Title: New transport properties of holographic superfluids

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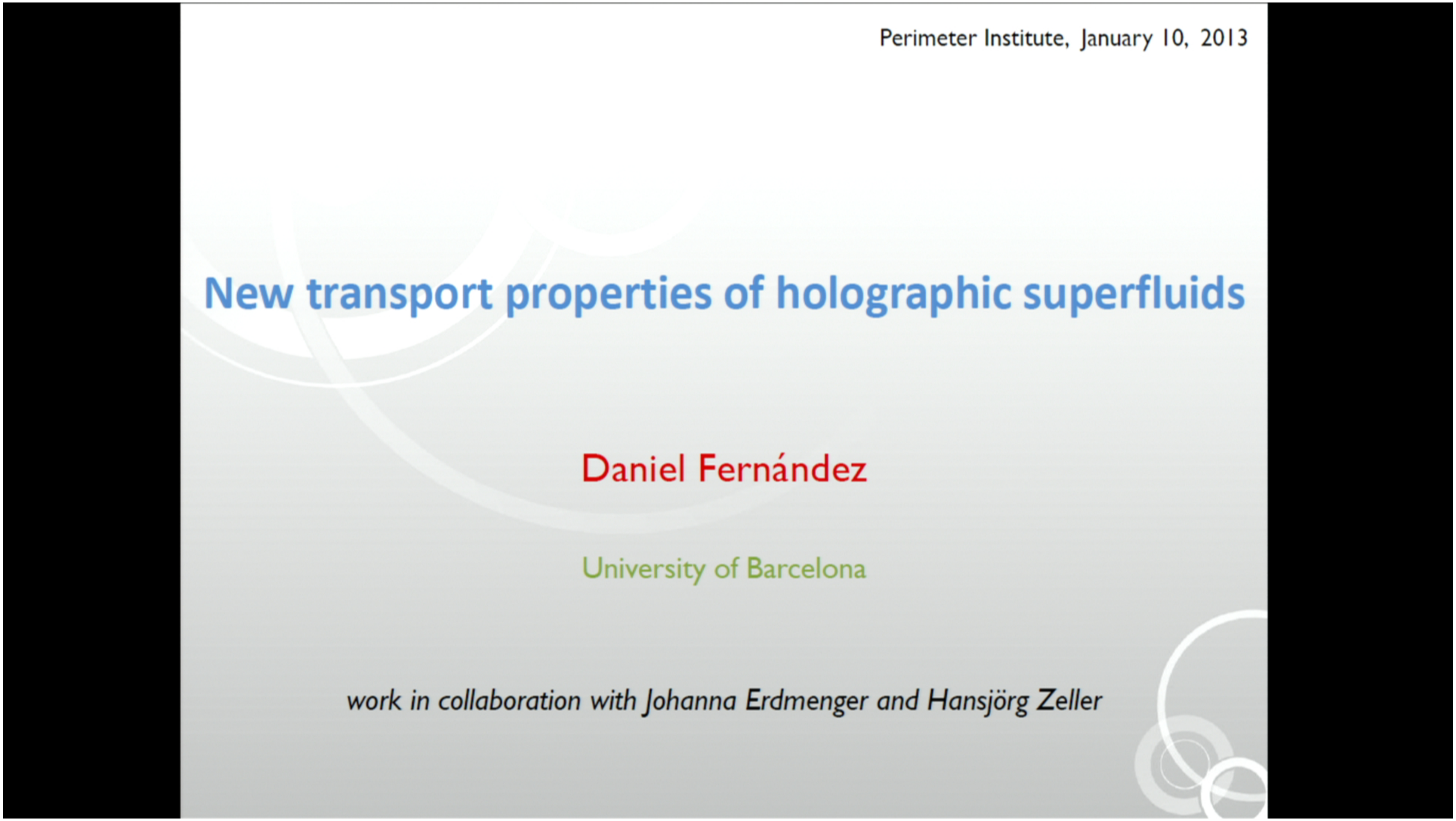
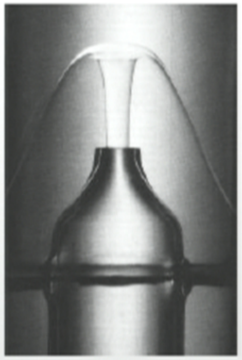
Abstract: In the context of holography applied to condensed matter theory, I will present an analysis of transport properties of p-wave superfluids by means of a gravity dual. Fluctuations modes in the SU(2) Einstein-Yang-Mills theory are considered, and phenomenological implications are derived. Due to the spatial anisotropy of the system, a non-universal shear viscosity is obtained, along with a new coefficient associated to normal stress differences. I will also discuss how the transport phenomena in this model is related to the thermoelectric, flexoelectric and piezoelectric effects (mixing of electric current, heat flux and mechanical stress), which have been observed in some superconductors and crystal liquids.


Perimeter Institute, January 10, 2013
New transport properties of holographic superfluids
Daniel Fernández
University of Barcelona
work in collaboration with Johanna Erdmenger and Hansjörg Zeller

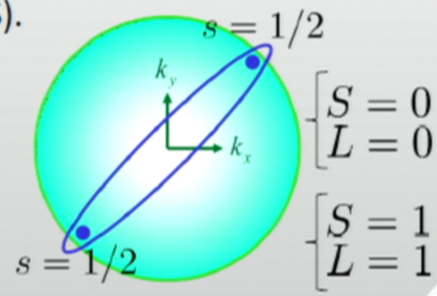
s-wave and p-wave superfluids

Superfluid:
- State of matter with zero viscosity at very low temperatures.
- Gauge theory with spontaneous breaking of global symmetry.



Conventional superfluids:
- Helium-4: Bose-Einstein condensation of atoms.
- New hydrodynamic mode: Superfluid velocity

"*p*-wave" SFs, like Helium-3:
- Cooper pairs of ions form bosonic states (like in BCS).
- Rotational symmetry is broken: more modes.
- Superconductivity with new pairing states.
- Much lower temperature than conventional.
- Several different phases.



Liquid crystals:
- Flow like liquids, but molecules are oriented.
- Related to high temperature SCs (*d*-wave).

[Lee, Osheroff, C. Richardson, Leggett]

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Hydrodynamics of superfluids

Condensed-matter analog of the Higgs phenomena

$$\Psi = |\Psi|\, e^{i\varphi}$$

Spontaneous Symmetry Breaking of continuous symmetry

→ Nambu-Goldstone boson in the spectrum

→New hydrodynamic mode

$$\{\mu,\ T,\ u_\mu\},\ v_\mu = \partial_\mu \varphi$$

(superfluid velocity)

- Bosons form a highly collective state.
- Wavefunction Ψ is expectation value. Phase φ, coherent superposition in condensate.
- In our case:

$$SU(2) \xrightarrow[\text{Expl.B}]{} U(1)_3 \xrightarrow[\text{SSB}]{} \mathbb{Z}_2$$

$$SO(3) \xrightarrow[\text{SSB}]{} SO(2)$$

- 3 Goldstone modes! We can expect different hydrodynamics.

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Outlining the duality

IIB Supergravity on AdS_5 ⟷ Conformal Field Theory at large N_c and strong coupling

- Energy Scale ⟷ Radial Coordinate
- Temperature ⟷ Black Hole
- Global currents J_μ ⟷ Gauge Fields A_μ

In particular,

$$\mu = \lim_{r\to\infty} A_t, \quad T = \frac{\kappa_{\rm BH}}{2\pi} \quad \text{and if} \quad A_\mu(x,r) \to A_\mu^{(0)}(x) + A_\mu^{(2)}(x)\frac{1}{r^2} + \dots,$$

- Expected Value $A_\mu^{(2)}(x) = \langle J_\mu(x) \rangle$
- Source $A_\mu^{(0)}(x)$

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And to be precise,

$$Z_{\rm SUGRA}[\phi(x,r)|_{r\to r_{\rm bdy}} = \phi_0(x)] = \left\langle e^{\int d^4x\, \phi_0(x)\mathcal{O}(x)} \right\rangle$$

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The Field/Operator Correspondence

Field-Operator dictionary: $\Phi(x,r) \leftrightarrow \mathcal{O}(x)$

If the action for bulk field is

$S \propto \int \mathrm{d}r\, \mathrm{d}^4x\, \sqrt{-g}\, \left(\partial_M \Phi \partial^M \Phi + m^2 \Phi^2\right) + \ldots$, the asymptotic solution is

$$\Phi(x,r) \to \phi_0(x)\, r^{\Delta-4} + \phi_2(x)\, r^{-\Delta}$$

where

$$\Delta = 2 + \sqrt{m^2 L^2 + 4}$$

Stability requires real Δ, otherwise exponential growth.
→ Mass term not "too negative" (BF bound)

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If $m^2L^2 \geq -3$,

- $\phi_0(x)$ is non-normalizable, enters boundary theory.

$$S_{\mathrm{bdry}} \to S_{\mathrm{bdry}} + \int \mathrm{d}^4x\, \phi_0(x)\mathcal{O}(x)$$

- $\phi_2(x)$ is normalizable, belongs to bulk Hilbert space.

Hilbert spaces of dual theories identified:
Normalizable modes ↔ states of bdry theory

$$\phi_2(x) \propto \langle \mathcal{O}(x) \rangle_{\phi_0}$$

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source: $\phi_0(x)$; vev: $\phi_2(x)$

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AdS/CFT and Green's functions

Retarded Green's function = Correlator:

$$G^{\mathrm{R}}_{\mathcal{O}_{\mathrm{A}}\mathcal{O}_{\mathrm{B}}} = -\mathrm{i}\int \mathrm{d}^{d-1}x\, dt\, e^{\mathrm{i}\omega t - \mathrm{i}kx}\,\theta(t)\,\langle[\mathcal{O}_{\mathrm{A}}(t,x),\, \mathcal{O}_{\mathrm{B}}(0,0)]\rangle$$

Time-dependent perturbation in the action includes a source for B:

$$S(t) = \ldots + \int \mathrm{d}^{d-1}x\, \phi_{\mathrm{B}(0)}(t,x)\, \mathcal{O}_{\mathrm{B}}(x)$$

Expectation value for observable A in its presence is

$$\langle\mathcal{O}_{\mathrm{A}}\rangle(t,x) = \mathrm{Tr}\,\rho(t)\,\mathcal{O}_{\mathrm{A}}(x) \quad \text{where} \quad \mathrm{i}\,\partial_t\rho = [H_0 + \delta H, \rho].$$

The increase due to a δH is $\delta\langle\mathcal{O}_{\mathrm{A}}\rangle$. The perturbation comes from the source:

$$\phi_{\mathrm{B}}(r) \to \phi_{\mathrm{B}}(r) + \delta\phi_{\mathrm{B}}(r)e^{-\mathrm{i}\omega t + \mathrm{i}kx}$$

Linear response around equilibrium:

$$\boxed{\delta\langle\mathcal{O}_{\mathrm{A}}\rangle(\omega,k) = G^{\mathrm{R}}_{\mathcal{O}_{\mathrm{A}}\mathcal{O}_{\mathrm{B}}}(\omega,k)\,\delta\phi_{\mathrm{B}(0)}(\omega,k)}$$

[Son, Starinets]

The correspondence allows for a simple calculation!

The gravity model

SU(2) Einstein-Yang-Mills theory

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[R - \Lambda - \frac{\alpha^2}{2} F^a_{MN} F^{aMN} \right] + S_{\text{bdy}}$$

$$\left(\alpha = \frac{\kappa_5}{\hat{g}},\ \Lambda = -\frac{12}{L^2} \right)$$

Ansatz for gauge field:

$$A = \phi(r)\, \tau^3\, dt + w(r)\, \tau^1\, dx$$

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Chemical potential
→ explicit breaking

Spontaneous value $w(r) \to w_1^b / r^2$ acquired in broken phase:

$$w_1^b \propto \langle J_1^x \rangle \neq 0$$

[Ammon, Erdmenger, Grass, Kerner, O'Bannon]

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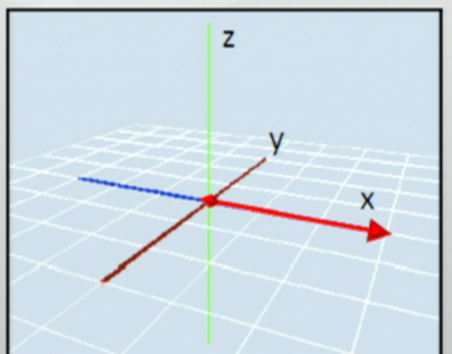

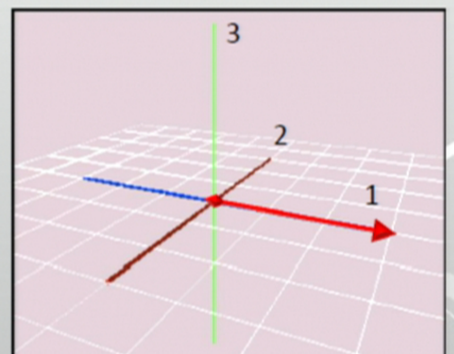

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Possible backgrounds

Ansatz for the metric:

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2 f(r)^{-4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

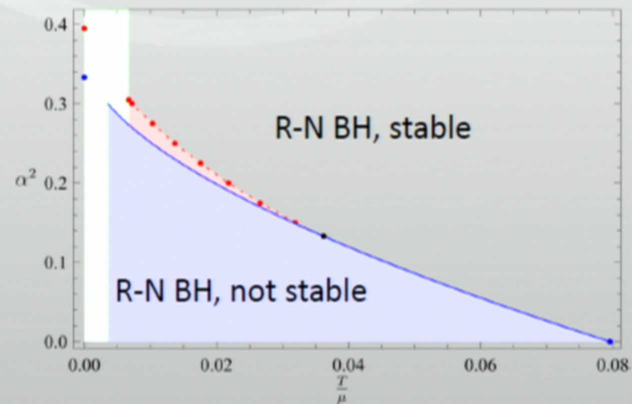
Solution 1

- Reissner–Nordström BH (asymptotically AdS)
- $w(r) = 0$
- Ground State for $\frac{\mu}{T} < \left(\frac{\mu}{T}\right)_c$

Solution 2

- Charged BH with vector hair (asymptotically AdS)
- $w(r) \neq 0$
- Ground State for $\frac{\mu}{T} > \left(\frac{\mu}{T}\right)_c$

Phase diagram:



[Erdmenger, Grass, Kerner, Hai Ngo]

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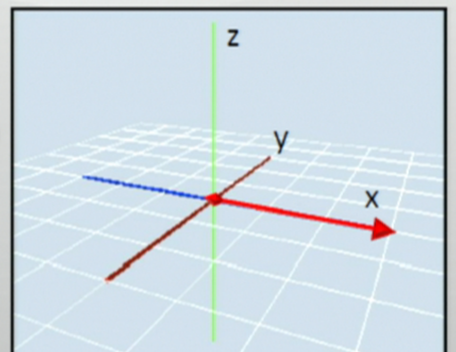
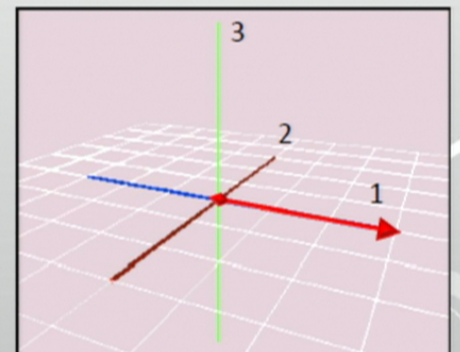
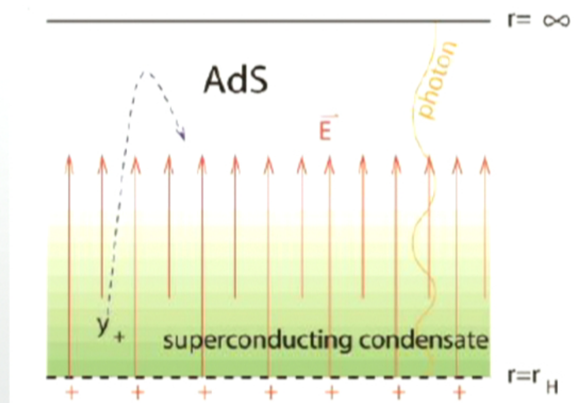




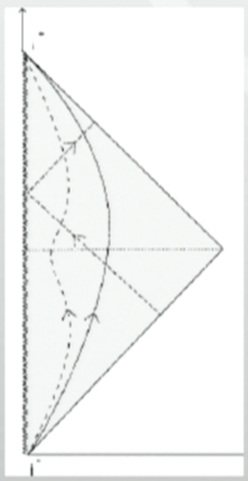
Vector hair

In solution 2,
a condensate layer floats above the horizon.

- In asympt. **flat** spacetime, Electrostatic repulsion sends it to infty.
- In asympt. **AdS** spacetime, Massive particles do not reach bdry.



[Gubser, Pufu]



Action for A_x^1:

$$S_{A_x^1} \sim \partial_M A_x^1 \partial^M A_x^1 + \underbrace{2g^{tt}g^{xx}\left(A_t^3\right)^2}_{m_{\text{eff}}^2}\left(A_x^1\right)^2$$

- Since $g_{tt}(r_{\text{H}}) = 0$, A_x^1 is tachyonic near the horizon...
- It condenses in a normalizable profile ($w = 0$ at bdry.)
- This translates into $\langle J_1^x \rangle \neq 0$ in the dual field theory.
- The action can be embedded into M-theory.

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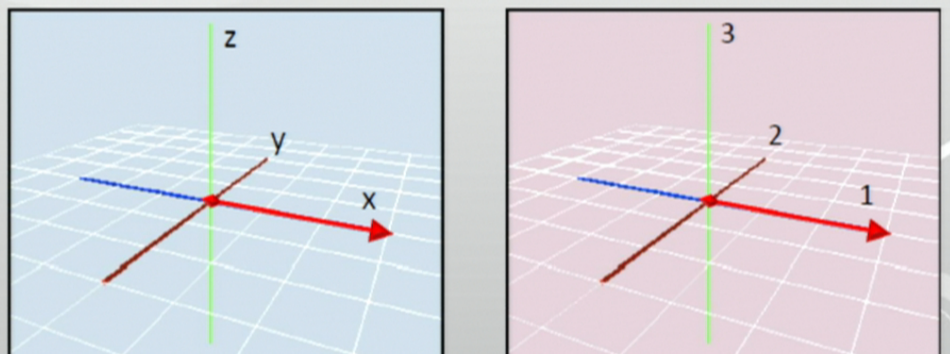

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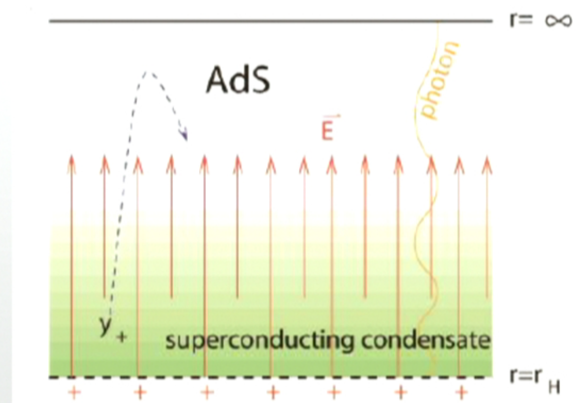

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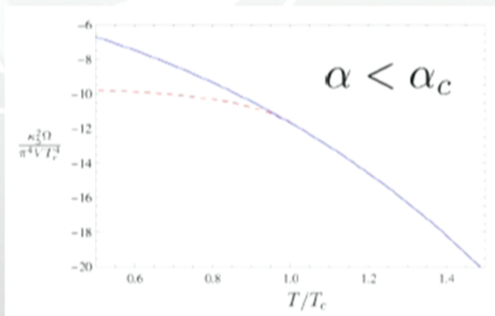
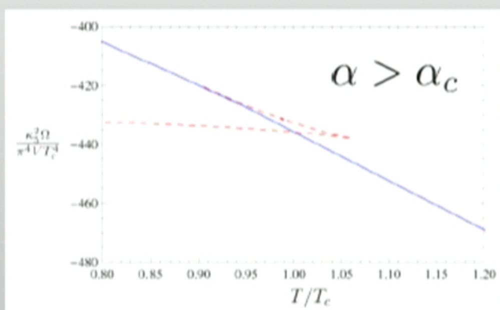
Thermodynamics

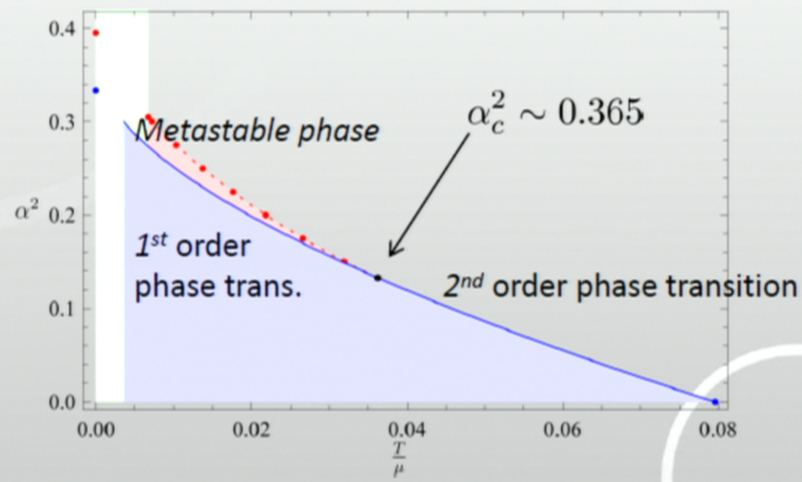
Solution to the EOM in gravity theory ⟷ Thermal equilibrium state in field theory

Central quantity: Free Energy $\Omega = I_{\text{on-shell}}/T$





Besides thermodynamic calculations, ask if solution stable under perturbations...



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9/23

Perturbations

$$\hat{g}_{MN}(t,\vec{x},r) = g_{MN}(r) + \int \frac{\mathrm{d}\omega\,\mathrm{d}^3\vec{k}}{(2\pi)^4}\, h_{MN}(\omega,\vec{k},r)e^{-\mathrm{i}\omega t + \mathrm{i}\vec{k}\cdot\vec{x}}$$

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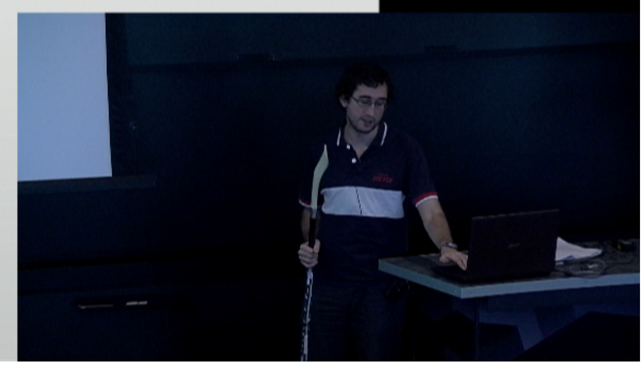

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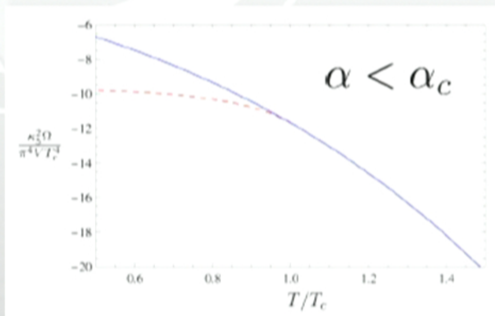

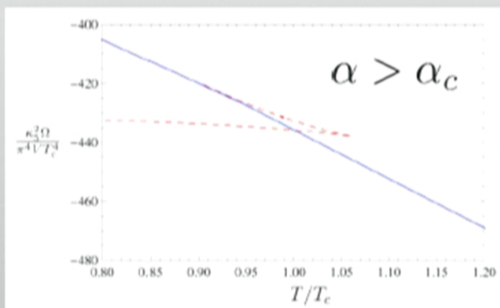

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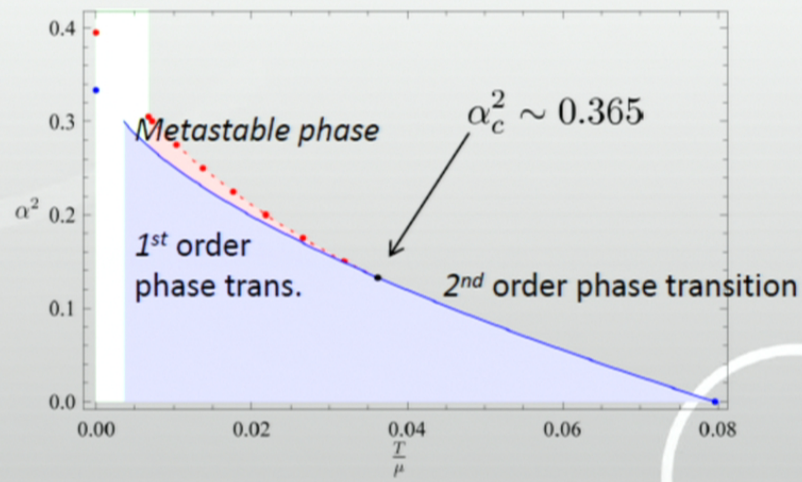

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- Gauge fixing:

$$h_{Mr} = 0 \,,\, a^a_r = 0$$

- Longitudinal momentum:

$$k^\mu = (\omega, k_{|}, k_\perp, 0)$$

so that perturbations preserve *SO(2)*.

The classification of perturbation fields

Helicity 2, helicity 1, helicity 0:

$$a^a_M = \begin{pmatrix} a^a_t \\ a^a_x \\ a^a_y \\ a^a_z \\ 0 \end{pmatrix}, \quad h_{MN} = \begin{pmatrix} h_{tt} & h_{xt} & h_{ty} & h_{tz} & 0 \\ h_{xt} & h_{xx} & h_{xy} & h_{xz} & 0 \\ h_{yt} & h_{xy} & h_{yy} & h_{yz} & 0 \\ h_{zt} & h_{xz} & h_{yz} & h_{zz} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{yy} = \frac{1}{2}\left(h_{yy} + h_{zz} + h_{yy} - h_{zz}\right)$$

$$\xi_y = g^{yy} h_{yy}, \quad \xi_x = g^{xx} h_{xx}, \quad \xi_t = g^{tt} h_{tt}, \quad \xi_{tx} = g^{xx} h_{tx}$$

Parity:

If k=0, also classifiable by change under $P_{||}$:

- $U(1)_3 \to \mathbb{Z}_2 \Longrightarrow$ flip sign index 2
- $A = \phi(r)\,\tau^3\,\mathrm{d}t + w(r)\,\tau^1\,\mathrm{d}x \Longrightarrow$ flip indices 1,x

even	*odd*
a^3_t	a^1_t, a^2_t
a^1_x, a^2_x	a^3_x
ξ_t, ξ_x, ξ_y	ξ_{tx}

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The Physical Fields

Helicity zero, k=0:

- There are 10 perturbation modes.
- Einstein's and Yang-Mills's eqs. give 10 DEs and 6 constraints → 14 d.o.f. at bdry.
- Ingoing condition (for retarded GF) at the horizon takes away 10 d.o.f.
- Remaining: 4 physical fields, invariant under residual gauge freedom.

$$\Phi_1(\omega,r) \longrightarrow \left(a_x^1\right)_0^b,$$
$$\Phi_2(\omega,r) \longrightarrow \left(a_x^2\right)_0^b,$$
$$\Phi_3(\omega,r) \longrightarrow (\xi_x)_0^b - (\xi_y)_0^b,$$
$$\Phi_4(\omega,r) \longrightarrow \left(a_x^3\right)_0^b .$$

It is convenient to change into:

$$a_x^{\pm} = a_x^1 \pm \mathrm{i}a_x^2$$
$$\xi_{p,m} = \xi_x \pm \xi_y$$

The action cannot be written in terms of physical fields only.

$$S_{\text{o.s.(I)}} = \int d^d x \left[\alpha_{\text{IJ}} \left(\Phi_{\text{I}}\right)_0^b \left(\Phi_{\text{J}}\right)_0^b + \beta_{\text{IJ}} \left(\Phi_{\text{I}}\right)_0^b \left(\varphi_{\text{J}}\right)_p^b + \zeta_{\text{IJ}} \left(\varphi_{\text{I}}\right)_p^b \left(\varphi_{\text{J}}\right)_p^b\right],$$

$$S_{\text{o.s.(II)}} = \int d^d x \left[\kappa_{\text{I}j} \left(\Phi_{\text{I}}\right)_0^b \left(\varphi_j\right)_0^b + \lambda_{ij} \left(\varphi_i\right)_0^b \left(\varphi_j\right)_0^b\right],$$

Replace those perturbations by physical fields, so that

$$S_{\text{o.s.1}} = \int \mathrm{d}^d k \left[\Phi_{\text{I}}(-k,r) A(k,r)_{\text{IJ}} \partial_r \Phi_{\text{J}}(k,r) + \Phi_{\text{I}}(-k,r) B(k,r)_{\text{IJ}} \Phi_{\text{J}}(k,r)\right]_{r=r_b}$$

Transverse thermoelectric effect

[Erdmenger, Kerner, Zeller]

Simultaneous transport of electric charge and heat:

$$\begin{pmatrix} \langle J^y \rangle \\ \langle Q^y \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{yy} & T\alpha^{yy} \\ T\alpha^{yy} & T\bar{\kappa}^{yy} \end{pmatrix} \begin{pmatrix} E_y \\ -(\nabla_y T)/T \end{pmatrix}$$

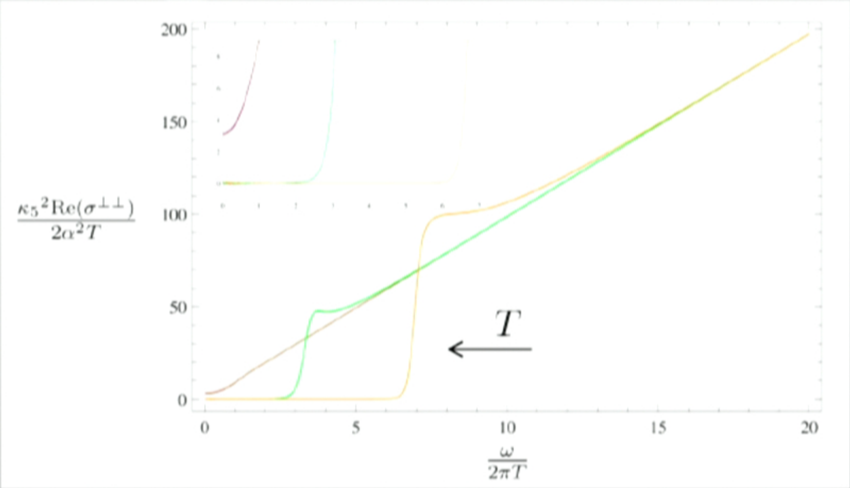
- Generation of electric current due to thermal gradient.
- Generation of thermal transport due to an external electric field.

<u>Heat flux</u>

$$Q^y = T^{ty} - \mu J^y$$

<u>Thermal gradient</u>

$$\nabla_y T = -\mathrm{i}\omega T \left(\xi_{ty}\right)_0^b$$



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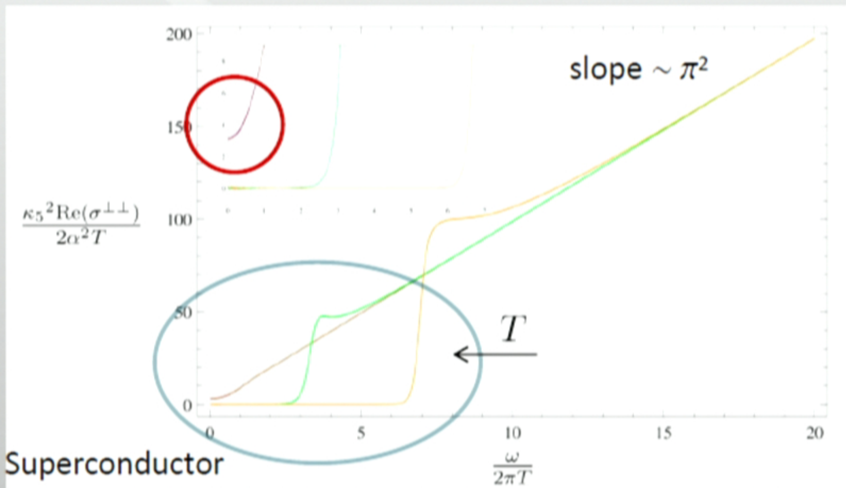
Heat flux: $Q^y = T^{ty} - \mu J^y$

Thermal gradient: $\nabla_y T = -\mathrm{i}\omega\, T\, (\xi_{ty})_0^b$

Electric field: $E_y = \mathrm{i}\omega \left((a_y^3)_0^b + \mu\, (\xi_{ty})_0^b \right)$



- Curves almost overlap for $T > T_c$
- Overlap of all curves asymptotically:

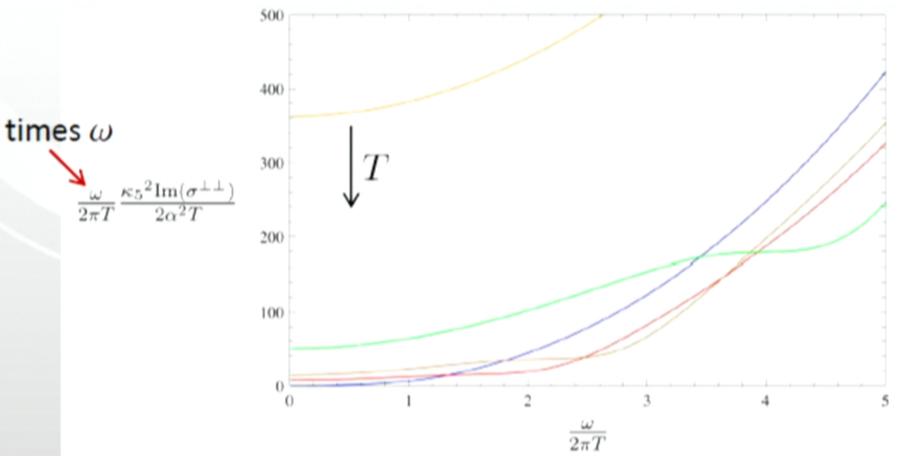
$$\mathrm{Re}(\sigma^{yy}) \propto \omega$$

- Consequence of conformal symmetry.

Transverse thermoelectric effect

[Erdmenger, Kerner, Zeller]

Imaginary part:



- Pole at the origin $\Longrightarrow$ Real part has delta peak (K-K relation)
- Delta peak due to sum rule, observed here.
- Anticipated behavior:

$$\omega\,\mathrm{Im}\,(\sigma) \simeq A_{\mathrm{D}}(\alpha, T) + A_s(\alpha)\left(1 - \frac{T}{T_c}\right)$$

Drude peak ∀T

Appears in superfluid phase

Transverse thermoelectric effect

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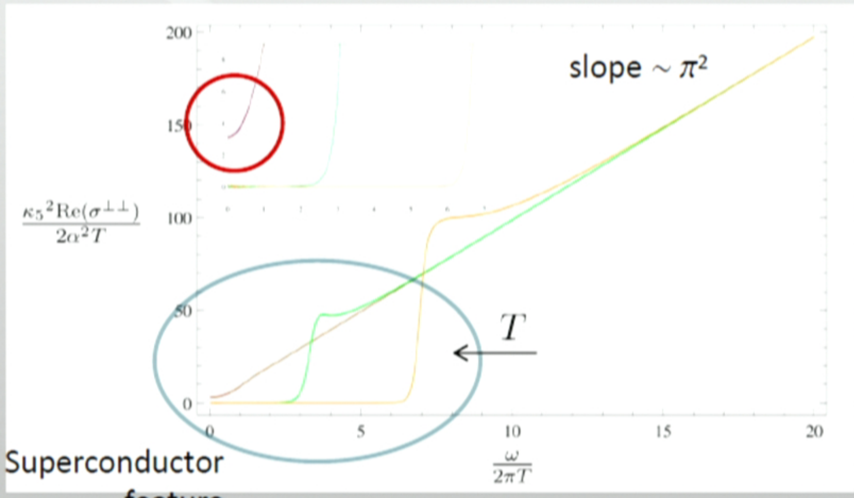

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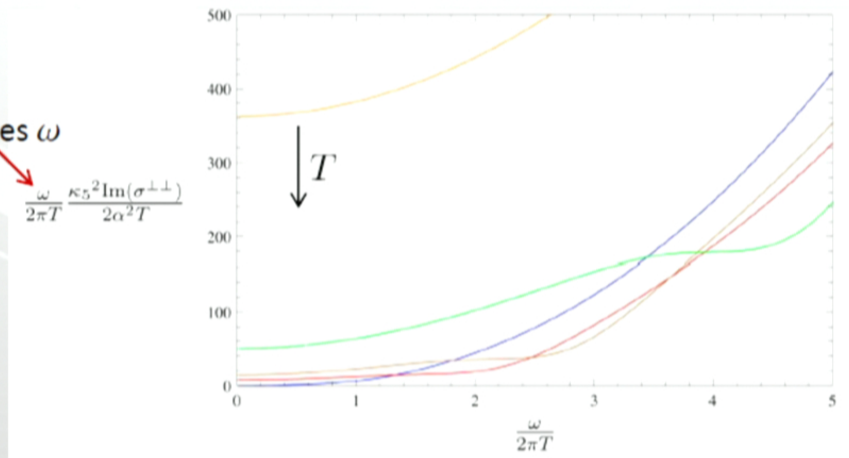
Transverse thermoelectric effect

[Erdmenger, Kerner, Zeller]

Imaginary part:

times ω



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Appears in superfluid phase

Longitudinal thermoelectric effect

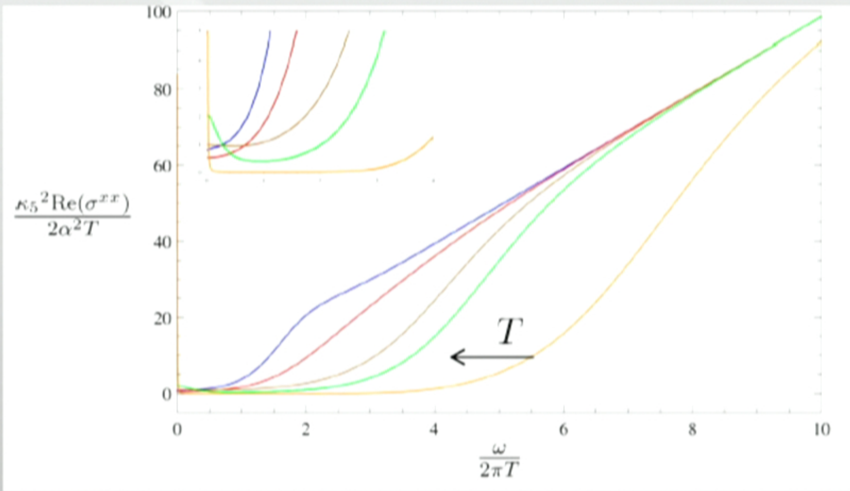
[Erdmenger, DF, Zeller]

$$\begin{pmatrix}\langle J^x\rangle\\ \langle Q^x\rangle\end{pmatrix} = \begin{pmatrix}\sigma^{xx} & T\alpha^{xx}\\ T\alpha^{xx} & T\bar{\kappa}^{xx}\end{pmatrix}\begin{pmatrix}E_x\\ -(\nabla_x T)/T\end{pmatrix}$$

Additional couplings:

$$\begin{pmatrix}\langle J_1^t\rangle\\ \langle J_2^t\rangle\\ \langle J^x\rangle\\ \langle Q^x\rangle\end{pmatrix} = \begin{pmatrix}\sigma_{1,1}^{t,t} & \sigma_{1,2}^{t,t} & \sigma_{1,3}^{t,x} & -\mu\sigma_{1,3}^{t,x}\\ \sigma_{2,1}^{t,t} & \sigma_{2,2}^{t,t} & \sigma_{2,3}^{t,x} & -\mu\sigma_{2,3}^{t,x}\\ \sigma_{3,1}^{x,t} & \sigma_{3,2}^{x,t} & \sigma^{xx} & T\alpha^{xx}\\ -\mu\sigma_{3,1}^{x,t} & -\mu\sigma_{3,2}^{x,t} & T\alpha^{xx} & T\bar{\kappa}^{xx}\end{pmatrix}\begin{pmatrix}i\omega a_t^1\\ i\omega a_t^2\\ E_x\\ -\frac{\nabla_x T}{T}\end{pmatrix}$$

Interpretation: a_t^1, a_t^2 rotate charge density in directions 1, 2 without changing its magnitude.



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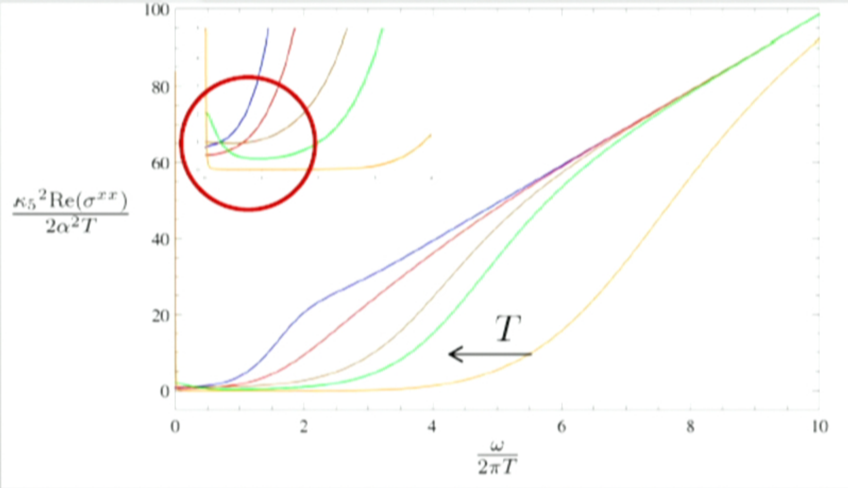
Interpretation: a_t^1, a_t^2 rotate charge density into directions 1, 2 without changing its total amount.



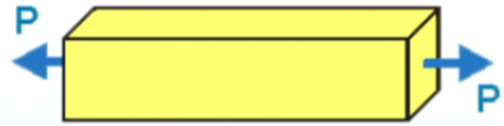
Differences:

- Decrease starts at larger ω.
- σ does not vanish for any frequency.
- In fact, it increases again.

Quasinormal mode

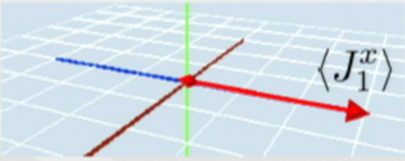
Piezoelectric effect

$$\begin{pmatrix} \langle J^x_\pm \rangle \\ \langle T^{xx}, T^{yy}, T^{tt} \rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} a^\pm_x \\ h_{xx}, h_{yy}, h_{tt} \end{pmatrix}$$



- Generation of electric current due to elongation/squeezing.
- Generation of mechanical strain due to an external electric field.

Intuitive picture:



$\langle J^x_1 \rangle \sim A^1_x$

Piezoelectric effect

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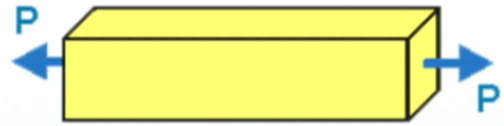

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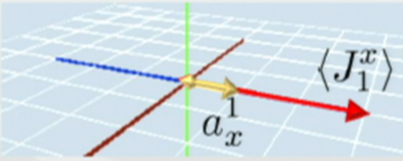


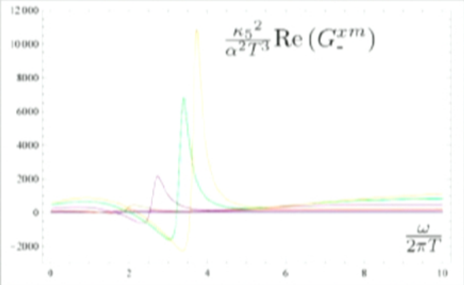
$\langle J^x_1 \rangle \sim A^1_x + a^1_x$

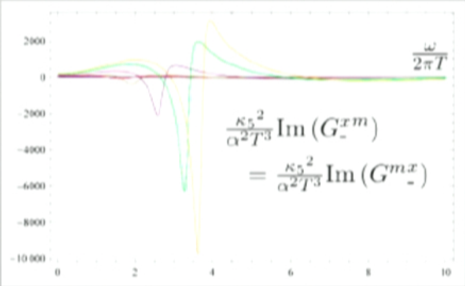
Background

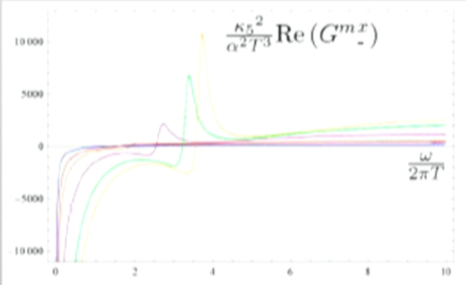
$\langle J^x_1 \rangle$

$\langle T^{xx}, T^{yy}, T^{tt} \rangle$

[Erdmenger, DF, Zeller]

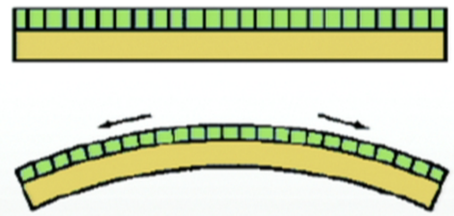






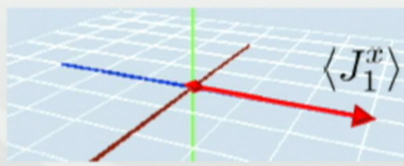
Flexoelectric effect

$$\begin{pmatrix} \langle J^y_\pm \rangle \\ \langle T^{xy} \rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} a^\pm_y \\ h_{xy} \end{pmatrix}$$



- Generation of electric current due to shear stress.
- Generation of shear deformation due to an external electric field.

Intuitive picture:



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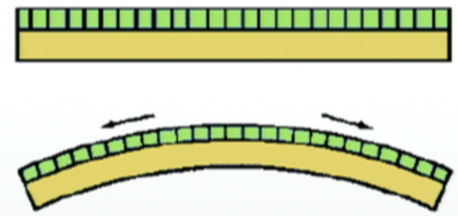

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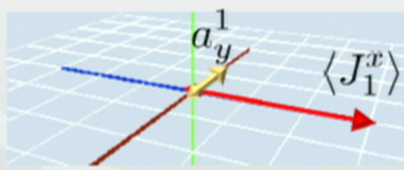
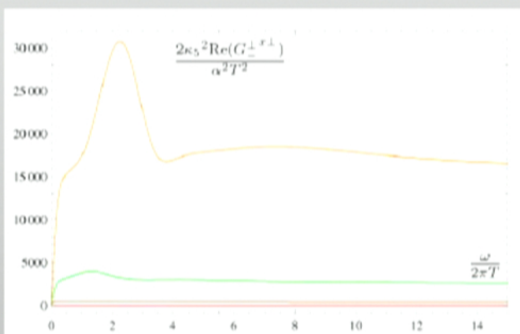
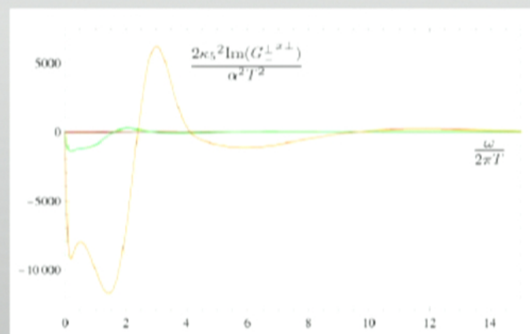


The system tries to cancel the new contribution.

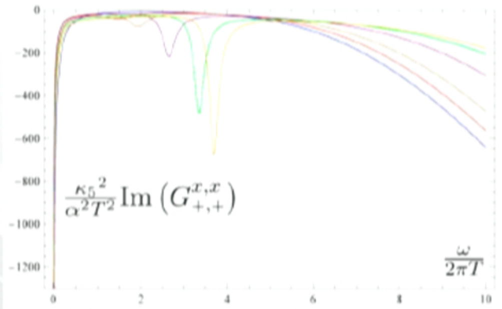
[Erdmenger, Kerner, Zeller]





Goldstone fields and Quasinormal modes

Condensate selects preferred direction $\Longrightarrow$ a_x^2 becomes Goldstone mode.



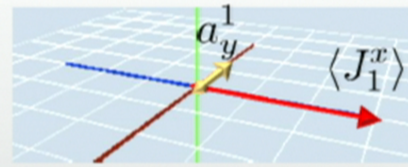
$$A_x^1 = w + \epsilon\, a_x^1,\;\; A_x^2 = \epsilon\, a_x^2$$

$$A_x^\pm = (w + \epsilon\, \rho) e^{\pm i\theta/w}$$

$$a_x^1 \sim \rho,\;\; a_x^2 \sim \theta$$

Other GS modes:

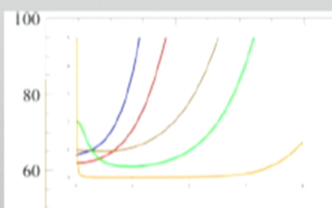
a_y^1, a_z^1

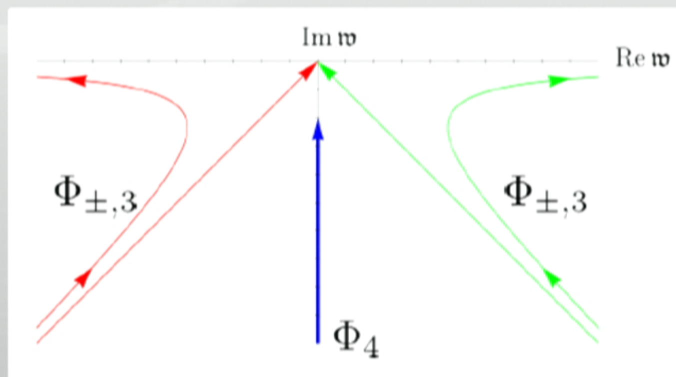


The poles at ω=0 reflect the formation of this massless mode.

Quasinormal modes behavior:

The quasinormal mode of the thermoelectric effect goes up the imaginary axis (ω=0)





The Physical Fields

Helicity zero, k=0:

- There are 10 perturbation modes.
- Einstein's and Yang-Mills's eqs. give 10 DEs and 6 constraints → 14 d.o.f. at bdry.
- Ingoing condition (for retarded GF) at the horizon takes away 10 d.o.f.
- Remaining: 4 physical fields, invariant under residual gauge freedom.

$$\Phi_1(\omega, r) \longrightarrow \left(a_x^1\right)_0^b ,$$
$$\Phi_2(\omega, r) \longrightarrow \left(a_x^2\right)_0^b ,$$
$$\Phi_3(\omega, r) \longrightarrow \left(\xi_x\right)_0^b - \left(\xi_y\right)_0^b ,$$
$$\Phi_4(\omega, r) \longrightarrow \left(a_x^3\right)_0^b .$$

It is convenient to change into:

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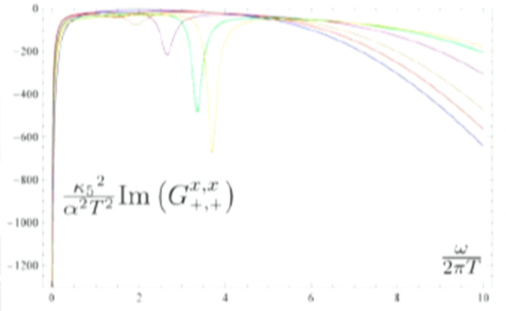

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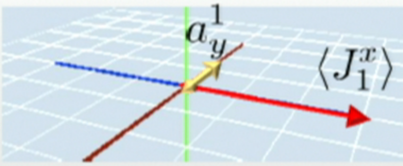

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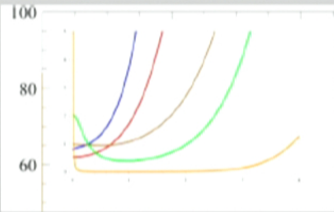

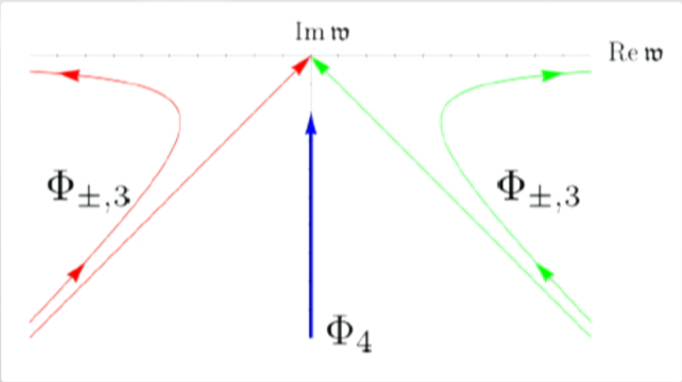

The viscosity tensor

[Landau, Lifshitz]

- Internal motion of a system causes dissipation of energy.
- Postulate dissipation function. Its velocity derivatives are frictional forces, linear in u_μ.
- Translation/rotation → No dissipation, so actually linear in $u_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu\right)$.

$$\Xi = \frac{1}{2}\eta^{\mu\nu\lambda\rho}u_{\mu\nu}u_{\lambda\rho} \Rightarrow T^{\mu\nu}_{\text{diss}} = -\frac{\partial \Xi}{\partial u_{\mu\nu}}$$

- For a transversely isotropic fluid,

$$\eta^{xxxx} = \zeta_x - 2\lambda\,, \qquad \eta^{yyyy} = \eta^{zzzz} = \zeta_y - \frac{\lambda}{2} + \eta_{yz}\,,$$

$$\eta^{xxyy} = \eta^{xxzz} = \lambda\,, \qquad \eta^{yyzz} = \zeta_y - \frac{\lambda}{2} - \eta_{yz}\,,$$

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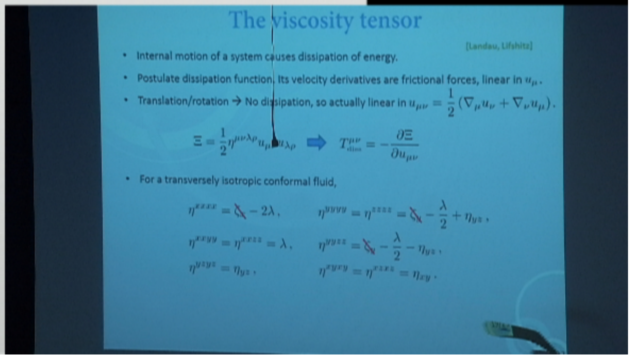

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$$\boxed{\eta^{yzyz} = \eta_{yz}\,,} \qquad \boxed{\eta^{xyxy} = \eta^{xzxz} = \eta_{xy}\,.}$$

Shear viscosities

$$\eta_{yz} = -\lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\,\langle T_{yz}T_{yz}\rangle \qquad \eta_{xy} = -\lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\,\langle T_{xy}T_{xy}\rangle$$

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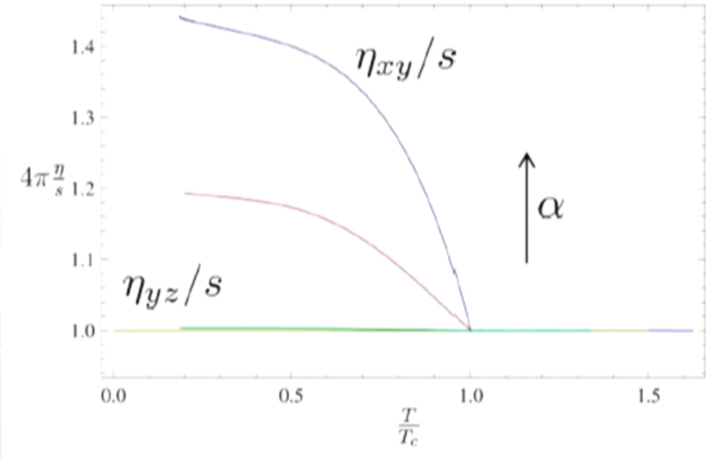
Shear viscosities



[Erdmenger, Kerner, Zeller]
[Kovtun, Son, Starinets, Buchel, Liu, Iqbal]

$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$$

$$\frac{\eta_{xy}}{s} \geq \frac{1}{4\pi}$$

- In the normal phase, they coincide with the universal value of an isotropic fluid.
- In the superfluid phase, they deviate but the viscosity bound is satisfied.

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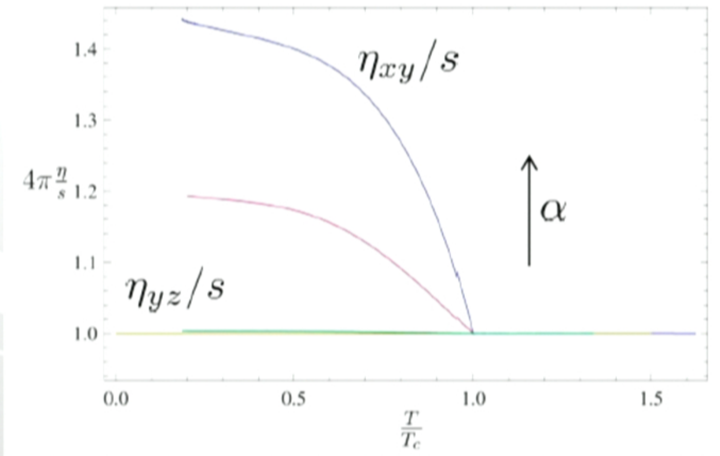
Shear viscosities

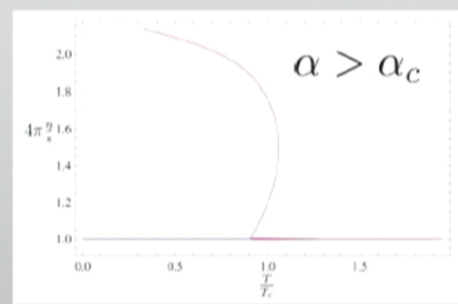
[Erdmenger, Kerner, Zeller]
[Kovtun, Son, Starinets, Buchel, Liu, Iqbal]



$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$$

$$\frac{\eta_{xy}}{s} \geq \frac{1}{4\pi}$$

- In the normal phase, they coincide with the universal value of an isotropic fluid.
- In the superfluid phase, they deviate but the viscosity bound is satisfied.



- In the *1st* order phase transition, it is multivalued.
- The presence of anisotropy makes it deviate.

The first normal stress difference

If we assume a conformal fluid,

$$\eta^{xxxx} = -2\lambda\,, \qquad \eta^{yyyy} = \eta^{zzzz} = -\frac{\lambda}{2} + \eta_{yz}\,,$$

$$\eta^{xxyy} = \eta^{xxzz} = \lambda, \quad \eta^{yyzz} = -\frac{\lambda}{2} - \eta_{yz}\,.$$

So that the dissipative part of the normal stress difference is:

$$T^{xx}_{\rm diss} - T^{yy}_{\rm diss} = 3\lambda\,(\nabla_x u_x - \nabla_y u_y)$$

Among the physical fields there is

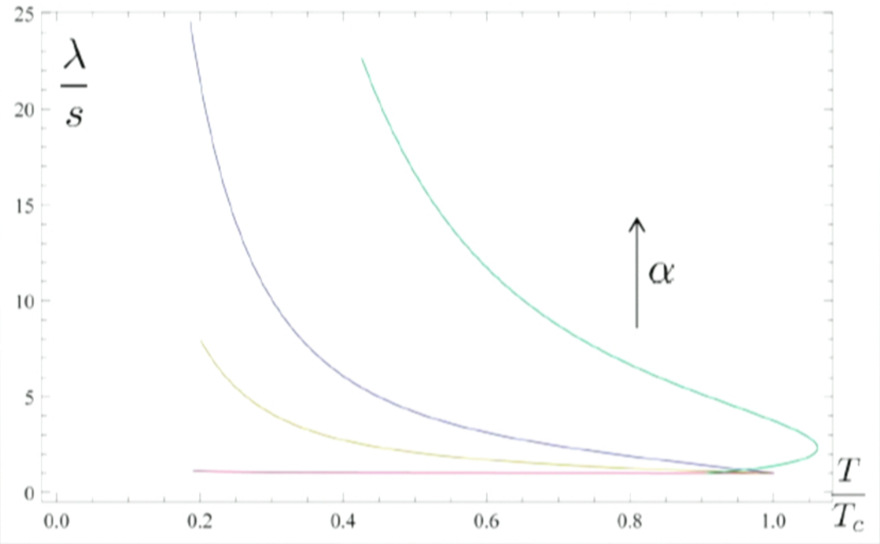
$\Phi_3(\omega, r) \longrightarrow (\xi_x)^b_0 - (\xi_y)^b_0\,,$ so its Green's function is identified with

$$G^{m,m}(\omega) = \lim_{|\vec{k}|\to 0} \int \mathrm{d}t\,\mathrm{d}^3x\, e^{-\mathrm{i}k_\mu x^\mu}\,\theta(t)\,\langle[T^x{}_x(t,\vec{x}) - T^y{}_y(t,\vec{x}), T^x{}_x(0,0) - T^y{}_y(0,0)]\rangle$$

Kubo formula: $$\lambda = -\lim_{\omega\to 0} \frac{1}{3\,\omega}\,\mathrm{Im}\,G^{m,m}(\omega)$$

The transport coefficient λ

[Erdmenger, DF, Zeller]



In the normal phase,

$$\frac{\lambda}{s} = \frac{1}{72\pi}$$

Similar to the $1/4\pi$ derivation: $h_{xx} - h_{yy}$ behaves as a minimally coupled scalar in gravity.

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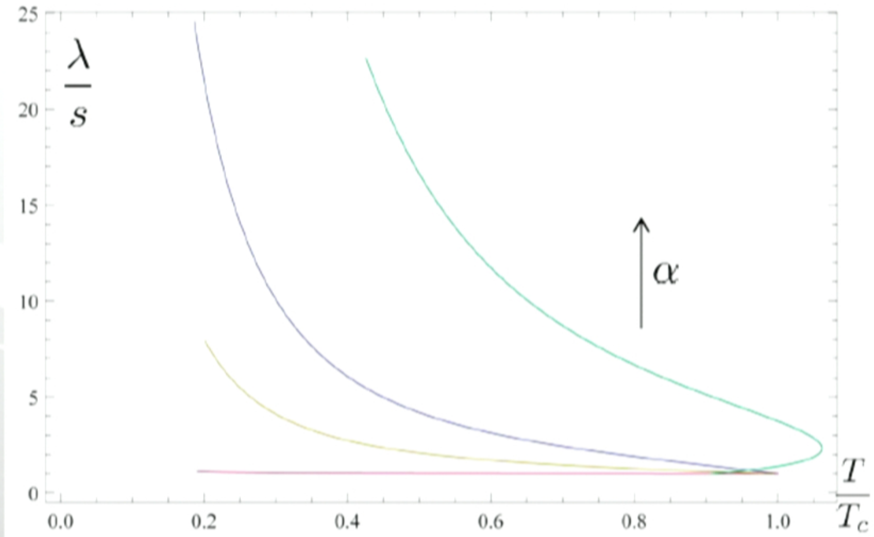
[Erdmenger, DF, Zeller]



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- Coventionally, normal stresses pull apart compressing surfaces.

<u>Weissenberg effect (crystal liquids):</u>

- Spinning rod in material $\Longrightarrow$ Fluid is attracted inwards: $\lambda > 0$.
- Effect more pronounced for lower T.
- At the level of symmetries, symilar to our system.

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Conclusions

✓ Check of the **universal bound** for the ratio η/s.

✓ Found **thermoelectric effect** favored by the condensate:

- Enhancement of conductivity for low T (high ω), suppression above T_c .
- Sudden increase due to a pole near ω=0, due to quasinormal mode.

✓ New phenomena: **Flexoelectric** and **Piezoelectric** effects.

- Bumps in correlators, related to possible bound states.

✓ In the ω=0 limit, found new component of **viscosity tensor**.

➢ Results valid as effective macroscopic description of transport properties near T_c .

Outlook:

- Covariant hydrodynamic description of anisotropic superfluids.
- Analysis at finite k: Dispersion relations and new instabilities.