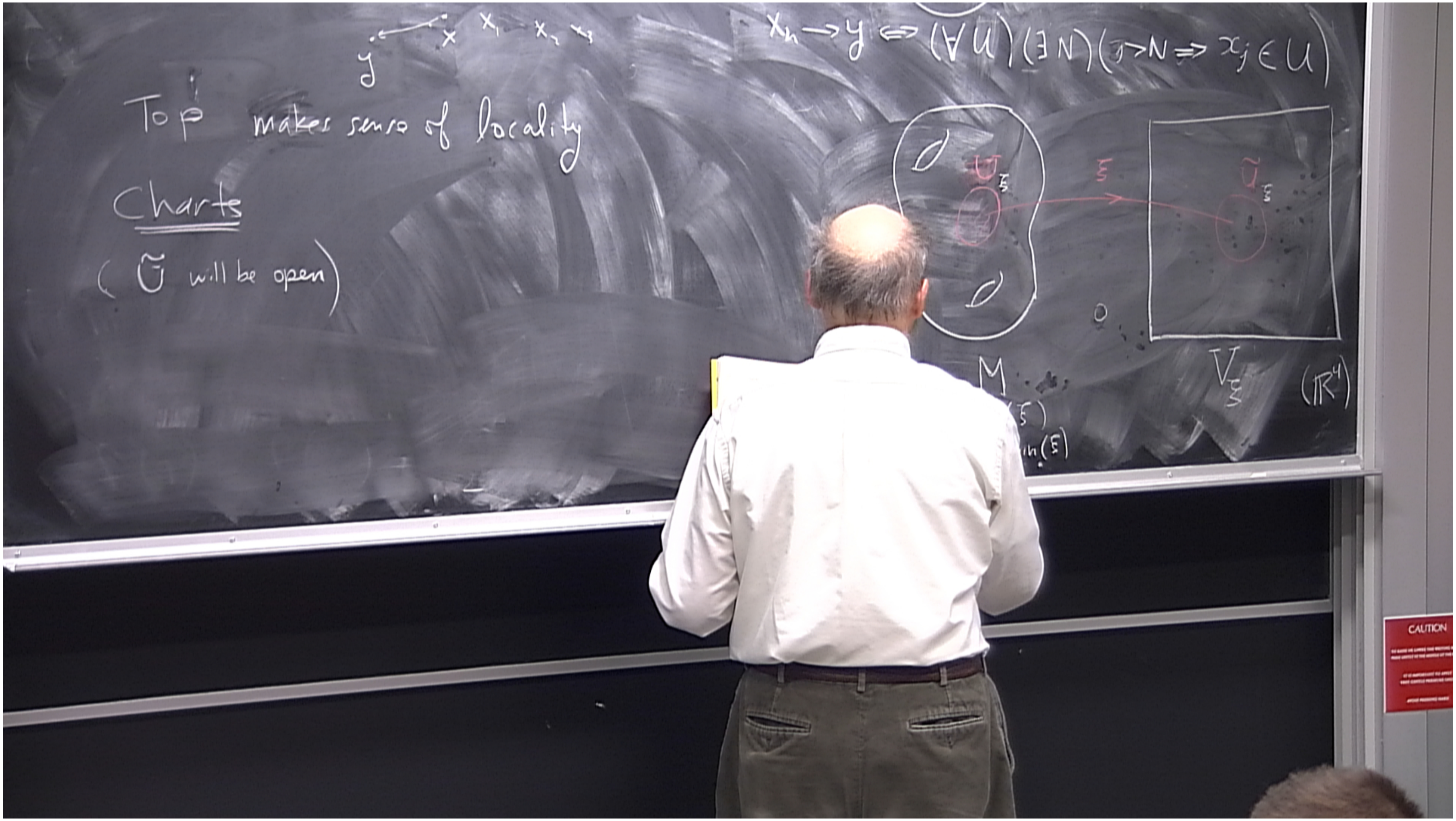
Top makes sense of locality

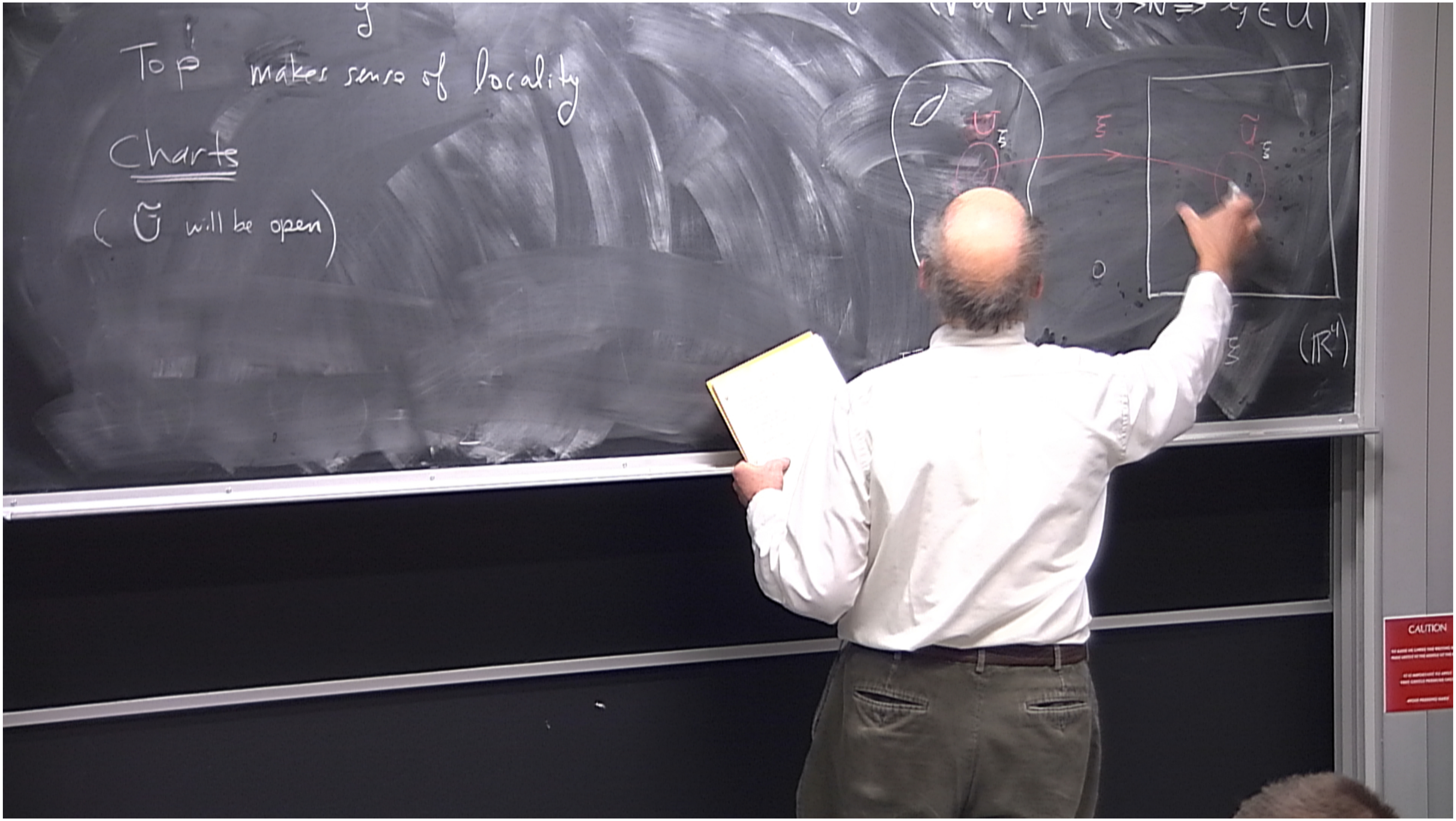
Charts

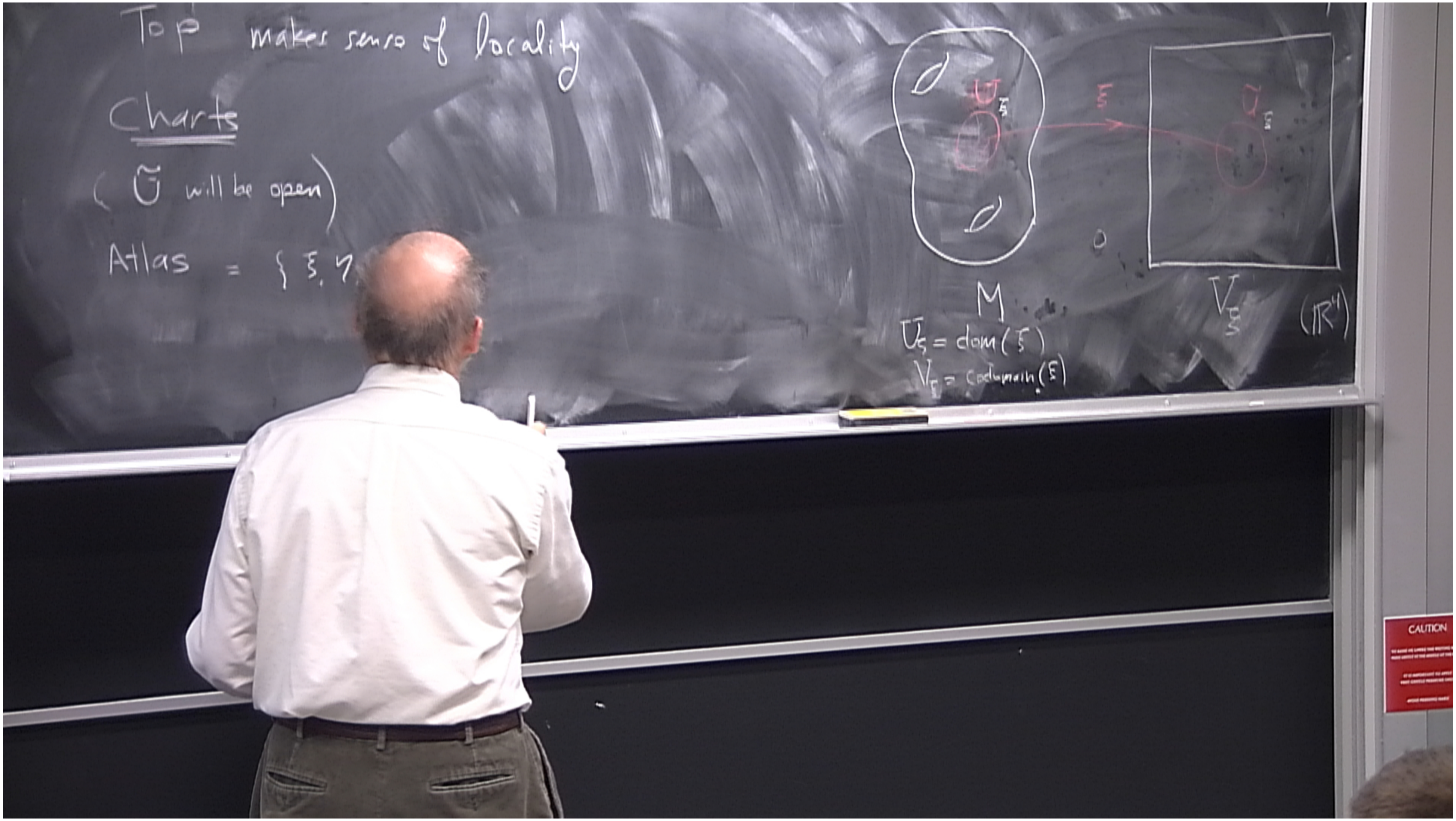
($\tilde{U}$ will be open)

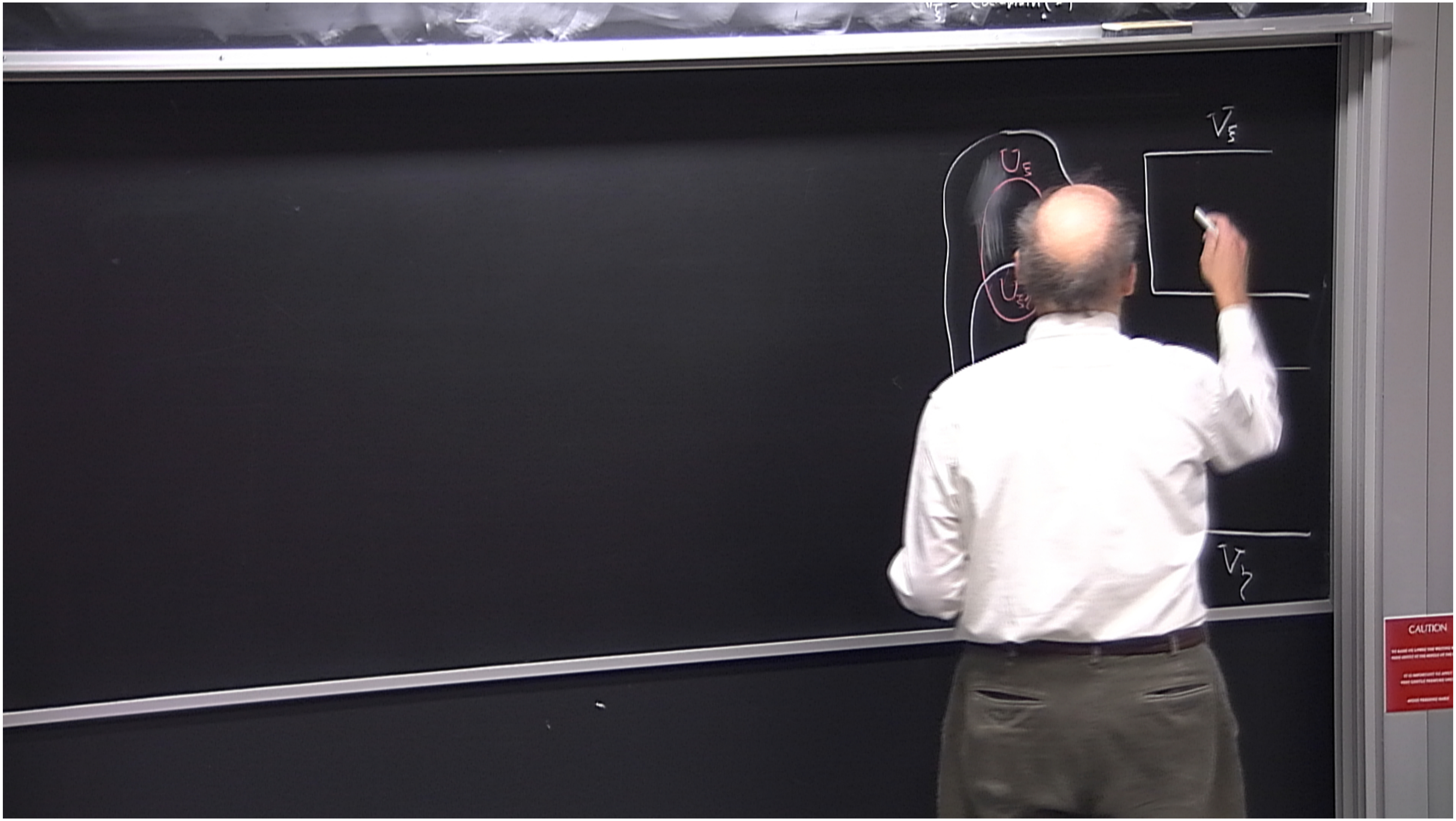


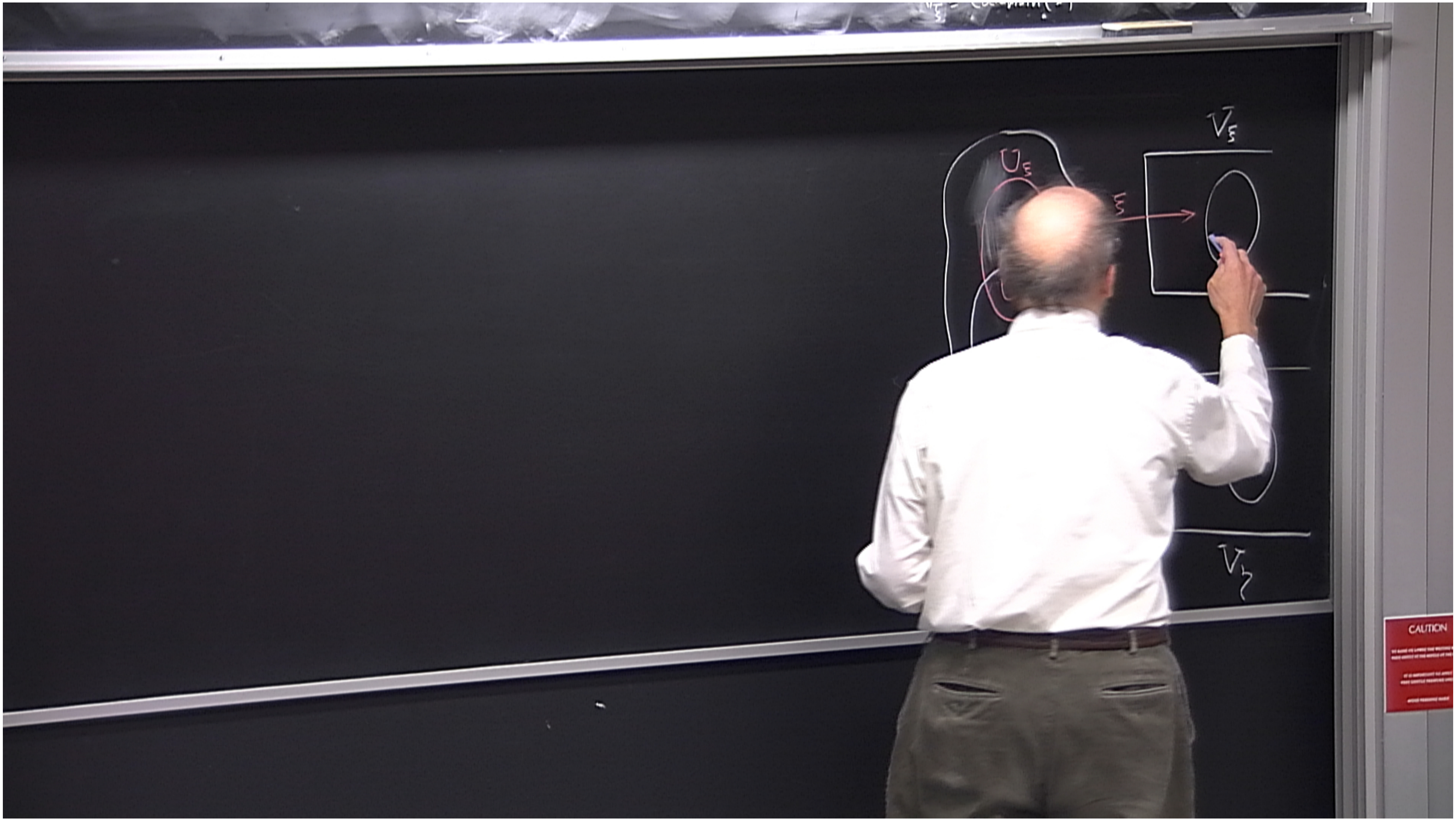

$x_n \to y \Leftrightarrow (\forall U)(\exists N)(j>N \Rightarrow x_j \in U)$
Top makes sense of locality
Charts
M
V_ξ
$(\mathbb{R}^4)$

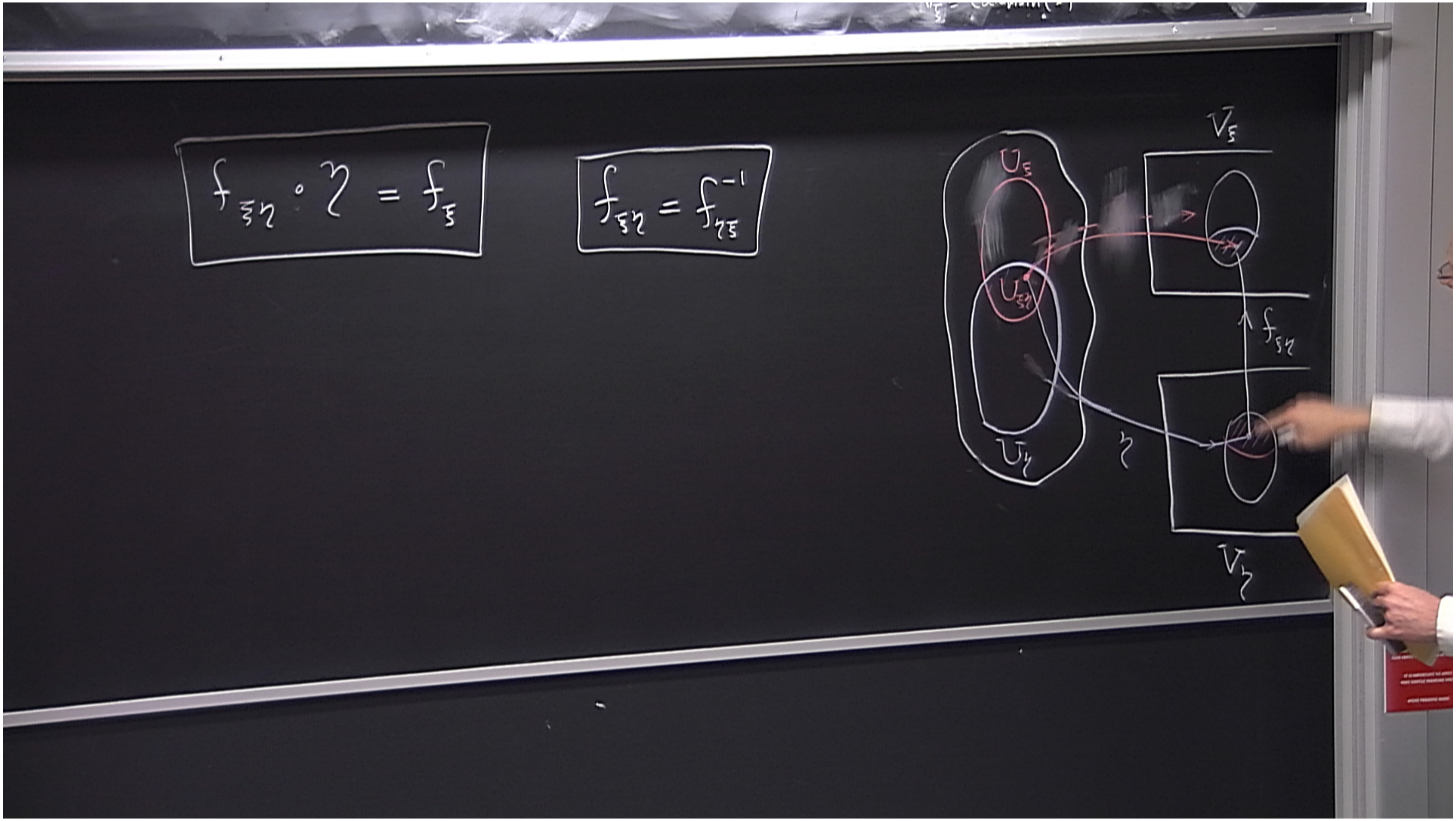

x_n → y ⟺ (∀U)(∃N)(j>N ⇒ x_j ∈ U)
Top makes sense of locality
Charts
(Ũ will be open)
M
V_ξ
(ℝ^4)

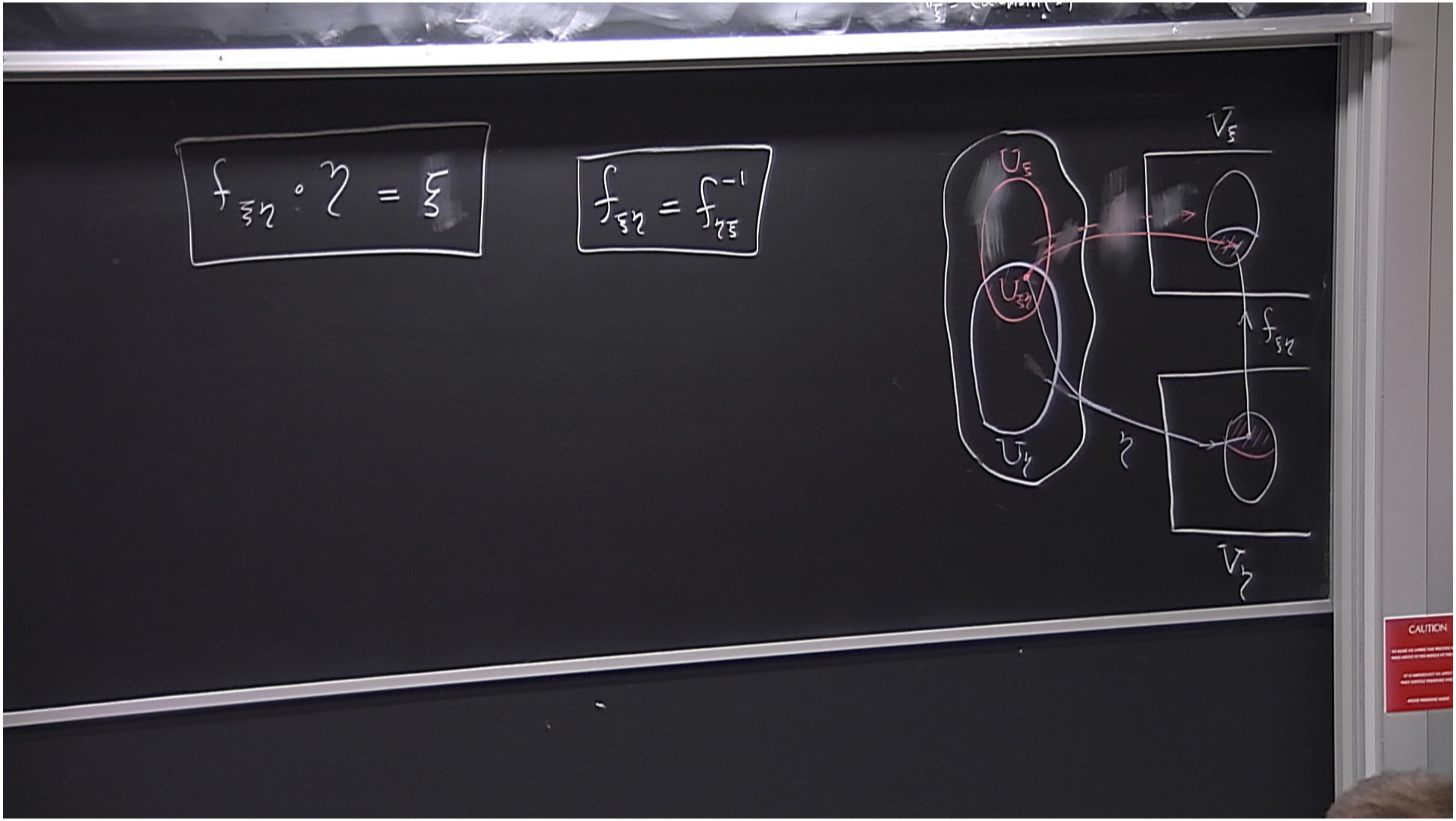

Top makes sense of locality
Charts
(Ũ will be open)
(ℝ⁴)

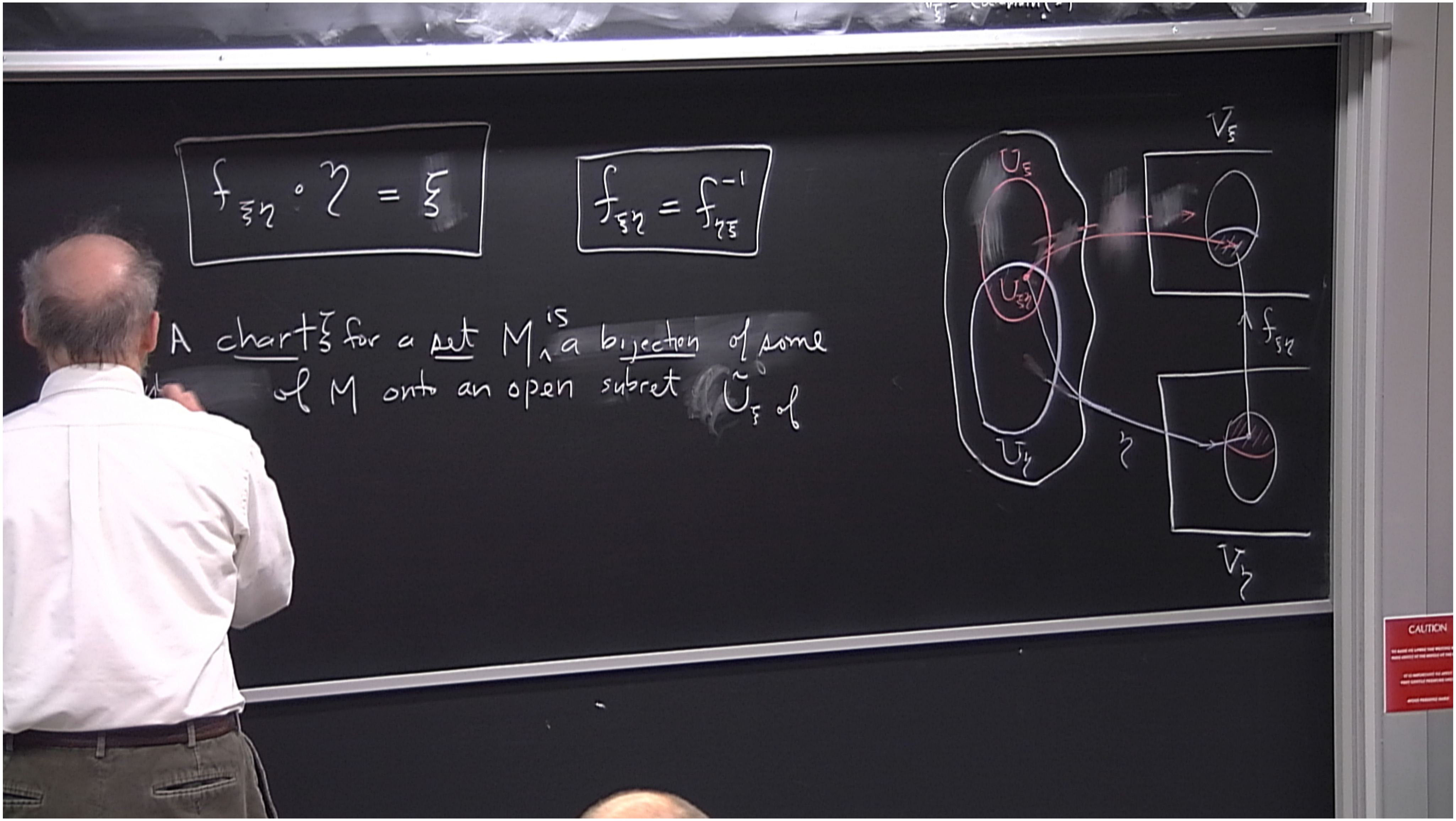

Top makes sense of locality
Charts
($\bar{U}$ will be open)
Atlas = { ξ, η
ξ
M
V_ξ
($\mathbb{R}^4$)
$\bar{U}_\xi$ = dom(ξ)
V_ξ = codomain(ξ)
CAUTION



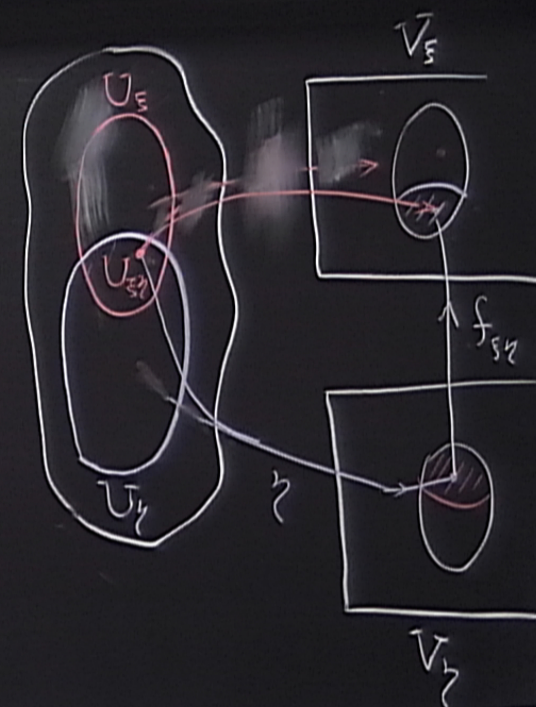

U_ξ
ξ
V_ξ
V_η
CAUTION


$f_{\xi\eta} \circ \eta = f_\xi$
$f_{\xi\eta} = f_{\eta\xi}^{-1}$
U_ξ
$U_{\xi\eta}$
U_η
η
V_ξ
$f_{\xi\eta}$
V_η


$f_{\xi\eta} \circ \eta = \xi$
$f_{\xi\eta} = f_{\eta\xi}^{-1}$
U_ξ
$U_{\xi\eta}$
U_η
η
$f_{\xi\eta}$
V_ξ
V_η

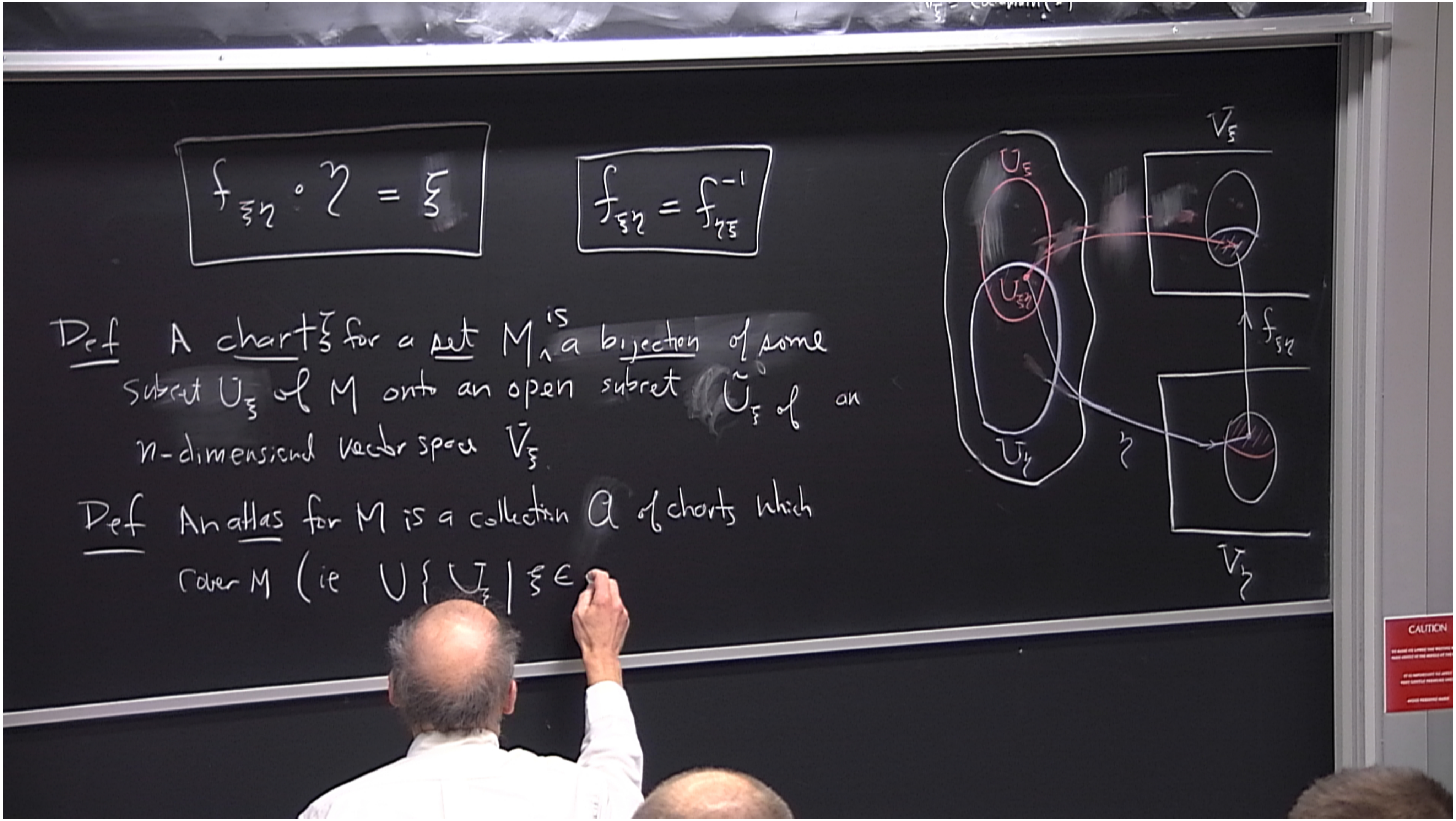

$f_{\xi\eta} \circ \eta = \xi$
$f_{\xi\eta} = f_{\eta\xi}^{-1}$
A chart ξ for a set M is a bijection of some
of M onto an open subset $\tilde{U}_\xi$ of
U_ξ
$U_{\xi\eta}$
U_η
V_ξ
V_η
$f_{\xi\eta}$

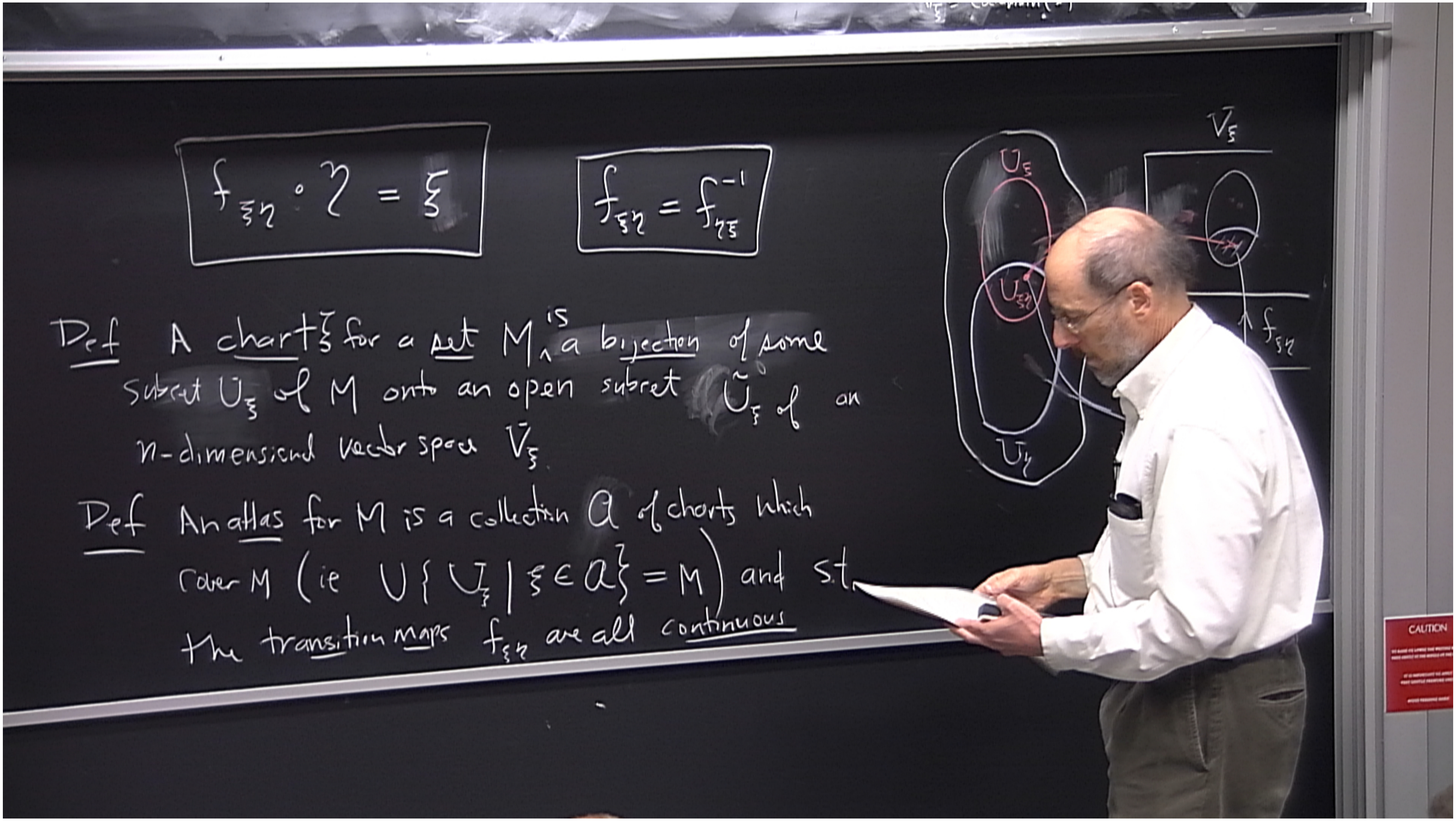
$$\boxed{f_{\xi\eta} \circ \eta = \xi} \qquad \boxed{f_{\xi\eta} = f_{\eta\xi}^{-1}}$$

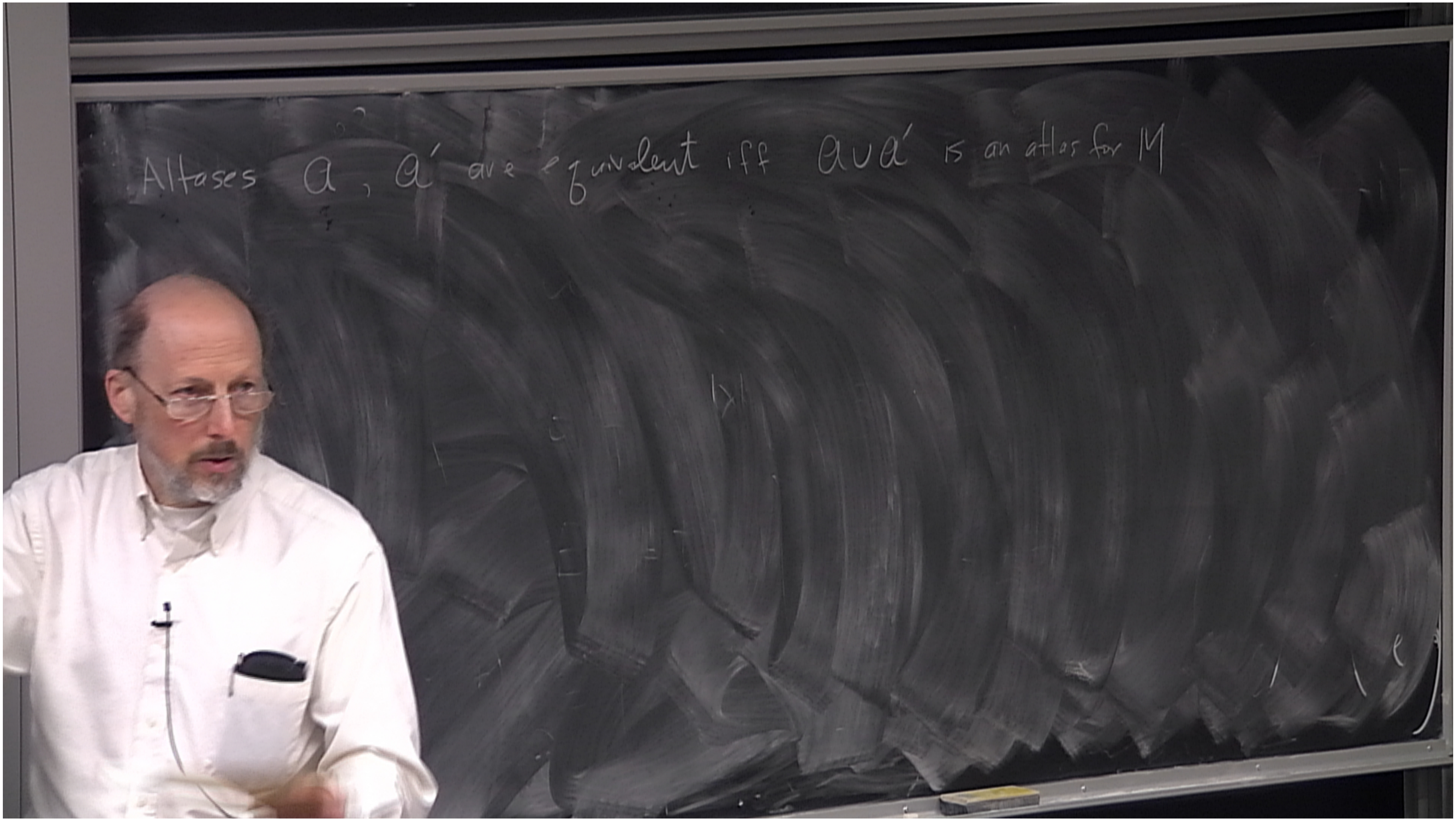


Def A chart ξ for a set M is a bijection of some subset U_ξ of M onto an open subset $\tilde{U}_\xi$ of an n-dimensional vector space V_ξ.

Def An atlas for M is a collection $\mathcal{A}$ of charts which cover M (ie


$f_{\xi\eta} \circ \eta = \xi$
$f_{\xi\eta} = f_{\eta\xi}^{-1}$
Def A chart ξ for a set M is a bijection of some subset U_ξ of M onto an open subset $\tilde{U}_\xi$ of an n-dimensional vector space $\bar{V}_\xi$
Def An atlas for M is a collection $\mathcal{A}$ of charts which cover M (ie $\cup\{U_\xi \mid \xi \in$
$\bar{V}_\xi$
U_ξ
$U_{\xi\eta}$
U_η
η
$f_{\xi\eta}$
$\bar{V}_\eta$


$f_{\xi\eta} \circ \eta = \xi$
$f_{\xi\eta} = f_{\eta\xi}^{-1}$
Def A chart ξ for a set M is a bijection of some subset U_ξ of M onto an open subset $\tilde{U}_\xi$ of an n-dimensional vector space $\bar{V}_\xi$.
Def An atlas for M is a collection $\mathcal{A}$ of charts which cover M (ie $\bigcup\{U_\xi \mid \xi \in \mathcal{A}\} = M$) and s.t. the transition maps $f_{\xi\eta}$ are all continuous
U_ξ
$U_{\xi\eta}$
U_η
$\bar{V}_\xi$
$f_{\xi\eta}$
CAUTION


Altases $\mathcal{A}$, $\mathcal{A}'$ are equivalent iff $\mathcal{A} \cup \mathcal{A}'$ is an atlas for M

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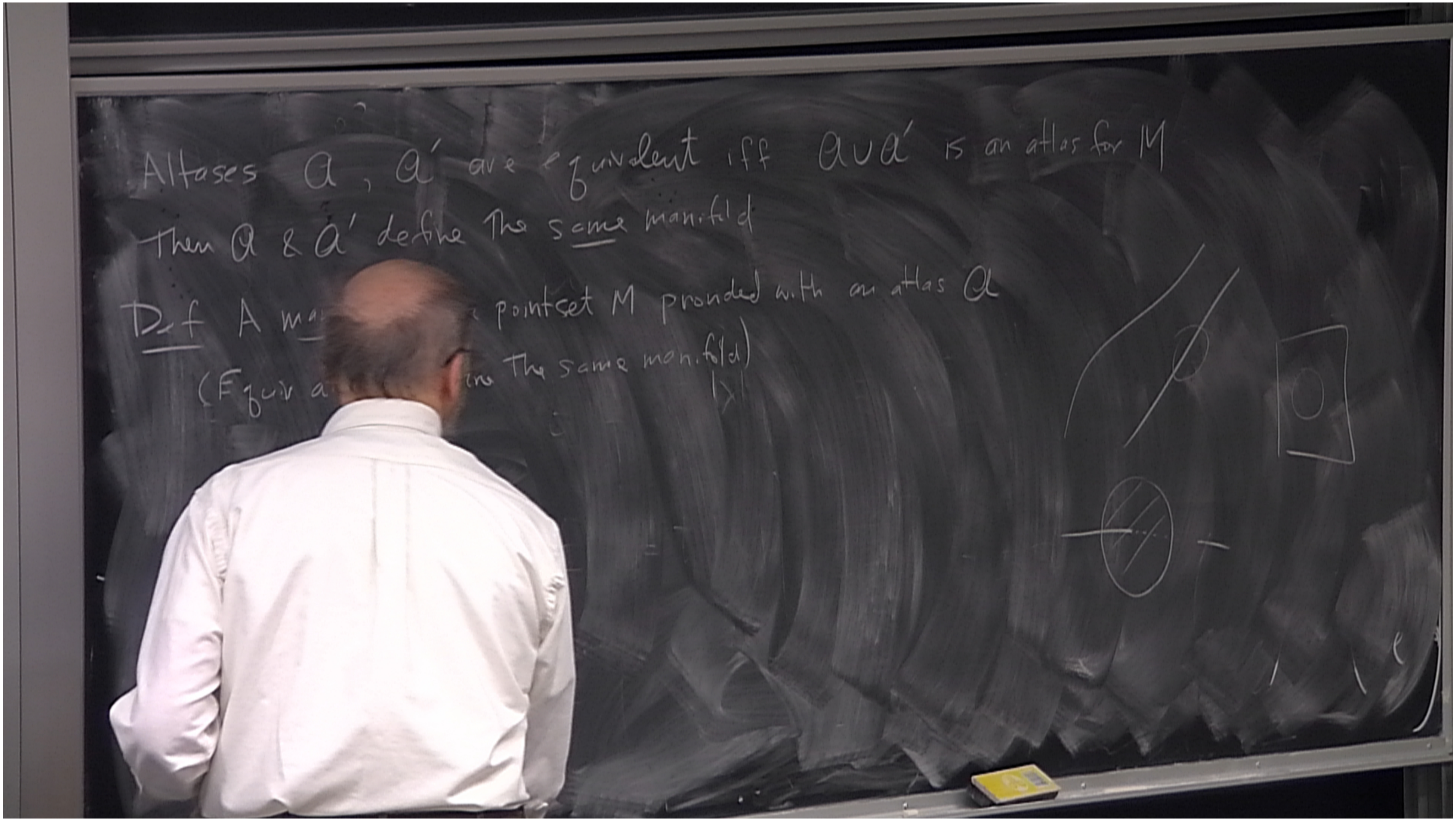
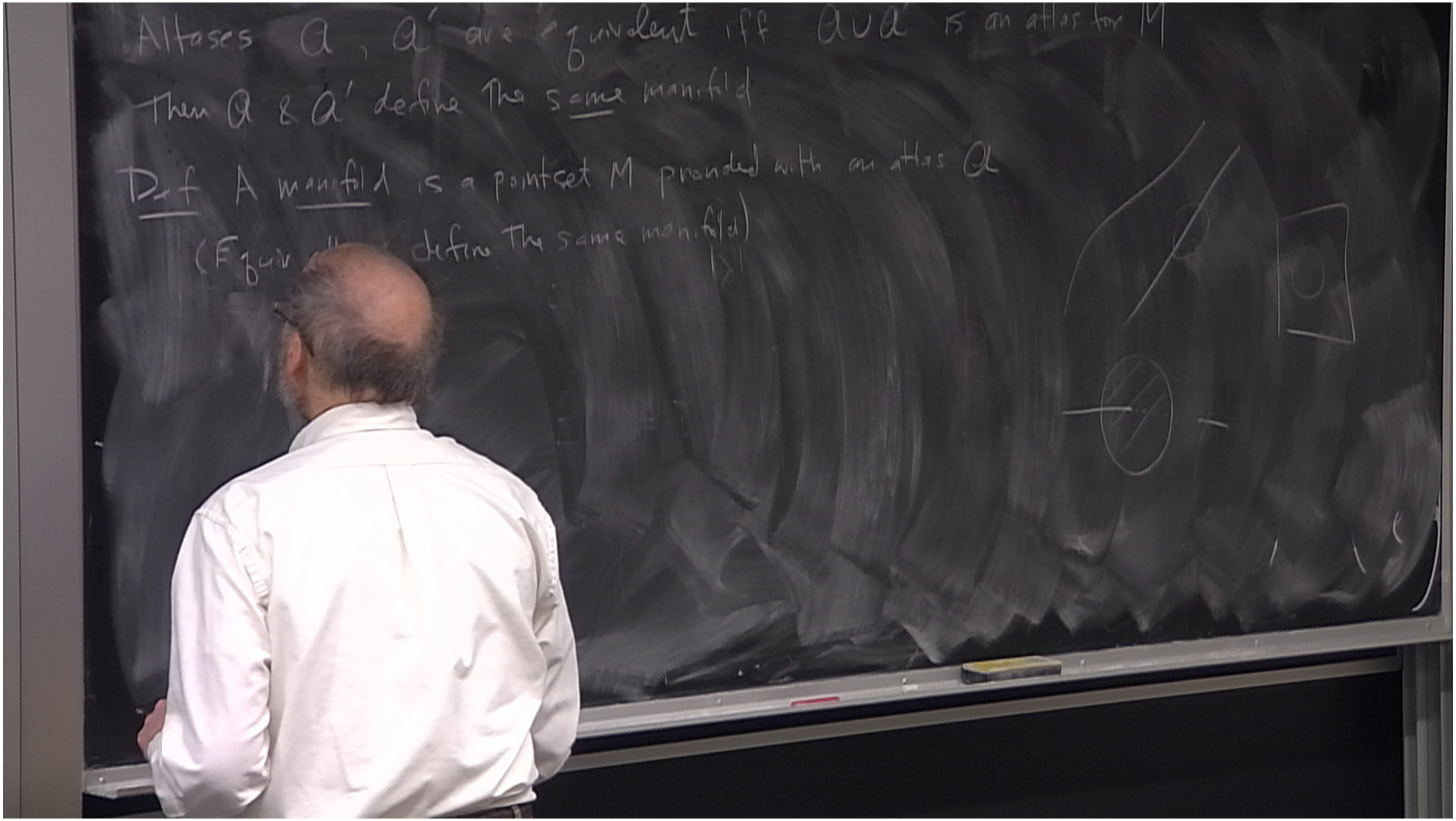
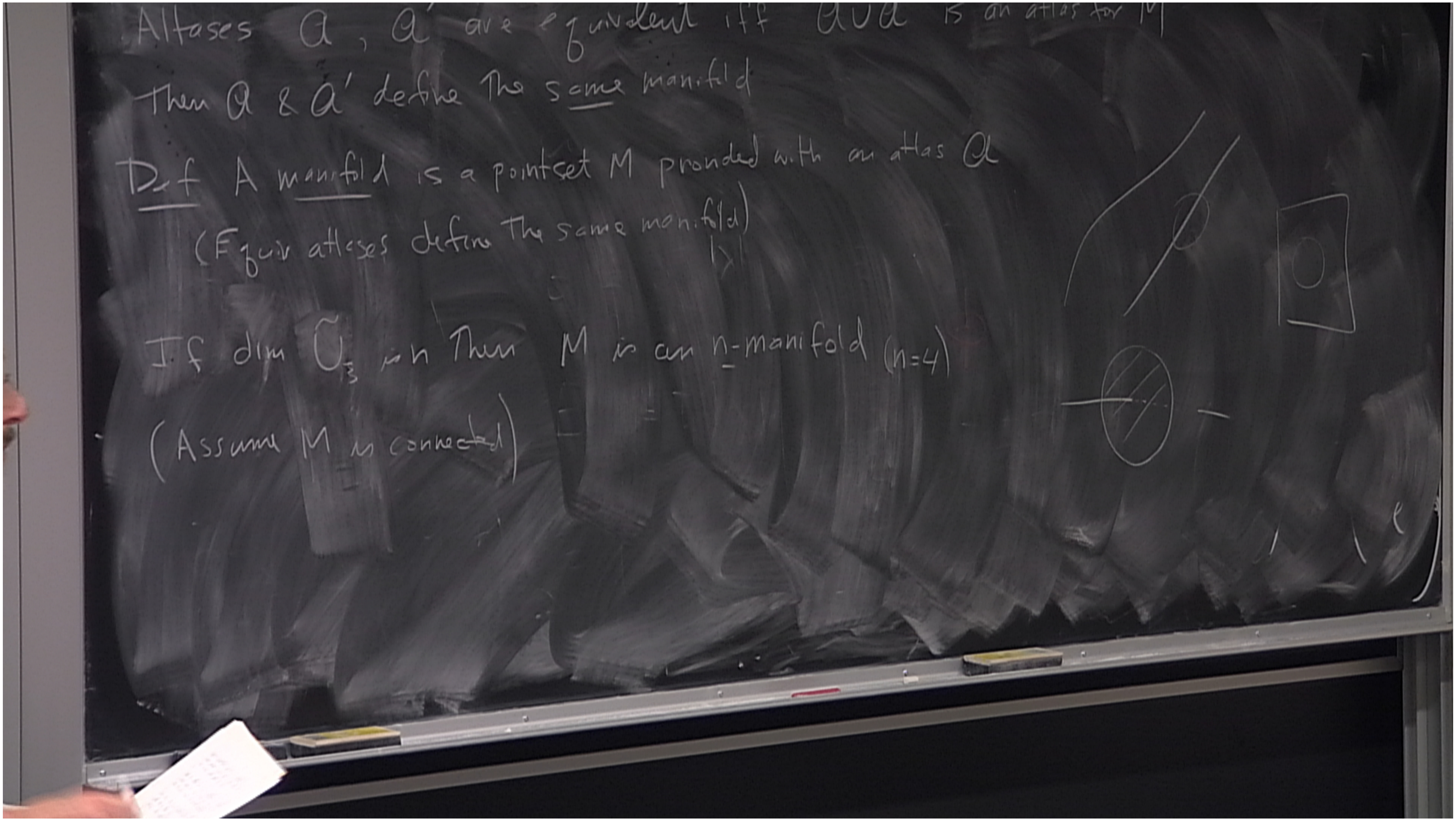
Then $\mathcal{A}$ & $\mathcal{A}'$ define the <u>same</u> manifold

<u>Def</u> A <u>manifold</u> is a pointset M provided with an atlas $\mathcal{A}$

(Equiv atlases define the same manifold)


Altases a, a′ are equivalent iff a∪a′ is an atlas for M
Then a & a′ define the same manifold
Def A
pointset M provided with an atlas a
the same manifold)


Altases $\mathcal{A}$, $\mathcal{A}'$ are equivalent iff $\mathcal{A} \cup \mathcal{A}'$ is an atlas for M
Then $\mathcal{A}$ & $\mathcal{A}'$ define the same manifold
Def A manifold is a pointset M provided with an atlas $\mathcal{A}$
define the same manifold)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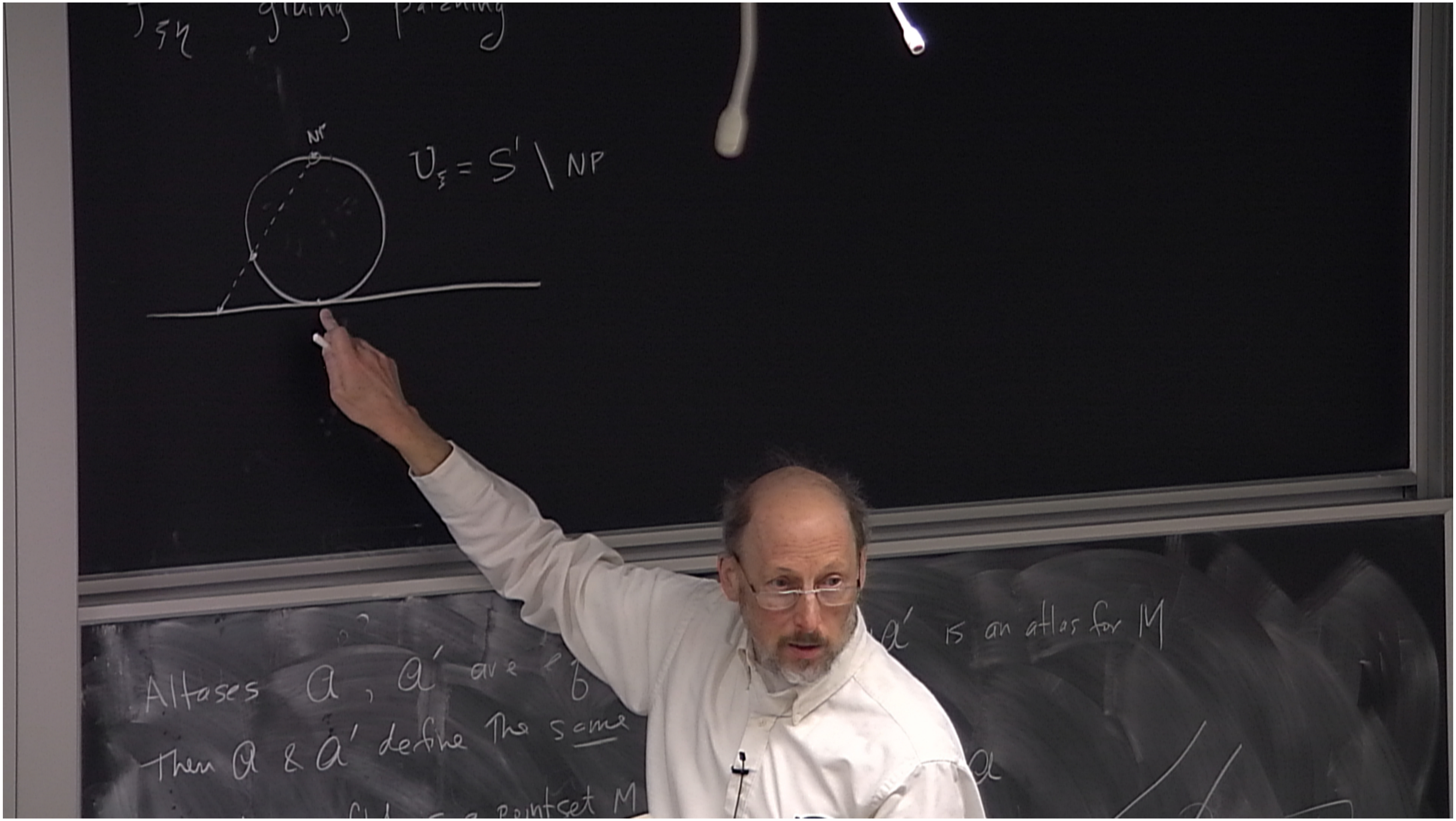
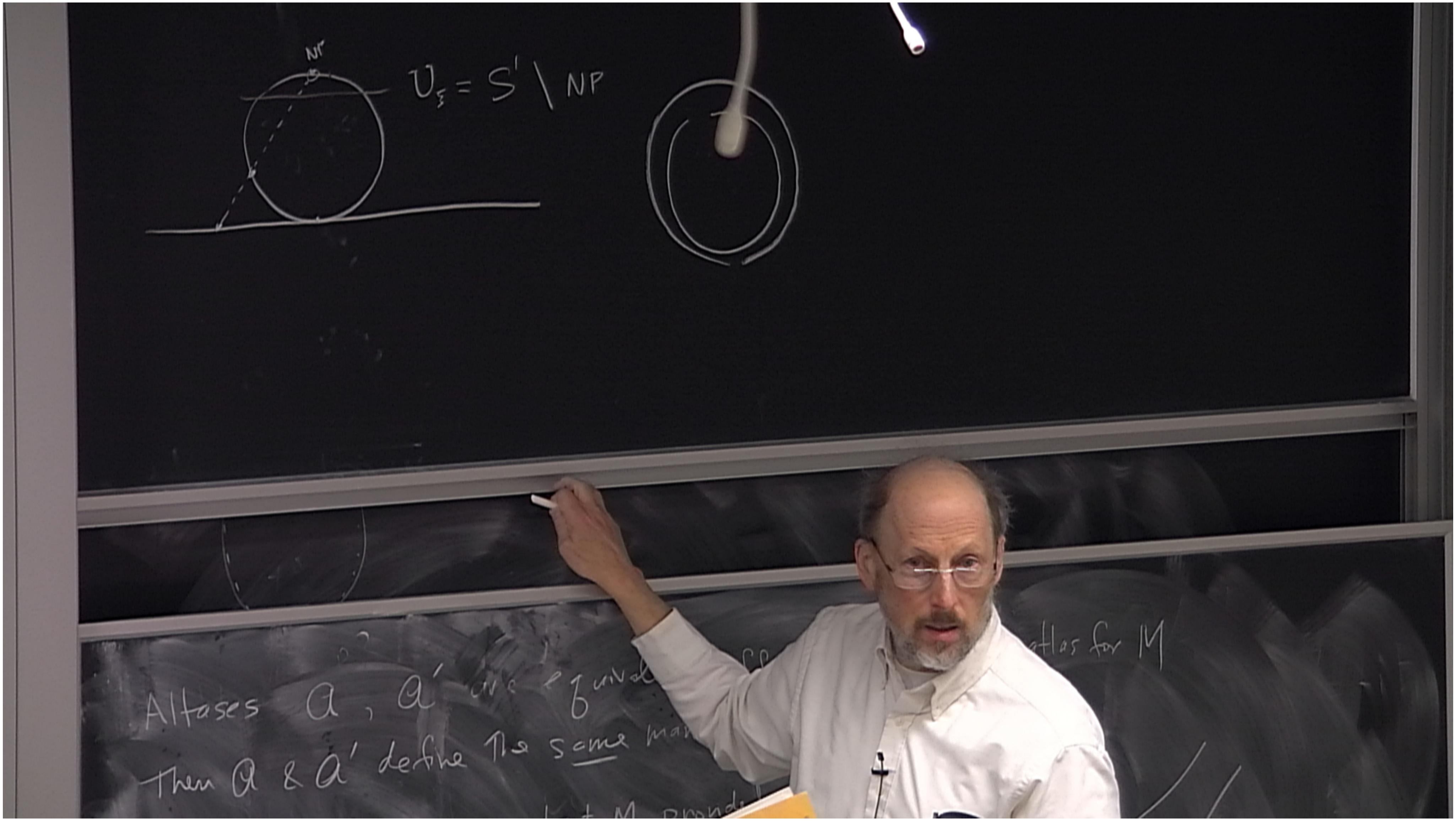
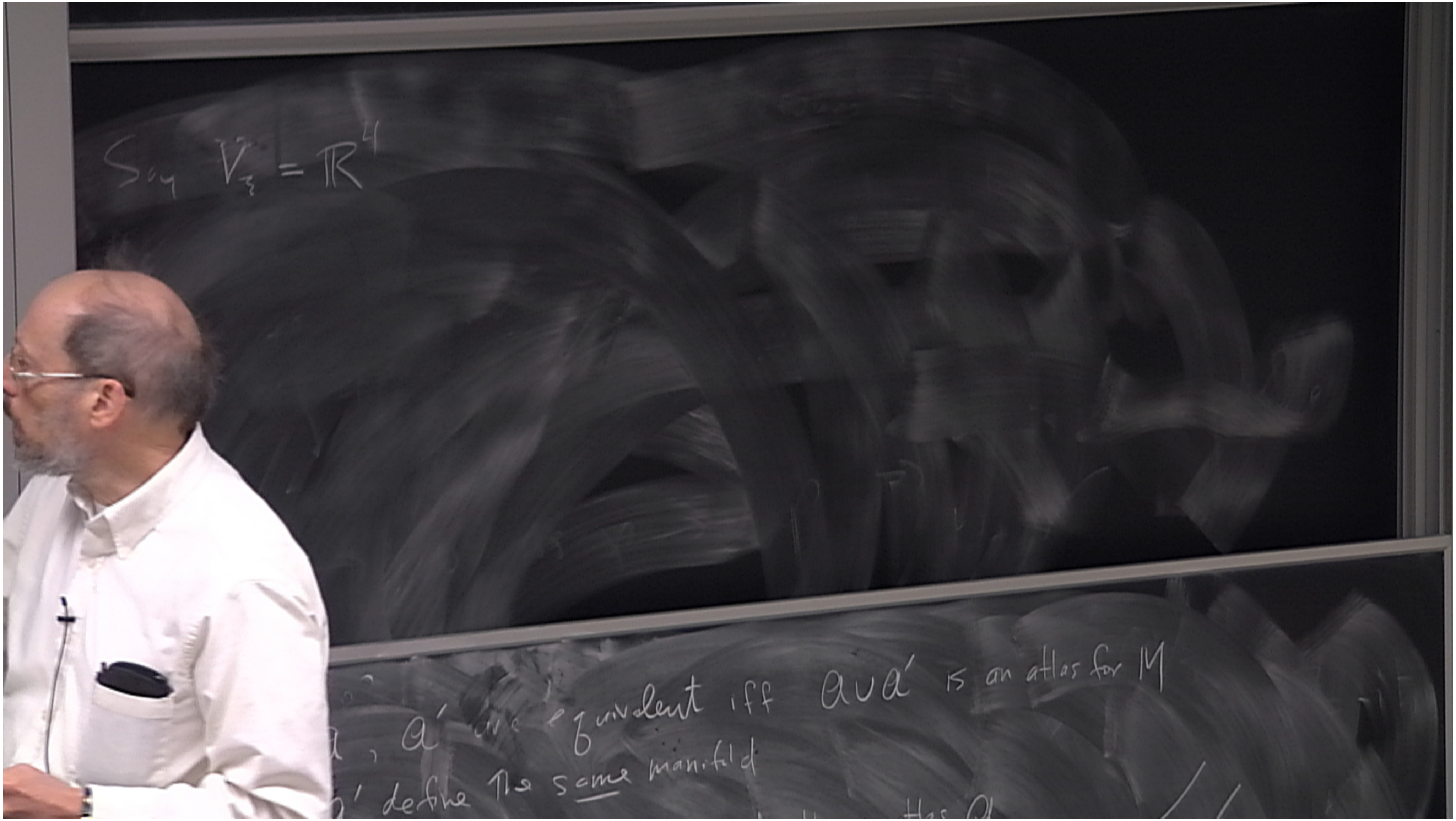
Def A manifold is a pointset M provided with an atlas $\mathcal{A}$
(Equiv atlases define the same manifold)
If dim U_β = n Then M is an n-manifold (n=4)
(Assume M is connected)

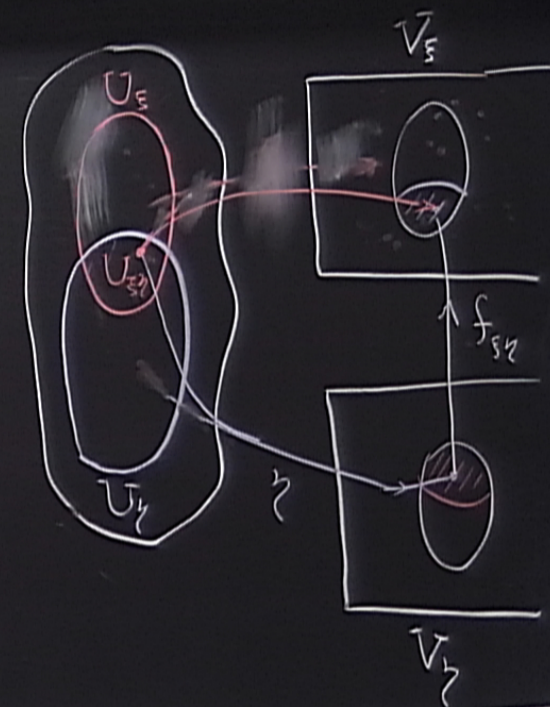

NP
U = S' \ NP
Altases a, a' are
Then a & a' define the same
is an atlas for M
pointset M


NP
$U_\xi = S^1 \setminus NP$
Altases $\mathcal{a}$, $\mathcal{a}'$
atlas for M
Then $\mathcal{a}$ & $\mathcal{a}'$ define the same


Say $V_\xi = \mathbb{R}^4$
$\mathcal{A}, \mathcal{A}'$ are equivalent iff $\mathcal{A} \cup \mathcal{A}'$ is an atlas for M
$\mathcal{A}'$ define the same manifold

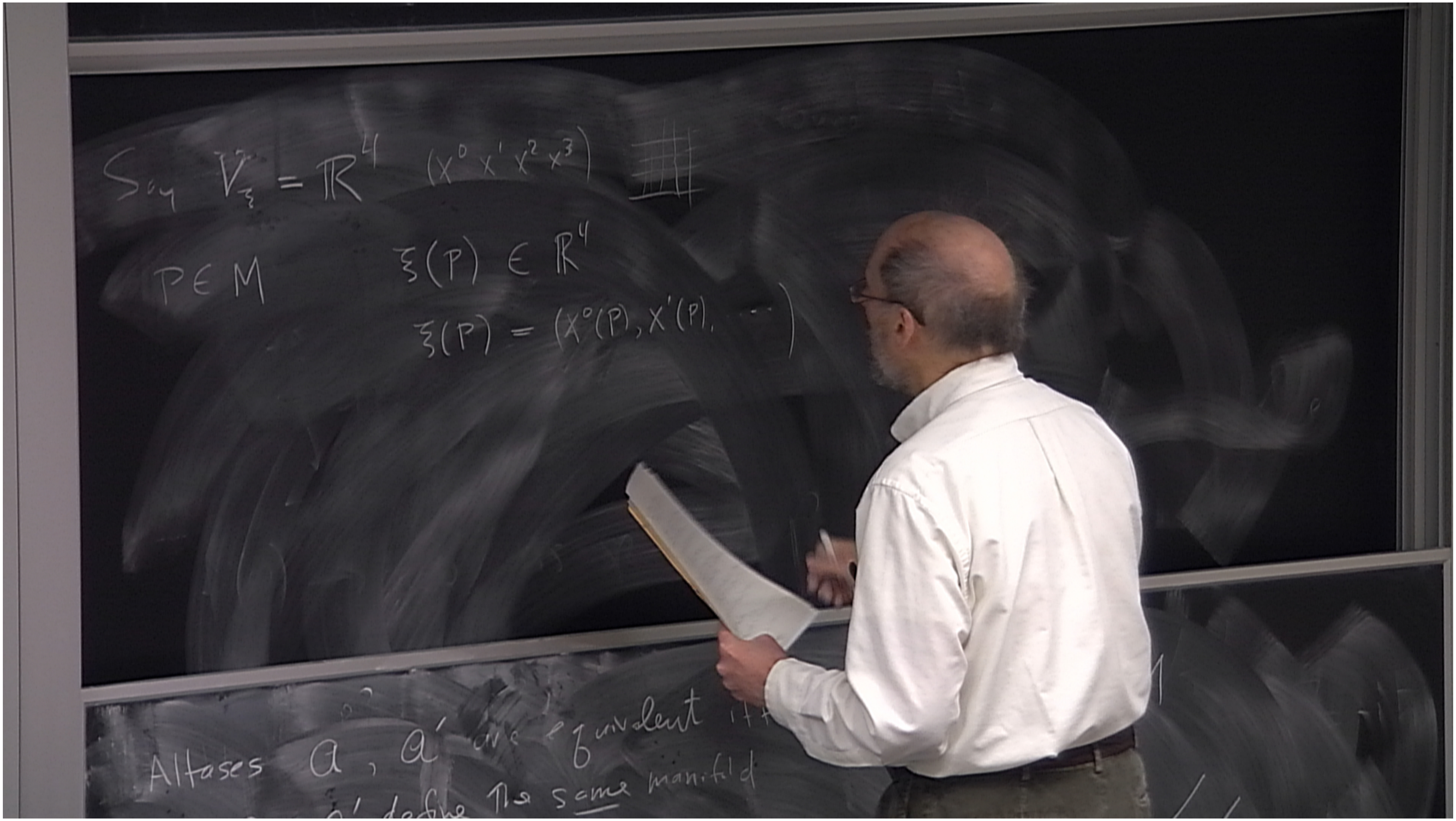
$$\boxed{f_{\xi\eta}\circ\eta = \xi}$$

$$\boxed{f_{\xi\eta} = f_{\eta\xi}^{-1}}$$



Def A chart ξ for a set M is a bijection of some subset U_ξ of M onto an open subset $\tilde{U}_\xi$ of an n-dimensional vector space $\tilde{V}_\xi$

Def An atlas for M is a collection $\mathcal{A}$ of charts which cover M (ie $\bigcup\{U_\xi \mid \xi\in\mathcal{A}\} = M$) and s.t. the transition maps $f_{\xi\eta}$ are all continuous


Say $V_\xi = \mathbb{R}^4$ $(x^0 x^1 x^2 x^3)$
$P \in M$ $\xi(P) \in \mathbb{R}^4$
$\xi(P) = (x^0(P), x^1(P), \dots)$
Altases a, a' are equivalent iff
the same manifold

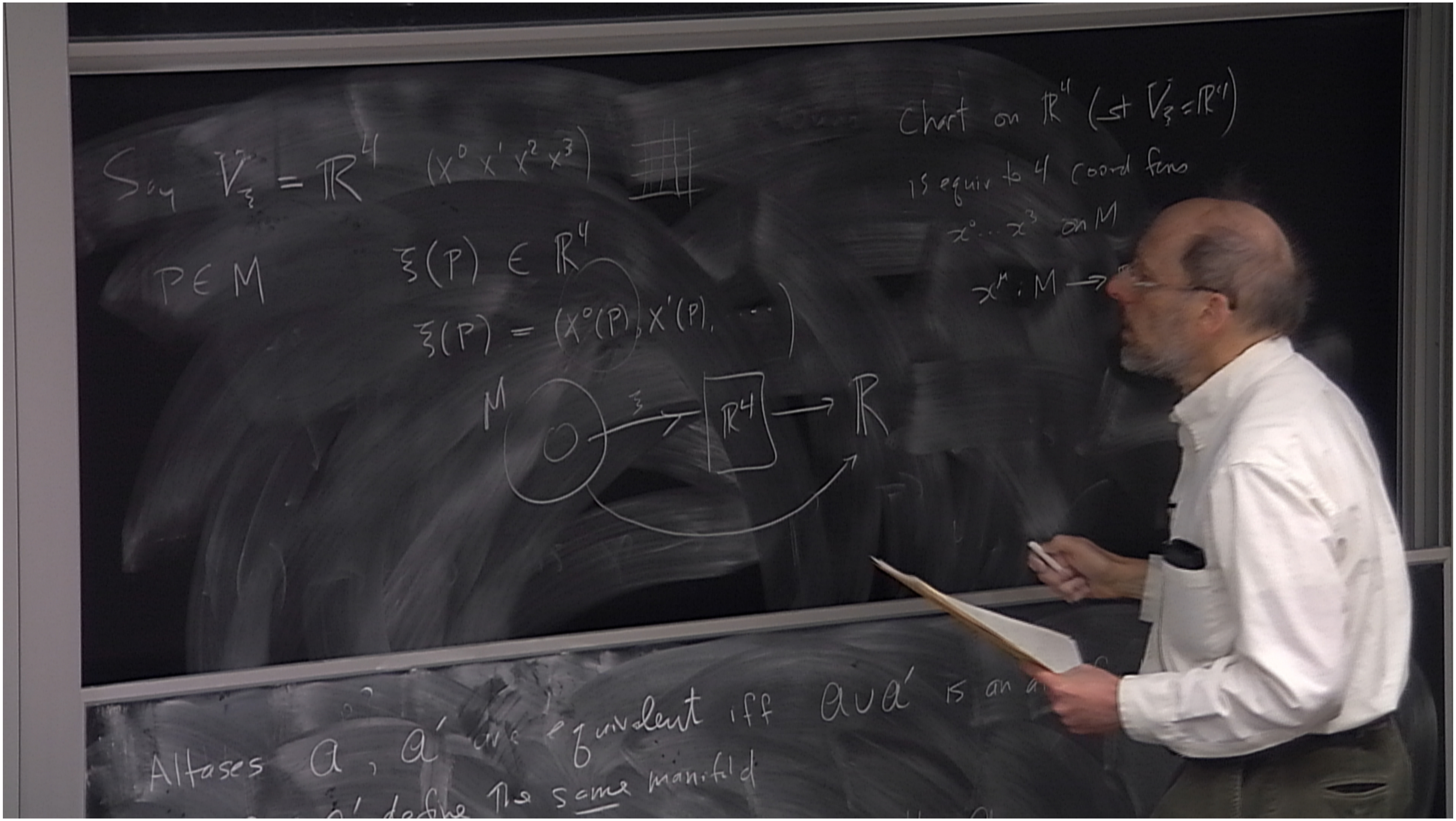

Say $V_\xi = \mathbb{R}^4$ $(x^0 x^1 x^2 x^3)$
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Chart on $\mathbb{R}^4$ (st $V_\xi = \mathbb{R}^4$)
is equiv to 4 coord fns
$x^0 \dots x^3$ on M
$x^\mu : M \rightarrow$
Altases a, a' are equivalent iff $a \cup a'$ is an
define the same manifold

Say $V_\xi = \mathbb{R}^4$ $(x^0\, x^1\, x^2\, x^3)$

$p \in M$ $\quad \xi(p) \in \mathbb{R}^4$

$\xi(p) = (x^0(p), x^1(p), \dots)$

$$\begin{tikzcd} M \supset O \arrow[r, "\xi"] \arrow[rr, bend right] & \boxed{\mathbb{R}^4} \arrow[r] & \mathbb{R} \end{tikzcd}$$

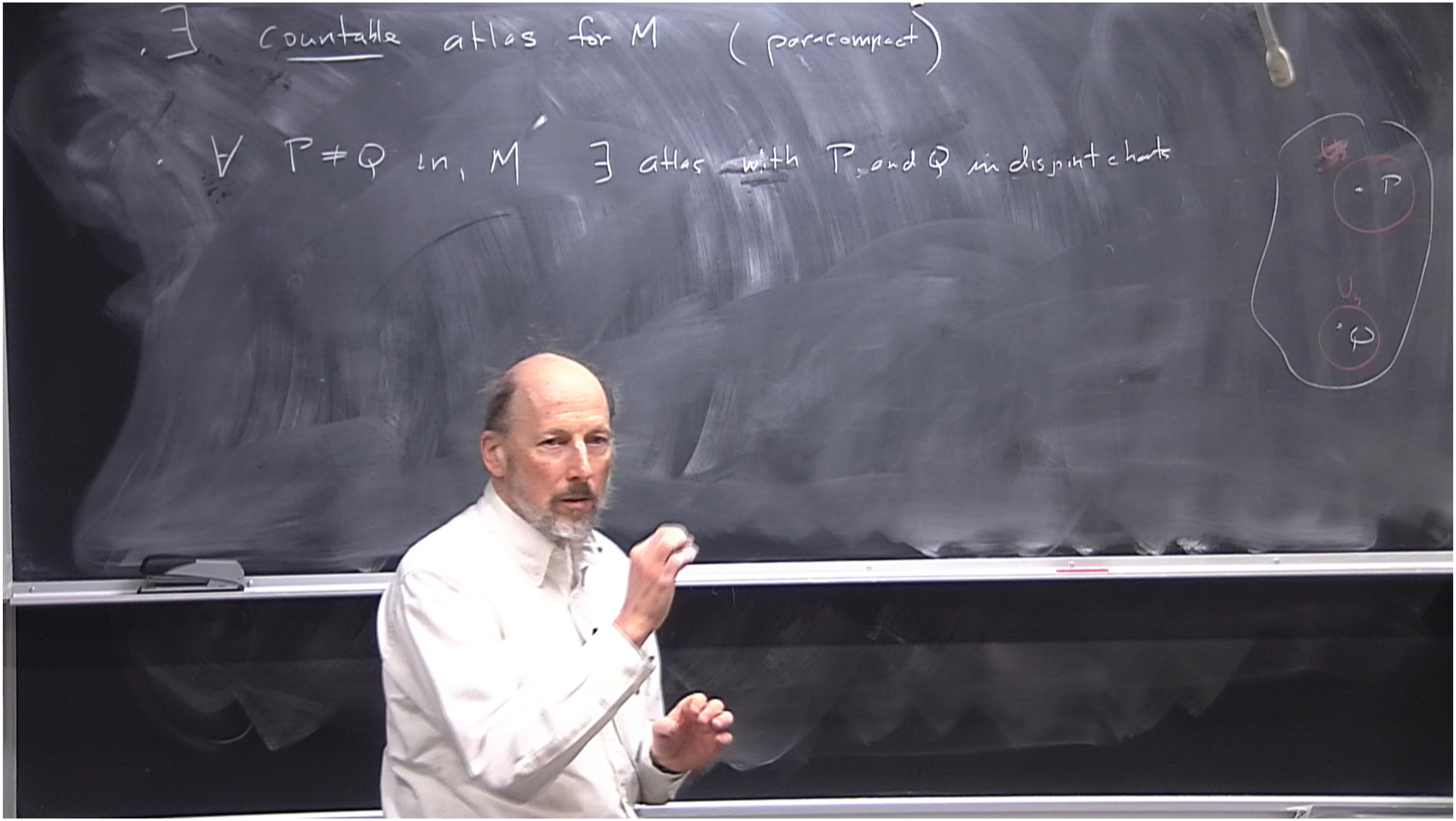
Chart on $\mathbb{R}^4$ (st $V_\xi = \mathbb{R}^4$)
is equiv to 4 coord fns
$x^0 \dots x^3$ on M

$x^\mu : M \to \mathbb{R}$

local coord system

Altases $\mathcal{A}, \mathcal{A}'$ are equivalent iff $\mathcal{A} \cup \mathcal{A}'$ is an atlas for M

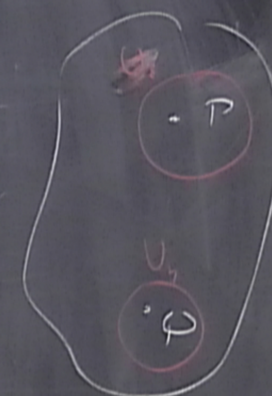
$\mathcal{A}'$ define the same manifold



∃ countable atlas for M (paracompact)
∀ P ≠ Q in M ∃ atlas with P and Q in disjoint charts

- $\exists$ <u>countable</u> atlas for M (paracompact)

- $\forall\ P \neq Q$ in M $\exists$ atlas with P and Q in disjoint charts



<u>Summary</u> M is a paracompact Hausdorff C^0-manifold (without boundary) of dim 4 (usually)