Title: If no information gain implies no disturbance, then any discrete physical theory is classical

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Abstract: (based on http://arxiv.org/abs/1210.0194)

It has been suggested that nature could be discrete in the sense that the underlying state space of a physical system has only a finite number of pure states. For example, the Bloch ball of a single qubit could be discretized into small patches and only appear round to us due to experimental limitations. Here, we present a strong physical argument for the quantum theoretical property that every state space (even the smallest possible one, the qubit) has infinitely many pure states. We propose a simple physical postulate which dictates that in fact the only possible discrete theory is classical mechanics. More specifically, we postulate that no information gain implies no disturbance, or read in the contrapositive, that disturbance leads to some form of information gain. In a theory like quantum mechanics where we already know that the converse holds, i.e. information gain does imply disturbance, this can be understood as postulating an equivalence between disturbance and information gain. What is more, we show that non-classical discrete theories are still ruled out even if we relax the postulate to hold only approximately in the sense that no information gain only causes a small amount of disturbance. Finally, our postulate also rules out popular generalizations such as the PR-box that allows non-local correlations beyond the limits of quantum theory.

If no information gain implies no disturbance, then any discrete physical theory is classical

Corsin Pfister (Speaker) & Stephanie Wehner

based on arXiv:1210.0194

Talk outline

1) Outline of the main result

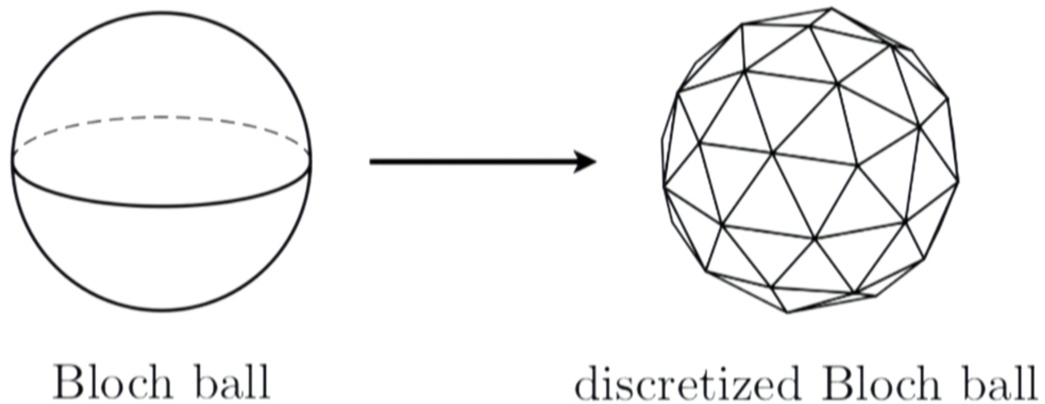
2) The framework

3) Result (and proof idea)

4) Discussion

1) Outline of the main result

- Quantum systems: continuously many pure state



- **Motivating question**: Could it be that physical state spaces are discrete (finitely many pure states)?
- Our main result is a strong argument for saying **no**.

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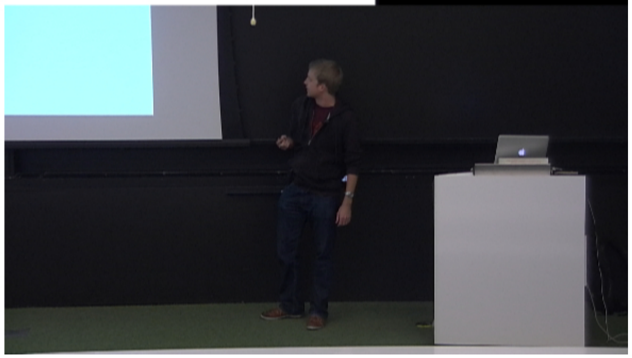

1) Outline of the main result

- Result formulated in a framework of generalized probabilistic theories (GPTs) / convex framework
- There: states form *any* convex set
- **Postulate:** Every (pure) measurement can be performed in a way which leaves the states with a definite outcome invariant
  (no information gain $\longrightarrow$ no disturbance)

1) Outline of the main result

- Result formulated in a framework of generalized probabilistic theories (GPTs) / convex framework

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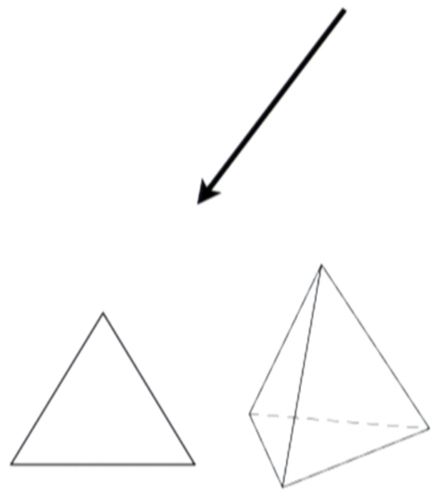
- **Postulate:** Every (pure) measurement can be performed in a way which leaves the states with a definite outcome invariant
  (no information gain $\longrightarrow$ no disturbance)

- **Main result:** A GPT with finitely many pure states is either classical or it violates the postulate.

1) Outline of the main result

Theories with finitely many pure states:

either



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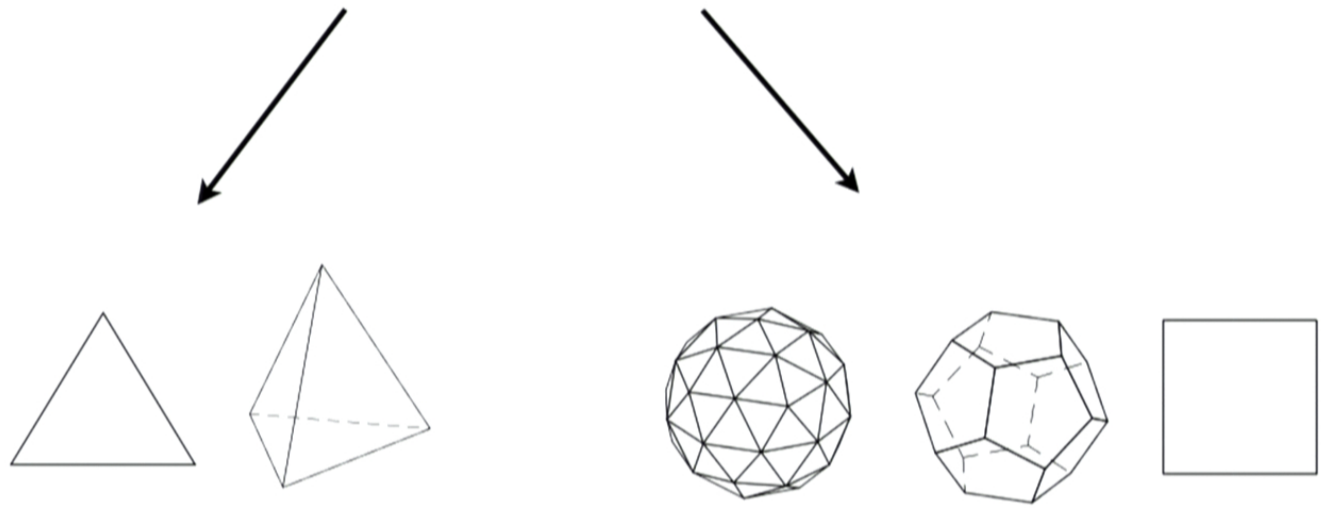

classical (simplex) violate postulate

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2) The framework

We work in the *abstract state space* formalism, which is based on four assumptions:

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- Assumption 1: states form convex subset Ω of a real vector space A
  (QT: density operators in $\mathrm{Herm}(\mathcal{H})$)
  Idea: probabilistic mixture $\simeq$ convex sum

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- Assumption 3: $\Omega \subset A$ is compact

2) The framework

States

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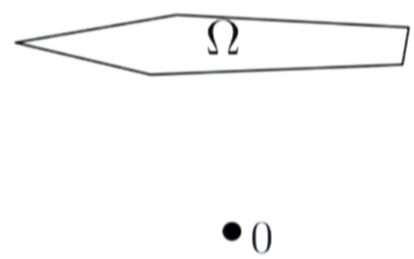
in a vector space A:

- compact convex subset of normalized states Ω



2) The framework

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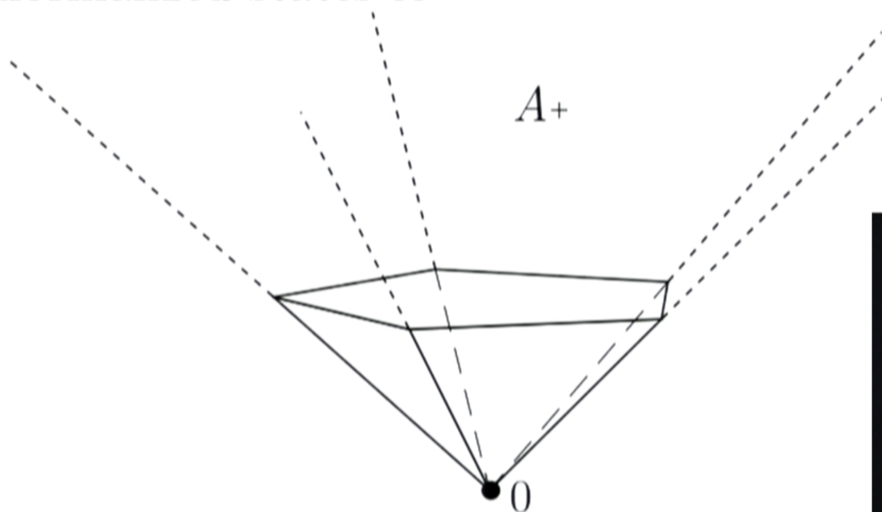
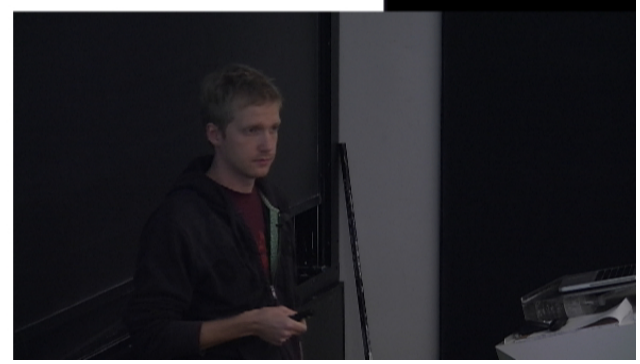
- subnormalized states $\Omega^{\leq 1}$

or

- closed cone A_+





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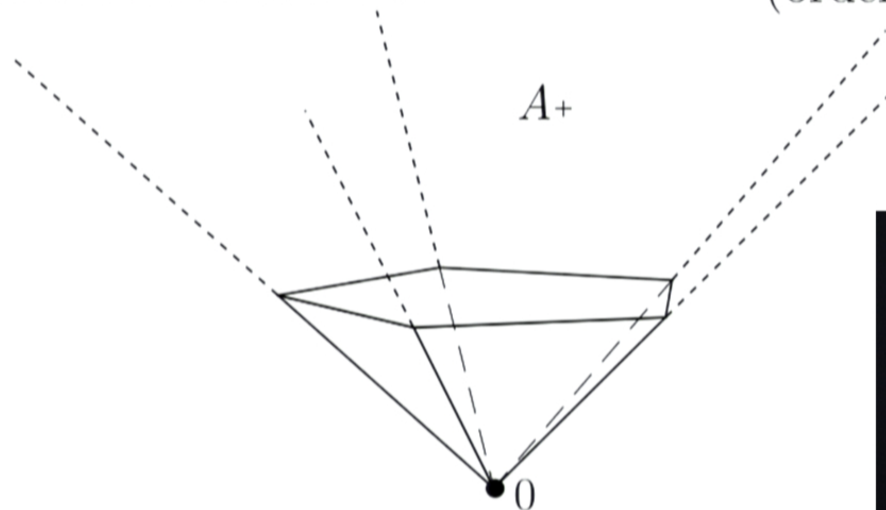

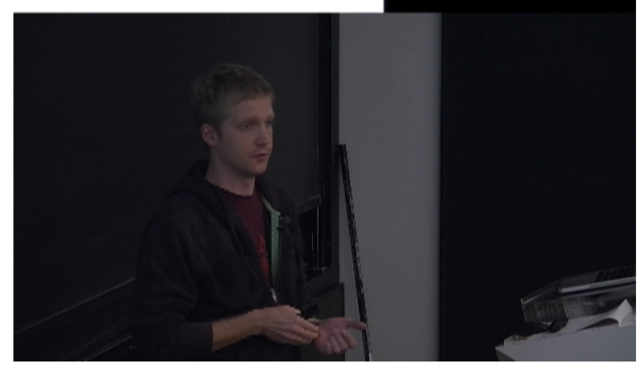

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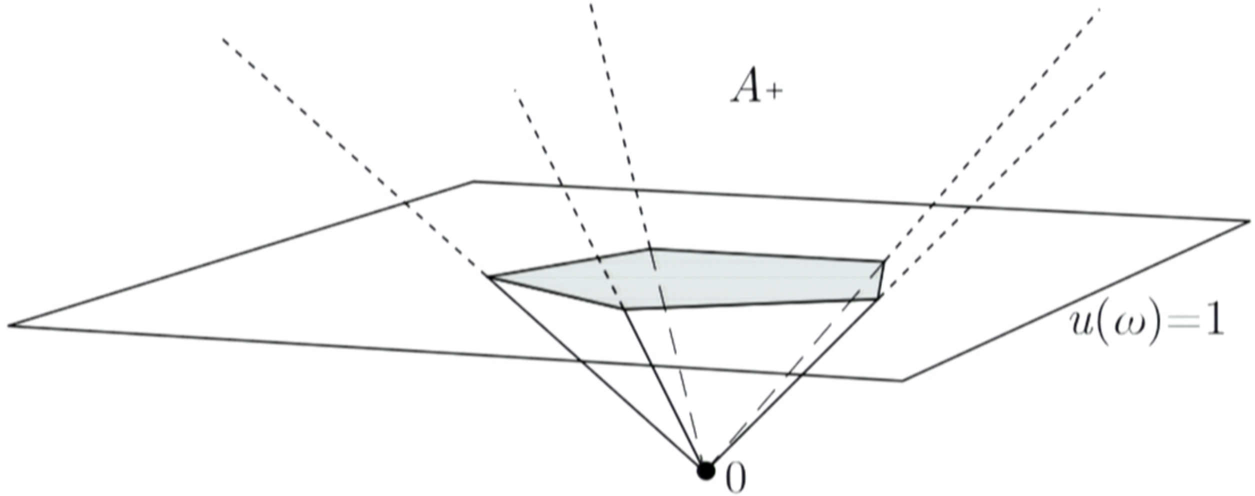

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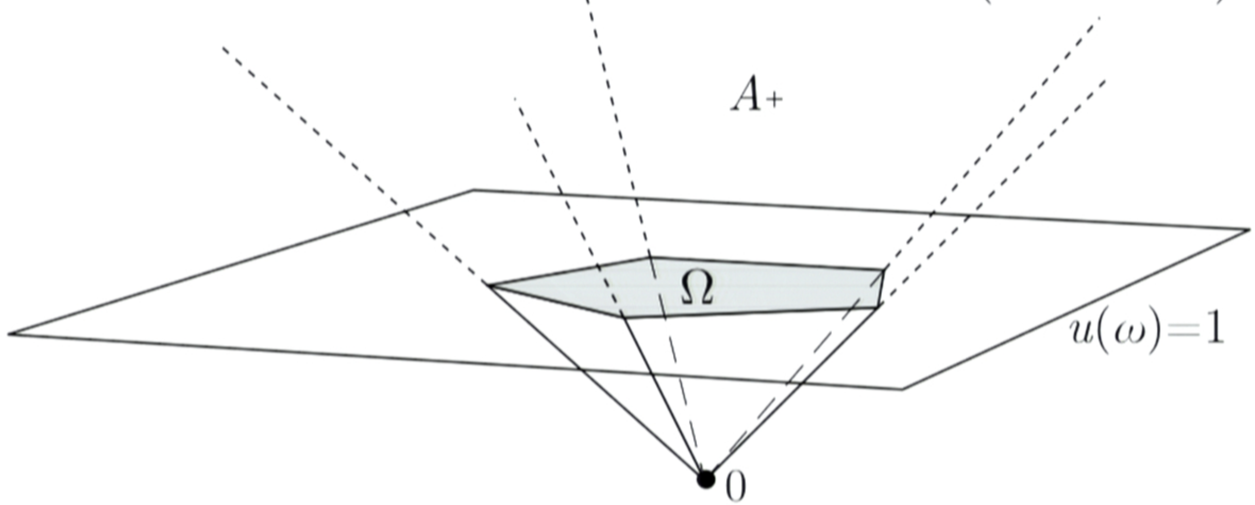

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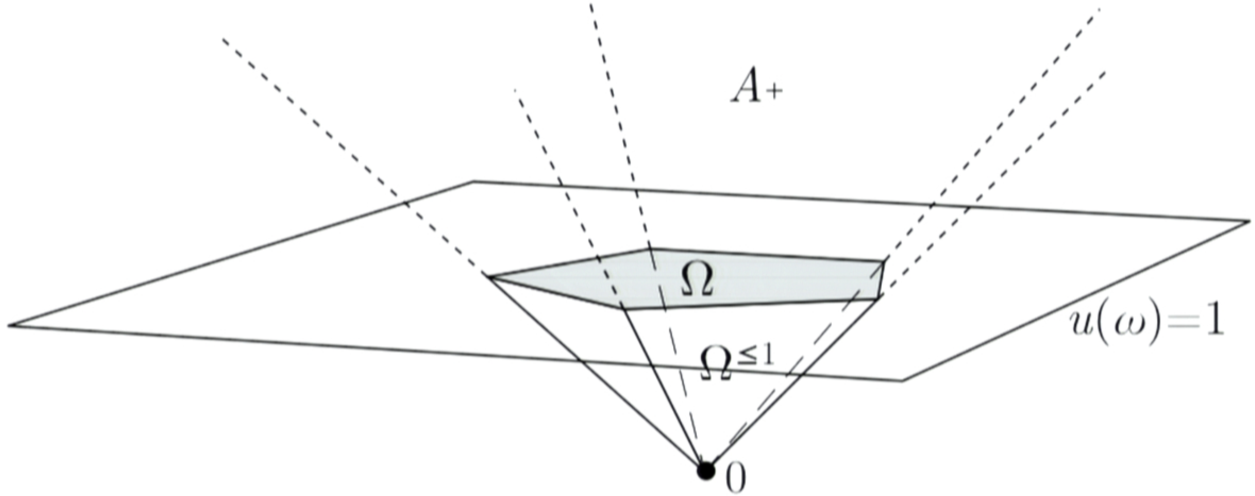

2) The framework

States

- Abstract state space: (A, A_+, u), where
  - A finite-dimensional real vector space
  - A_+ cone in A (closed and generating)
  - u linear functional (order unit)

2) The framework

Measurements

- Assumption 4: Every mathematically well-defined measurement is allowed

  $\longrightarrow$ triple $(A,\ A_+,\ u)$ fully determines the structure of one-shot measurement statistics (POVMs in QT)

2) The framework

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- effects: linear functionals in A^* which are
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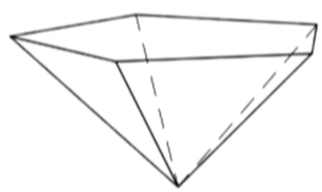

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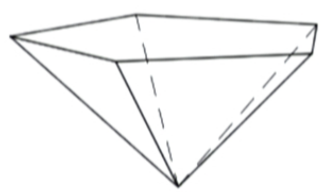

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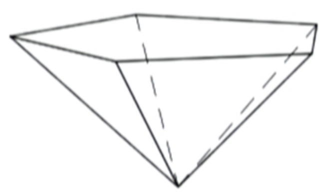

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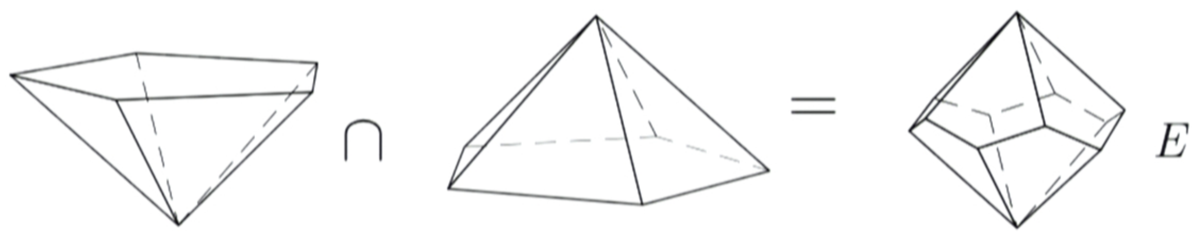

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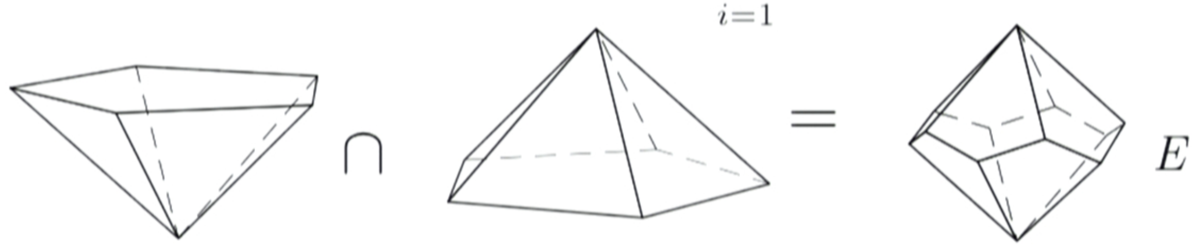
- this gives the *set of effects* E
- measurement $\{f_1, ..., f_n\}$ $\rightarrow$ $\sum_{i=1}^{n} f_i(\omega) = 1 \quad \forall \omega \in \Omega$



2) The framework

Examples

- Quantum Theory:
  - $A = \mathrm{Herm}(\mathcal{H})$ for some Hilbert space $\mathcal{H}$
  - A_+ cone of positive operators
  - u trace
  - Ω set of density operators
  - effects f are induced by POVM elements M
$$f_M : \rho \mapsto \mathrm{tr}(M\rho)$$
  - measurement $\{M_1, \ldots, M_n\} \Rightarrow \sum_{i=1}^{n} M_i = \mathrm{Id}_{\mathcal{H}}$

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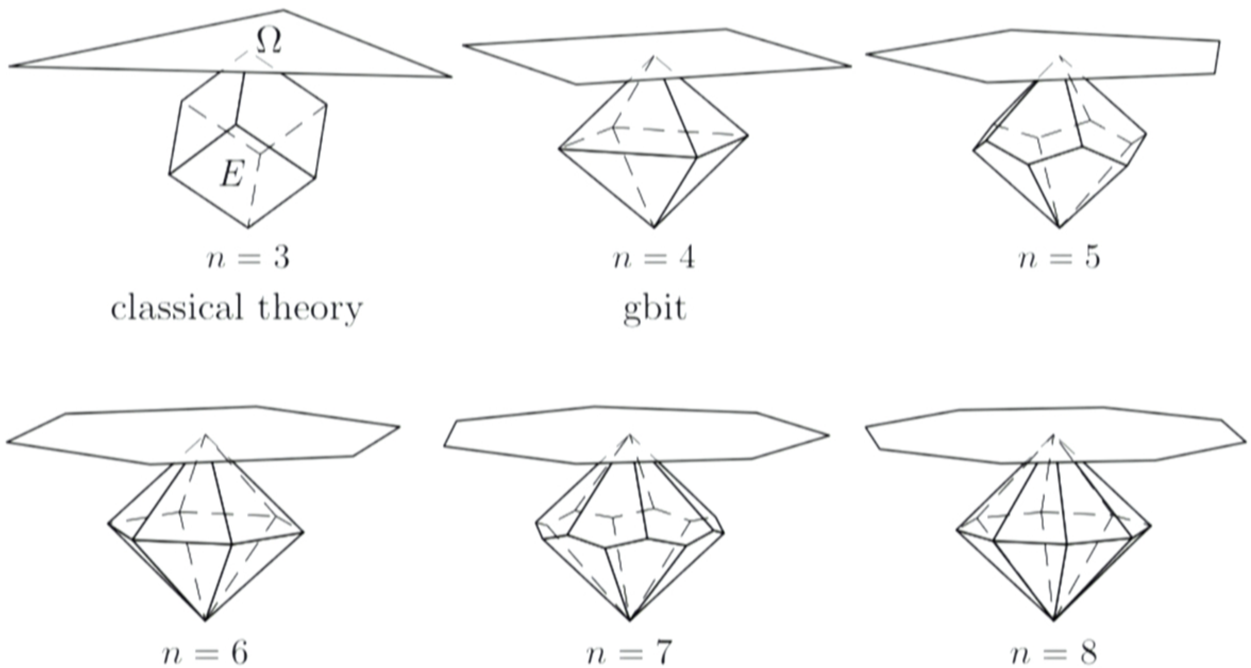
- Polygon models: $A = \mathbb{R}^3 \cong (\mathbb{R}^3)^* = A^*$



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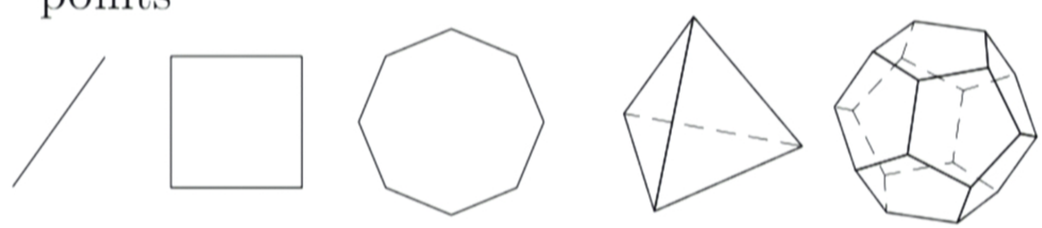
Examples

- Polytopic theories ("discrete" theories):
  - Ω polytope, i.e. convex hull of finitely many points



  - Examples of polytopic theories:
    - polygon models
    - box g-bit, PR-box
    - classical theories

2) The framework

Examples

- Classical theories:
  - Ω is a simplex (the convex hull of affinely independent points)

  - Point in a simplex: *unique* convex combination of the extreme points
    $\longrightarrow$ probability distribution

2) The framework

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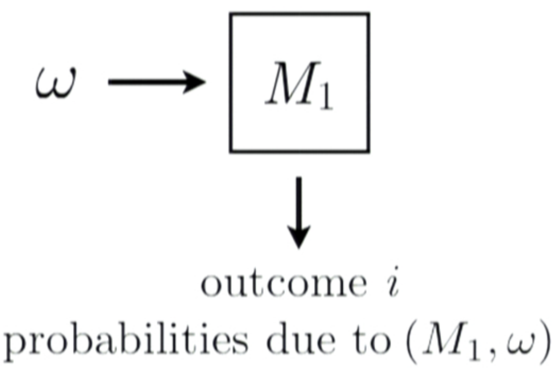

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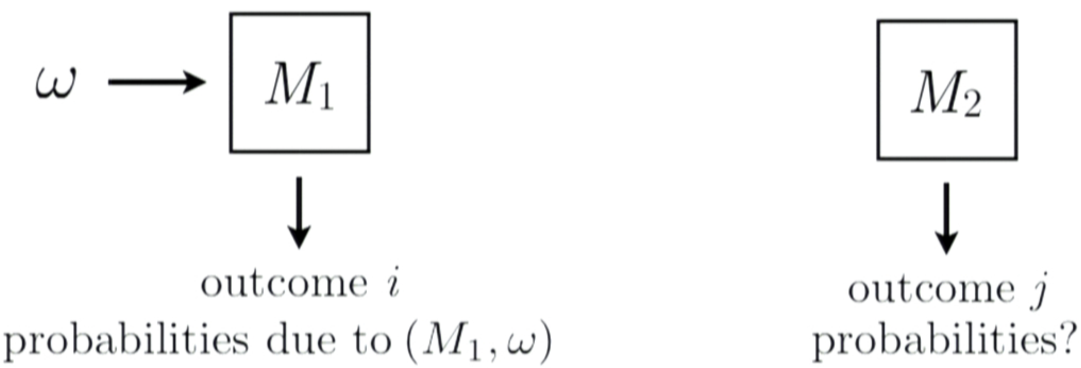

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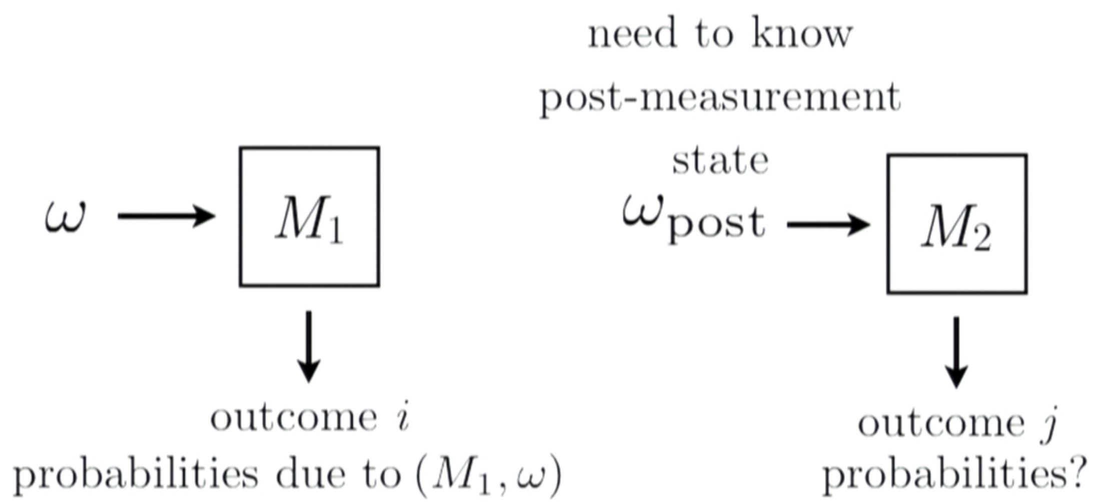

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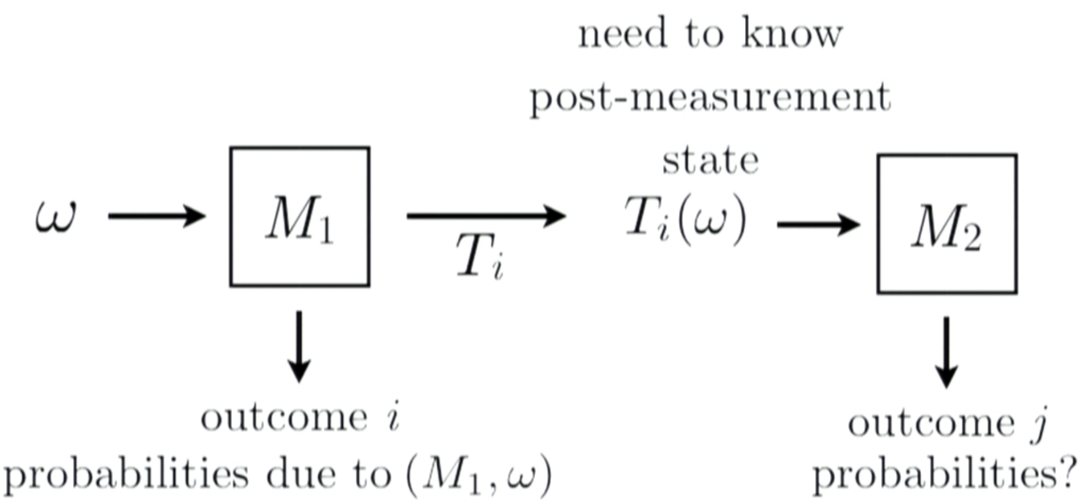

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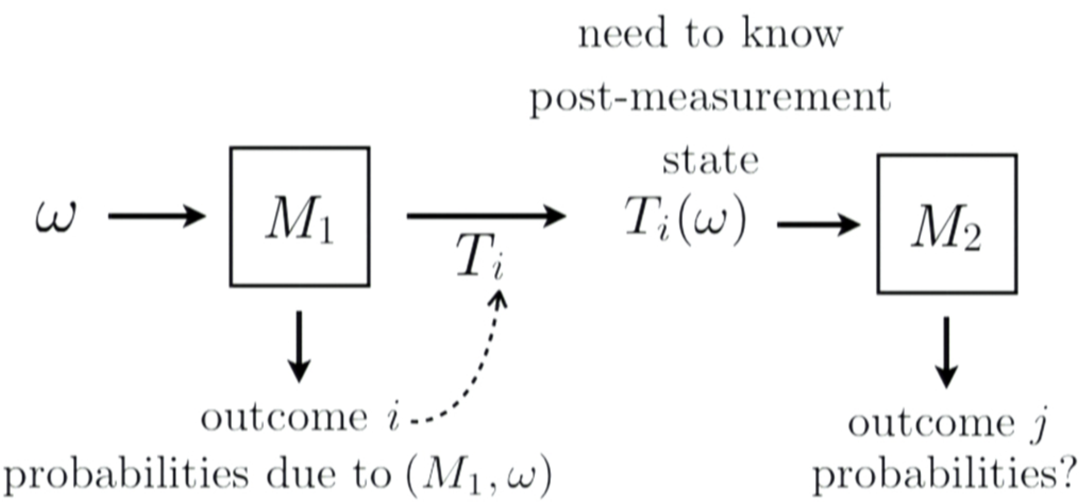
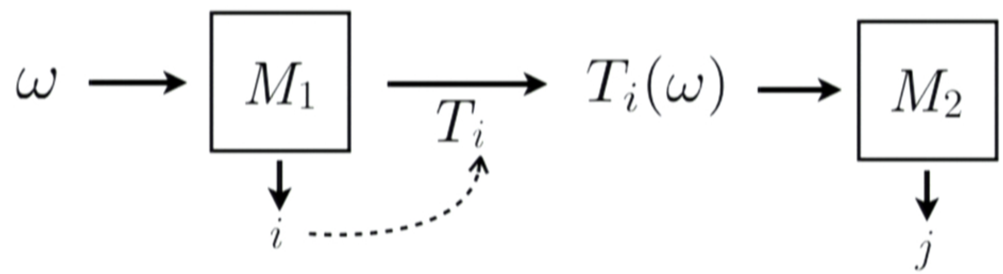


2) The framework

Transformation satisfies:



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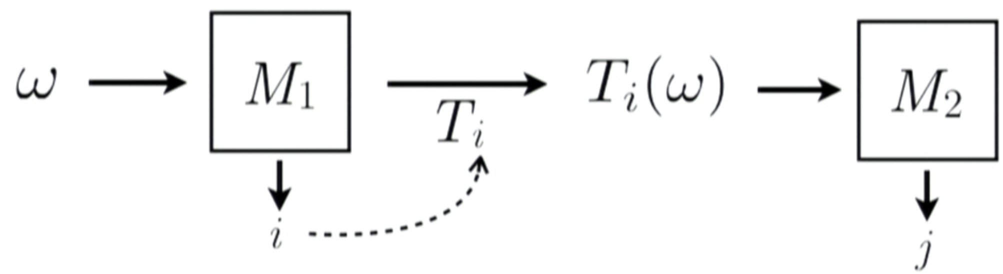
Transformation satisfies:

- compatibility with mixtures $\longrightarrow$ T_i linear



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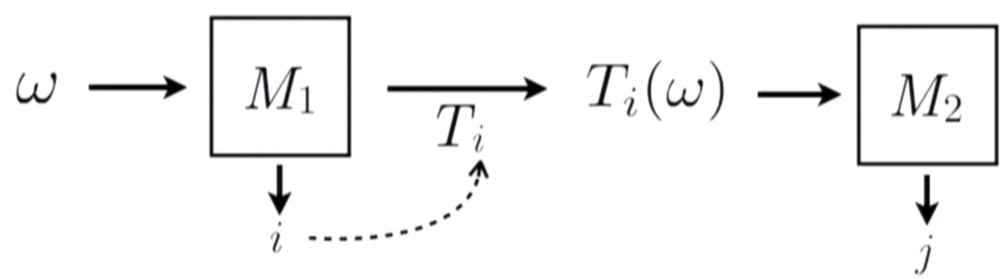
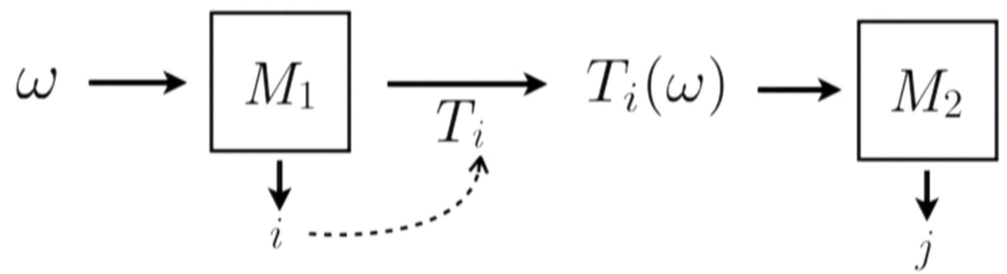


2) The framework

Transformation satisfies:

- compatibility with mixtures $\longrightarrow$ T_i linear
- maps states to states $\longrightarrow$ T_i positive
- induces the effect: $u \circ T_i = f_i$
  motivation: post-measurement normalization
  = probability for outcome



2) The framework

Example: quantum measurement

- measurement outcome i: orth. projector P_i
- transformation $T_i : \rho \mapsto P_i \rho P_i$
  satisfies
  - linearity
  - positivity
  - induces the effect: $u \circ T_i = f_i$

$u = \mathrm{tr}$ $\qquad$ $\mathrm{tr}(P_i \rho P_i) = \mathrm{tr}(P_i \rho) = f_i(\rho)$

$T_i : \rho \mapsto P_i \rho P_i$

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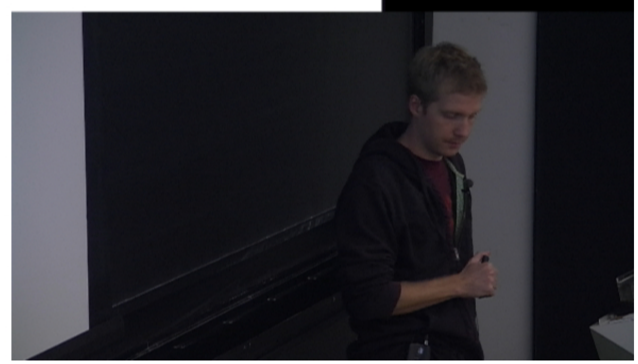
3) Result and proof idea

4) Discussion

3) Result and proof idea

Postulate: Every pure measurement can be performed in a way which leaves the states with a definite outcome invariant, i.e.:



3) Result and proof idea

Postulate: Every pure measurement can be performed in a way which leaves the states with a definite outcome invariant, i.e.:

For every pure effect f, there is a $T : A \to A$

- linear
- positive
- induces the effect, $u \circ T = f$
- leaves states with probability one invariant,

$$f(\omega) = 1 \quad \Rightarrow \quad T(\omega) = \omega$$

3) Result and proof idea

Postulate: Every pure measurement can be performed in a way which leaves the states with a definite outcome invariant

This postulate is satisfied by

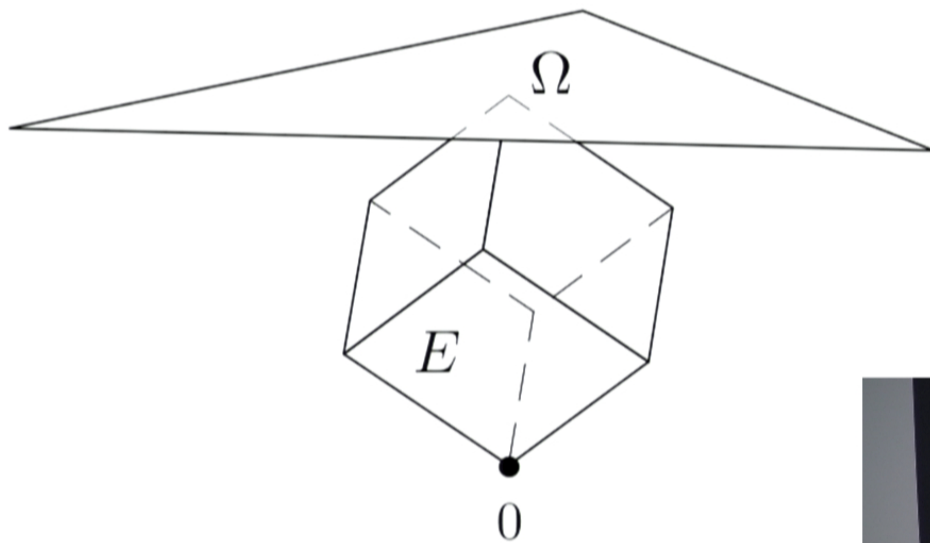
- Quantum theory:

$$\mathrm{tr}(P_i \rho) = 1 \quad \Rightarrow \quad P_i \rho P_i = \rho$$

- Classical theory

3) Result and proof idea

The postulate in classical theory:



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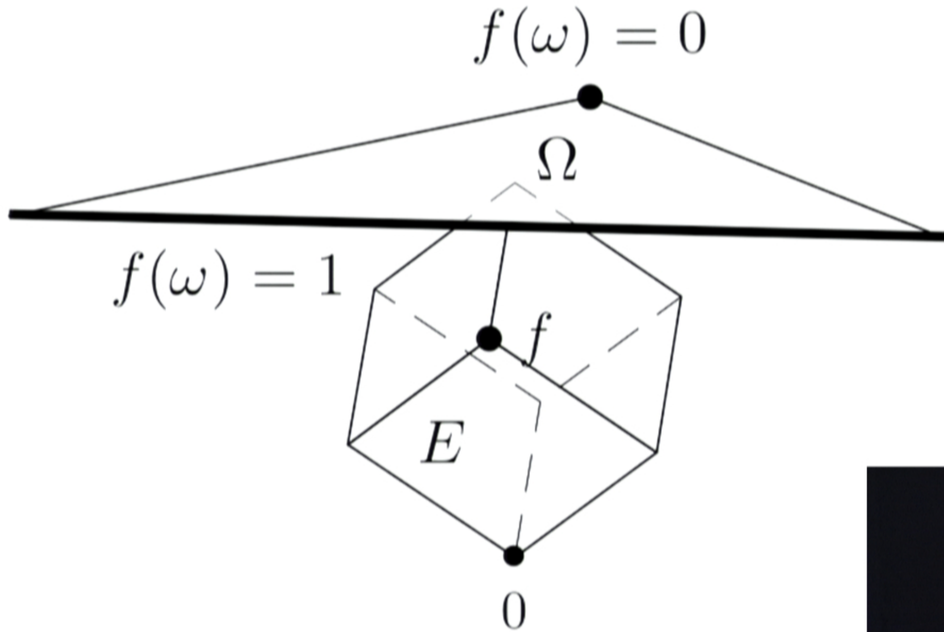

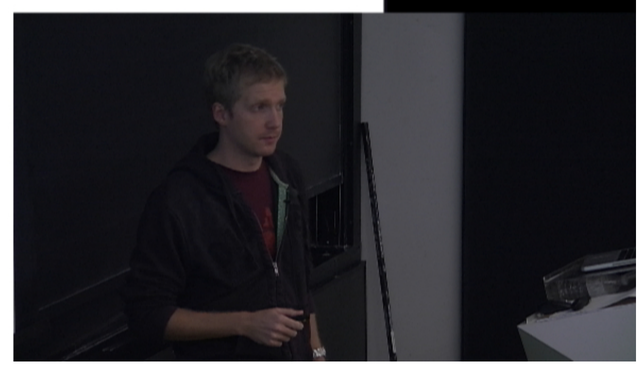

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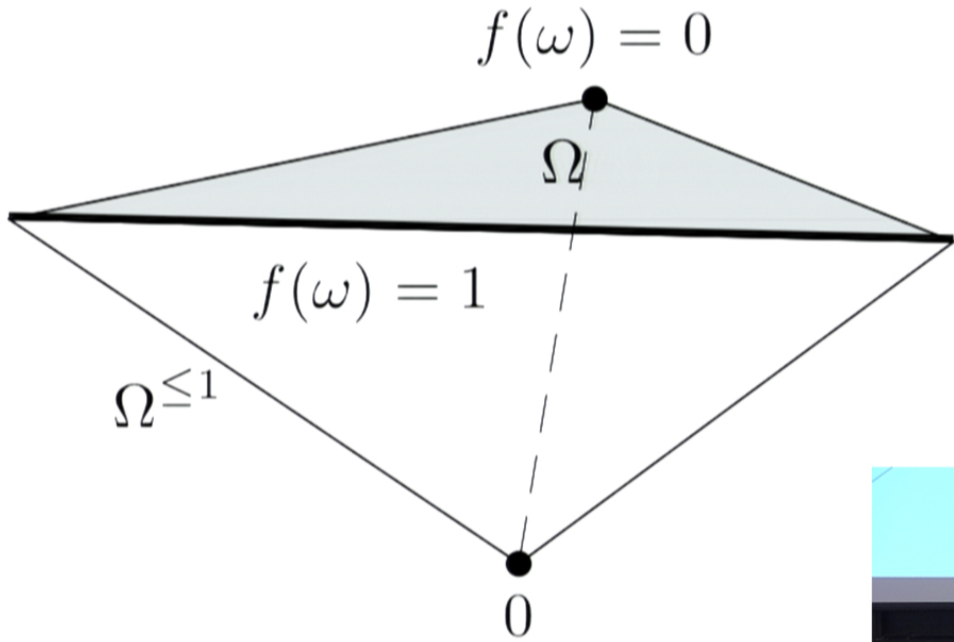

3) Result and proof idea

- However, *many* theories violate the postulate!
- **Theorem:** Let (A, A_+, u) be a discrete theory (i.e. Ω is a polytope) such that the postulate is satisfied. Then Ω is a simplex, i.e. (A, A_+, u) is a classical theory.

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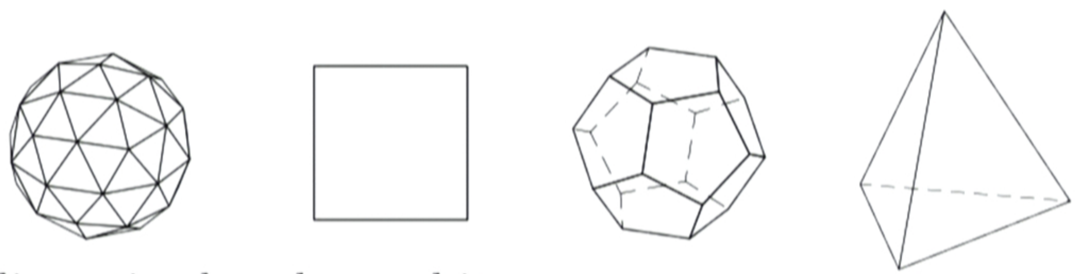

discretized Bloch sphere — box g-bit — classical

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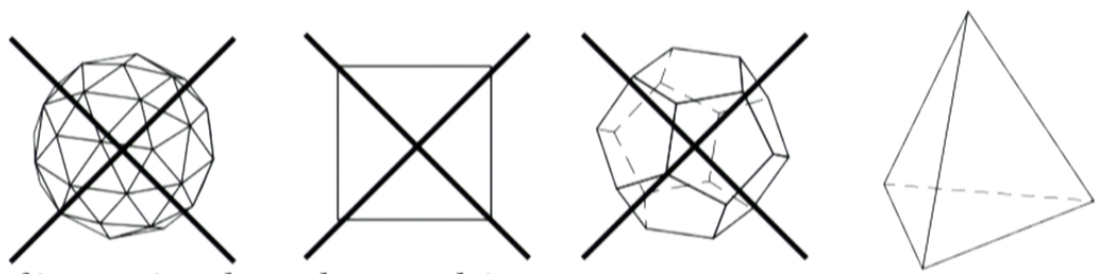

discretized Bloch sphere   box g-bit   classical

3) Result and proof idea

- Proof is mainly based on the following lemma:

  Lemma Let (A, A_+, u_A) be an abstract state space, let $f \in E_A$ be a pure effect. If there exists a transformation $T : A \to A$ such that $u_A \circ T = f$ and $T(\omega) = \omega$ for every $\omega \in F_f$, then

  (a) $\dim F_f + \dim \overline{F}_f \leq \dim \Omega_A - 1$ and

  (b) $\mathrm{aff}(F_f \cup \overline{F}_f) \cap \Omega_A = \mathrm{conv}(F_f \cup \overline{F}_f)$,

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- (a) violated $\longrightarrow$ dimension mismatch

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- (a) violated $\longrightarrow$ dimension mismatch
- (b) violated $\longrightarrow$ shape mismatch

3) Result and proof idea

Some intuition

- Two basic examples:
  - the square  ⟶ dimension mismatch
  - the pentagon  ⟶ shape mismatch
- For every non-classical polytope, dimension or shape mismatch occurs

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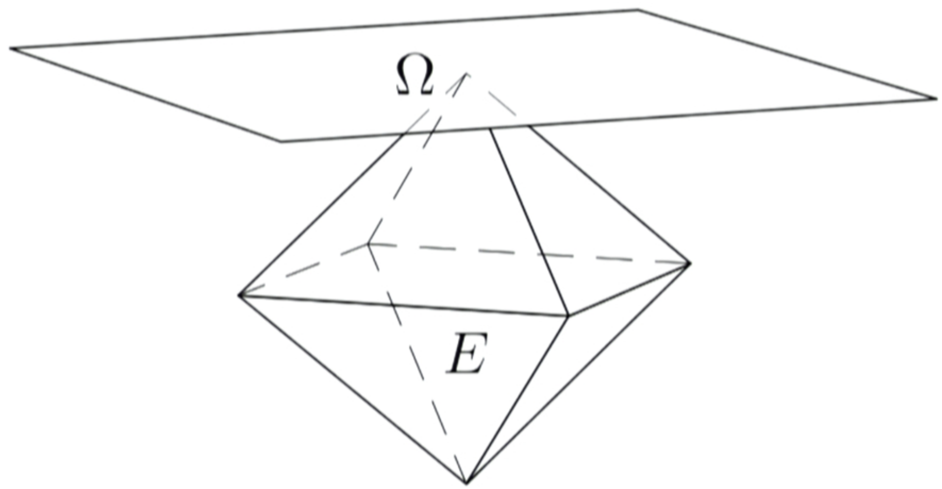
The square (box-gbit) violates the postulate:



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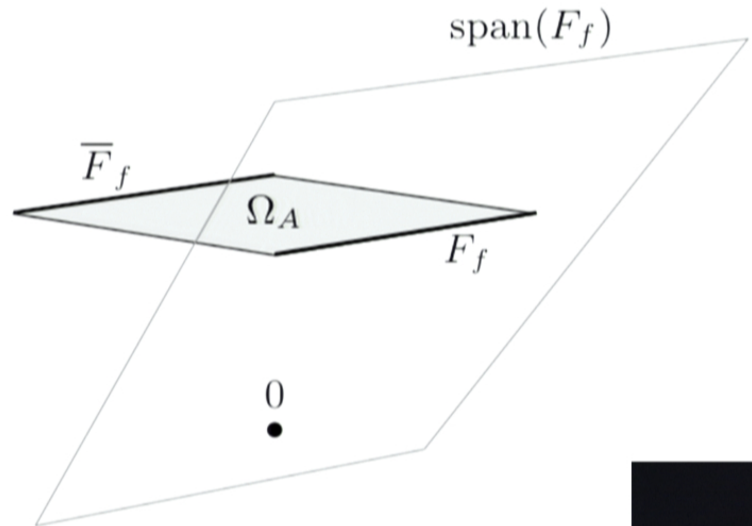

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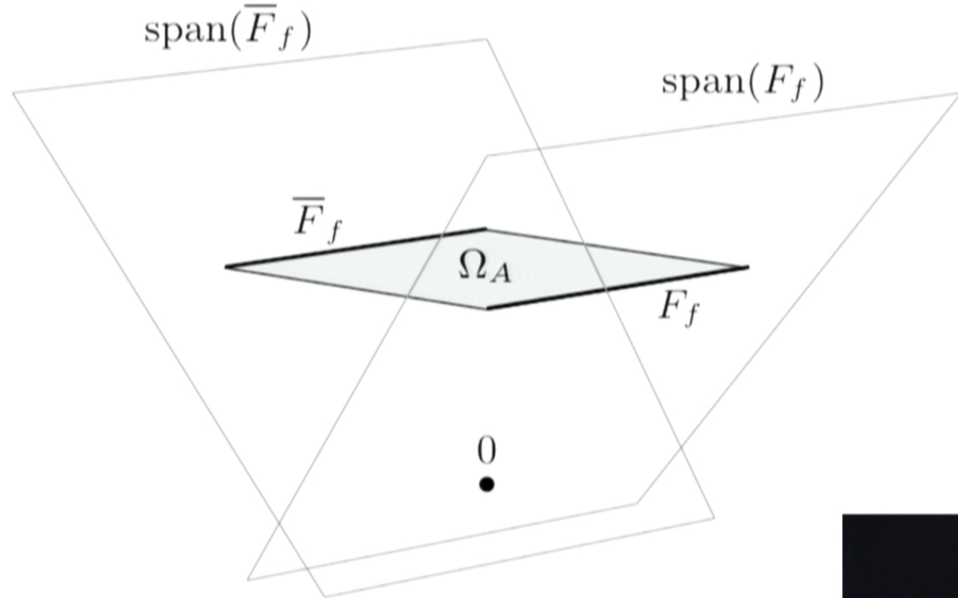

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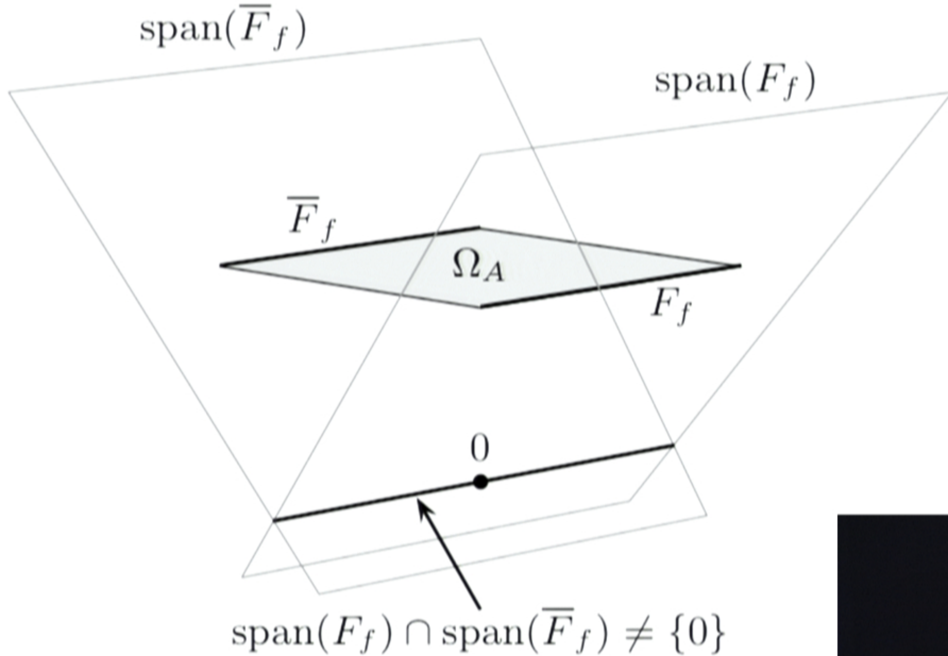

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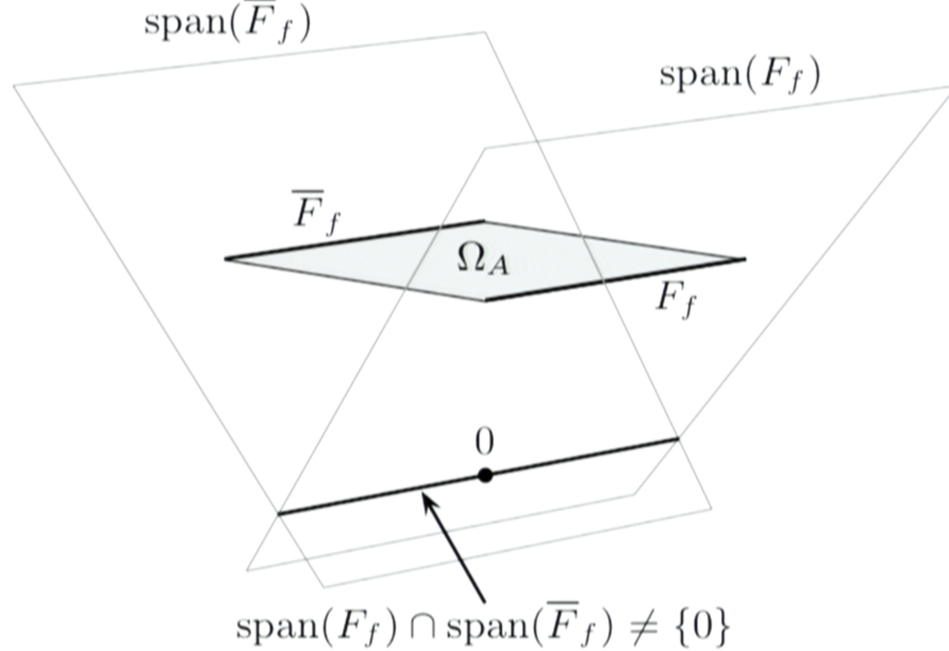

Postulate & linearity $\longrightarrow$ dimension mismtach

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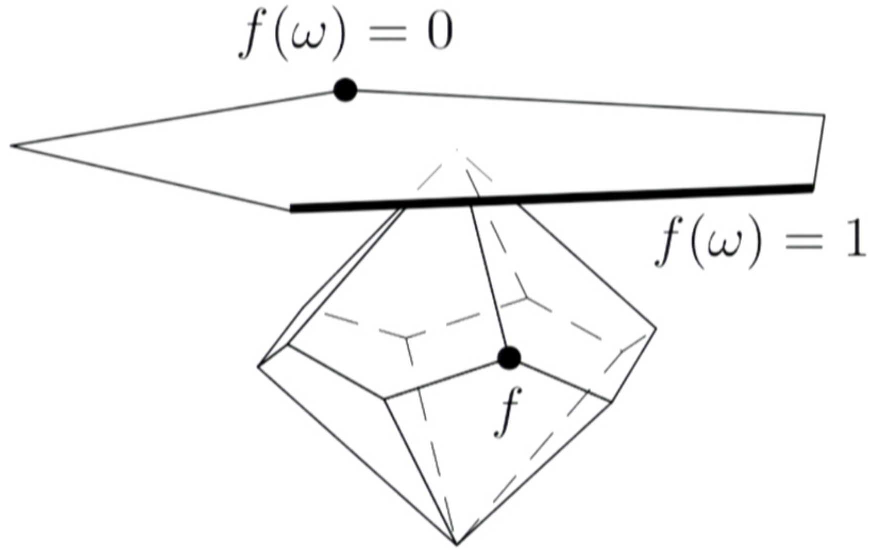

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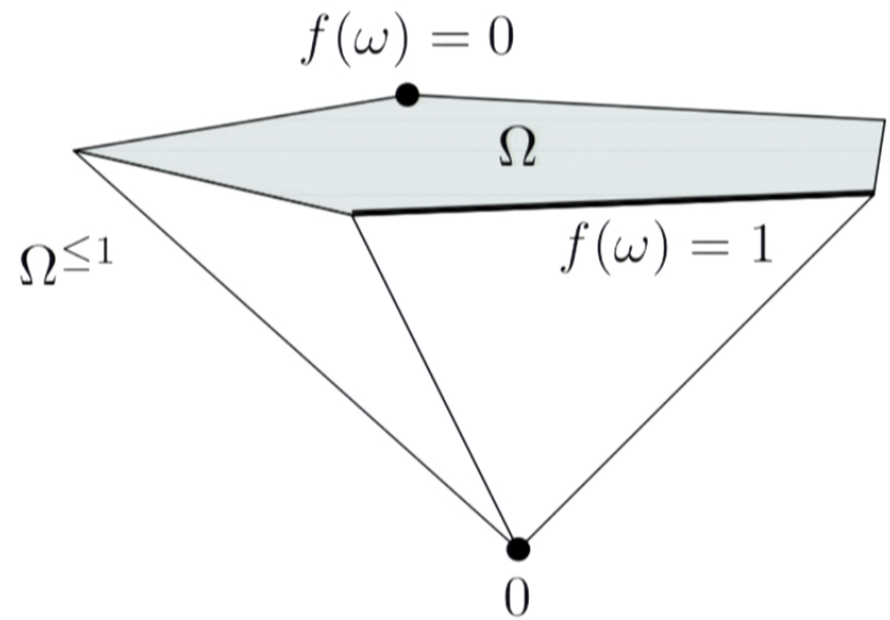

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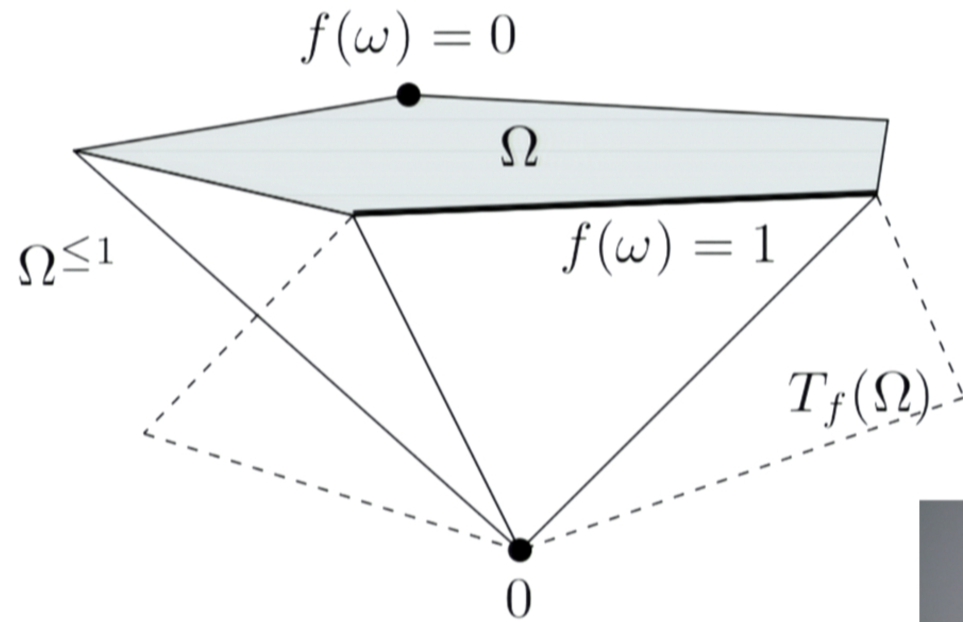

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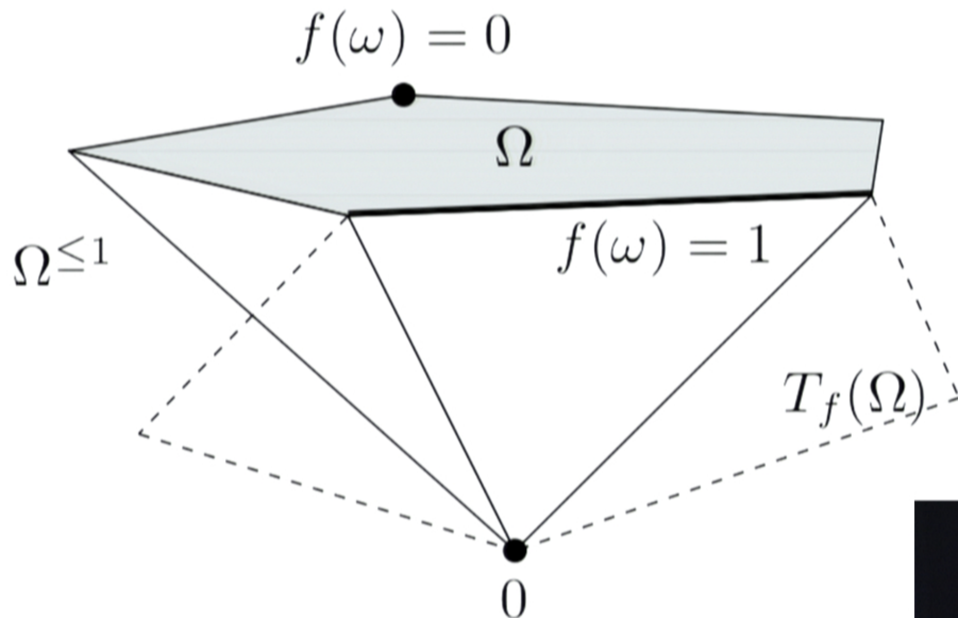


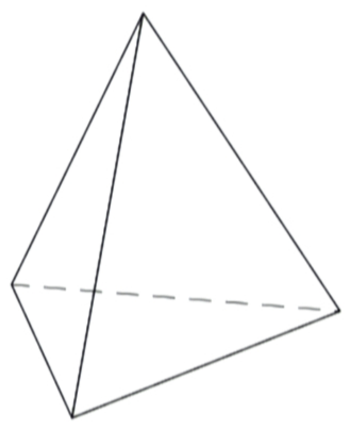
Postulate, linearity & positivity

$\longrightarrow$ shape mismatch

3) Result and proof idea

The proof idea

If neither of the mismatches occurs, then:
For every facet F of the polytope Ω, there is a $\omega \in \Omega$ such that $\Omega = \text{conv}(F \cup \{\omega\})$.



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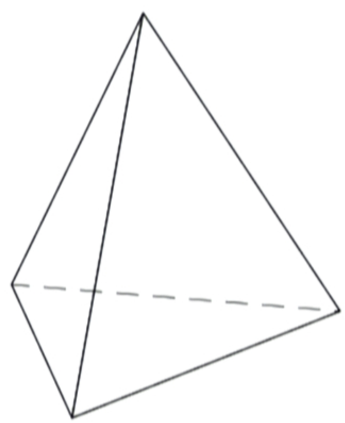

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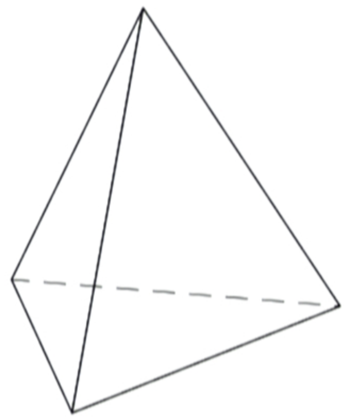

However, every polytope with this property is a simplex.

1) Outline of the main result

2) The framework

3) Result and proof idea

4) Discussion

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Robustness of the theorem

- Possible objection to the result:
  Verification of the condition
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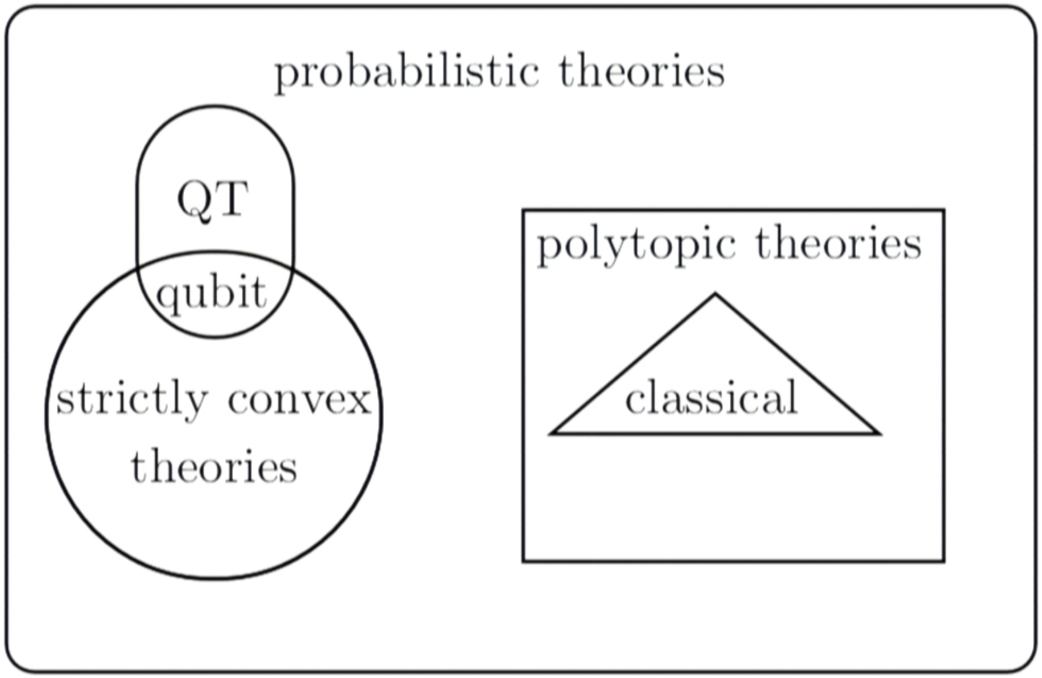
Ovierview over excluded theories



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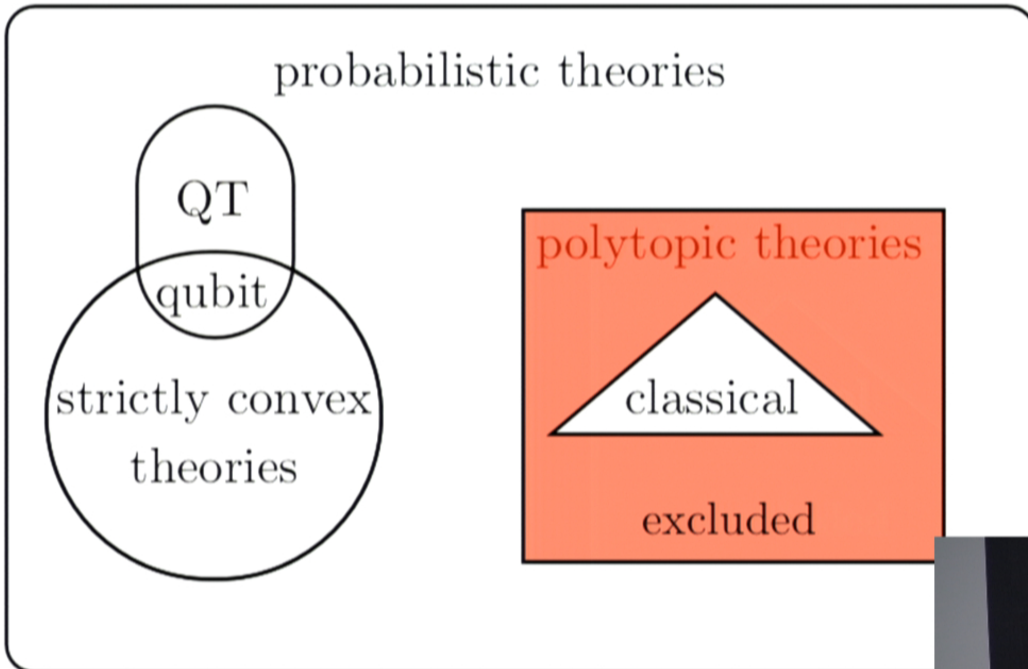

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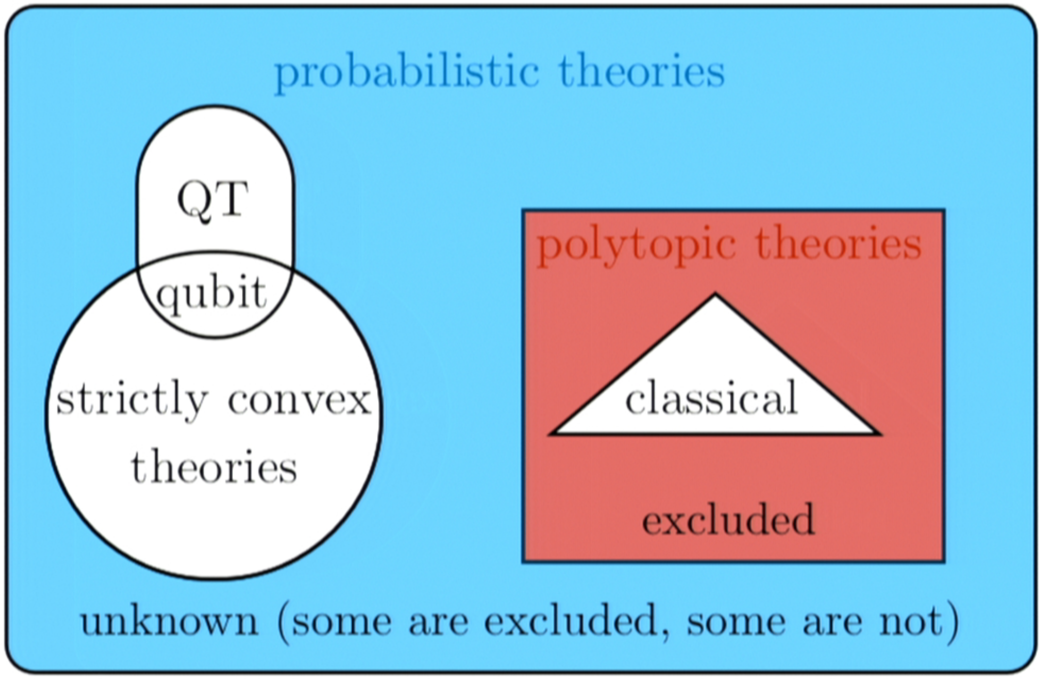

postulate

4) Discussion

Starting point for physical derivation of QT?

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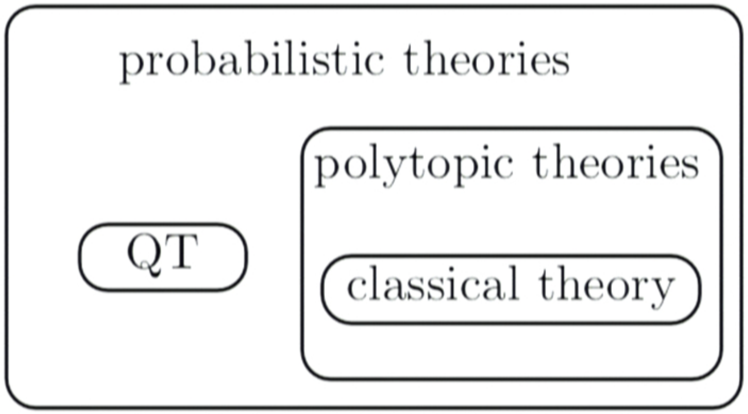

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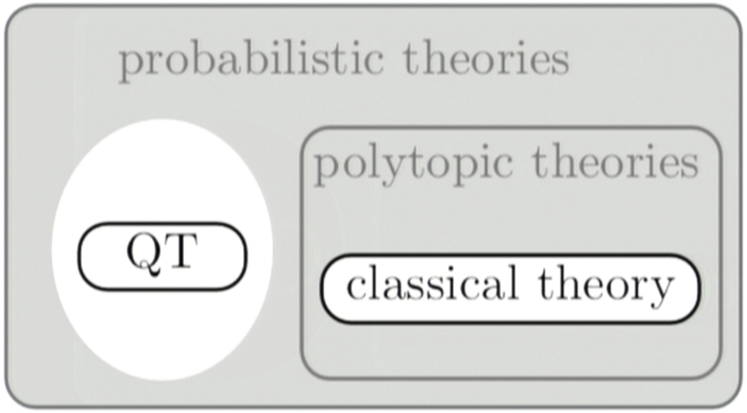

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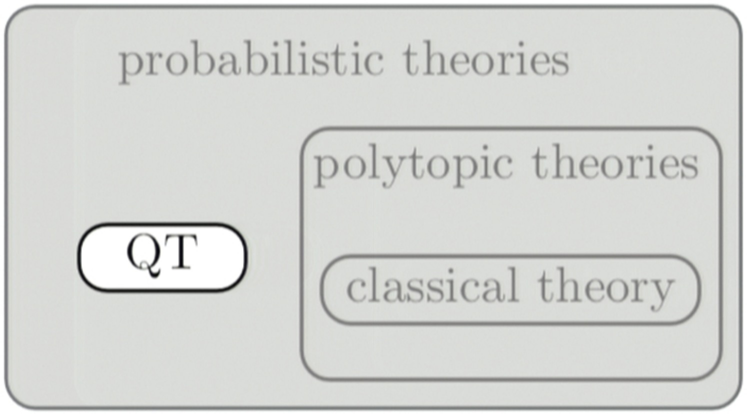

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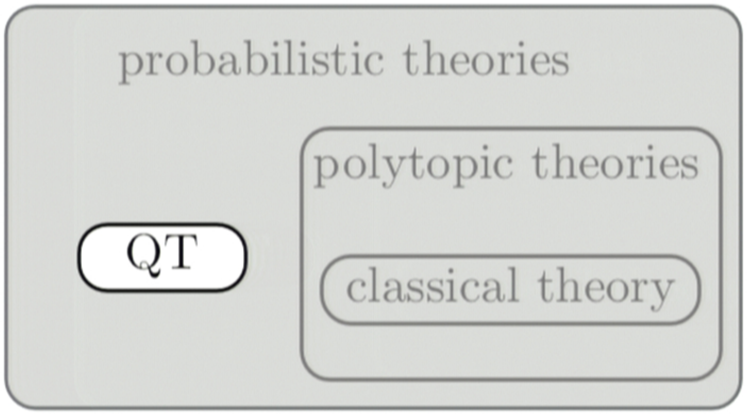

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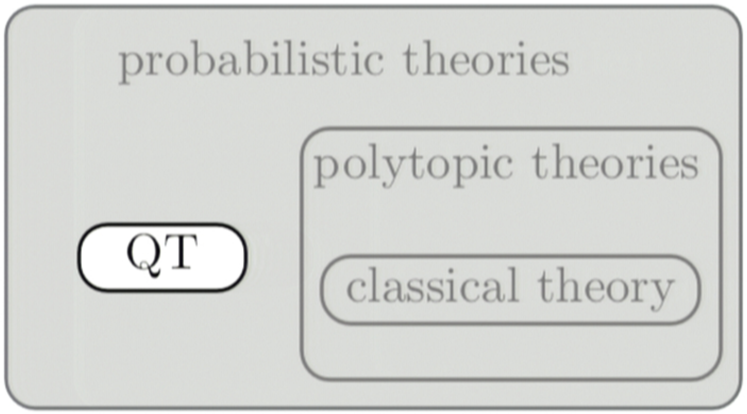

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Thanks for your attention!

arXiv:1210.0194

Check it out!

(revised version soon)