Title: Local topological order inhibits thermal stability in 2D

Date: Dec 05, 2012 04:00 PM

URL: http://pirsa.org/12120010

Abstract: We study the robustness of quantum information stored in the degenerate ground space of a local, frustration-free Hamiltonian with commuting terms on a 2D spin lattice. On one hand, a macroscopic energy barrier separating the distinct ground states under local transformations would protect the information from thermal fluctuations. On the other hand, local topological order would shield the ground space from static perturbations.

Here we demonstrate that local topological order implies a constant energy barrier, thus inhibiting thermal stability. Joint work with David Poulin.

arXiv:1209.5750

Local topological order inhibits thermal stability in 2D

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Olivier Landon-Cardinal
joint work with David Poulin

Département de Physique, Université de Sherbrooke, (Québec) Canada

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PhD work (Sept. 2009-)

TQO inhibits thermal stability

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Practical characterization of quantum systems
➡ extract information with a scalable amount of resources
(number of experimental copies/measurements, classical processing)

Certification: distance between the experimental state and the target state?
➡ Monte Carlo fidelity estimation

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M. da Silva, OLC and D. Poulin, PRL **107**, 210404 (2011)
S.T. Flammia and Y.-K. Liu, PRL **106**, 230501 (2011)
I. Fedorov, a., Steffen, L., Baur, M., da Silva, M.P. & Wallraff, a. Nature (2011)

Variational tomography: identify a state among a variational family
➡ MPS tomography

➡MERA tomography

Self-correcting memory

Self-correcting memory = physical system which encode (quantum) information

- reliably
- for a macroscopic period of time
- letting the memory interact with its environment (thermal noise)
- *without* active error correction

Encoding $|\psi_i\rangle$ —Noise→

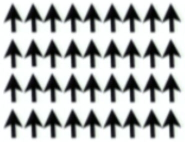
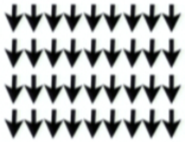
→ $|\psi_i'\rangle$ —Decoding→ $|\psi_f\rangle$

Code = degenerate groundspace of a local Hamiltonian of spin particles on a (2D) lattice.

Self-correcting classical memories

2D ferromagnetic Ising model

$$H_{\text{Ising2D}} = - \sum_{\langle i,j \rangle} \sigma_z^i \otimes \sigma_z^j$$

 0  1

- thermally stable: for T<TCurie, no macroscopic error droplets
- contrasts with 1D case : point-like excitations which diffuse freely

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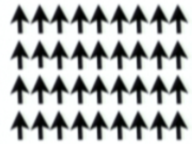

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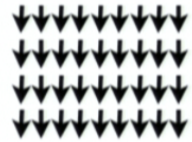

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(Archetypical) example : Kitaev's toric code (1997)

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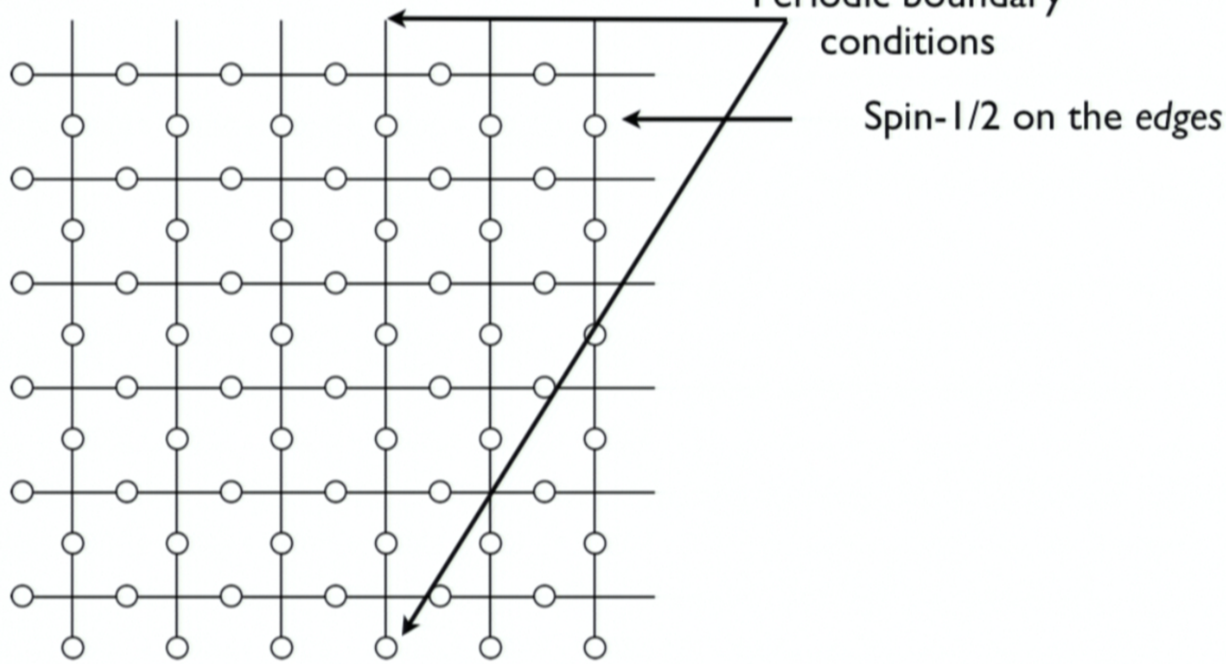

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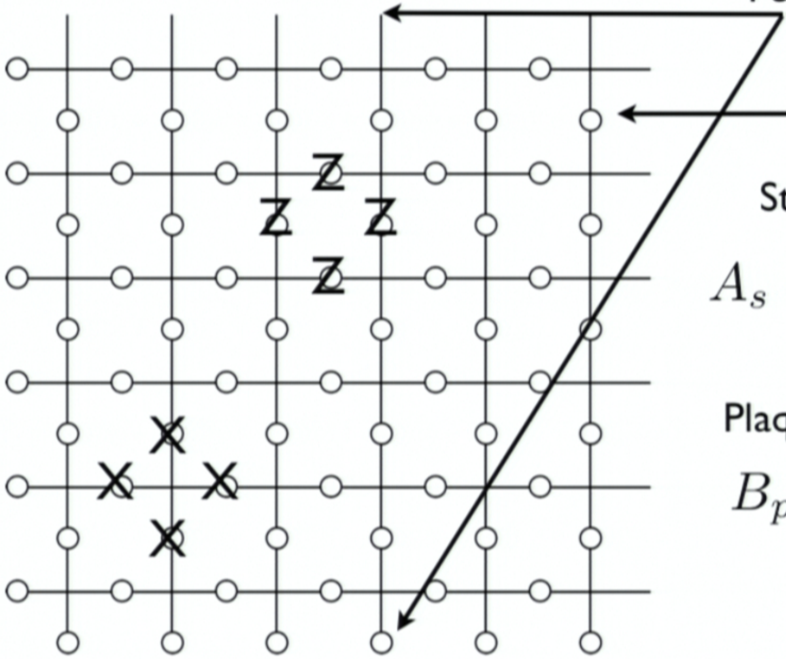


Periodic boundary conditions

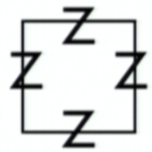
Spin-1/2 on the *edges*

Star operator

$$A_s = \bigotimes_{i \in \mathcal{N}(s)} \sigma_x^i$$

Plaquette operator

$$B_p = \bigotimes_{i \in p} \sigma_z^i$$



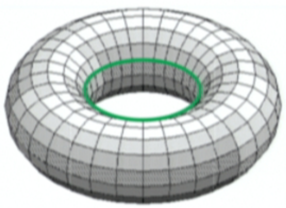
Unstability of Kitaev's toric code

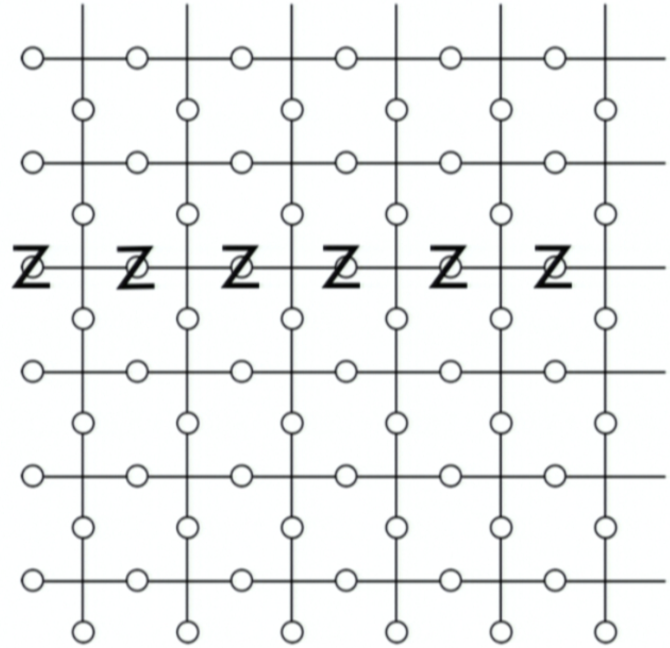
Groundstates

$$\forall s \; A_s|\psi\rangle = +|\psi\rangle$$

$$\forall p \; B_p|\psi\rangle = +|\psi\rangle$$

Logical operator : string of Z





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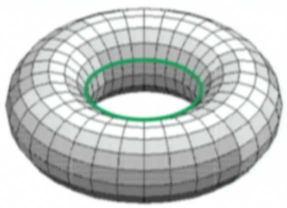
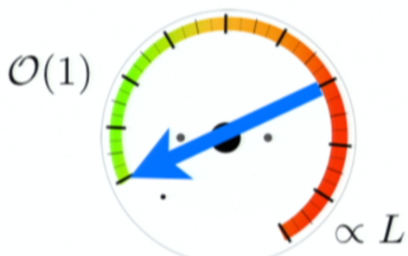
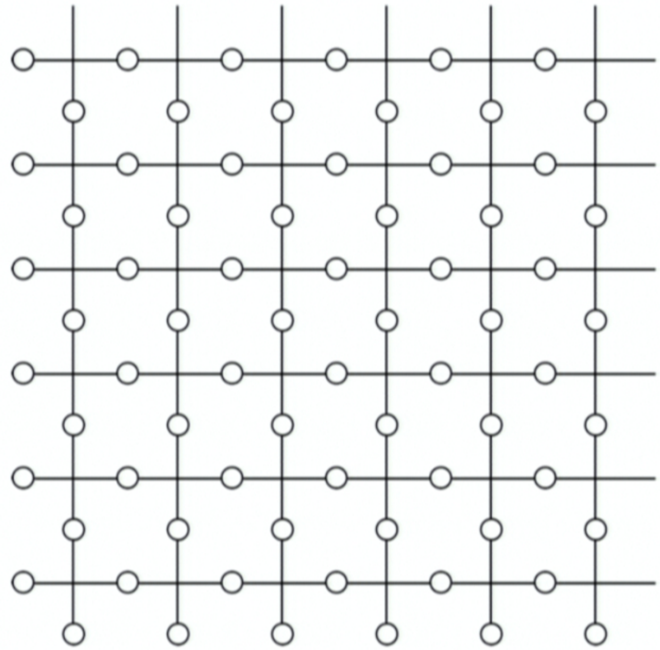


Excitations

Energy meter





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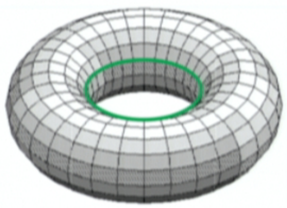

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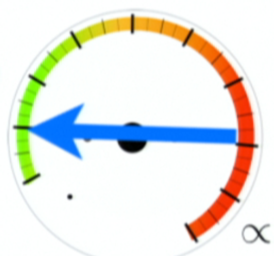

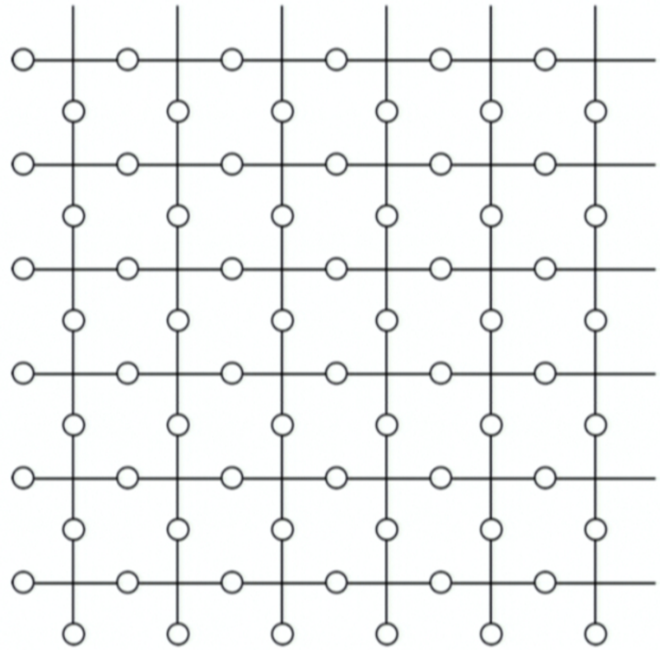

No energy for anyon propagation.

TQO inhibits thermal stability

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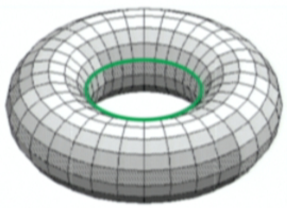

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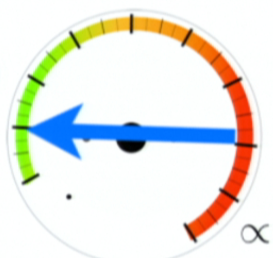

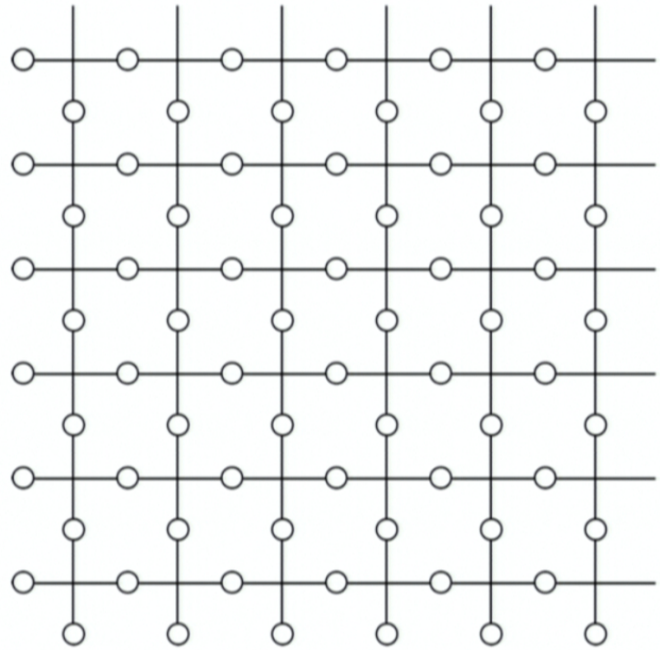

No energy for anyon propagation.

Thermal fluctuations can accumulate and corrupt the information.

Broad class of 2D codes: LCPCs

N finite dimensional spins located on the vertices of a 2D lattice (V, E).

$$H = -\sum_{X \subset V} P_X$$

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- bounded strentgh $\|P_X\| \leq 1$
- terms commute $[P_X, P_Y] = 0$
- local $\mathrm{diam(X)} \geq w \Rightarrow P_X = 0$
- frustration-free $\forall X \; P_X|\psi\rangle = +|\psi\rangle$

We are interested in the groundspace of H and scaling of the energy gap.
Without loss of generality, P_X = projector

Local commuting projector codes (LCPCs)

$[P_X, P_Y] = 0$

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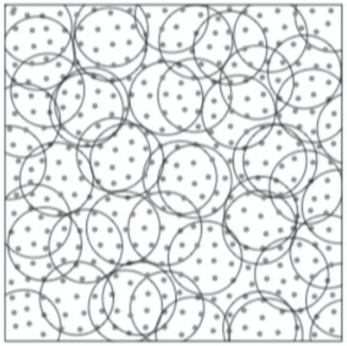
$(P_X)^2 = P_X$

Code projector $P = \prod P_X$



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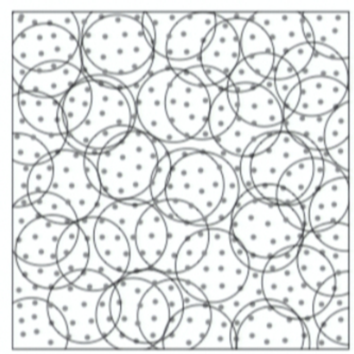

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Spectrum stability: local topological order

Spectrum of LCPC Hamiltonian is stable if the Hamiltonian has local topological quantum order (LTQO).

Bravyi, Hastings, Michalakis (2010)

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- local indistinguishability: local operators cannot discriminate groundstates.
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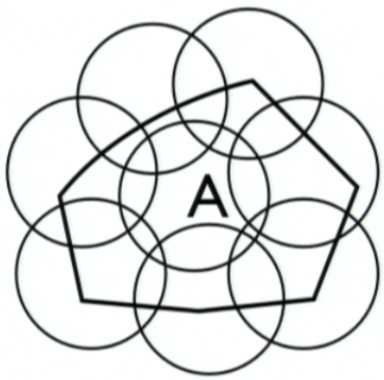

Formal definition of self-correction

Thermalization requires detailed knowledge of system dynamics.

Simplified model for thermalization

- penalize high energy states (Boltzmann factor) $\propto e^{-E/k_B T}$
- local moves in noise model

Encoding $|\psi_i\rangle$ —— Noise ——>  ——> $|\psi_i'\rangle$ —— Decoding ——> $|\psi_f\rangle$

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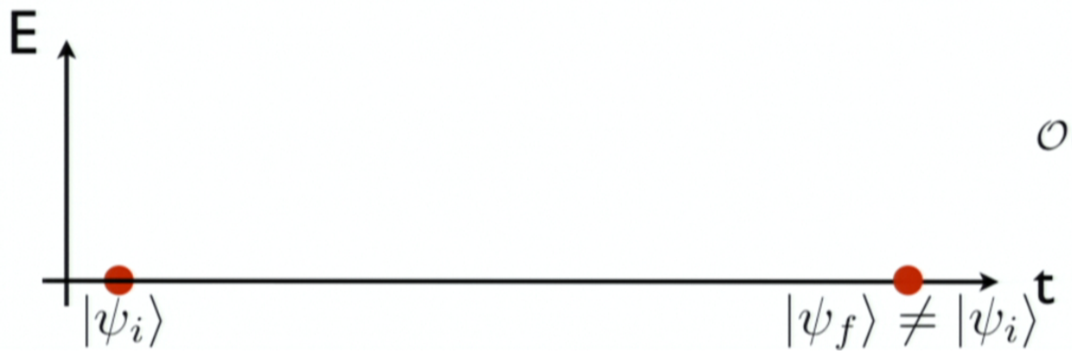
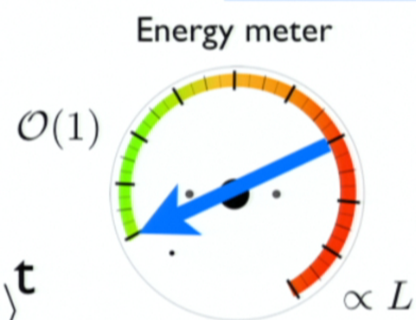
Logical operator : operator that maps groundstate to gs. $[K, P] = 0$

↳ Sequence of local moves (CPTP maps) that implements logical op?





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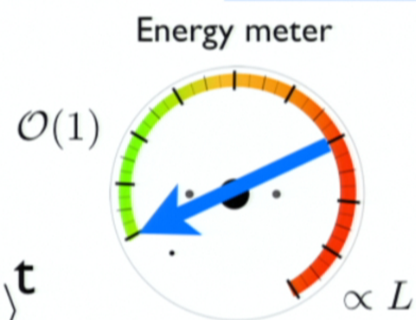
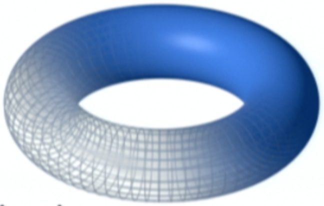


Known results: stabilizer codes & LCPCs



Instability in Kitaev's toric code

Key features

- logical operator is supported on a string of particles
- logical operator is a tensor product of single-body unitaries

General result for stabilizer codes

➡ cleaning lemma (Bravyi & Terhal '09)

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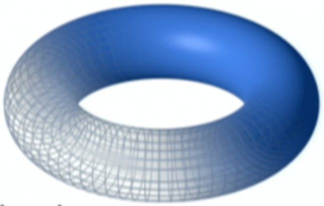

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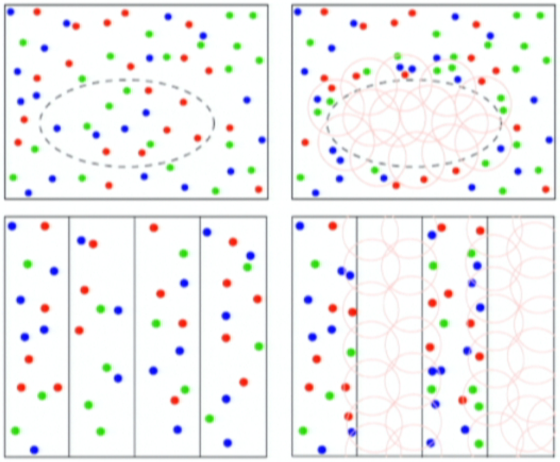
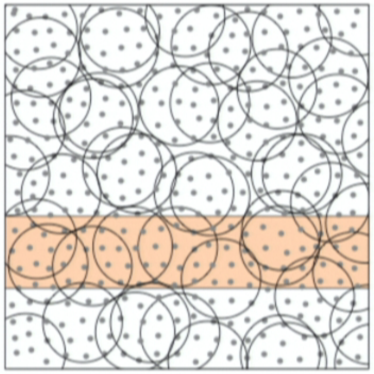


General result for stabilizer codes
➡ cleaning lemma (Bravyi & Terhal '09)

Generalization to LCPCs
➡ disentangling lemma
Bravyi, Poulin & Terhal '10
➡ Haah & Preskill '12

LCPCs : logical operator is supported on a strip, but not a tensor product.

Sketch of the proof (1): coarse-graining



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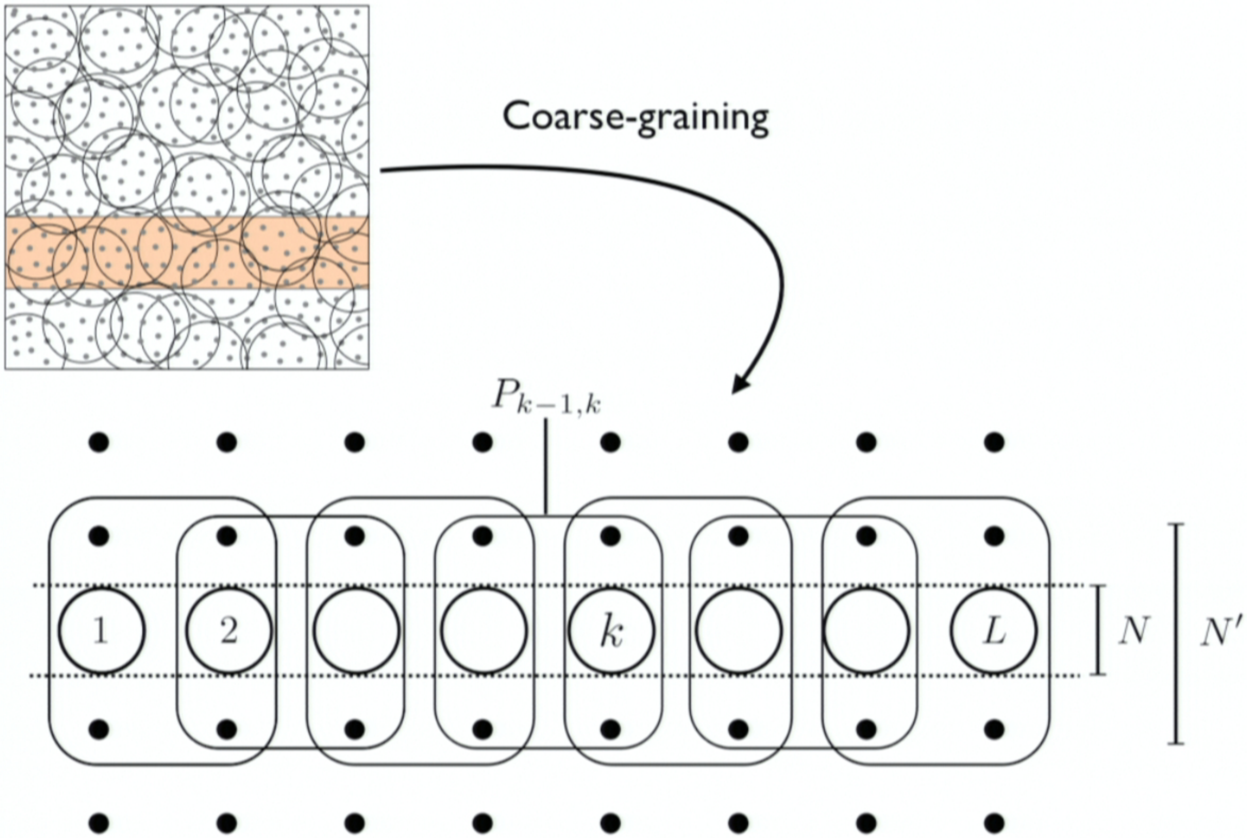

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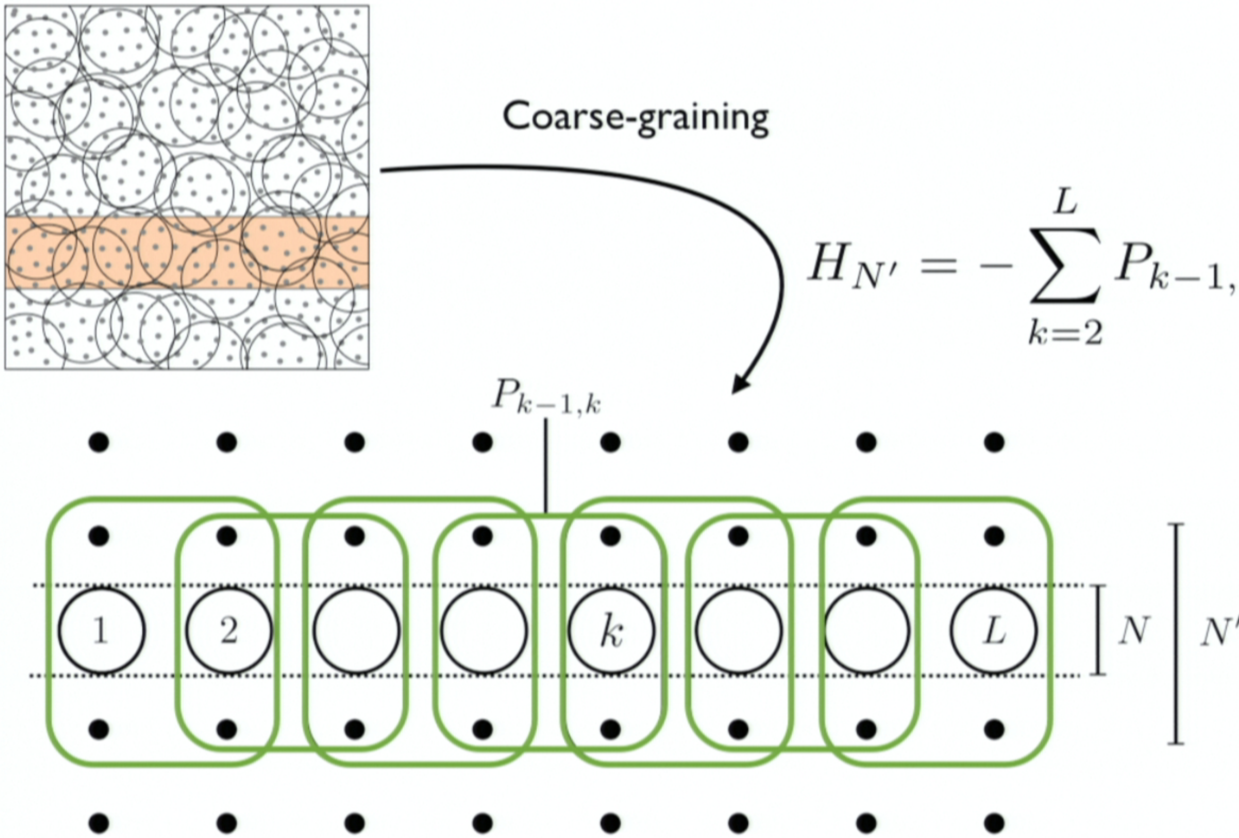


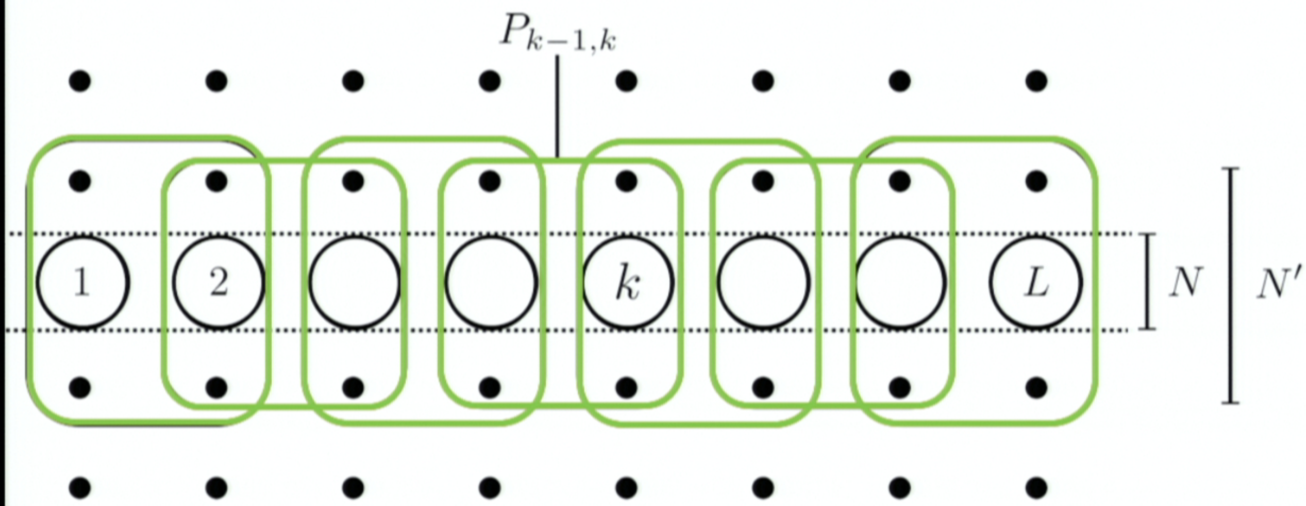
$$H_{N'} = -\sum_{k=2}^{L} P_{k-1,k}$$

- *Sites* on the strip
- Local constraints

Sketch of the proof (II): fortuitous model

Fortuitous model

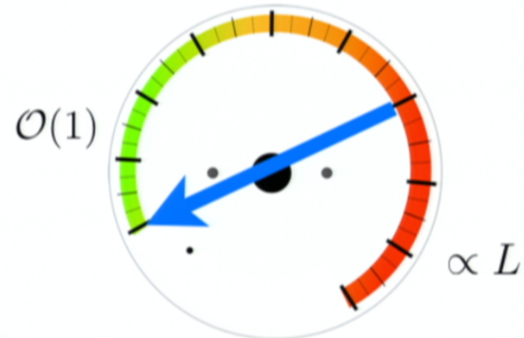
- depolarize every site on the strip N
- project back onto the code

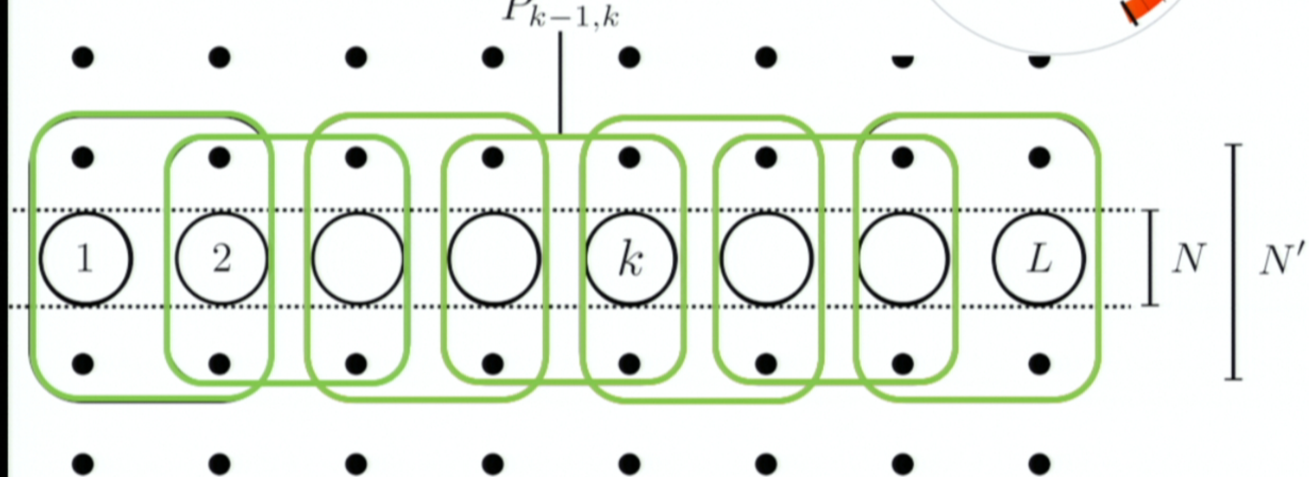


Sketch of the proof (III): iterative randomization model

Iterative randomization model

For every site k (iteration),
- apply random trial unitary
- measure $P_{k-1,k}$





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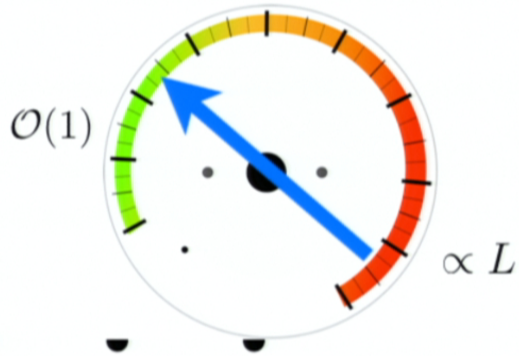

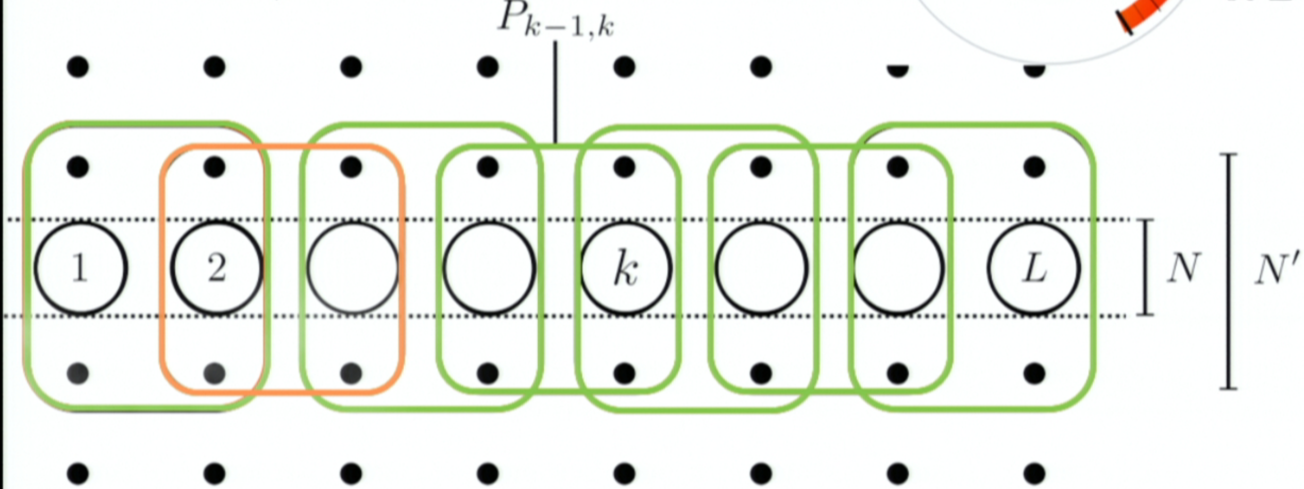

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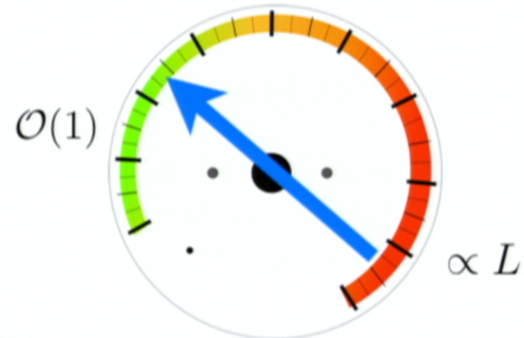

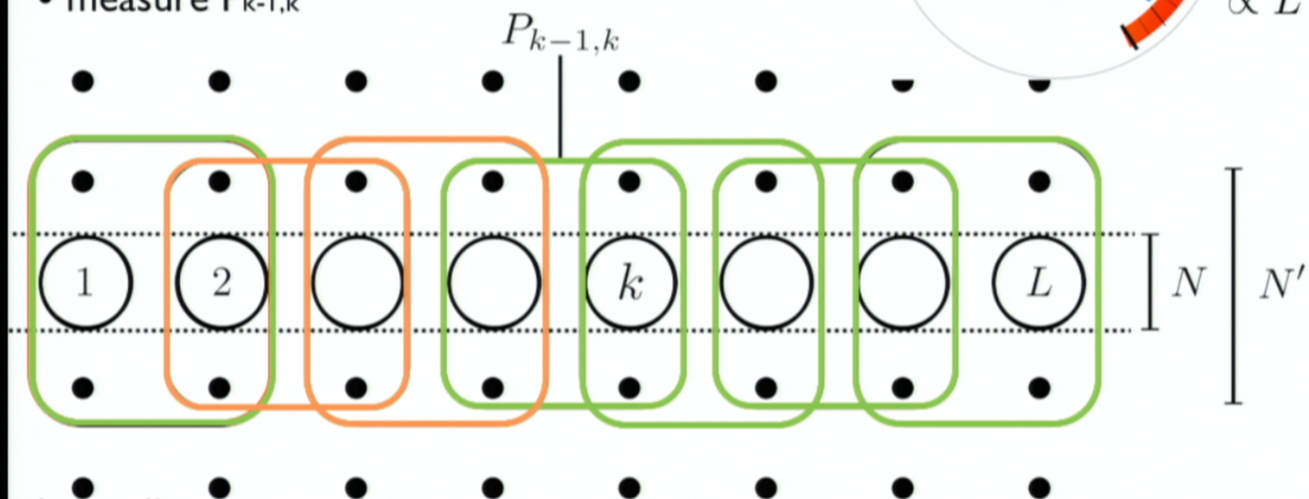

Immediate properties
- at any step, the energy is constant above the gs energy
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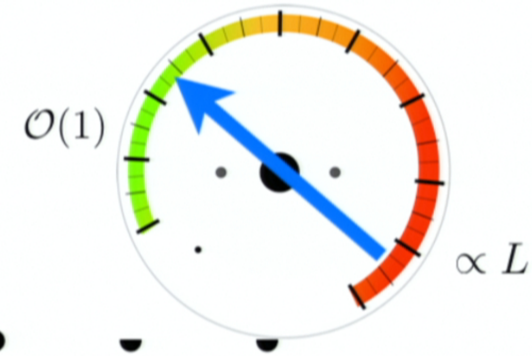

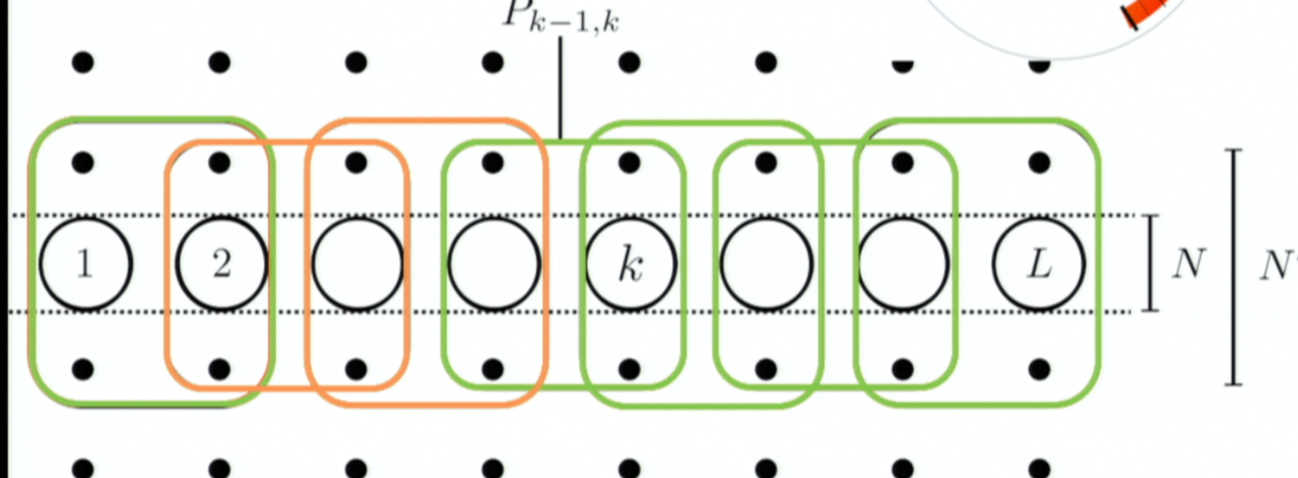

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To show
- no dead-end and expected number of trials at each iteration is constant
- effect of iterative randomization model = effect of fortuitous model

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Sketch of the proof (IV): no dead-end

Iterative randomization model

For every site k,
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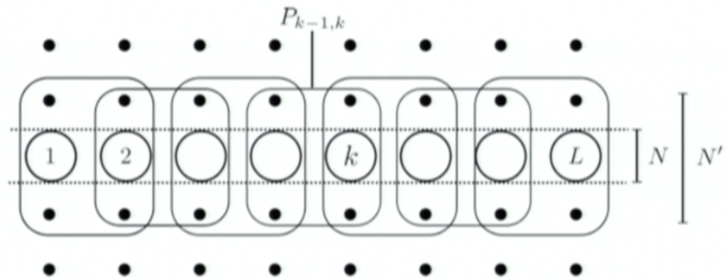

Dead-end = impossible to find eligible unitary at a given iteration.
State of the strip, yet consistent with previous constraints, can't be extended.

Sketch of the proof (IV): no dead-end

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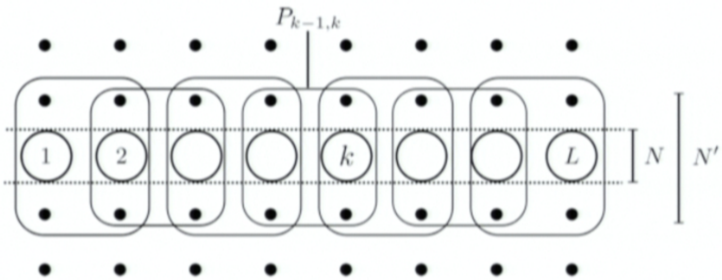

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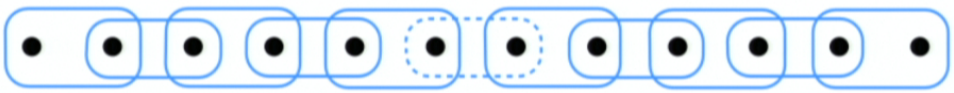
Simple example: chain of qutrits



$P_{i,i+1} = |00\rangle\langle 00| + |11\rangle\langle 11| + |22\rangle\langle 22|$

$P^*_{i,i+1} = |00\rangle\langle 00| + |11\rangle\langle 11|$

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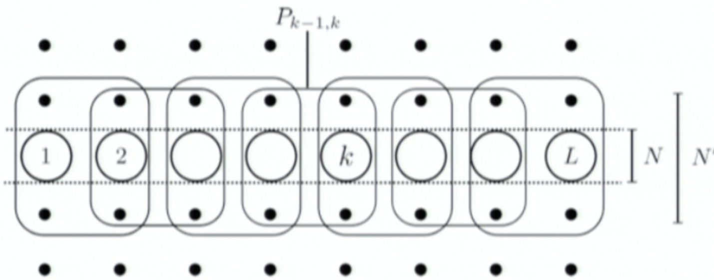
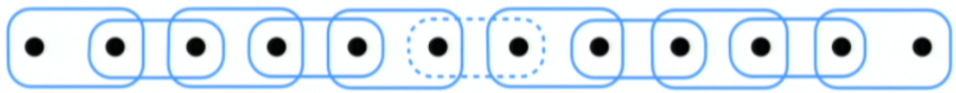
For every site k,
- apply random trial unitary
- measure $P_{k-1,k}$



Dead-end = impossible to find eligible unitary at a given iteration.
State of the strip, yet consistent with previous constraints, can't be extended.

Simple example: chain of qutrits



$P_{i,i+1} = |00\rangle\langle 00| + |11\rangle\langle 11| + |22\rangle\langle 22|$

$P^*_{i,i+1} = |00\rangle\langle 00| + |11\rangle\langle 11|$ $\qquad P = |0\ldots 0\rangle\langle 0\ldots 0| + |1\ldots 1\rangle\langle 1\ldots 1|$

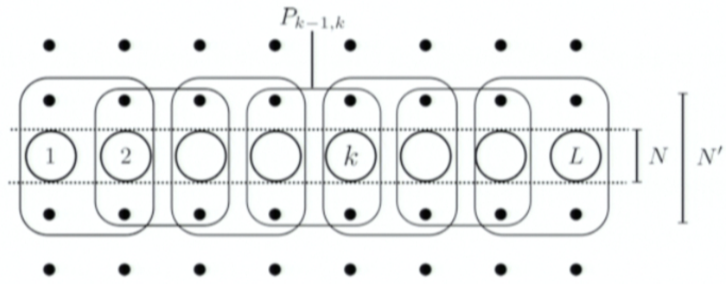
Dead-end: start preparing all 2 state...

Violates local consistency: look at any site k far from defect

Sketch of the proof (IV): no dead-end

Iterative randomization model

For every site k,
- apply random trial unitary
- measure $P_{k-1,k}$



Proposition Local topological order implies that, at any iteration k, there exists an eligible unitary.

Proof (contrapositive).

Dead end at step k $\quad \forall U_k \; P_{k-1,k} U_k |\psi\rangle = 0$

TQO inhibits thermal stability

Olivier Landon-Cardinal

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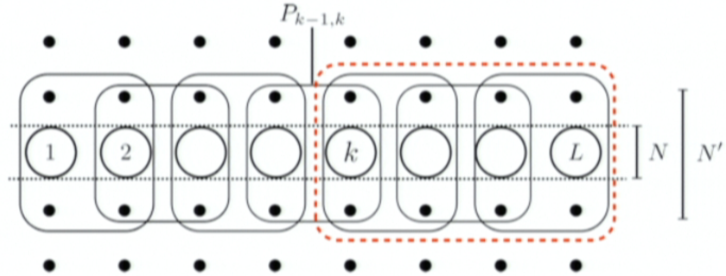

Proposition Local topological order implies that, at any iteration k, there exists an eligible unitary.

Proof (contrapositive).

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Average over Haar measure $\quad P_{k-1,k} \left(\mathrm{Tr}_k [\psi] \otimes I_k / D \right) = 0$

Trace out region at the right of site k $\quad \mathrm{Tr}_k [P_{k-1,k}] \, \mathrm{Tr}_{R_k} [\psi] = 0$

Exists state in image of $P_{i-1,i}$ for i<k and in kernel of $\mathrm{Tr}_k P_{k-1,k}$
Violation of local consistency for site k-2.

□

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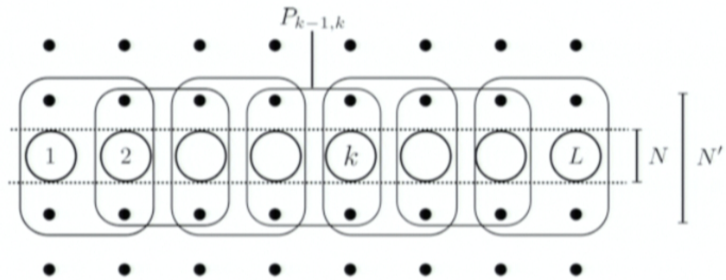
Sketch of the proof (IV): expected number of trials

Iterative randomization model

For every site k,

- apply random trial unitary
- measure $P_{k-1,k}$

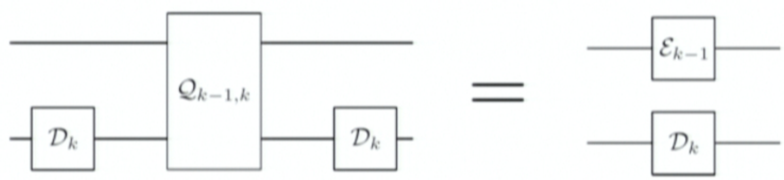


Proposition Local topological order implies that, the expected # of trials at iteration k is a constant.

Proof. Introduce maps

- successful measurement of $P_{k-1,k}$ $\mathcal{P}_{k-1,k}$
- failed measurement of $P_{k-1,k}$ $\mathcal{Q}_{k-1,k}$
- depolarizing of site k $\mathcal{D}_k$

Biasing map



Sketch of the proof (IV): expected number of trials

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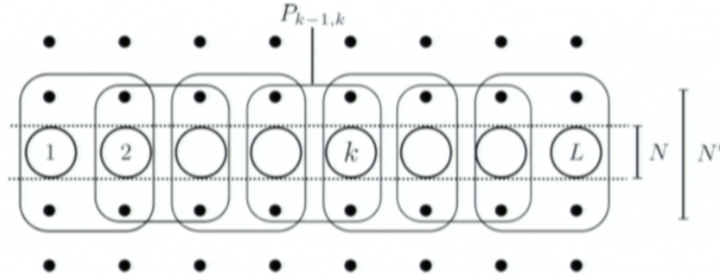

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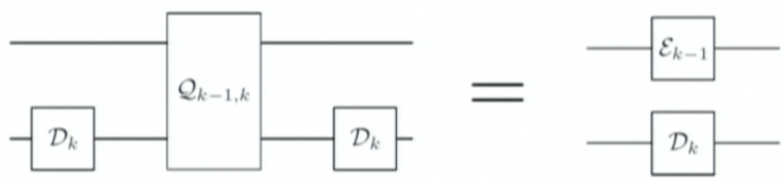

Succes after m failed trials $\mathcal{P}_{k-1,k}\mathcal{D}_k\left(\mathcal{Q}_{k-1,k}\mathcal{D}_k\right)^m = \mathcal{P}_{k-1,k}\left(\mathcal{E}_{k-1}^m \otimes \mathcal{D}_k\right)$

Expected # of trials
$$A_k(\psi) = \sum_{m=1}^{\infty}(m+1)\mathrm{Tr}\left[\mathcal{P}_{k-1,k}\left(\mathcal{E}_{k-1}^m \otimes \mathcal{D}_k\right)[\psi]\right]$$
$$= \mathrm{Tr}\left[\mathcal{P}_{k-1,k}\left(\left(\mathcal{I}_{k-1} - \mathcal{E}_{k-1}\right)^{-2} \otimes \mathcal{D}_k\right)[\psi]\right]$$

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Conclusion

Main result (arXiv:1209.5750)
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- noise process occurring in parallel (rather than sequentially)
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Hope for self-correcting quantum memories

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Non-zero temperature: minimization of free energy E-TS

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3D Codes with scalable energy barrier
➡ Haah's cubic code Haah, PRA, **83** (2011) Bravyi & Haah, PRL, **107** (2011)