Title: The Spectrum of Strings on Warped AdS3

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Abstract: Warped AdS3 has isometry SL(2,R) x U(1). It is closed related to Kerr/CFT, non local dipole theories and AdS/CMT. In this talk I will derive the spectrum of string theory on
Warped AdS3. This is possible because the worldsheet theory can be mapped to the worldsheet on AdS3 by a nonlocal field redefinition.

The Spectrum of Strings on Warped $AdS_3 \times S^3$

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Harvard University

T. Azeyanagi, D. Hofman, W.S, and A. Strominger, arXiv: 1207.5050

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Motivations

- String spectrum is only known in a few backgrounds, for example, flat space, pp wave background, and $AdS_3 \times S^3 \times \mathcal{M}^4$ with NS-NS flux $\cdots$
  Today I will show you another example.
- Warped AdS_3 appears in various physics context, like Kerr/CFT, AdS/CMT on Shrödinger space, and non local dipole field theory.

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Outline

1. Introduction to Warped AdS_3
   - Warped AdS_3
   - Dipole deformations
   - Kerr/CFT
2. String theory on W$AdS_3 \times S^3$
   - String theory on AdS_3
   - String theory on TsT background
3. String Spectrum
   - The massive spectrum
   - The boundary modes

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Warped AdS_3 ($WAdS_3$) *D. Anninos, W. Li, M. Padi, W.S. and A. Strominger*

Warped AdS_3 is a fiberation of S^1 over AdS_2.

- the isometry group is $SL(2,\mathbb{R}) \times U(1)$
- the signature of the $U(1)$ decides whether it is spacelike, timelike, or null warped
- the length of the $U(1)$ decides whether it is stretched or squashed, or ordinary AdS_3

Metric for spacelike and timelike warped AdS_3

$$ds^2 = \ell^2(\sigma_1^2 + \sigma_2^2 + (1+\lambda^2)\sigma_3^2)$$

Null warped AdS_3,

$$ds_3^2 = \ell^2(d\rho^2 + e^{2\rho} d\gamma d\bar{\gamma} - \lambda^2 e^{4\rho} d\bar{\gamma}^2)$$

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Warped AdS_3 appears in many theories.
In 3d, Einstein gravity with matter, topologically massive gravity, new massive gravity, etc.
In 6d, minimal supergravity, which can be obtained from IIB string theory,etc
Lots of work has been done on the dictionary of holography
2d CFT? boundary conditions? asymptotic symmetry group?
One peculiar property in the dictionary of holography, for massive scalars, the conformal weight is

$$h_R = \frac{1}{2}(1 + \sqrt{1 + m^2\ell^2 + 4\lambda^2 p^2})$$

Dipole deformations *Bergman, Dasgupta, Ganor, Karczmarek, Rajesh*

In a dipole deformed field theory, the ordinary product of fields is replaced by a nonlocal $\star$ product. For two functions f and g which are momentum and charge eigenstates one then has

$$f \star g = e^{iL^{\mu}(q_f p^{g}_{\mu} - i q_g p^{f}_{\mu})} fg.$$

In the bulk, when the background spacetime has two $U(1)$ isometries, a dipole deformation is dual to applying TsT transformations. When the two $U(1)$ directions are

- one transverse, one parallel $\Longrightarrow$ dipole deformation
- both parallel $\Longrightarrow$ noncommutative deformation
- both transverse $\Longrightarrow$ β- deformation

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Dipole deformation of D1-D5 CFT

In the bulk, $AdS_3 \times S^3 \to WAdS_3 \times$ squashed S^3

- the NS ground state ($T = 0$ BTZ)$\Longrightarrow$null $WAdS_3$ (Shrödinger space)
- finite T BTZ $\Longrightarrow$ spacelike squshed $WAdS_3$

One can choose a consistent boundary so that the asymptotic symmetry group is generated by

- a left moving Virasoro algebra with central charge $c_L = 6Q^2$
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Microscopic entropy

- The dipole deformed black holes have the same entropy and temperatures as the original BTZ black holes. Cardy's formula $S = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = S_{BH}$. *W.S and Strominger*
- $S = -\frac{4\pi i P_0 P_0^{vac}}{k} + 4\pi\sqrt{-(L_0^{vac} - \frac{(P_0^{vac})^2}{k})(L_0 - \frac{P_0^2}{k})} = S_{BH}$.
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Kerr/CFT *M. Guica, T. Hartmn, W.S and Strominger*

Kerr/CFT is the conjecture that quantum gravity at the near horizon region of extremal Kerr black holes (NHEK) is holographically dual to a two dimensional CFT with central charge $c_L = 12J$. The original proposal applies to 4d Kerr black holes in Einstein gravity. Later it was generalized to almost any rotating black holes in any dimensions. In particular, some generalization of Kerr/CFT can be embedded in string theory. The main idea is to find a family of solutions in string theory interpolating NHEK and $AdS_3 \times \mathcal{M}$.

5d NHEK uplifted to 6d $\rightarrow AdS_3 \times S^3$=D1/D5 CFT *M. Guica and Strominger*

4d NHEK$\times S^1 \rightarrow AdS_3 \times S^2$=MSW CFT *G. Compere, W.S and A. Virmani*

$WAdS_3$/dipole-CFT vs Kerr/CFT

- NHEK always contains $WAdS_3$ as a subspace
- Momentum dependent conformal weight
- Similar deformation properties from $AdS_3 \times S^3$
  $SL(2,R)_L \times SU(2)_R \times U(1)^2$ invariant, and reduced to $WAdS_3$ in 3d
  conformal weights (2,1) operator
  D-brane constructions—d1-d5 or ns1-ns5 dipole charge densities on D1-D5 *W.S and Strominger*
- $T = 0$ of NHEK at infinitesimal deformation *El-Showk and Guica*
- the first step of the U duality transformations from $AdS_3 \times S^3$ to NHEK *Bena, Guica and W.S*

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- No consistent boundary conditions to allow two Virasoro generators so far
- explanation of $h_R = \frac{1}{2}(1 + \sqrt{1 + m^2 Q + 4\lambda^2 p^2})$
- details of the dual CFT

Going to string theory?

The background before deformations

We start with the solution of string theory $AdS_3 \times S^3 \times \mathcal{M}^4$ supported with NS-NS flux, where $\mathcal{M}^4$ (T^4 or K^3) is a four dimensional compact manifold. The six dimensional part of the solution is

$$\begin{aligned} ds^2 &= Q(d\rho^2 + e^{2\rho} d\gamma d\bar{\gamma} + d\Omega_3) \\ B &= -\frac{Q}{4}(\cos\theta d\phi \wedge d\psi + 2e^{2\rho} d\gamma \wedge d\bar{\gamma}) \end{aligned}$$

where $d\Omega_3$ is the metric on a unit three sphere. We work in Lorenzian signature, therefore γ and $\bar{\gamma}$ should be thought as independent coordinates, instead of being complex conjugate to each other. This solution can also be obtained as the near horizon of the NS1-NS5 configuration, with Q NS1 branes and Q NS5 branes. Isometry is $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R \times SU(2)_L \times SU(2)_R$.

WZW model

The world sheet theory on AdS_3 is $SL(2,\mathbb{R})$ WZW model. The B field provides the WZ term, which makes the theory conformal invariant.

$$\begin{aligned}\mathcal{L} &= \frac{Q}{2\pi}\left(e^{2\rho}\partial\bar{\gamma}\bar{\partial}\gamma + \partial\rho\bar{\partial}\rho\right) \\ &= \frac{Q}{2\pi}\left(\partial\rho\bar{\partial}\rho + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}e^{-2\rho}\bar{\partial}\bar{\gamma}\right)\end{aligned}$$

At the boundary of AdS_3, both spacetime and string worldsheet theory is weakly coupled. The theory is approximately a free field theory, including a free field ρ, a left moving β, γ pair, and a right moving $\bar{\beta}, \bar{\gamma}$ pair. We use bar to denote right moving quantities, while unbarred quantities are left moving.

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Vertex operators

The primary operator with conformal weight (h, h) under the $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ is

$$\Phi_h(x,\bar{x};z,\bar{z}) = \frac{1}{\pi}\left(\frac{1}{(\gamma - x)(\bar{\gamma} - \bar{x})e^{\rho} + e^{-\rho}}\right)^{2h}$$

Fourier transform to partial momentum space, we have

$$\begin{aligned}\Phi_h(p,\bar{x};z,\bar{z}) &\equiv \int \frac{dx}{\sqrt{2\pi}} e^{ipx}\Phi_{h,\bar{h}}(x,\bar{x}) \\ &\to \frac{(-1)^h p^{2h-1}}{\pi\Gamma(2h)(\bar{\gamma} - \bar{x})^{2h} e^{2h\rho}} e^{ip\gamma}\end{aligned}$$

as $\rho \to \infty$.

$AdS_3 \xrightarrow{TsT} WAdS_3$

After T_γ, followed by $\psi \to \psi - 2\lambda\gamma$ and then T_γ again, and the gravity solution becomes S^3 fiberation over AdS_3

$$
\begin{aligned}
ds^2 &= Q\left(d\rho^2 + e^{2\rho} d\gamma d\bar{\gamma} + d\Omega_3 + \lambda e^{2\rho} d\bar{\gamma}(d\psi + \cos\theta d\phi)\right) \\
B &= -\frac{Q}{4}[\cos\theta d\phi \wedge d\psi + 2e^{2\rho} d\gamma \wedge d\bar{\gamma} \\
&\quad +2\lambda e^{2\rho}(d\psi + \cos\theta d\phi) \wedge d\bar{\gamma}]
\end{aligned}
$$

The new background has isometry $U(1)_L \times SL(2,\mathbb{R})_R \times SU(2)_L \times U(1)_R$. Dimensional reduction on the three sphere gives a null warped AdS_3 (Shrödinger space) in three dimensions, with the metric

$$ds_3^2 = Q(d\rho^2 + e^{2\rho} d\gamma d\bar{\gamma} - \lambda^2 e^{4\rho} d\bar{\gamma}^2)$$

Scalar wave equations

scalar waves on the deformed background can be expanded as

$$\Phi = e^{i(\bar{p}\bar{\gamma}+p\gamma+i\bar{q}\psi+iq\phi)}R(\rho)\Theta(\theta)$$

The right moving conformal weight is changed to

$$\bar{h}_{sugra} = \frac{1}{2}\left(1+\sqrt{1+(M^2+K)Q-8\lambda p\bar{q}+4\lambda^2p^2}\right)$$

Field redefinition *Frolov; Alday, Arutyunov and Frolov*

One can check that the EOM and the constraints become that of AdS_3 upon the following field redefinitition

$$\begin{aligned} \partial\hat{\gamma} &= \partial\gamma - \lambda^2 e^{2\rho}\partial\bar{\gamma}, \quad \bar{\partial}\hat{\gamma} = \bar{\partial}\gamma + \lambda(\bar{\partial}\psi + \cos\theta\bar{\partial}\phi) \\ \partial\hat{\psi} &= \partial\psi + 2\lambda e^{2\rho}\partial\bar{\gamma}, \quad \bar{\partial}\hat{\psi} = \bar{\partial}\psi \end{aligned}$$

The integrated form of the field redefinition can be written as

$$\hat{\gamma} = \gamma - 2\frac{\lambda^2}{Q}\mu + \lambda\bar{\varphi} \quad \hat{\psi} = \psi + 4\lambda\frac{\mu}{Q}$$

where

$$\partial\mu = j^- = Q\frac{1}{2}e^{2\rho}\partial\bar{\gamma} = \frac{1}{2}\beta \quad - i\frac{Q}{4}\bar{\partial}\bar{\varphi} = -i\frac{Q}{4}(\bar{\partial}\psi + \cos\theta\bar{\partial}\phi)$$

Twisted boundary conditions

The field redefinition leads to a new boundary condition

$$\hat{\gamma}(\sigma + 2\pi) \sim \hat{\gamma}(\sigma) - 2\pi\frac{\lambda}{Q}(\bar{q} - \lambda p),$$
$$\hat{\psi}(\sigma + 2\pi) \sim \hat{\psi}(\sigma) + 4\pi\frac{\lambda}{Q}p$$

- To recapitulate, string theory on the TsT background is the same as string theory on $AdS_3 \times S^3$ with the above twisted boundary conditions

Vertex operators

In the free field approximation to the $AdS_3 \times S^3$ model, we have the leading OPEs

$$\begin{aligned}\mu(z)\hat{\gamma}(w) &\sim -\ln(z-w),\\ \rho(z,\bar{z})\rho(w,\bar{w}) &\sim -\frac{1}{2(Q-2)}\ln((z-w)(\bar{z}-\bar{w})),\\ \bar{\varphi}(\bar{z})\bar{\varphi}(\bar{w}) &\sim -\frac{2}{Q}\ln(\bar{z}-\bar{w}).\end{aligned}$$

We consider the vertex operators $V_{p,\bar{q}}$ creating states of definite $(p,\bar{q})$. Then by definition,

$$\oint \frac{j^-}{2\pi} V_{p,\bar{q}} \;=\; pV_{p,\bar{q}} \quad Q\oint \frac{1}{2\pi} 2\bar{k}_3 V_{p,\bar{q}} = \bar{q}\, V_{p,\bar{q}}\,.$$

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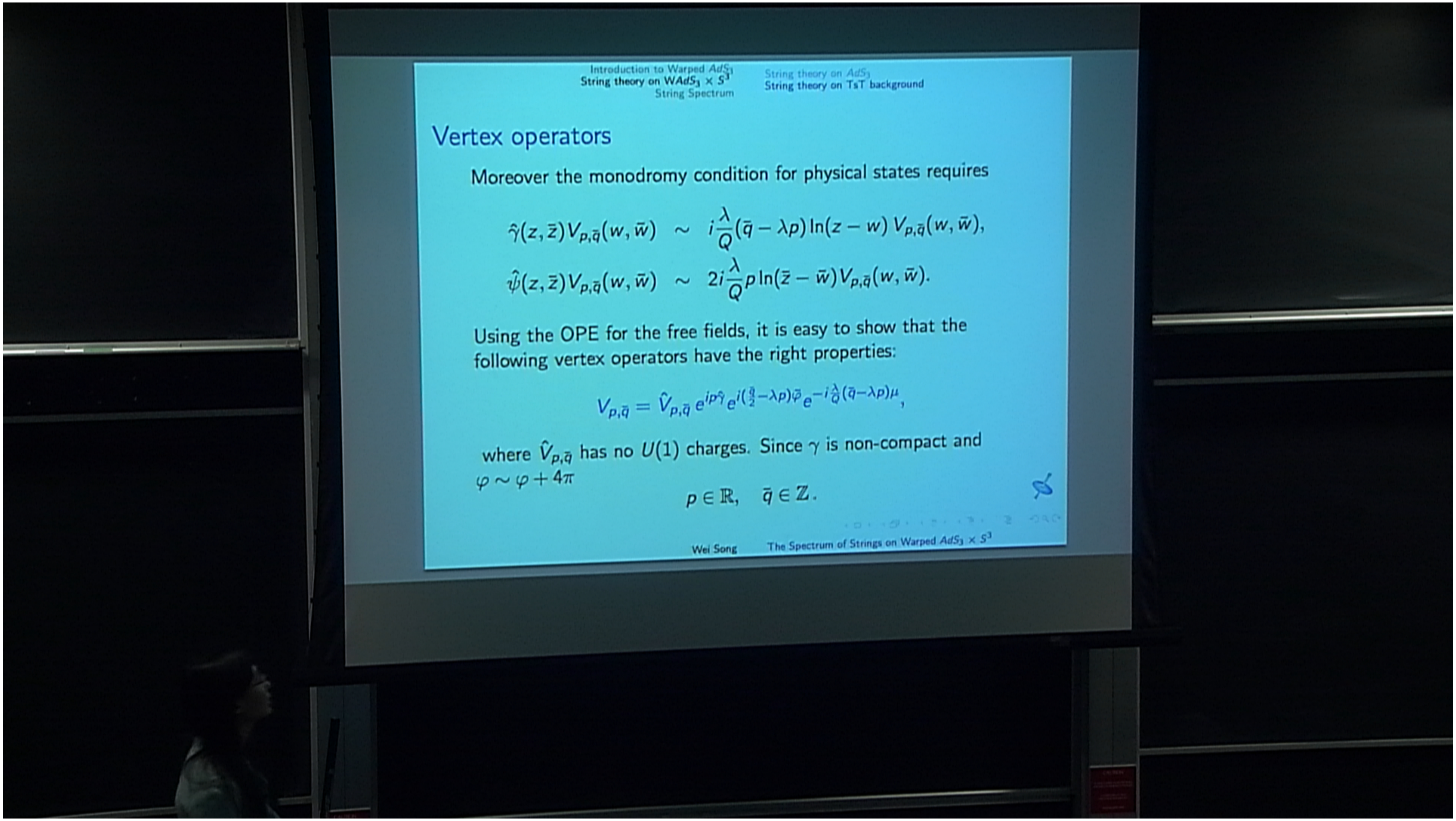
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Moreover the monodromy condition for physical states requires
$\hat{\gamma}(z,\bar{z})V_{p,\bar{q}}(w,\bar{w}) \sim i\frac{\lambda}{Q}(\bar{q}-\lambda p)\ln(z-w)\,V_{p,\bar{q}}(w,\bar{w}),$
$\hat{\psi}(z,\bar{z})V_{p,\bar{q}}(w,\bar{w}) \sim 2i\frac{\lambda}{Q}p\ln(\bar{z}-\bar{w})V_{p,\bar{q}}(w,\bar{w}).$
Using the OPE for the free fields, it is easy to show that the following vertex operators have the right properties:
$V_{p,\bar{q}} = \hat{V}_{p,\bar{q}}\, e^{ip\hat{\gamma}} e^{i(\frac{\bar{q}}{2}-\lambda p)\bar{\varphi}} e^{-i\frac{\lambda}{Q}(\bar{q}-\lambda p)\mu},$
where $\hat{V}_{p,\bar{q}}$ has no $U(1)$ charges. Since γ is non-compact and $\varphi \sim \varphi + 4\pi$
$p \in \mathbb{R}, \quad \bar{q} \in \mathbb{Z}.$
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$$p \in \mathbb{R}, \quad \bar{q} \in \mathbb{Z}\,.$$

Locality

We need to show that the deformed vertex operators are mutually local. It is easily seen that the OPE between two operators $V_{p,\bar{q}}(z)$ and $V_{p',\bar{q}'}(w)$ acquires the λ-dependent prefactor

$$((\bar{z}-\bar{w})(z-w))^{-\frac{\lambda}{Q}((\bar{q}-\lambda p)p'+(\bar{q}'-\lambda p')p)}$$

which has no branch cuts. Hence, the deformation does not disturb mutual locality of the vertex operators. The spectral flow can be also viewed as a $SO(2,1)$ rotation on the Narain lattice, rotating $\hat{\gamma}$, $\hat{\psi}$ to γ, ψ.

If $V_{p,\bar{q}}$ is the Fourier transform with respect to $\hat{\gamma}$ of a boundary primary, before the deformation it contains a factor $\Phi_{(0)h}(p,\bar{x};z,\bar{z})$. After the deformation, it becomes

$$\Phi_h(p,\bar{x};z,\bar{z}) \quad = \quad \Phi_{(0)h}(p,\bar{x};z,\bar{z})\mathcal{U}.$$

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The massive spectrum

For the highest weight representation, the on shell condition from spectral flow is

$$\frac{-\bar{h}(\bar{h}-1)+\bar{J}(\bar{J}+1)}{Q-2}+\frac{(\lambda p)^2-\lambda p\bar{q}}{Q-2}+\bar{N}-\bar{a}=0$$

Therefore the conformal weights will be modified to

$$\bar{h}=\frac{1}{2}(1+\sqrt{1+Qm^2})$$

where m is the mass in three dimensions,

$$Qm^2=4Q(\bar{N}-\bar{a})+\bar{q}(\bar{q}+2)-4\lambda p\bar{q}+4\lambda^2p^2.$$

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The right moving boundary gravitons

Boundary gravitons are created by nontrivial diffeomorphisms. The weight $(1,1)$ vertex operator for a general infinitesimal diffeomorphism ζ^μ and gauge transformation Λ_μ is, to leading order in the α' sigma model expansion,

$$V_{\zeta,\Lambda} = [\mathcal{L}_\zeta(g_{\mu\nu} + B_{\mu\nu}) + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu]\partial X^\mu \bar{\partial} X^\nu .$$

For the nontrivial right-moving diffeomorphisms parameterized by $e^{i\bar{p}\bar{\gamma}}$, the diffeomorphisms are

$$\zeta^\mu \partial_\mu = e^{i\bar{p}\bar{\gamma}} \left(\partial_{\bar{\gamma}} - \frac{i\bar{p}}{2}\partial_\rho + \frac{\bar{p}^2}{2} e^{-2\rho} \partial_\gamma \right), \quad \Lambda_\mu = 0.$$

These vertex operators carry neither left momentum nor right charge and are unaffected by the deformation. Hence the tower of right-moving massless gravitons remains intact.

The right-moving boundary stress tensor is

$$\bar{T}_B(\bar{p}) = \frac{Q}{2\pi} \oint (e^{2\rho}\bar{\partial}\hat{\gamma} - i\bar{p}\bar{\partial}\rho)e^{i\bar{p}\bar{\gamma}}.$$

One can check using the OPEs that it indeed generates the Virasoro algebra

$$[\bar{T}_B(\bar{p}), \bar{T}_B(\bar{p}')] = -i(\bar{p} - \bar{p}')\,\bar{T}_B(\bar{p} + \bar{p}') + \frac{\bar{p}'^2\bar{p} - \bar{p}'\bar{p}^2}{4}\bar{I}$$

with the central term

$$\bar{I} = \frac{Q}{2\pi} \oint \bar{\partial}\bar{\gamma}\, e^{i(\bar{p}+\bar{p}')\bar{\gamma}},$$

independent of λ. In the classical limit is

$$c_R = \frac{6\langle \bar{I} \rangle}{\delta(\bar{p} + \bar{p}')} = 6Q^2.$$

The two point functions imply $\bar{T}_B(\bar{p})$ has $(h, \bar{h}) = (0, 2)$. This suggests that the right-moving conformal symmetry of the boundary theory is unaffected by the deformation.

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The left moving boundary gravitons

The left moving gravitons are generated by $\xi = e^{ip\gamma}(\partial_\gamma - \frac{ip}{2}\partial_\rho)$, which has integer falloff, and therefore cannot be obtained from the string spectrum. The reason is that we are using covariant formalism here, which corresponds to impose covariant gauge on the pure gauge modes. For example, in the spin one case, $A_\mu = \partial_\mu \Lambda$, covariant gauge means $\Box_\lambda \Lambda = 0$. That means that Λ satisfy the on shell condition we just found, which has momentum dependent falloff. However, in the BRST formalism, we should still be able to write down the vertex operator of the left moving graviton. Whether it should be included in the physical spectrum depends on the choice of boundary conditions.

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Table: $AdS_3 \times S^3$

Isometry	ASG
$SL(2,\mathbb{R})_L$	Virasoro_L
$SL(2,\mathbb{R})_R$	Virasoro_R
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Table: $WAdS_3 \times S^3$ with Brown-Henneaux boundary conditions

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New boundary conditions on AdS_3

Comepere, W.S and Strominger, to appear

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