Title: Mathematica - Lecture 4

Date: Dec 14, 2012 09:00 AM

URL: http://pirsa.org/12120022

Abstract:

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Fri 10:11 AM

Lecture December 14.nb

In[6]:= $\Sigma[6] = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix};$

$\Sigma[5] = \begin{pmatrix} 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & -I & 0 \end{pmatrix};$

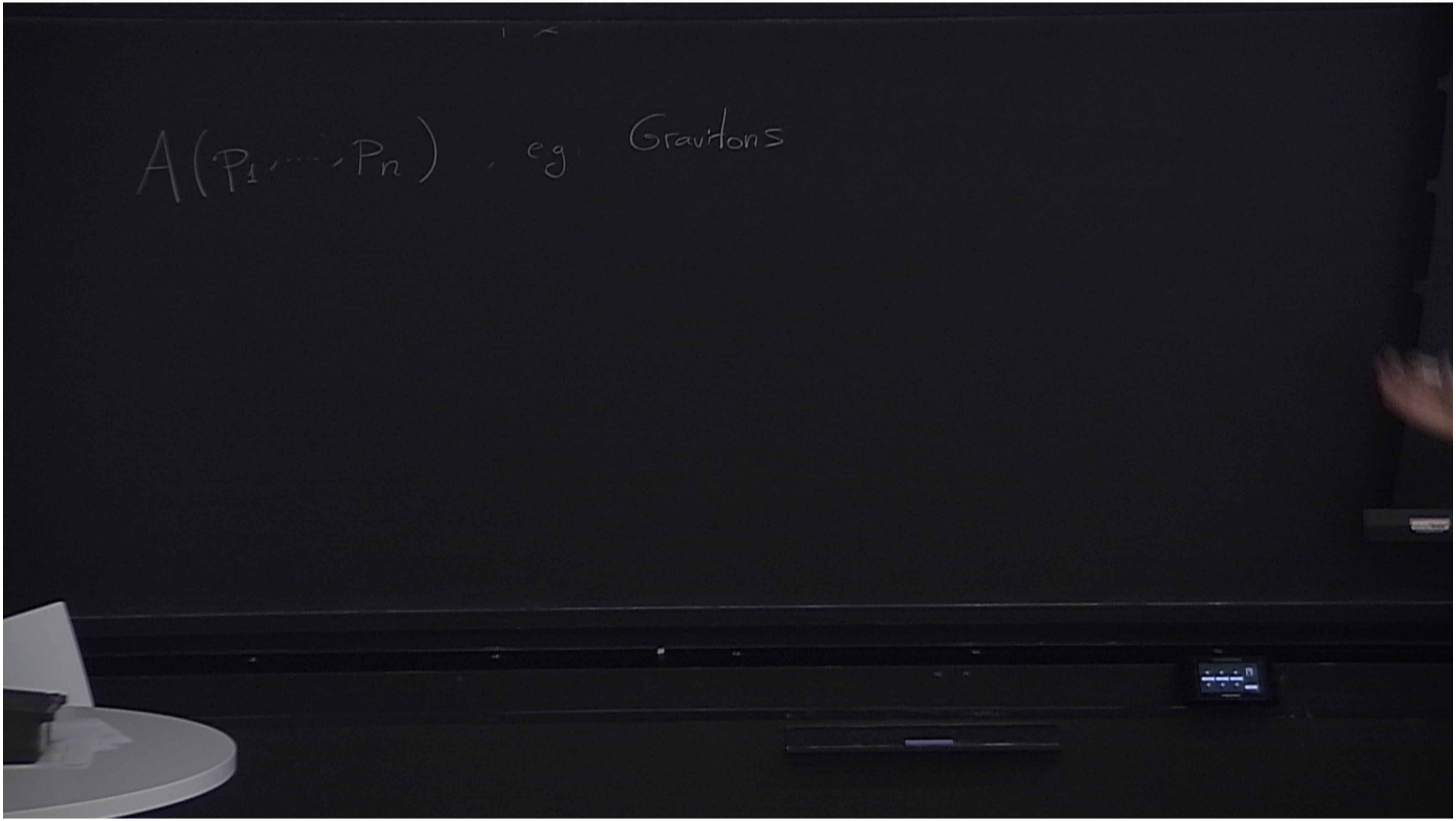
$\Sigma[4] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix};$

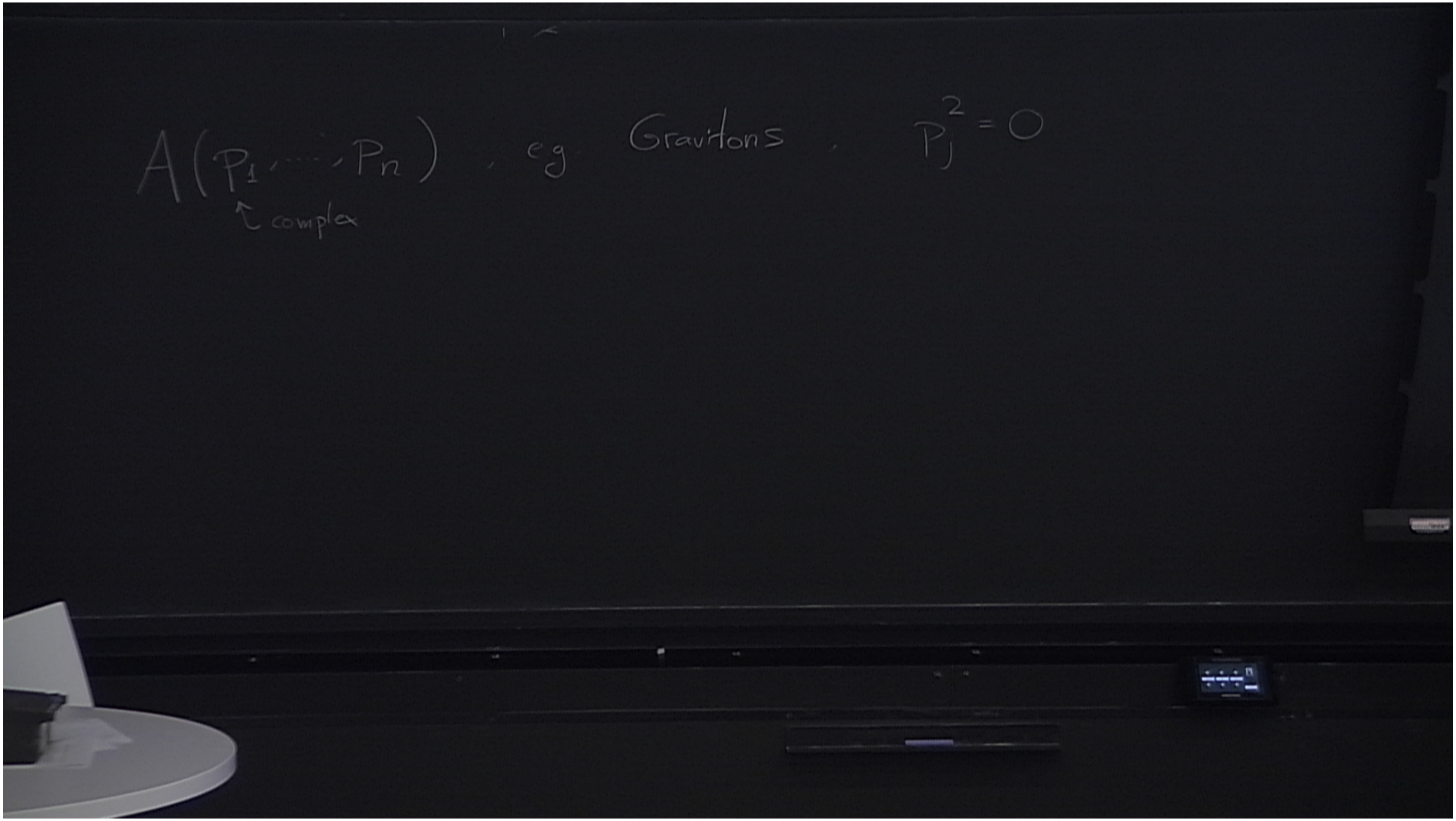
$\Sigma[3] = \begin{pmatrix} 0 & 0 & -I & 0 \\ 0 & 0 & 0 & I \\ I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \end{pmatrix};$

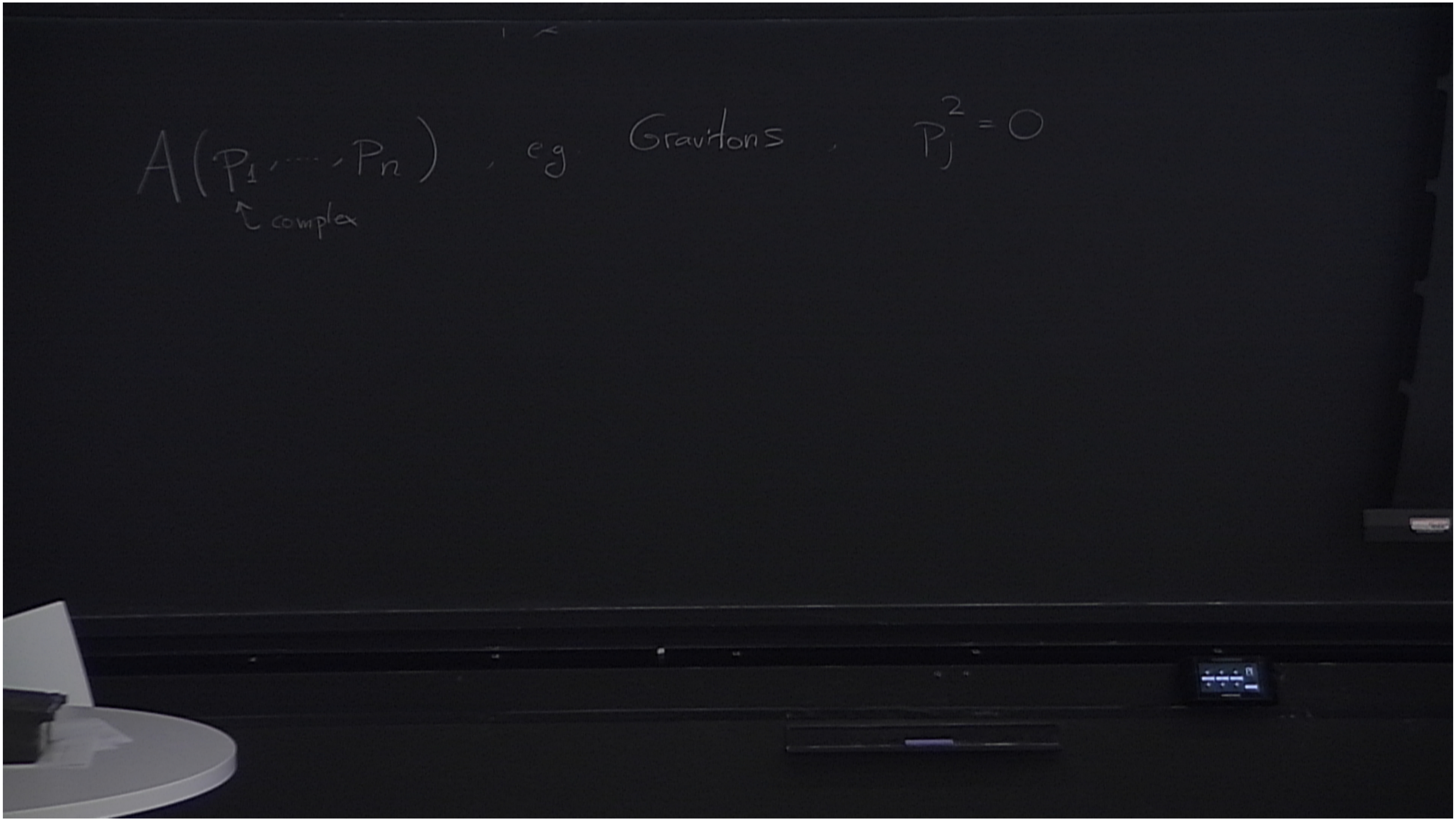
$\Sigma[2] = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix};$

$\Sigma[1] = \begin{pmatrix} 0 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \\ 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \end{pmatrix};$

200%


A(p1, ..., pn) , eg Gravitons

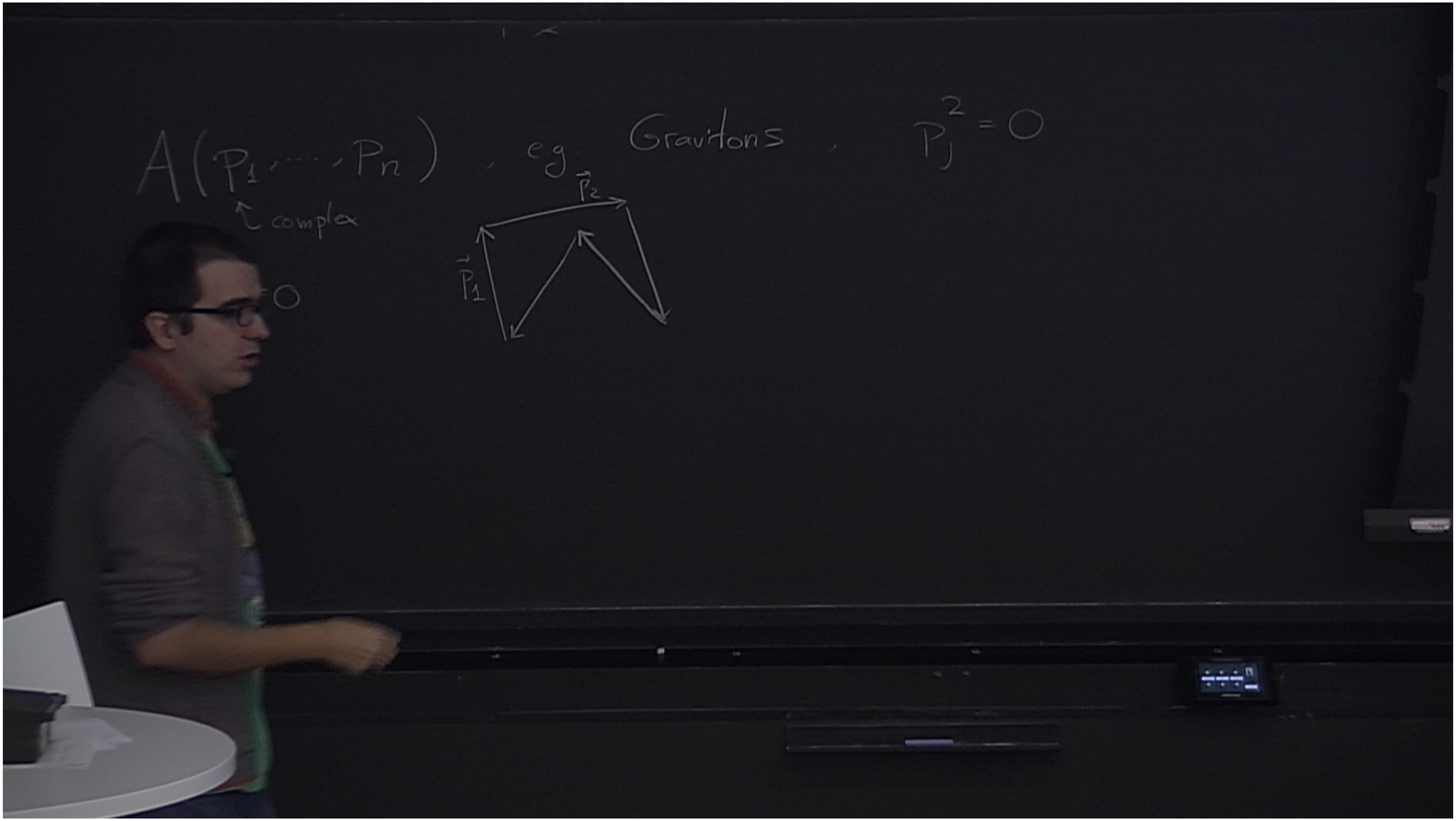

$A(p_1, \ldots, p_n)$, e.g. Gravitons, $p_j^2 = 0$
complex

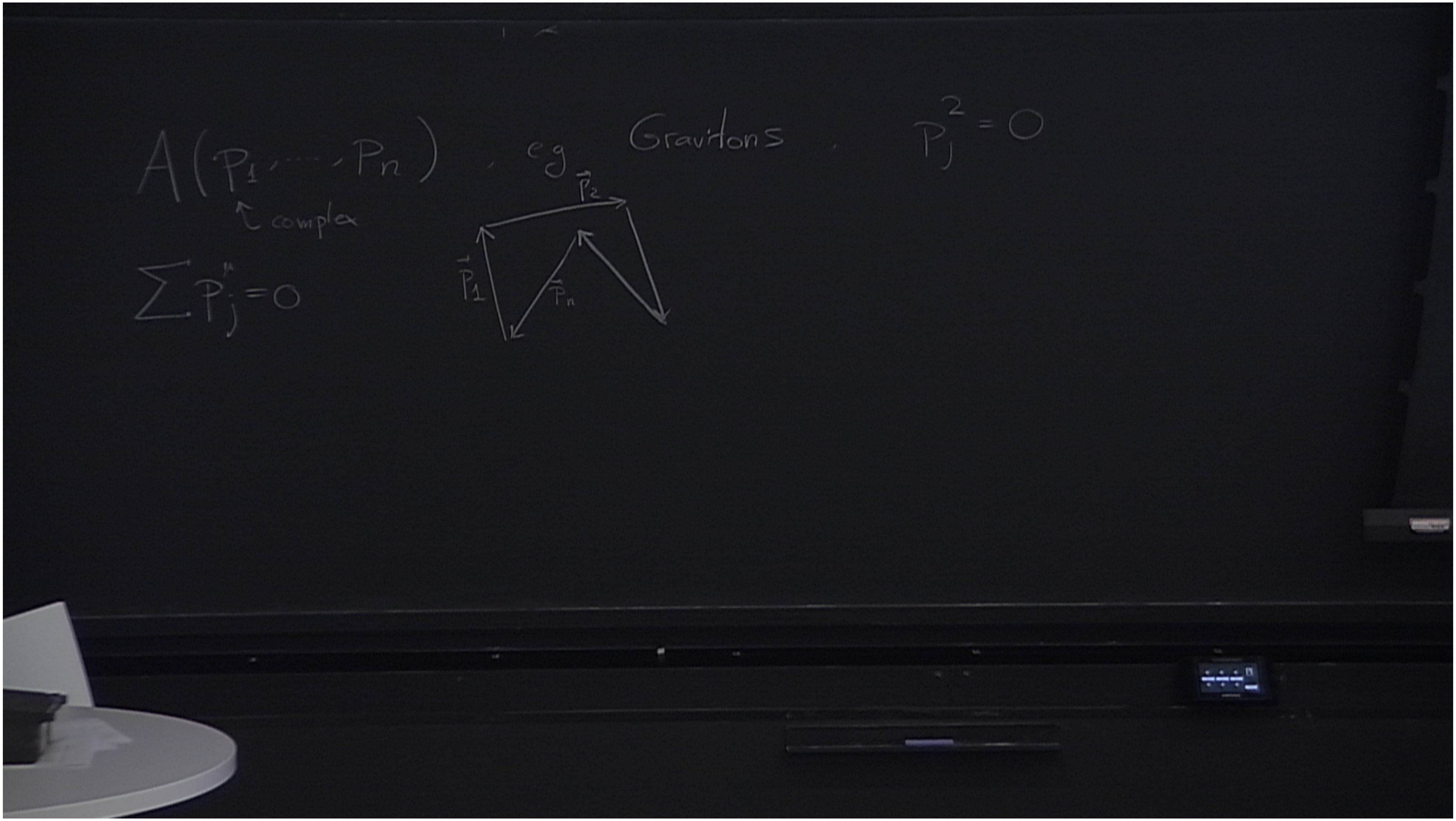

A(p1, ..., pn) , e.g. Gravitons , pj2 = 0
complex

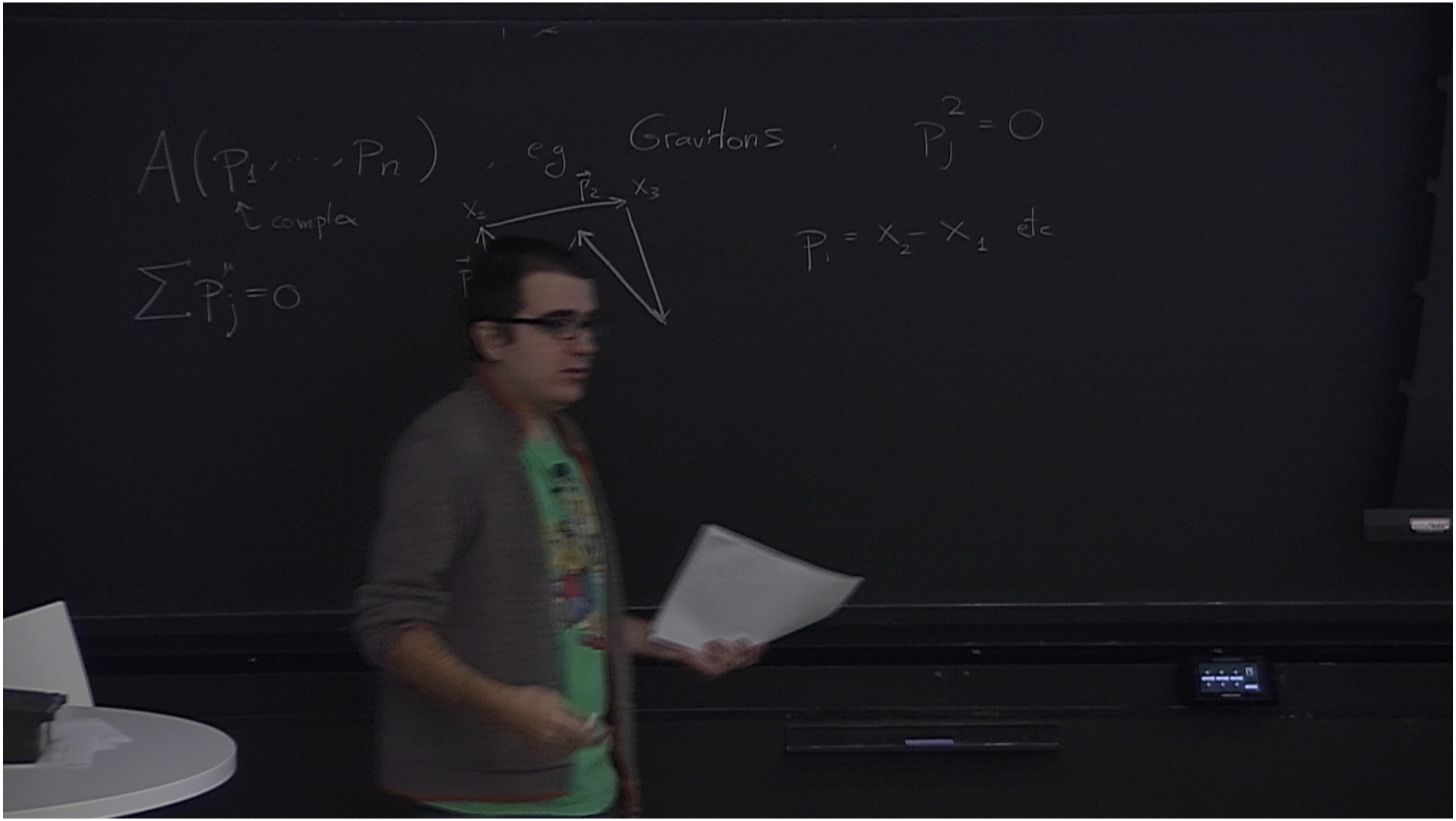
$A(p_1, \ldots, p_n)$, e.g. Gravitons , $p_j^2 = 0$

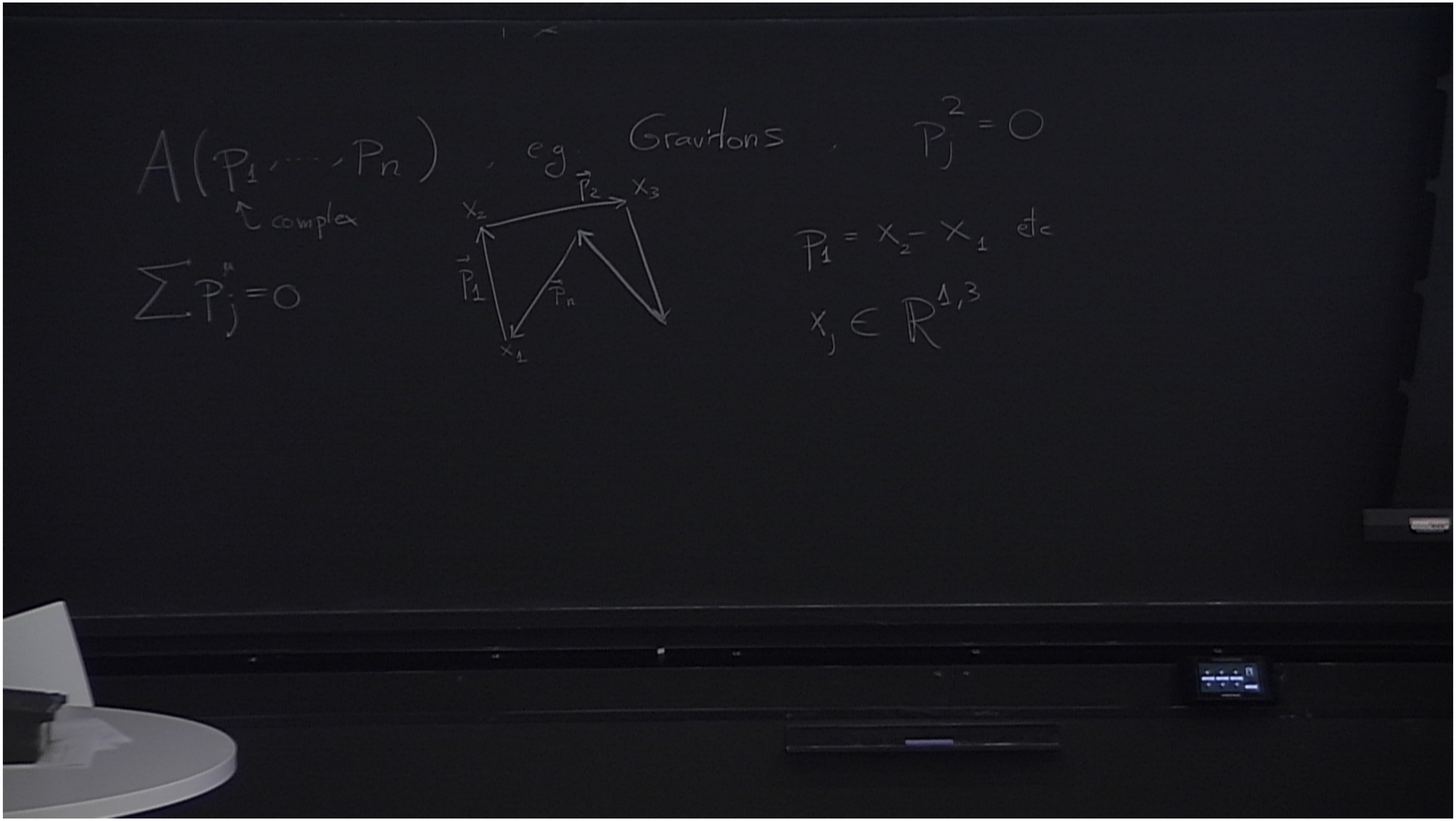
p_1 ↑ complex

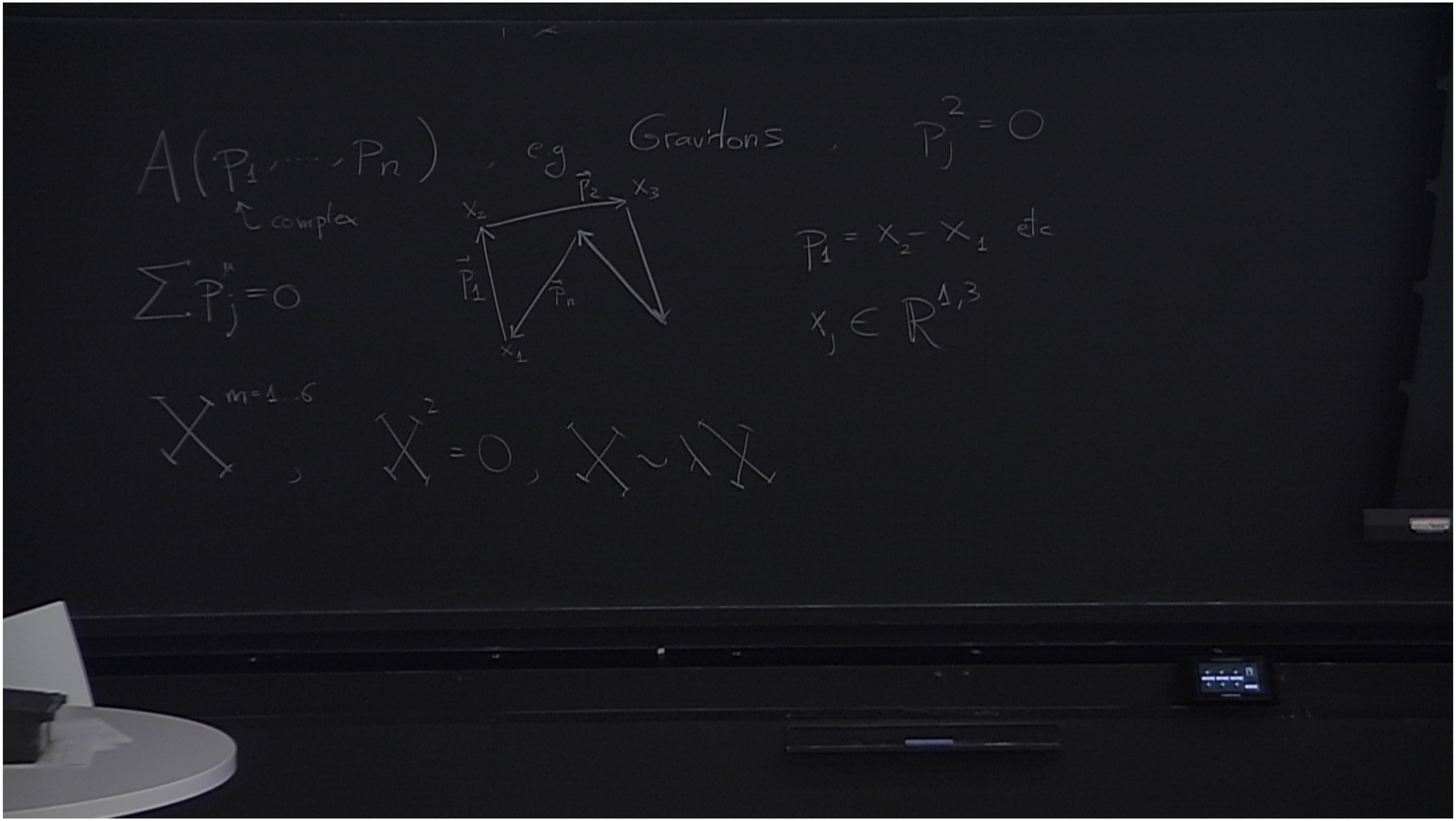
$\sum p_j^\mu = 0$


A(p1, ..., pn) , e.g. Gravitons , pj² = 0
complex
p2
p1


A(p1, ..., pn)
e.g. Gravitons
pj² = 0
complex
Σ pj^μ = 0
p1
p2
pn


$A(p_1, \dots, p_n)$, e.g. Gravitons , $p_j^2 = 0$
↑ complex
$\sum p_j^\mu = 0$
x_2
$\vec{p}_2$
x_3
$p_1 = x_2 - x_1$ etc

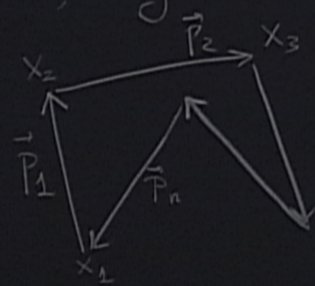

A(p1, ..., pn), e.g. Gravitons, pj^2 = 0
complex
∑ pj^μ = 0
x2
p2
x3
p1
pn
x1
p1 = x2 − x1 etc
xj ∈ R^1,3


A(p1, ..., pn) , e.g. Gravitons , pj^2 = 0
complex
Σ pj^μ = 0
x2 p2 x3
p1 pn
x1
p1 = x2 − x1 etc
xj ∈ R^1,3
X^m=1...6 , X^2 = 0 , X ~ λX

$A(p_1, \ldots, p_n)$, e.g. Gravitons, $p_j^2 = 0$

↑ complex

$\sum p_j^\mu = 0$

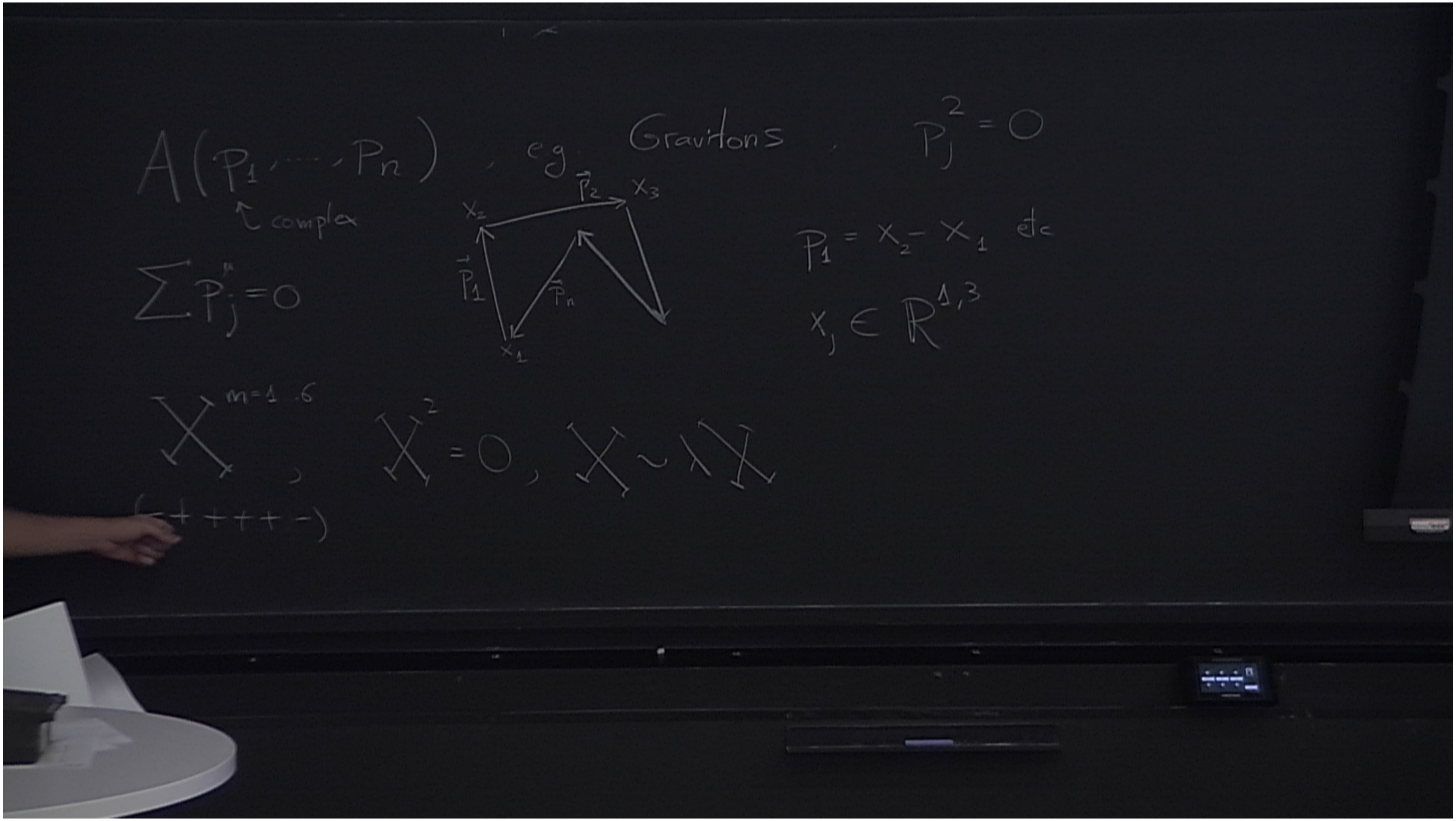


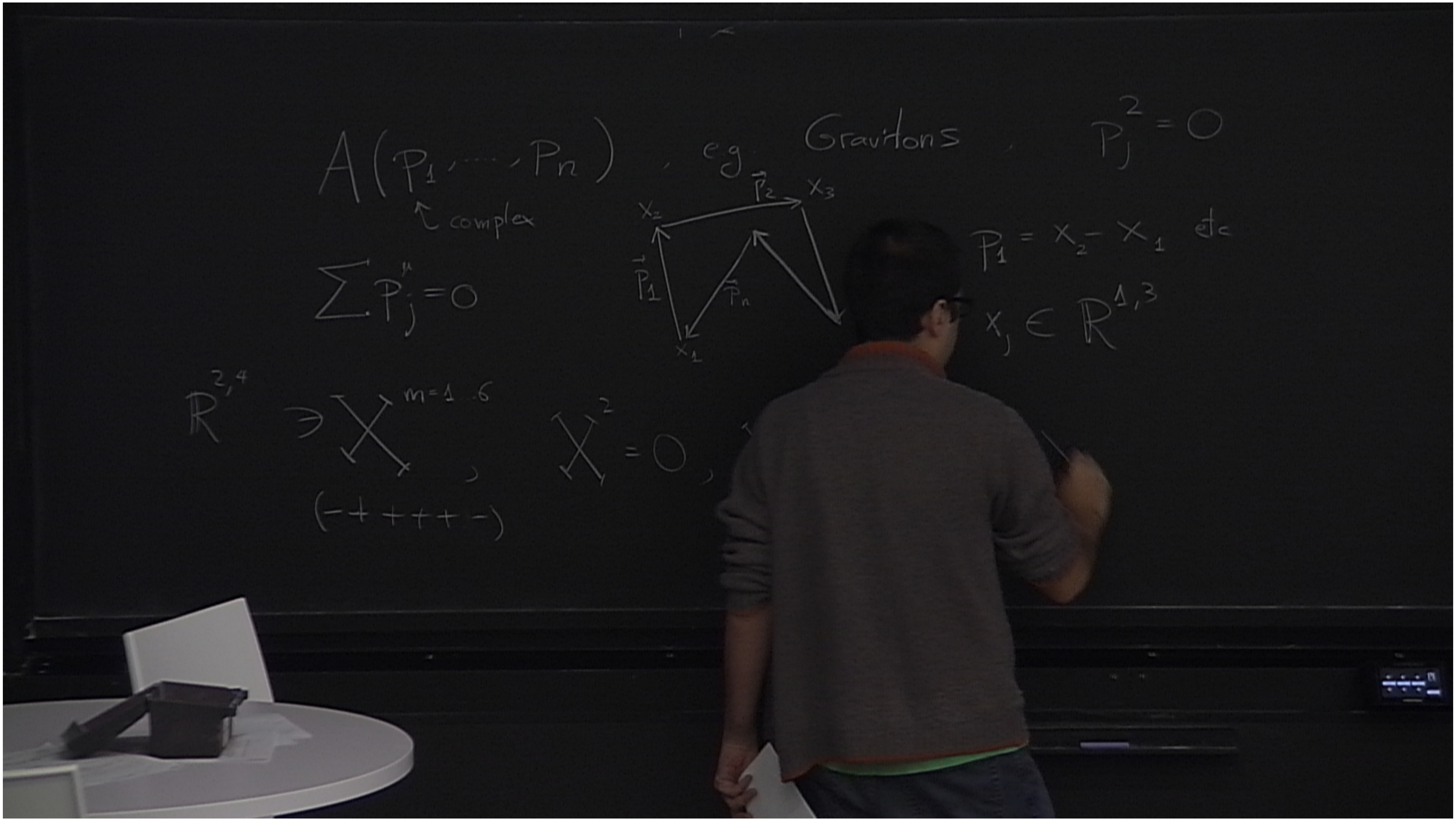
$p_1 = x_2 - x_1$ etc

$x_j \in \mathbb{R}^{1,3}$

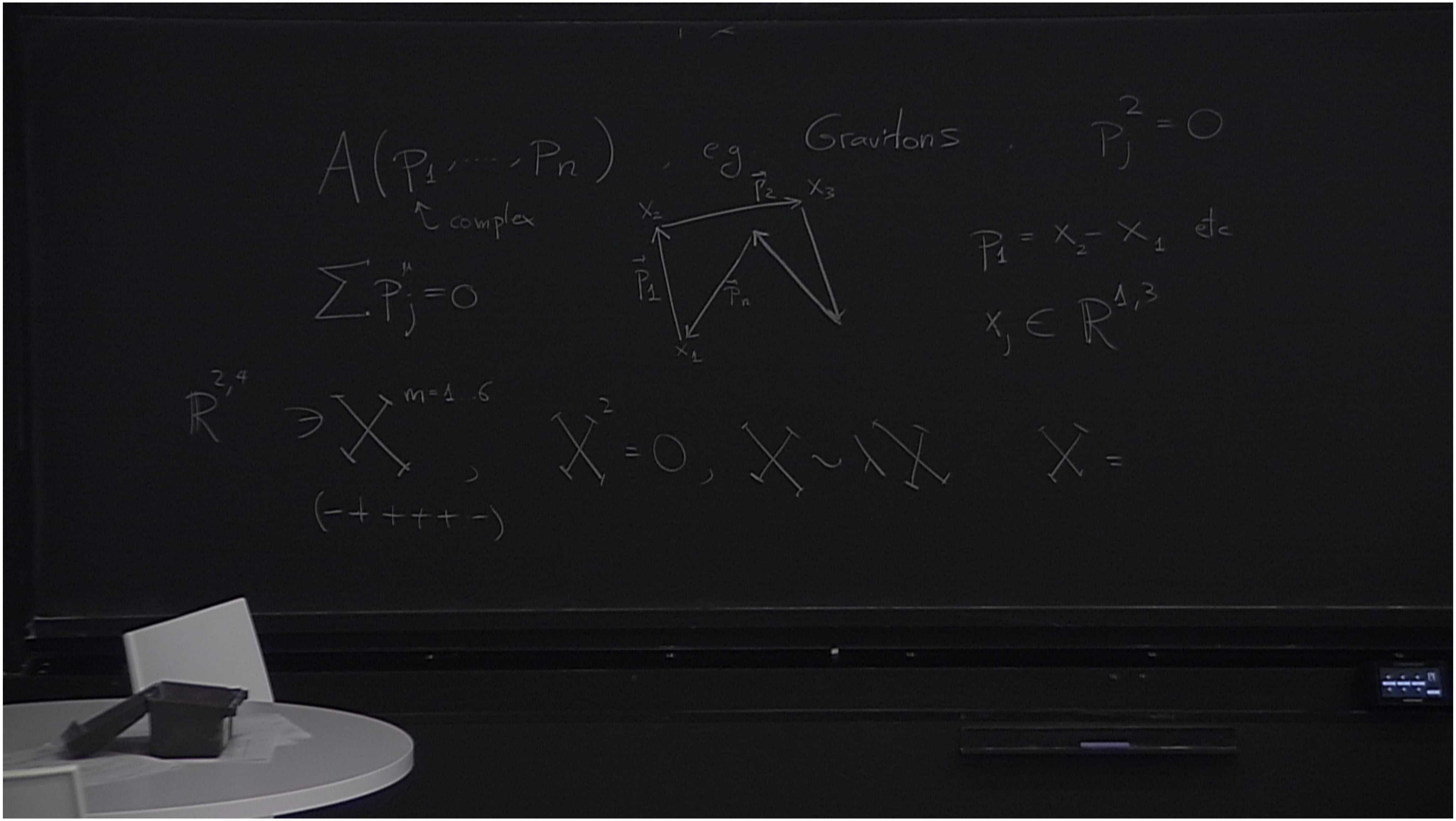
$X^{m=1..6}$, $(--++++)$

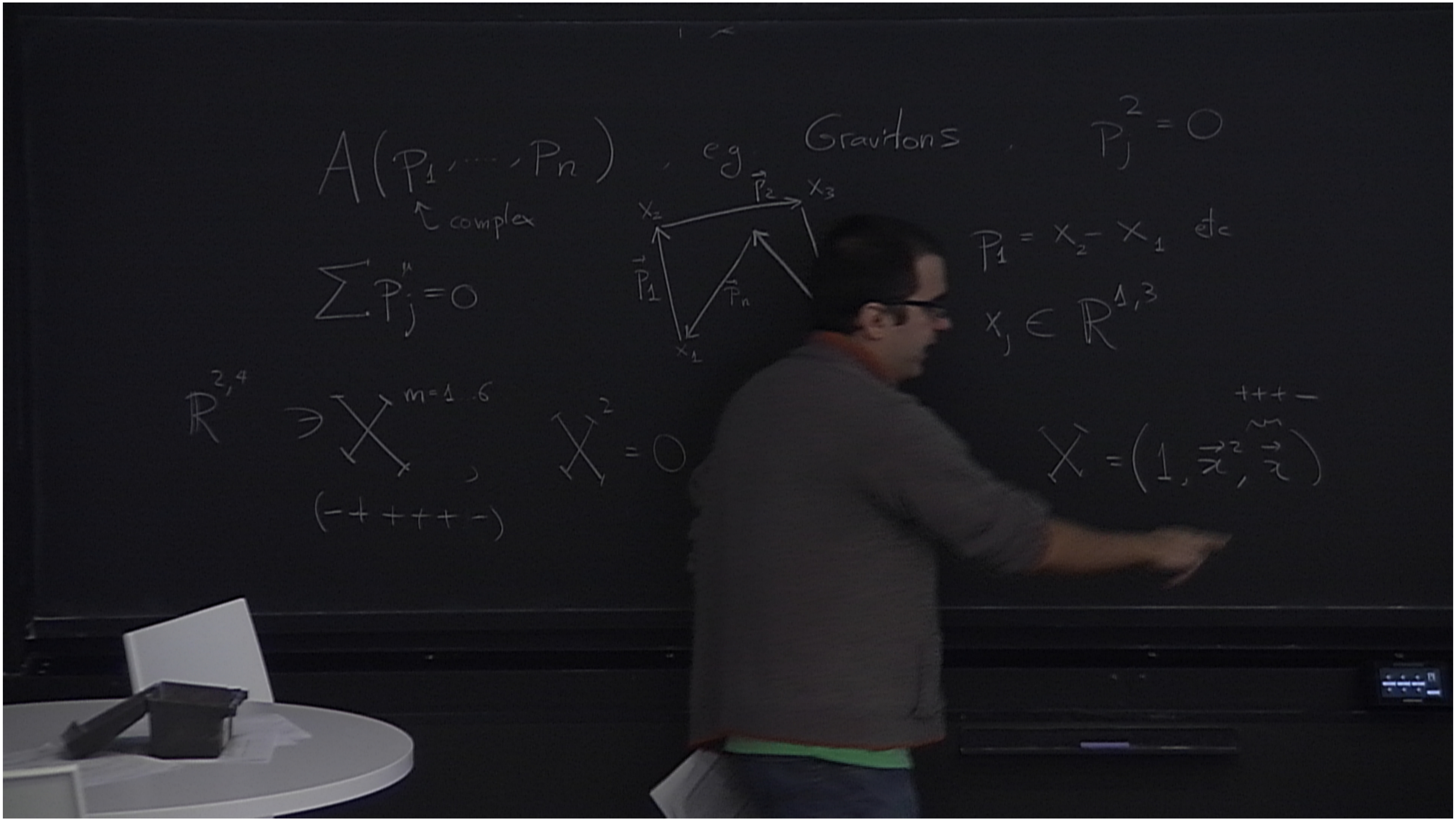
$X^2 = 0$, $X \sim \lambda X$

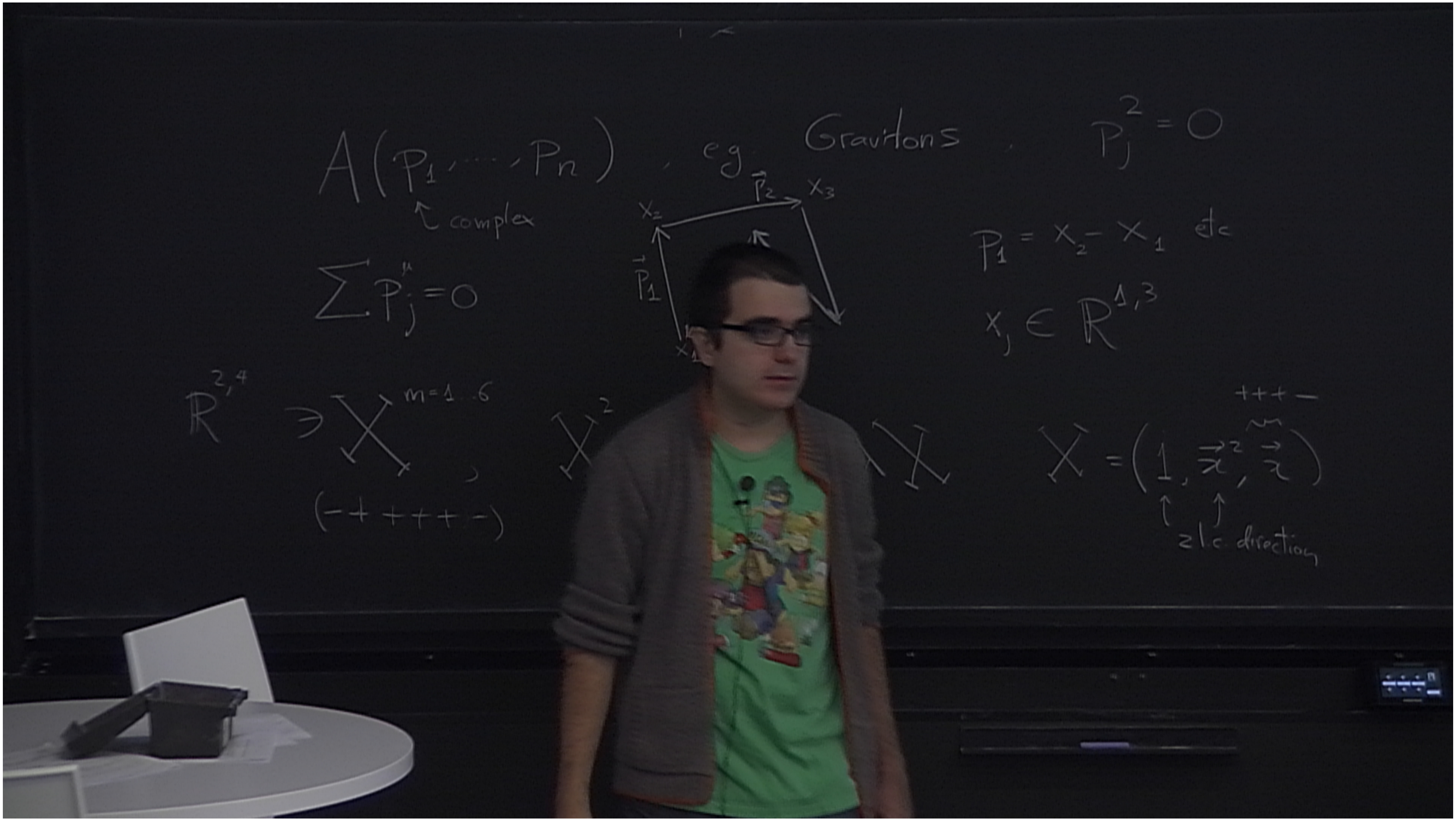

A(P1, ..., Pn), e.g. Gravitons, Pj2 = 0
complex
Σ Pj = 0
x2
P2
x3
P1
Pn
x1
P1 = x2 − x1 etc
xj ∈ R1,3
X m=1..6
X2 = 0, X ~ λX
(−++++−)

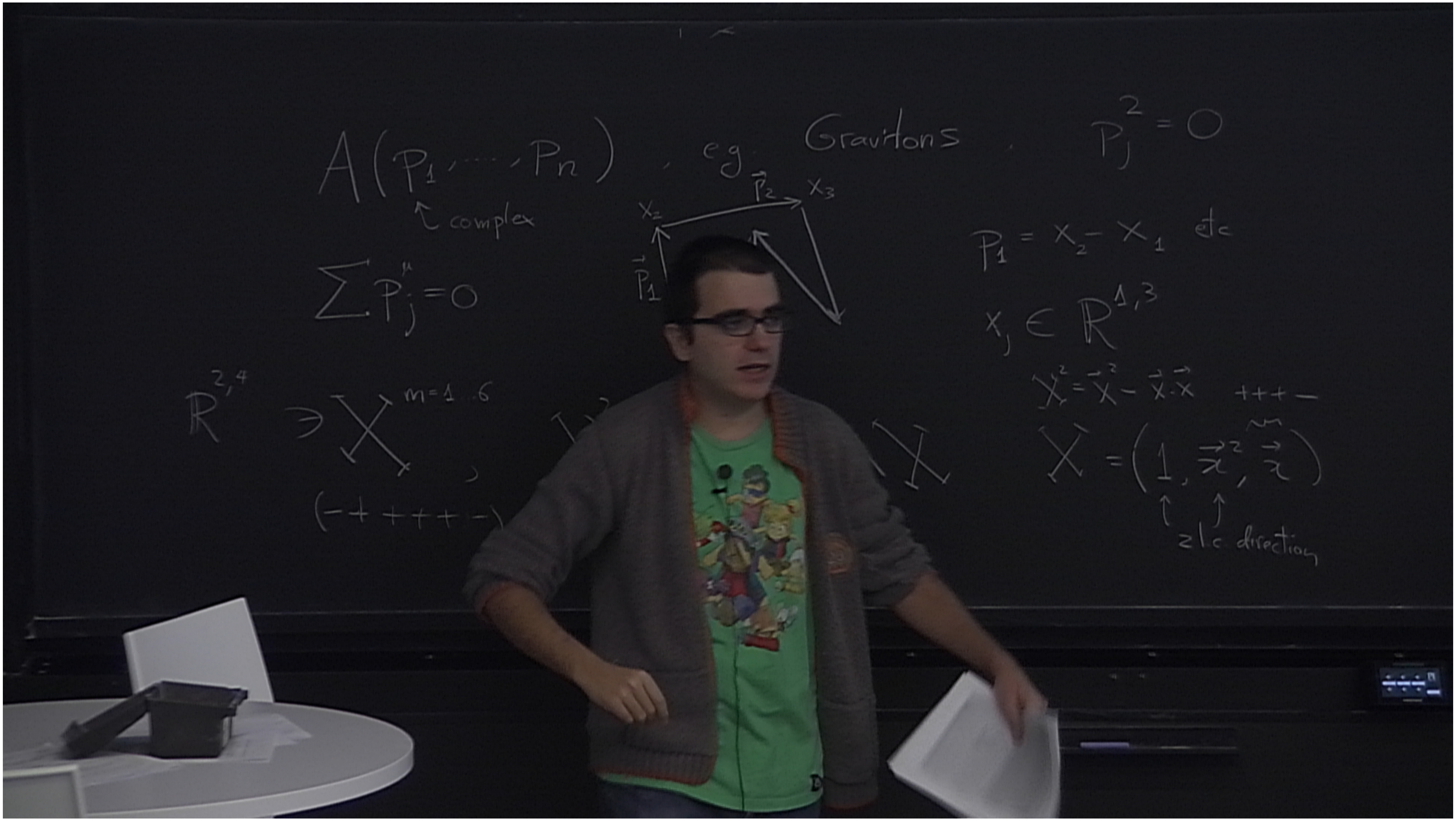


$A(p_1, \dots, p_n)$, e.g. Gravitons, $p_j^2 = 0$
complex
$\sum p_j^\mu = 0$
x_1 x_2 x_3 $\vec{p}_1$ $\vec{p}_2$ $\vec{p}_n$
$p_1 = x_2 - x_1$ etc
$x_j \in \mathbb{R}^{1,3}$
$\mathbb{R}^{2,4} \ni X^{m=1..6}$, $X^2 = 0$,
$(-++++-)$

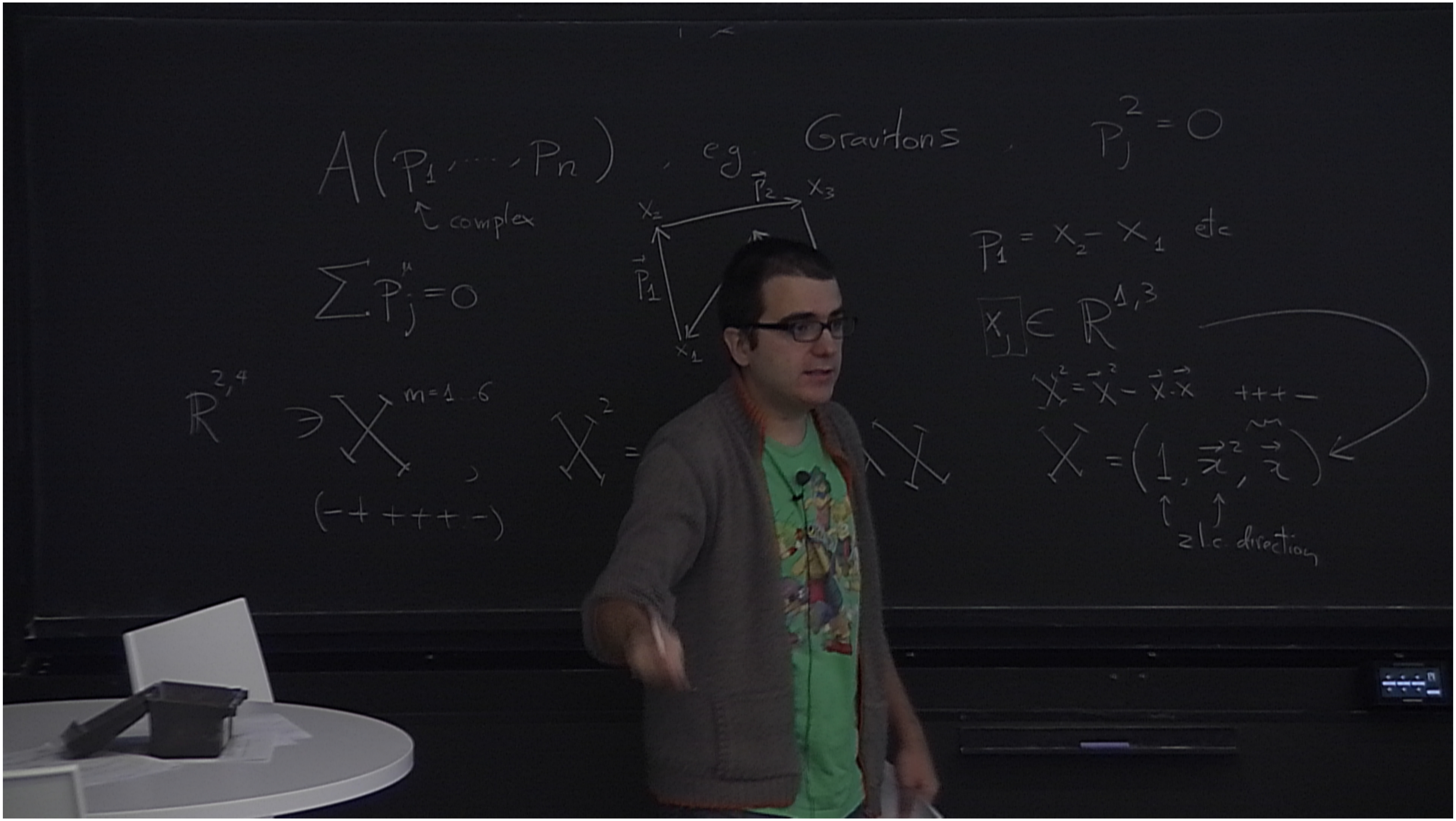

$A(p_1, \ldots, p_n)$, e.g. Gravitons, $p_j^2 = 0$
complex
$\sum p_j^\mu = 0$
x_1, x_2, x_3, $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_n$
$p_1 = x_2 - x_1$ etc
$x_j \in \mathbb{R}^{1,3}$
$\mathbb{R}^{2,4} \ni X^{m}$ $m=1..6$, $X^2 = 0$, $X \sim \lambda X$ $X =$
$(-++++-)$

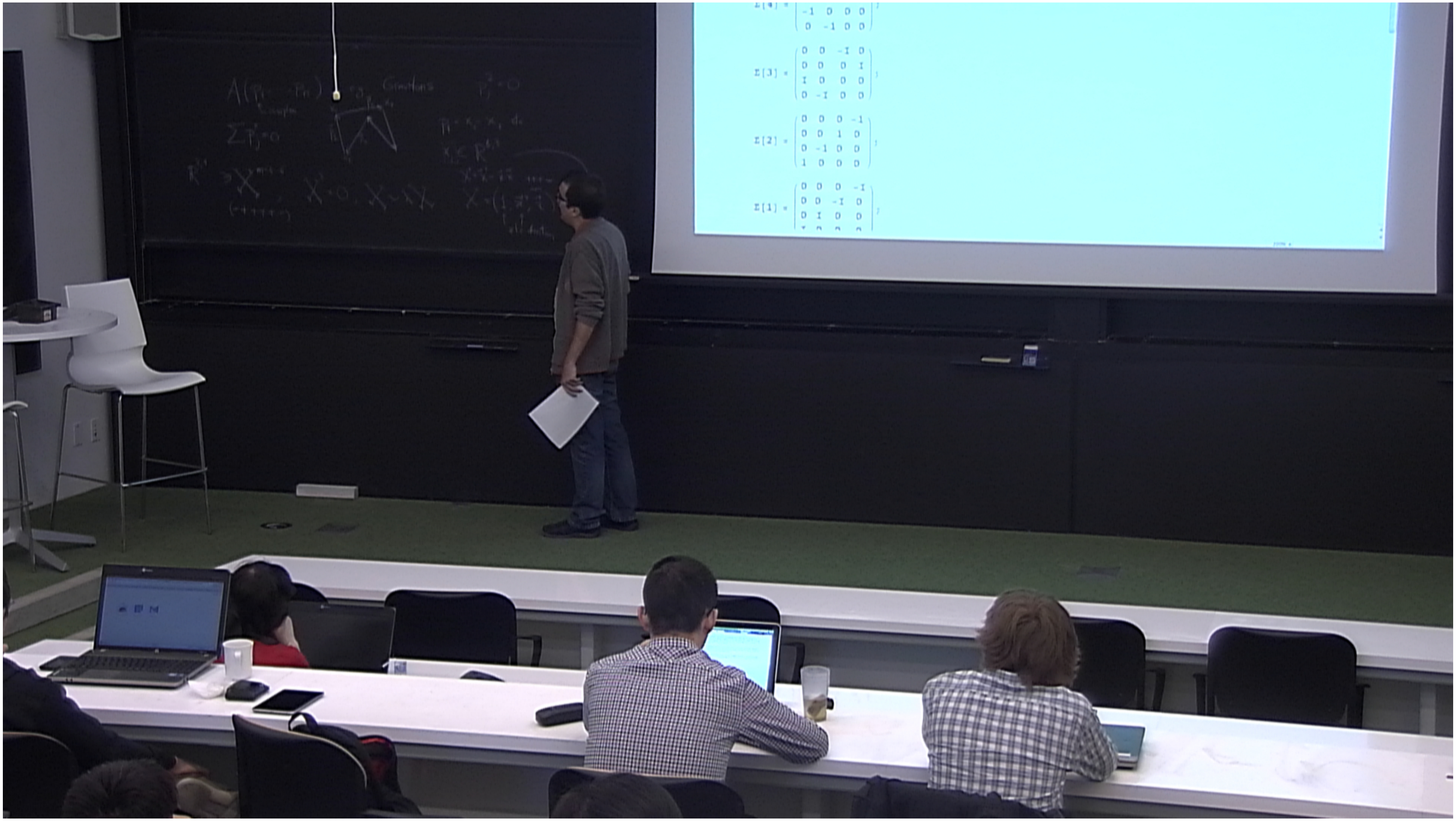

A(P1, ..., Pn)
↑ complex
eg. Gravitons
Pj^2 = 0
ΣPj^μ = 0
P1 = X2 − X1 etc
xj ∈ R^1,3
R^2,4 ∋ X^m, m=1..6
(−+++++−)
X^2 = 0
X = (1, x^2, x)
+++−

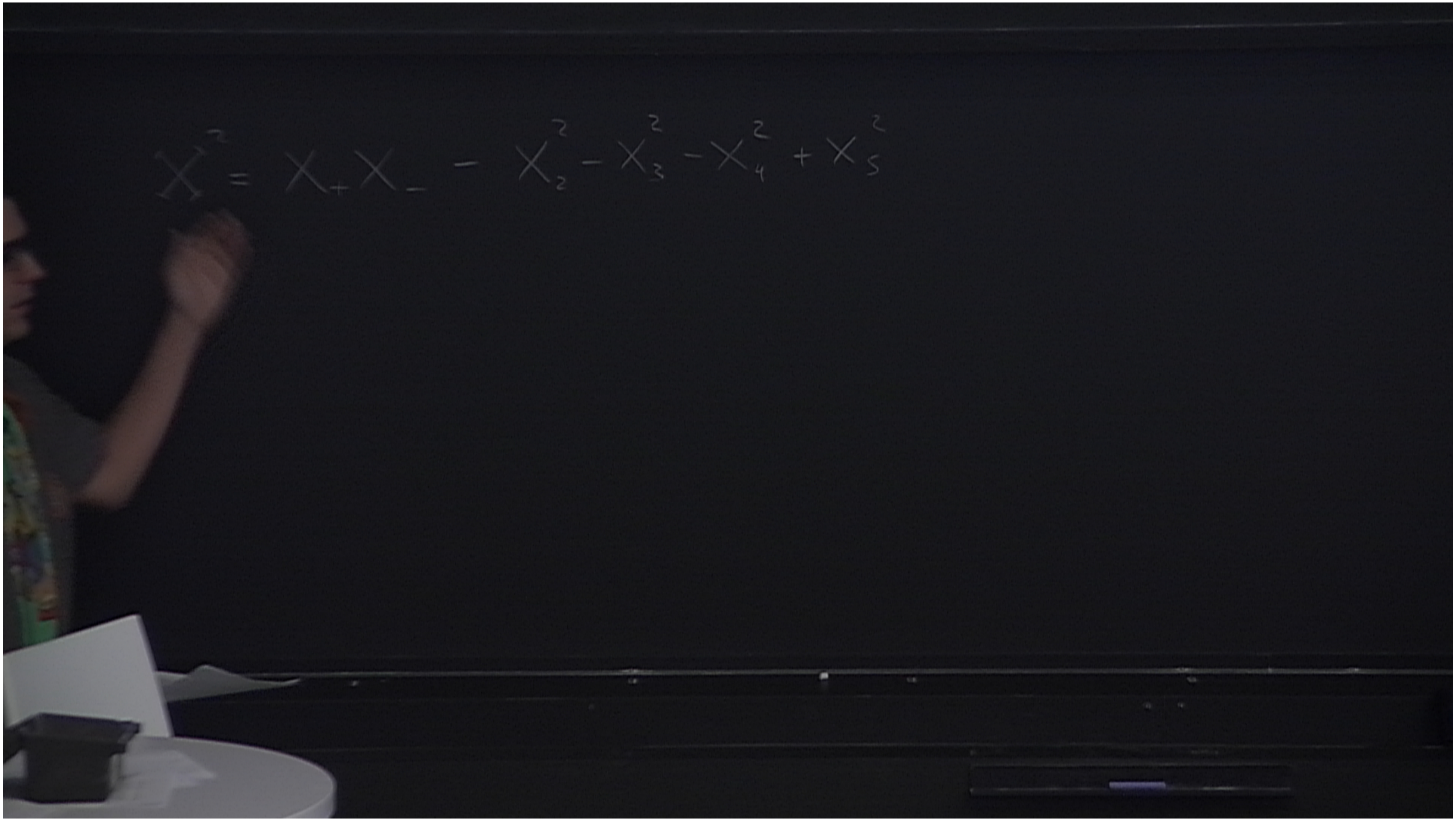

A(p1, ..., pn) , e.g. Gravitons , pj² = 0
complex
Σ pj^μ = 0
p1 = x2 − x1 etc
xj ∈ R^{1,3}
R^{2,4} ∋ X^{m=1..6}
(−++++−)
+++−
X = (1, x², x)
z|c direction

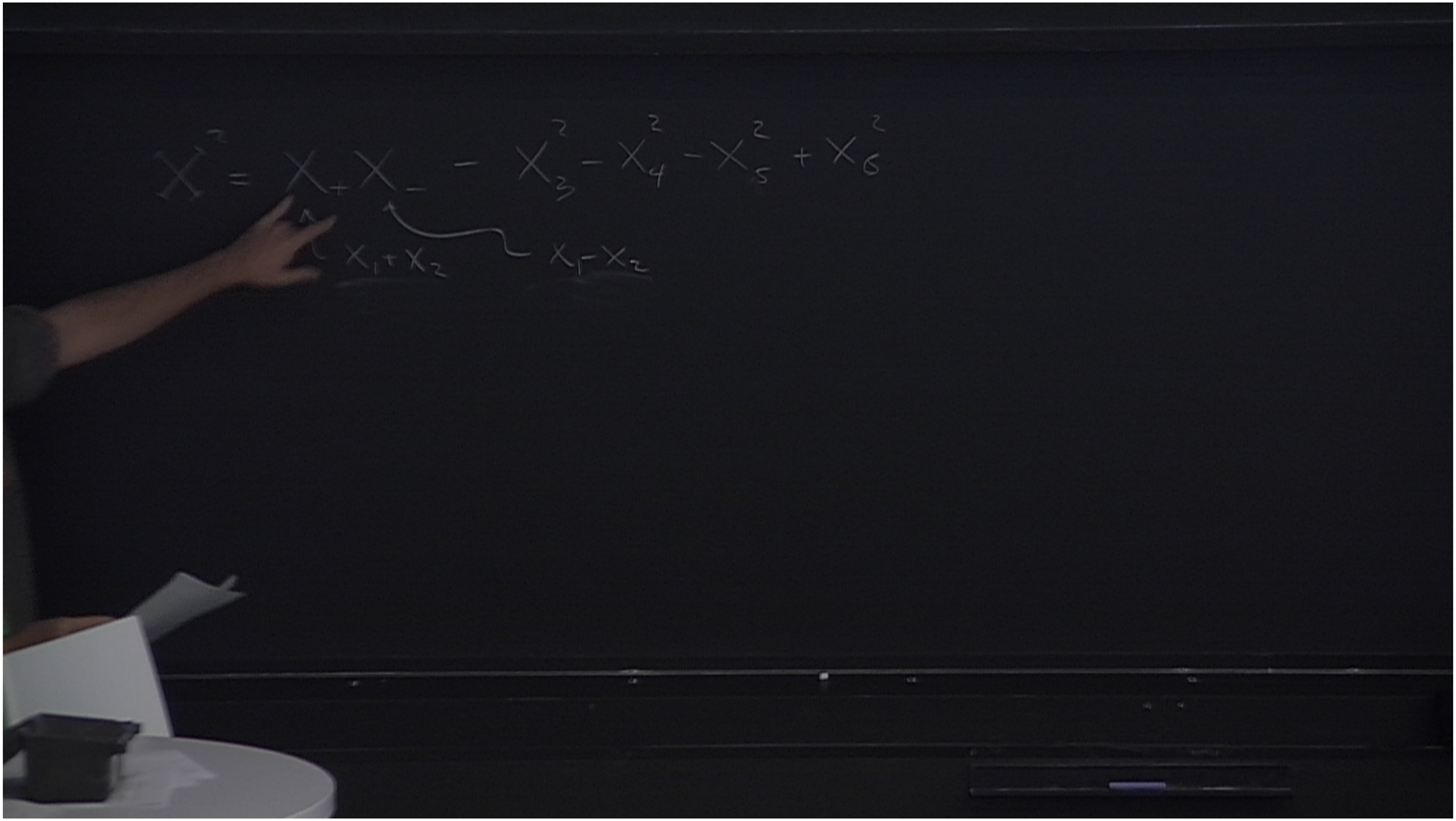


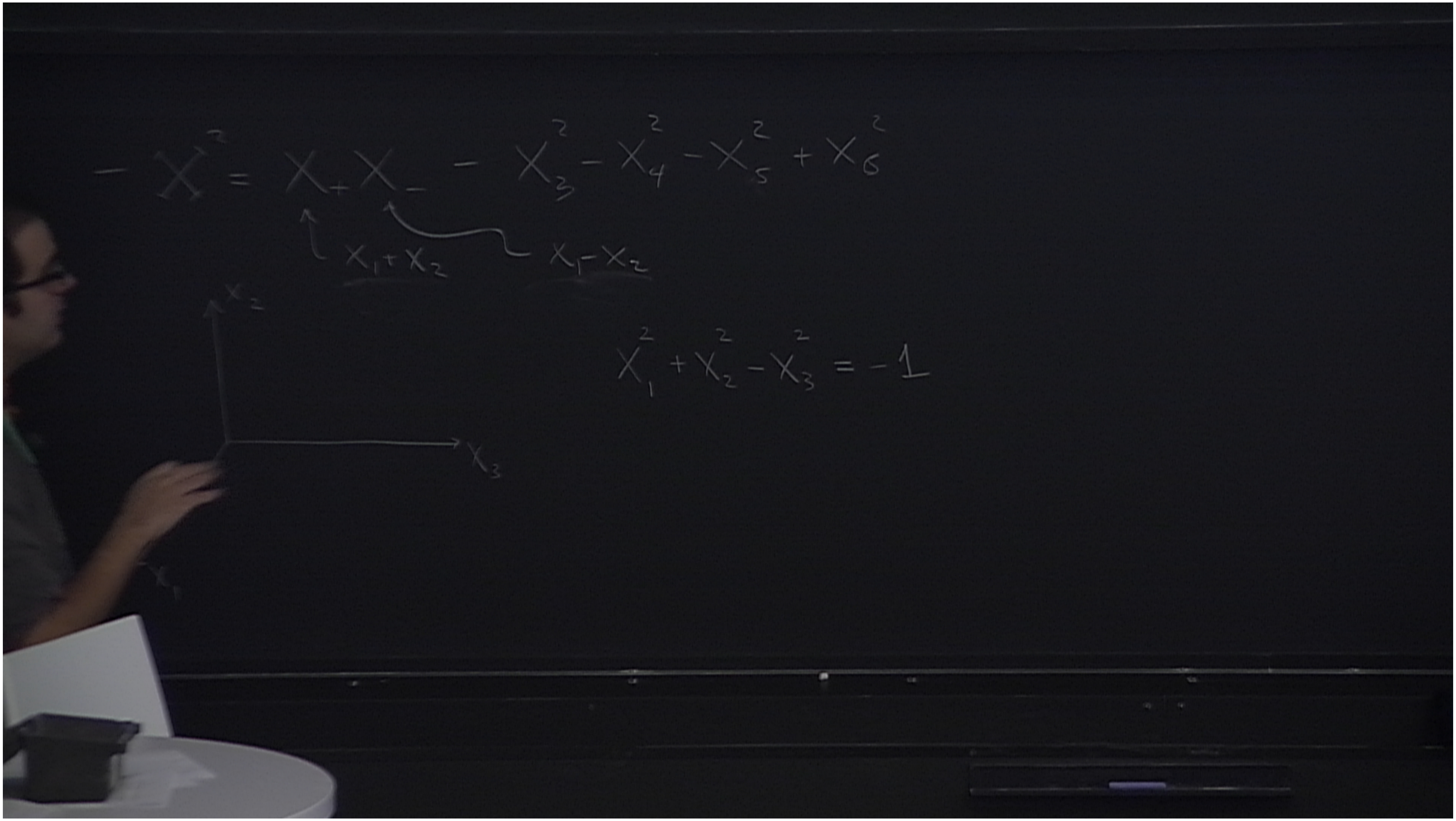
A(p1, ..., pn) , e.g. Gravitons , pj^2 = 0
↑ complex
Σ pj^μ = 0
p2 x3
x2
p1
p1 = x2 − x1 etc
xj ∈ R^1,3
X^2 = X^2 − X·X +++ −
X = (1, x^2, x)
z l.c. direction
R^2,4 ∋ X^m=1..6
(− + + + + −)

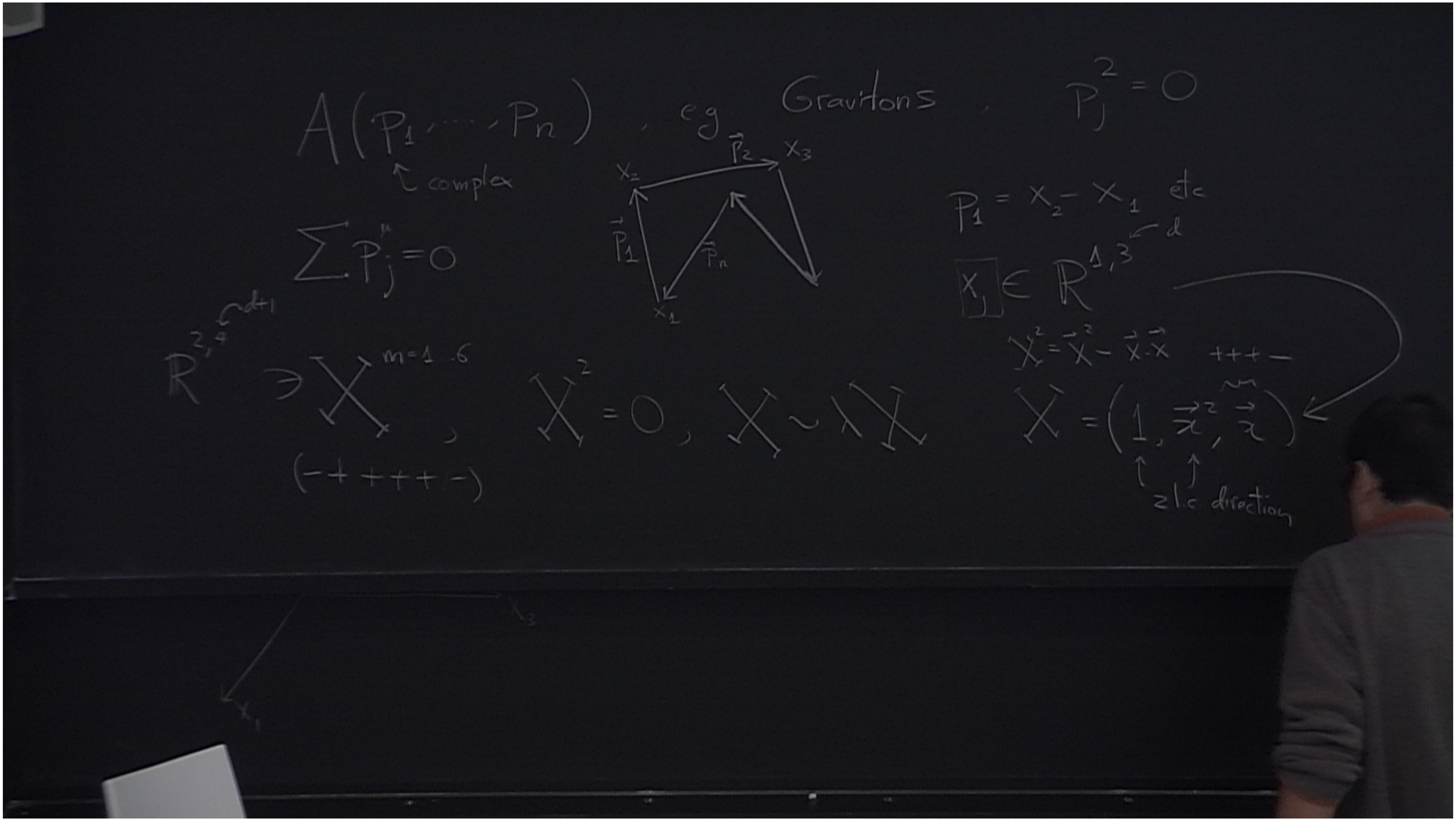

A(p1, ..., pn) , e.g. Gravitons , pj^2 = 0
complex
Σ pj^μ = 0
x2 p2 x3
p1
x1
p1 = x2 − x1 etc
xj ∈ R^{1,3}
X^2 = X^2 − x·x +++−
X = (1, x^2, x)
z l.c. direction
R^{2,4} ∋ X^{m=1..6}
(−+ +++ −)

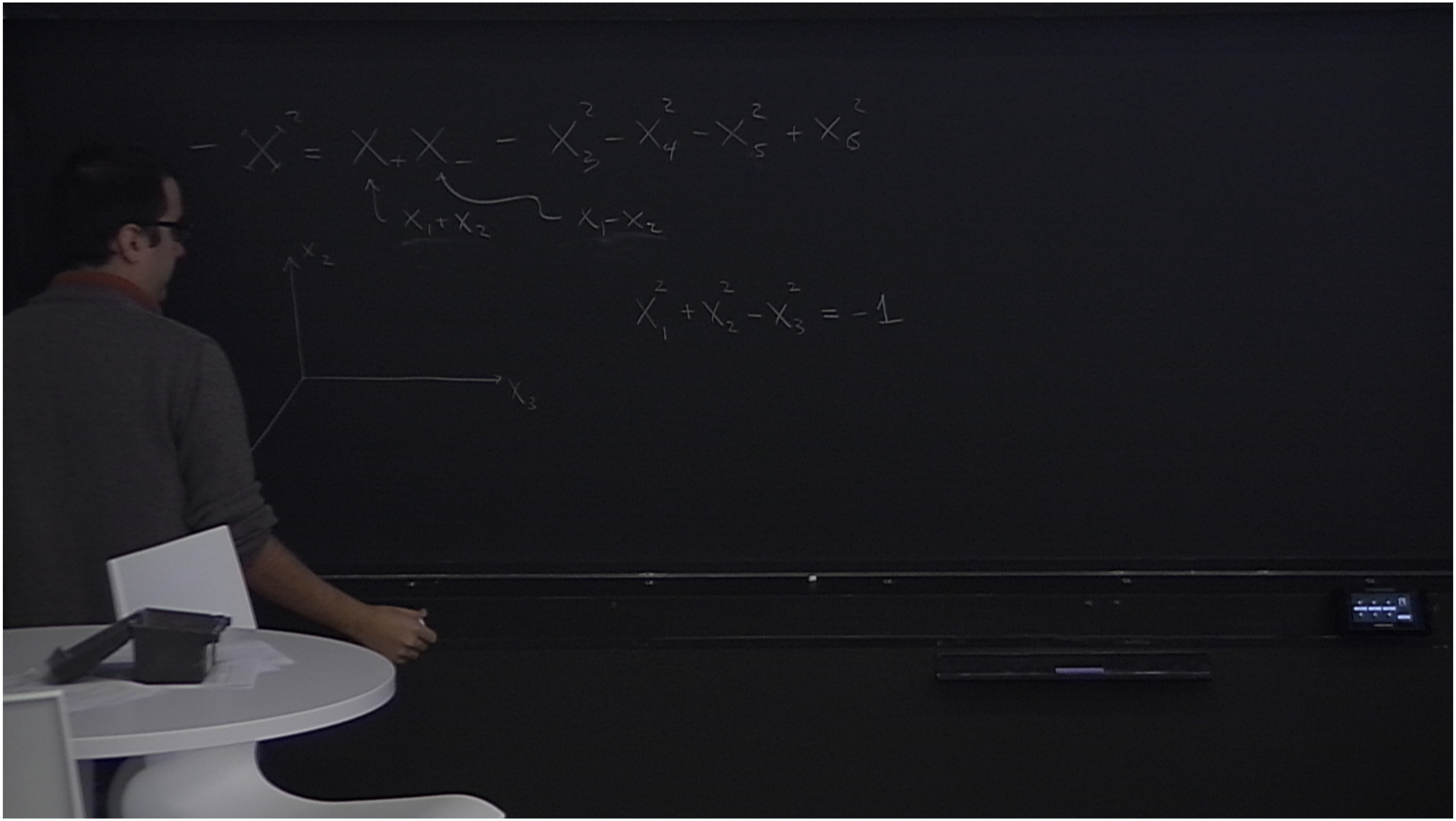

-1 0 0 0
0 -1 0 0
Σ[3] =
0 0 -I 0
0 0 0 I
I 0 0 0
0 -I 0 0
Σ[2] =
0 0 0 -1
0 0 1 0
0 -1 0 0
1 0 0 0
Σ[1] =
0 0 0 -I
0 0 -I 0
0 I 0 0

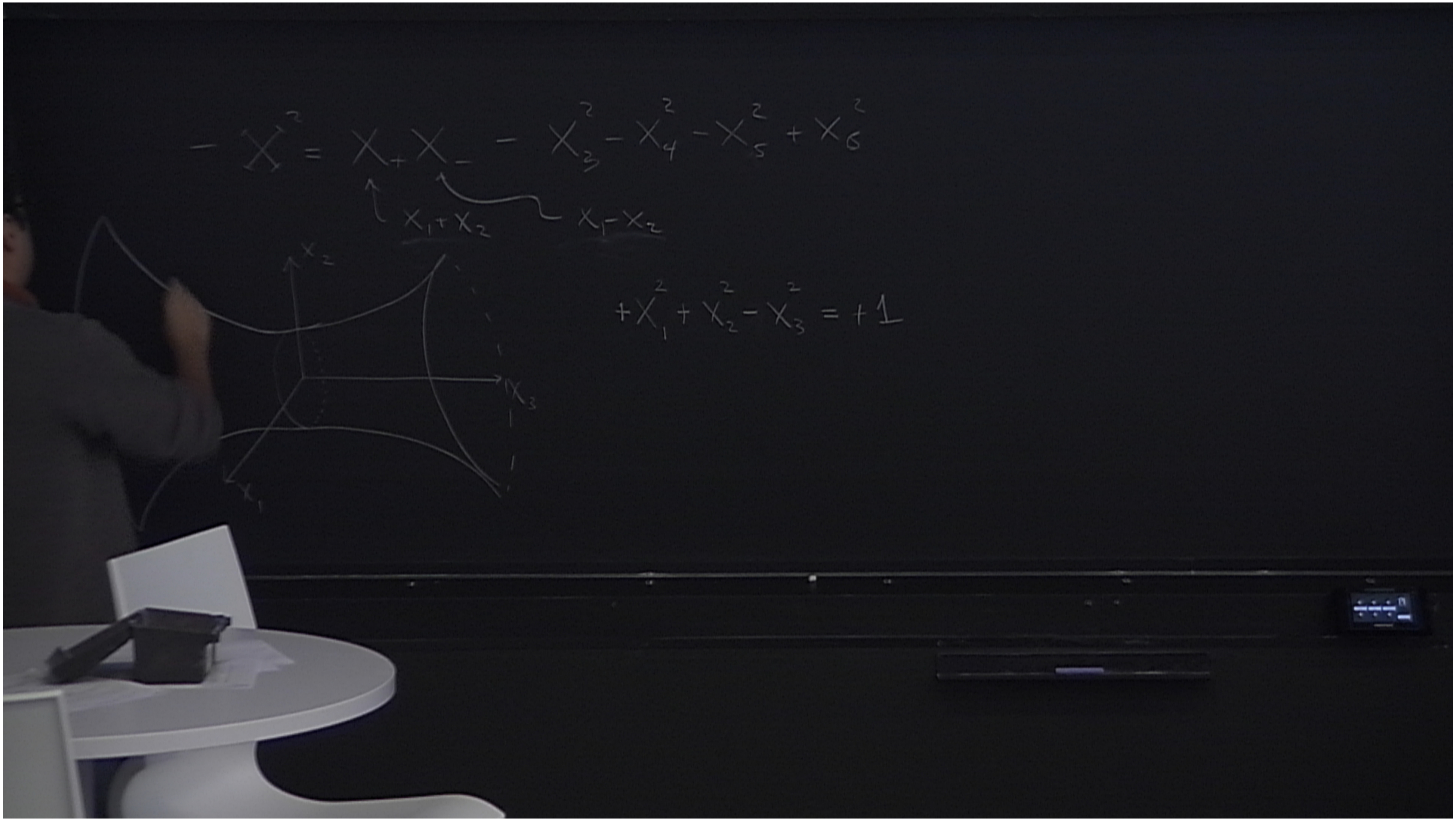

$X^2 = X_+ X_- - X_2^2 - X_3^2 - X_4^2 + X_5^2$

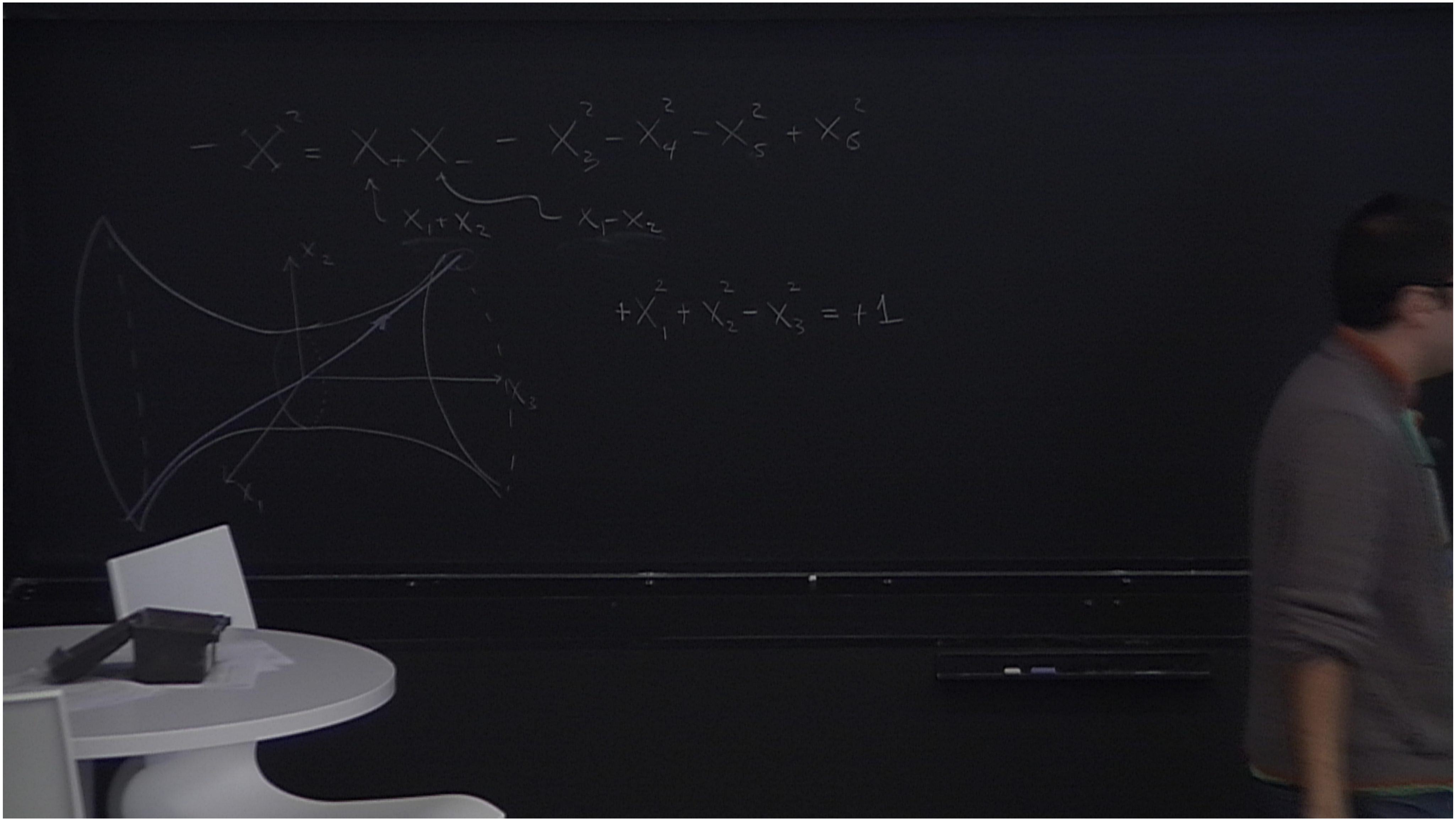

$X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2$
$X_1 + X_2$
$X_1 - X_2$

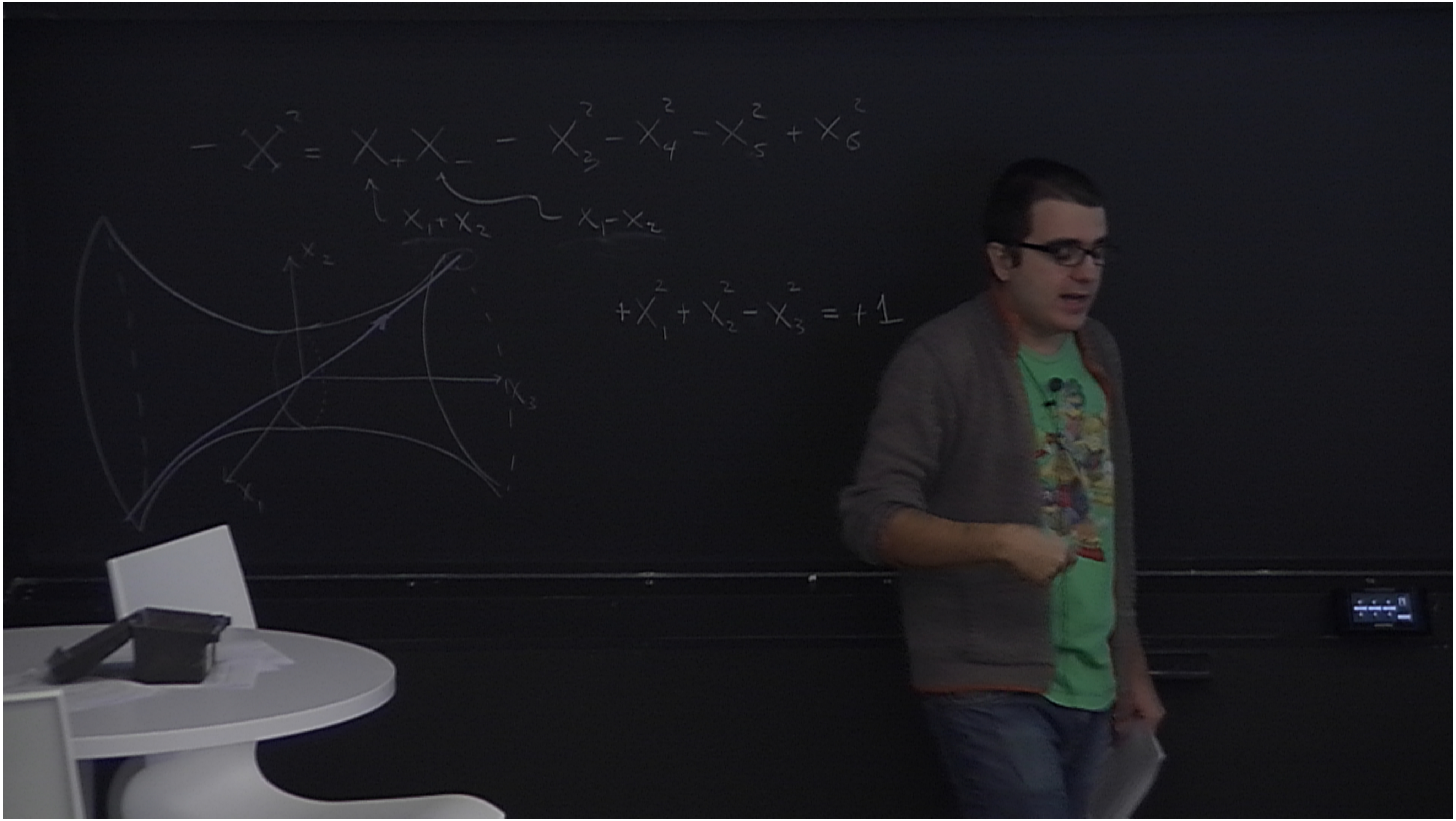

- X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2
X_1 + X_2
X_1 - X_2
X_1^2 + X_2^2 - X_3^2 = -1
X_2
X_3
X_1

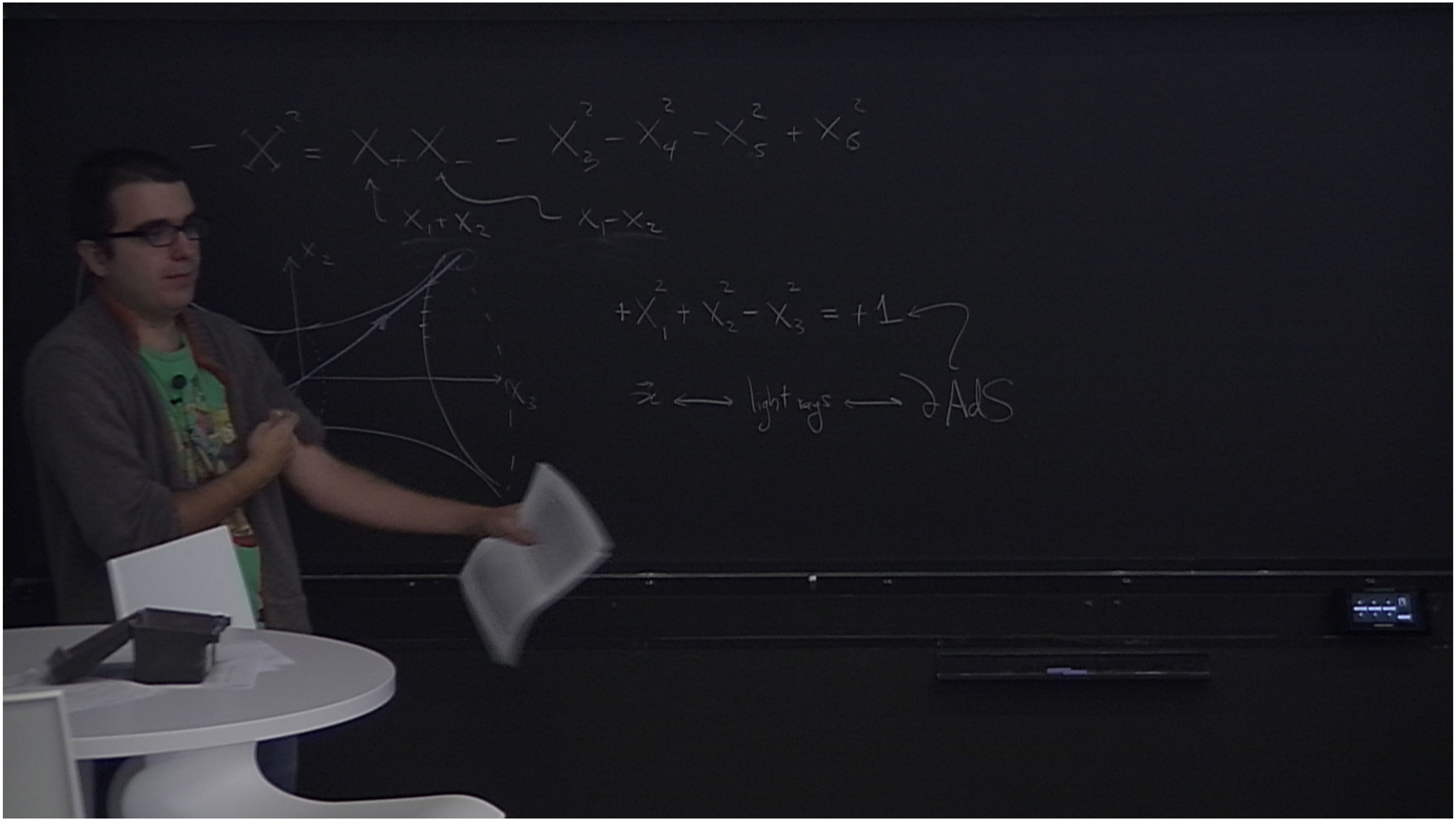

A(p1, ..., pn), e.g. Gravitons, pj^2 = 0
complex
Σ pj^μ = 0
x1 x2 x3
p1 p2 pn
p1 = x2 − x1 etc
xj ∈ R^1,3
d
d+1
R^2,4 ∋ X^m=1..6, X^2 = 0, X ~ λX
(−+++++−)
+++−
X = (1, x^2, x)
l.c. direction

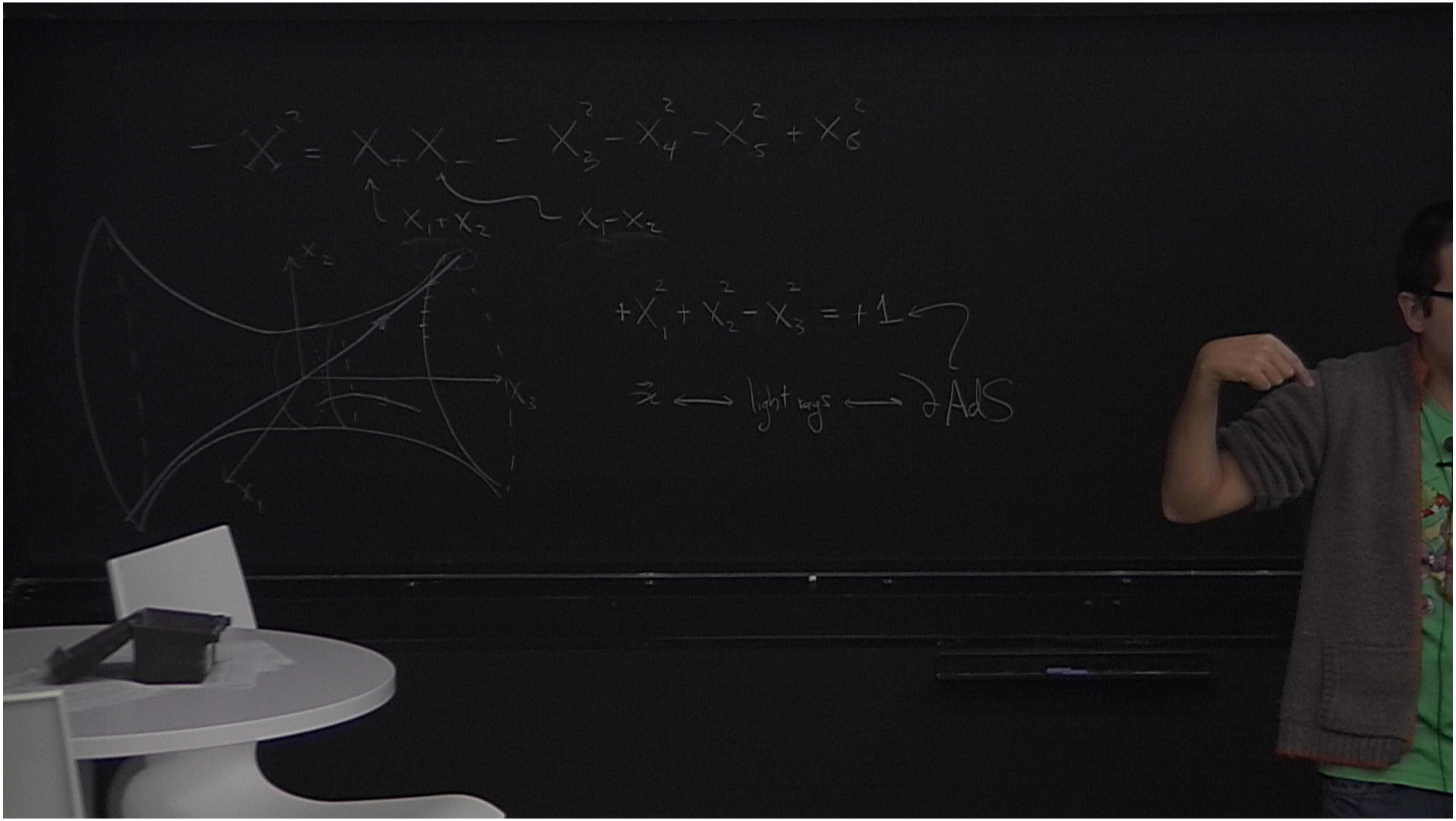

$-X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2$
$X_1 + X_2$
$X_1 - X_2$
X_2
X_3
$X_1^2 + X_2^2 - X_3^2 = -1$

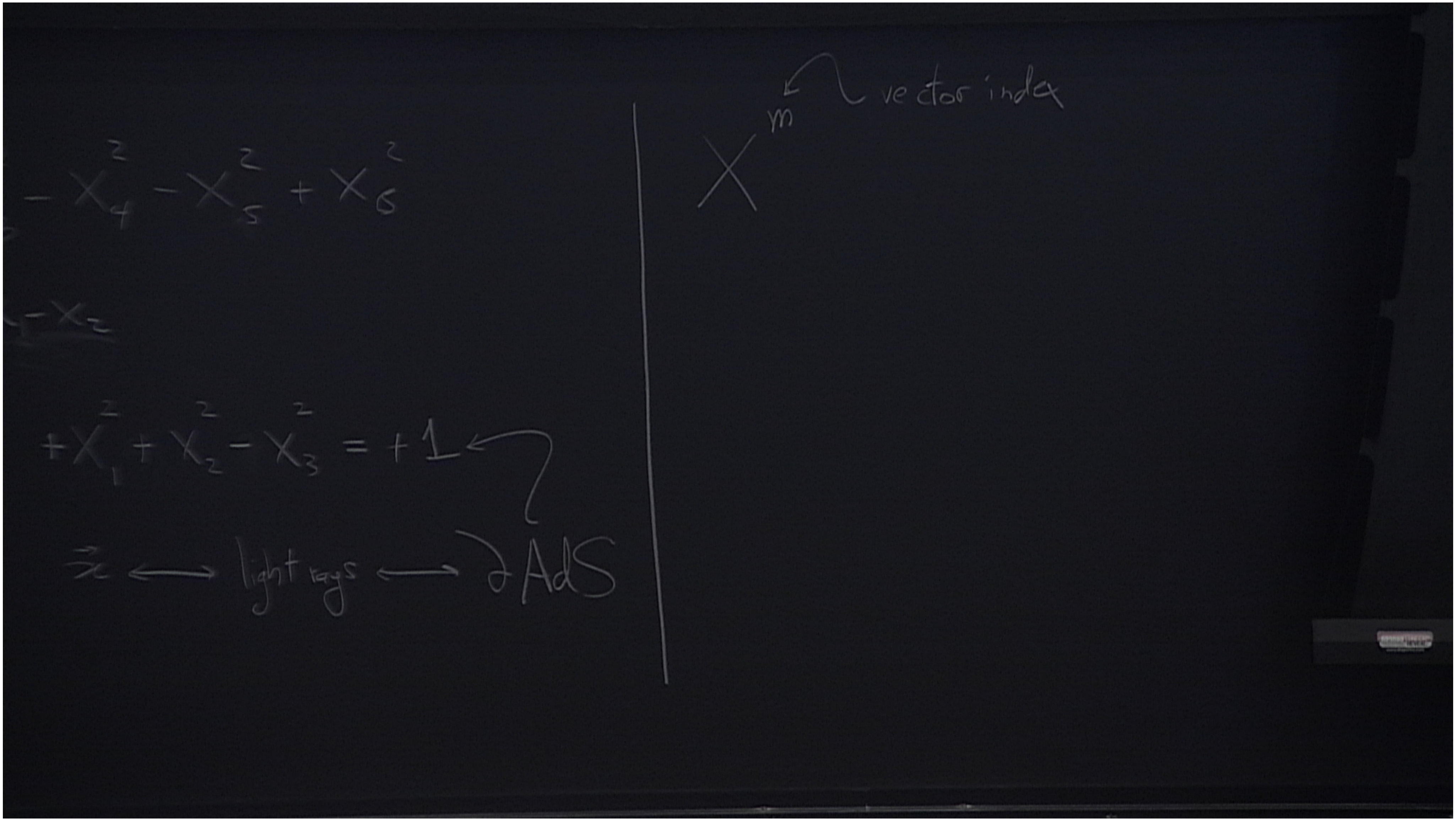

$-X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2$
$X_1 + X_2$
$X_1 - X_2$
$+X_1^2 + X_2^2 - X_3^2 = +1$
X_2
X_3
X_1

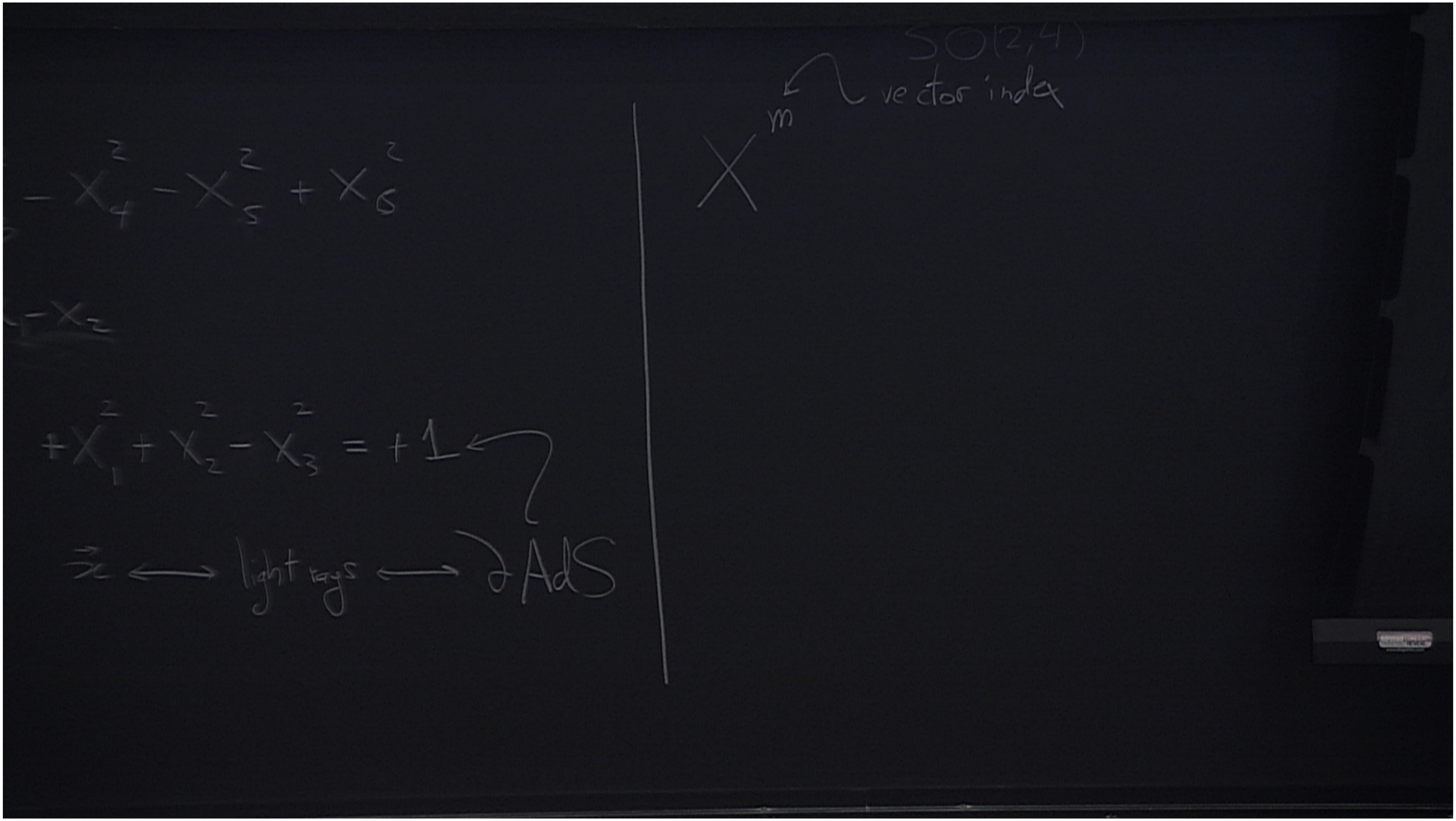

$-X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2$
$X_1 + X_2$
$X_1 - X_2$
$+X_1^2 + X_2^2 - X_3^2 = +1$
X_2
X_3
X_1


$-X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2$
$X_1 + X_2$
$X_1 - X_2$
$+X_1^2 + X_2^2 - X_3^2 = +1$
X_2
X_3
X_1

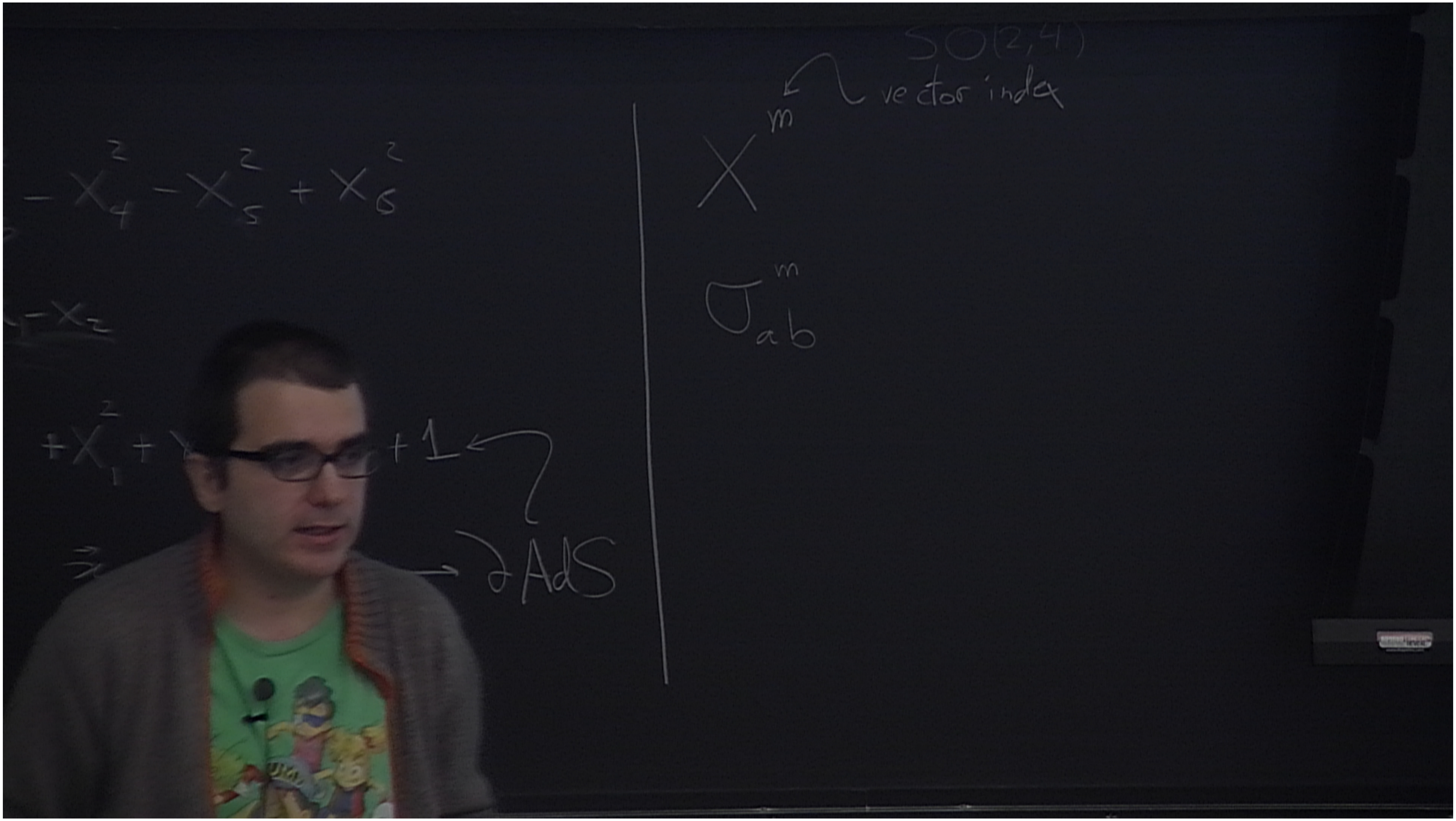

− X² = X₊X₋ − X₃² − X₄² − X₅² + X₆²
X₁+X₂
X₁−X₂
X₂
X₃
+X₁² + X₂² − X₃² = +1
light rays
∂AdS

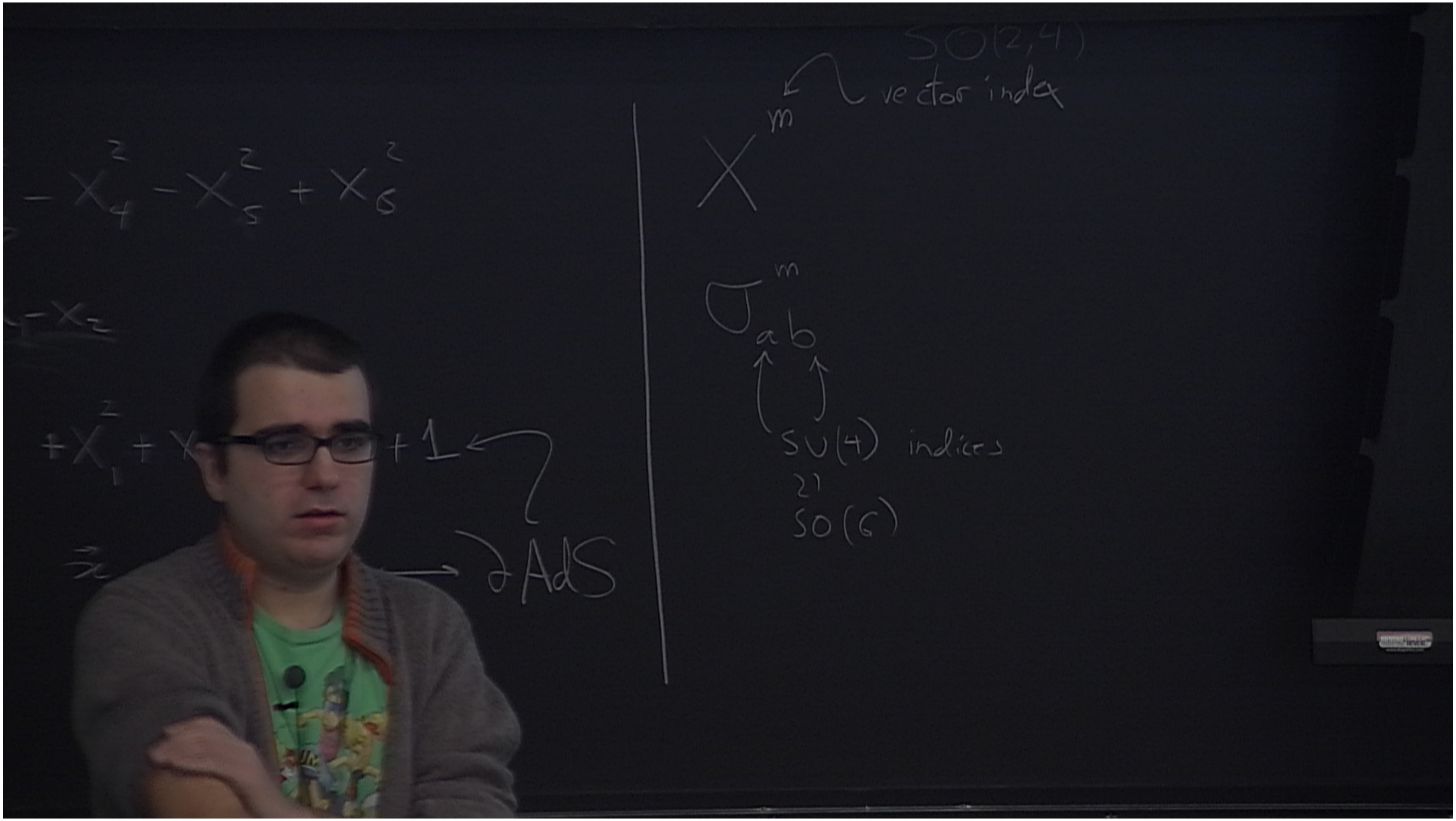

$-X^2 = X_+ X_- - X_3^2 - X_4^2 - X_5^2 + X_6^2$
$X_1 + X_2$
$X_1 - X_2$
X_2
X_3
X_1
$+X_1^2 + X_2^2 - X_3^2 = +1$
$\vec{x} \longleftrightarrow$ light rays $\longleftrightarrow \partial AdS$

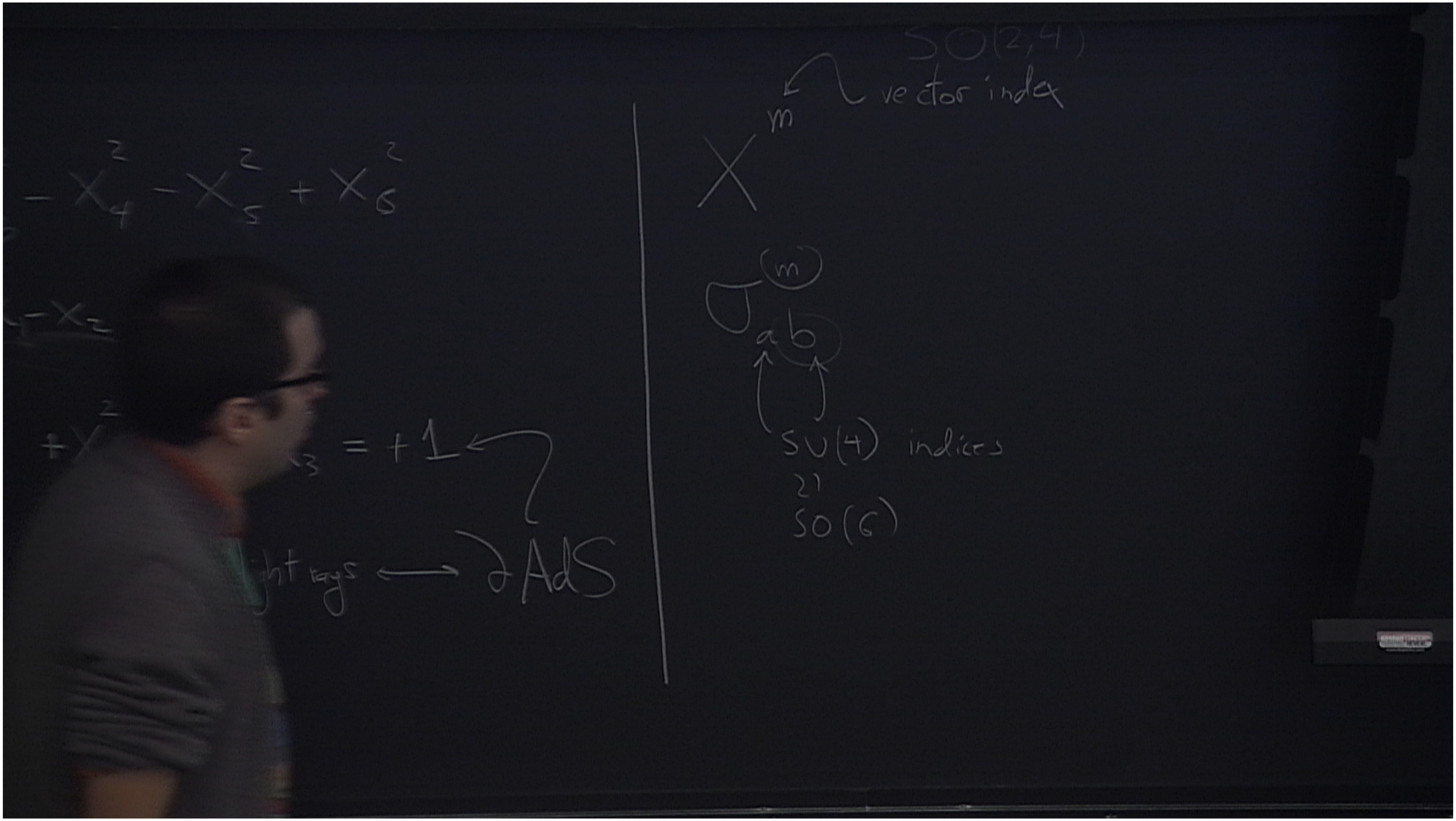

- X4² - X5² + X6²
-X2
+X1² + X2² - X3² = +1
x ⟷ light rays ⟷ ∂AdS
X^m
vector index



− X4² − X5² + X6²
+X1² + X2² − X3² = +1
x ⟷ light rays ⟶ ∂AdS
SO(2,4)
vector index
X^m


SO(2,4)
vector index
∂AdS


SO(2,4)
vector index
SU(4) indices
SO(6)
∂AdS

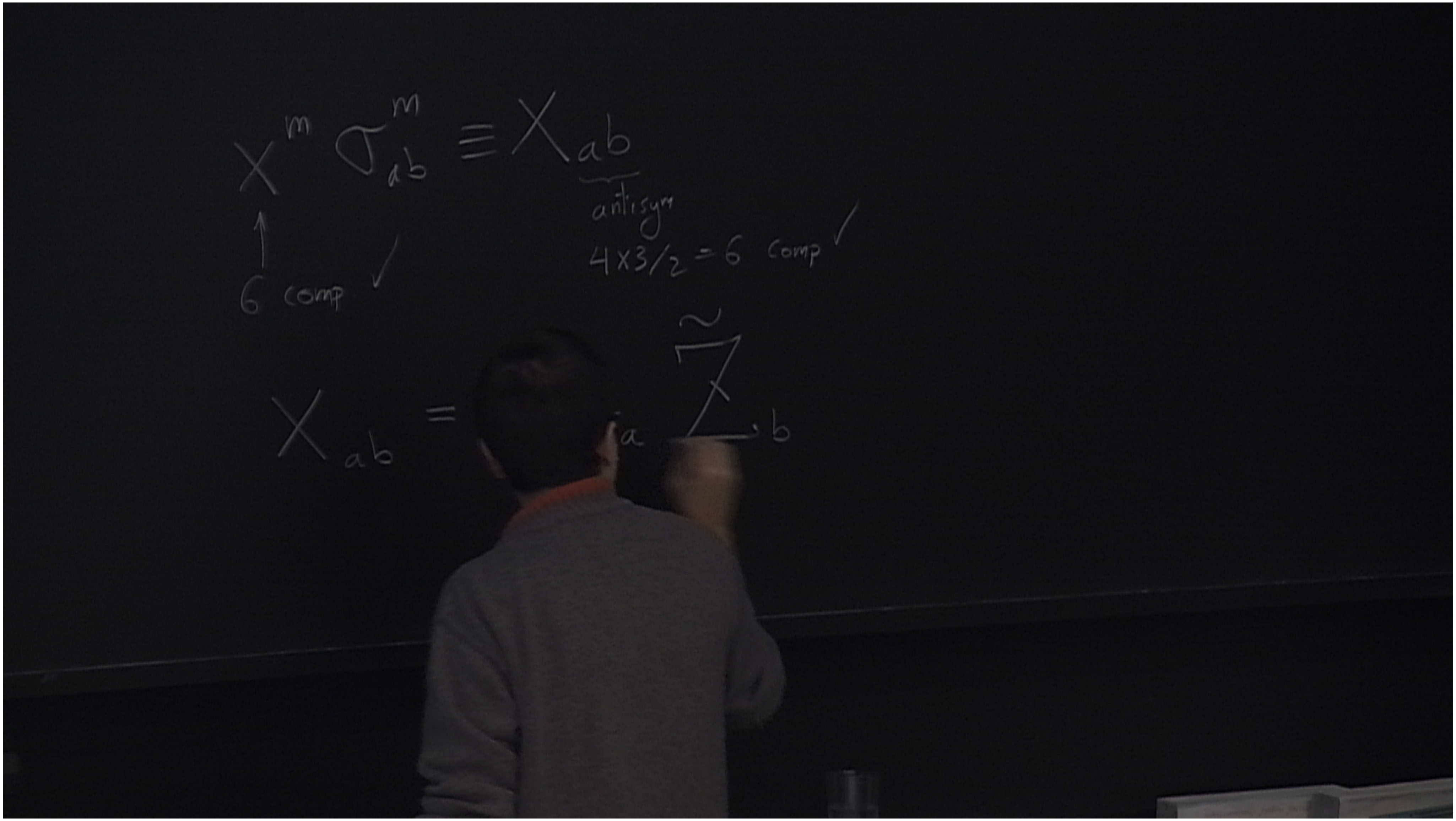

SO(2,4)
vector index
SU(4) indices
SO(6)
∂AdS

$$X^m \sigma^m_{ab} \equiv X_{ab}$$

$\uparrow$ 6 comp ✓

ab: antisym

$4 \times 3/2 = 6$ comp ✓



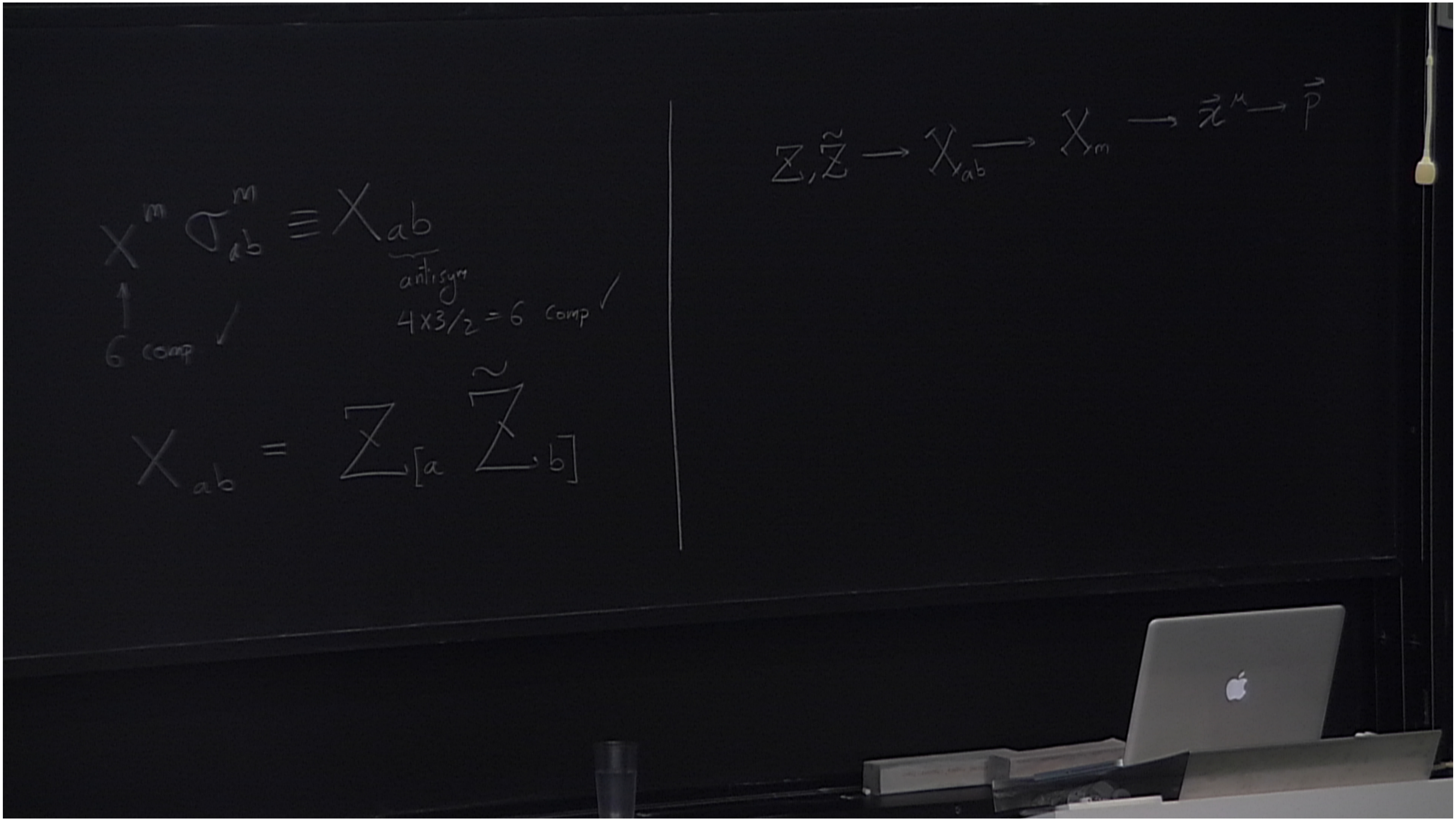
X^m σ^m_ab ≡ X_ab
antisym
4×3/2 = 6 comp
6 comp
X_ab =
a
b

$$X^m \sigma^m_{ab} \equiv X_{\underbrace{ab}_{\text{antisym}}}$$

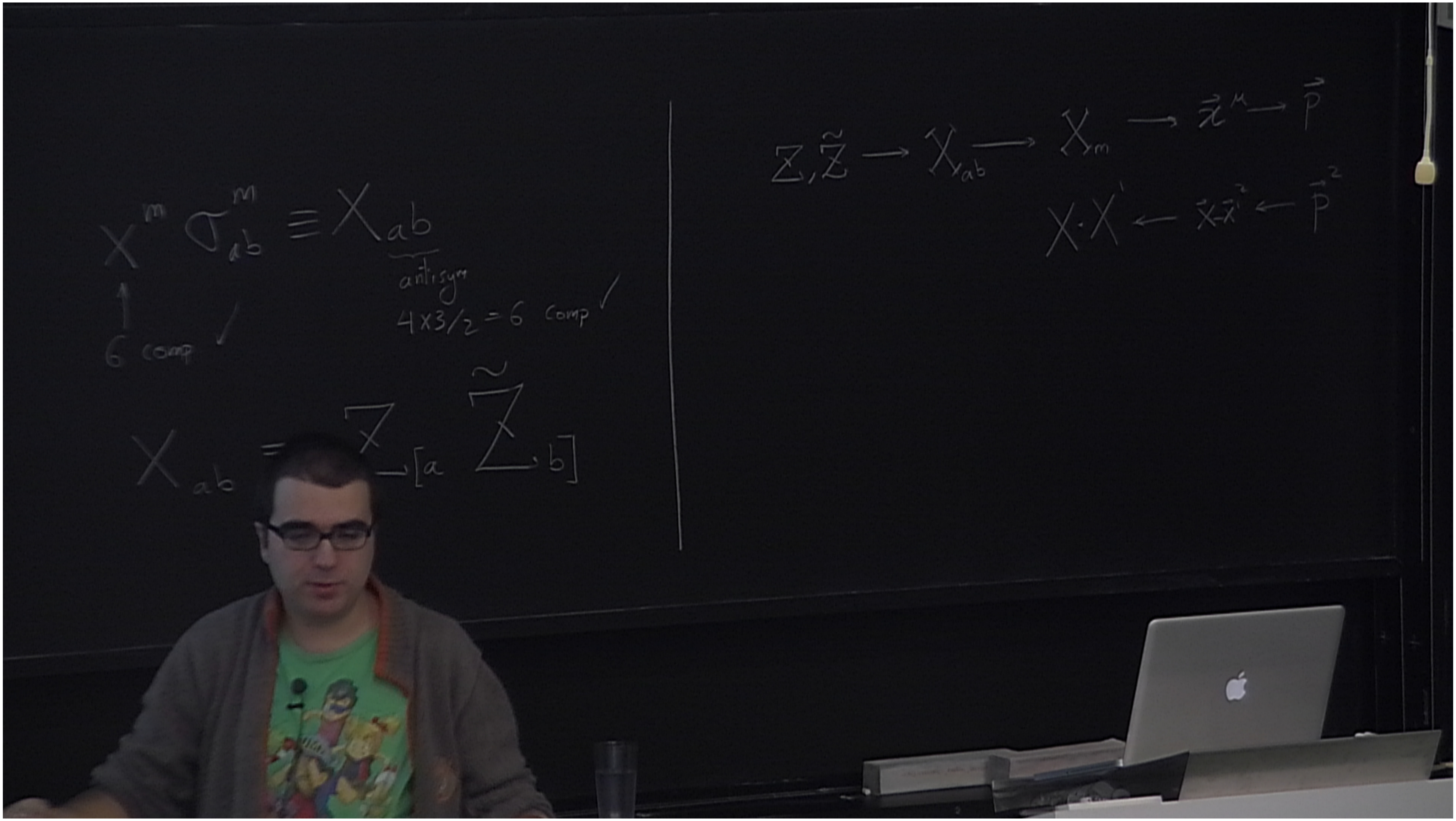
↑ 6 comp ✓

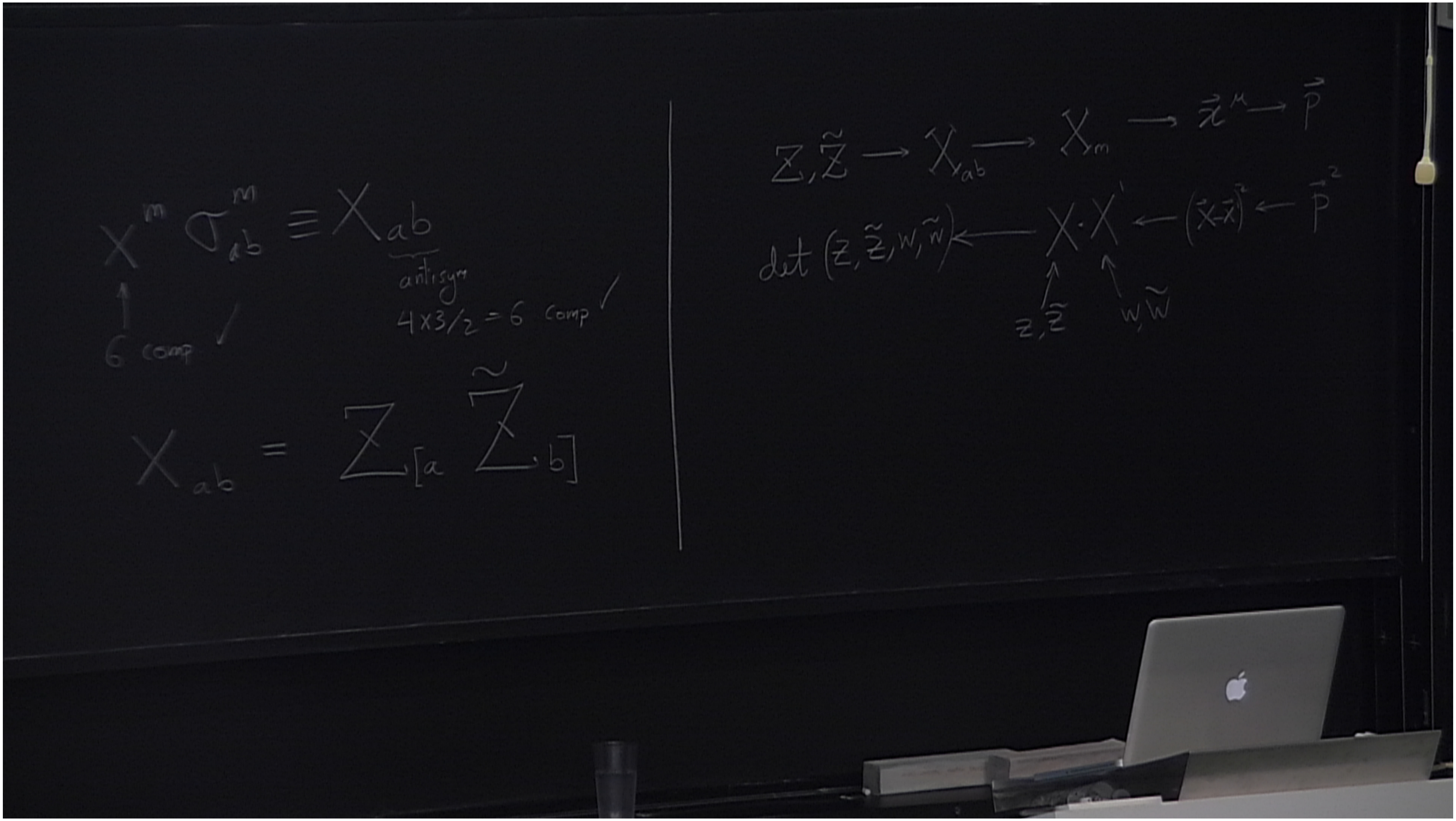
antisym $4 \times 3/2 = 6$ comp ✓

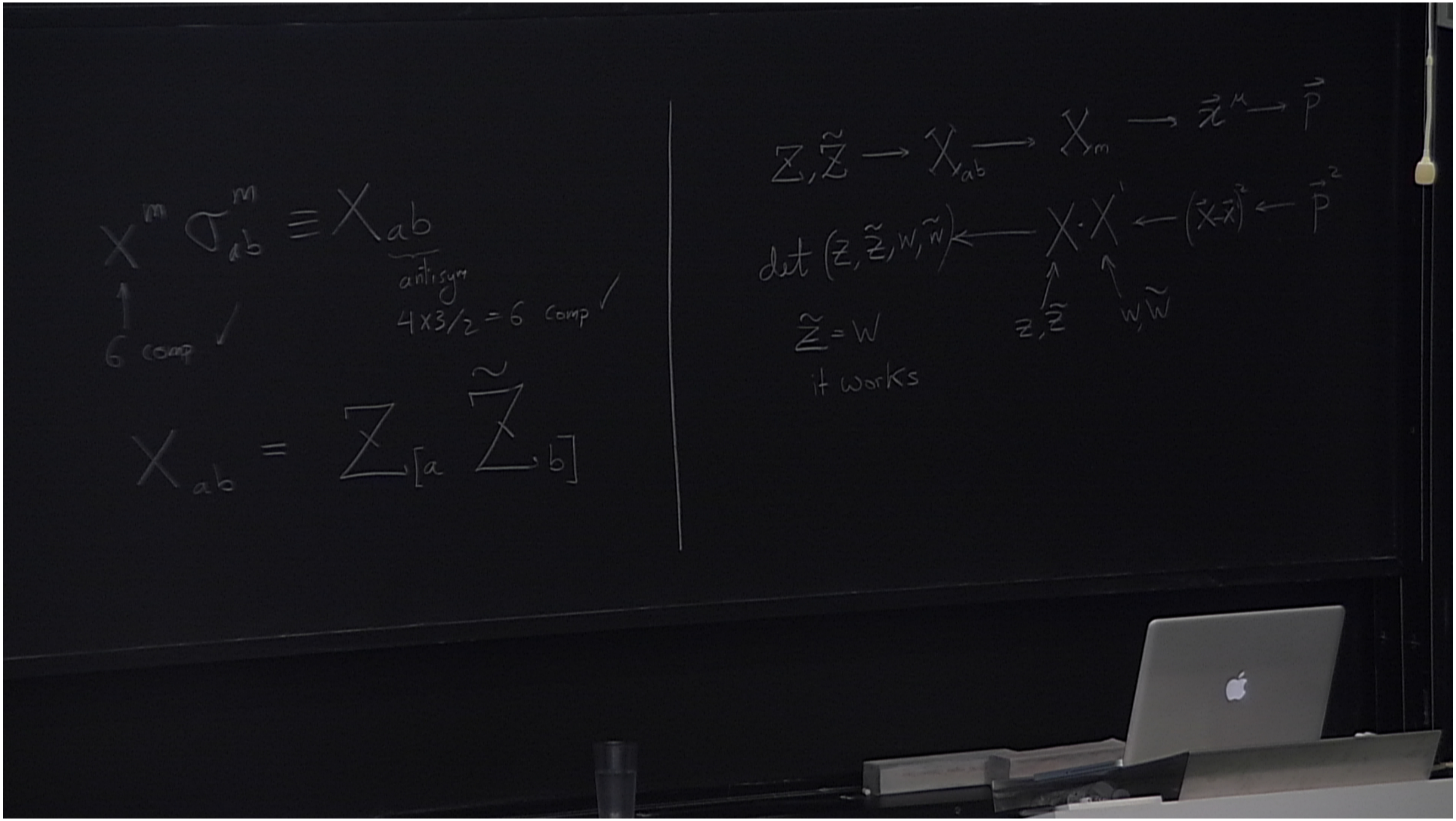
$$X_{ab} = Z_{[a} \tilde{Z}_{b]}$$



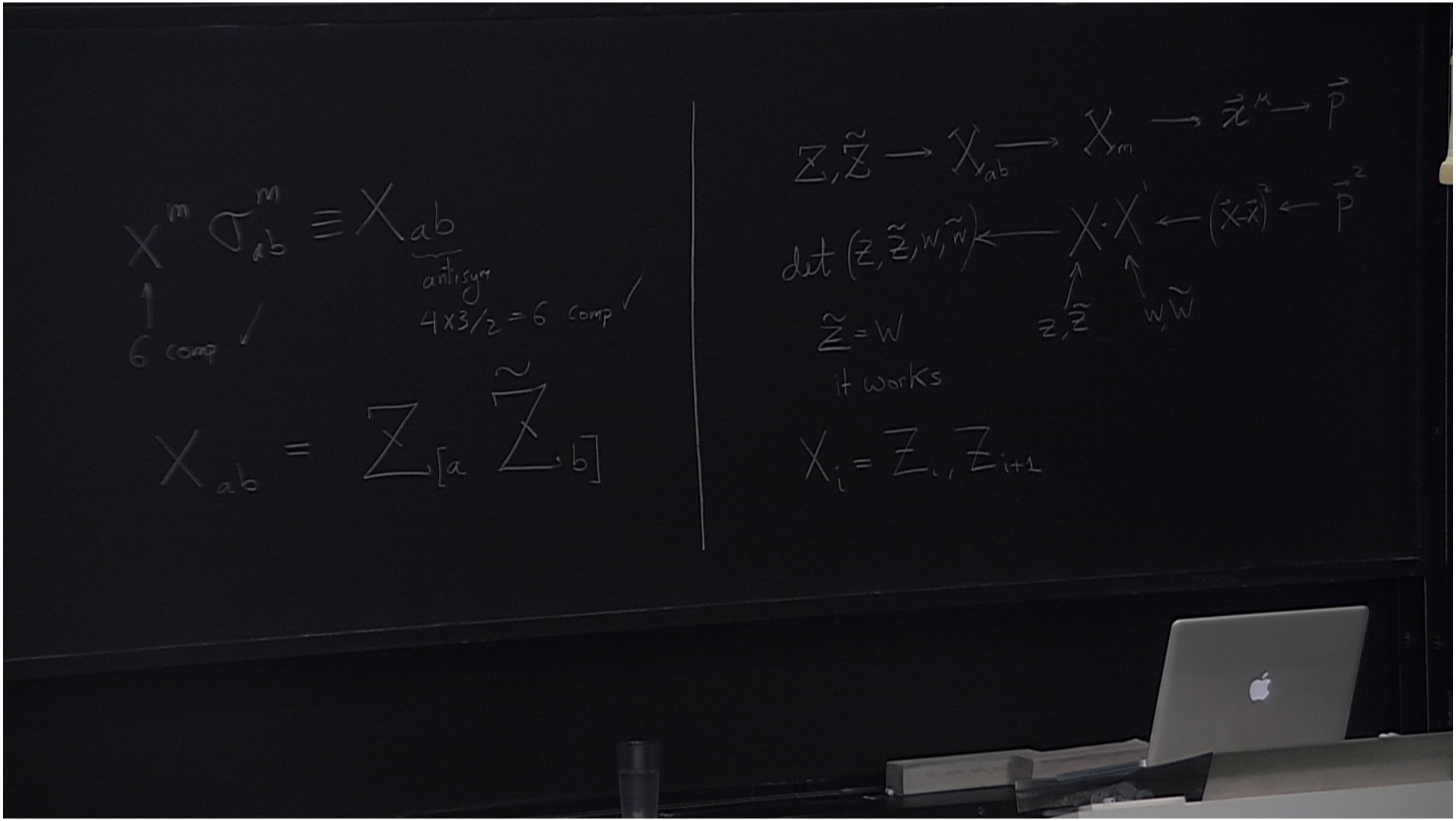
x^m σ^m_ab ≡ X_ab
antisym
4×3/2 = 6 comp
6 comp
X_ab = Z_[a Z̃_b]
Z, Z̃ → X_ab → X_m → x^μ → p


antisym
4x3/2 = 6 comp
6 comp

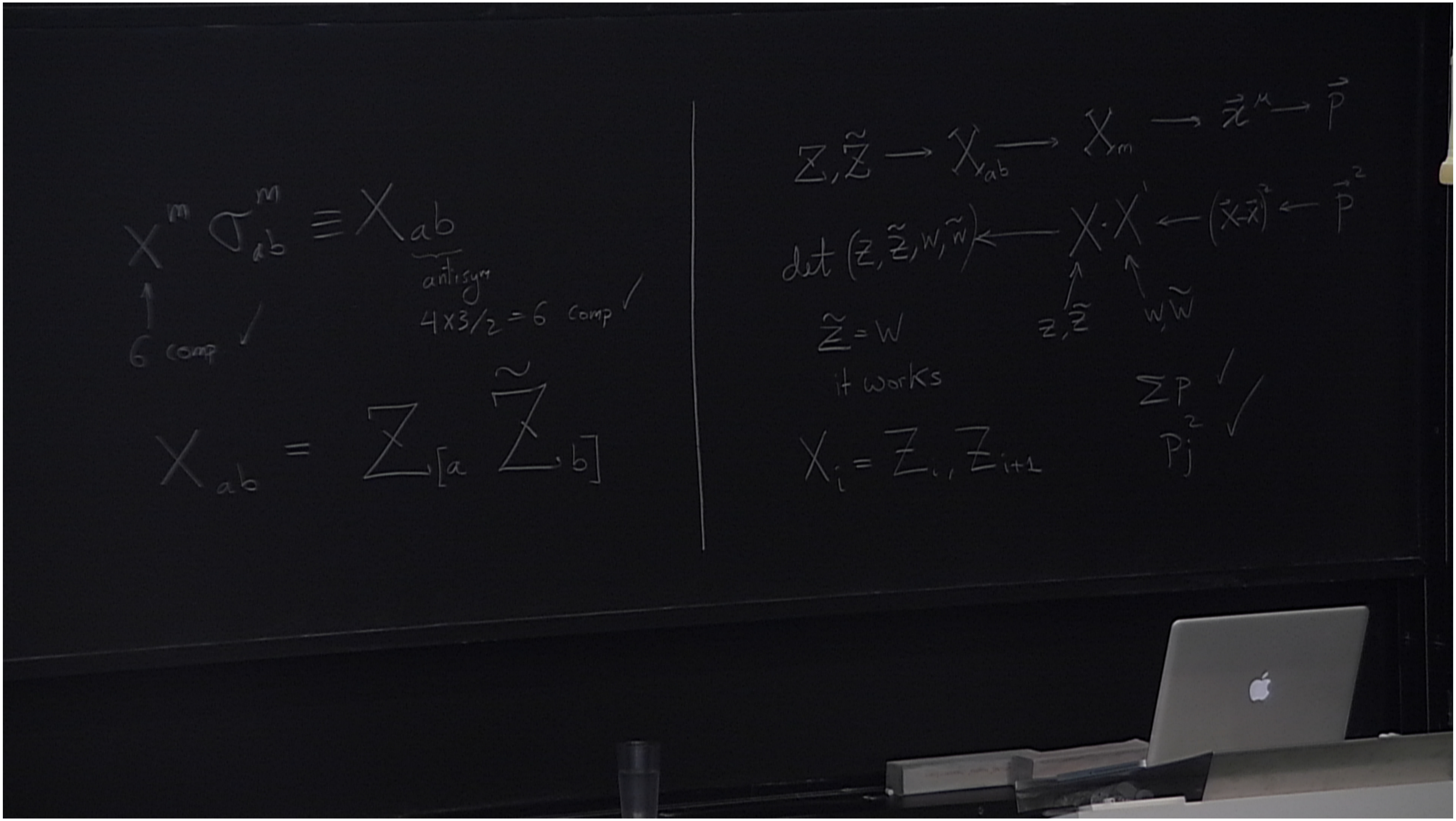

$X^m \sigma^m_{ab} \equiv X_{ab}$
antisym
$4 \times 3/2 = 6$ comp ✓
6 comp ✓
$X_{ab} = Z_{[a} \tilde{Z}_{b]}$
$Z, \tilde{Z} \rightarrow X_{ab} \rightarrow X_m \rightarrow \vec{x}^\mu \rightarrow \vec{p}$
$\det(Z, \tilde{Z}, W, \tilde{W}) \leftarrow X \cdot X' \leftarrow (\vec{x} - \vec{x}')^2 \leftarrow \vec{p}^2$
$Z, \tilde{Z}$
$W, \tilde{W}$



X^m σ^m_ab ≡ X_ab
antisym
4x3/2 = 6 comp
6 comp
X_ab = Z_[a Z̃_b]
Z, Z̃ → X_ab → X_m → x^μ → p
det (Z, Z̃, W, W̃) ← X·X' ← (x-x')² ← p²
Z̃ = W
it works
Z, Z̃
W, W̃



X^m σ^m_ab ≡ X_ab
antisym
4x3/2 = 6 comp
6 comp
X_ab = Z_[a Z̃_b]
Z, Z̃ → X_ab → X_m → x^μ → p
det (Z, Z̃, W, W̃) ← X·X' ← (x-x')² ← p²
Z̃ = W
it works
Z, Z̃
W, W̃
X_i = Z_i, Z_i+1


$X^m \sigma^m_{ab} \equiv X_{ab}$
antisym
$4 \times 3/2 = 6$ comp ✓
6 comp ✓
$X_{ab} = Z_{[a} \tilde{Z}_{b]}$
$Z, \tilde{Z} \longrightarrow X_{ab} \longrightarrow X_m \longrightarrow \vec{x}^\mu \longrightarrow \vec{p}$
$\det(Z, \tilde{Z}, W, \tilde{W}) \longleftarrow X \cdot X' \longleftarrow (\vec{x} - \vec{x}')^2 \longleftarrow \vec{p}^2$
$Z, \tilde{Z}$
$W, \tilde{W}$
$\tilde{Z} = W$
it works
Σp ✓
p_j^2 ✓
$X_i = Z_i, Z_{i+1}$

$$\Sigma[6]=\begin{pmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0 & -I & 0 & 0\\ I & 0 & 0 & 0\\ 0 & 0 & 0 & I\\ 0 & 0 & -I & 0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0 & 0 & -I & 0\\ 0 & 0 & 0 & I\\ I & 0 & 0 & 0\\ 0 & -I & 0 & 0\end{pmatrix};$$

$$\Sigma[2]=\begin{pmatrix}0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix};\Sigma[1]=\begin{pmatrix}0 & 0 & 0 & -I\\ 0 & 0 & -I & 0\\ 0 & I & 0 & 0\\ I & 0 & 0 & 0\end{pmatrix};$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:41) Fri 10:45 AM

Lecture December 14.nb

In[30]:=

```
n = 6;
```

Generating Momenta

In[31]:=

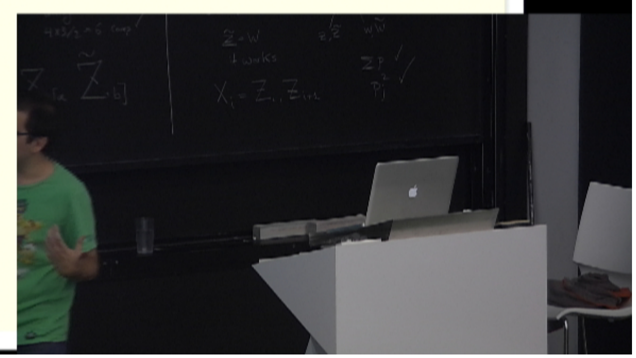
$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[39]:=

```
RandomInteger[{-400, 400}]
```

Out[39]=

-170



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:41) Fri 10:46 AM

Lecture December 14.nb

In[30]:=

```
n = 6;
```

Generating Momenta

In[31]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[40]:=

```
RandomInteger[{-400, 400}, {n, 4}]
```

Out[40]=

```
{{-231, 55, 231, 364}, {391, 171, 214, -264}, {272, 184, 21, 306},
 {-377, -76, -145, -326}, {299, 151, 126, -363}, {348, 73, 96, 322}}
```

Lecture December 14.nb

In[30]:=

```
n = 6;
```

Generating Momenta

In[31]:=

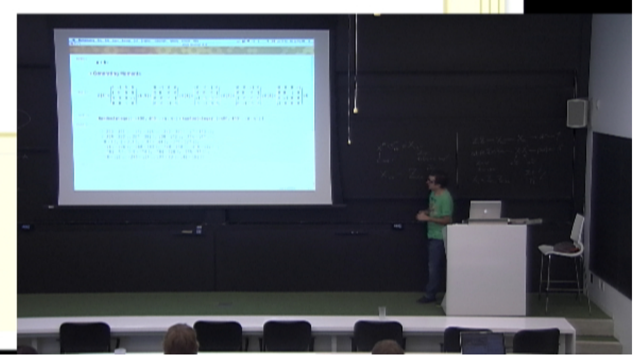
$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[41]:=

```
RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I
```

Out[41]=

```
{{-270 - 371 i, -151 + 216 i, -290 - 350 i, 17 + 363 i},
 {-329 + 212 i, 200 - 351 i, 208 - 172 i, -374 - 197 i},
 {8 + 7 i, 1 + 313 i, -187 + 144 i, -375 - 227 i},
 {-261 - 226 i, -268 - 183 i, -348 - 260 i, -208 + 112 i},
 {-381 - 5 i, 315 + 378 i, -384 - 228 i, -255 + 98 i},
 {-8 + 111 i, -295 - 129 i, 107 - 83 i, 141 - 161 i}}
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Lecture December 14.nb

In[30]:=
```
n = 6;
```

Generating Momenta

In[31]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[43]:=
```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[44]:=
```
Zs[[1]]
```

Out[44]=
```
{169 - 230 i, 8 + 194 i, -103 + 216 i, -65 + 137 i}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:40) Fri 10:48 AM

Lecture December 14.nb

In[30]:=
```
n = 6;
```

Generating Momenta

In[31]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[43]:=
```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

```
KroneckerProduct
```

In[44]:=
```
Zs[[1]]
```

Out[44]=
```
{169 - 230 i, 8 + 194 i, -103 + 216 i, -65 + 137 i}
```

```
Z[a] Zt[b]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help — Fri 10:48 AM

Lecture December 14.nb

In[30]:=
```
n = 6;
```

Generating Momenta

In[31]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[43]:=
```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[45]:=
```
KroneckerProduct[Zs[[1]], Zs[[2]]]
```

Out[45]=
```
{{-23 122 - 99 159 i, 31 520 + 12 535 i, 35 543 + 9470 i, -53 553 + 51 674 i},
 {54 430 + 42 840 i, -22 070 + 6740 i, -22 904 + 10 078 i, 6480 - 50 210 i},
 {34 640 + 78 025 i, -27 930 - 5365 i, -30 761 - 2208 i, 35 975 - 50 980 i},
 {22 047 + 49 399 i, -17 705 - 3365 i, -19 495 - 1361 i, 22 733 - 32 349 i}}
```

In[44]:=
```
Zs[[1]]
```

Out[44]=
```
{169 - 230 i, 8 + 194 i, -103 + 216 i, -65 + 137 i}
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:39) Fri 10:49 AM

Lecture December 14.nb

In[30]:=
```
n = 6;
```

Generating Momenta

In[31]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma$$

In[43]:=
```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[46]:=
```
Table[Tr[Σ[m].KroneckerProduct[Zs[[1]], Zs[[2]]]], {m, 6}]
```

Out[46]=
```
{-17 718 - 70 574 i, -80 626 - 13 168 i, 21 710 - 23 282 i,
 -25 088 + 115 400 i, -19 314 - 78 380 i, 32 560 - 79 924 i}
```

In[44]:=
```
Zs[[1]]
```

Out[44]=
```
{169 - 230 i, 8 + 194 i, -103 + 216 i, -65 + 137 i}
```

200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:39) Fri 10:51 AM

Lecture December 14.nb

In[30]:=

```
n = 6;
```

Generating Momenta

In[47]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=I\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};$$

In[48]:=

```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

$$X1 == \frac{\#}{\#[[1]] + I\,\#[[2]]}$$ `&@Table[Tr[Σ[m].KroneckerProduct[Zs[[1]], Zs[[2]]]], {m, 6}]`

Out[50]=

{279 300 + 96 308 i, -171 006 + 147 986 i, 193 980 - 136 362 i,
-180 146 - 86 496 i, 5250 + 364 211 i, -58 330 + 126 267 i}

In[44]:=

```
Zs[[1]]
```

Out[44]=

{169 - 230 i, 8 + 194 i, -103 + 216 i, -65 + 137 i}

200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:38) Fri 10:52 AM

Lecture December 14.nb

In[30]:=
```
n = 6;
```

Generating Momenta

In[47]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=I\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};$$

In[48]:=
```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[51]:=
```
X1 == Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[1]], Zs[[2]]]], {m, 6}], -4]
```

Out[51]=

$$X1 == \left\{\frac{8914564599}{5705789450}-\frac{854080407\, i}{5705789450},\ -\frac{4298653409}{5705789450}-\frac{6203670413\, i}{5705789450},\ -\frac{13258217389}{11411578900}+\frac{24109083877\, i}{11411578900},\ -\frac{8545718993}{11411578900}+\frac{6111745249\, i}{11411578900}\right\}$$

In[44]:=
```
Zs[[1]]
```

Out[44]=

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:38) Fri 10:52 AM

Lecture December 14.nb

In[47]:=

$$\Sigma[6]=\begin{pmatrix}1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=\begin{pmatrix}I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma[1$$

In[48]:=

```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[57]:=

```
X[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
   {m, 6}], -4]
```

In[58]:=

```
X[6]
```

Out[58]=

$$\left\{-\frac{1\,949\,700\,477}{1\,347\,361\,040}-\frac{1\,232\,862\,161\,i}{1\,347\,361\,040},\ \frac{7\,418\,175}{269\,472\,208}+\frac{453\,429\,767\,i}{269\,472\,208},\ \frac{221\,816\,475}{269\,472\,208}-\frac{62\,483\,691\,i}{269\,472\,208},\ -\frac{527\,888\,663}{1\,347\,361\,040}+\frac{13\,675\,581\,i}{1\,347\,361\,040}\right\}$$

In[44]:=

```
Zs[[1]]
```

Out[44]=

{169 - 230 i, 8 + 194 i, -103 + 216 i, -65 + 137 i}

200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:37) Fri 10:53 AM

Lecture December 14.nb

In[47]:=

$$\Sigma[6]=\begin{pmatrix}1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=I\begin{pmatrix}I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};\Sigma[1$$

In[48]:=

```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

```
x[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
    {m, 6}], -4]

P[a_] := x[Mod[a + 1, n, 1]] - x[a];
```

```
P[a_] := x[Mod[a + 1, n, 1]] - x[a];
```

In[60]:=

```
x[6]
```

Out[60]=

$$\left\{-\frac{1949700477}{1347361040}-\frac{1232862161\,i}{1347361040},\ \frac{7418175}{269472208}+\frac{453429767\,i}{269472208},\right.$$

Lecture December 14.nb

In[65]:=

```
x[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
    {m, 6}], -4]

P[a_] := x[Mod[a + 1, n, 1]] - x[a];

η = DiagonalMatrix[{1, 1, 1, -1}];
```

In[68]:=

```
P[1]
```

Out[68]=

$$\left\{\frac{364\,051\,109\,821\,387\,157}{6\,522\,605\,915\,352\,627\,400}+\frac{7\,211\,807\,094\,074\,696\,549\,i}{6\,522\,605\,915\,352\,627\,400},\ -\frac{9\,171\,280\,442\,114\,261\,587}{6\,522\,605\,915\,352\,627\,400}+\frac{2\,254\,750\,298\,739\,590\,391\,i}{6\,522\,605\,915\,352\,627\,400},\ \frac{2\,423\,560\,892\,557\,642\,299}{6\,522\,605\,915\,352\,627\,400}+\frac{5\,303\,798\,523\,393\,058\,993\,i}{6\,522\,605\,915\,352\,627\,400},\ \frac{2\,862\,447\,046\,539\,989\,963}{6\,522\,605\,915\,352\,627\,400}-\frac{1\,816\,418\,669\,910\,588\,259\,i}{6\,522\,605\,915\,352\,627\,400}\right\}$$

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Lecture December 14.nb

```
P[a_] := x[Mod[a + 1, n, 1]] - x[a];

η = DiagonalMatrix[{1, 1, 1, -1}];

In[74]:= Table[P[m].η.P[m], {m, 6}]
         Sum[P[m], {m, 6}]

Out[74]= {0, 0, 0, 0, 0, 0}

Out[75]= {0, 0, 0, 0}

In[70]:= P[1][[1]]^2 + P[1][[2]]^2 + P[1][[3]]^2 - P[1][[4]]^2

Out[70]= 0
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:38) Fri 10:55 AM

Lecture December 14.nb

```
(0  0  0 -1)   (0  0  0  I)   (-1  0  0  0)   (I  0  0  0)   (0 -1  0  0)
(0  0  1  0)   (0  0 -I  0)   (0  -1  0  0)   (0 -I  0  0)   (1  0  0  0)
```

In[48]:=

```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[65]:=

```
x[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
    {m, 6}], -4]

P[a_] := x[Mod[a + 1, n, 1]] - x[a];

η = DiagonalMatrix[{1, 1, 1, -1}];
```

In[74]:=

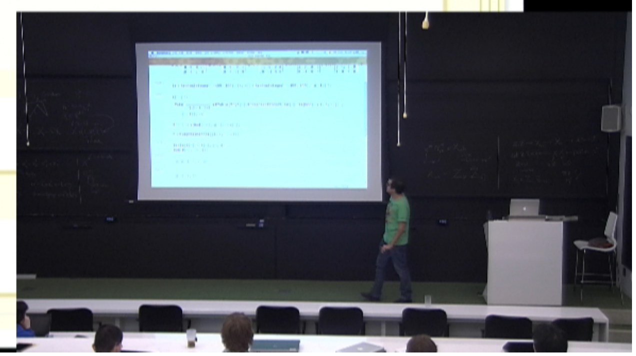
```
Table[P[m].η.P[m], {m, 6}]
Sum[P[m], {m, 6}]
```

Out[74]=

```
{0, 0, 0, 0, 0, 0}
```

Out[75]=

```
{0, 0, 0, 0}
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:36) Fri 10:57 AM

Lecture December 14.nb

In[30]:=
```
n = 6;
```

Generating Momenta

In[47]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=I\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};$$

In[48]:=
```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[88]:=
```
x[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
    {m, 6}], -4]

P[a_] := x[Mod[a + 1, n, 1]] - x[a];

η = DiagonalMatrix[{1, 1, 1, -1}];
```

In[86]:=
```
Table[P[m].η.P[m], {m, 6}]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:35) Fri 10:58 AM

Lecture December 14.nb

Generating Momenta

In[47]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=I\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};$$

In[48]:=

```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[88]:=

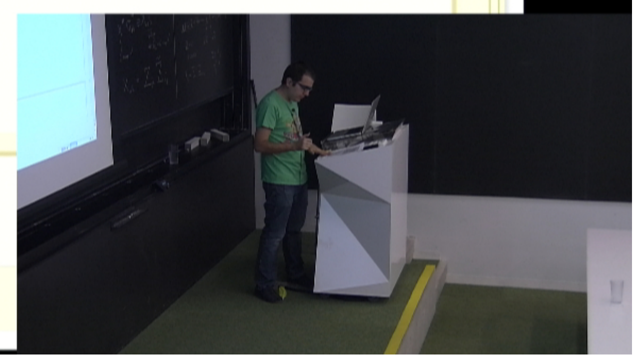
```
x[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
   {m, 6}], -4]

P[a_] := x[Mod[a + 1, n, 1]] - x[a];

η = DiagonalMatrix[{1, 1, 1, -1}];
```

In[95]:=

```
(* Table[P[m].η.P[m],{m,6}]
 Sum[P[m],{m,6}] *)
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:35) Fri 10:58 AM

Lecture December 14.nb

Generating Momenta

In[47]:=

$$\Sigma[6]=\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix};\Sigma[5]=I\begin{pmatrix}0&-I&0&0\\I&0&0&0\\0&0&0&I\\0&0&-I&0\end{pmatrix};\Sigma[4]=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix};\Sigma[3]=\begin{pmatrix}0&0&-I&0\\0&0&0&I\\I&0&0&0\\0&-I&0&0\end{pmatrix};\Sigma[2]=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix};$$

In[48]:=

```
Zs = RandomInteger[{-400, 400}, {n, 4}] + RandomInteger[{-400, 400}, {n, 4}] I;
```

In[88]:=

```
x[a_] :=
 Take[#/(#[[1]] + I #[[2]]) &@Table[Tr[Σ[m].KroneckerProduct[Zs[[a]], Zs[[Mod[a + 1, n, 1]]]]],
    {m, 6}], -4]

P[a_] := x[Mod[a + 1, n, 1]] - x[a];

η = DiagonalMatrix[{1, 1, 1, -1}];
```

In[95]:=

```
(* Table[P[m].η.P[m],{m,6}]
 Sum[P[m],{m,6}] *)
```

200%

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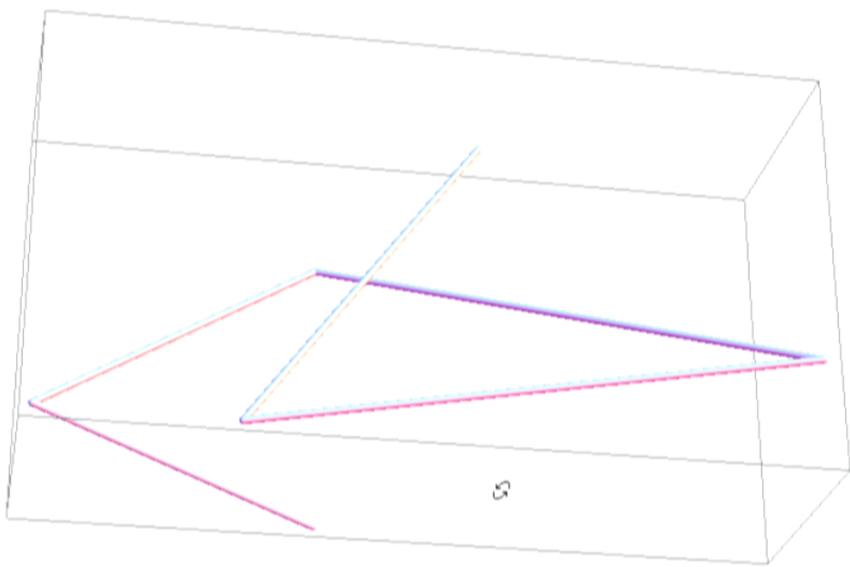
Lecture December 14.nb

In[103]:=

```
Graphics3D@Tube[xs]
```

Out[103]=



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Lecture December 14.nb

In[106]:=

```
xs = Table[Drop[#, 1] &@Re@x[m], {m, n}] ~Join~ {Drop[#, 1] &@Re@x[1]};
```

In[109]:=

```
Graphics3D[{Red, Tube[xs, 0.02]}]
```

Out[109]=

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Lecture December 14.nb

Spinors

```
P /@ Range[n] // MatrixForm // N
```

trixForm=

-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

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Lecture December 14.nb

Spinors

```
P /@ Range[n] // MatrixForm // N
```

$$\begin{pmatrix} -3.84492-2.15797\,i & -2.52759-0.381257\,i & -1.39375+1.57057\,i & -4.30614-1.64228\,i \\ 2.76744+1.03884\,i & 1.20857+0.685531\,i & 0.148157-1.4913\,i & 2.661+1.30871\,i \\ 4.15343+0.877485\,i & 1.0882-1.66795\,i & 1.51628-0.960114\,i & 4.03348+0.0926522\,i \\ -2.56658-0.703406\,i & -0.91044+3.36372\,i & -2.83749-1.62773\,i & -1.97574-1.70143\,i \\ 1.56671-2.32637\,i & 3.33835-1.3097\,i & -0.802002+1.63612\,i & -3.43636+2.71485\,i \\ -2.07608+3.27143\,i & -2.19709-0.690338\,i & 3.36882+0.872462\,i & 3.02375-0.772502\,i \end{pmatrix}$$

```
P[1]
```

$$\left\{-\frac{75\,309\,677\,858\,411\,265}{19\,586\,795\,378\,651\,272}-\frac{21\,133\,837\,653\,262\,225\,i}{9\,793\,397\,689\,325\,636},\ -\frac{321\,797\,445\,962\,849\,115}{127\,314\,169\,961\,233\,268}-\frac{97\,078\,752\,943\,405\,405\,i}{254\,628\,339\,922\,466\,536},\ -\frac{177\,444\,516\,909\,824\,245}{127\,314\,169\,961\,233\,268}+\frac{49\,988\,856\,312\,045\,813\,i}{31\,828\,542\,490\,308\,317},\ -\frac{548\,233\,091\,824\,957\,689}{127\,314\,169\,961\,233\,268}-\frac{52\,271\,368\,224\,246\,560\,i}{31\,828\,542\,490\,308\,317}\right\}$$

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Lecture December 14.nb

```
P /@ Range[n] // MatrixForm // N
```

-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

$$\lambda[a] = \left\{\frac{P[a][[1]] - P[a][[2]]\, I}{P[a][[4]] - P[a][[3]]}, 1\right\};\ \lambda t[a] = \left\{\frac{P[a][[1]] - P[a][[2]]\, I}{P[a][[4]] - P[a][[3]]}, 1\right\},$$

$$\left\{-\frac{75\,309\,677\,858\,411\,265}{19\,586\,795\,378\,651\,272} - \frac{21\,133\,837\,653\,262\,225\, i}{9\,793\,397\,689\,325\,636},\right.$$
$$-\frac{321\,797\,445\,962\,849\,115}{127\,314\,169\,961\,233\,268} - \frac{97\,078\,752\,943\,405\,405\, i}{254\,628\,339\,922\,466\,536},$$
$$-\frac{177\,444\,516\,909\,824\,245}{127\,314\,169\,961\,233\,268} + \frac{49\,988\,856\,312\,045\,813\, i}{31\,828\,542\,490\,308\,317},$$
$$\left.-\frac{548\,233\,091\,824\,957\,689}{127\,314\,169\,961\,233\,268} - \frac{52\,271\,368\,224\,246\,560\, i}{31\,828\,542\,490\,308\,317}\right\}$$

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Lecture December 14.nb

```
P /@ Range[n] // MatrixForm // N
```

Out[141]//MatrixForm=

-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

In[143]:=

$$\text{Do}\left[\lambda[a] = \left\{\frac{P[a][[1]] - P[a][[2]]\, I}{P[a][[4]] - P[a][[3]]},\ 1\right\};\ \lambda t[a] = \{P[a][[1]] + P[a][[2]]\, I,\ P[a][[4]] - P[a][[3]]\},\ \{a, n\}\right]$$

In[146]:=

```
λt[2]
```

Out[146]=

$$\left\{\frac{4\,028\,832\,500\,288\,932\,545}{1\,935\,159\,204\,795\,899\,218} + \frac{4\,349\,082\,254\,409\,601\,219\, i}{1\,935\,159\,204\,795\,899\,218},\ \frac{2\,431\,379\,513\,506\,499\,299}{967\,579\,602\,397\,949\,609} + \frac{2\,709\,231\,198\,295\,452\,456\, i}{967\,579\,602\,397\,949\,609}\right\}$$

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Lecture December 14.nb

In[143]:=

```
Do[λ[a] = {(P[a][[1]] - P[a][[2]] I)/(P[a][[4]] - P[a][[3]]), 1}; λt[a] = {P[a][[1]] + P[a][[2]] I, P[a][[4]] - P[a][[3]]},
 {a, n}]
```

$$\left\{\frac{P1 - P2\, I}{P4 - P3}, 1\right\}$$

```
{P1 + P[a][[2]] I, P[a][[4]] - P[a][[3]]}
```

Out[147]=

$$\left\{\frac{P1 - i\, P2}{-P3 + P4}, 1\right\}$$

Part::pspec : Part specification Mod[1 + a, 6, 1] is neither an integer nor a list of integers. ≫

Part::pspec : Part specification Mod[1 + Mod[1 + a, 6, 1], 6, 1] is neither an integer nor a list of integers. ≫

Part::pspec : Part specification Mod[1 + a, 6, 1] is neither an integer nor a list of integers. ≫

General::stop : Further output of Part::pspec will be suppressed during this calculation. ≫

Out[148]=

A very large output was generated. Here is a sample of it:

```
{<<1>> . - Tr[{{0,-1,0,0},{1,0,0,0},{0,0,0,-1},{0,0,1,0}}.<<1>>] +
```

200%

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Lecture December 14.nb

$$\text{Do}\left[\lambda[a] = \left\{\frac{P[a][[1]] - P[a][[2]]\,I}{P[a][[4]] - P[a][[3]]},\ 1\right\};\ \lambda t[a] = \{P[a][[1]] + P[a][[2]]\,I,\ P[a][[4]] - P[a][[3]]\},\ \{a, n\}\right]$$

In[151]:=

$$\lambda = \left\{\frac{P1 - P2\,I}{P4 - P3},\ 1\right\}$$

$$\lambda t = \{P1 + P2\,I,\ P4 - P3\}$$

Out[151]=

$$\left\{\frac{P1 - i\,P2}{-P3 + P4},\ 1\right\}$$

Out[152]=

$$\{P1 + i\,P2,\ -P3 + P4\}$$

In[153]:=

```
Table[λ[a] λt[b], {a, 2}, {b, 2}]
```

Out[153]=

$$\left\{\left\{\{P1 + i\,P2,\ -P3 + P4\}[1]\left\{\frac{P1 - i\,P2}{-P3 + P4},\ 1\right\}[1],\ \{P1 + i\,P2,\ -P3 + P4\}[2]\left\{\frac{P1 - i\,P2}{-P3 + P4},\ 1\right\}[1]\right\},\right.$$
$$\left.\left\{\{P1 + i\,P2,\ -P3 + P4\}[1]\left\{\frac{P1 - i\,P2}{-P3 + P4},\ 1\right\}[2],\ \{P1 + i\,P2,\ -P3 + P4\}[2]\left\{\frac{P1 - i\,P2}{-P3 + P4},\ 1\right\}[2]\right\}\right\}$$

200%

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Lecture December 14.nb

In[154]:=

```
λ = {(P1 - P2 I)/(P4 - P3), 1}
λt = {P1 + P2 I, P4 - P3}
```

$$\lambda = \left\{\frac{P1 - P2\, I}{P4 - P3},\ 1\right\}$$

$$\lambda t = \{P1 + P2\, I,\ P4 - P3\}$$

Out[154]=

$$\left\{\frac{P1 - i\, P2}{-P3 + P4},\ 1\right\}$$

Out[155]=

$$\{P1 + i\, P2,\ -P3 + P4\}$$

In[159]:=

```
Table[λ[[a]] λt[[b]], {a, 2}, {b, 2}] // FullSimplify[#, {P1^2 + P2^2 + P3^2 - P4^4 == 0}] &
```

Out[159]=

$$\left\{\left\{\frac{P1^2 + P2^2}{-P3 + P4},\ P1 - i\, P2\right\},\ \{P1 + i\, P2,\ -P3 + P4\}\right\}$$

```
P1.
```

200%

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Lecture December 14.nb

▪ Spinors

1]:=

```
P /@ Range[n] // MatrixForm // N
```

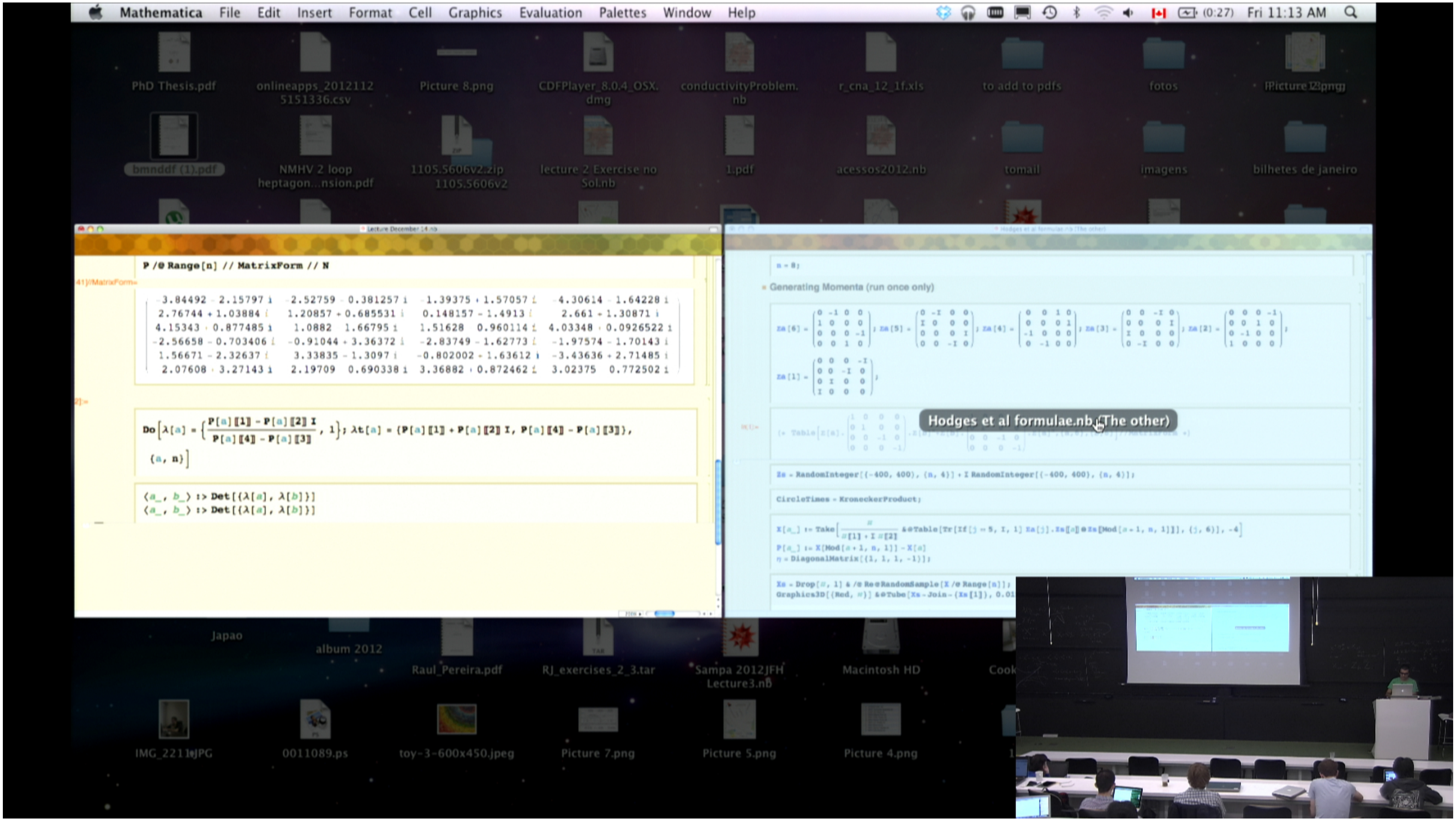
41]//MatrixForm=

-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

2]:=

$$\text{Do}\left[\lambda[a] = \left\{\frac{P[a][[1]] - P[a][[2]]\, I}{P[a][[4]] - P[a][[3]]}, 1\right\};\ \lambda t[a] = \{P[a][[1]] + P[a][[2]]\, I,\ P[a][[4]] - P[a][[3]]\},\ \{a, n\}\right]$$

```
Det[{λ[a], λ[b]}]
```



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Fri 11:13 AM
P /@ Range[n] // MatrixForm // N
Generating Momenta (run once only)
Hodges et al formulae.nb (The other)
CircleTimes = KroneckerProduct;

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:27) Fri 11:13 AM

Lecture December 14.nb

```
P /@ Range[n] // MatrixForm // N
```

41]//MatrixForm=

-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

2]:=

$$\text{Do}\left[\lambda[a] = \left\{\frac{P[a][[1]] - P[a][[2]]\,I}{P[a][[4]] - P[a][[3]]},\ 1\right\};\ \lambda t[a] = \{P[a][[1]] + P[a][[2]]\,I,\ P[a][[4]] - P[a][[3]]\},\ \{a, n\}\right]$$

3]:=

```
rep = {〈a_, b_〉 :> Det[{λ[a], λ[b]}], ‖a_, b_‖ :> Det[{λt[a], λt[b]}]};
```

```
〈a_, b_〉 :> Det[{λ[a], λ[b]}]
〈a_, b_〉 :> Det[{λ[a], λ[b]}]
```

200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:26) Fri 11:14 AM

Lecture December 14.nb

//MatrixForm=

-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

```
Do[λ[a] = {(P[a][[1]] - P[a][[2]] I)/(P[a][[4]] - P[a][[3]]), 1}; λt[a] = {P[a][[1]] + P[a][[2]] I, P[a][[4]] - P[a][[3]]},
  {a, n}]
```

```
rep = {⟨a_, b_⟩ :> Det[{λ[a], λ[b]}], ‖a_, b_‖ :> Det[{λt[a], λt[b]}]};
```

Gluons

$$\frac{1}{\square}$$

200%

//MatrixForm=

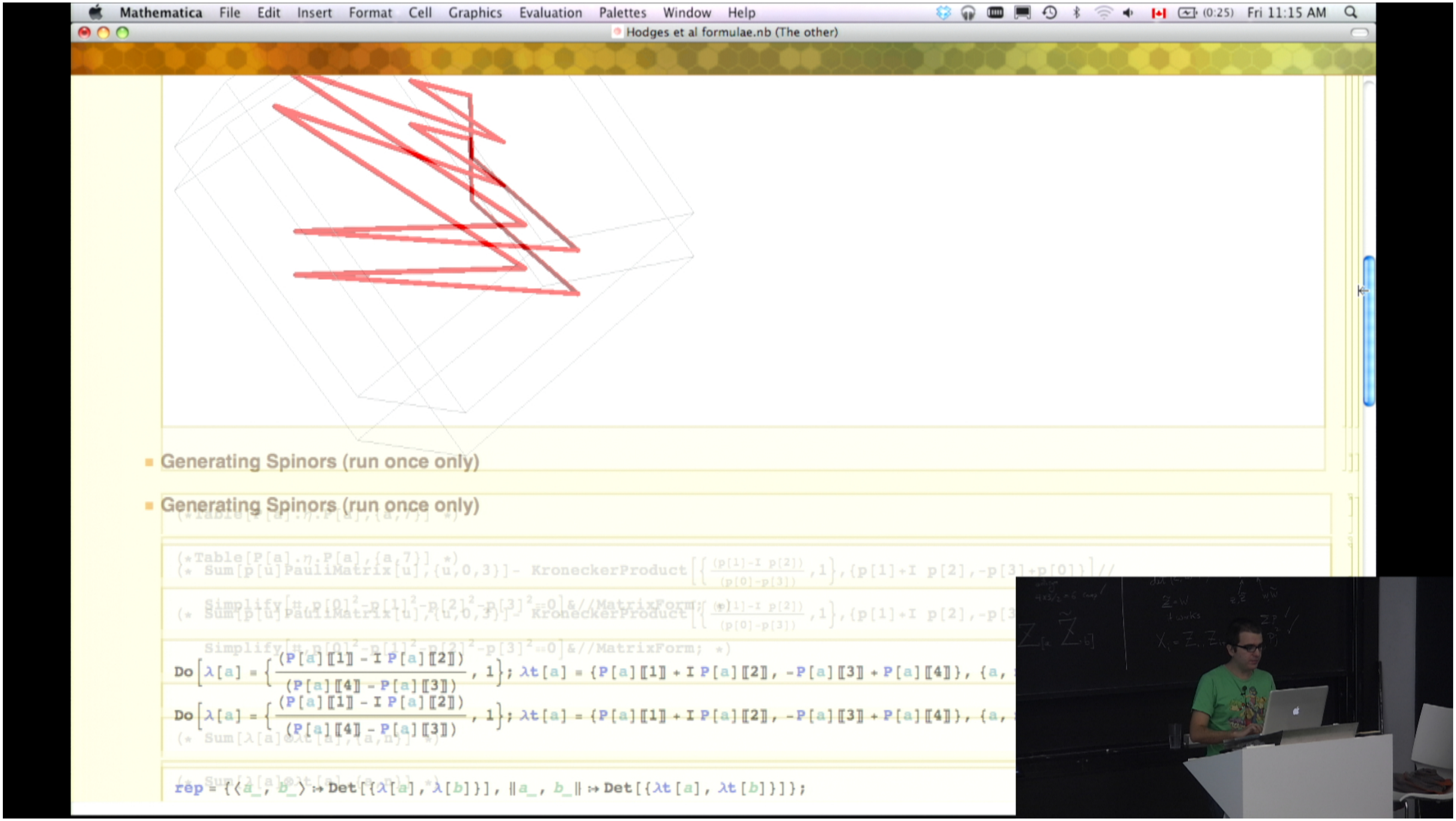
-3.84492 - 2.15797 i	-2.52759 - 0.381257 i	-1.39375 + 1.57057 i	-4.30614 - 1.64228 i
2.76744 + 1.03884 i	1.20857 + 0.685531 i	0.148157 - 1.4913 i	2.661 + 1.30871 i
4.15343 + 0.877485 i	1.0882 - 1.66795 i	1.51628 - 0.960114 i	4.03348 + 0.0926522 i
-2.56658 - 0.703406 i	-0.91044 + 3.36372 i	-2.83749 - 1.62773 i	-1.97574 - 1.70143 i
1.56671 - 2.32637 i	3.33835 - 1.3097 i	-0.802002 + 1.63612 i	-3.43636 + 2.71485 i
-2.07608 + 3.27143 i	-2.19709 - 0.690338 i	3.36882 + 0.872462 i	3.02375 - 0.772502 i

```
Do[λ[a] = {(P[a][[1]] - P[a][[2]] I)/(P[a][[4]] - P[a][[3]]), 1}; λt[a] = {P[a][[1]] + P[a][[2]] I, P[a][[4]] - P[a][[3]]},
  {a, n}]
```

```
rep = {⟨a_, b_⟩ :> Det[{λ[a], λ[b]}], ‖a_, b_‖ :> Det[{λt[a], λt[b]}]};
```

Gluons

$$\frac{1}{\langle 1, 2\rangle \langle 2, 3\rangle \,..\, \langle n, 1\rangle}$$


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:25) Fri 11:15 AM
Hodges et al formulae.nb (The other)
Generating Spinors (run once only)
Generating Spinors (run once only)
(*Table[P[a].η.P[a],{a,7}] *)
(* Sum[p[u]PauliMatrix[u],{u,0,3}]- KroneckerProduct[{{(p[1]-I p[2])/(p[0]-p[3]),1},{p[1]+I p[2],-p[3]+p[0]}}]//
Simplify[#,p[0]^2-p[1]^2-p[2]^2-p[3]^2==0]&//MatrixForm; *)
Do[λ[a] = {(P[a][[1]] - I P[a][[2]])/(P[a][[4]] - P[a][[3]]), 1}; λt[a] = {P[a][[1]] + I P[a][[2]], -P[a][[3]] + P[a][[4]]}, {a,
(* Sum[λ[a]⊗λt[a],{a,n}] *)
rep = {⟨a_, b_⟩ ⧴ Det[{λ[a], λ[b]}], ‖a_, b_‖ ⧴ Det[{λt[a], λt[b]}]};

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Lecture December 14.nb

```
Do[λ[a] = {(P[a][[1]] - P[a][[2]] I)/(P[a][[4]] - P[a][[3]]), 1}; λt[a] = {P[a][[1]] + P[a][[2]] I, P[a][[4]] - P[a][[3]]},
 {a, n}]
```

```
rep = {⟨a_, b_⟩ :→ Det[{λ[a], λ[b]}], ‖a_, b_‖ :→ Det[{λt[a], λt[b]}]};
```

Gluons

$$\frac{1}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle}$$

Hodges

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/8};
```

$$\Phi = \text{Table}\left[\text{If}\left[i == j, -\sum_{k=1}^{n} \text{If}\left[k == i, 0, \frac{\|i, k\| \langle k, r\rangle \langle k, l\rangle}{\langle i, k\rangle \langle i, r\rangle \langle i, l\rangle}\right], \frac{\|i, j\|}{\langle i, j\rangle}\right], \{i, n\}, \{j, n\}\right];$$

```
ϕ[c1_, c2_, c3_][r1_, r2_, r3_] :=
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:25) Fri 11:17 AM

Lecture December 14.nb

```
rep = {⟨a_, b_⟩ :→ Det[{λ[a], λ[b]}], ‖a_, b_‖ :→ Det[{λt[a], λt[b]}]};
```

Gluons

$$\frac{1}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle}$$

Hodges

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/8};
```

$$\Phi = \text{Table}\left[\text{If}\left[i == j, -\sum_{k=1}^{n} \text{If}\left[k == i, 0, \frac{\| i, k\| \langle k, r\rangle \langle k, l\rangle}{\langle i, k\rangle \langle i, r\rangle \langle i, l\rangle}\right], \frac{\| i, j\|}{\langle i, j\rangle}\right], \{i, n\}, \{j, n\}\right];$$

$$\phi[c1\_, c2\_, c3\_][r1\_, r2\_, r3\_] := \left(\frac{\text{Det@Delete}[\text{Delete}[\Phi \text{ /. rep}, \{\{c1\}, \{c2\}, \{c3\}\}]^{T}, \{\{r1\}, \{r2\}, \{r3\}\}]}{\langle c1, c2\rangle \langle c2, c3\rangle \langle c3, c1\rangle \langle r1, r2\rangle \langle r2, r3\rangle \langle r3, r1\rangle \text{ /. rep}}\right);$$

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify) // N[#, 20] &
(ϕ[1, 2, 5][2, 4, 5] // Simplify) // N[#, 20] &
```

200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Fri 11:17 AM

Lecture December 14.nb

Gluons

$$\frac{1}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle}$$

Hodges

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/8};
```

$$\Phi = \text{Table}\left[\text{If}\left[i == j, -\sum_{k=1}^{n}\text{If}\left[k == i, 0, \frac{\|i, k\|\langle k, r\rangle\langle k, l\rangle}{\langle i, k\rangle\langle i, r\rangle\langle i, l\rangle}\right], \frac{\|i, j\|}{\langle i, j\rangle}\right], \{i, n\}, \{j, n\}\right];$$

$$\phi[c1\_, c2\_, c3\_][r1\_, r2\_, r3\_] := \left(\frac{\text{Det@Delete}[\text{Delete}[\Phi \text{ /. rep}, \{\{c1\}, \{c2\}, \{c3\}\}]^{T}, \{\{r1\}, \{r2\}, \{r3\}\}]}{\langle c1, c2\rangle\langle c2, c3\rangle\langle c3, c1\rangle\langle r1, r2\rangle\langle r2, r3\rangle\langle r3, r1\rangle \text{ /. rep}}\right);$$

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify) // N[#, 20] &
(ϕ[1, 2, 5][2, 4, 5] // Simplify) // N[#, 20] &
```

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Lecture December 14.nb

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/8};
```

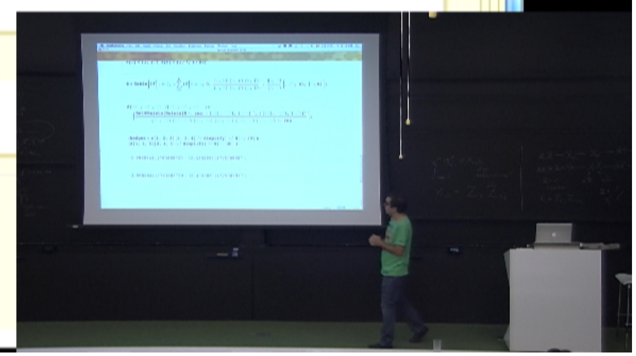
```
Φ = Table[If[i == j, -Sum[If[k == i, 0, (‖i, k‖ ⟨k, r⟩ ⟨k, l⟩)/(⟨i, k⟩ ⟨i, r⟩ ⟨i, l⟩)], {k, 1, n}], ‖i, j‖/⟨i, j⟩], {i, n}, {j, n}];
```

```
ϕ[c1_, c2_, c3_][r1_, r2_, r3_] :=
  (Det@Delete[Delete[Φ /. rep, {{c1}, {c2}, {c3}}]ᵀ, {{r1}, {r2}, {r3}}] /
     (⟨c1, c2⟩ ⟨c2, c3⟩ ⟨c3, c1⟩ ⟨r1, r2⟩ ⟨r2, r3⟩ ⟨r3, r1⟩ /. rep));
```

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify) // N[#, 20] &
(ϕ[1, 2, 5][2, 4, 5] // Simplify) // N[#, 20] &
```

```
-3.9980844117838857 29 + 32.430289114725088987 i
```

```
-3.9980844117838857 29 + 32.430289114725088987 i
```



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Lecture December 14.nb

Hodges

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/18};
```

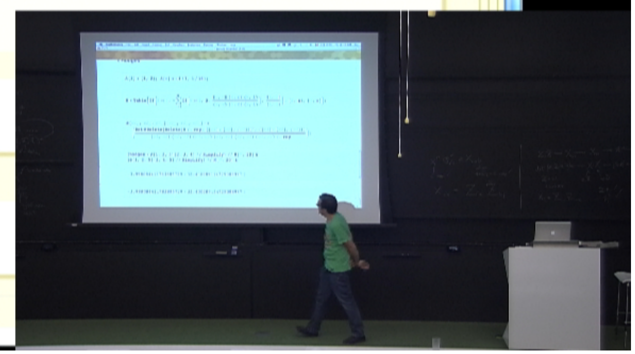
```
Φ = Table[If[i == j, -Sum[If[k == i, 0, (‖i, k‖ ⟨k, r⟩ ⟨k, l⟩)/(⟨i, k⟩ ⟨i, r⟩ ⟨i, l⟩)], {k, 1, n}], ‖i, j‖/⟨i, j⟩], {i, n}, {j, n}];
```

```
ϕ[c1_, c2_, c3_][r1_, r2_, r3_] :=
  (Det@Delete[Delete[Φ /. rep, {{c1}, {c2}, {c3}}]ᵀ, {{r1}, {r2}, {r3}}] /
   ⟨c1, c2⟩ ⟨c2, c3⟩ ⟨c3, c1⟩ ⟨r1, r2⟩ ⟨r2, r3⟩ ⟨r3, r1⟩ /. rep);
```

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify) // N[#, 20] &
(ϕ[1, 2, 5][2, 4, 5] // Simplify) // N[#, 20] &
```

```
-3.9980844117838885729 + 32.430289114725088987 i
```

```
-3.9980844117838885729 + 32.430289114725088987 i
```



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Lecture December 14.nb

Hodges

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/18};
```

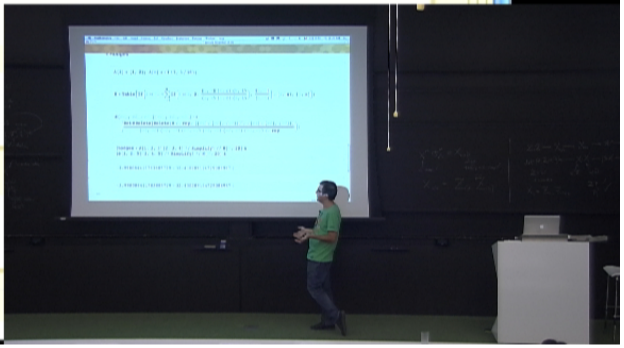
$$\Phi = \mathrm{Table}\left[\mathrm{If}\left[i == j, -\sum_{k=1}^{n} \mathrm{If}\left[k == i, 0, \frac{\|i,k\| \langle k, r\rangle \langle k, l\rangle}{\langle i, k\rangle \langle i, r\rangle \langle i, l\rangle}\right], \frac{\|i, j\|}{\langle i, j\rangle}\right], \{i, n\}, \{j, n\}\right];$$

$$\phi[c1\_, c2\_, c3\_][r1\_, r2\_, r3\_] := \left(\frac{\mathrm{Det@Delete}[\mathrm{Delete}[\Phi \mathbin{/.} \mathrm{rep}, \{\{c1\}, \{c2\}, \{c3\}\}]^{\mathsf{T}}, \{\{r1\}, \{r2\}, \{r3\}\}]}{\langle c1, c2\rangle \langle c2, c3\rangle \langle c3, c1\rangle \langle r1, r2\rangle \langle r2, r3\rangle \langle r3, r1\rangle \mathbin{/.} \mathrm{rep}}\right);$$

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify) // N[#, 20] &
(ϕ[1, 2, 5][2, 4, 5] // Simplify) // N[#, 20] &
```

```
-3.9980844117838857 29 + 32.430289114725088987 i
```

```
-3.9980844117838857 29 + 32.430289114725088987 i
```



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Lecture December 14.nb

Hodges

```
λ[l] = {1, 2}; λ[r] = {1/7, 1/18};
```

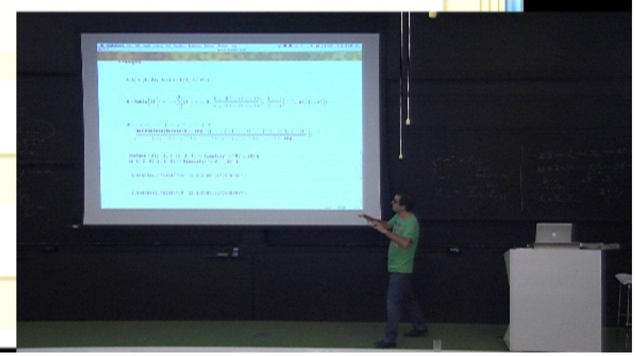
```
Φ = Table[If[i == j, -Sum[If[k == i, 0, (‖i, k‖ ⟨k, r⟩ ⟨k, l⟩)/(⟨i, k⟩ ⟨i, r⟩ ⟨i, l⟩)], {k, 1, n}], ‖i, j‖/⟨i, j⟩], {i, n}, {j, n}];
```

```
ϕ[c1_, c2_, c3_][r1_, r2_, r3_] :=
  (Det@Delete[Delete[Φ /. rep, {{c1}, {c2}, {c3}}]ᵀ, {{r1}, {r2}, {r3}}] /
    ⟨c1, c2⟩ ⟨c2, c3⟩ ⟨c3, c1⟩ ⟨r1, r2⟩ ⟨r2, r3⟩ ⟨r3, r1⟩ /. rep);
```

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify) // N[#, 20] &
(ϕ[1, 2, 5][2, 4, 5] // Simplify) // N[#, 20] &
```

```
-3.9980844117838857 29 + 32.430289114725088987 i
```

```
-3.9980844117838857 29 + 32.430289114725088987 i
```



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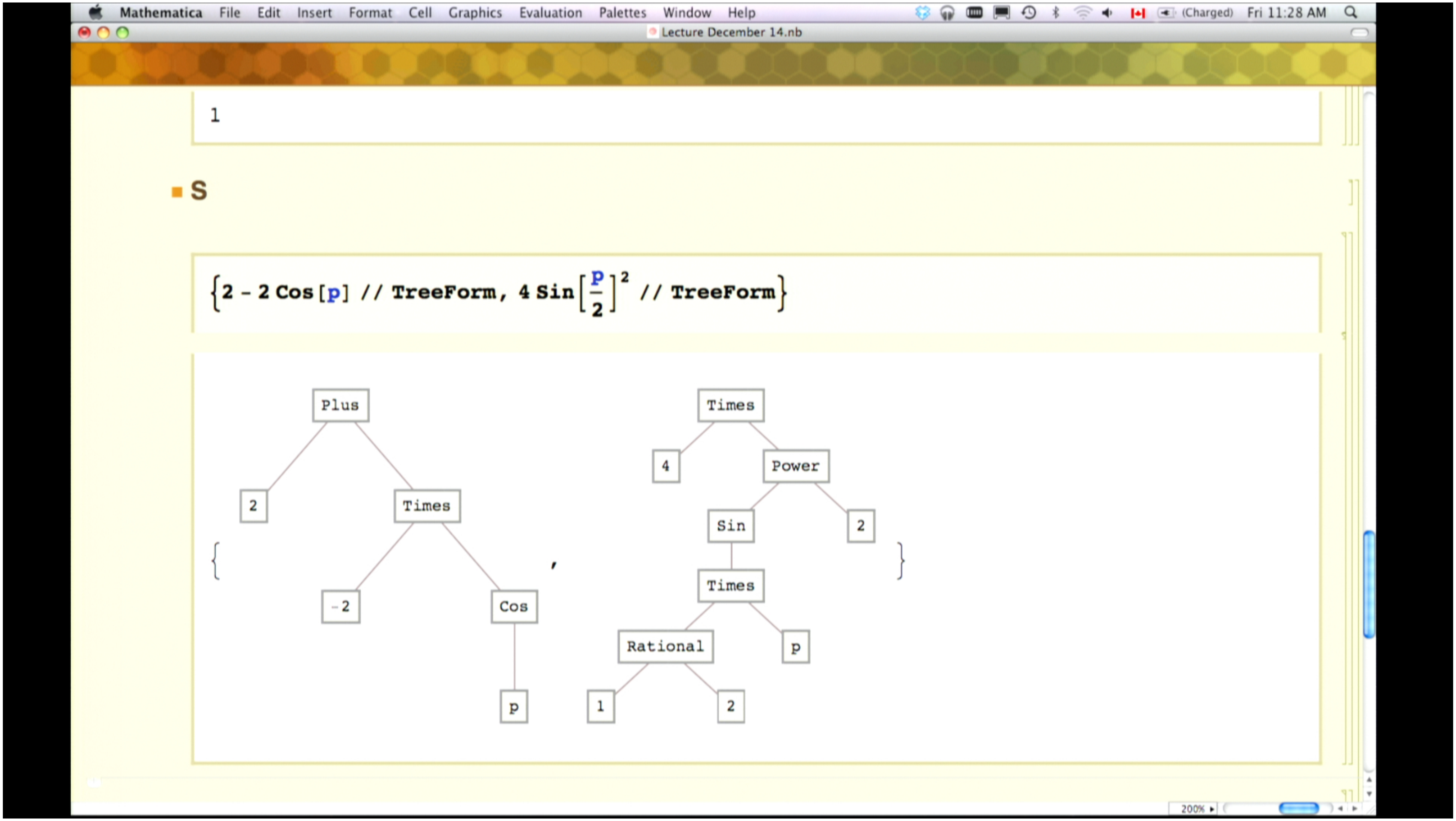
Lecture December 14.nb

```
φ[c1_, c2_, c3_][r1_, r2_, r3_] :=
  (Det@Delete[Delete[Φ /. rep, {{c1}, {c2}, {c3}}]ᵀ, {{r1}, {r2}, {r3}}]
   / (⟨c1, c2⟩ ⟨c2, c3⟩ ⟨c3, c1⟩ ⟨r1, r2⟩ ⟨r2, r3⟩ ⟨r3, r1⟩ /. rep));
```

```
(hodges = ϕ[1, 2, 3][2, 3, 4] // Simplify)
```

```
-11 071 281 553 505 106 047 642 501 132 308 383 954 594 880 902 893 879 410 509 745 431 558 062 993 \
   537 541 476 440 602 325 442 658 770 609 670 463 358 688 720 271 596 004 736 232 689 465 391 399 \
   885 657 185 951 726 865 950 642 087 136 440 674 498 686 462 440 159 815 114 009 783 915 754 391 \
   331 543 552 368 414 277 365 034 385 400 507 399 813 601 170 470 775 983 218 131 986 710 386 577 \
   981 495 086 200 689 396 918 548 297 632 052 236 390 229 836 662 163 286 408 898 093 929 578 609 \
   298 570 049 889 999 793 053 916 908 842 645 753 609 007 330 957 557 633 491 995 137 513 881 875 \
   608 265 121 858 081 390 318 030 084 807 832 603 749 894 783 316 606 976 /
  2 769 146 524 489 027 754 611 650 156 534 680 982 744 548 431 649 589 279 040 617 781 442 685 \
   145 882 738 845 080 610 817 336 569 995 045 804 397 435 522 246 771 750 043 022 350 130 024 \
   066 422 920 318 390 875 341 034 292 642 968 992 543 724 464 221 436 746 268 649 374 882 852 \
   795 748 136 815 135 601 173 171 233 432 230 648 396 847 482 224 565 212 142 369 660 709 320 \
   944 156 274 771 095 242 404 036 058 943 623 413 389 961 895 659 111 932 278 736 079 000 876 \
   111 487 399 503 039 463 766 473 223 146 031 770 311 685 808 445 642 396 834 247 203 494 896 \
   859 436 101 288 851 751 056 134 135 403 335 459 839 260 125 976 108 467 725 326 866 290 514 \
   225 +
 (269 412 667 170 645 986 550 001 367 269 583 463 156 547 004 907 237 100 379 465 458 912 554 716 \
    193 577 269 310 772 637 409 313 487 519 886 897 956 444 387 265 290 385 199 035 738 342 868 \
    932 476 795 796 438 800 002 137 153 750 234 558 634 359 348 992 152 511 738 648 033 561 572 \
```

200%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(Charged) Fri 11:28 AM
Lecture December 14.nb
1
S
{2 - 2 Cos[p] // TreeForm, 4 Sin[p/2]^2 // TreeForm}
Plus
2
Times
-2
Cos
p
Times
4
Power
Sin
2
Times
Rational
p
1
2
200%

Lecture December 14.nb

```
matrix = RandomInteger[{-10, 10}, {4, 4}] + I RandomInteger[{-10, 10}, {4, 4}];
```

$$\texttt{matrixH} = \texttt{N@}\frac{\texttt{matrix} + \texttt{matrix}^{H}}{2};$$

```
U = Eigenvectors@matrixH
```

```
{{-0.224504 + 0.48112 i, -0.091056 - 0.346092 i, -0.337933 + 0.643377 i, -0.248834},
 {-0.227489 + 0.353964 i, 0.648485 - 0.331543 i, 0.491079 - 0.0927447 i, 0.206747},
 {0.488369 - 0.319517 i, 0.297601 - 0.253025 i, -0.260798 + 0.386457 i, 0.538007},
 {-0.348898 + 0.280632 i, -0.40705 + 0.152309 i, -0.0582067 - 0.0368044 i, 0.778386}}
```

```
U.matrixH.IverTranspose[U]
```

```
{{7.01446 + 7.77431 i, 4.70823 + 0.374689 i, -2.71361 - 0.532728 i, 0.558654 + 0.614004 i},
 {5.96128 + 0.47441 i, -5.35194 + 7.12518 i, 0.967337 + 1.46304 i, 0.497898 - 0.16062 i},
 {4.82676 + 0.947577 i, -1.35895 - 2.05533 i, 2.74419 - 4.93896 i, -1.35722 - 1.82939 i},
 {1.55069 + 1.70433 i, 1.09154 - 0.352128 i, 2.11799 + 2.85483 i, -3.78015 + 1.50339 i}}
```

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Lecture December 14.nb

```
matrixH = N@ (matrix + matrix) / 2 ;
```

```
U = Transpose@Eigenvectors@matrixH;
```

```
Inverse[U].matrixH.U
```

$$\{\{-13.2253 - 1.82589\times10^{-15}\,i,\ -8.88178\times10^{-16} + 1.20606\times10^{-15}\,i,$$
$$3.55271\times10^{-15} - 5.05048\times10^{-16}\,i,\ -3.55271\times10^{-15} - 2.44286\times10^{-17}\,i\},$$
$$\{-1.55431\times10^{-15} - 3.69556\times10^{-16}\,i,\ -10.4454 + 8.26252\times10^{-16}\,i,$$
$$-7.32747\times10^{-15} + 1.17112\times10^{-15}\,i,\ 3.55271\times10^{-15} - 1.09357\times10^{-15}\,i\},$$
$$\{6.10623\times10^{-15} - 8.65819\times10^{-16}\,i,\ -6.21725\times10^{-15} - 3.70422\times10^{-16}\,i,$$
$$7.43529 - 3.03452\times10^{-17}\,i,\ -1.5099\times10^{-14} + 3.88372\times10^{-16}\,i\},$$
$$\{-3.21965\times10^{-15} - 2.86313\times10^{-16}\,i,\ 2.66454\times10^{-15} + 1.18504\times10^{-16}\,i,$$
$$-2.08722\times10^{-14} - 7.7172\times10^{-16}\,i,\ -4.76458 - 1.79299\times10^{-16}\,i\}\}$$

200%

```
matrix = RandomInteger[{-10, 10}, {4, 4}] + I RandomInteger[{-10, 10}, {4, 4}];
```

$$\texttt{matrixH} = \texttt{N@}\frac{\texttt{matrix} + \texttt{matrix}^{H}}{2};$$

```
U = Transpose@Eigenvectors@matrixH;
```

```
Inverse[U].matrixH.U // Chop
```

```
{{-13.2253, 0, 0, 0}, {0, -10.4454, 0, 0}, {0, 0, 7.43529, 0}, {0, 0, 0, -4.76458}}
```

```
matrix = RandomInteger[{-10, 10}, {4, 4}] + I RandomInteger[{-10, 10}, {4, 4}];
```

```
matrix^H - Transpose@Conjugate@matrix;
```

$$\text{matrixH} = \text{N@}\frac{\text{matrix} + \text{matrix}^{H}}{2};$$

```
U = Transpose@Eigenvectors@matrixH;
```

```
Inverse[U].matrixH.U // Chop
```

```
{{-13.2253, 0, 0, 0}, {0, -10.4454, 0, 0}, {0, 0, 7.43529, 0}, {0, 0, 0, -4.76458}}
```