Title: Accidental Supersymmetry and the Renormalization of Co-dimension 2 Branes

Date: Dec 14, 2012 01:00 PM

URL: http://pirsa.org/12120039

Abstract: In this talk, I'll give a brief summary of how one-loop
bulk eï¬€ects
renormalize both bulk and brane eï¬€ective interactions for geometries sourced by codimension-two
branes. I'll then discuss what these results imply for a six-dimensional
supergravity model which aims to capture the features that make extra-dimensional
physics attractive for understanding naturalness issues in particle physics.
I'll also emphasize the role that brane back-reaction plays in yielding
unexpected results, and present a one-loop contribution to the 4D vacuum energy
whose size is set by the KK scale.

Accidental Supersymmetry and the Renormalization of Co-dimension 2 Branes

Matt Williams

December 14, 2012

McMaster University

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Based on hep-th/1210.3753, hep-th/1210.5405
with C.P. Burgess, L. van Nierop, S. Parameswaran, and A. Salvio.

Motivation

- If Vacuum Energy $\leftrightarrow$ Space-time Curvature
  then $\rho_V \simeq (3 \times 10^{-3}\ \mathrm{eV})^4 \implies$ fine-tuning.

Motivation

- If Vacuum Energy $\leftrightarrow$ Space-time Curvature
  then $\rho_V \simeq (3 \times 10^{-3}\ \mathrm{eV})^4 \implies$ fine-tuning.
- Extra dimensions seem already ruled out:

$$E_{cc} = 3 \times 10^{-3}\ \mathrm{eV} = \frac{\hbar c}{L} \implies L \simeq 65\ \mu\mathrm{m}\ \mathrm{X}\,;$$

  but loop factors can make a significant difference:

$$E_{cc} = (4\pi) 3 \times 10^{-3}\ \mathrm{eV} = \frac{\hbar c}{L} \implies L \simeq 5\ \mu\mathrm{m}\ \checkmark\,.$$

- Two micron-sized extra dimensions give $M_g \sim 10\ \mathrm{TeV}$
  $\implies$ readily testable phenomenology.

Motivation

- If Vacuum Energy $\leftrightarrow$ Space-time Curvature
  then $\rho_V \simeq (3 \times 10^{-3}\ \mathrm{eV})^4 \implies$ fine-tuning.
- Extra dimensions seem already ruled out:

$$E_{cc} = 3 \times 10^{-3}\ \mathrm{eV} = \frac{\hbar c}{L} \implies L \simeq 65\ \mu\mathrm{m}\ \mathrm{X}\,;$$

  but loop factors can make a significant difference:

$$E_{cc} = (4\pi) 3 \times 10^{-3}\ \mathrm{eV} = \frac{\hbar c}{L} \implies L \simeq 5\ \mu\mathrm{m}\ \checkmark\,.$$

- Two micron-sized extra dimensions give $M_g \sim 10\ \mathrm{TeV}$
  $\implies$ readily testable phenomenology.
- A new purpose for supersymmetry: taming extra-dimensional contributions to the 4D vacuum energy.

Today's Agenda

- Introduce a particular model for studying supersymmetric codimension-2 systems.

Today's Agenda

- Introduce a particular model for studying supersymmetric codimension-2 systems.
- Present the 1-loop 4D vacuum energy due to an extra-dimensional hypermultiplet in six dimensions. (Similar methods hold for other multiplets.)
- Identify the relevant bulk, brane counterterms needed to regularize divergences.
- Test whether the renormalized, back-reacted 4D vacuum energy remains small (as advertised).

Take-home Message

1. We can do loop calculations with co-dimension 2 branes
2. We are using these techniques to test the naturalness of a particular co-dimension 2 model
3. Brane back-reaction is crucial to understanding extra-dimensional supersymmetry

6D Chiral, Gauged Supergravity

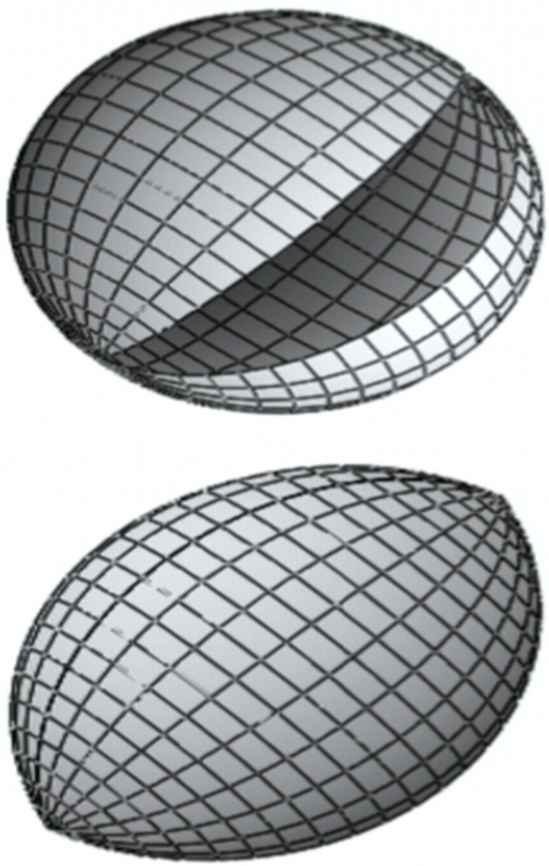
Gravity Sector:

- graviton (g_{MN}), gravitino (ψ_M), antisymmetric Kalb-Ramond field (G_{MNP}), dilatino (χ) and dilaton (ϕ).

Matter Sector:

- Gauge multiplet: gauge potential (A^a_M) and gaugino (λ^a);
  - Includes $U(1)_R \rightarrow$ stabilizing flux
- Hypermultiplet: four complex scalars (Φ^I) and hyperino (Ψ^I);

Rugby-Ball Background



$$S_b = -\int \mathrm{d}^4x \sqrt{-g_4}\, L_b\,, \quad b = \pm 1\,.$$

Background Metric:

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + r^2\left(\mathrm{d}\theta^2 + \alpha^2 \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

where

$$\alpha = 1 - \frac{\kappa^2 L_+}{2\pi} = 1 - \frac{\kappa^2 L_-}{2\pi}\,.$$

Deficit angle: $\delta = 2\pi(1-\alpha)$.
If $L_b = 0$ then $\alpha = 1 \to$ sphere.

Stabilizing $U(1)_R$ Flux

Ansatz: $F_{\mu\nu} = F_{\mu m} = 0$; $F_{mn} = f\,\epsilon_{mn}$. ($\epsilon_{mn} \to$ 2D volume form)

$$S_b = -\int \mathrm{d}^4x\sqrt{-g_4}\Big(T_b - \underbrace{\frac{\mathcal{A}_b}{2g^2}\epsilon^{mn}F_{mn}}_{\uparrow} + \mathcal{O}(\partial^2)\Big)$$

Brane-localized flux has two effects:

Stabilizing $U(1)_R$ Flux

Ansatz: $F_{\mu\nu} = F_{\mu m} = 0$; $F_{mn} = f\,\epsilon_{mn}$. ($\epsilon_{mn} \rightarrow$ 2D volume form)

$$S_b = -\int \mathrm{d}^4x\sqrt{-g_4}\Big(T_b - \underbrace{\frac{\mathcal{A}_b}{2g^2}\epsilon^{mn}F_{mn}}_{\uparrow} + \mathcal{O}(\partial^2)\Big)$$

Brane-localized flux has two effects:

1. alters gauge field boundary conditions ($b = \pm 1 \leftarrow$ N/S pole):

$$A_\varphi(\cos\theta = b) = b\frac{\mathcal{A}_b\, e^{\phi}}{2\pi} := b\,\Phi_b\,;$$

2. changes flux quantization condition ($N \in \mathbb{Z}$, $\Phi = \sum_b \Phi_b$):

$$f = \frac{N - \Phi}{2\alpha r^2}\,, \quad \text{or} \quad F_{\theta\phi} = \frac{N-\Phi}{2}\sin\theta\,.$$

Stabilizing $U(1)_R$ Flux

Ansatz: $F_{\mu\nu} = F_{\mu m} = 0$; $F_{mn} = f\,\epsilon_{mn}$. ($\epsilon_{mn} \to$ 2D volume form)

$$S_b = -\int \mathrm{d}^4x \sqrt{-g_4}\Big(T_b - \underbrace{\frac{\mathcal{A}_b}{2g^2}\epsilon^{mn}F_{mn}}_{\uparrow} + \mathcal{O}(\partial^2)\Big)$$

Brane-localized flux has two effects:

1. alters gauge field boundary conditions ($b = \pm 1 \leftarrow$ N/S pole):

$$A_\varphi(\cos\theta = b) = b\frac{\mathcal{A}_b\,e^\phi}{2\pi} := b\,\Phi_b\,;$$

2. changes flux quantization condition ($N \in \mathbb{Z}$, $\Phi = \sum_b \Phi_b$):

$$f = \frac{N - \Phi}{2\alpha r^2}\,, \quad \text{or} \quad F_{\theta\phi} = \frac{N-\Phi}{2}\sin\theta\,.$$

EOM: $f = \pm 1/2r^2 \implies N = \pm 1,\ \Phi = \pm(1-\alpha)$.

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M\phi)\Gamma^M\epsilon + \frac{1}{12}\,e^{-\phi}G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\,g_R}\,e^{-\phi/2}F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\,g_R}{\kappa^2}\,e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\,D_M\epsilon + \frac{1}{24}\,e^{-\phi}\,G_{PQR}\Gamma^{PQR}\Gamma_M\epsilon$$

These should vanish at a supersymmetric background.

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M\phi)\Gamma^M\epsilon + \frac{1}{12}\,e^{-\phi}G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\,g_R}\,e^{-\phi/2}F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\,g_R}{\kappa^2}\,e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\,D_M\epsilon + \frac{1}{24}\,e^{-\phi}\,G_{PQR}\Gamma^{PQR}\Gamma_M\epsilon$$

These should vanish at a supersymmetric background.

- $\delta\chi \to$ check!
- $\delta\lambda \to \epsilon = \begin{pmatrix}\epsilon_{4\pm}\\ 0\end{pmatrix}$, $\gamma_5\epsilon_{4\pm} = \pm\epsilon_{4\pm}$ EOM: $\left(e^\phi = \frac{\kappa^2}{4g_R^2r^2}\right)$

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M\phi)\Gamma^M\epsilon + \frac{1}{12}\,e^{-\phi}G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\,g_R}\,e^{-\phi/2}F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\,g_R}{\kappa^2}\,e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\,D_M\epsilon + \frac{1}{24}\,e^{-\phi}\,G_{PQR}\Gamma^{PQR}\Gamma_M\epsilon$$

These should vanish at a supersymmetric background.

- $\delta\chi \to$ check!
- $\delta\lambda \to \epsilon = \begin{pmatrix}\epsilon_{4\pm}\\ 0\end{pmatrix}$, $\gamma_5\epsilon_{4\pm} = \pm\epsilon_{4\pm}$ EOM: $\left(e^{\phi} = \frac{\kappa^2}{4g_R^2 r^2}\right)$

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M\phi)\Gamma^M\epsilon + \frac{1}{12}\,e^{-\phi}G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\,g_R}\,e^{-\phi/2}F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\,g_R}{\kappa^2}\,e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\,D_M\epsilon + \frac{1}{24}\,e^{-\phi}\,G_{PQR}\Gamma^{PQR}\Gamma_M\epsilon$$

These should vanish at a supersymmetric background.

- $\delta\chi \to$ check!
- $\delta\lambda \to \epsilon = \begin{pmatrix}\epsilon_{4\pm}\\ 0\end{pmatrix}$, $\gamma_5\epsilon_{4\pm} = \pm\epsilon_{4\pm}$ EOM: $\left(e^{\phi} = \frac{\kappa^2}{4g_R^2 r^2}\right)$
- $\delta\psi_M \to \Phi_+ = \Phi_- = \pm(1-\alpha)/2$. (Before: $\Phi = \pm(1-\alpha)$.)

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M \phi)\Gamma^M \epsilon + \frac{1}{12}\, e^{-\phi} G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\, g_R}\, e^{-\phi/2} F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\, g_R}{\kappa^2}\, e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\, D_M\epsilon + \frac{1}{24}\, e^{-\phi}\, G_{PQR}\Gamma^{PQR}\Gamma_M\epsilon$$

These should vanish at a supersymmetric background.

- $\delta\chi \to$ check!
- $\delta\lambda \to \epsilon = \begin{pmatrix} \epsilon_{4\pm} \\ 0 \end{pmatrix}$, $\gamma_5\epsilon_{4\pm} = \pm\epsilon_{4\pm}$ EOM: $\left(e^{\phi} = \frac{\kappa^2}{4g_R^2 r^2}\right)$
- $\delta\psi_M \to \Phi_+ = \Phi_- = \pm(1-\alpha)/2$. (Before: $\Phi = \pm(1-\alpha)$.)

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M \phi)\Gamma^M \epsilon + \frac{1}{12}\, e^{-\phi} G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\, g_R}\, e^{-\phi/2} F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\, g_R}{\kappa^2}\, e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\, D_M \epsilon + \frac{1}{24}\, e^{-\phi}\, G_{PQR}\Gamma^{PQR}\Gamma_M \epsilon$$

These should vanish at a supersymmetric background.

- $\delta\chi \to$ check!
- $\delta\lambda \to \epsilon = \begin{pmatrix} \epsilon_{4\pm} \\ 0 \end{pmatrix}$, $\gamma_5 \epsilon_{4\pm} = \pm\epsilon_{4\pm}$ $\quad$ EOM: $\left(e^{\phi} = \frac{\kappa^2}{4 g_R^2 r^2}\right)$
- $\delta\psi_M \to \Phi_+ = \Phi_- = \pm(1-\alpha)/2$. (Before: $\Phi = \pm(1-\alpha)$.)

Accidental SUSY: The Easy Way

SUSY transformations:

$$\delta\chi = \frac{1}{\kappa\sqrt{2}}(\partial_M\phi)\Gamma^M\epsilon + \frac{1}{12}\,e^{-\phi}G_{MNP}\Gamma^{MNP}\epsilon$$

$$\delta\lambda = \frac{1}{2\sqrt{2}\,g_R}\,e^{-\phi/2}F_{MN}\Gamma^{MN}\epsilon - \frac{i\sqrt{2}\,g_R}{\kappa^2}\,e^{\phi/2}\epsilon$$

$$\delta\psi_M = \frac{\sqrt{2}}{\kappa}\,D_M\epsilon + \frac{1}{24}\,e^{-\phi}\,G_{PQR}\Gamma^{PQR}\Gamma_M\epsilon$$

These should vanish at a supersymmetric background.

- $\delta\chi \rightarrow$ check!
- $\delta\lambda \rightarrow \epsilon = \begin{pmatrix}\epsilon_{4\pm}\\ 0\end{pmatrix}$, $\gamma_5\epsilon_{4\pm} = \pm\epsilon_{4\pm}$ EOM: $\left(e^{\phi} = \frac{\kappa^2}{4g_R^2r^2}\right)$
- $\delta\psi_M \rightarrow \Phi_+ = \Phi_- = \pm(1-\alpha)/2$. (Before: $\Phi = \pm(1-\alpha)$.)

Why is this an interesting model?

Because loop corrections are the leading contribution to 4D vacuum energy, ρ_V.
In hep-th/1108.0345, L. van Nierop & C.P. Burgess show

$$\frac{\partial L_b}{\partial \phi} = 0 \implies \Lambda\Big|_{\text{class.}} = 0\,.$$

Back-reaction is important here, too.

Why is this an interesting model?

Because loop corrections are the leading contribution to 4D vacuum energy, ρ_V.
In hep-th/1108.0345, L. van Nierop & C.P. Burgess show

$$\frac{\partial L_b}{\partial \phi} = 0 \implies \Lambda\Big|_{\text{class.}} = 0\,.$$

Back-reaction is important here, too.
This is a sketch of their argument:

- 4D curvature: $\hat{R}_4 := \hat{g}^{\mu\nu}\hat{R}_{\mu\nu}\Big|_{\text{class.}} \propto \Box\phi \quad \leftarrow$ total derivative;
- $\int d^2y\, \hat{R}_4 = \lim_{\rho\to 0}\left(2\pi\rho\,\partial_\rho\phi\right)$ ($\rho \to$ proper distance);

Why is this an interesting model?

Because loop corrections are the leading contribution to 4D vacuum energy, ρ_V.
In hep-th/1108.0345, L. van Nierop & C.P. Burgess show

$$\frac{\partial L_b}{\partial \phi} = 0 \implies \Lambda\Big|_{\text{class.}} = 0\,.$$

Back-reaction is important here, too.
This is a sketch of their argument:

- 4D curvature: $\hat{R}_4 := \hat{g}^{\mu\nu}\hat{R}_{\mu\nu}\Big|_{\text{class.}} \propto \Box\phi \quad \leftarrow$ total derivative;
- $\int d^2y\, \hat{R}_4 = \lim_{\rho\to 0}(2\pi\rho\,\partial_\rho\phi)$ ($\rho \to$ proper distance);
- Matching:

$$\lim_{\rho\to 0}(2\pi\rho\,\partial_\rho\phi) = \kappa^2\frac{\partial L_b}{\partial \phi}\,;$$

Why is this an interesting model?

Because loop corrections are the leading contribution to 4D vacuum energy, ρ_V.
In hep-th/1108.0345, L. van Nierop & C.P. Burgess show

$$\frac{\partial L_b}{\partial \phi} = 0 \implies \Lambda\Big|_{\text{class.}} = 0\,.$$

Back-reaction is important here, too.
This is a sketch of their argument:

- 4D curvature: $\hat{R}_4 := \hat{g}^{\mu\nu}\hat{R}_{\mu\nu}\Big|_{\text{class.}} \propto \Box\phi \quad \leftarrow$ total derivative;
- $\int d^2y\, \hat{R}_4 = \lim_{\rho\to 0}(2\pi\rho\,\partial_\rho\phi)$ ($\rho \to$ proper distance);
- Matching:

$$\lim_{\rho\to 0}(2\pi\rho\,\partial_\rho\phi) = \kappa^2\frac{\partial L_b}{\partial\phi}\,;$$

- Therefore, $\Lambda\Big|_{\text{class.}} = 0$ if $\frac{\partial L_b}{\partial \phi} = 0$.

Sketch of a Hyperscalar Loop Calculation

Action: $S = S_B + \sum_b S_b$ where

$$S_B \supset -\int \mathrm{d}^6X\sqrt{-g}\left(\frac{1}{2}\,\mathcal{G}_{IJ}(\Phi)\,g^{MN}D_M\Phi^I D_N\Phi^J + \frac{2g_R^2}{\kappa^4}\,e^\phi\,\mathcal{U}(\Phi)\right)$$

$$S_b = -\int \mathrm{d}^4x\sqrt{-g_4}\left(T_b - \frac{\mathcal{A}_b}{2g^2}\epsilon^{mn}F_{mn} + \mathcal{O}(\partial^2)\right)$$

where

$$D_M\phi = \partial_M\phi - iqA_M\phi\,, \quad \mathcal{U}(\Phi) = 1 + \frac{\kappa^2}{2}\,\mathcal{G}_{IJ}(0)\Phi^I\Phi^J + \dots\,.$$

EOM:

$$e^\phi = \frac{\kappa^2}{4g_R^2r^2} \implies m^2 = \frac{1}{2r^2}\,.$$

We want:

$$\Sigma_{1L} = -\int d^4x\,V_{1L} = \frac{i}{2}\ln\mathrm{Det}(-D^MD_M + m^2)$$

Sketch of a Hyperscalar Loop Calculation

Action: $S = S_B + \sum_b S_b$ where

$$S_B \supset -\int \mathrm{d}^6X\sqrt{-g}\left(\frac{1}{2}\mathcal{G}_{IJ}(\Phi)\, g^{MN} D_M\Phi^I D_N\Phi^J + \frac{2g_R^2}{\kappa^4}\, e^{\phi}\,\mathcal{U}(\Phi)\right)$$

$$S_b = -\int \mathrm{d}^4x\sqrt{-g_4}\left(T_b - \frac{\mathcal{A}_b}{2g^2}\epsilon^{mn}F_{mn} + \mathcal{O}(\partial^2)\right)$$

where

$$D_M\phi = \partial_M\phi - iqA_M\phi\,, \quad \mathcal{U}(\Phi) = 1 + \frac{\kappa^2}{2}\,\mathcal{G}_{IJ}(0)\Phi^I\Phi^J + \dots .$$

EOM:

$$e^{\phi} = \frac{\kappa^2}{4g_R^2 r^2} \implies m^2 = \frac{1}{2r^2}\,.$$

We want:

$$\Sigma_{1L} = -\int d^4x\, V_{1L} = \frac{i}{2}\ln \mathrm{Det}(-D^M D_M + m^2)$$

Sketch of a Hyperscalar Loop Calculation

Action: $S = S_B + \sum_b S_b$ where

$$S_B \supset -\int d^6X\sqrt{-g}\left(\frac{1}{2}\mathcal{G}_{IJ}(\Phi)\,g^{MN}D_M\Phi^I D_N\Phi^J + \frac{2g_R^2}{\kappa^4}\,e^{\phi}\,\mathcal{U}(\Phi)\right)$$

$$S_b = -\int d^4x\sqrt{-g_4}\left(T_b - \frac{\mathcal{A}_b}{2g^2}\epsilon^{mn}F_{mn} + \mathcal{O}(\partial^2)\right)$$

where

$$D_M\phi = \partial_M\phi - iqA_M\phi\,, \quad \mathcal{U}(\Phi) = 1 + \frac{\kappa^2}{2}\,\mathcal{G}_{IJ}(0)\Phi^I\Phi^J + \dots\,.$$

EOM:

$$e^{\phi} = \frac{\kappa^2}{4g_R^2r^2} \implies m^2 = \frac{1}{2r^2}\,.$$

We want:

$$\Sigma_{1L} = -\int d^4x\,V_{1L} = \frac{i}{2}\ln\mathrm{Det}(-D^MD_M + m^2)$$

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$V_{1L} = \frac{1}{2}\,\mu^{2\varepsilon} \sum_{jn} \int \frac{\mathrm{d}^d k_E}{(2\pi)^d} \, \ln\left(\frac{k_E^2 + m^2 + m_{jn}^2}{\mu^2}\right) .$$

- Heat kernel expansion;
- Solve for scalar's KK spectrum, $m_{jn}^2(\alpha, N, \Phi_b)$;
- Sum over KK spectrum using Poisson resummation;
- Take limit $d \to 4$:

$$V_{1L} = \frac{\mathcal{C}}{(4\pi)^2}\left[\frac{1}{4-d} + \ln\left(\frac{\mu}{m}\right)\right] + \mathcal{V}_f \, ,$$

where

$$\mathcal{C} := \frac{s_{-1}}{6} m^6 r^2 - \frac{s_0}{2} m^4 + s_1 \frac{m^2}{r^2} - \frac{s_2}{r^4}$$

and where. . .

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$V_{1L} = \frac{1}{2}\,\mu^{2\varepsilon} \sum_{jn} \int \frac{\mathrm{d}^d k_E}{(2\pi)^d}\, \ln\left(\frac{k_E^2 + m^2 + m_{jn}^2}{\mu^2}\right).$$

- Heat kernel expansion;
- Solve for scalar's KK spectrum, $m^2_{jn}(\alpha, N, \Phi_b)$;
- Sum over KK spectrum using Poisson resummation;
- Take limit $d \to 4$:

$$V_{1L} = \frac{\mathcal{C}}{(4\pi)^2}\left[\frac{1}{4-d} + \ln\left(\frac{\mu}{m}\right)\right] + \mathcal{V}_f\,,$$

where

$$\mathcal{C} := \frac{s_{-1}}{6} m^6 r^2 - \frac{s_0}{2} m^4 + s_1 \frac{m^2}{r^2} - \frac{s_2}{r^4}$$

and where. . .

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$V_{1L} = \frac{1}{2}\,\mu^{2\varepsilon} \sum_{jn} \int \frac{\mathrm{d}^d k_E}{(2\pi)^d}\, \ln\left(\frac{k_E^2 + m^2 + m_{jn}^2}{\mu^2}\right) .$$

- Heat kernel expansion;
- Solve for scalar's KK spectrum, $m_{jn}^2(\alpha, N, \Phi_b)$;
- Sum over KK spectrum using Poisson resummation;
- Take limit $d \to 4$:

$$V_{1L} = \frac{\mathcal{C}}{(4\pi)^2}\left[\frac{1}{4-d} + \ln\left(\frac{\mu}{m}\right)\right] + \mathcal{V}_f\ ,$$

where

$$\mathcal{C} := \frac{s_{-1}}{6} m^6 r^2 - \frac{s_0}{2} m^4 + s_1 \frac{m^2}{r^2} - \frac{s_2}{r^4}$$

and where. . .

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$V_{1L} = \frac{1}{2}\,\mu^{2\varepsilon} \sum_{jn} \int \frac{\mathrm{d}^d k_E}{(2\pi)^d}\, \ln\left(\frac{k_E^2 + m^2 + m_{jn}^2}{\mu^2}\right) .$$

- Heat kernel expansion;
- Solve for scalar's KK spectrum, $m_{jn}^2(\alpha, N, \Phi_b)$;
- Sum over KK spectrum using Poisson resummation;
- Take limit $d \to 4$:

$$V_{1L} = \frac{\mathcal{C}}{(4\pi)^2}\left[\frac{1}{4-d} + \ln\left(\frac{\mu}{m}\right)\right] + \mathcal{V}_f\ ,$$

where

$$\mathcal{C} := \frac{s_{-1}}{6} m^6 r^2 - \frac{s_0}{2} m^4 + s_1 \frac{m^2}{r^2} - \frac{s_2}{r^4}$$

and where. . .

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$V_{1L} = \frac{1}{2}\,\mu^{2\varepsilon} \sum_{jn} \int \frac{\mathrm{d}^d k_E}{(2\pi)^d}\, \ln\left(\frac{k_E^2 + m^2 + m_{jn}^2}{\mu^2}\right) .$$

- Heat kernel expansion;
- Solve for scalar's KK spectrum, $m_{jn}^2(\alpha, N, \Phi_b)$;
- Sum over KK spectrum using Poisson resummation;
- Take limit $d \to 4$:

$$V_{1L} = \frac{\mathcal{C}}{(4\pi)^2}\left[\frac{1}{4-d} + \ln\left(\frac{\mu}{m}\right)\right] + \mathcal{V}_f \,,$$

where

$$\mathcal{C} := \frac{s_{-1}}{6} m^6 r^2 - \frac{s_0}{2} m^4 + s_1 \frac{m^2}{r^2} - \frac{s_2}{r^4}$$

and where. . .

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$V_{1L} = \frac{1}{2}\,\mu^{2\varepsilon} \sum_{jn} \int \frac{\mathrm{d}^d k_E}{(2\pi)^d} \,\ln\left(\frac{k_E^2 + m^2 + m_{jn}^2}{\mu^2}\right) .$$

- Heat kernel expansion;
- Solve for scalar's KK spectrum, $m_{jn}^2(\alpha, N, \Phi_b)$;
- Sum over KK spectrum using Poisson resummation;
- Take limit $d \to 4$:

$$V_{1L} = \frac{\mathcal{C}}{(4\pi)^2}\left[\frac{1}{4-d} + \ln\left(\frac{\mu}{m}\right)\right] + \mathcal{V}_f \,,$$

where

$$\mathcal{C} := \frac{s_{-1}}{6} m^6 r^2 - \frac{s_0}{2} m^4 + s_1 \frac{m^2}{r^2} - \frac{s_2}{r^4}$$

and where. . .

Sketch of a Hyperscalar Loop Calculation (cont'd)

$$
\begin{aligned}
s^{\rm hs}_{-1} &= \alpha\,, \\
s^{\rm hs}_0(\omega, N, \Phi_b) &= \alpha\left[-\frac{1}{3} + \frac{\omega^2}{6}(1-3F)\right]\,, \\
s^{\rm hs}_1(\omega, N, \Phi_b) &= \alpha\left[\frac{17}{360} - \frac{\mathcal{N}^2}{24} - \frac{\omega^2}{36}(1-3F) - \frac{\omega^3\mathcal{N}}{12}\sum_b \Phi_b\, G(|\Phi_b|) + \frac{\omega^4}{180}\left(1-15F^{(2)}\right)\right], \\
s^{\rm hs}_2(\omega, N, \Phi_b) &= \alpha\left[-\frac{1}{210} + \frac{\mathcal{N}^2}{180} + \left(\frac{1}{240} - \frac{\mathcal{N}^2}{144}\right)(1-3F)\omega^2 - \frac{\omega^4\mathcal{N}^2}{360}\left(1-15F^{(2)}\right)\right. \\
&\qquad \left. - \frac{\omega^5\mathcal{N}}{120}\sum_b \Phi_b\, G(|\Phi_b|)(1+3F_b) + \left(\frac{1}{1260} - \frac{F^{(2)}}{120} - \frac{F^{(3)}}{60}\right)\omega^6\right]
\end{aligned}
$$

with

$$
\omega := \alpha^{-1}\,, \quad \mathcal{N} := \frac{N-\Phi}{\alpha}\,, \quad F_b := |\Phi_b|\,(1-|\Phi_b|)\,, \quad F^{(n)} := \sum_b F_b^n\,,
$$

$$
F^{(1)} := F\,, \quad G(|\Phi_b|) := (1-|\Phi_b|)(1-2|\Phi_b|)\,.
$$

"Bulk Renormalization is Independent of BC's"

Sphere, no BLF's ($\alpha = 1, \Phi_b = 0$):

$$s_{-1}^{\rm sph} = 1\,,\ s_0^{\rm sph} = -\frac{1}{6}\,,\ s_1^{\rm sph,0} = \frac{1}{40}\,,\ s_1^{\rm sph,2} = -\frac{N^2}{24}\,,\ s_2^{\rm sph,0} = \frac{1}{5040}\,,\ \text{and}\ s_2^{\rm sph,2} = -\frac{N^2}{240}\,.$$

Required bulk counterterms:

$$V_{1L{\rm ct}} = -(4\pi r^2)\left[U + \frac{1}{2\kappa^2}\,R + \frac{\zeta_{R^2}}{\kappa}\,\overline{R}^2 + \frac{1}{4g_R^2}\,F_{MN}F^{MN} + \zeta_{R^3}\,\overline{R}^3 + \frac{\kappa\zeta_{AR}}{8g_R^2}\,R\,F_{MN}F^{MN}\right]$$

so that $V_{\rm ct} = -\int d^2y\,\mathcal{L}_{\rm ct}\Big|_{\rm bkgd}$ cancels all UV divergences.

"Bulk Renormalization is Independent of BC's"

Sphere, no BLF's ($\alpha = 1, \Phi_b = 0$):

$$s_{-1}^{\text{sph}} = 1\,,\; s_0^{\text{sph}} = -\frac{1}{6}\,,\; s_1^{\text{sph},0} = \frac{1}{40}\,,\; s_1^{\text{sph},2} = -\frac{N^2}{24}\,,\; s_2^{\text{sph},0} = \frac{1}{5040}\,,\text{ and } s_2^{\text{sph},2} = -\frac{N^2}{240}\,.$$

Required bulk counterterms:

$$V_{1L\text{ct}} = -(4\pi r^2)\left[U + \frac{1}{2\kappa^2}\,R + \frac{\zeta_{R^2}}{\kappa}\,\overline{R}^2 + \frac{1}{4g_R^2}\,F_{MN}F^{MN} + \zeta_{R^3}\,\overline{R}^3 + \frac{\kappa\zeta_{AR}}{8g_R^2}\,R\,F_{MN}F^{MN}\right]$$

so that $V_{\text{ct}} = -\int d^2y\,\mathcal{L}_{\text{ct}}\Big|_{\text{bkgd}}$ cancels all UV divergences.

"Brane Renormalization depends only on itself"

- Bulk renormalization is independent of brane physics — contribution to V_{1L} scales with α (volume factor);
- Brane renormalization vanishes in the limit $\alpha \to 1$, $\Phi_b \to 0$.

$$s_i = \alpha s_i^{\mathrm{sph}} + \sum_b \delta s_{i(b)}$$

"Brane Renormalization depends only on itself"

- Bulk renormalization is independent of brane physics — contribution to V_{1L} scales with α (volume factor);
- Brane renormalization vanishes in the limit $\alpha \to 1$, $\Phi_b \to 0$.

$$s_i = \alpha s_i^{\rm sph} + \sum_b \delta s_{i(b)}$$

Brane counterterms:

$$\mathcal{L}_{b,{\rm ct}} = -\sqrt{-g_4}\left(T_{b,{\rm ct}} - \frac{\mathcal{A}_{b,{\rm ct}}}{2g_R^2}\,\epsilon\cdot F + \frac{\zeta_{R\,b}}{\kappa}\,R + \frac{\kappa\zeta_{A\,b}}{4g_R^2}\,F^2 + \frac{\kappa\zeta_{\tilde{A}R\,b}}{2g_R^2}\,R\,\epsilon\cdot F + \zeta_{R^2 b}\,\overline{R}^2\right)\,.$$

Explicit Example of Cancellation

Similar story for the hyperino (but they have $q_R = 0$) — upshot is:

$$\left(\mu\,\frac{\partial T_b}{\partial \mu}\right)_{\rm f} = \frac{m^4}{2(4\pi)^2\omega}\left(\frac{\delta\omega}{3} + \frac{\delta\omega^2}{6}\right).$$

Compare with

$$4 \times \left(\mu\,\frac{\partial T_b}{\partial \mu}\right)_{\rm hs} = \frac{m^4}{2(4\pi)^2\omega}\left(\frac{2\,\delta\omega}{3} + \frac{\delta\omega^2}{3} - 2\,\omega^2|\Phi_b|(1-|\Phi_b|)\right)$$

when $\Phi_b = \pm\frac{1}{2}(1-\alpha) \quad (= \pm\frac{1}{2}(\omega-1)/\omega)$.
Net result:

$$\left(\mu\,\frac{\partial T_b}{\partial \mu}\right)_{\rm hm} = 0\,.$$

In general, individual brane terms renormalize, but their net contribution to $\mu(\partial V/\partial\mu)$ vanishes.

Explicit Example of Cancellation

Similar story for the hyperino (but they have $q_R = 0$) — upshot is:

$$\left(\mu \frac{\partial T_b}{\partial \mu}\right)_{\rm f} = \frac{m^4}{2(4\pi)^2\omega}\left(\frac{\delta\omega}{3} + \frac{\delta\omega^2}{6}\right).$$

Compare with

$$4 \times \left(\mu \frac{\partial T_b}{\partial \mu}\right)_{\rm hs} = \frac{m^4}{2(4\pi)^2\omega}\left(\frac{2\,\delta\omega}{3} + \frac{\delta\omega^2}{3} - 2\,\omega^2|\Phi_b|(1 - |\Phi_b|)\right)$$

when $\Phi_b = \pm\frac{1}{2}(1-\alpha) \quad (= \pm\frac{1}{2}(\omega - 1)/\omega)$.
Net result:

$$\left(\mu \frac{\partial T_b}{\partial \mu}\right)_{\rm hm} = 0\,.$$

In general, individual brane terms renormalize, but their net contribution to $\mu(\partial V/\partial\mu)$ vanishes.

Not quite the full story...

Back-reaction:

- Relevant thing: e^{ϕ} dependence — enters from terms $\propto m^4$, m^2.

Not quite the full story...

Back-reaction:

- Relevant thing: e^{ϕ} dependence — enters from terms $\propto m^4$, m^2.
- Massive multiplet ($\eta = |\Delta\Phi|/(2\alpha)$, $m^2 = M^2 e^{\phi}$):

$$\rho_V = \frac{\eta}{(4\pi r^2)^2}\left[\left(\frac{\kappa M}{2\,g_R}\right)^4 - \left(\frac{\eta}{3} + \frac{(\omega^2-1)}{2\omega}\eta^2 - \frac{\eta^3}{3}\right)\left(\frac{\kappa M}{2\,g_R}\right)^2\right]\ln\left(\frac{M_s}{M}\right)$$

Take-home Message

1. We can do loop calculations with co-dimension 2 branes
2. We are using these techniques to test the naturalness of a particular co-dimension 2 model
3. Brane back-reaction is crucial to understanding extra-dimensional supersymmetry

Thank you for your attention!

Thank you for your attention!