Title: What is motion? What is time?

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Abstract: I give an account of the Machian approach to dynamics, from Mach's critique of Newton to the work of Barbour, Bertotti, York and O'Murchadha, which culminated in the theory of Shape Dynamics, a new and original way of thinking about General Relativity. I conclude commenting on the present research lines in Shape Dynamics, and the opportunity it offers to solve the problem of time in quantum gravity.

Support for this colloquium is provided by The Templeton Frontiers Program.

WHAT IS MOTION?
WHAT IS TIME?

FLAVIO MERCATI

PERIMETER INSTITUTE - JAN 2013

NEWTON'S LAW OF INERTIA

EVERY BODY PERSEVERES IN ITS STATE OF REST, OR OF UNIFORM MOTION IN A RIGHT LINE, UNLESS IT IS COMPELLED TO CHANGE THAT STATE BY FORCES IMPRESS'D THEREON.

THE MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY (1687)

REST, UNIFORM MOTION AND RIGHT LINE
WITH RESPECT TO WHAT?

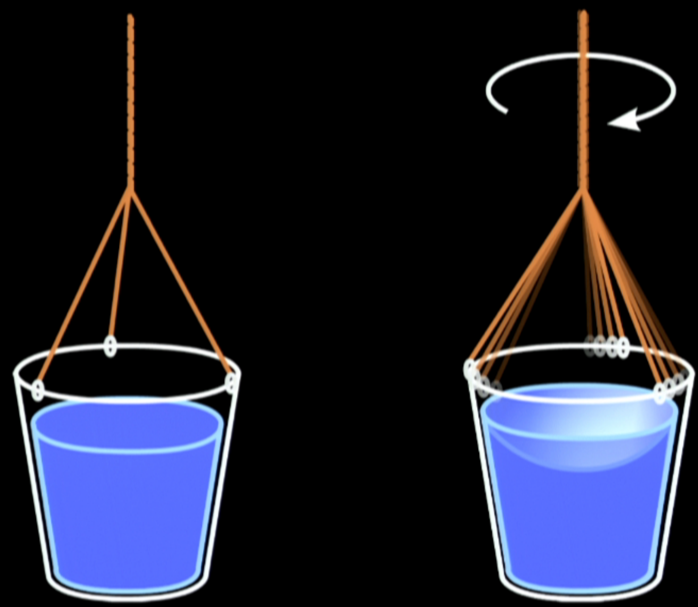
NEWTON'S ANSWER

ABSOLUTE, TRUE, AND MATHEMATICAL TIME, OF ITSELF, AND FROM ITS OWN NATURE, FLOWS EQUABLY WITHOUT REGARD TO ANYTHING EXTERNAL, AND BY ANOTHER NAME IS CALLED DURATION.

ABSOLUTE SPACE, IN ITS OWN NATURE, WITHOUT REGARD TO ANYTHING EXTERNAL, REMAINS ALWAYS SIMILAR AND IMMOVEABLE.

LEIBNIZ: THOSE ARE NOT OBSERVABLE

NEWTON'S BUCKET



THE PROOF OF ABSOLUTE SPACE

(AND AN OPPORTUNITY TO MAKE FUN OF DESCARTES)

THE SCHOLIUM PROBLEM

TO DEDUCE THE TRUE MOTIONS
(IN ABSOLUTE SPACE AND TIME)
FROM THE OBSERVABLE MOTIONS

...BUT HOW WE ARE TO OBTAIN THE TRUE MOTIONS FROM THEIR CAUSES, EFFECTS, AND APPARENT DIFFERENCES, AND THE CONVERSE, SHALL BE EXPLAINED MORE AT LARGE IN THE FOLLOWING TREATISE. FOR TO THIS END IT WAS THAT I COMPOSED IT.

...NEWTON NEVER DID IT!

AND WHAT ABOUT HIS SUCCESSORS?

...AFTER 200 YEARS OF THIS ATTITUDE

ERNST MACH

THE DISCOVERER OF SHOCKWAVES

MACH NUMBERS (AND FRENCH *SHAVING RAZORS*) ARE NAMED AFTER HIM

MACH'S PRINCIPLE

…NEVER FORMULATED BY MACH!

MACH JUST OBSERVED THAT EXPERIMENTALLY, NEWTON'S LAWS WERE NOT VERIFIED RELATIVE TO INVISIBLE SPACE BUT TO THE DISTANT STARS, WITH THE EARTH'S ROTATION PROVIDING A CLOCK

NO VISIBLE EFFECT SHOULD HAVE AN INVISIBLE CAUSE: THE SAME STARS SHOULD EXPLAIN THE WATER'S BEHAVIO

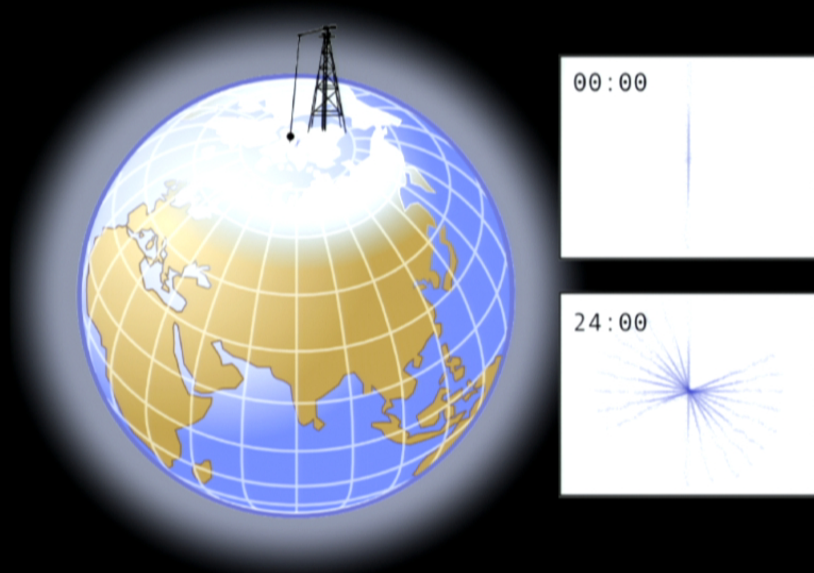
MACH'S KEY REMARK:

NO ONE IS COMPETENT TO SAY HOW THE EXPERIMENT WOULD TURN OUT IF THE SIDES OF THE VESSEL INCREASED IN THICKNESS AND MASS UNTIL THEY WERE ULTIMATELY SEVERAL LEAGUES THICK.

THE SCIENCE OF MECHANICS (1883)

....THIS STRUCK A LOT OF PEOPLE (INCLUDING EINSTEIN)

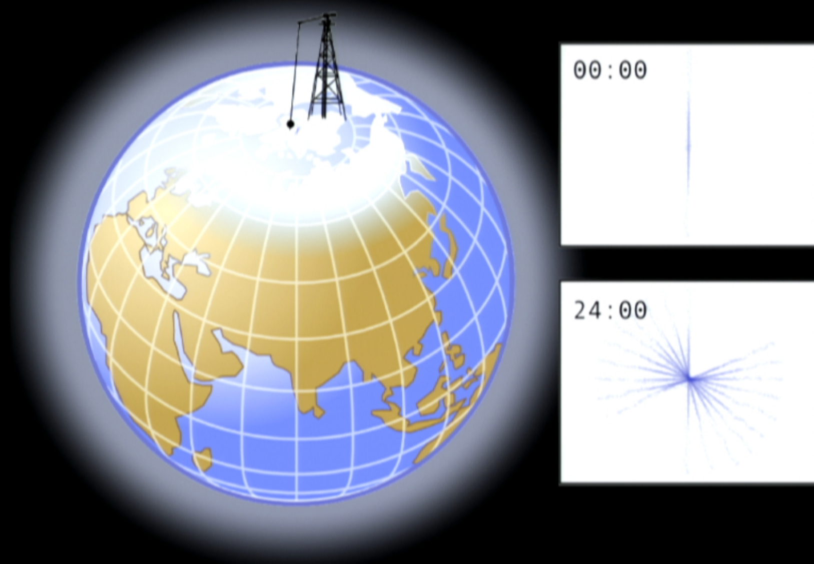
HOFMANN'S THOUGHT EXPERIMENT



PROPOSED IN 1904

OBSERVED IN THE 2000'S BY THE LAGEOS AND GRAVITY PROBE B SATELLITES

HOFMANN'S THOUGHT EXPERIMENT



PROPOSED IN 1904

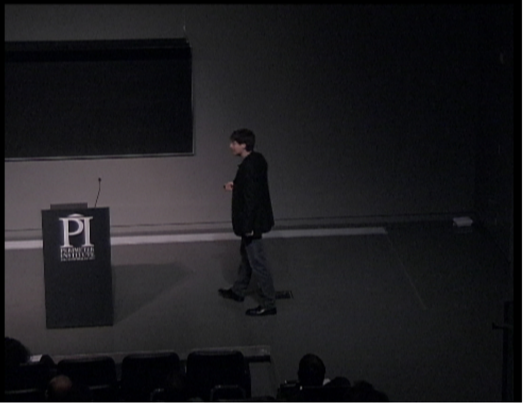
OBSERVED IN THE 2000'S BY THE LAGEOS AND GRAVITY PROBE B SATELLITES

EINSTEIN AND MACH

MACH WAS A STIMULUS TO THE CREATION OF GENERAL RELATIVITY

EINSTEIN TRIED TO FIND A PRECISE FORMULATION OF MACH'S PRINCIPLE, CAME UP WITH SEVERAL FORMULATIONS, AND ABANDONED THEM ALL.

THE PROBLEM IS THAT MACH WAS NEVER TOO CLEAR ABOUT IT.

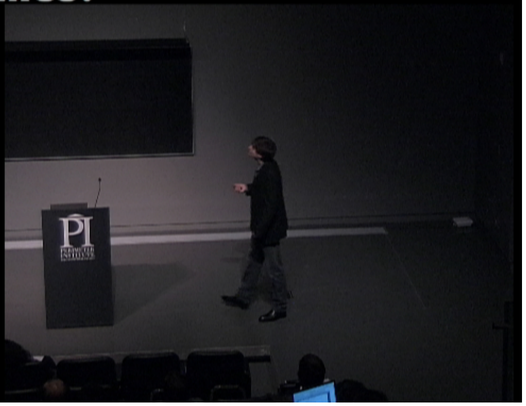


B&B REDISCOVER POINCARÉ

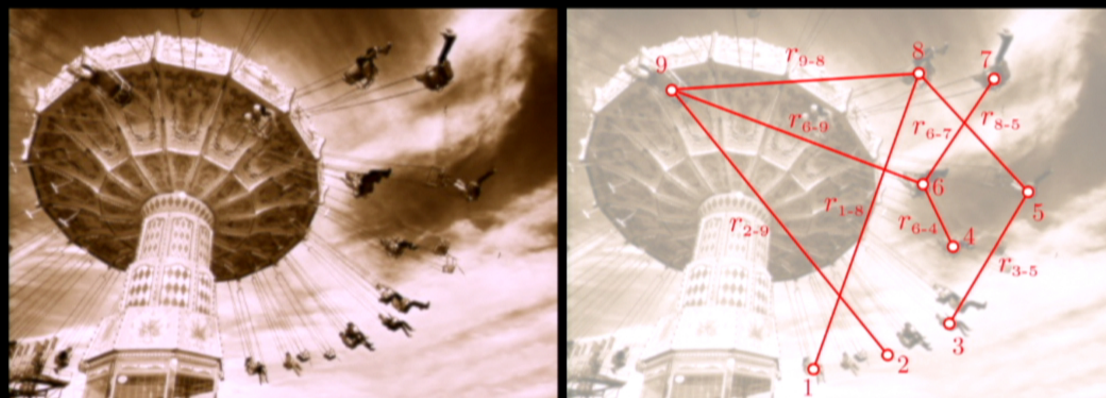
H. POINCARÉ INADVERTENTLY GAVE A PRECISE DEFINITION OF MACH'S PRINCIPLE IN "SCIENCE AND HYPOTHESIS" (1902)

PASSED UNDER THE RADARS UNTIL 1982, WHEN J. BARBOUR AND B. BERTOTTI REALIZED ITS RELEVANCE FOR MACH'S PRINCIPLE

POINCARÉ: WHAT'S THE PROBLEM WITH NEWTONIAN DYNAMICS? WHY DOES HE HAVE TO INTRODUCE ABSOLUTE SPACE?



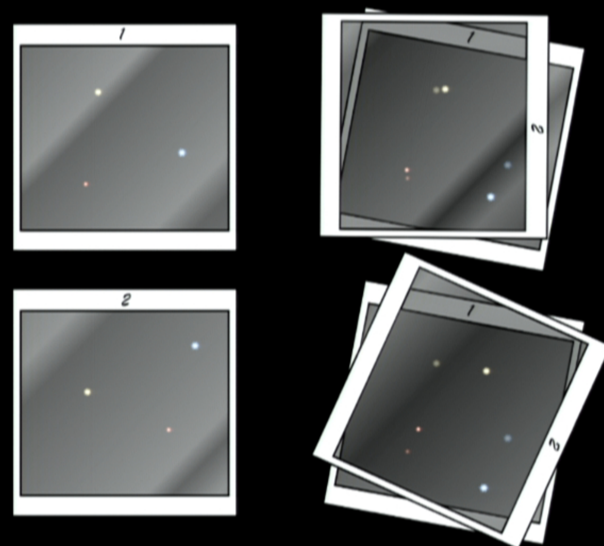
ANSWER: BECAUSE OF ANGULAR MOMENTUM.



CAN'T DETERMINE FUTURE EVOLUTION FROM OBSERVABLE INITIAL DATA ALONE

r_{ab} , $\frac{dr_{ab}}{dt}$ $\qquad$ $r_{ab} = ||\vec{r}_a - \vec{r}_b||$

THE PROBLEM OF EQUILOCALITY



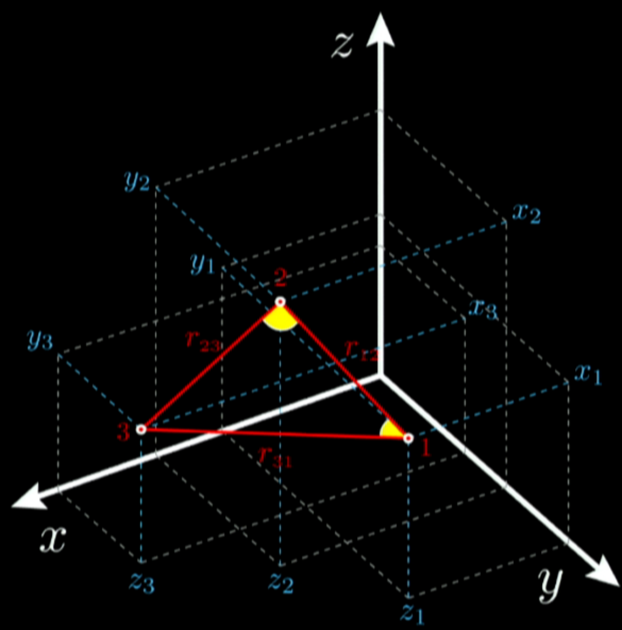
WHAT DOES IT MEAN TO BE
AT THE SAME PLACE AT DIFFERENT TIMES?

THE MACH–POINCARÉ PRINCIPLE

THE DYNAMICS SHOULD BE SUCH THAT **OBSERVABLE** INITIAL DATA (AND THEIR FIRST DERIVATIVES) ALONE SHOULD UNIQUELY DETERMINE THE EVOLUTION.

...BUT **WHAT** IS OBSERVABLE?

CONFIGURATION SPACES



$3N$ **CARTESIAN COORDINATES** (x_a, y_a, z_a) **(3-body: 9 coordinates)**

$\frac{N(N-1)}{2}$ **INTER-PARTICLE SEPARATIONS** r_{ab} **(3-body: 3 separations)**

$\frac{N(N-1)}{2} - 1$ **RATIOS** **(3-body: triangle - 2 angles)**

CONFIGURATION SPACES

$Q^N =$ **SPACE OF CARTESIAN COORDINATES** ($3N$-**DIMENSIONAL**)

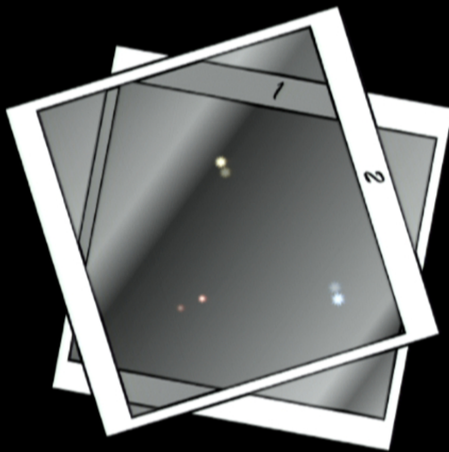
QUOTIENTING BY THE EUCLIDEAN GROUP
Eucl **OF ROTATIONS AND TRANSLATIONS:**

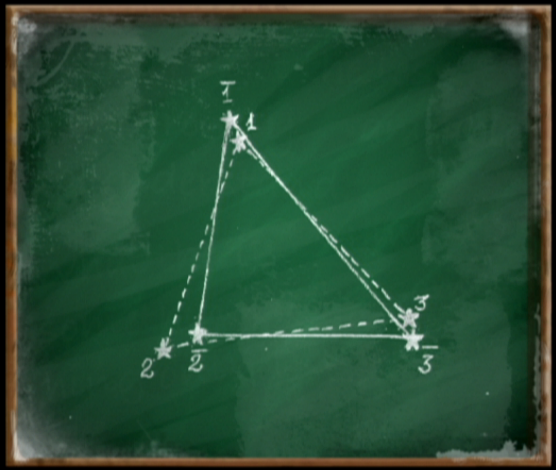
$Q_R = Q^N/\text{Eucl} =$ **RELATIVE CONFIGURATION SPACE** ($3N-6$-**DIM.**)

QUOTIENTING BY THE SIMILARITY GROUP
Sim **OF ROTATIONS, TRANSLATIONS AND RESCALINGS:**

$\mathsf{S} = Q^N/\text{Sim} =$ ***SHAPE SPACE*** ($3N-7$-**DIMENSIONAL**)

BEST MATCHING





$$d\vec{r}_{\rm a} = \vec{r}_{\rm a}{}' - \vec{r}_{\rm a}\,, \qquad \vec{r}_{\rm a} \to \vec{r}_{\rm a} + d\vec{a} + d\vec{\omega}\times\vec{r}_{\rm a}$$

$$\delta\vec{r}_{\rm a} = d\vec{r}_{\rm a} - d\vec{a} - d\vec{\omega}\times\vec{r}_{\rm a}$$

$$\mathscr{L} = \sqrt{\sum_{\rm a}\delta\vec{r}_{\rm a}\cdot\delta\vec{r}_{\rm a}}\,, \qquad \delta_{\rm bm}\vec{r}_{\rm a} = \left\{\delta\vec{r}_{\rm a}\ \text{ s.t. }\ \mathscr{L} = \min_{da,d\omega}\sqrt{\sum_{\rm a}\delta\vec{r}_{\rm a}\cdot\delta\vec{r}_{\rm a}}\right\}$$

CONFIGURATION SPACES

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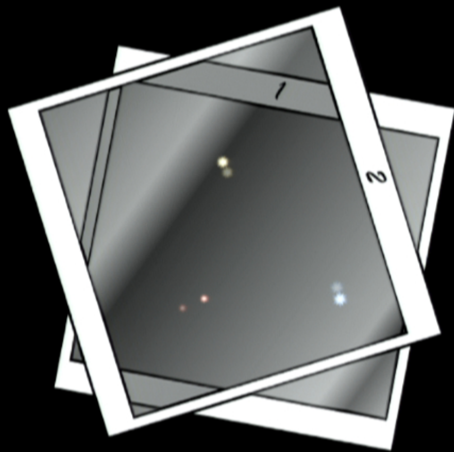
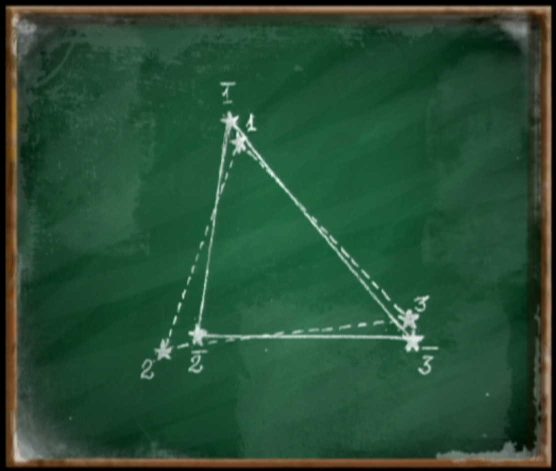
BEST MATCHING





$$d\vec{r}_{\mathrm{a}} = \vec{r}_{\mathrm{a}}{}' - \vec{r}_{\mathrm{a}}, \qquad \vec{r}_{\mathrm{a}} \to \vec{r}_{\mathrm{a}} + d\vec{a} + d\vec{\omega} \times \vec{r}_{\mathrm{a}}$$

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$$\mathscr{L} = \sqrt{\sum_{\mathrm{a}} \delta\vec{r}_{\mathrm{a}} \cdot \delta\vec{r}_{\mathrm{a}}}, \qquad \delta_{\mathrm{bm}}\vec{r}_{\mathrm{a}} = \left\{ \delta\vec{r}_{\mathrm{a}} \ \text{ s.t. } \ \mathscr{L} = \min_{da,d\omega} \sqrt{\sum_{\mathrm{a}} \delta\vec{r}_{\mathrm{a}} \cdot \delta\vec{r}_{\mathrm{a}}} \right\}$$

BEST MATCHING

$\delta_{\rm bm}\vec{r}_{\rm a}$ IS A MEASURE OF THE INTRINSIC CHANGE
(CHANGE NOT DUE TO INVISIBLE MOTION IN ABSOLUTE SPACE)

$\delta_{\rm bm}\vec{r}_{\rm a}$ CAN BE 'DRESSED', *E.G.:*

GIVING MORE WEIGHT TO HEAVIER PARTICLES:

$$\mathscr{L} = \sqrt{\sum_{\rm a} m_{\rm a}\, \delta\vec{r}_{\rm a}\cdot\delta\vec{r}_{\rm a}}$$

AND GIVING DIFFERENT WEIGHT TO DIFFERENT CONFIGURATIONS
(POTENTIAL ENERGY):

$$\mathscr{L} = \sqrt{U(\vec{r})\sum_{\rm a} m_{\rm a}\, \delta\vec{r}_{\rm a}\cdot\delta\vec{r}_{\rm a}}\,, \qquad \text{E.G.} \qquad U(\vec{r}) = E_{\rm tot} - \sum_{\rm a<b}\frac{m_{\rm a}\, m_{\rm b}}{||\vec{r}_{\rm a}-\vec{r}_{\rm b}||}$$

NOTE THAT $U(\vec{r})$ HAS TO BE A FUNCTION ON Q_R!


WHAT ABOUT TIME?

BACK TO MACH

"IT IS UTTERLY BEYOND OUR POWER TO MEASURE THE CHANGES OF THINGS BY TIME. QUITE THE CONTRARY, TIME IS AN ABSTRACTION AT WHICH WE ARRIVE BY MEANS OF THE CHANGES OF THINGS"

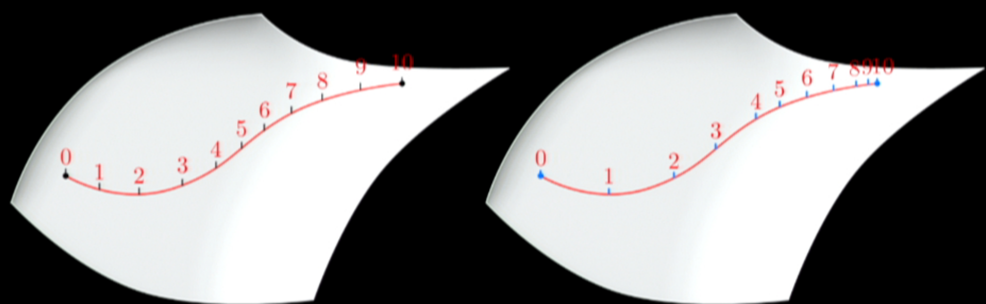
THE SCIENCE OF MECHANICS (1883)

GEODESIC PRINCIPLE

PARAMETRIZED CURVE $x^i(\lambda)$ **MINIMIZING** $S = \int d\lambda \sqrt{g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}}$,

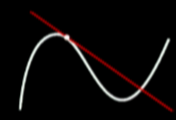
INVARIANT UNDER $\lambda \to \lambda'(\lambda)$, **(** λ' **MONOTONIC)**

$$S = \int d\lambda \sqrt{g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}} \equiv \int d\lambda' \sqrt{g_{ij} \frac{dx^i}{d\lambda'} \frac{dx^j}{d\lambda'}},$$

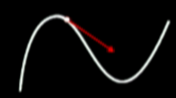


WEAK AND STRONG POINCARÉ PRINCIPLES

A GEODESIC IS FIXED GIVEN A
POINT AND A TANGENT LINE



STRONGER THAN THE POINCARÉ PRINCIPLE
REQUIRING A POINT AND A TANGENT VECTOR



STRONG POINCARÉ PRINCIPLE:

"A POINT AND A DIRECTION IN OBSERVABLE CONFIGURATION SPACE UNIQUELY DETERMINE THE EVOLUTION"

WEAK POINCARÉ PRINCIPLE:

"A POINT AND A TANGENT VECTOR IN OBSERVABLE CONFIGURATION SPACE UNIQUELY DETERMINE THE EVOLUTION"

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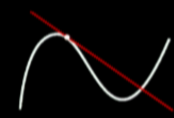

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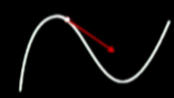

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"A POINT AND A DIRECTION IN OBSERVABLE CONFIGURATION SPACE UNIQUELY DETERMINE THE EVOLUTION"

WEAK POINCARÉ PRINCIPLE:
"A POINT AND A TANGENT VECTOR IN OBSERVABLE CONFIGURATION SPACE UNIQUELY DETERMINE THE EVOLUTION"

(JACOBI ACTION)

$$S=\int d\lambda \sqrt{U(r)\sum_a m_a \frac{\delta \vec{r}_a}{d\lambda}\cdot\frac{\delta \vec{r}_a}{d\lambda}}=\int d\lambda \mathscr{L}\,, \qquad \frac{\delta \vec{r}_a}{d\lambda}=\dot{\vec{r}}_{\mathrm{a}}-\dot{\vec{a}}-\dot{\vec{\omega}}\times\vec{r}_{\mathrm{a}}\,, \quad \dot{\vec{r}}_{\mathrm{a}}=\frac{d\vec{r}_a}{d\lambda}$$

MOMENTA AND QUADRATIC CONSTRAINT

$$\vec{p}_a=\frac{\delta\mathscr{L}}{\delta\dot{\vec{r}}_{\mathrm{a}}}=m_a\frac{\delta \vec{r}_a}{d\lambda}\sqrt{\frac{U(r)}{\sum_b \frac{\delta \vec{r}_b}{d\lambda}\cdot\frac{\delta \vec{r}_b}{d\lambda}}} \quad\Rightarrow\quad \boxed{\sum_a \frac{\vec{p}_a\cdot\vec{p}_a}{2m_a}-U(r)=H=0}$$

$$\dot{\vec{q}}_a=\frac{\partial H}{\partial \vec{p}_a}=\frac{\vec{p}_a}{m_a}\,, \qquad \dot{\vec{p}}_a=-\frac{\partial H}{\partial \vec{q}_a}=-\frac{\partial U(r)}{\partial \vec{q}_a}$$

BEST-MATCHING WRT ROTATIONS AND TRANSLATIONS

$$\frac{\delta\mathscr{L}}{\delta\dot{\vec{a}}}=-\sum_a \vec{p}_a=\boxed{\vec{P}_{\mathrm{tot}}=0} \qquad \frac{\delta\mathscr{L}}{\delta\dot{\vec{\omega}}}=-\sum_a \vec{r}_a\times\vec{p}_a=\boxed{\vec{L}_{\mathrm{tot}}=0}$$

TOTAL MOMENTUM AND ANGULAR MOMENTUM OF THE UNIVERSE MUST BE ZERO

(OF COURSE DOESN'T MEAN THAT SUBSYSTEMS CAN'T HAVE MOMENTUM OR ANGULAR MOMENTUM)

A MACHIAN MODEL

$\vec{P}_{\text{tot}} = 0$ **AND** $\vec{L}_{\text{tot}} = 0$ **ARE** ***GAUGE CONSTRAINTS.*** **(TECHN. LINEAR AND FIRST-CLASS) THEY SAY THAT CERTAIN DIRECTIONS IN PHASE SPACE ARE UNPHYSICAL**

THE DYNAMICAL PROBLEM IS WELL-POSED WHEN AN EQUAL NUMBER OF GAUGE FIXINGS $\vec{G}_P = 0$ **AND** $\vec{G}_L = 0$ **ARE IMPOSED.**

E.G.: $\vec{G}_P = \sum_a m_a \vec{r}_a = 0$ **(CENTER-OF-MASS COORDINATES)**

GIVEN $6N$ $\vec{r}_a$ **AND** $\vec{p}_a$ **AT** $\lambda = 0$**,**
MINUS 12 $\vec{P}_{\text{tot}} = 0$**,** $\vec{L}_{\text{tot}} = 0$**,** $\vec{G}_P = 0$ **AND** $\vec{G}_L = 0$
SPECIFY A POINT AND A TANGENT VECTOR IN THE
$3N-6$**–DIMENSIONAL** Q_R **(WEAK P.P.)**

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SPECIFY A POINT AND A TANGENT VECTOR IN THE
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GEOMETRODYNAMICS

CONFIGURATION SPACES

START WITH A 3-MANIFOLD $\mathscr{M}$

Riem = **SPACE OF RIEMANNIAN 3-METRICS $g_{ij}(x)$ ON $\mathscr{M}$**
(6 D.O.F'S PER POINT)

QUOTIENT BY 3-DIFFEOMORPHISMS $g_{ij} \to g_{ij} + \nabla_i \xi_j + \nabla_j \xi_i$
(COORDINATE CHANGES) -3 D.O.F.'S PER POINT

Superspace = Riem/diff **SPACE OF 3-GEOMETRIES**
(3 D.O.F'S PER P.)

QUOTIENT BY CONFORMAL TRANSFORMATIONS $g_{ij} \to \phi^4 g_{ij}$
(RESCALINGS) -1 D.O.F. PER POINT

S = Superspace/conf **SPACE OF CONFORMAL 3-GEOMETRIES**
(2 D.O.F'S PER P.)

LET'S TRY WITH Riem

$$S = \int d\lambda \sqrt{U(g)\,(g^{ik}g^{j\ell} - g^{ij}g^{k\ell})\,\frac{\delta g_{ij}}{d\lambda}\frac{\delta g_{k\ell}}{d\lambda}}$$

(BAIERLEIN-SHARP-WHEELER ACTION FOR GR)

$$\frac{\delta g_{ij}}{d\lambda} = \dot{g}_{ij} - \nabla_i \dot{\xi}_j - \nabla_j \dot{\xi}_i\,, \qquad U(g) = \det g\left({}^{(3)}R - 2\Lambda\right)$$

MOMENTA AND QUADRATIC CONSTRAINT

$$p^{ij} = \frac{\delta \mathscr{L}}{\delta \dot{g}_{ij}} = \frac{\delta g_{k\ell}}{d\lambda}\sqrt{\frac{U(g)}{\frac{\delta g}{d\lambda} : \frac{\delta g}{d\lambda}}} \quad \Rightarrow \quad \boxed{p^{ij}p_{ij} - \frac{1}{2}\mathrm{tr}p^2 - U(g) = H = 0}$$

BEST-MATCHING WRT DIFFEOS

$$\frac{\delta \mathscr{L}}{\delta \dot{\xi}_i} = -2\,\boxed{\nabla_j p^{ij} = \Pi^i = 0}$$

THE MOMENTUM IS A *TRANSVERSE* TENSOR

LET'S TRY WITH Riem

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SPECIAL RELATIVITY AND GAUGE THEORY

THEORY CONSISTENT ONLY IF CONSTRAINTS ARE COMPATIBLE
(TECHN. IF THEY CLOSE A POISSON ALGEBRA)

ADDING SCALAR FIELD WITH ARBITRARY PROPAGATION SPEED

$$S = \int d\lambda \sqrt{g\,\left(R - 2\Lambda - \mathbf{c}\, g^{ij}\nabla_i\varphi\nabla_j\varphi\right)\ \left[(g^{ik}g^{j\ell} - g^{ij}g^{k\ell})\frac{\delta g_{ij}}{d\lambda}\frac{\delta g_{k\ell}}{d\lambda} + \left(\dot{\varphi} - \dot{\xi}_i\nabla^i\varphi\right)^2\right]}$$

THEORY CONSISTENT ONLY IF $\mathbf{c} = 1$

ADDING VECTOR FIELD WITH ALL SORTS OF COUPLINGS

$$U \to g(R - 2\Lambda) + \alpha\,\nabla_i A_j \nabla^i A^j + \beta\,\nabla_i A_j \nabla^j A^i + \gamma(\nabla_i A^i)^2 + f(A_i A^i)$$

THEORY CONSISTENT ONLY IF $\alpha = -\beta = \frac{1}{4},\ \ \gamma = f = 0$
THIS IS MAXWELL'S THEORY

SIMILARLY, FOR A VECTOR FIELD WITH AN INTERNAL INDEX A_i^a,
CONSISTENT ONLY IF COUPLINGS = YANG-MILLS

LET'S TRY WITH Riem

$$S = \int d\lambda \sqrt{U(g)\,(g^{ik}g^{j\ell} - g^{ij}g^{k\ell})\,\frac{\delta g_{ij}}{d\lambda}\frac{\delta g_{k\ell}}{d\lambda}}$$

(BAIERLEIN-SHARP-WHEELER ACTION FOR GR)

$$\frac{\delta g_{ij}}{d\lambda} = \dot{g}_{ij} - \nabla_i \dot{\xi}_j - \nabla_j \dot{\xi}_i\,, \qquad U(g) = \det g\left({}^{(3)}R - 2\Lambda\right)$$

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$$S = \int d\lambda \sqrt{g\,\left(R - 2\Lambda - \mathbf{c}\, g^{ij}\nabla_i\varphi\nabla_j\varphi\right)\;\left[\left(g^{ik}g^{j\ell} - g^{ij}g^{k\ell}\right)\frac{\delta g_{ij}}{d\lambda}\frac{\delta g_{k\ell}}{d\lambda} + \left(\dot{\varphi} - \dot{\xi}_i\nabla^i\varphi\right)^2\right]}$$

THEORY CONSISTENT ONLY IF $\mathbf{c} = 1$

ADDING VECTOR FIELD WITH ALL SORTS OF COUPLINGS

$$U \to g(R - 2\Lambda) + \alpha\,\nabla_i A_j \nabla^i A^j + \beta\,\nabla_i A_j \nabla^j A^i + \gamma(\nabla_i A^i)^2 + f(A_i A^i)$$

THEORY CONSISTENT ONLY IF $\alpha = -\beta = \frac{1}{4},\ \gamma = f = 0$

THIS IS MAXWELL'S THEORY

SIMILARLY, FOR A VECTOR FIELD WITH AN INTERNAL INDEX A_i^a, CONSISTENT ONLY IF COUPLINGS = YANG-MILLS

SPECIAL RELATIVITY AND GAUGE THEORY

THEORY CONSISTENT ONLY IF CONSTRAINTS ARE COMPATIBLE
(TECHN. IF THEY CLOSE A POISSON ALGEBRA)

ADDING SCALAR FIELD WITH ARBITRARY PROPAGATION SPEED

$$S = \int d\lambda \sqrt{g\,\left(R - 2\Lambda - \mathbf{c}\, g^{ij}\nabla_i\varphi\nabla_j\varphi\right)\;\left[\left(g^{ik}g^{j\ell} - g^{ij}g^{k\ell}\right)\frac{\delta g_{ij}}{d\lambda}\frac{\delta g_{k\ell}}{d\lambda} + \left(\dot{\varphi} - \dot{\xi}_i\nabla^i\varphi\right)^2\right]}$$

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CONSISTENT ONLY IF COUPLINGS = YANG-MILLS

LET'S TRY WITH Riem

$$S = \int d\lambda \sqrt{U(g)\,(g^{ik}g^{j\ell} - g^{ij}g^{k\ell})\,\frac{\delta g_{ij}}{d\lambda}\frac{\delta g_{k\ell}}{d\lambda}}$$

(BAIERLEIN-SHARP-WHEELER ACTION FOR GR)

$$\frac{\delta g_{ij}}{d\lambda} = \dot{g}_{ij} - \nabla_i \dot{\xi}_j - \nabla_j \dot{\xi}_i \,, \qquad U(g) = \det g \left({}^{(3)}R - 2\Lambda \right)$$

MOMENTA AND QUADRATIC CONSTRAINT

$$p^{ij} = \frac{\delta \mathscr{L}}{\delta \dot{g}_{ij}} = \frac{\delta g_{k\ell}}{d\lambda} \sqrt{\frac{U(g)}{\frac{\delta g}{d\lambda} : \frac{\delta g}{d\lambda}}} \quad \Rightarrow \quad \boxed{p^{ij}p_{ij} - \frac{1}{2}\mathrm{tr}p^2 - U(g) = H = 0}$$

BEST-MATCHING WRT DIFFEOS

$$\frac{\delta \mathscr{L}}{\delta \dot{\xi}_i} = -2 \boxed{\nabla_j p^{ij} = \Pi^i = 0}$$

THE MOMENTUM IS A *TRANSVERSE* TENSOR

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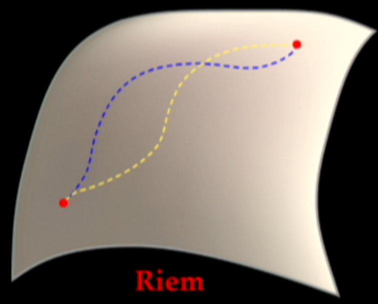
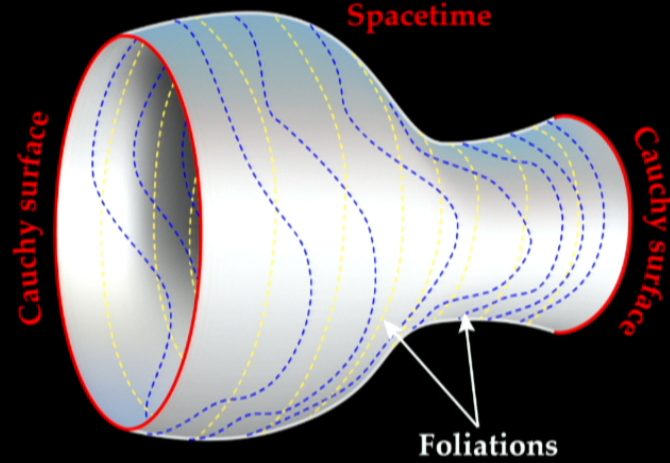
WHAT ABOUT POINCARÉ'S PRINCIPLE?





$\mathscr{H} = \int d^3x N(x) H(x)$ **GENERATES A CURVE IN** Riem **FOR EACH** $N(x)$

THE POINCARÉ PRINCIPLE IS NOT SATISFIED

JAMES W. YORK (CA. 1972)

TRYING TO SOLVE THE INITIAL VALUE PROBLEM, THAT IS TO FIND INITIAL g_{ij} AND p^{ij} SUCH THAT

$$p^{ij}p_{ij} - \frac{1}{2}\mathrm{tr}p^2 - gR + 2\Lambda = 0\,, \qquad \nabla_j p^{ij} = 0$$

HE DISCOVERED THAT IT IS SUFFICIENT TO FIX AN INITIAL CONFORMAL 3-GEOMETRY PLUS THE MOMENTUM CONJUGATE TO THE VOLUME $P = \int d^3x\, \mathrm{tr}\, p$

$$g_{ij} \quad \text{modulo} \quad g_{ij} \to g_{ij} + \nabla_i \xi_j + \nabla_j \xi_i \ , \ g_{ij} \to \phi^4 g_{ij}$$

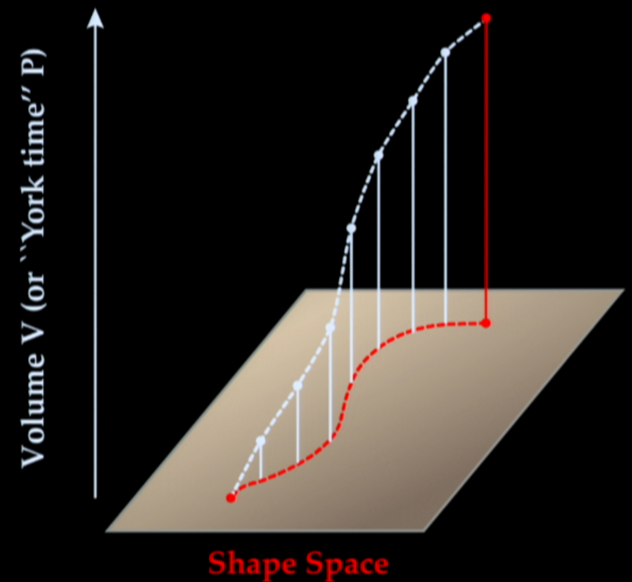
$$\text{with} \int d^3x\, \phi^6 \sqrt{g} = \int d^3x \sqrt{g}$$

...AND THE INITIAL-VALUE PROBLEM CAN BE SOLVED.

BARBOUR (2005): THIS SUGGESTS CONFIGURATION SPACE OF GR IS SHAPE SPACE PLUS VOLUME

SHAPE DYNAMICS

GR AS A THEORY ON SHAPE SPACE+VOLUME SATISFYING WEAK P.P.



J. BARBOUR, N. O'MURCHADHA, E. ANDERSON, B. FOSTER, H. GOMES, S. GRYB, B. KELLEHER, T. KOSLOWSKI

HOW TO INTERPRET SHAPE DYNAMICS?

EITHER A REPARAMETRIZATION-INVARIANT THEORY ON SHAPE SPACE + VOLUME

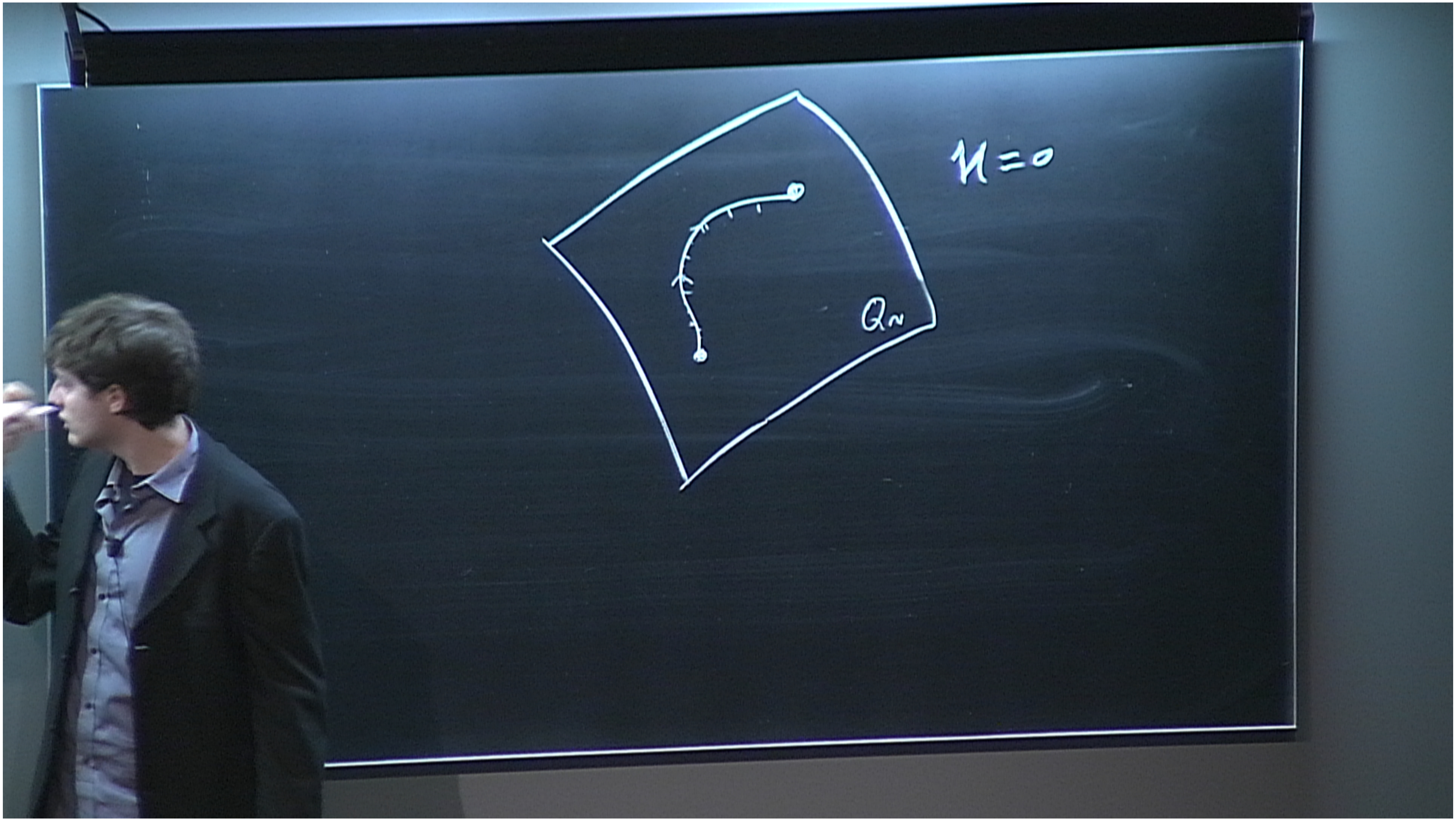
$$H_{\mathrm{SD}}[g^{\mathrm{conf}}, p_{\mathrm{TT}}, V, P] = 0$$

OR A THEORY ON SHAPE SPACE ALONE WITH PHYSICAL TIME P AND TIME-DEPENDENT HAMILTONIAN:

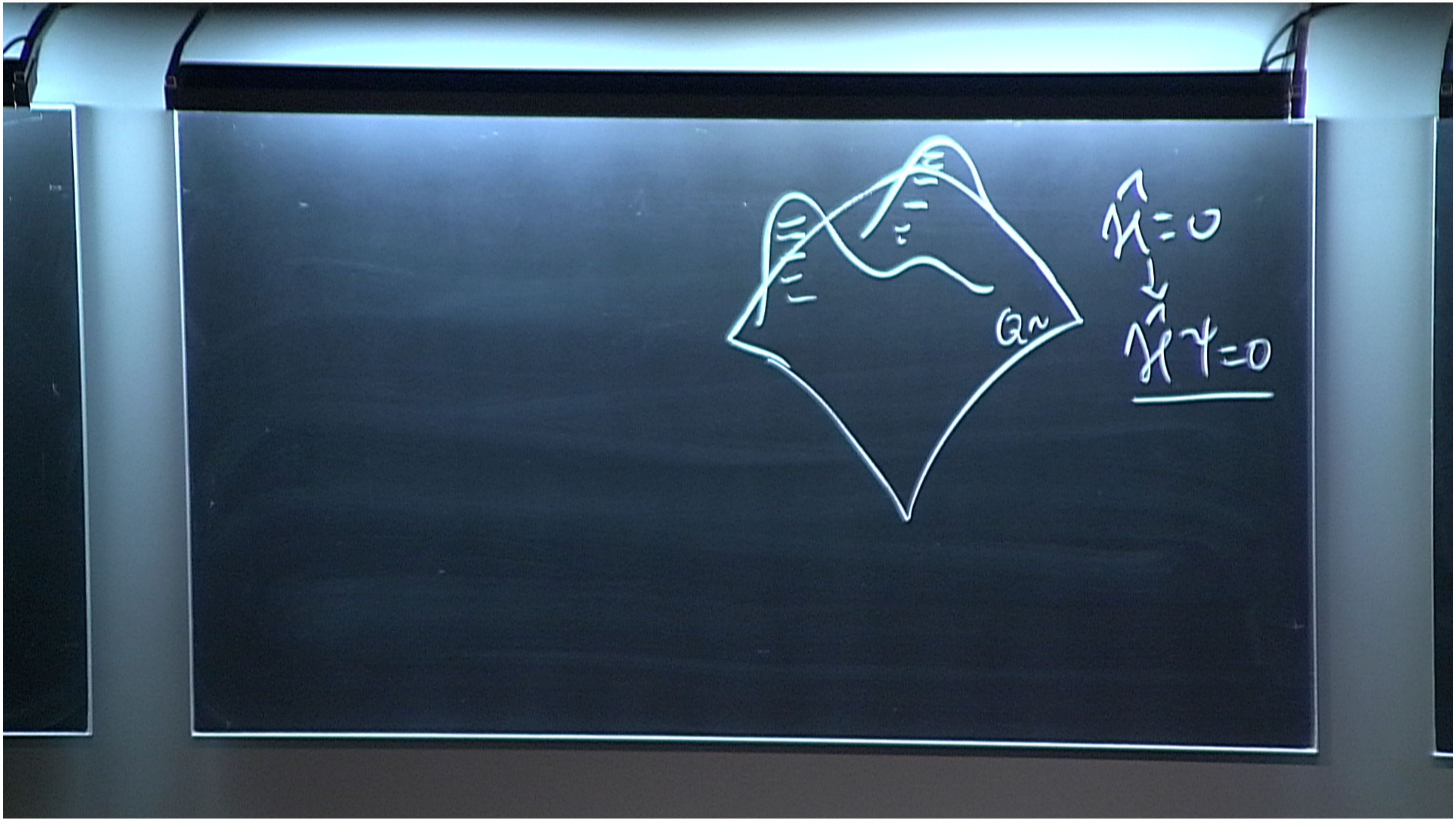
$$V = V[g^{\mathrm{conf}}, p_{\mathrm{TT}}, P]$$

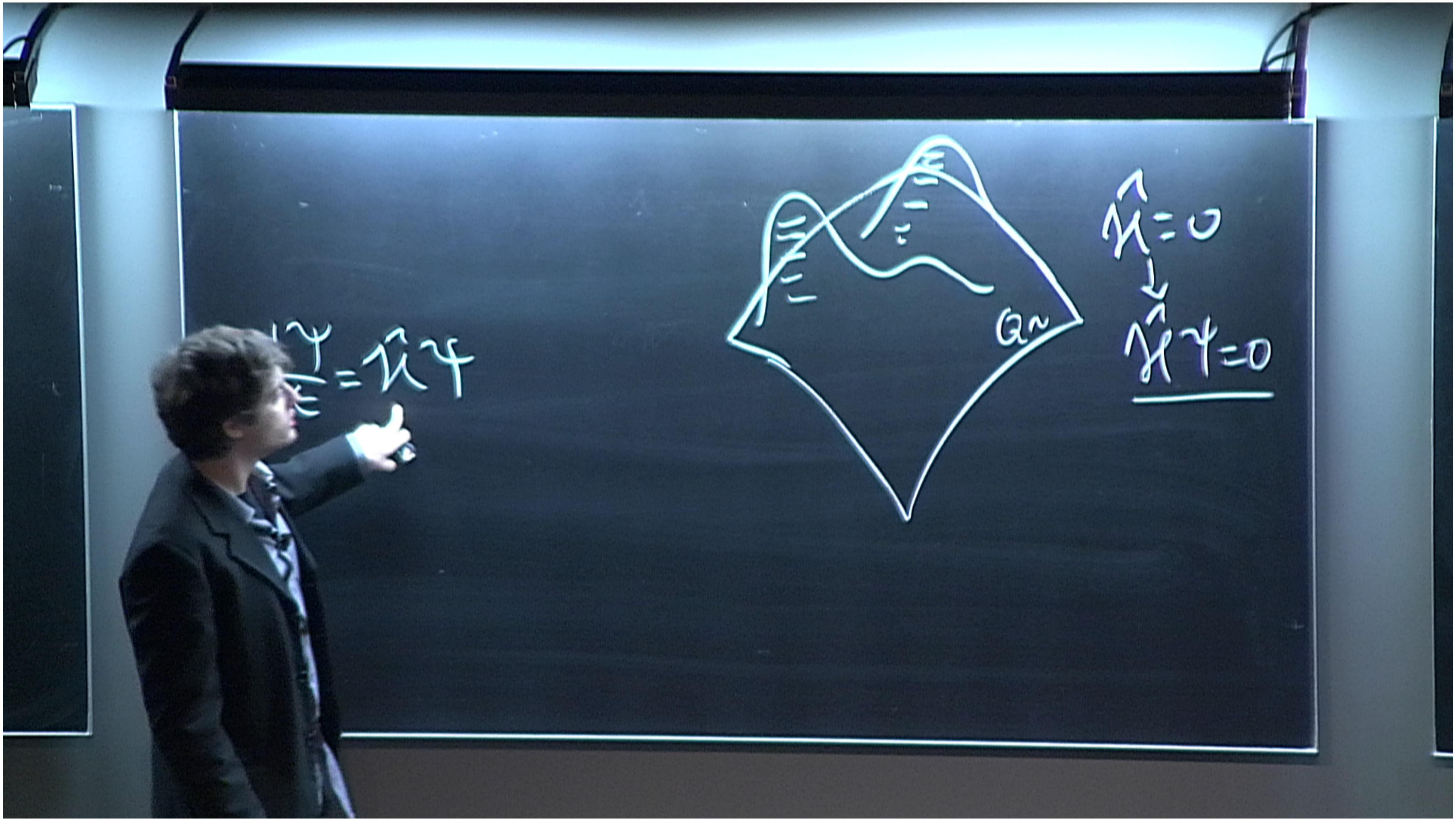
CON: WHAT ABOUT THE "TIME FROM CHANGE" SLOGAN?
PRO: SOLVES THE PROBLEM OF TIME IN QUANTUM MECHANICS

(UPCOMING PAPER WITH J. BARBOUR AND T. KOSLOWSKI)



Qn


$\hat{\mathcal{H}}=0$
$\hat{\mathcal{H}}\psi=0$



TIME FROM QUANTUM ANOMALY?

IF WE INSIST ON FULL CONFORMAL INVARIANCE AT THE FOUNDATIONS OF OUR THEORY A SCALE ANOMALY EMERGES UPON QUANTIZATION

THE CLASSICAL, SCALE INVARIANT THEORY CAN AT MOST BE A FIXED POINT OF AN RG FLOW THAT DOESN'T PRESERVE SCALE INVARIANCE

THIS RG FLOW MIGHT BE IDENTIFIED WITH TIME EVOLUTION (S. GRYB'S INPUT)

(UPCOMING PAPER WITH J. BARBOUR AND M. LOSTAGLIO)