Title: Superhorizon fluctuations in de Sitter and cosmological spacetimes

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Abstract: I will discuss superhorizon fluctuations in de Sitter space. The first part of the talk will focus on computing entanglement entropies of field theories in a fixed de Sitter background. Those computations are done for free theories and also theories with gravity duals. If time permits, I will also discuss superhorizon fluctuations in cosmological backgrounds. In particular, I focus on showing that subhorizon fluctuations can not produce any significant backreaction on superhorizon modes. If those late time effects existed, one in principle could not trust the scale invariant spectrum of inflationary theories to be the source of the spectrum of thermal fluctuations of the CMB.

Superhorizon Fluctuations in dS and in Cosmology

Guilherme Pimentel
Princeton University

Outline

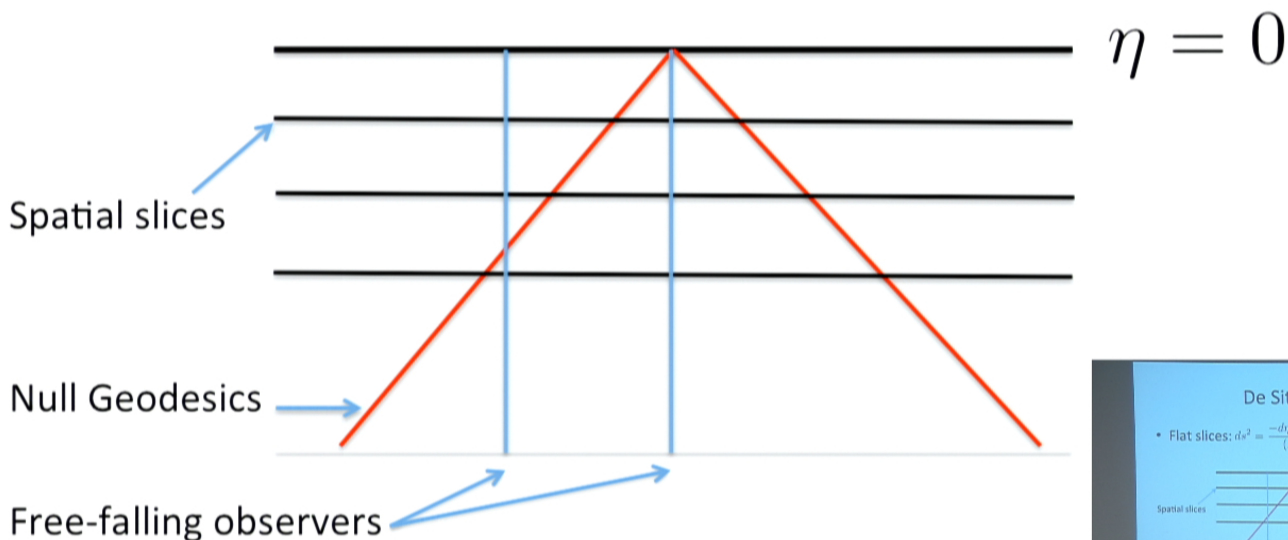
- Superhorizon Fluctuations and the Wavefunction
- Entanglement entropy in de Sitter space
- Inflationary correlators and IR effects

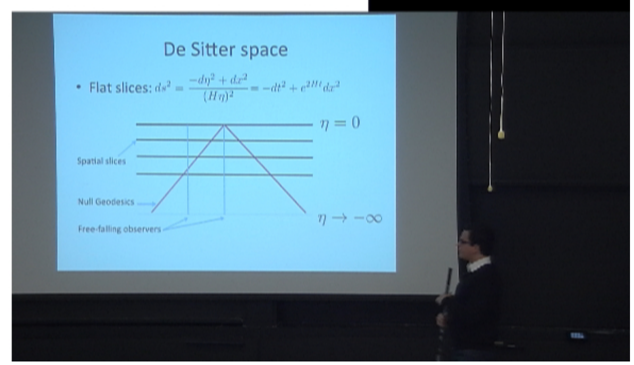
Superhorizon Fluctuations

- dS is the most symmetric expanding solution to Einstein's equations – nice toy model for Cosmology
- In flat space we study scattering amplitudes
  - Observables encoded in the S matrix
  - What about dS?

De Sitter space

- Flat slices: $ds^2 = \frac{-d\eta^2 + dx^2}{(H\eta)^2} = -dt^2 + e^{2Ht}dx^2$





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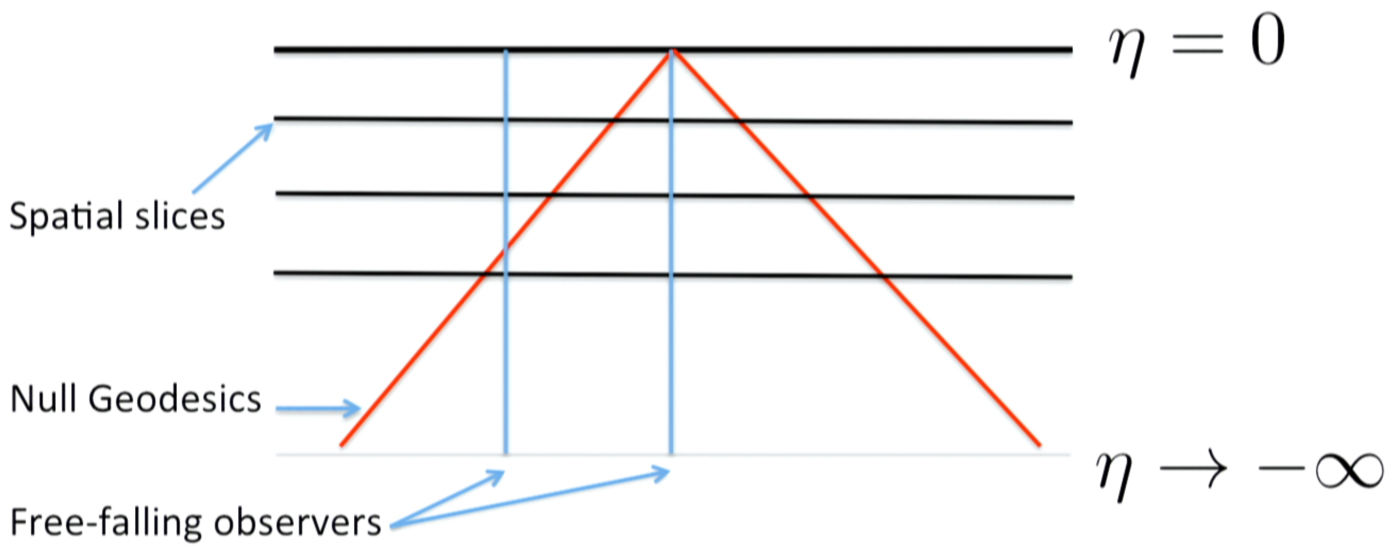
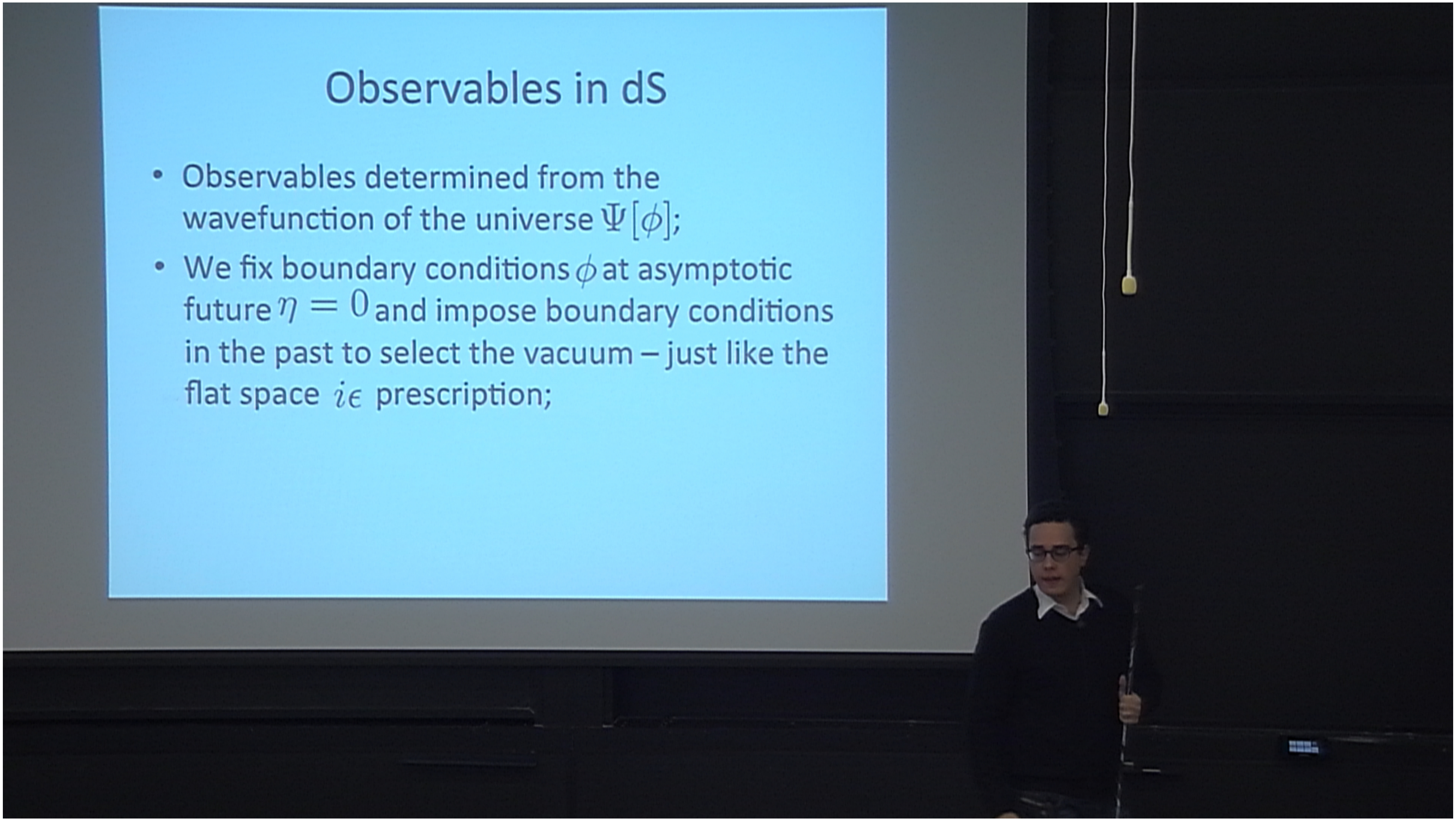


De Sitter space

- In flat slices, translation symmetry along the spatial directions – momentum modes;
- Fixed comoving distance corresponds to physical distance that expands exponentially in time $\frac{R}{\eta}$;
- Physical wavelength of modes of constant comoving momentum get stretched;


Observables in dS
• Observables determined from the wavefunction of the universe $\Psi[\phi]$;
• We fix boundary conditions ϕ at asymptotic future $\eta = 0$ and impose boundary conditions in the past to select the vacuum – just like the flat space $i\epsilon$ prescription;

Bunch-Davies vacuum

- If one follows a comoving wavenumber k well into the past, at some point its physical wavelength is very small – field feels as if in flat space – BD/Hartle-Hawking/Chernikov-Tagirov vacuum; $\phi \sim e^{ik\eta}, \eta \to -\infty$
- Different combinations of positive and negative frequency modes correspond to α vacua

Mottola-Allen

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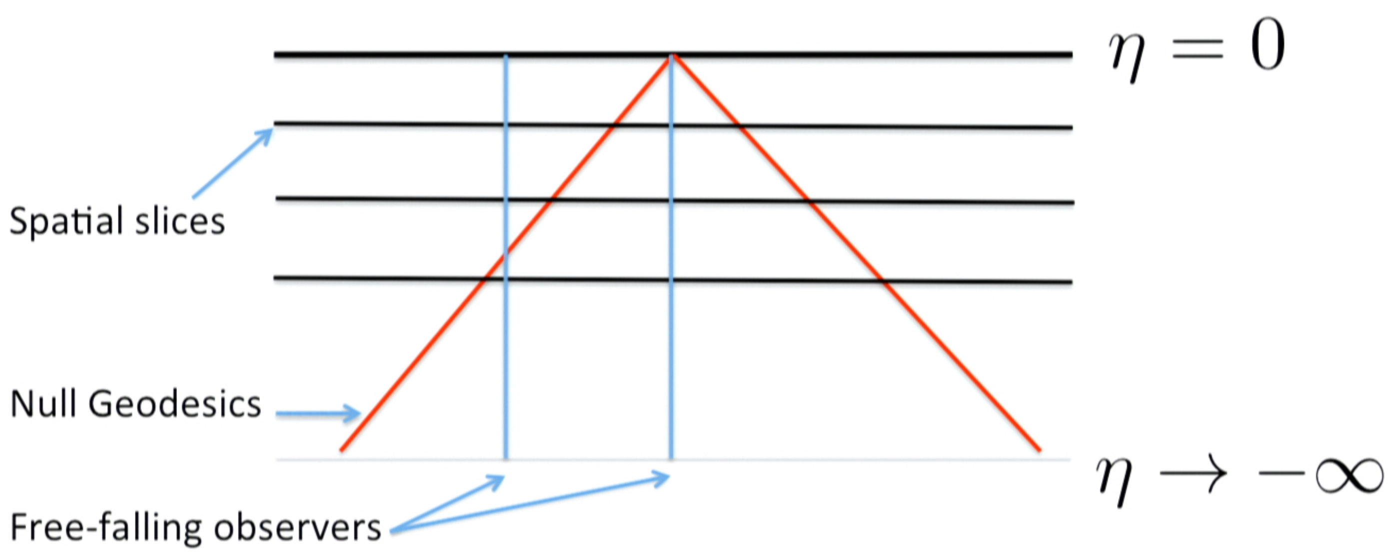

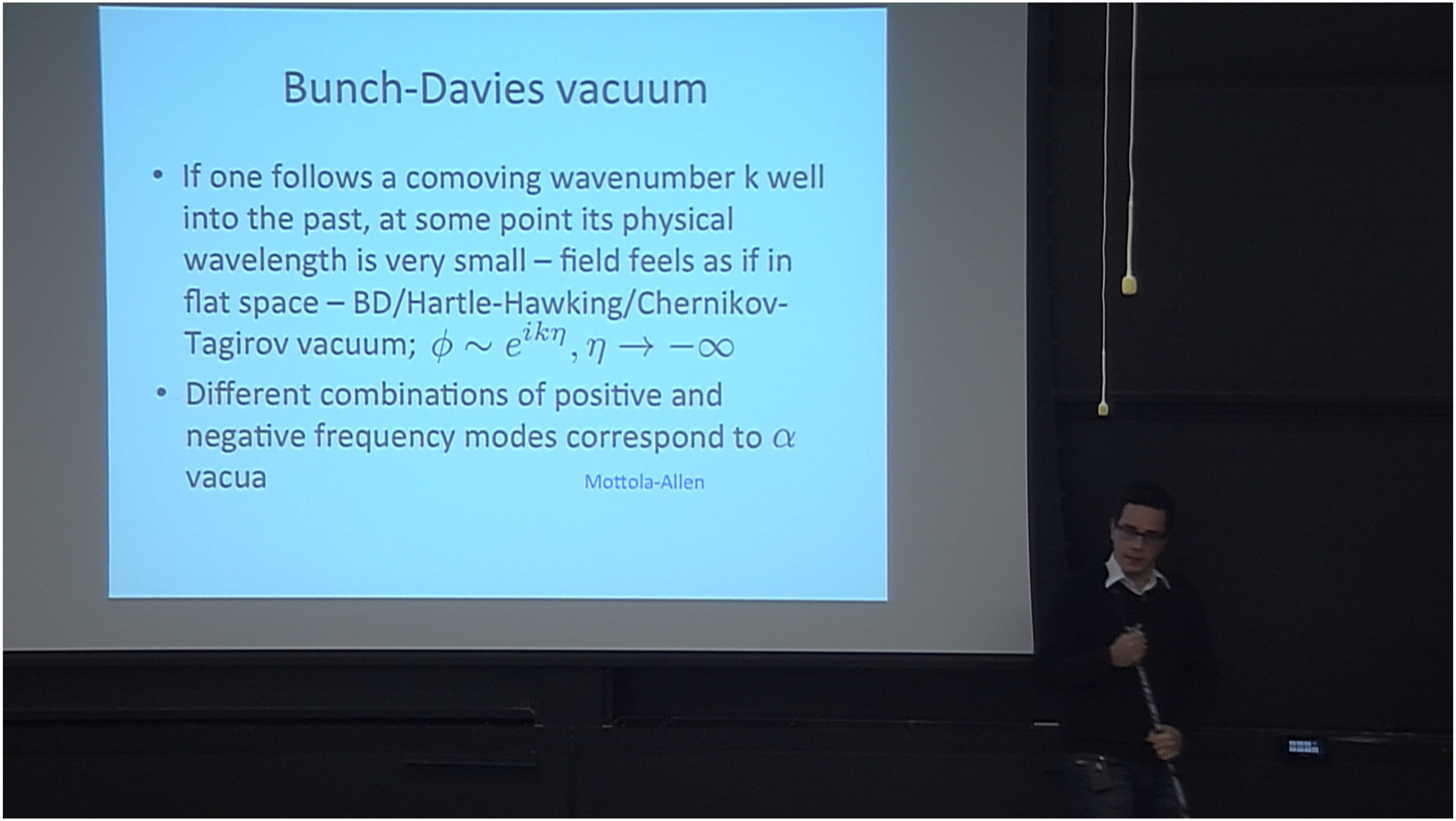
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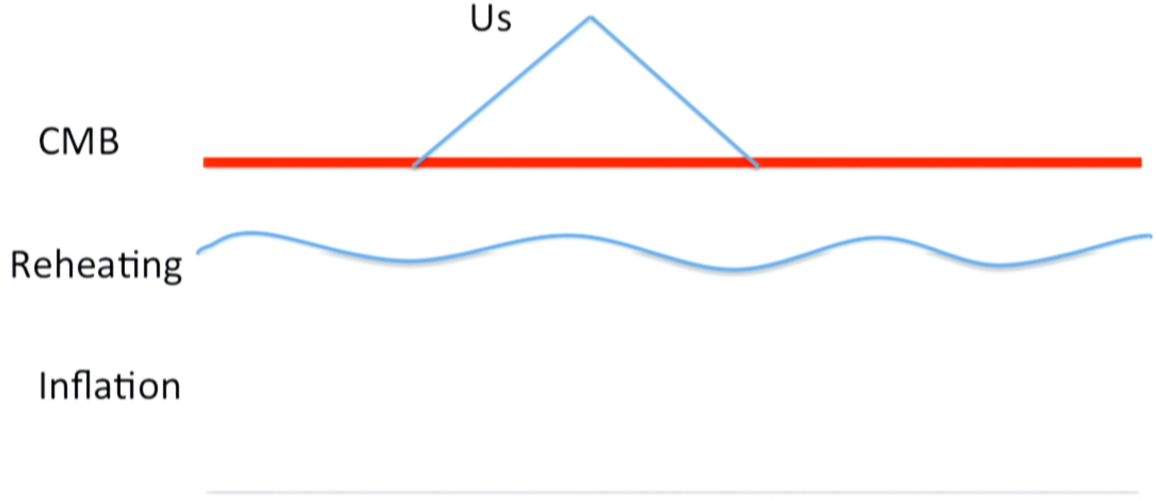
Superhorizon modes

- We are computing the wavefunction at late times;
- Comoving wavelength modes are all stretched beyond the Hubble radius – modes have "exited the horizon";
- The wavefunction of the universe computes quantities that can NOT be measured by a local observer in dS;

So why bother computing it??

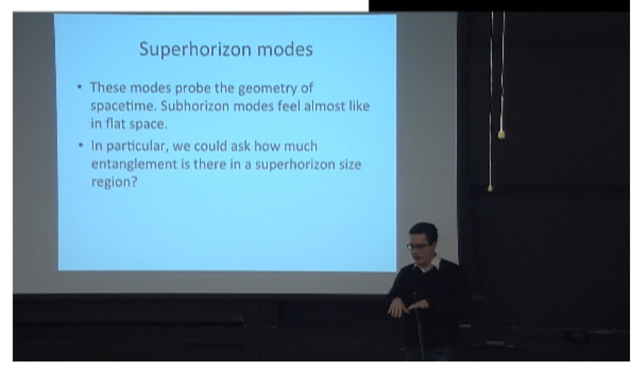
We observe superhorizon fluctuations in the CMB!

- Great triumph of inflationary theory (not by design): generates spectrum of anisotropies in the CMB temperature;
- After inflation is over, we can observe a larger part of reheating surface, so we have access to superhorizon modes.



Superhorizon modes

- These modes probe the geometry of spacetime. Subhorizon modes feel almost like in flat space.
- In particular, we could ask how much entanglement is there in a superhorizon size region?



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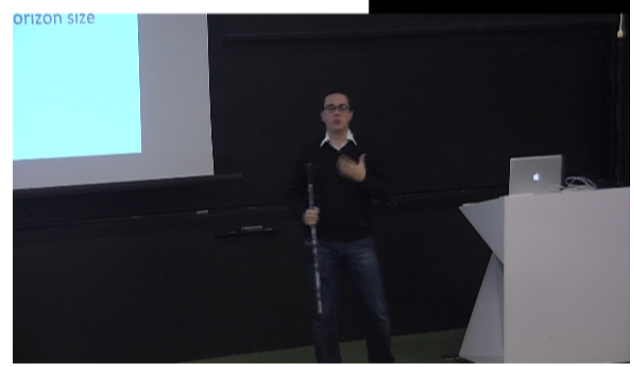
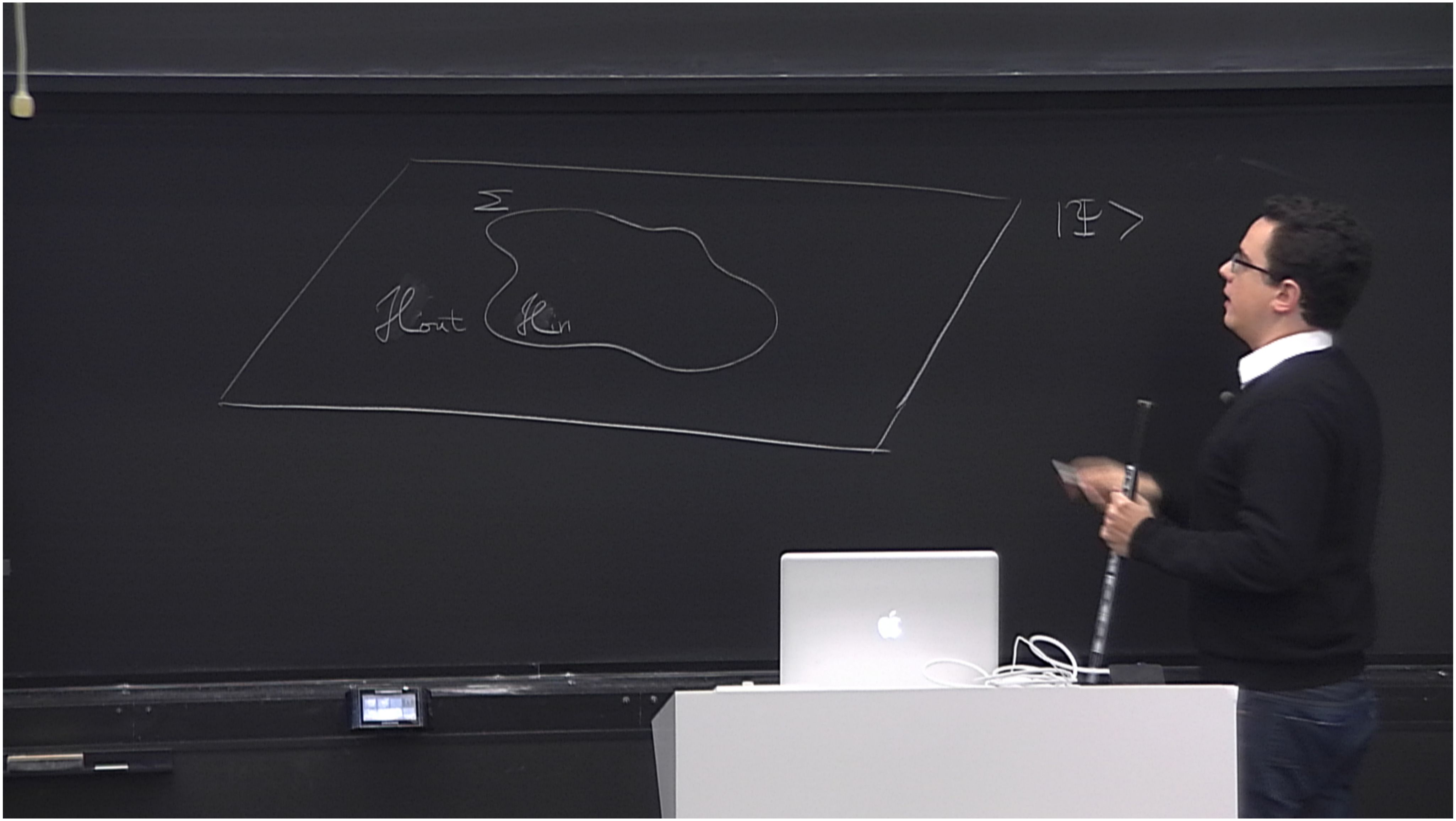
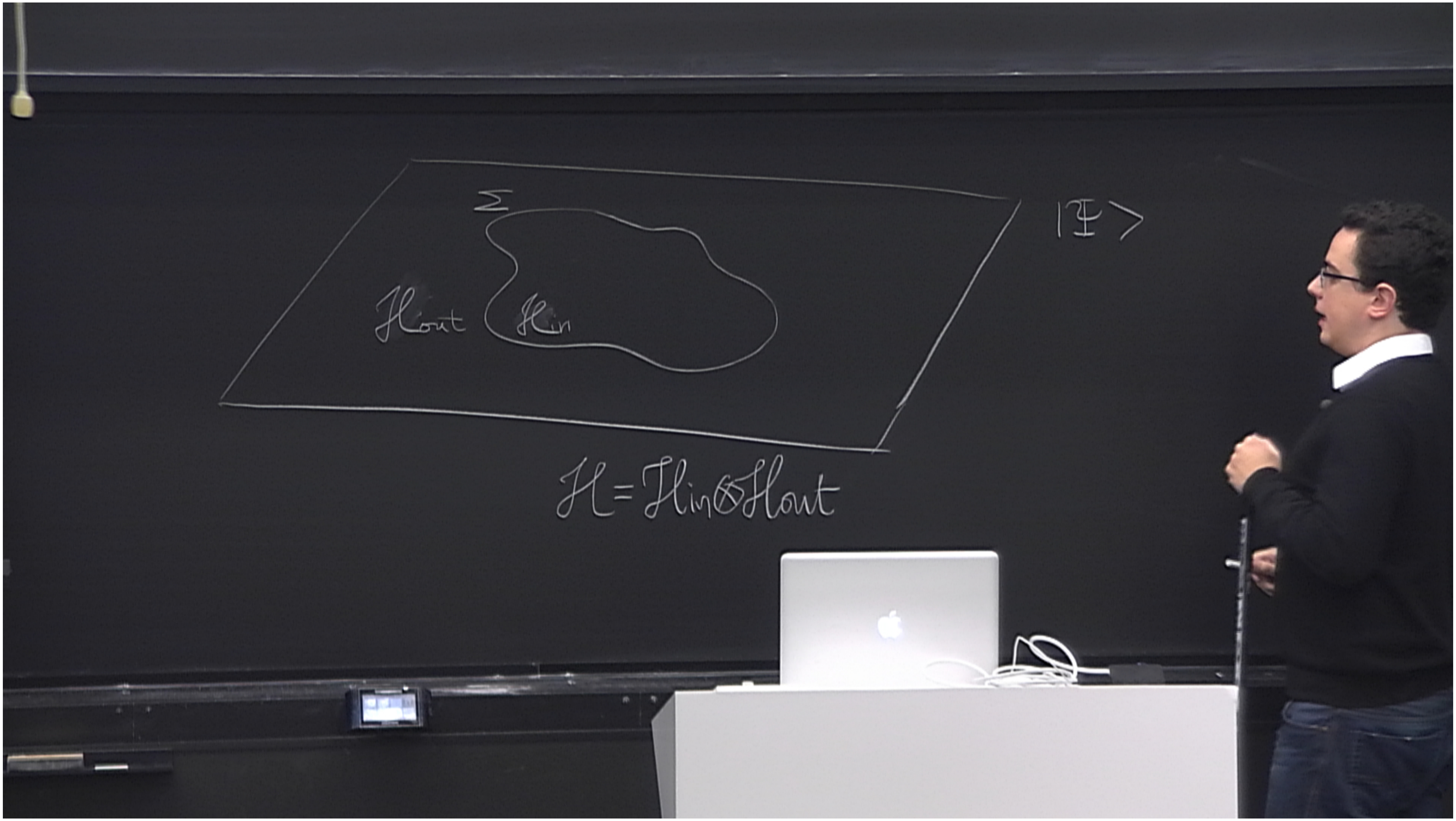


Entanglement Entropy

- EE = Von Neumann entropy for density matrix associated to an entangling surface and a given state;
- There is a need to introduce some UV regulator to make sense of this entropy. In principle it is infinite for QFTs;

't Hooft
Bombelli, Koul, Lee, Sorkin
Srednicki
Callan, Larsen, Wilczek


Σ
$\mathcal{H}_{out}$
$\mathcal{H}_{in}$
$|\Psi\rangle$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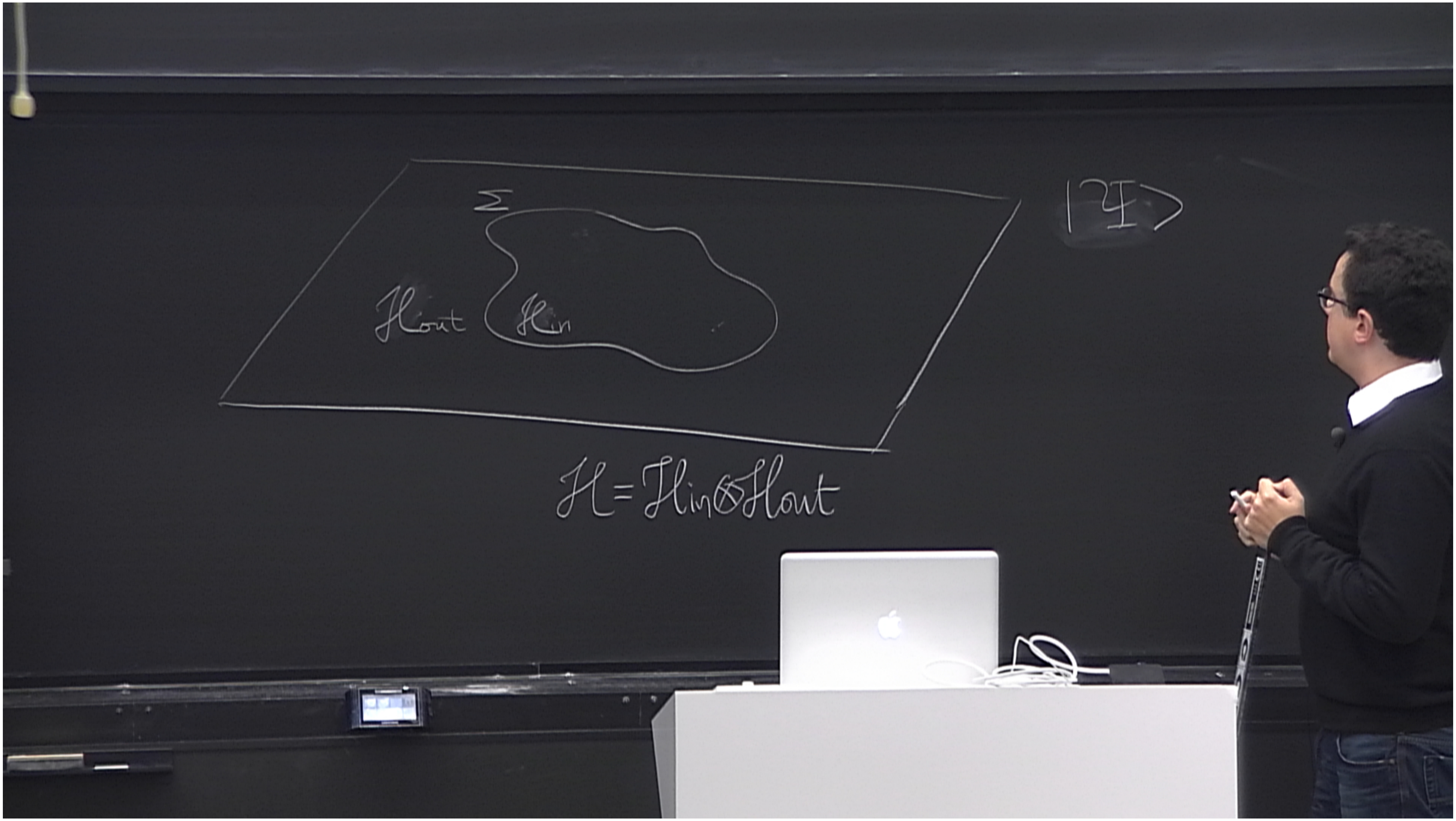
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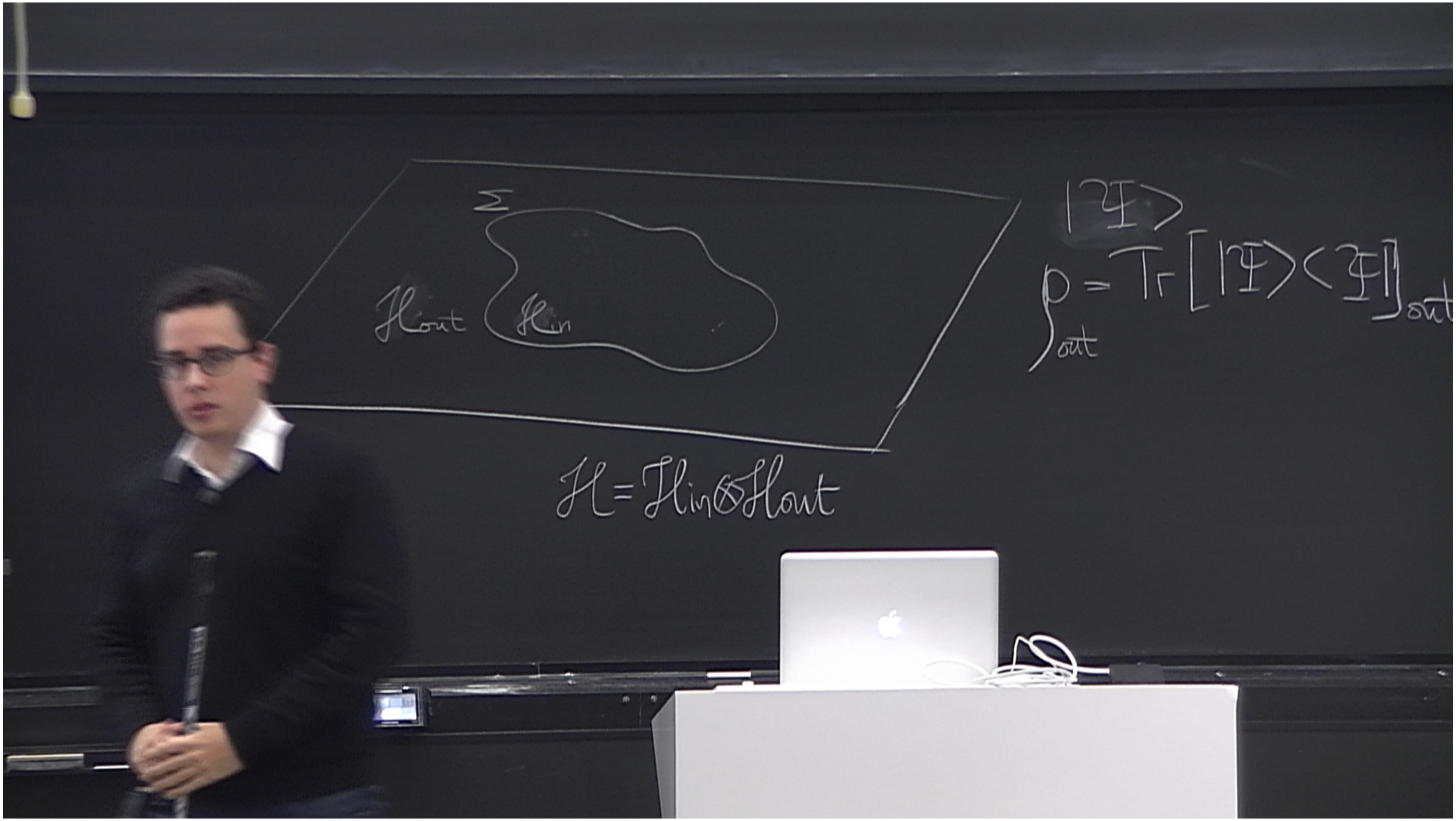
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$|\Psi\rangle$
$\rho_{out} = \mathrm{Tr}[|\Psi\rangle\langle\Psi|]_{out}$

Entanglement Entropy in dS

Maldacena , GP

- Goal is to understand superhorizon correlations of fields in the BD-HH-CT vacuum of dS;
- Take a spherical region of fixed comoving radius, such that the physical region is much bigger than dS radius, and find EE for the HH-BD-CT vacuum;

Entanglement Entropy in dS

- Compute the entropy for this sphere for different theories:
  - Free scalars;
  - Theories with holographic duals (CFTs and slightly deformed CFTs).
- Important: Metric does NOT fluctuate throughout these calculations (QFT in a curved background). We always take dS to be 4-dimensional.

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General Remarks on EE

- Structure of UV divergences is the same regardless of background being dS or flat space:

$$S = c_1 \frac{A}{\epsilon^2} + c_2 \log \epsilon H + S_{intr}$$

- We focus on the “interesting” part of the entropy.

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Interesting Entropy

- For massive theories in flat space

Hertzberg, Wilczek

$$S_{intr} \sim \text{Area}$$

- For theories at finite temperature

$$S_{intr} \sim \text{Volume}$$

- dS is seen as a thermal state in static coordinates. What is the form of the entropy?

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Interesting Entropy in dS

- Recall general form of the entropy

$$S = c_1 \frac{A}{\epsilon^2} + c_2 \log \epsilon H + S_{intr}$$

- For dS, it turns out that

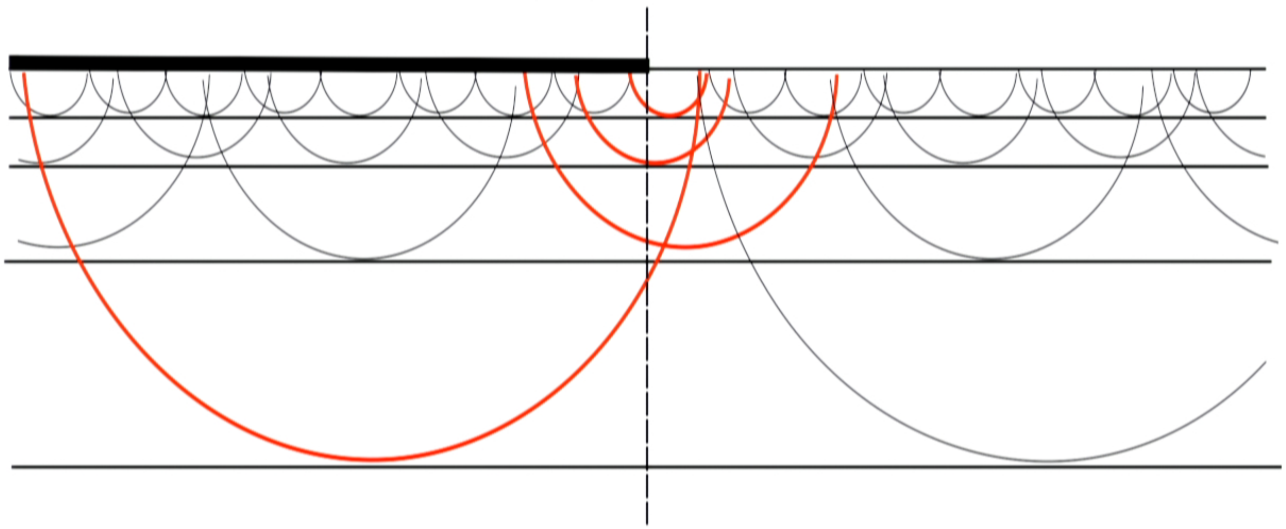
$$S_{intr} = d_1 \frac{A_c}{\eta^2} + d_2 \log \eta + \text{finite}$$

. Why ???

- The structure of the terms is the same but the coefficients are different. We focus on computing d_2

Why???

Wavefunction becomes time-independent at late times. Only time dependence for EE has to come from local terms. We wrote all possible local terms just as in UV divergent piece.
Entanglement had already been established itself in the past. Pair is created and one particle is outside of entangling surface at late times.



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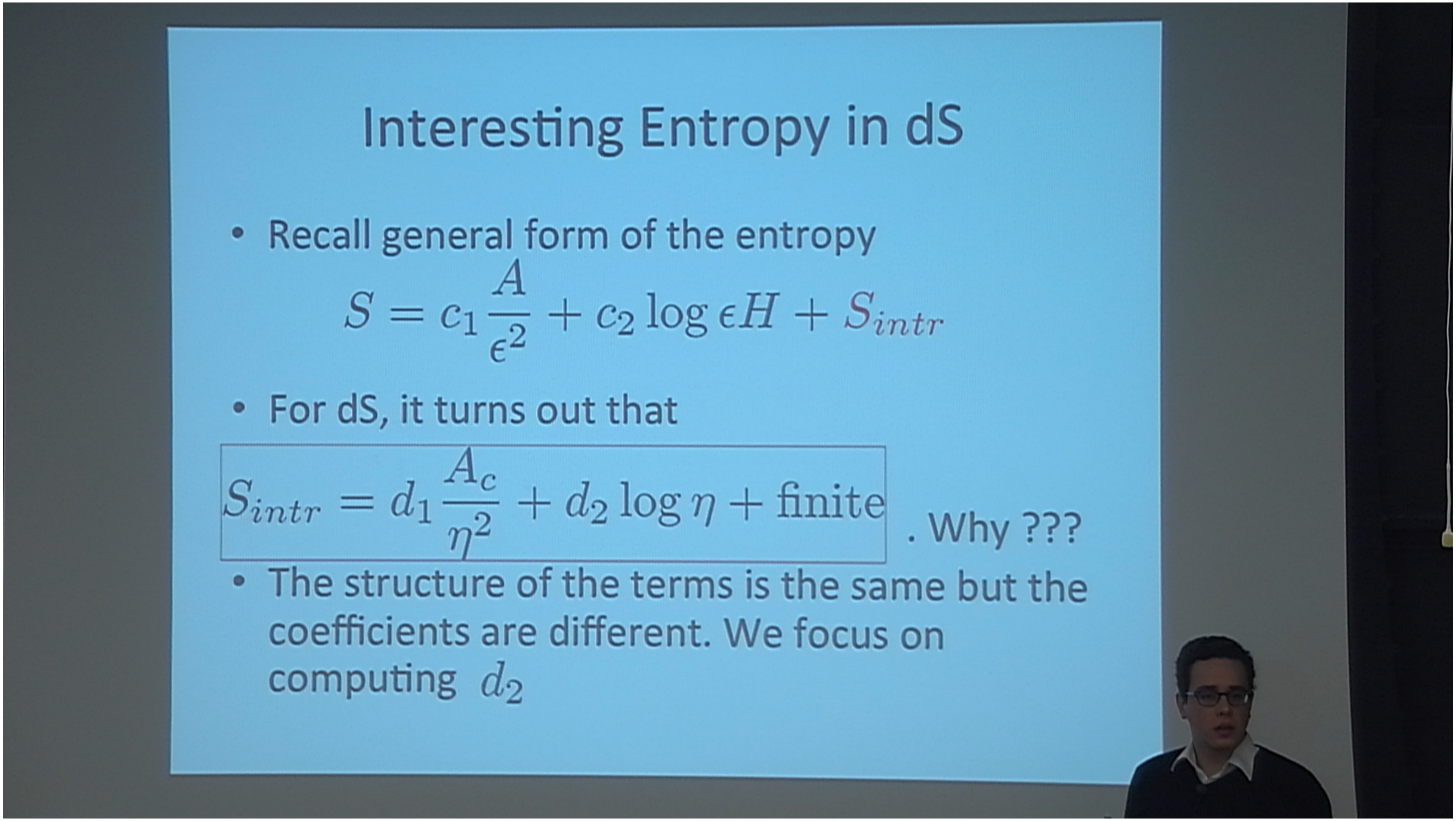
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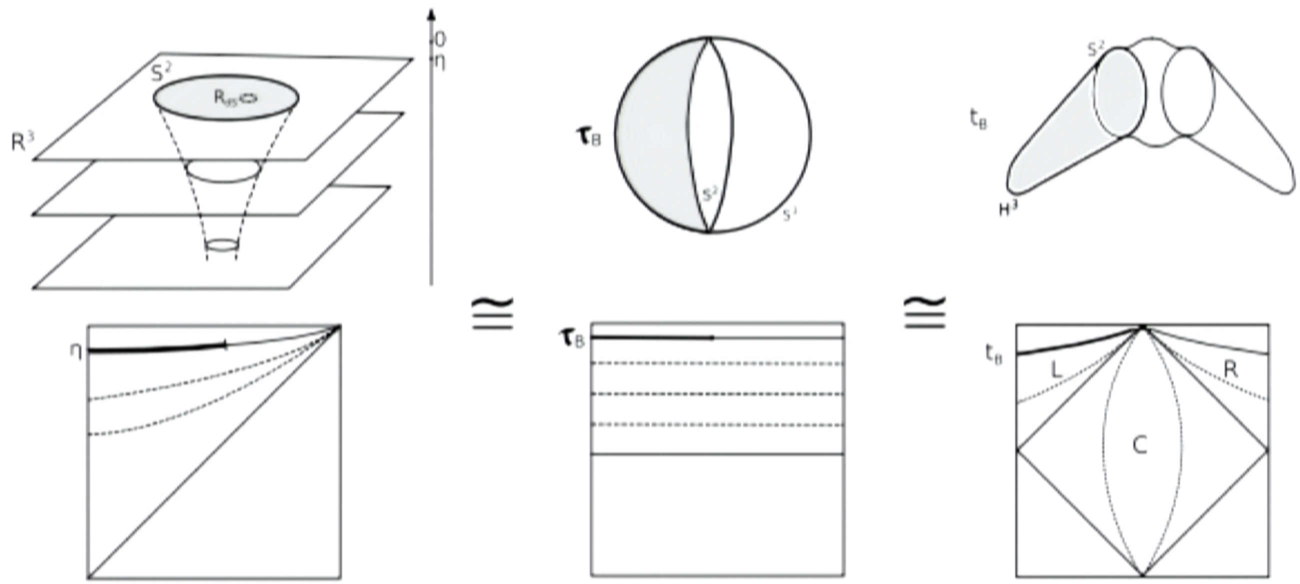
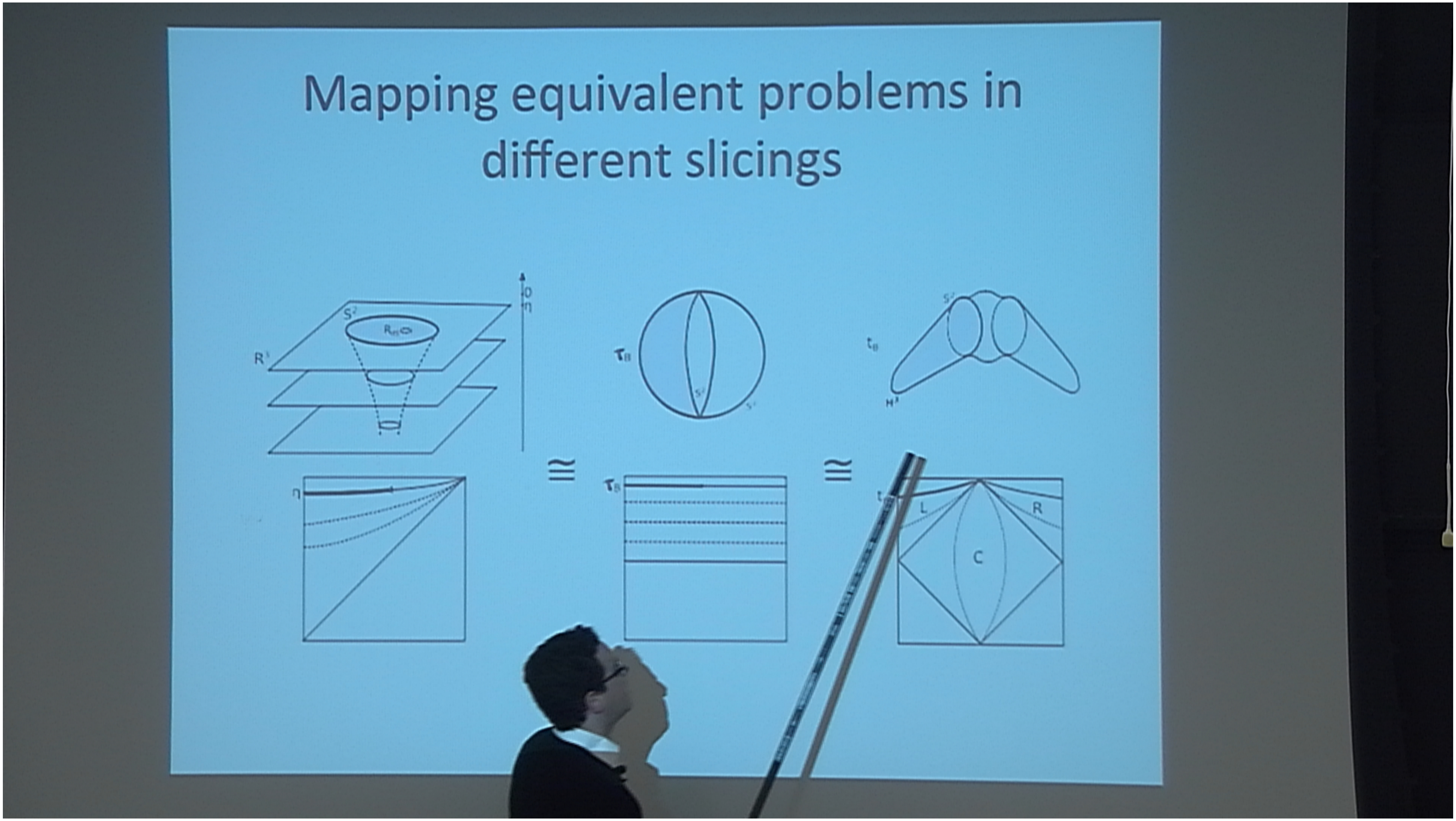
Free Theories

- dS isometry group is SO(4,1). Entangling surface leaves an SO(3,1) invariance at late times.
- We exploit this isometry to calculate the density matrix for the free theory.

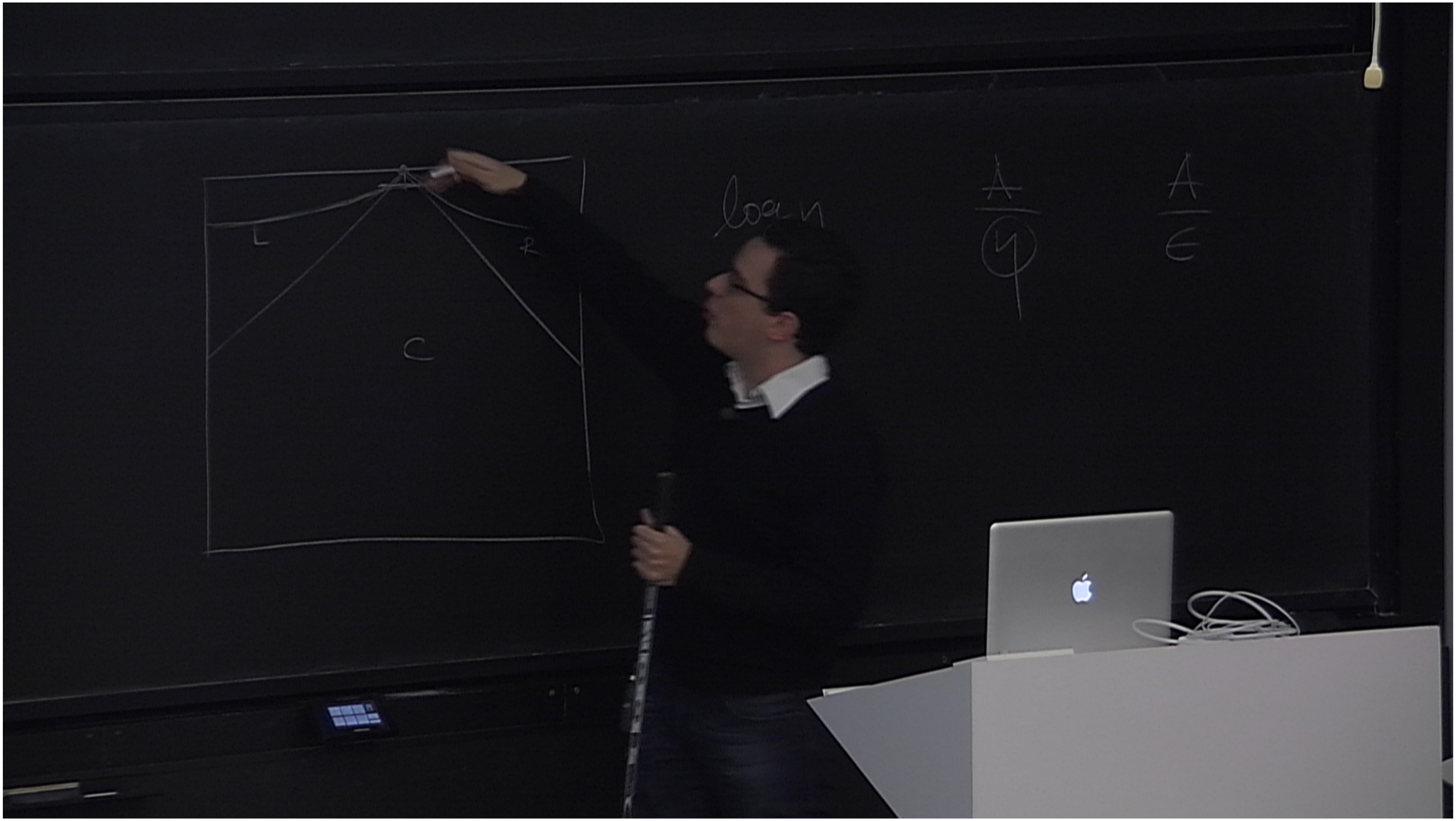
Mapping equivalent problems in different slicings



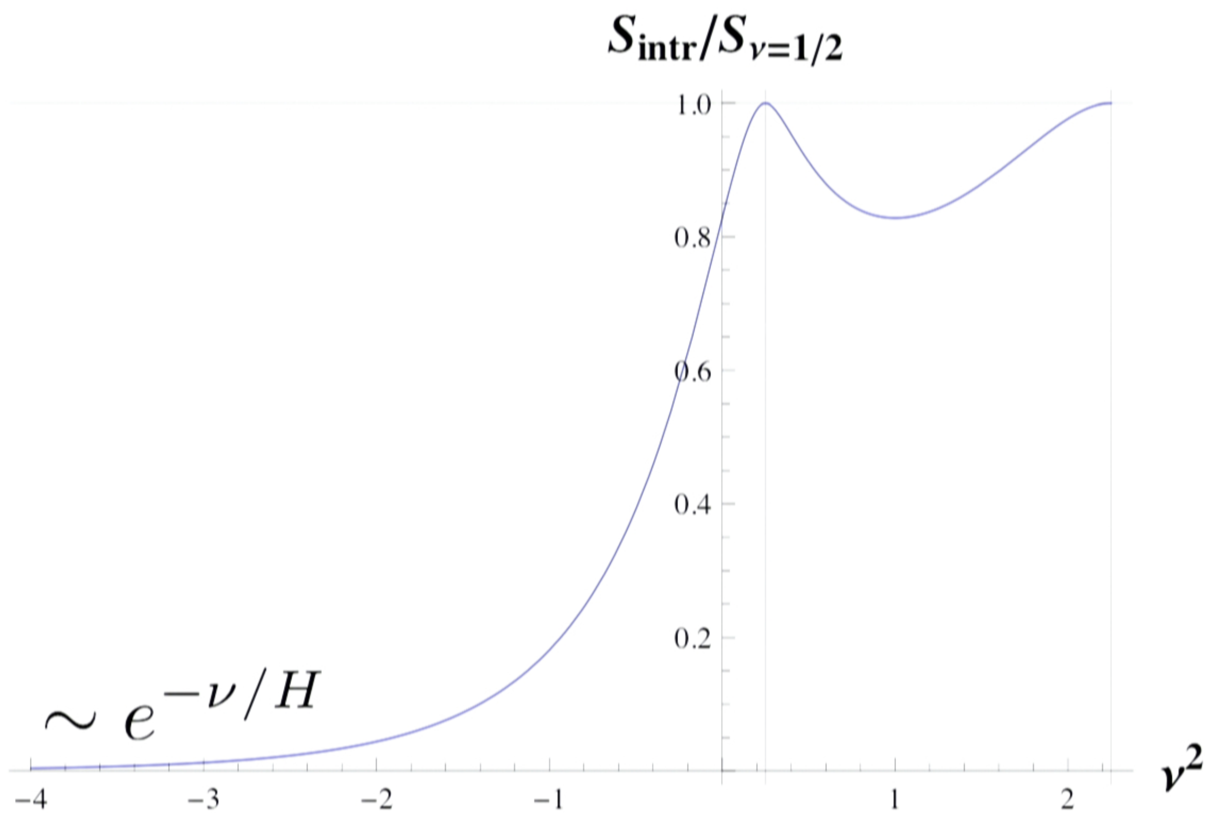

Mapping equivalent problems in different slicings
S²
R³
η
τ_B
S³
t_B
H³
≅
≅
L
R
C

Free Theories

- Hyperbolic slicing has explicit SO(3,1) symmetry of the slices;
- Wavefunctions were determined in the context of open inflation; Sasaki, Tanaka ,Yamamoto Bucher, Goldhaber, Turok
- Due to a mixing of the UV and IR cutoffs in hyperbolic coordinates, d_1 is not captured by this procedure, but we can extract the coefficient of $d_2 \log \eta$.


L
R
C
loan
A
A

Free Theories (Result for d_2)



EE in Holographic Duals of dS QFTs

- EE for a theory with an AdS holographic dual is given by solution to a minimal area problem in the bulk geometry.

  Ryu, Takayanagi
  Hubeny, Rangamani, Takayanagi

- For CFTs in dS, result is the same as in flat space.

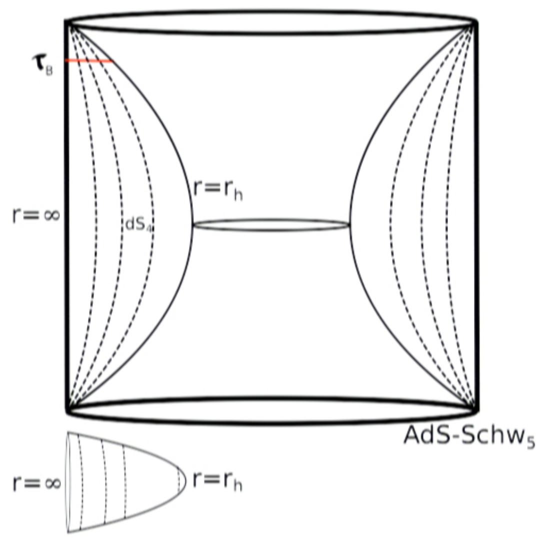
EE in Holographic Duals of dS QFTs

- For non-CFTs in dS, there are two distinct cases:
  - "Small" deformations – Coleman-de Luccia geometry;
  - "Large" deformations – Cigar geometry; (deformation parameter compared to Hubble)

Strominger, Maldacena, Hawking
Buchel, Langfelder, Walcher
Aharony, Fabinger, Horowitz, Silverstein,
Balasubramanian, Ross, Cai, Titchener,
Alishahiha,Karch,Tong,Larjo, Simon,
Hirayama,He, Rozali, Hutasoit, Kumar, Rafferty,
Marolf, Rangamani, Van Raamsdonk
Hertog, Horowitz

Cigar Geometry

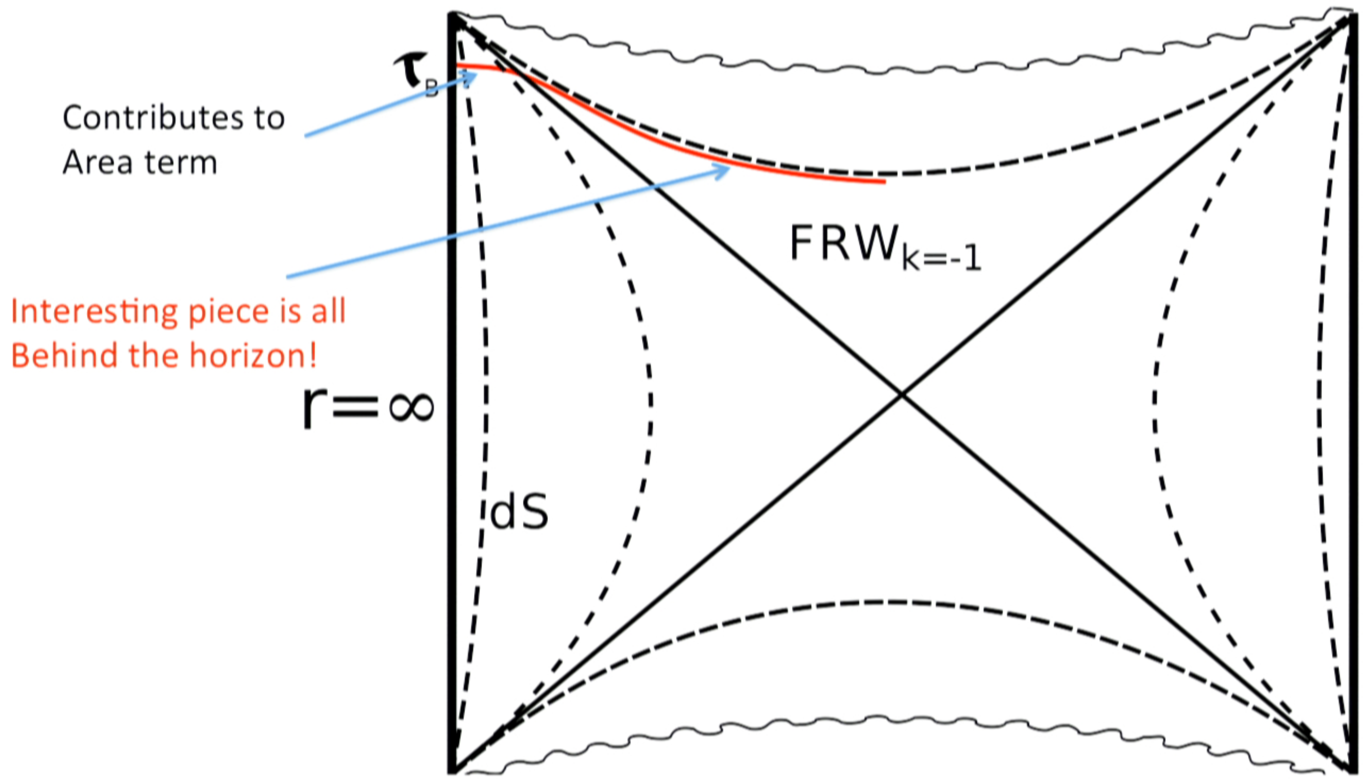
- "Bubble of nothing" – Area law

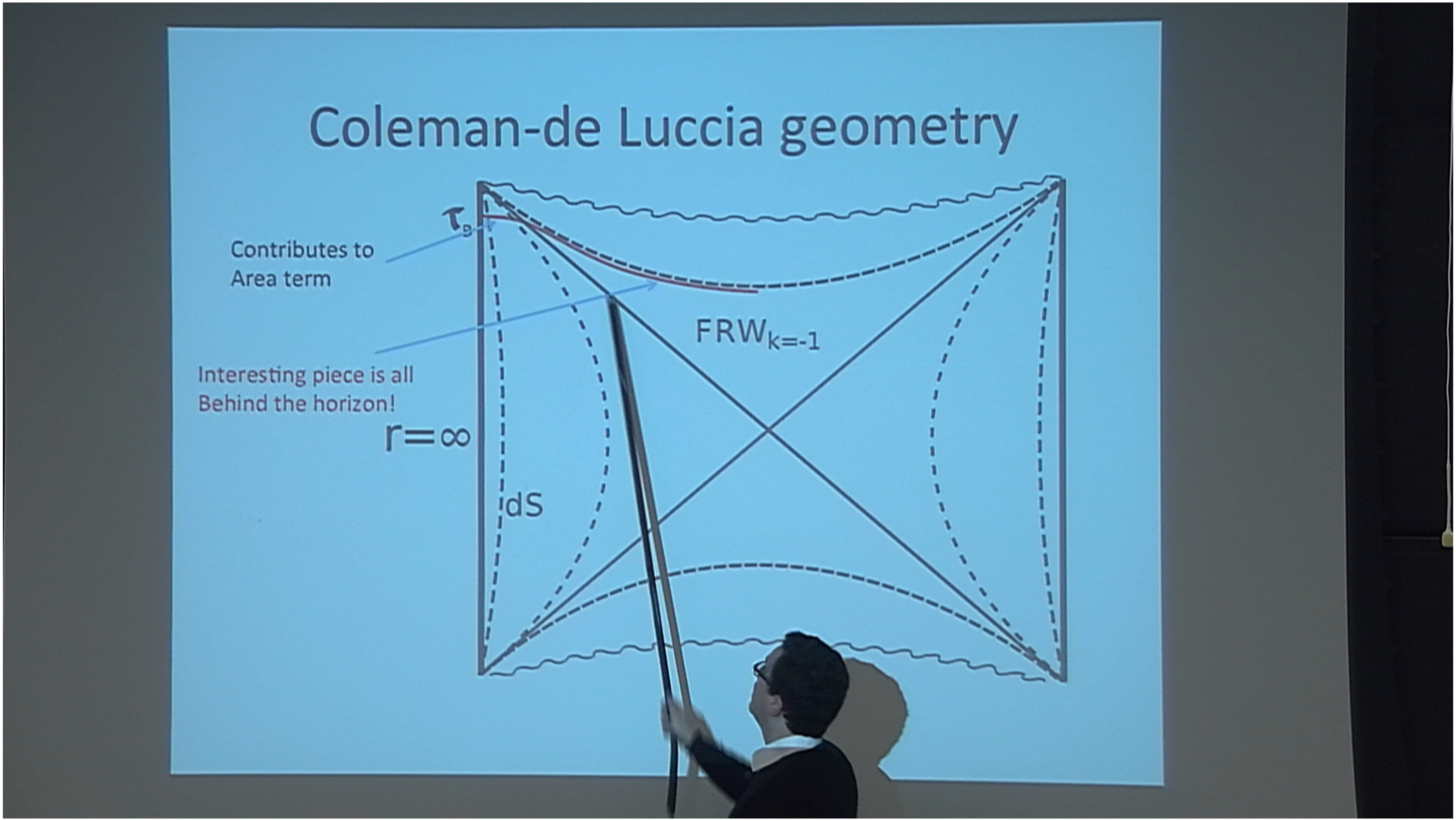


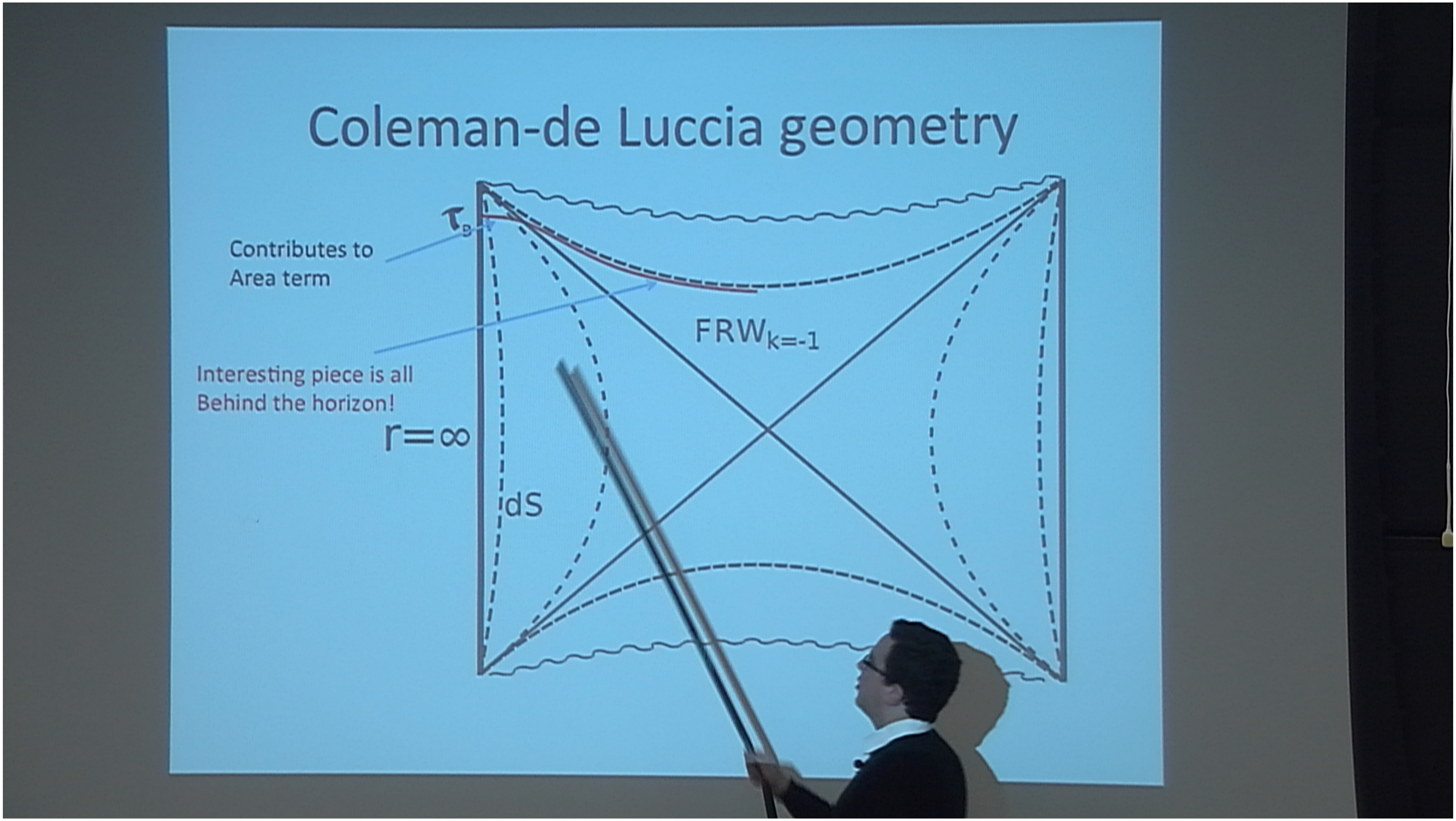
Hubeny, Rangamani, Takayanagi
Klebanov, Kutasov, Murugan

- To leading order in N, no logarithmic term.

Coleman-de Luccia geometry




Coleman-de Luccia geometry
τ_B
Contributes to
Area term
$FRW_{k=-1}$
Interesting piece is all
Behind the horizon!
$r=\infty$
dS



Coleman-de Luccia geometry
Contributes to
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Behind the horizon!
r=∞
dS
FRW k=-1

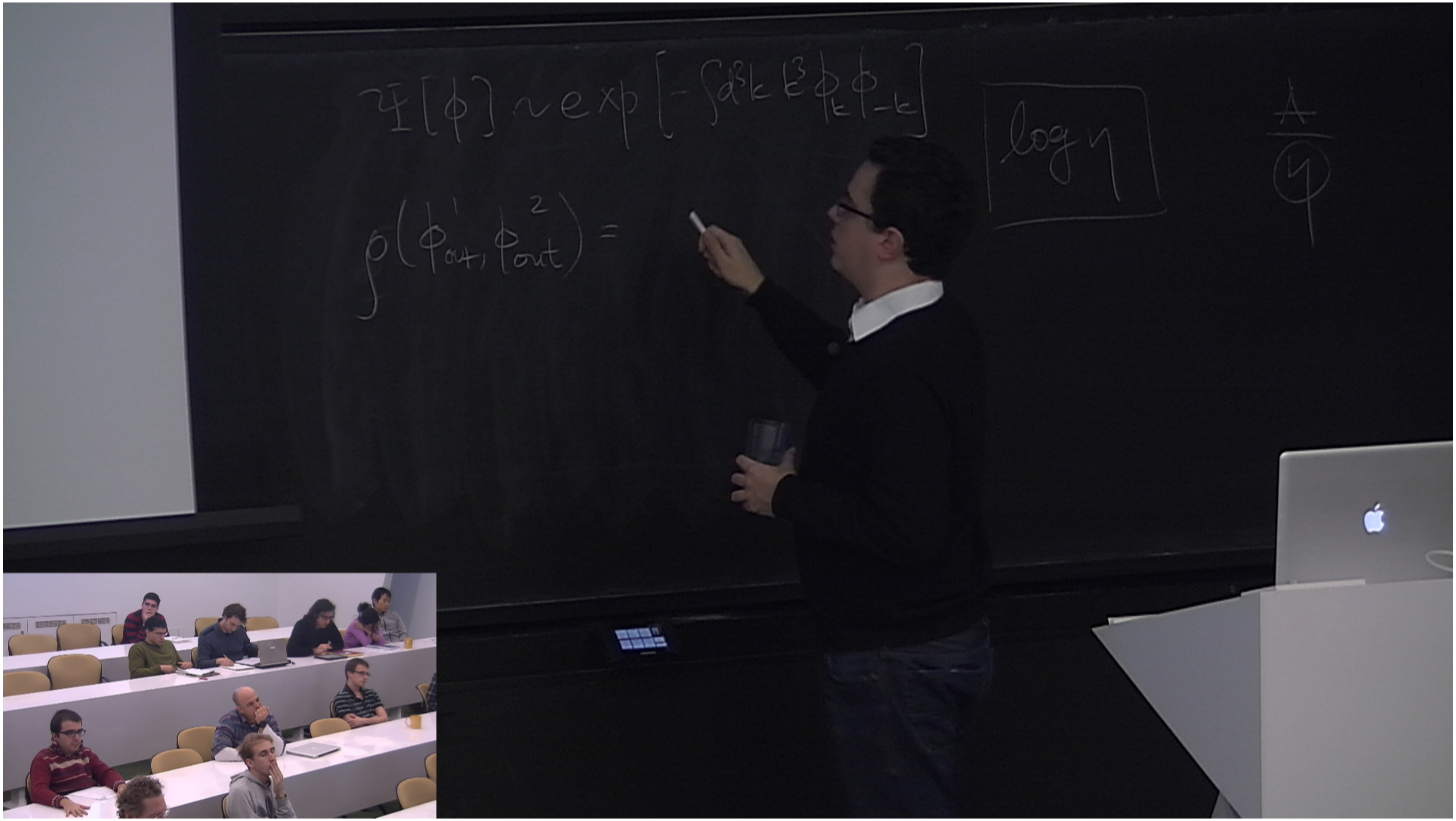
Coleman-de Luccia geometry

- Interesting piece of the EE coming solely from region behind the horizon (of the bulk geometry)!
- Superhorizon correlations in dS are probing the cosmology of the FRW universe

Conclusions

- EE in dS has new UV finite pieces that can be computed by standard methods;
- In 4D: Area law and logarithmic piece in conformal time;
- Log piece has long range entanglement information;
- For holographic duals, logarithmic piece in EE is all due to region behind the horizon…


L
R
C
log η



$\Psi[\phi] \sim \exp\left[-\int d^3k\, k^3 \phi_k \phi_{-k}\right]$
log η
$\rho(\phi^1_{out}, \phi^2_{out}) =$