Title: All one-loop amplitudes in N=6 Chern-Simons matter theories

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Abstract: In this talk we will present results on all one-loop scattering amplitudes in N=6 Chern-Simons matter theories. Especially we will discuss connection between certain triple-cut diagrams and tree-level recursive diagrams, and a general formula capturing the multi-particle factorization of arbitrary one-loop amplitudes in the theories is obtained from this connection. Furthermore a recursion relation for the supercoefficients of one-loop amplitudes will be derived, which leads the solution for all one-loop amplitudes.

All one-loop amplitudes in ABJM

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Brandhuber, Travaglini, C.W. arxiv: 1205.6705, 1207.6908

Perimeter Institute, 12/18/2012

Outline

- Review of amplitudes in ABJM theory
- Connections to N=4 sYM.
- All one-loop amplitudes

i. Intriguing relations between tree and loop
ii. Particular triple cuts and BCFW diagrams
iii. Non-trivial factorizations
iv. Recursion relations for supercoefficients

- Summary and questions

ABJM thoery (Aharony,Bergman,Jafferis,Maldacena)

- Matter contents:

1. 4 complex bosons & 4 complex fermions

$$(\phi^A, \psi^\alpha_A)^I_{\bar{I}} \ , (\bar{\phi}_A, \bar{\psi}^A_\alpha)^{\bar{I}}_I$$

   - Transform in bi-fundamental of U(N)xU(N).

2. Gauge fields $(A, \hat{A})$

   - have Chern-Simons level k and –k correspondingly.
   - only appear as internal states.
   - zero-momentum mode plays interesting roles.

- Dual to M-theory on $AdS_4 \times S^7/Z_k$
- Large k limit, with $\lambda := N/k$ fixed, dual to IIA string theory on $AdS_4 \times CP^3$.

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Amplitudes

- Tree-level:

1. Four-point (Agarwal,Beisert,McLoughlin)
2. BCFW recursion relations (Gang,Huang,Koh,Lee,Lipstein)

- One-loop:

1. Four-point vanishes (Agarwal,Beisert,McLoughlin)
2. Six-point (Bargheer et al;Bianchi et al)
3. All one-loop amplitudes (Brandhuber,Travaglini,CW)

- Two-loop:

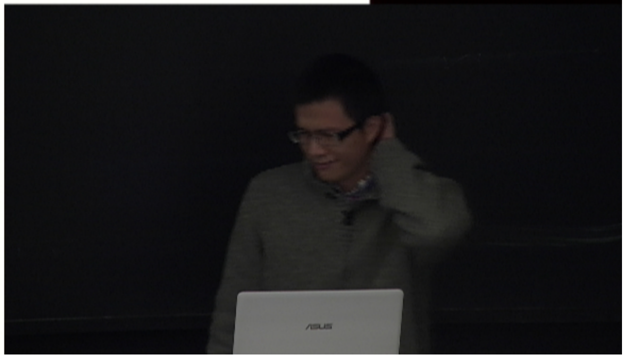
1. Four-point (Chen,Huang; Bianchi et al)
2. Six-point (Caron-Huot,Huang)

Spinor helicity formalism in 3d

- On-shell momentum:

$$p_{\alpha\beta} = \lambda_\alpha \lambda_\beta$$

1. Little group $\lambda \longrightarrow -\lambda$, no helicity.
2. Can be real(E>0) or imaginary(E<0)
3. Lorentz invariants $\langle i\, j \rangle \;:=\; \lambda_{i\alpha}\lambda_{j\beta}\,\epsilon^{\alpha\beta}$



Superamplitudes

- On-shell superfield: N=3 superspace

1. η^A with A=1,2,3 are fermionic variables.
2. Superspace (λ, η)
3. Superfield:

   - Bosonic: superfield for the particles

$$\Phi(\lambda,\eta) \;=\; \phi^4(\lambda) + \eta^A\psi_A(\lambda) + \frac{1}{2}\epsilon_{ABC}\eta^A\eta^B\phi^C(\lambda) + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\psi_4(\lambda)$$

   -Fermionic: superfield for the anti-particles

$$\bar{\Phi}(\lambda,\eta) \;=\; \bar{\psi}^4(\lambda) + \eta^A\bar{\phi}_A(\lambda) + \frac{1}{2}\epsilon_{ABC}\eta^A\eta^B\bar{\psi}^C(\lambda) + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\bar{\phi}_4(\lambda)$$

- Superamplitudes:

$$\mathcal{M} = \mathcal{M}(\bar{\Phi}_1, \Phi_2, \bar{\Phi}_3, \ldots, \bar{\Phi}_{n-1}, \Phi_n)$$

-The same number of particles and anti-particles to have gauge invariance.

- Φ and $\bar{\Phi}$ alternate.

1. Colour-decomposition

$$\mathcal{M}_n\left(\bar{\Phi}_1{}^{\bar{A}_1}_{A_1}, \Phi_2{}^{B_2}_{\bar{B}_2}, \ldots, \Phi_n{}^{B_n}_{\bar{B}_n}\right) = \sum_\sigma \mathcal{M}_n\left(\bar{\Phi}_{\sigma_1}, \Phi_{\sigma_2}, \ldots, \Phi_{\sigma_n}\right) \delta^{\bar{A}_{\sigma_1}}_{\bar{B}_{\sigma_2}} \delta^{B^{\sigma_2}}_{A_{\sigma_3}} \delta^{\bar{A}_{\sigma_3}}_{\bar{B}_{\sigma_4}} \cdots \delta^{B^{\sigma_n}}_{A_{\sigma_1}}$$

2. Supersymemtric and superconformal generators:

$$Q^{\alpha A} = \lambda^\alpha \eta^A \qquad Q^\alpha_A = \lambda^\alpha \frac{\partial}{\partial \eta^A}$$

$$\mathcal{S}^A_\alpha = \eta^A \frac{\partial}{\partial \lambda^\alpha} \qquad \mathcal{S}_{A\alpha} = \frac{\partial}{\partial \eta^A} \frac{\partial}{\partial \lambda^\alpha}$$

- Simple properties---all about signs

1. Little Group:

$$\mathcal{M}_n(\cdots;-\lambda_i,-\eta_i;\cdots) \;=\; (-)^i\,\mathcal{M}_n(\cdots;\lambda_i,\eta_i;\cdots)$$

2. Cyclic symmetry:

$$\mathcal{M}_n(\bar{1},2,\bar{3},\ldots,n) \;=\; (-)^{\frac{n-2}{2}}\mathcal{M}_n(\bar{3},4,\bar{5},\ldots,\bar{1},2)$$

3. KK-relation:

$$\sum_P \mathcal{M}_{2k}(\bar{1},P(2),\bar{3},\cdots,P(2k))=0$$

4. Reflection:

$$\mathcal{M}_n^{(l)}(\bar{1},2,\bar{3},\cdots,n)=(-)^{n(n-2)/8+l}\mathcal{M}_n^{(l)}(\bar{1},n,\overline{n-1},\cdots,2)$$

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Example: four-point amplitudes

- Four-point amplitude at tree-level

$$\mathcal{M}(\bar{1},2,\bar{3},4) = \frac{\delta^{(3)}\left(\sum_{i=1}^{4}\lambda_i\lambda_i\right)\,\delta^{(6)}\left(\sum_{i=1}^{4}\lambda_i\eta_i\right)}{\langle 12\rangle\langle 23\rangle}$$

- Four-point amplitude at one-loop vanishes.
- Four-point amplitude at two-loop matches with four-point amplitude of N=4 at one-loop.
- Four-point amplitudes are the only amplitudes reminiscent of MHV amplitudes in d=4.

Similarity to N=4 sYM

- Two-loop four-point amplitude in ABJM matches with one-loop four-point amplitude in N=4 sYM. (Chen, Huang;Bianchi et al)
- Conjectured Wilson-loop/correlation/amplitude dualities at four-point. (Bianchi et al) (Henn et al; Wiegandt)
- Yangian symmetries, Grassmannian, twistor string-like formalism. (Lee&Huang;Gang et al;Bargheer et al)
- Color-Kinematics duality. (Bargheer, He, McLoughlin&Huang, Johansson)

Dissimilarity to N=4 sYM

- n-point amplitudes have Grassmann degree 3n/2:
1. No MHV amplitudes, no helicity
2. No amplitudes with odd number particles
- All one-loop light-like Wilson loop/correlation function vanish.
- Whereas all one-loop amplitudes are non-vanishing, except four-point.

(Bargheer,Beisert,Loebbert,McLoughlin;Bianchi,Leoni,Mauri,Penati,Santambrogio; Branduhuber,Travaglini,CW)

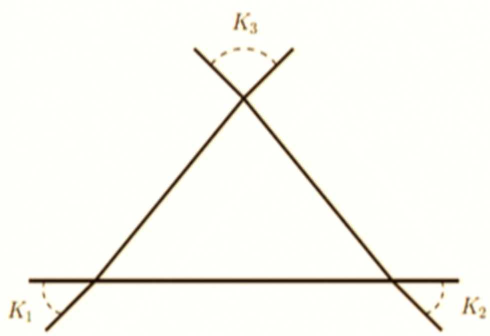
One-loop amplitudes in ABJM

- Only scalar triangle because of dual conformal

1. One-mass and two-mass triangles vanish at d=3

$$\mathcal{M}_n^{(1)} = \sum_{K_1,K_2,K_3} \mathcal{C}_{K_1,K_2,K_3}\, \mathcal{I}^{3m}(K_1,K_2,K_3)$$

2. Three-mass triangles are finite



$$= \frac{-i\,\pi^3}{\sqrt{-(K_1^2+i\varepsilon)}\sqrt{-(K_2^2+i\varepsilon)}\sqrt{-(K_3^2+i\varepsilon)}}$$

No IR-divergence at One-loop

- Triangle in d=3 is finite.
- No amplitudes with odd-number particles guarantee the absence of IR-divergence at one-loop.

  -The virtual IR divergence of one-loop 2n-point amplitude must be cancelled by IR-divergence of (2n+1)-point amplitude at tree-level.

  -But there is no (2n+1)-point amplitude.

Six-point amplitude

- Tree-level: $\mathcal{M}^{(0)}(\bar{1},2,\bar{3},4,\bar{5},6) \;=\; Y^{(1)}_{12;4} \;+\; Y^{(2)}_{12;4}$
  - Y-function

$$Y^{(1)}_{12;4} \;=\; \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{P^2_{24}}\,\frac{\delta^{(3)}(\epsilon_{ijk}\langle j\,k\rangle\eta_i - i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\langle\bar{j}\,\bar{k}\rangle\eta_{\bar{i}})}{(\langle 2|P_{34}|5\rangle + i\langle 3\,4\rangle\langle 6\,1\rangle)(\langle 1|P_{23}|4\rangle + i\langle 2\,3\rangle\langle 5\,6\rangle)} \qquad i,j=2,3,4$$

$$Y^{(2)}_{12;4} \;=\; \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{P^2_{24}}\,\frac{\delta^{(3)}(\epsilon_{ijk}\langle j\,k\rangle\eta_i + i\,\epsilon_{\bar{i}\bar{j}\bar{k}}\langle\bar{j}\,\bar{k}\rangle\eta_{\bar{i}})}{(\langle 2|P_{34}|5\rangle - i\langle 3\,4\rangle\langle 6\,1\rangle)(\langle 1|P_{23}|4\rangle - i\langle 2\,3\rangle\langle 5\,6\rangle)} \qquad \bar{i},\bar{j}=5,6,1$$

- One-loop:

$$\mathcal{M}^{(1)}(\bar{1},2,\bar{3},4,\bar{5},6) \;=\; i\pi^3\mathcal{S}(Y^{(1)}_{12;4} \;-\; Y^{(2)}_{12;4})$$

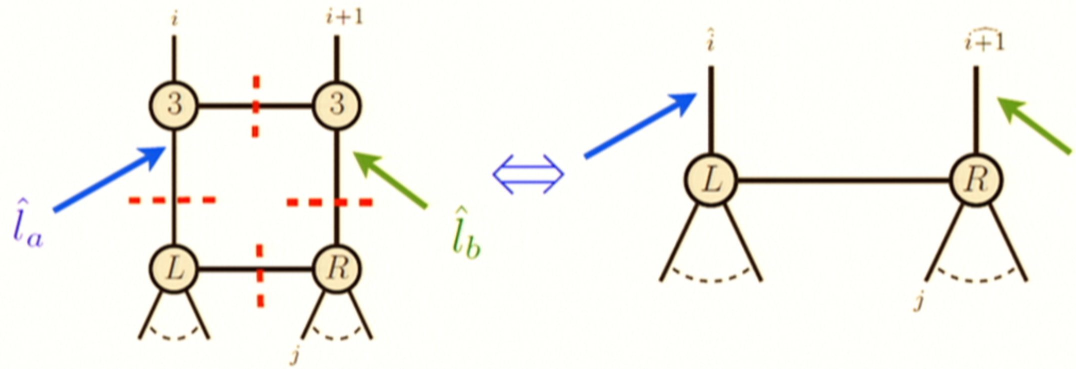
  - $\mathcal{S} \;=\; \mathrm{sgn}(\langle 1\,2\rangle)\mathrm{sgn}(\langle 3\,4\rangle)\mathrm{sgn}(\langle 5\,6\rangle) + \mathrm{sgn}(\langle 2\,3\rangle)\mathrm{sgn}(\langle 4\,5\rangle)\mathrm{sgn}(\langle 6\,1\rangle)$
  - $\mathrm{sgn}\,(\langle m\,n\rangle) \;:=\; i\,\dfrac{\langle m\,n\rangle}{\sqrt{-(\langle m\,n\rangle^2 + i\varepsilon)}}$

- We wish to explain this intriguing structure and possibly to extend it to all one-loop.
- We will find a connection between BCFW diagrams and certain triple-cut diagrams in 3d.
  - Allow us to determine one-loop amplitudes up to ten points.
  - Recursion relations for the amplitudes beyond ten points.

BCFW and one-loop in d=4

- Two-mass hard box and BCFW bridge

(Roiban,Spradlin,Volovich;Britto,Cachazo,Feng)



- What is the analogue in ABJM?
- The answer is not clear. Since in d=4 it's the IR relation, one-loop amplitudes in ABJM are finite.

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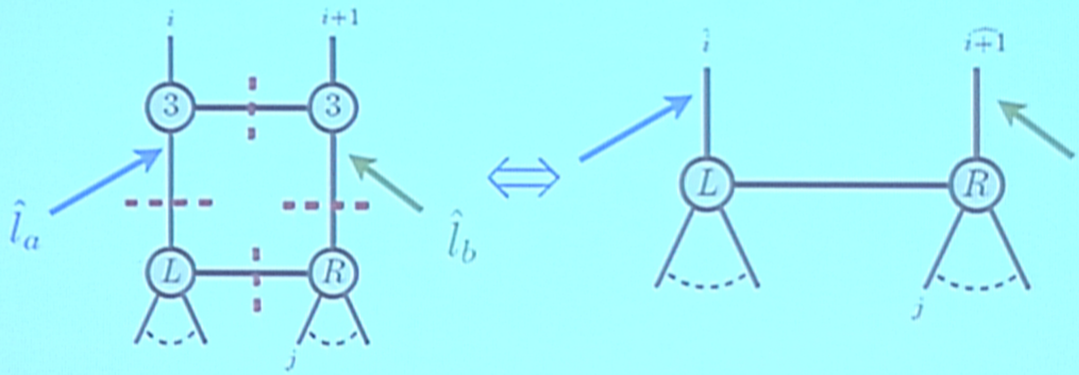

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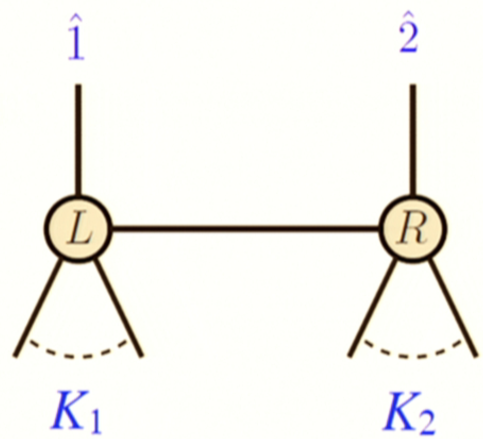
BCFW in d=3 (Gang,Huang,Koh,Lee,Lipstein)

- Linear BCFW shifts are not enough in d=3.

(Arkani-Hamed,Kaplan)

- Non-linear BCFW shifts



$$\begin{pmatrix} \hat{\lambda}_1 \\ \hat{\lambda}_2 \end{pmatrix} = R(z) \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

$$\begin{pmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \end{pmatrix} = R(z) \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

From momentum conservation $P_{\hat{1}} + P_{\hat{2}} = P_1 + P_2$

$$R(z) = \begin{pmatrix} \frac{1}{2}(z+z^{-1}) & -\frac{1}{2i}(z-z^{-1}) \\ \frac{1}{2i}(z-z^{-1}) & \frac{1}{2}(z+z^{-1}) \end{pmatrix} \quad R(z=1) \to 1$$

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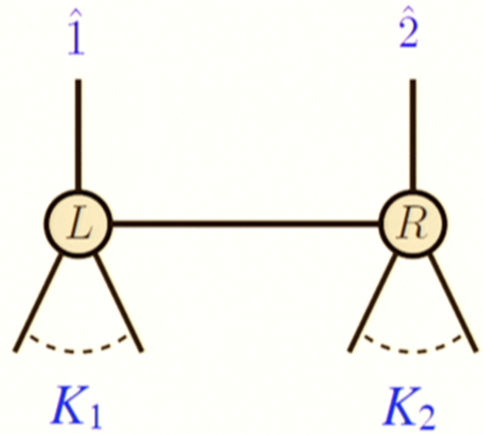

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BCFW in d=3

- On-shell solutions:

$$z_1^2 = \frac{K_1 \cdot K_2 + \sqrt{K_1^2 K_2^2}}{4(q \cdot K_1)}, \quad z_2^2 = \frac{K_1 \cdot K_2 - \sqrt{K_1^2 K_2^2}}{4(q \cdot K_1)}$$

where $q^{\alpha\beta} := \frac{1}{4}(\lambda_1 + i\lambda_2)^\alpha (\lambda_1 + i\lambda_2)^\beta$

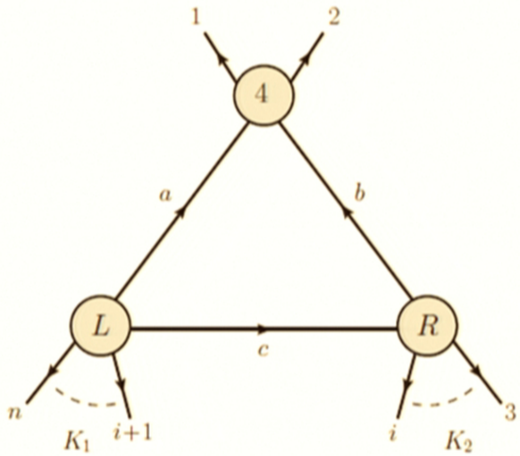
- Recursion relation

$$\mathcal{M}_n = \int d^3\eta_c \, \frac{H(z_1, z_2)}{p_f^2} \left[\mathcal{M}_R(\bar{3} \ldots, i, -\bar{c}, \hat{2})\, \mathcal{M}_L(\overline{i+1}, \ldots, n, \bar{\hat{1}}, c) \right]_{z=z_1} + (z_1 \leftrightarrow z_2)$$

where

$$H(z_1, z_2) := \frac{z_1(z_2^2 - 1)}{z_1^2 - z_2^2}$$

One-loop amplitudes v.s. BCFW



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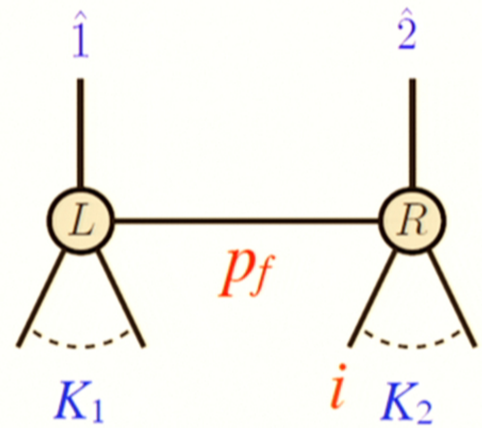


- Easy to check that cut solutions are the same as BCFW shifts in d=3.
- We then check the recursion relations:

- Supercoefficient:

$$\mathcal{C}_{12;i}(z) = \int d^3\eta_a d^3\eta_b d^3\eta_c\, \mathcal{M}_4(\bar{1}, 2, -\bar{b}, -a)\, \mathcal{M}_R(\bar{3}, \ldots, i, -\bar{c}, b)\mathcal{M}_L(\overline{i+1}, \ldots, n, \bar{a}, c)$$

- Plug in four-point amplitude, we find

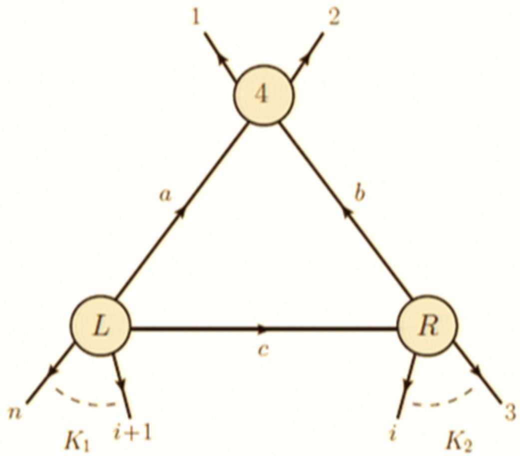
$$\mathcal{C}_{12;i} = \int d^3\eta_c \frac{G(z_1)}{p_f^2}\Big[\mathcal{M}_R(\bar{3}\ldots, i, -\bar{c}, \hat{2})\, \mathcal{M}_L(\overline{i+1}, \ldots, n, \hat{\bar{1}}, c)\Big]_{z=z_1} + (z_1 \leftrightarrow z_2)$$

$$\mathcal{R}_{12;i} = \int d^3\eta_c \frac{H(z_1, z_2)}{p_f^2}\Big[\mathcal{M}_R(\bar{3}\ldots, i, -\bar{c}, \hat{2})\, \mathcal{M}_L(\overline{i+1}, \ldots, n, \hat{\bar{1}}, c)\Big]_{z=z_1} + (z_1 \leftrightarrow z_2)$$

where $G(z) = \dfrac{\langle 1\, 2\rangle}{z - z^{-1}}$ and

$$\frac{1}{p_f^2}\frac{H(z_1, z_2)}{G(z_1)} = -\frac{1}{p_f^2}\frac{H(z_2, z_1)}{G(z_2)} = \frac{1}{\langle 1\, 2\rangle \sqrt{K_1^2\, K_2^2}}$$

One-loop amplitudes v.s. BCFW



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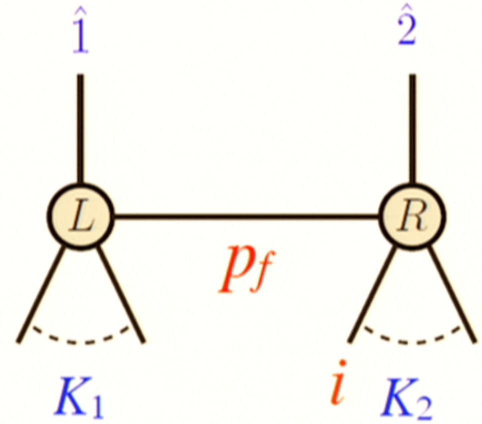

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- Summary:
  - Recursion: $\mathcal{R}_{12;i} = Y^{(1)}_{12;i} + Y^{(2)}_{12;i}$
  - Supercoeffcient:

$$\mathcal{C}_{12;i} = -\langle 12\rangle \sqrt{K_1^2 K_2^2}\left(Y^{(1)}_{12;i} - Y^{(2)}_{12;i}\right)$$

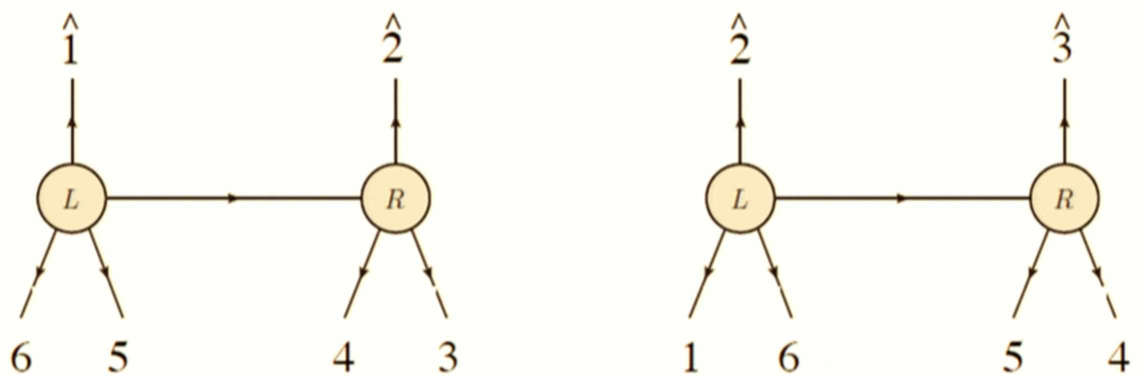
- Coefficient x massive triangle:

$$\mathcal{C}_{12;i}\,\mathcal{I}_{12,K_1,K_2} = -i\frac{\pi^3}{4}\frac{\langle 12\rangle}{\sqrt{-(P_{12}^2+i\varepsilon)}}\frac{\langle \xi\mu\rangle}{\sqrt{-(K_1^2+i\varepsilon)}}\frac{\langle \xi'\mu'\rangle}{\sqrt{-(K_2^2+i\varepsilon)}}\left(Y^{(1)}_{12;i} - Y^{(2)}_{12;i}\right)$$

  - $K_{1\,ab} := \xi_{(a}\mu_{b)}\,,\ K_{2\,ab} := \xi'_{(a}\mu'_{b)}$
  - Sign functions in pre-factor
  - Individual Y-function is dual conformal invariant.

- Six-point cut diagrams

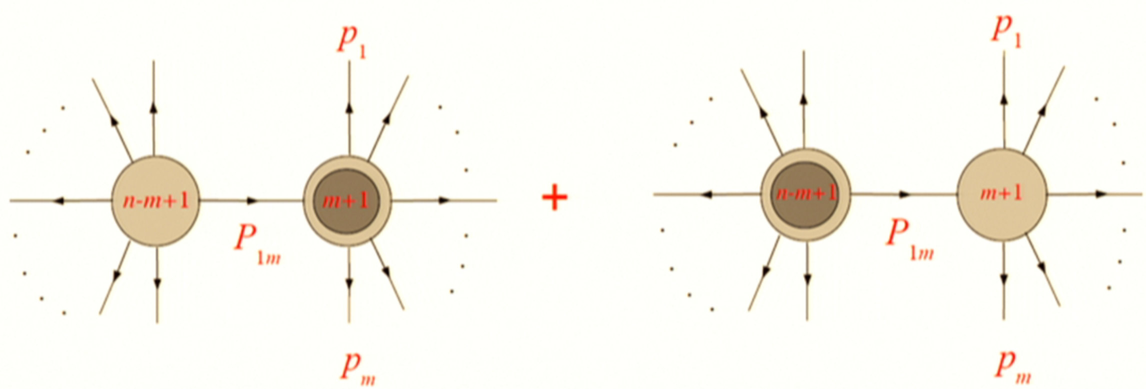
- Corresponding BCFW diagrams



- Explains the result of six-point amplitude.
- Complete solution for the amplitudes up to ten points, because they only involve diagrams with four-point amplitude at one corner.
- We will derive a recursion relation for higher-point amplitudes.

Factorization at one loop

- Factorization is usually trivial if there were no IR-divergence. (Bern, Chalmers)
- One might expect simple factorization for ABJM at one-loop



Puzzles

- Six-point one-loop amplitude has non-trivial multiple particle poles (Bargheer, Beisert, Loebber, McLouglin)

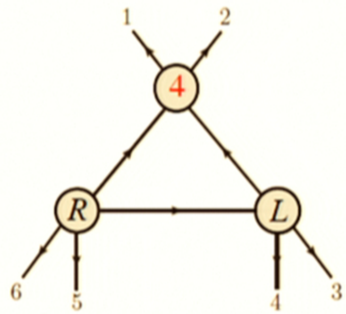
$$\mathcal{M}_6^{(1)} \sim \mathcal{M}_4^{(0)} \frac{\text{sign}}{P^2} \mathcal{M}_4^{(0)}$$

- Even four-point amplitude at tree-level has a pole

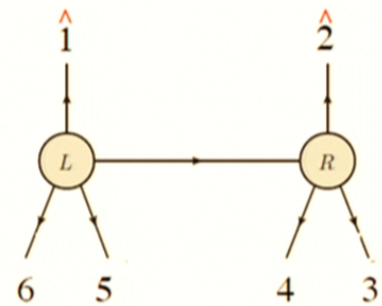
$$\mathcal{M}_4^{(0)} \sim \frac{1}{\langle 12 \rangle}, \qquad p_1 + p_2 \to 0$$

Seeing poles from BCFW diagrams

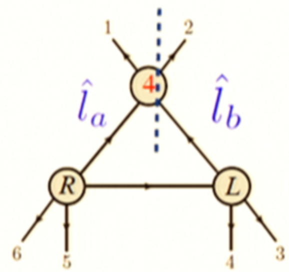
- The connection

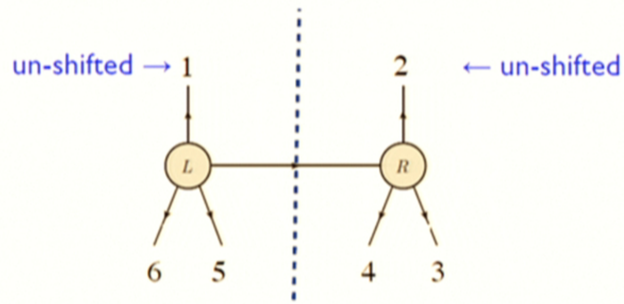


$\Longleftrightarrow$

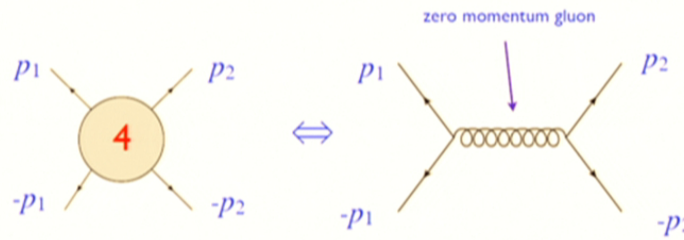


Under factorization limit $P_{234}^2 \to 0$: we loose hats





- zero-momentum mode



- This contributes exactly the right form

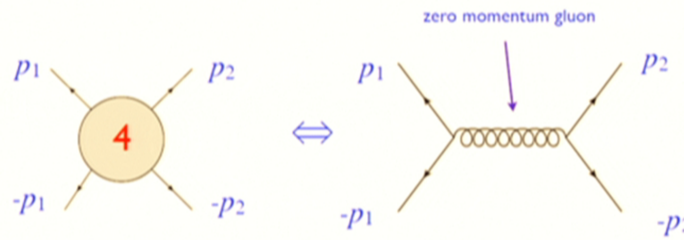
$$\mathcal{M}_6^{(1)} \sim \mathcal{M}_4^{(0)} \frac{\text{sign}}{P^2} \mathcal{M}_4^{(0)}$$

- Generally one loop factorization should include extra term

$$\mathcal{M}_{m+1}^{(0)} \, \frac{\mathcal{F}_n}{P_{1m}^2} \, \mathcal{M}_{n-m+1}^{(0)}$$

- Where $\mathcal{F}_n$ involves momenta both from $\mathcal{M}_{m+1}^{(0)}$ and $\mathcal{M}_{n-m+1}^{(0)}$, so it is an non-factorizing term.

- zero-momentum mode



- This contributes exactly the right form

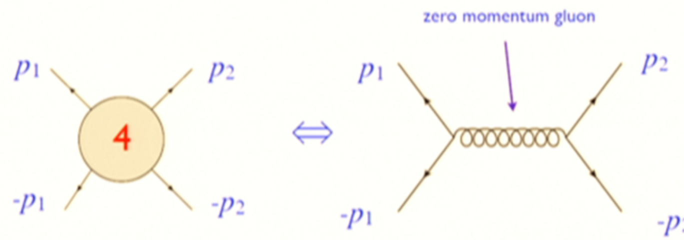
$$\mathcal{M}_6^{(1)} \sim \mathcal{M}_4^{(0)} \frac{\text{sign}}{P^2} \mathcal{M}_4^{(0)}$$

- Generally one loop factorization should include extra term

$$\mathcal{M}_{m+1}^{(0)} \frac{\mathcal{F}_n}{P_{1m}^2} \mathcal{M}_{n-m+1}^{(0)}$$

- Where $\mathcal{F}_n$ involves momenta both from $\mathcal{M}_{m+1}^{(0)}$ and $\mathcal{M}_{n-m+1}^{(0)}$, so it is an non-factorizing term.

- zero-momentum mode



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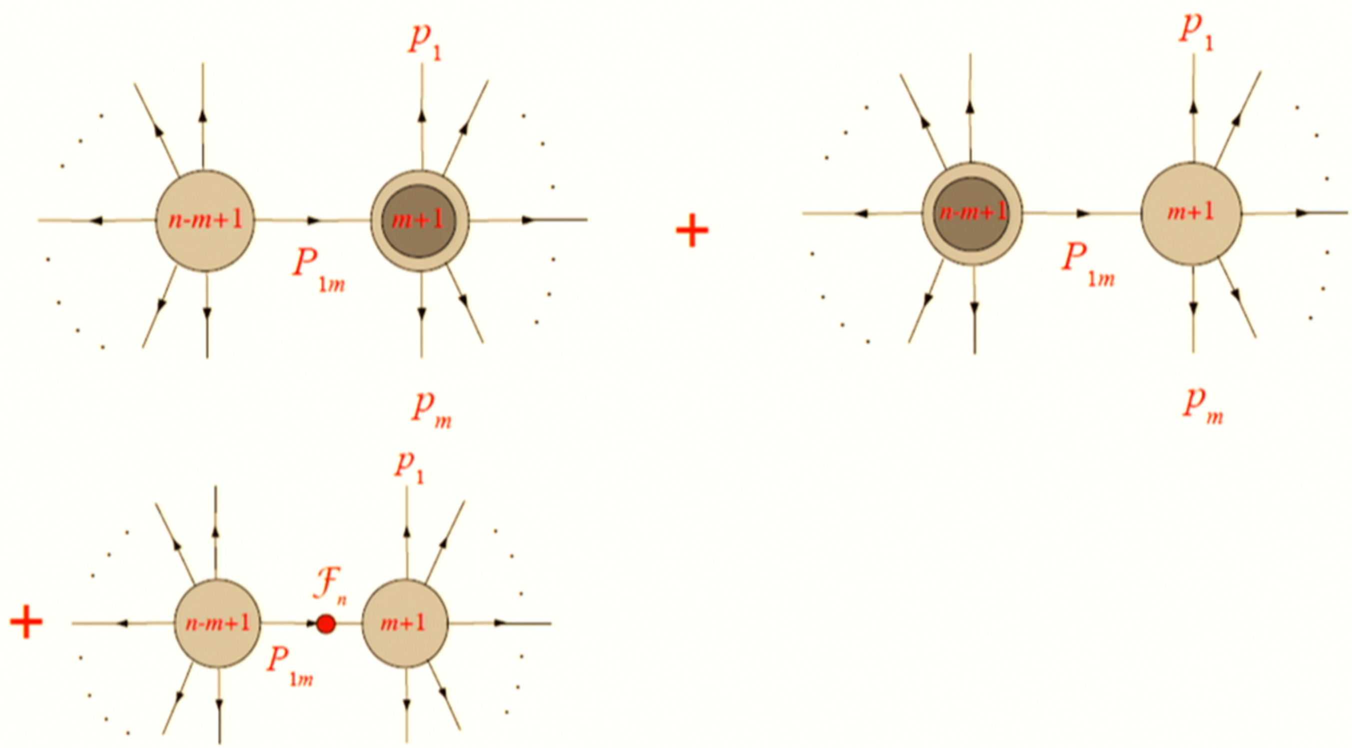
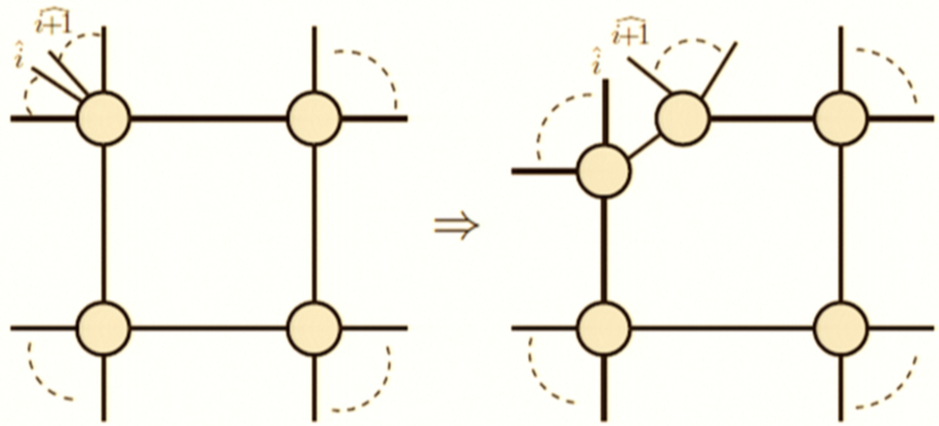
- General factorization at one-loop

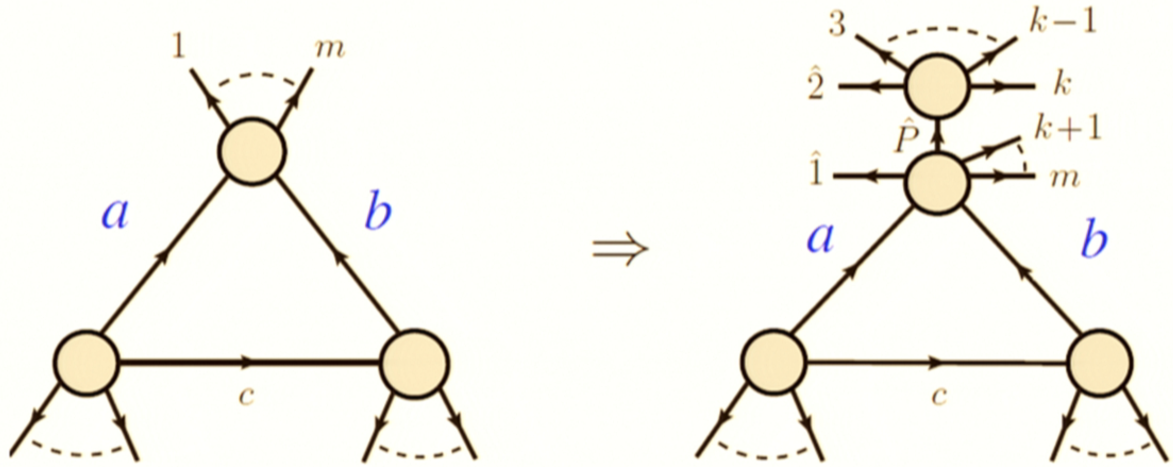


Recursion relations for coefficients

- The idea is BCFW deforming two legs at the same corner of a cut diagram. (Bern,Bjerrum-Bohr,Dunbar,Ita)
- May lead to "bad" diagrams

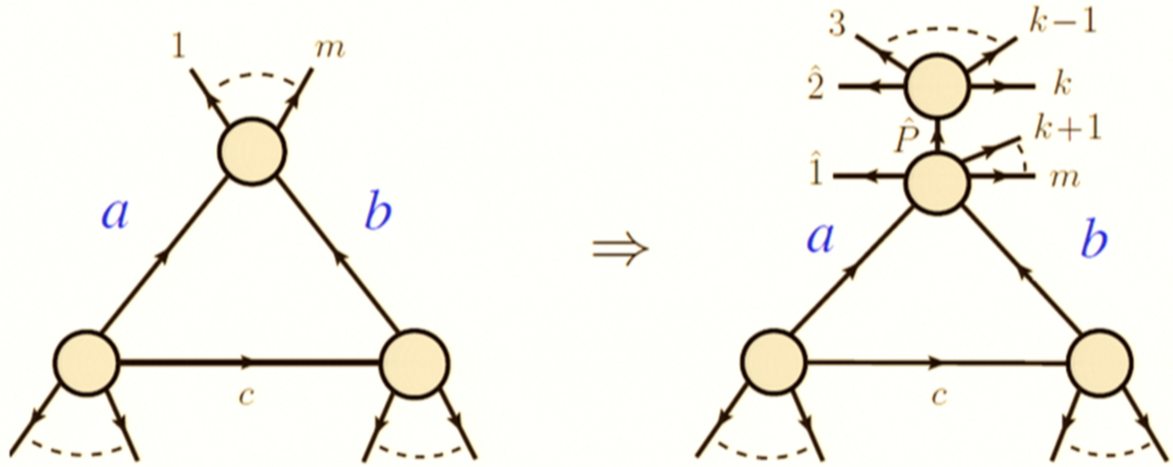


- The “bad” diagrams can always be avoided in ABJM because the amplitudes with odd number particles vanish.



- One can always reduce one corner to four-point.

- The “bad” diagrams can always be avoided in ABJM because the amplitudes with odd number particles vanish.



- One can always reduce one corner to four-point.

General one-loop amplitudes

- Supercoefficient of a general diagram x massive triangle

$$
\begin{aligned}
\mathcal{C}_{n;1,2,\ldots,m;i}\mathcal{I}_{1,m;m+1,i;j+1,n} &= \mathcal{S}_{1,m;m+1,i;j+1,n} \\
&\times \Big[\sum_{\alpha,\beta=1}^{2} (-)^{\alpha+\beta+1} \big(Y^{(\alpha;\beta)}_{n;1\,2;1_1\,P_1;m;i} + Y^{(\alpha;\beta)}_{n;m-1\,m;-P_1\,m_1;n;i} \big) \\
&+ \sum_{j,\alpha,k} (-)^{\frac{k_1+\ldots+k_{j-1}}{2}+\frac{m(j+1)}{2}+\alpha_0+\alpha_{j+1}} Y^{(\alpha_0;\alpha_1;\ldots;\alpha_{j+1})}_{n;12;1_1 P_1;\ldots;1_{j+1} P_{j+1};k_0;k_1;\ldots;k_{j-1};m;i} \Big]
\end{aligned}
$$

where $$\mathcal{S}_{1,m;m+1,i;j+1,n} = \frac{\pi^3}{8} \frac{\langle \xi_{1m}\mu_{1m}\rangle \langle \xi_{m+1i}\mu_{m+1i}\rangle \langle \xi_{i+1n}\mu_{i+1n}\rangle}{\sqrt{-(P^2_{1,m}+i\varepsilon)}\sqrt{-(P^2_{m+1,i}+i\varepsilon)}\sqrt{-(P^2_{i+1,n}+i\varepsilon)}}$$

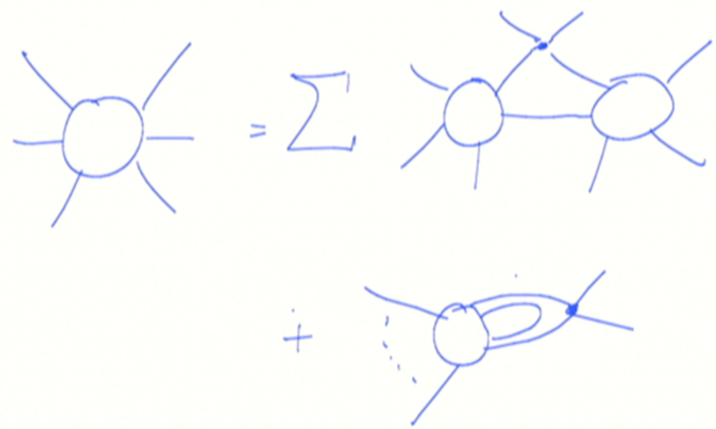
and Y-functions are tree-level BCFW terms.

Summary and Questions

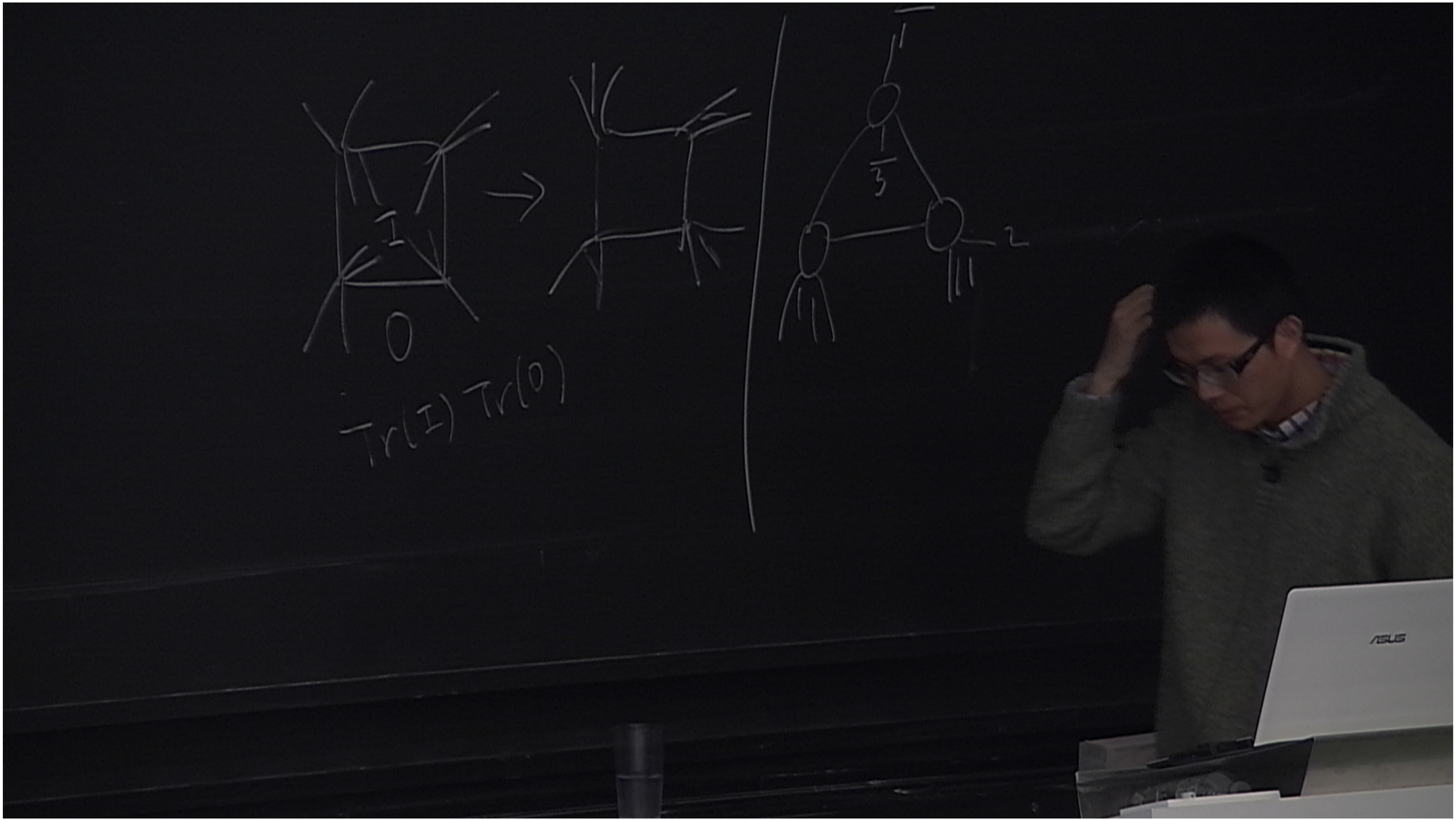
- Intriguing structures of ABJM amplitudes
- Connection between certain triple cut diagrams and BCFW diagrams
- Non-trivial factorization at one-loop.
- Solving all one-loop amplitudes in terms of tree-level BCFW terms.

Summary and Questions

- Higher-loop/point amplitudes and correlation functions.
- Non-planar amplitudes.
- Recursion relation for integrands. (Arkani-Hamed string 2012)




Thank you!


I
O
Tr(I) Tr(O)
3
2