Title: Nonlinear Extensions for the Inflationary Power Spectra

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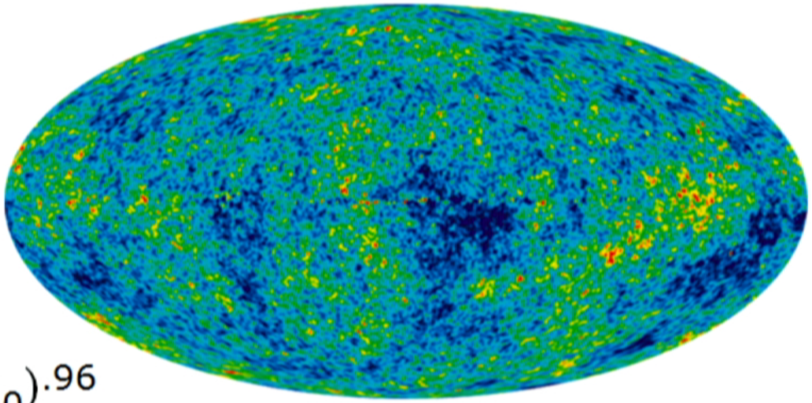
Abstract: We expound several principles in an attempt to clarify the debate over infrared loop corrections to the primordial scalar and tensor power spectra from inflation. Among other things we note that existing proposals for nonlinear extensions of the scalar fluctuation field ζ introduce new ultraviolet divergences which no one understands how to renormalize. Loop corrections and higher correlators of these putative observables would also be enhanced by inverse powers of the slow roll parameter ϵ. We propose an extension which might be better behaved.

Nonlinear Extensions of the Inflationary Power Spectra

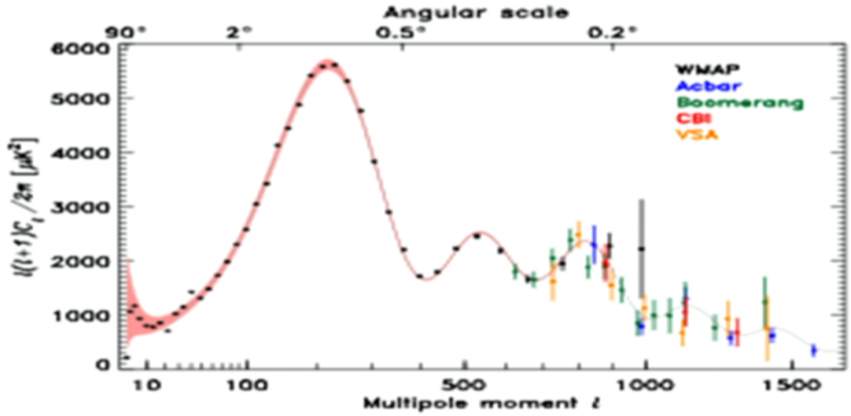
S.P. Miao
ITF, University of Utrecht

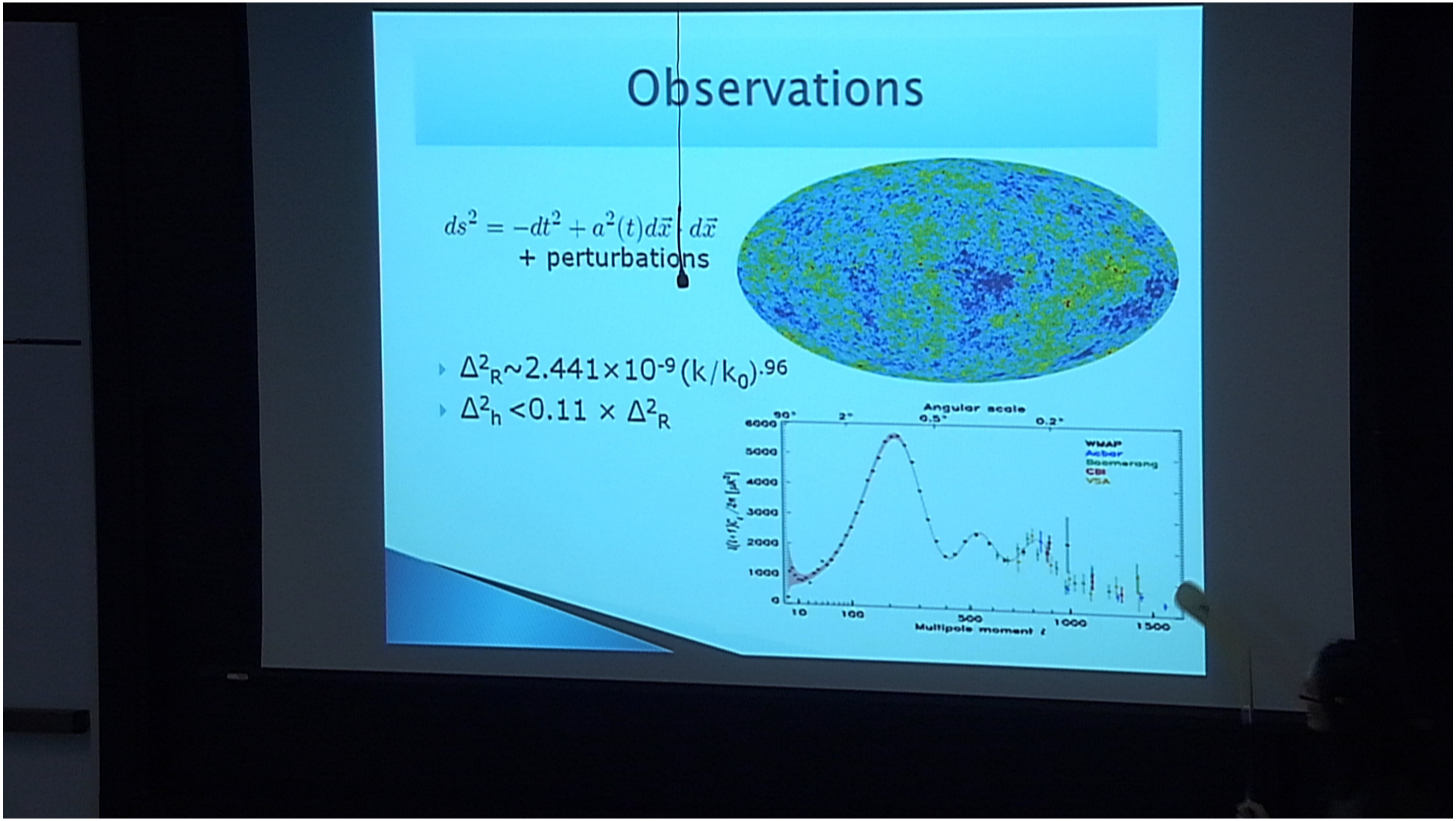
Observations

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x}$$
+ perturbations



- $\Delta^2_R \sim 2.441 \times 10^{-9} (k/k_0)^{.96}$
- $\Delta^2_h < 0.11 \times \Delta^2_R$

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4000
3000
2000
1000
0
WMAP
Acbar
Boomerang
CBI
VSA
10
100
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Multipole moment l

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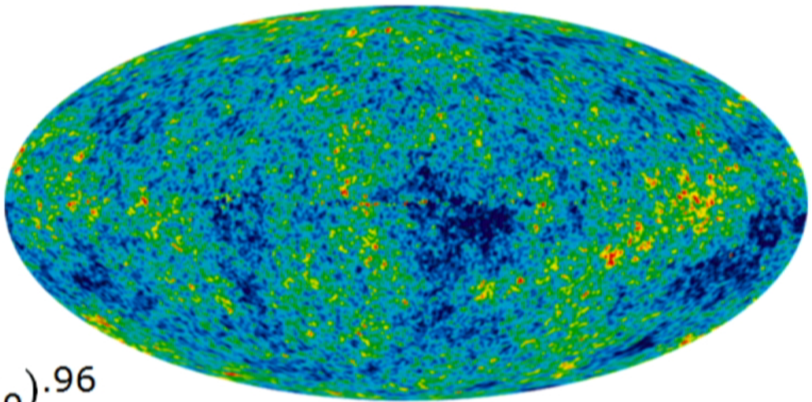

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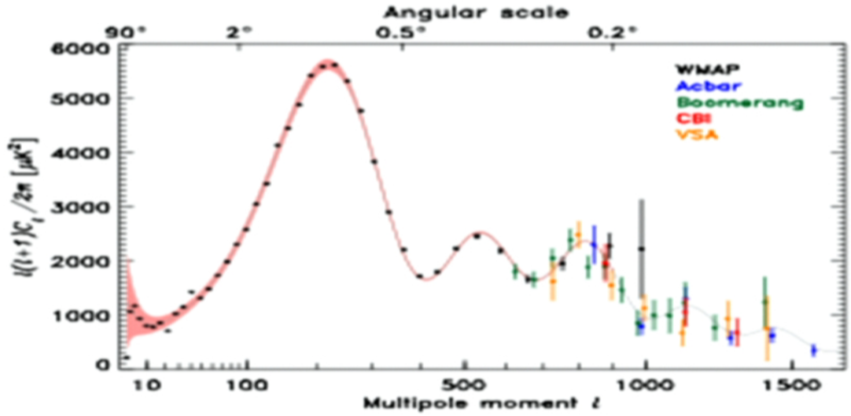

What theoretical quantities represent Δ^2_R & Δ^2_h ?

- At tree order:

$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3x\, e^{-i\vec{k}\cdot\vec{x}} \Big\langle \Omega \Big| \zeta(t,\vec{x})\zeta(t,\vec{0}) \Big| \Omega \Big\rangle$$

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- How to deal with IR divergent loops, i.e. $\ln\big[LH_1 a(t)\big]$
  - Big L ➔ big loop corrections
  - No physical principle fix IR cutoff
  - But loop corrections to <ζζ> & <hh> same order as from nonlinear extensions of ζζ & hh
- The nonlinear proposal by Tanaka & Urakawa
  - get rid of IR divergence without IR cutoff
  - Many potential issues occur ➔ A better fix

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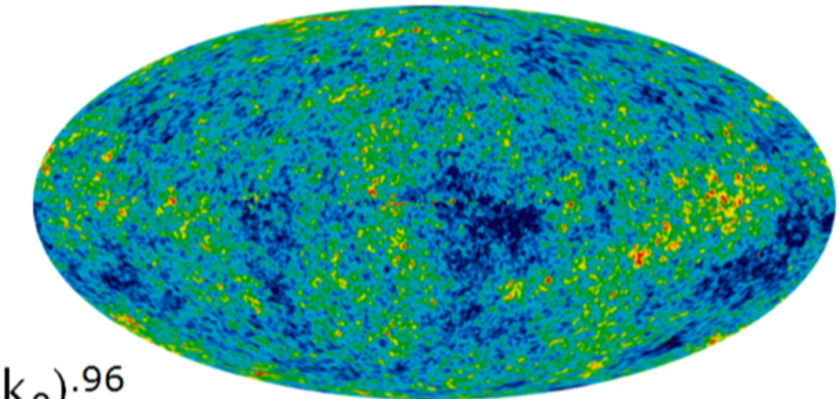

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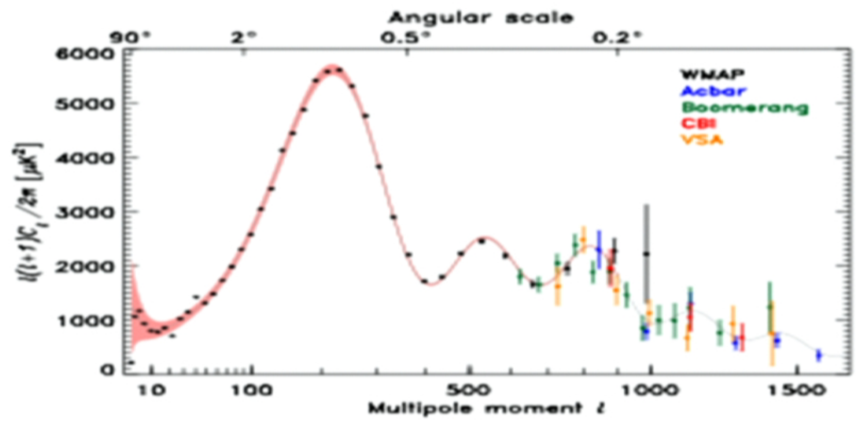

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Single Scalar Inflation

$$\mathcal{L} = \frac{1}{16\pi G}\mathrm{R}\sqrt{-\mathrm{g}} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi \mathrm{g}^{\mu\nu}\sqrt{-\mathrm{g}} - V(\varphi)\sqrt{-\mathrm{g}}$$

- Slow roll parameter: $\epsilon(t) \equiv -\frac{\dot{H}}{H^2}$,$0<\epsilon<1$
- 1^{st} horizon crossing: $k/a(t_k) = H(t_k)$
- Convection of Maldacena & Weinberg :

$$g_{ij}(t,\vec{x}) \equiv a^2(t)e^{2\zeta(t,\vec{x})}\tilde{g}_{ij}(t,\vec{x})$$

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Primordial Power Spectra

- Scalar: $$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \Big\langle \Omega \Big| \zeta(t,\vec{x}) \zeta(t,\vec{0}) \Big| \Omega \Big\rangle$$
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- Gauges: $$G_0(t,\vec{x}) \equiv \varphi(t,\vec{x}) - \varphi_0(t) = 0$$ $$G_i(t,\vec{x}) \equiv \partial_j h_{ij}(t,\vec{x}) = 0$$
- Quadratic (fixed & constrained) action:

$$\mathcal{L}_{\zeta^2} = \frac{(D-2)\,\epsilon\, a^{D-1}}{16\pi G} \left\{ \dot{\zeta}^2 - \frac{1}{a^2} \partial_k \zeta \partial_k \zeta \right\}$$

$$\mathcal{L}_{h^2} = \frac{a^{D-1}}{64\pi G} \left\{ \dot{h}_{ij} \dot{h}_{ij} - \frac{1}{a^2} \partial_k h_{ij} \partial_k h_{ij} \right\}$$

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Primordial Power Spectra Tree Order

- D=4,

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \left\{ 4\pi G \times |u_\zeta(t,k)|^2 + O(G^2) \right\}$$

$$\Delta^2_h(k) = \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \left\{ 32\pi G \times 2 \times |u(t,k)|^2 + O(G^2) \right\}$$

- If ε=const,

$$u_\zeta(t,k) = \frac{u(t,k)}{\sqrt{\epsilon}} \qquad \Delta^2_{\mathcal{R}} = C^2(\epsilon) \times \frac{GH^2(t_k)}{\pi\epsilon(t_k)} + O(G^2)$$

$$u(t,k) = -iC(\epsilon) \times \frac{H(t_k)}{\sqrt{2k^3}} \qquad \Delta^2_h = C^2(\epsilon) \times \frac{16GH^2(t_k)}{\pi} + O(G^2)$$

- The Tensor-to-scalar ratio: $r \equiv \frac{\Delta^2_h}{\Delta^2_{\mathcal{R}}} = 16\epsilon$

Primordial Power Spectra
Compare predictions with the data

- South Pole Telescope (95% confidence): $r=16\epsilon<0.11 \rightarrow \varepsilon<0.007 \rightarrow (1/\epsilon)>145$
  - very close to de Sitter ($\epsilon = 0$)
  - $1/\epsilon$ corrections are large (cf: $i\Delta_\zeta \sim GH^2/\epsilon$)
- $\Delta^2_R \sim 2.441\times10^{-9}$ & $r=\Delta^2_h/\Delta^2_R < 0.11 \rightarrow$
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  $\mathcal{L}_{\zeta h^2} \sim \epsilon \zeta h^2$, $\mathcal{L}_{\zeta^2 h} \sim \epsilon \zeta^2 h$, $\mathcal{L}_{\zeta^2 h^2} \sim \epsilon \zeta^2 h^2$
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  - No $1/\epsilon$'s in loops & non-Gaussianity
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IR Divergence versus IR Logs

- IR ∞ from $h_{ij}(t,x) \sim$ constant
- IR Logs from continual horizon crossing
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- Gauge dependent ≠ unphysical
  - S-matrix from gauge dep. Green's funcs.
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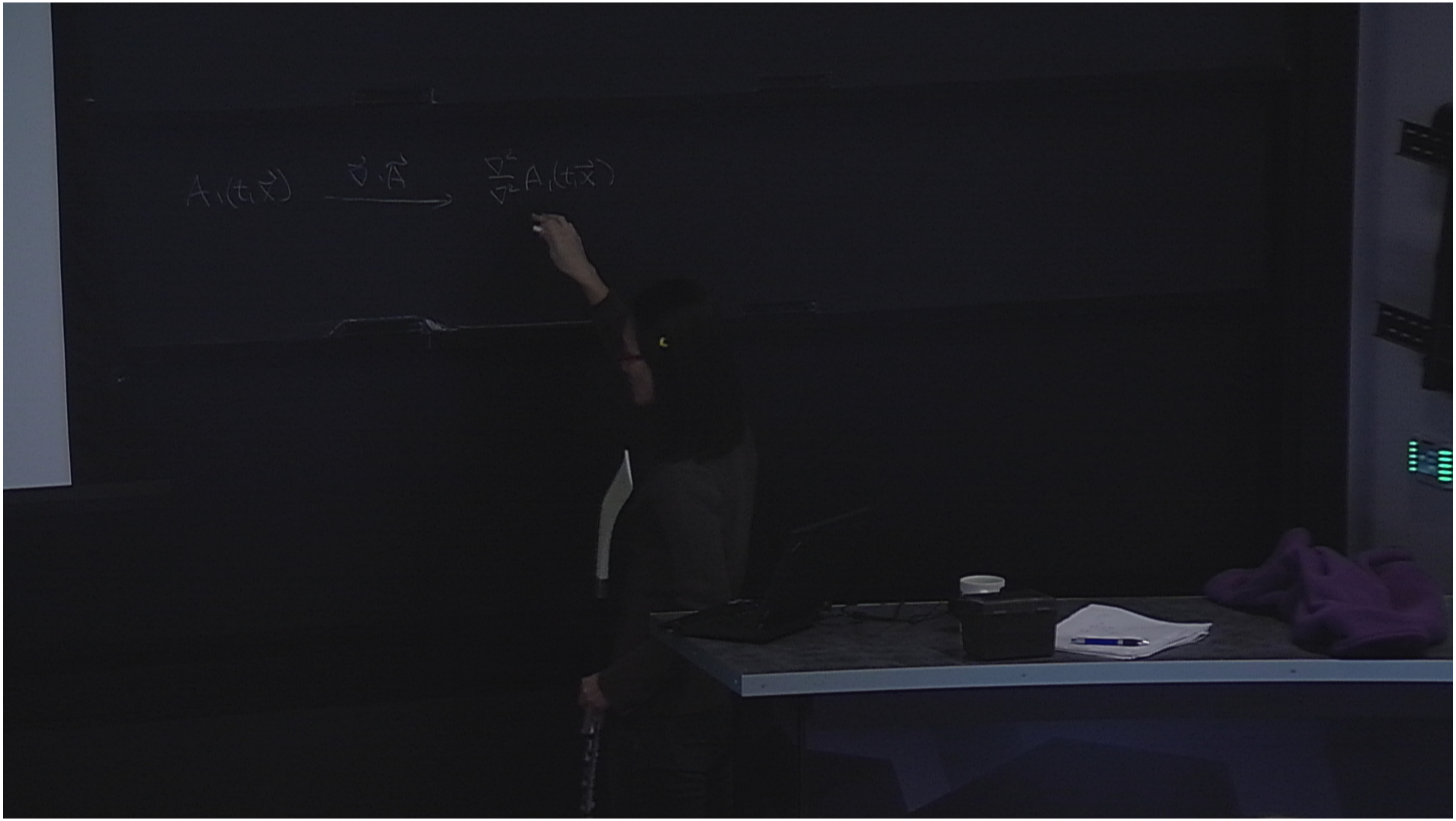
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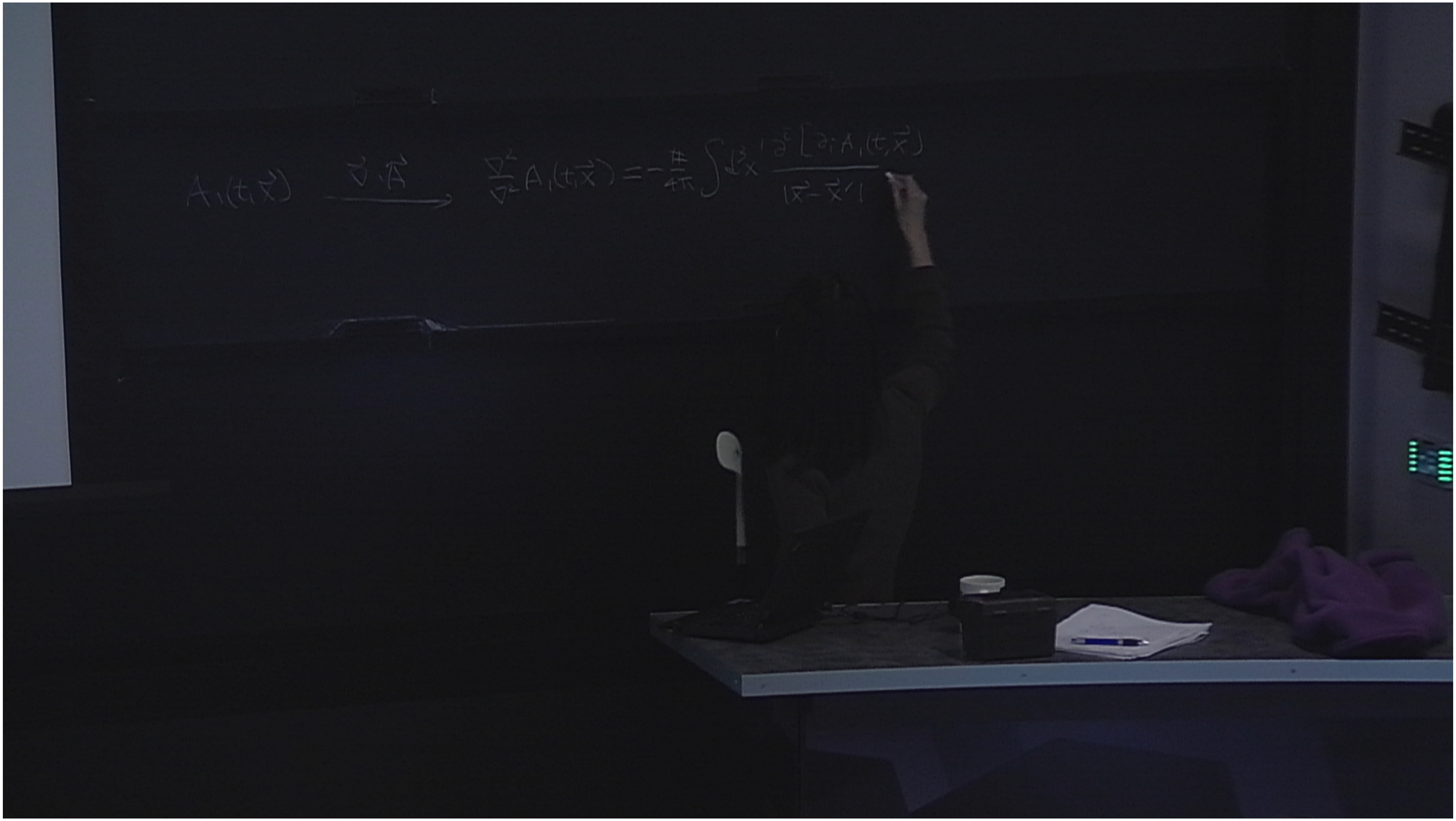

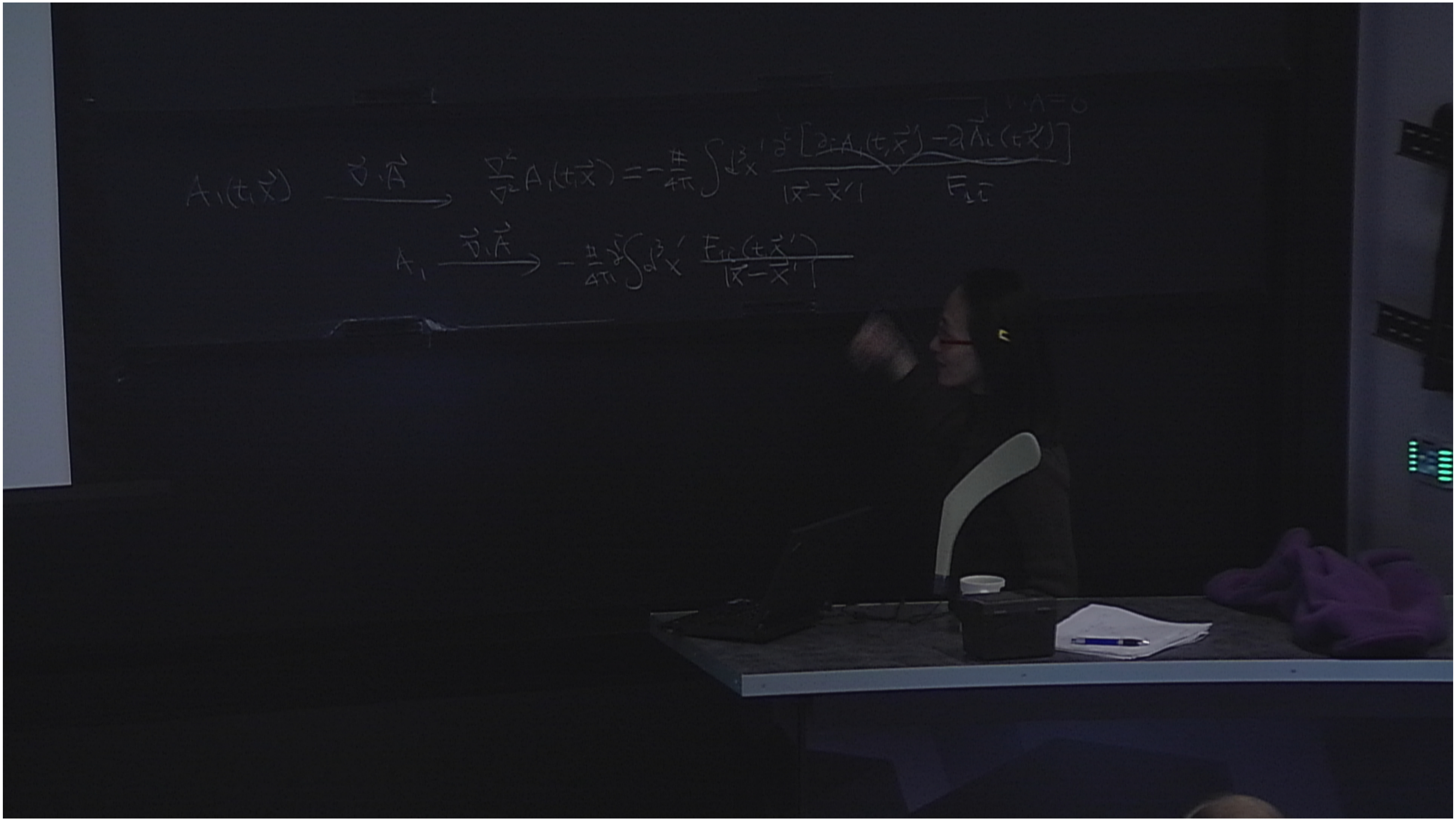

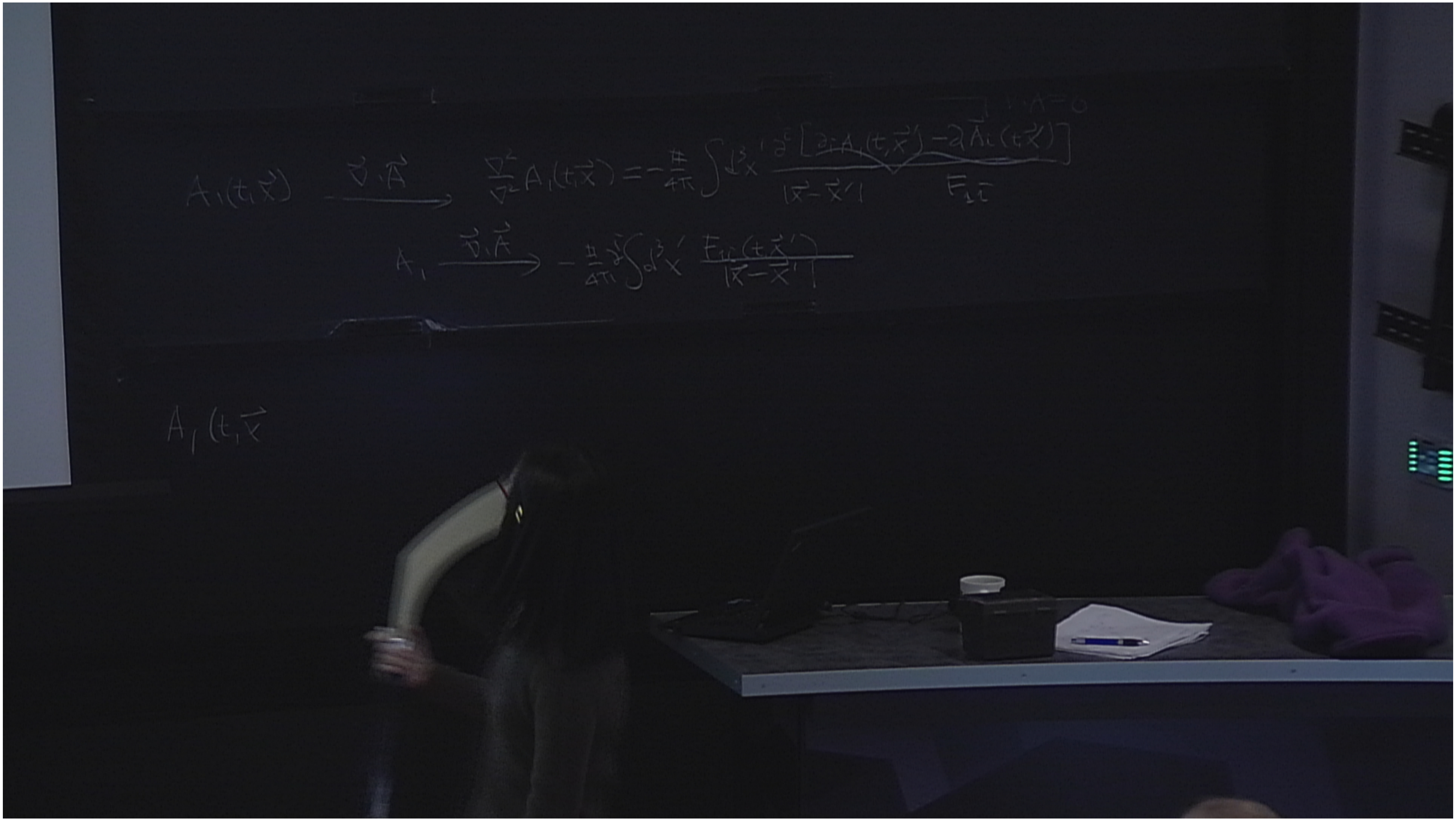


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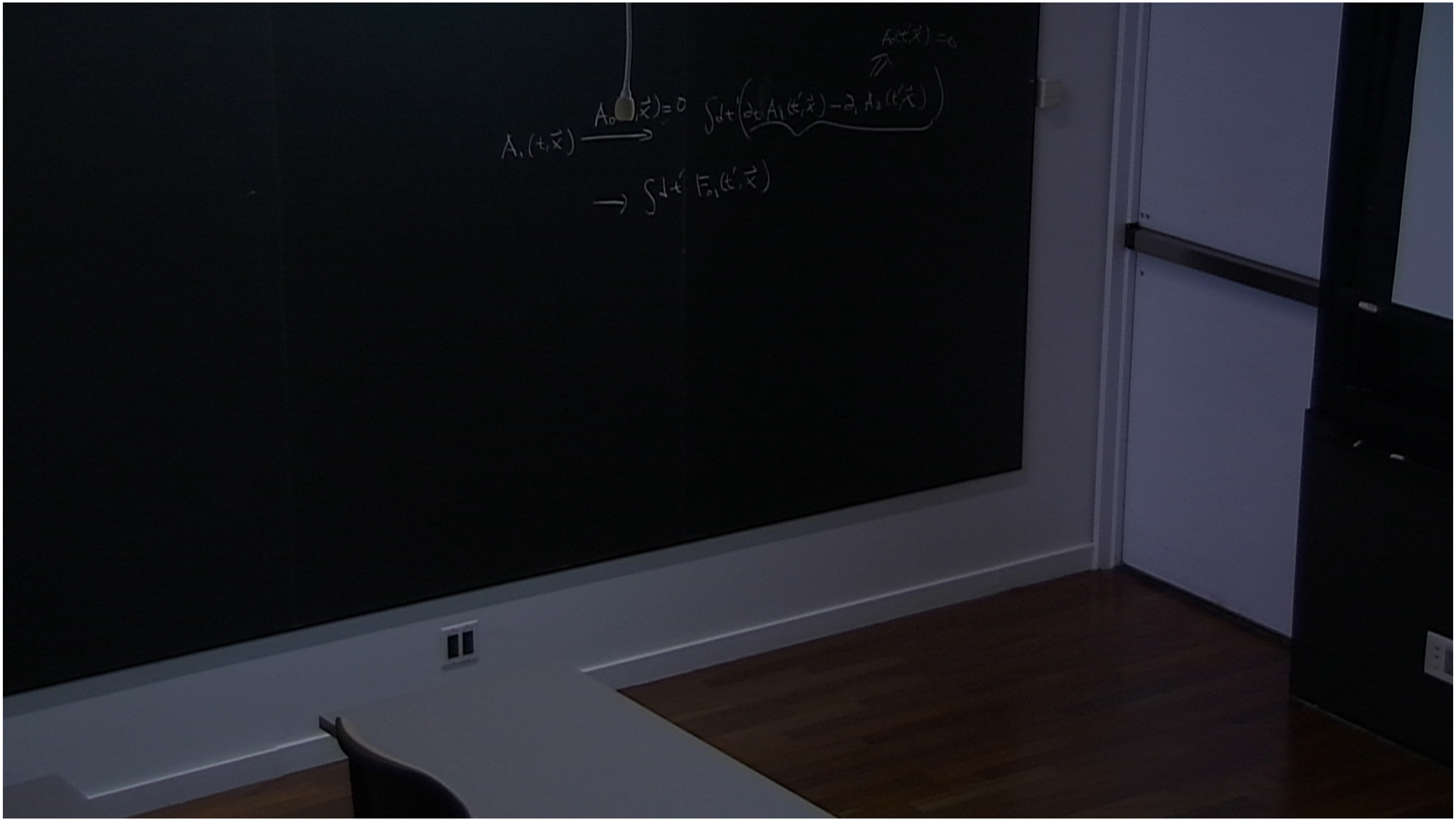

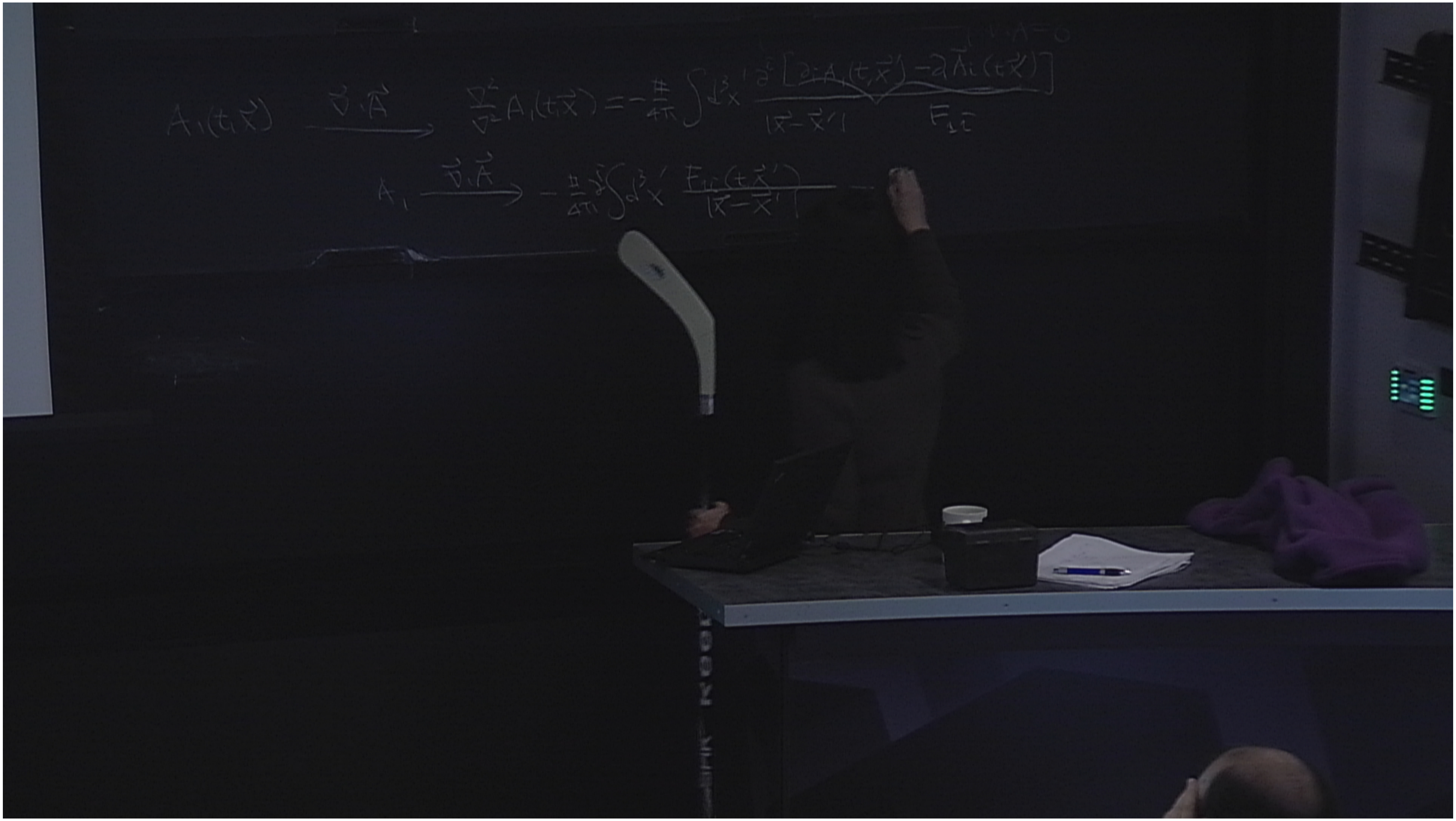

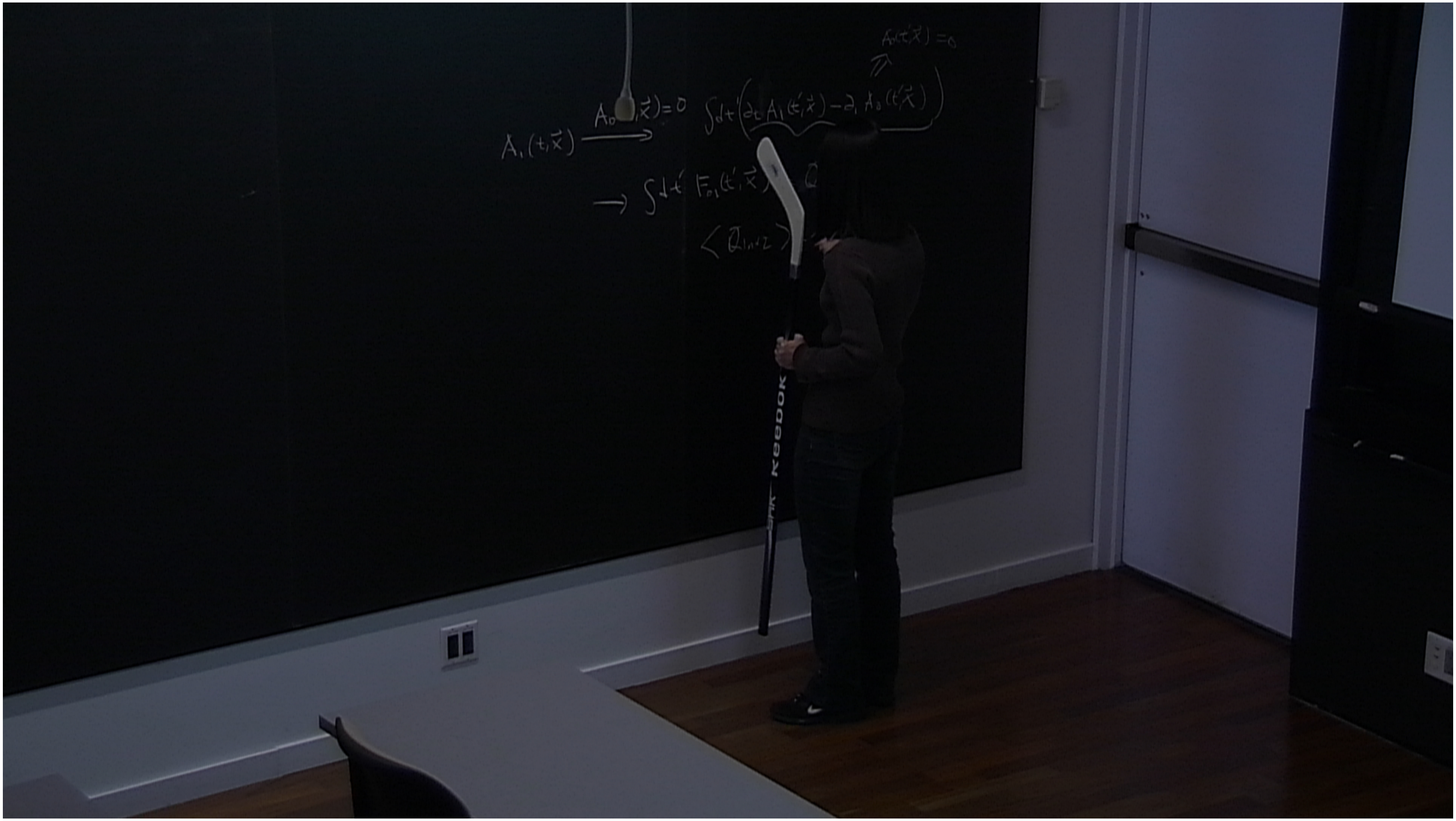

Danger of gauge invariant operators

- No local gauge invariants in GR
- Nonlocal field's redefinitions can change physics!
  - E.g. $\mathcal{D}\varphi = I[\varphi]$
  - Yang-Feldman: $\varphi = \varphi_0 + \mathcal{D}^{-1} I[\varphi]$
  - Free field: $\varphi_0[\varphi] = \varphi - \mathcal{D}^{-1} I[\varphi]$
- Relates any 2 theories w. same DOF's
  - E.g., GR=EM!

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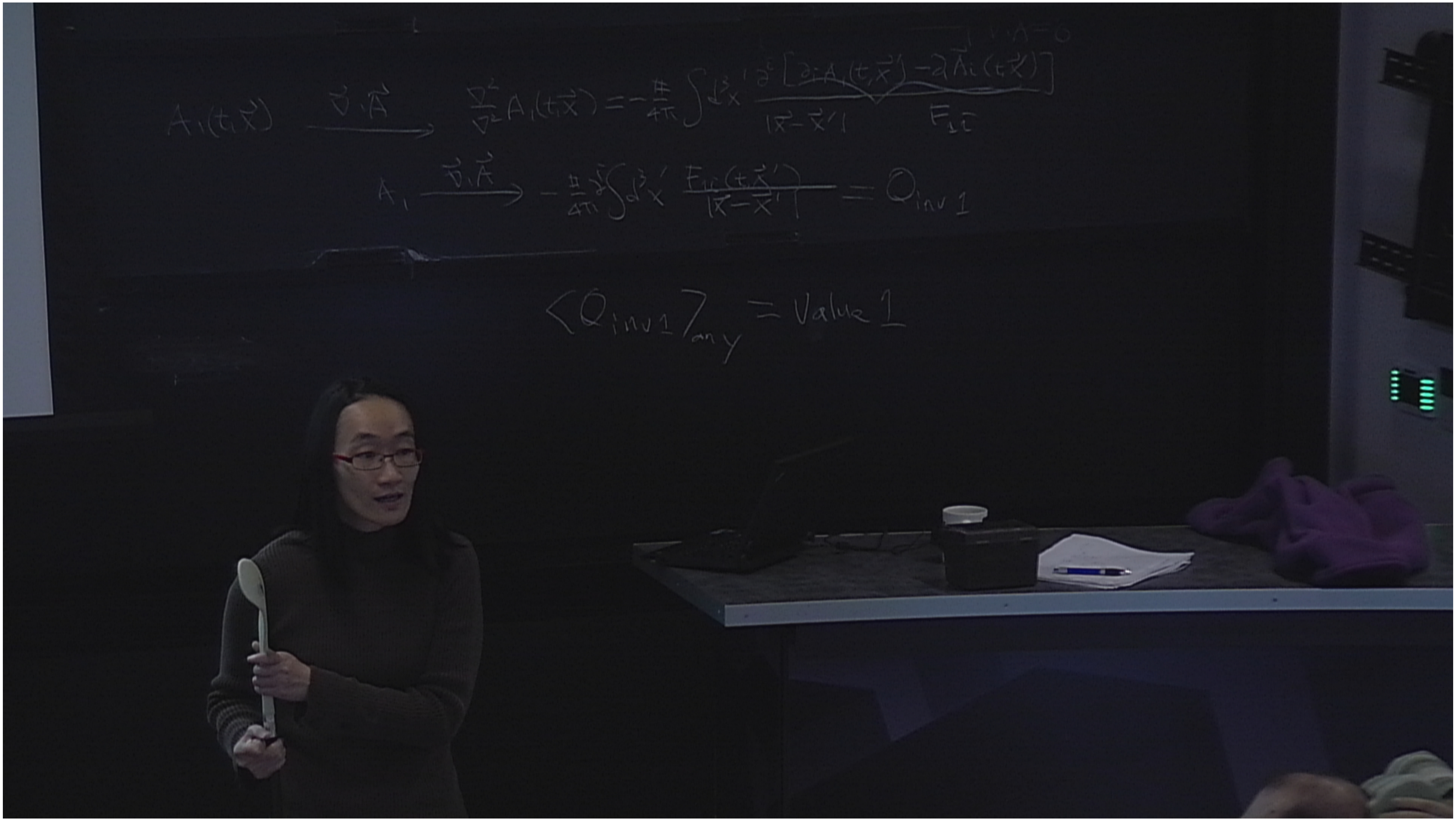

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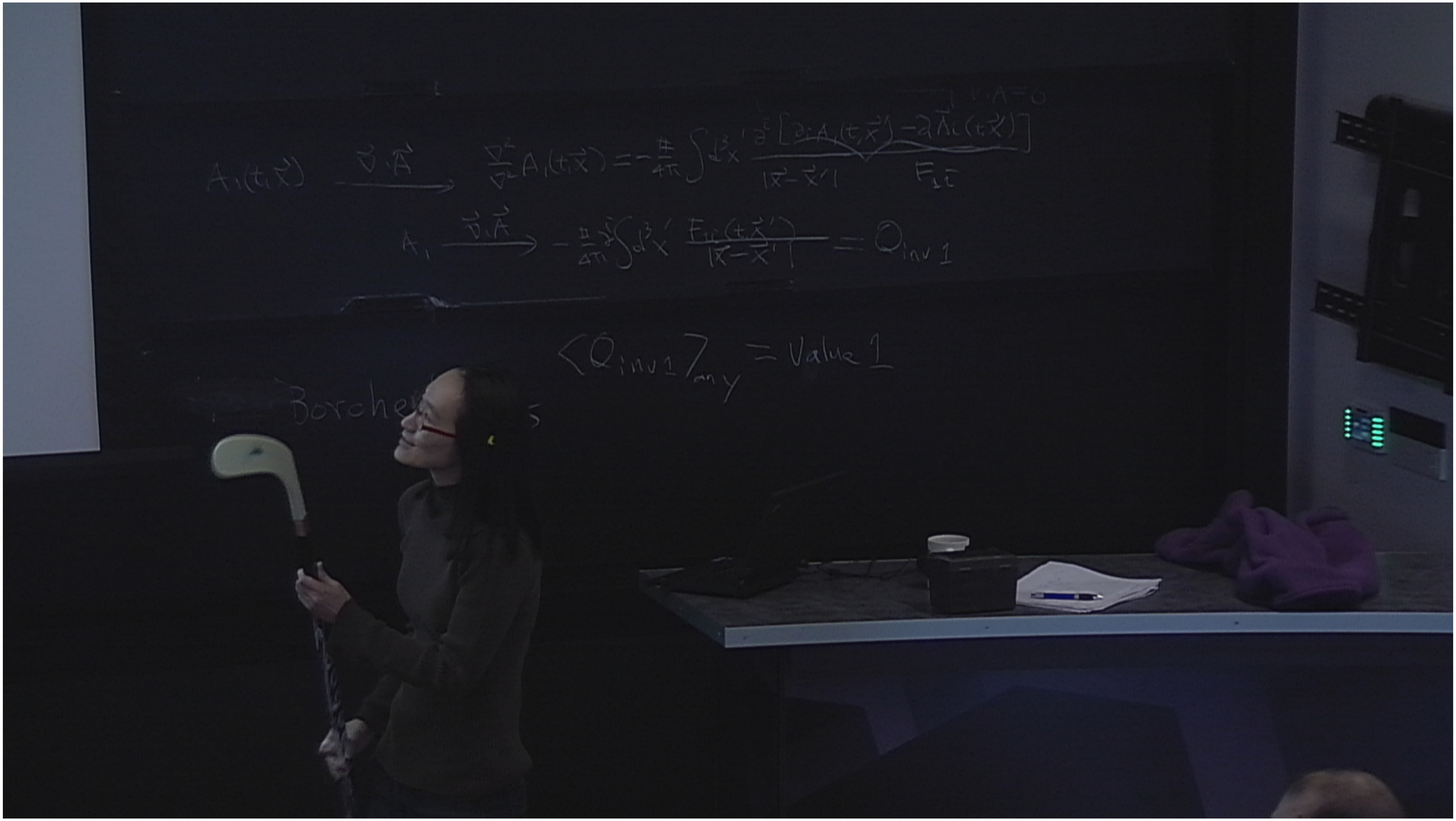
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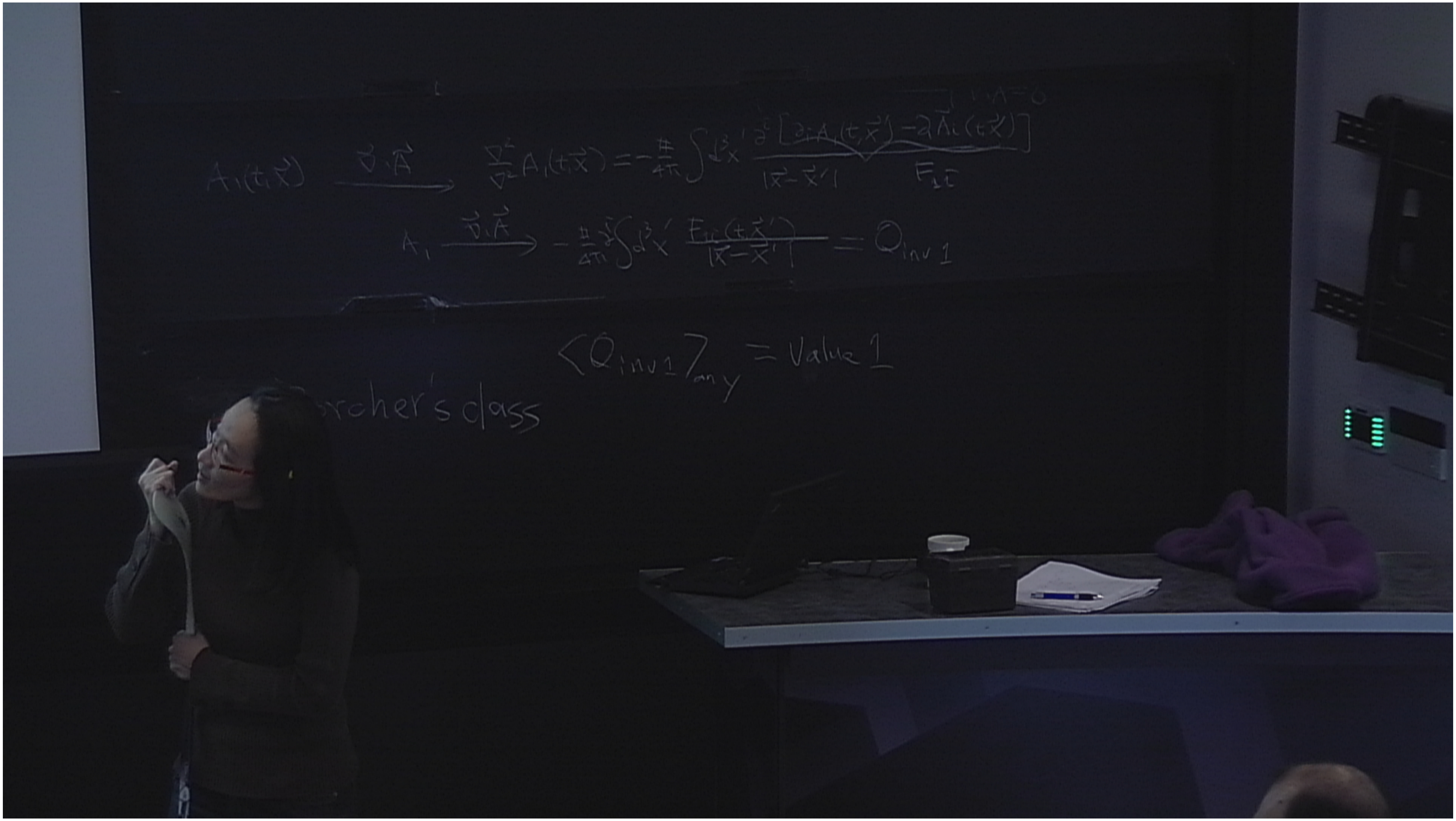
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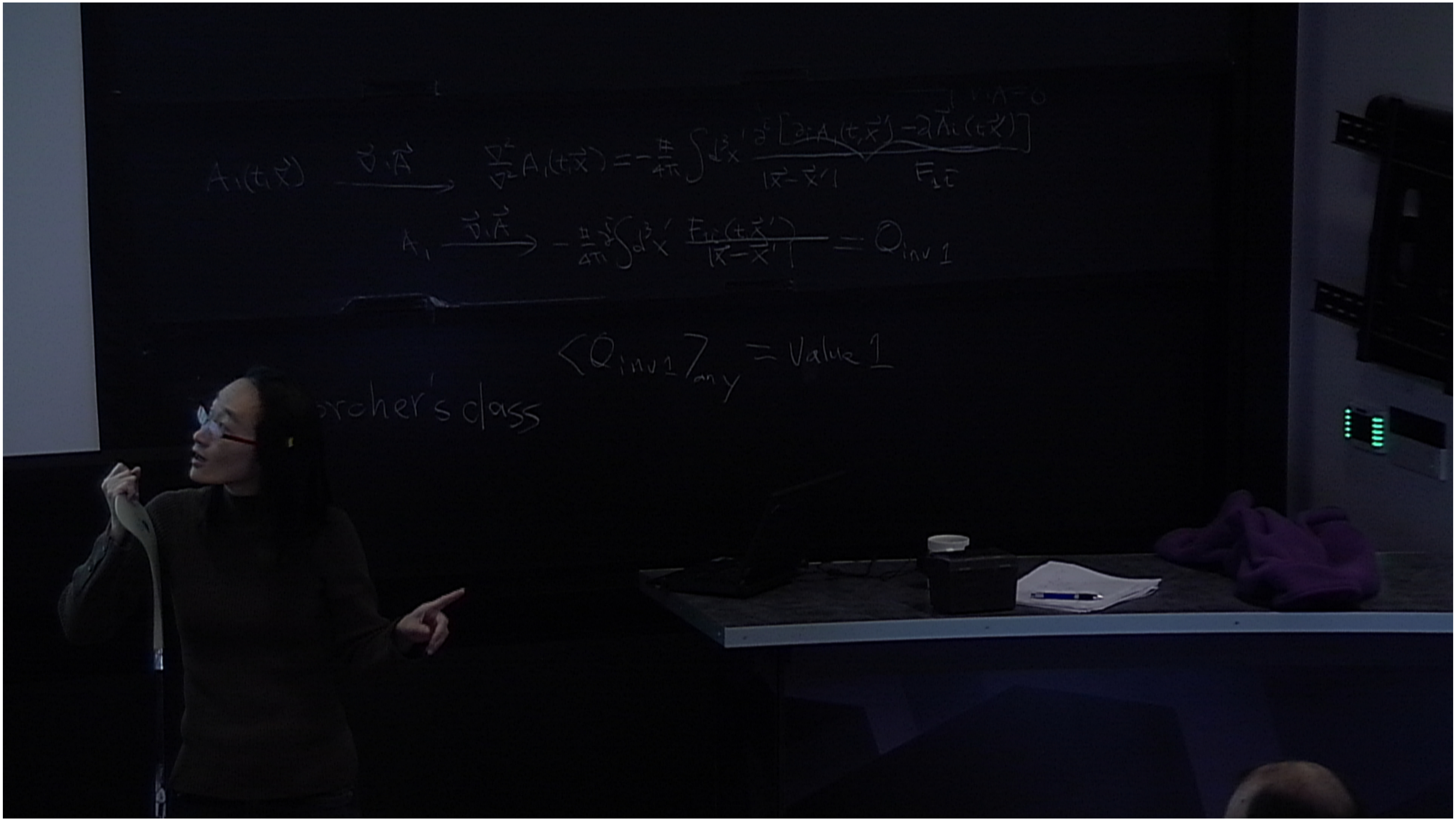
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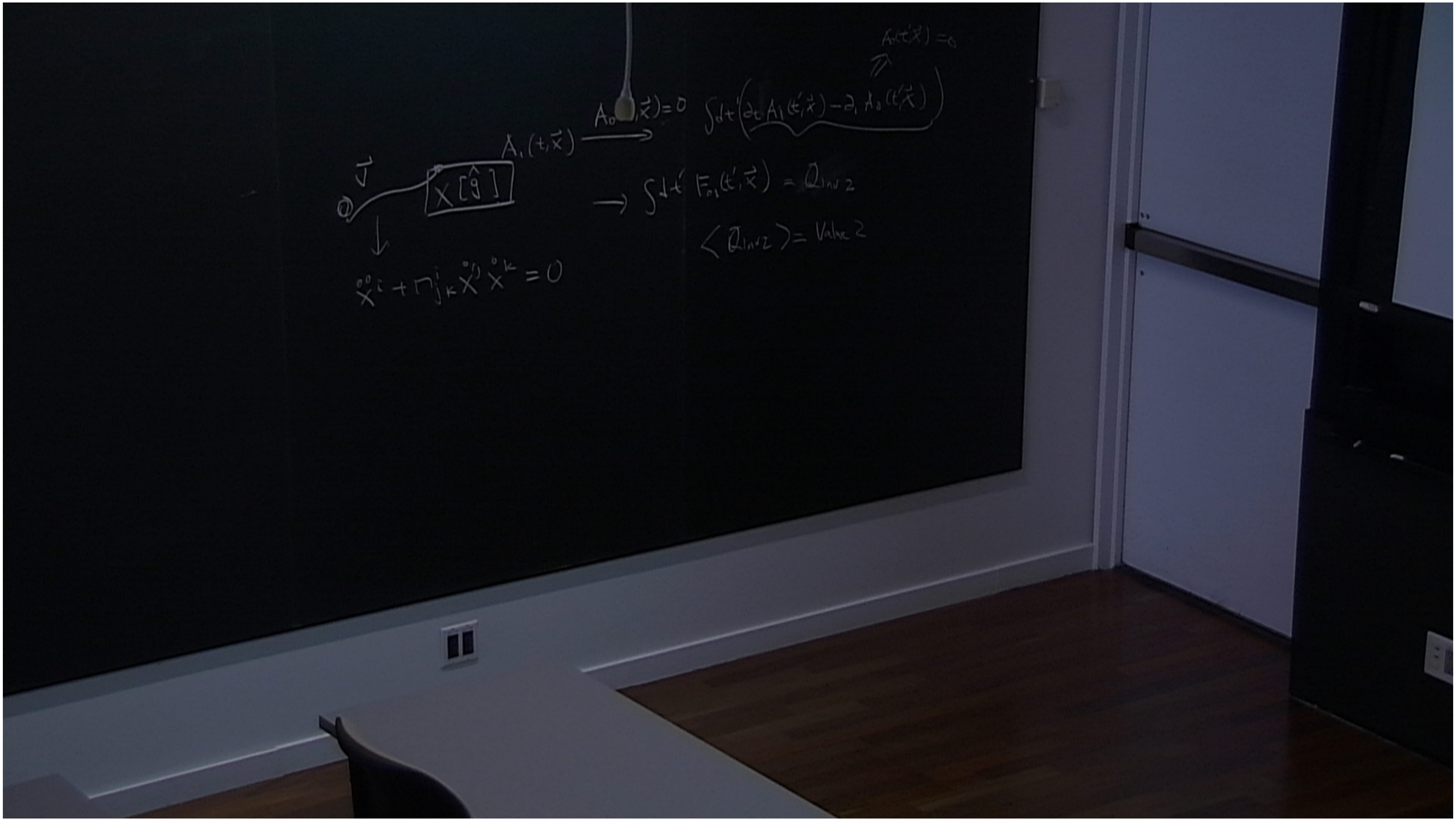

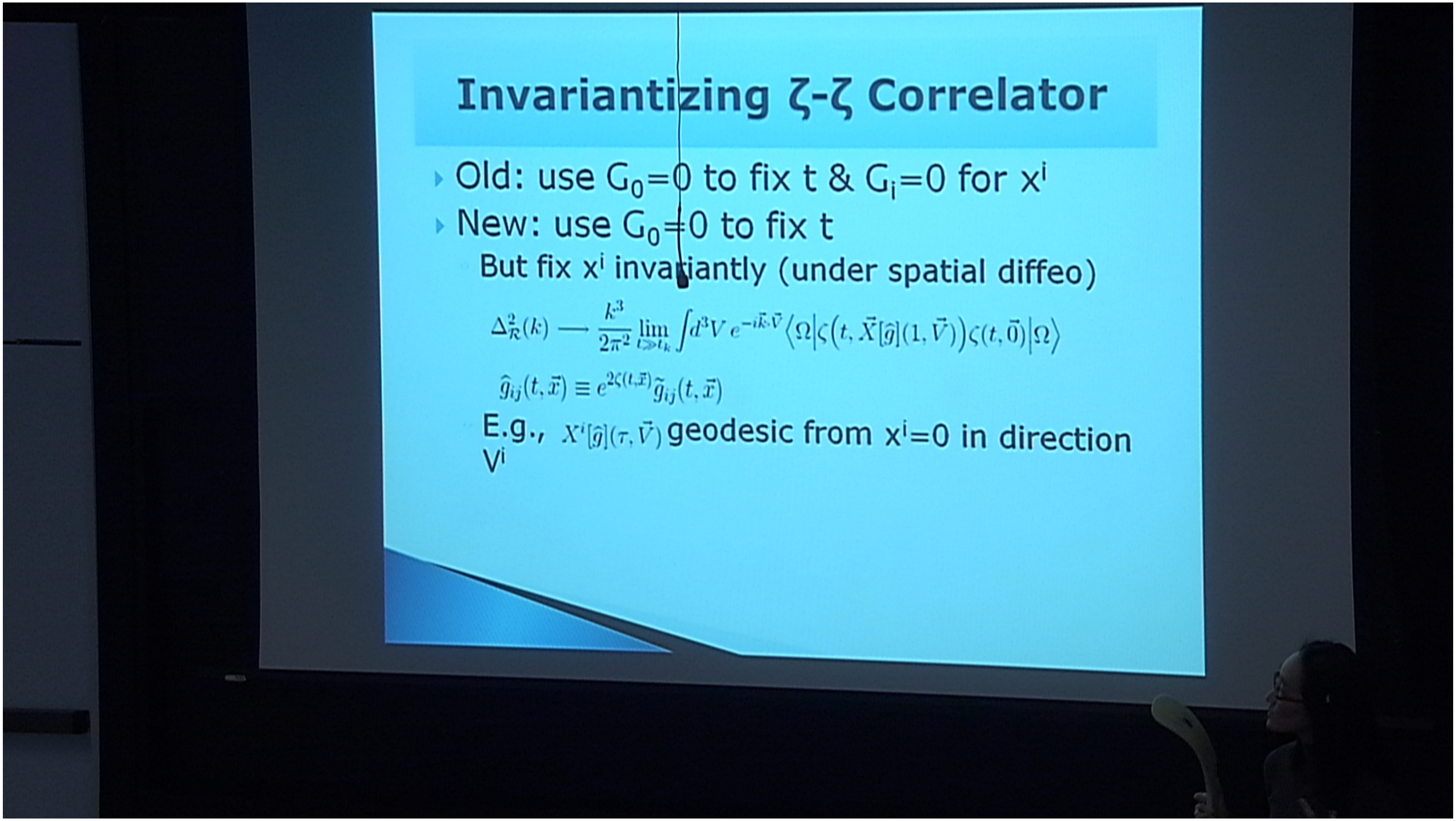
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  - More UV divergences (1/(D-4) ➔ 1/(D-4)^2)

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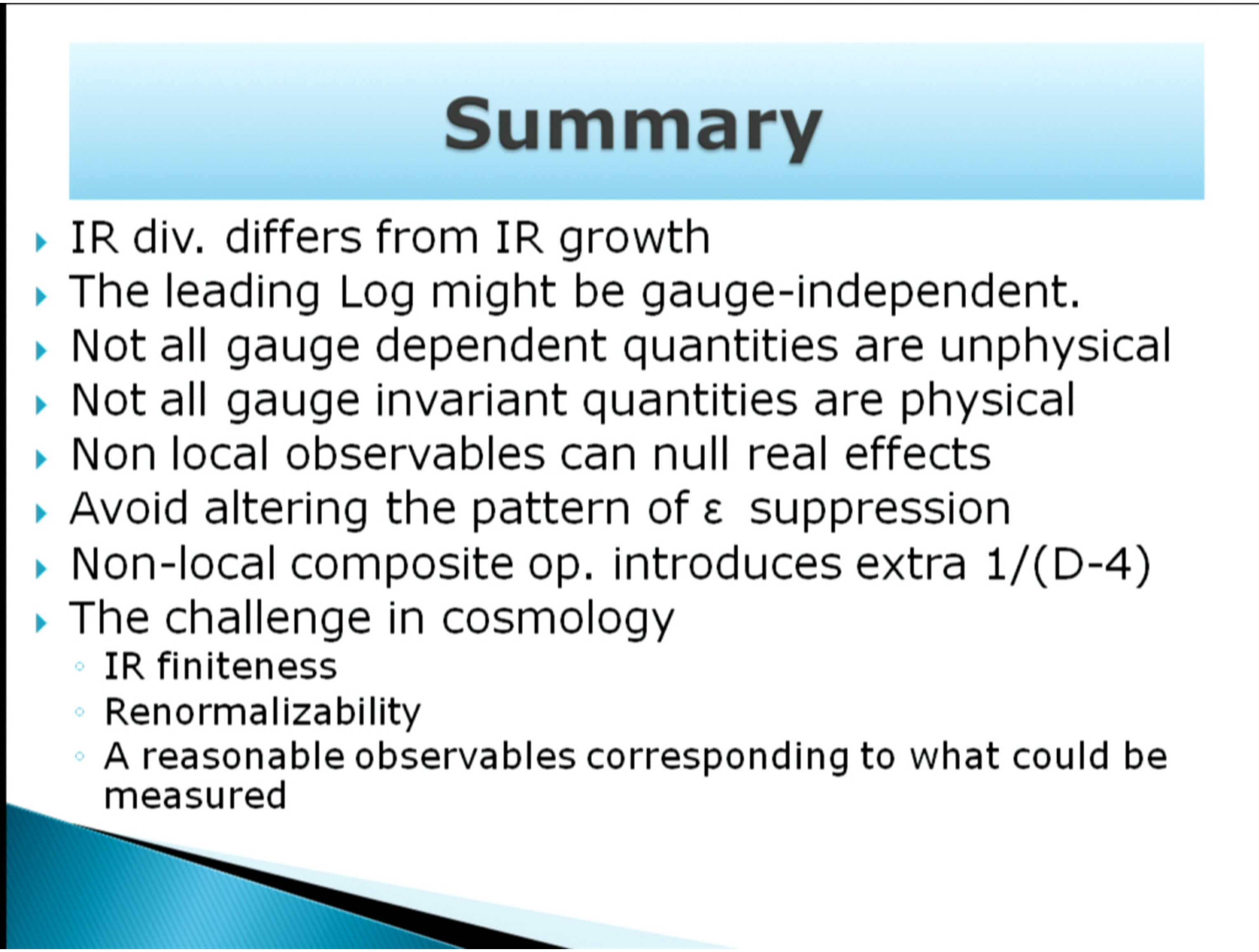
A Spatially Invariant ζ-ζ Correlator

- Original definition
  - Depend on gauges
  - Tree orders & Loops corrections depend on infrared cutoff
- New definition
  - Do not depend on the spatial gauge
  - Do not depend on the infrared cutoff
  - But need to deal with some potential dangers
    - Extra UV (renormalize non-local composite op.)
    - Peculiar ε pattern
    - Field redefinition might null real effects

Summary

- IR div. differs from IR growth
- The leading Log might be gauge-independent.
- Not all gauge dependent quantities are unphysical
- Not all gauge invariant quantities are physical
- Non local observables can null real effects
- Avoid altering the pattern of ε suppression
- Non-local composite op. introduces extra 1/(D-4)
- The challenge in cosmology
  - IR finiteness
  - Renormalizability
  - A reasonable observables corresponding to what could be measured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